New ideas in SM predictions for the LHC

Christian Bauer

The Search for New States and Forces of Nature

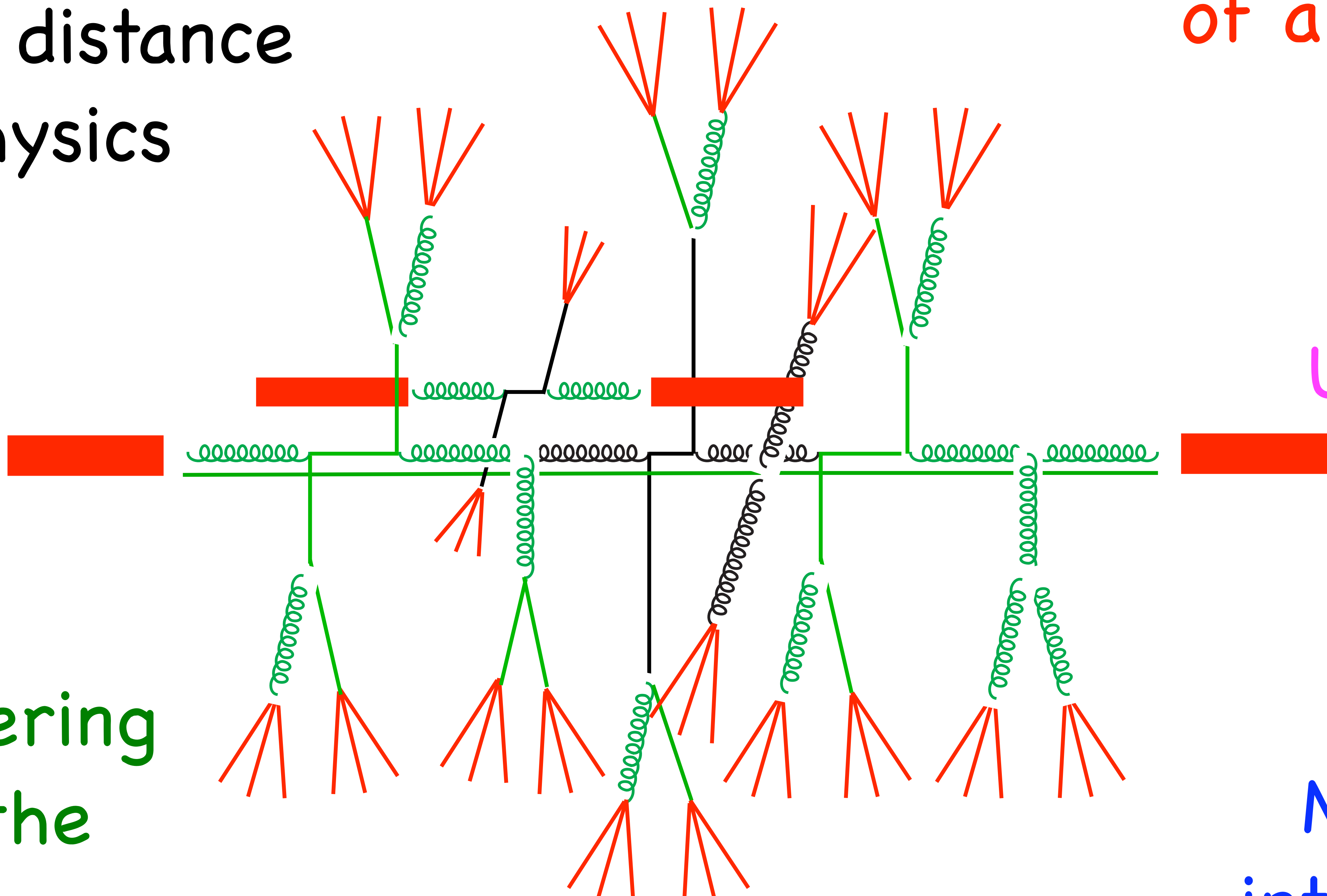
GGI 10/27/2009

Work discussed here in collaboration with
Jesse Thaler and Frank Tackmann

Complicated interactions

underlying short distance physics

hadronization of all partons

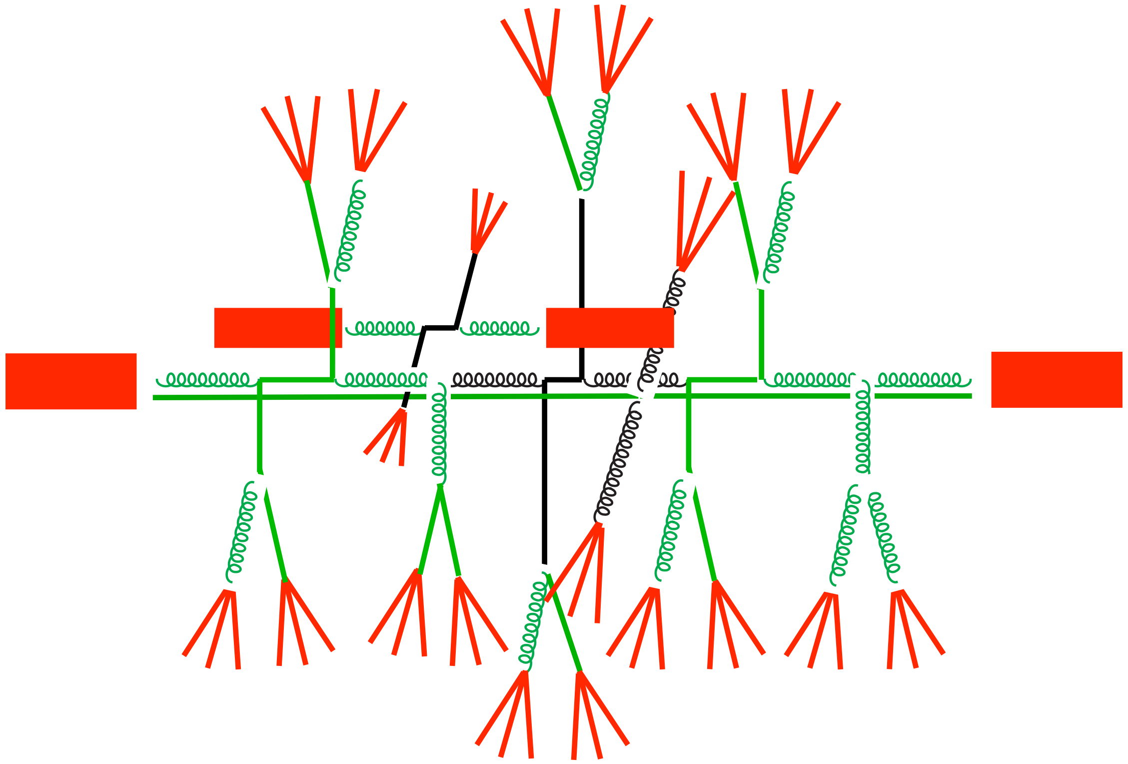


Underlying event

showering of the partons

Multiple interactions

Different distance scales



- Distance scale set by d~1/s
- Distance scale set by d~1/m_J
- Distance scale set by d~1/Λ
- Various scales involved

- Distance scale d~1/Λ governed by non-perturbative physics
- Not known how to calculate
- Gluons can couple different distance scales to one another
- Not clear how to isolate the perturbative piece

How can anything be calculated?

Factorization

- Separating physics from different distance scales crucial to allow theoretical predictions for any process
- This has been major effort over past 30 years
- Most trivial factorization separates partonic scattering from the parton distribution functions
- Rigorous proofs exist only for the simplest processes (Drell-Yan ...)
- Much work still invested in this direction
- Effective field theory (SCET) has allowed new angle at this problem

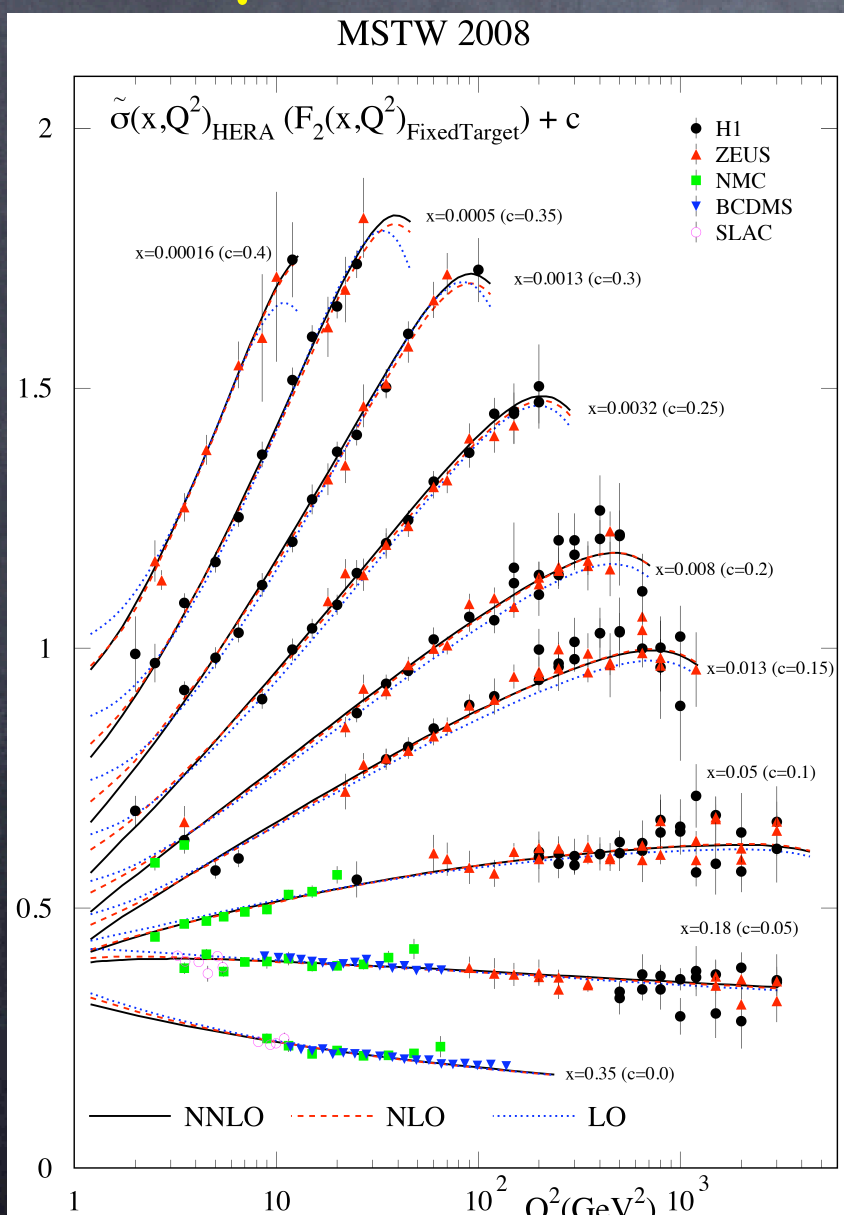
Obtain the required ingredients

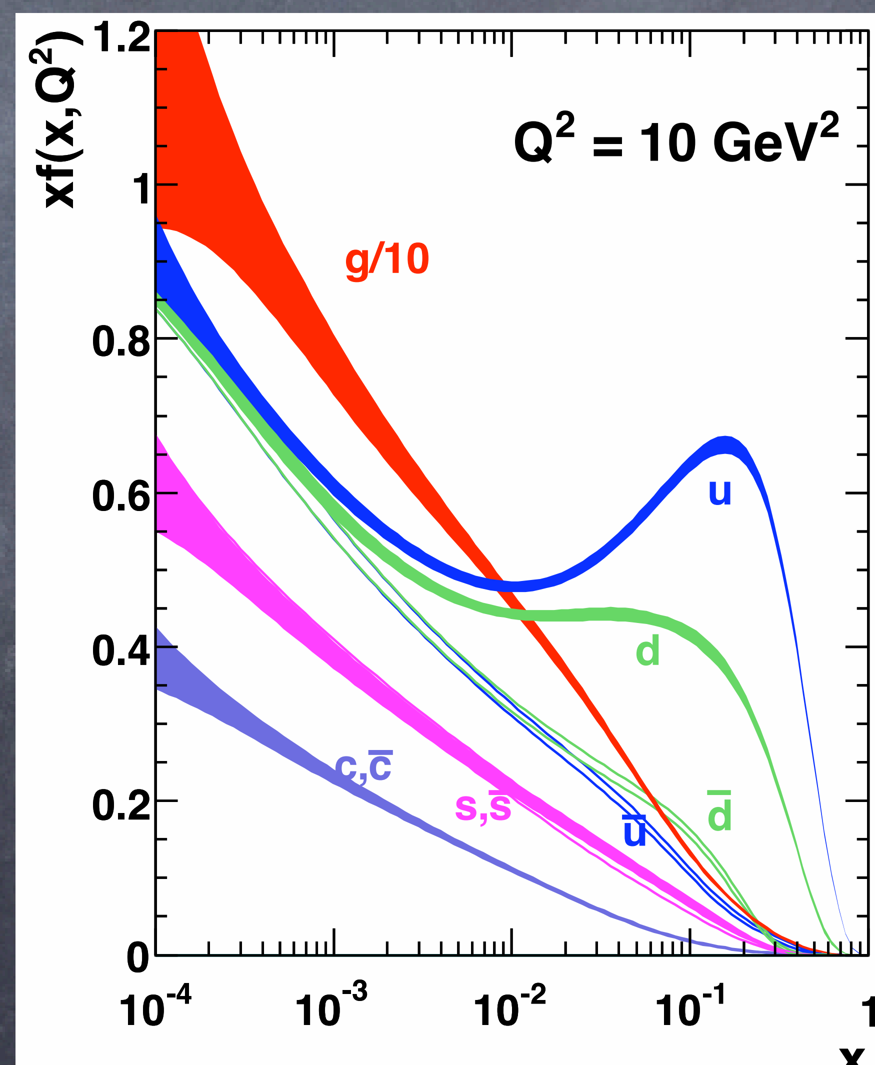
Data	Calculations	Models
Extract expressions from data	Calculate using expansion of QCD	Use more or less inspired models of QCD
PDF's Hadron decays	hard process QCD radiation	Hadronization Underlying event Multi scattering
Best	Good	Worst

Using Data

- Measured BR's used for unstable particles decay
- PDF's extracted using simple well understood processes

$\sigma(p+e^- \rightarrow X+e^-) \quad = \quad f_q \quad \otimes \quad \sigma(q+e^- \rightarrow q+e^-)$





Partonic cross section calculated to NNLO in perturbation theory

Models

- Hadronization, underlying event and multiple proton scatterings are all non-perturbative effects that can not be calculated
- Not universal enough to extract from data
- Need more or less motivated models that capture some basic properties of QCD and adjust free parameters in the model to measured distributions
- For best tuning, make sure that data samples are not biased ⇒ min bias data

Very important to describe the physics, but need to remember that large uncertainties present

Calculations

$$\sigma = \int dPS \, |M|^2$$

To obtain cross sections, need to integrate $|M|^2$ over phase space of final state particles.

Calculation of the square of the amplitude. At higher orders need to perform loop integrals.

This is what I want to focus on in this talk

Phase space integration

Phase space integration

Phase space integrals require high dimensional integrals

Only for very low multiplicity can perform by hand

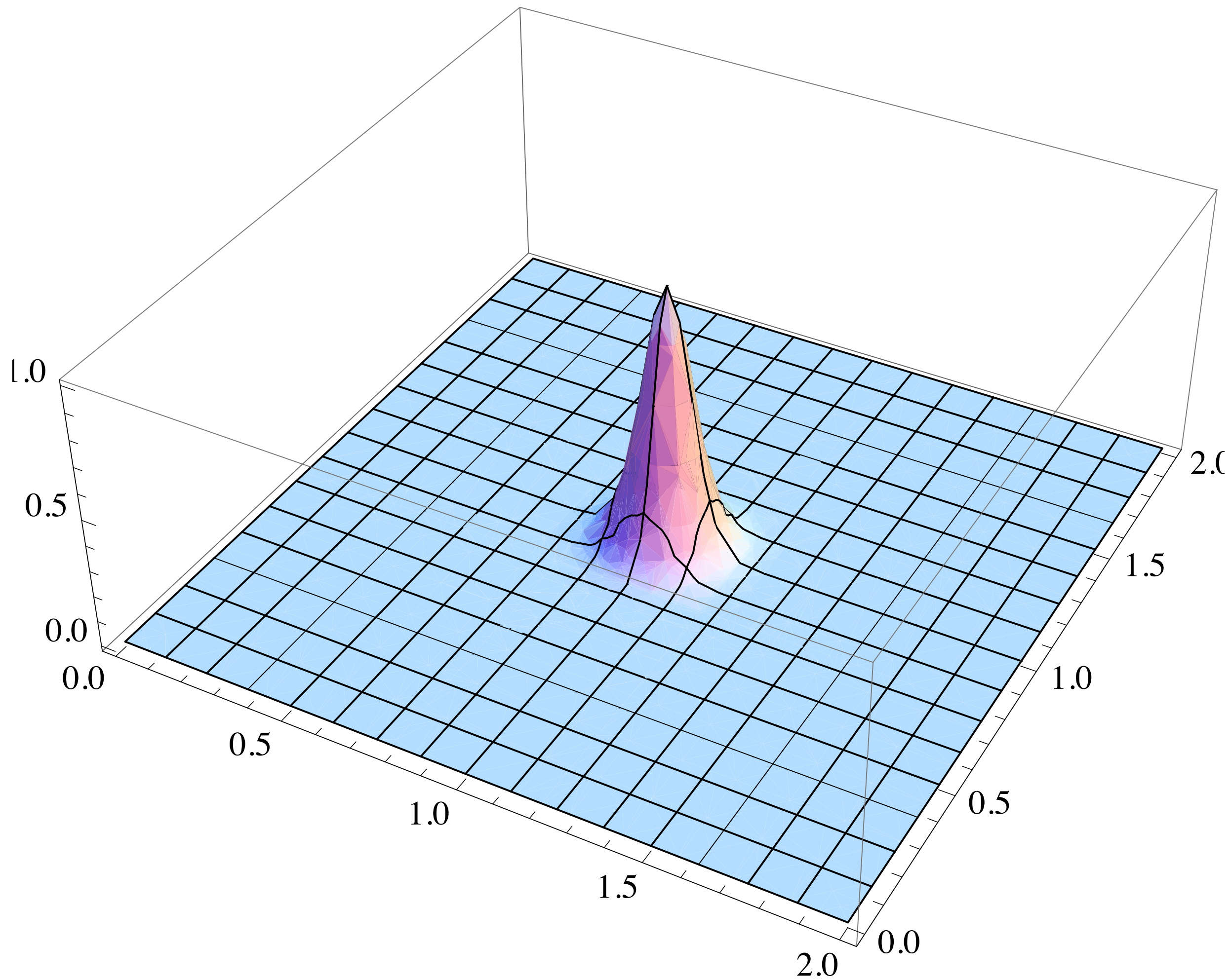
Need numerical tools
$\Rightarrow$ Monte Carlo integration

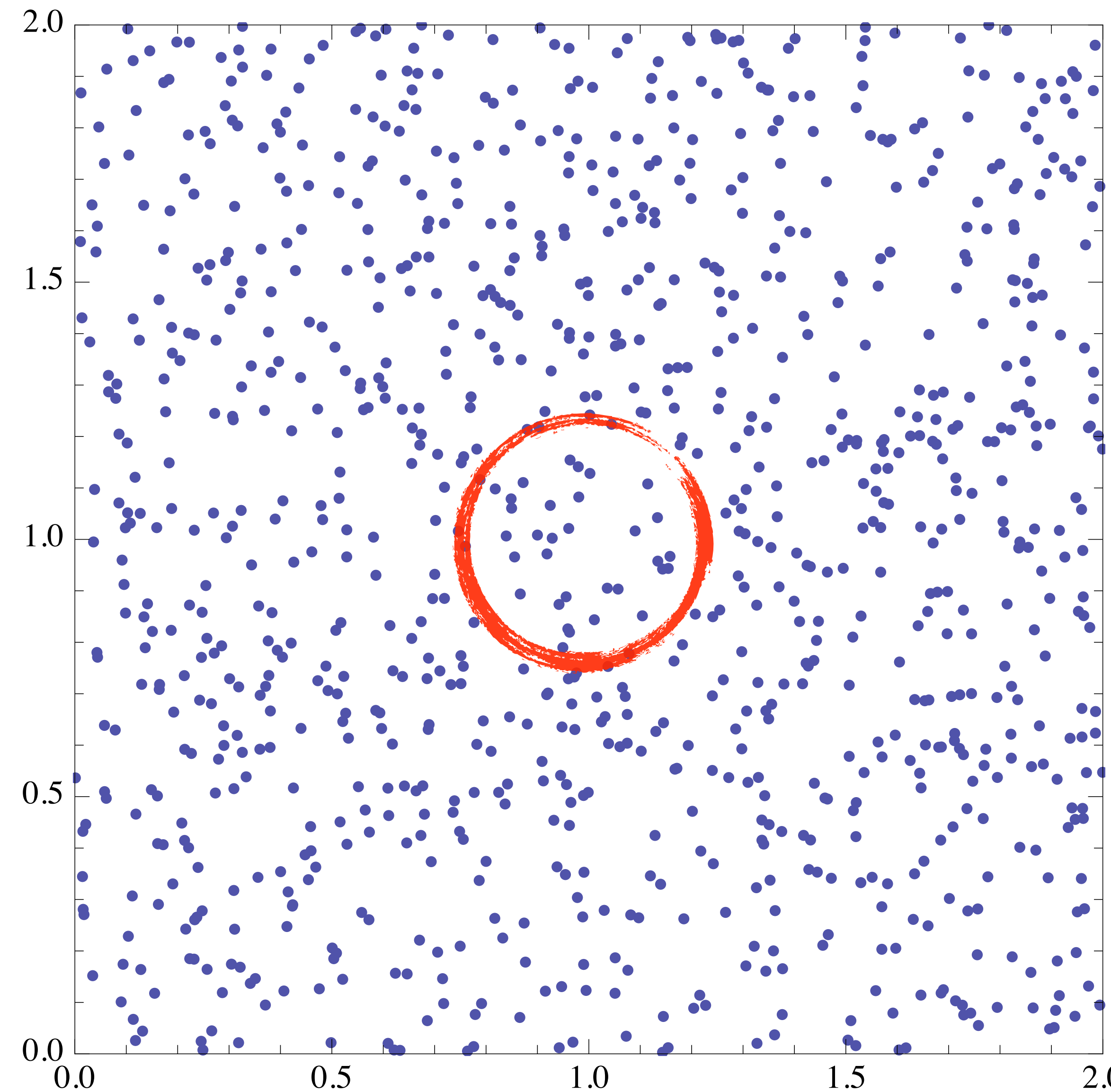
Main bottleneck in calculations is the efficiency of the Monte Carlo integration

Singularities in $|M|^2$ make MC integration inefficient

Phase space integration

How to integrate simple 2-dim gaussian distribution

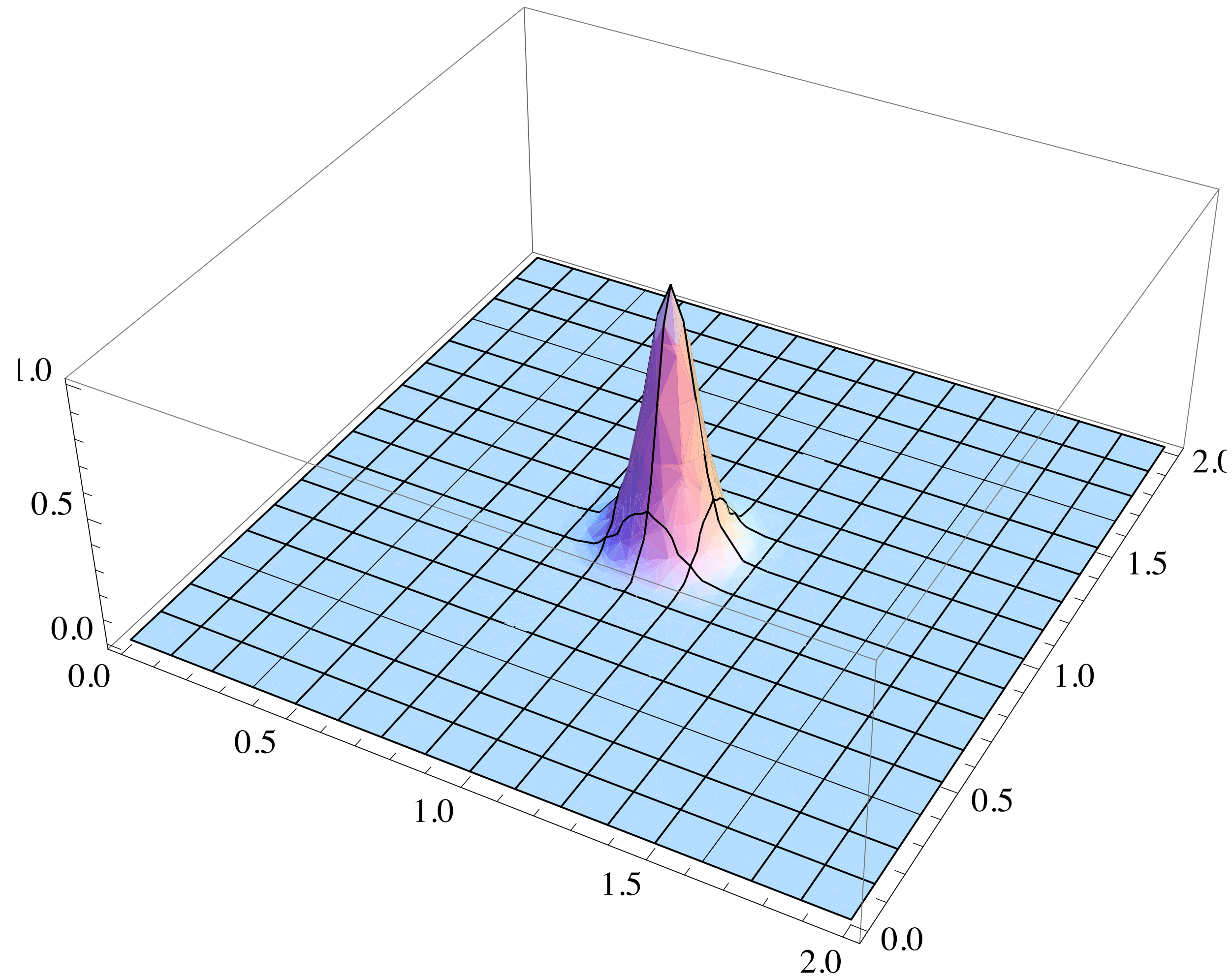


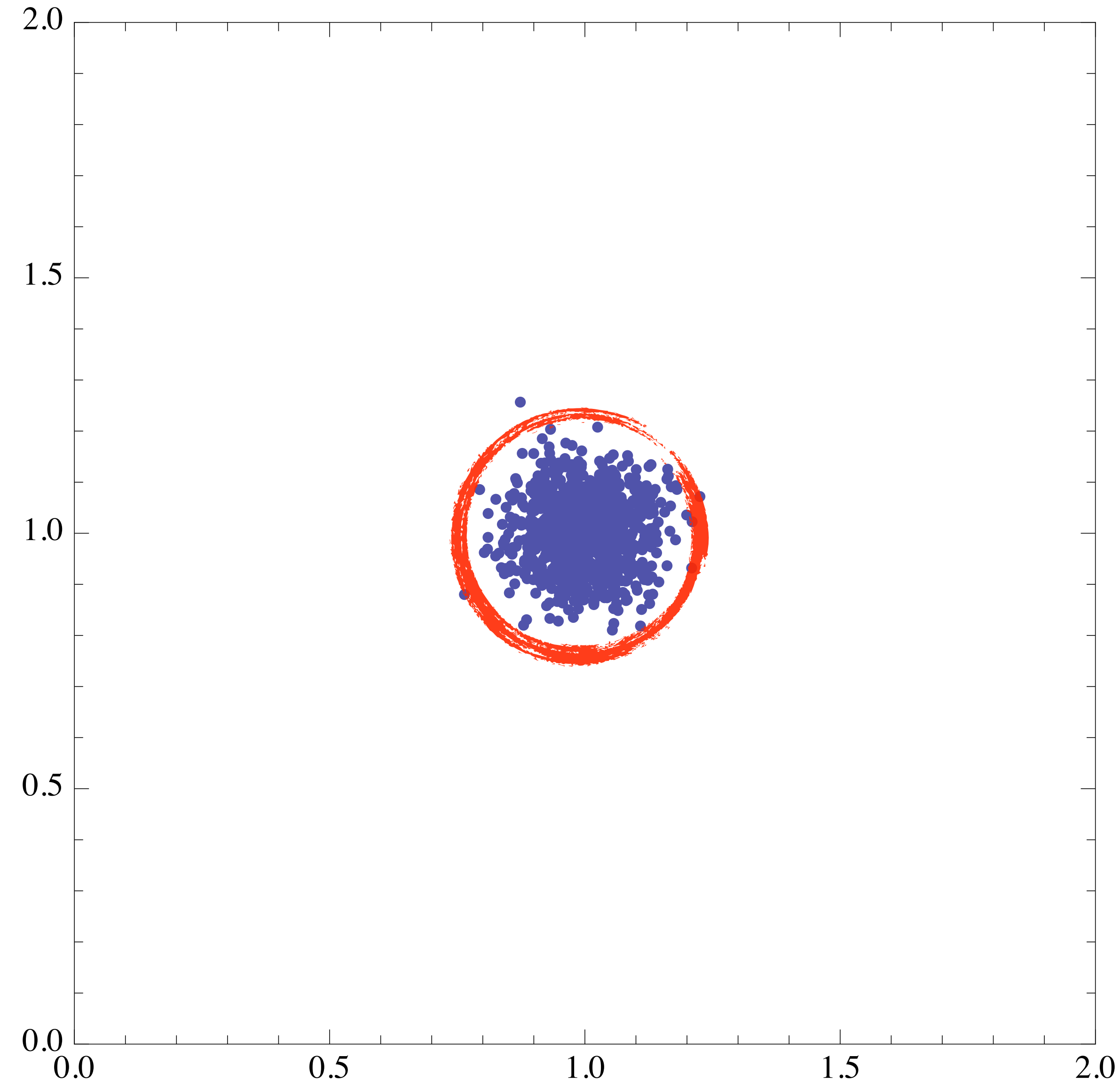


Very inefficient way of doing the integration

Phase space integration

How to integrate simple 2-dim gaussian distribution



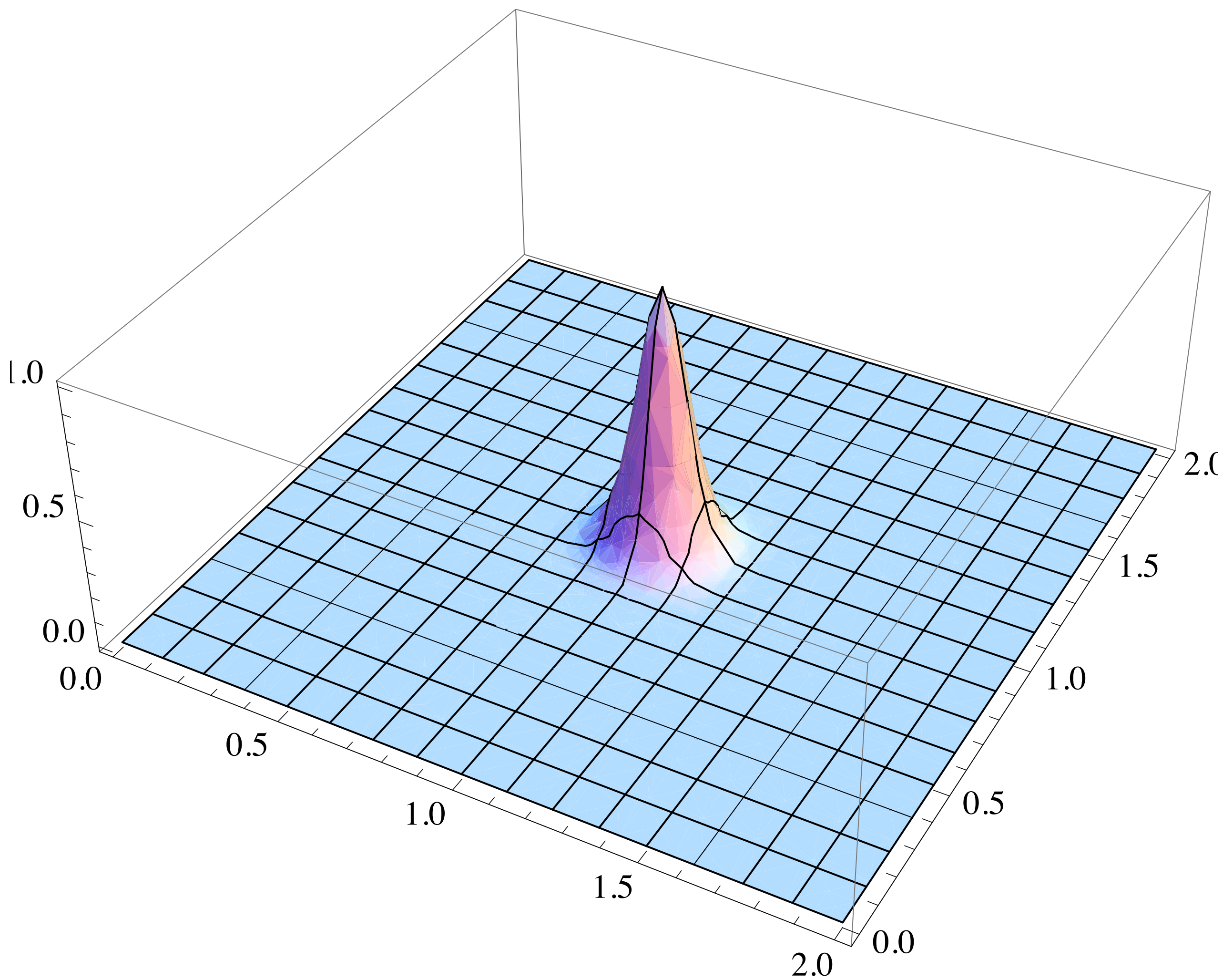


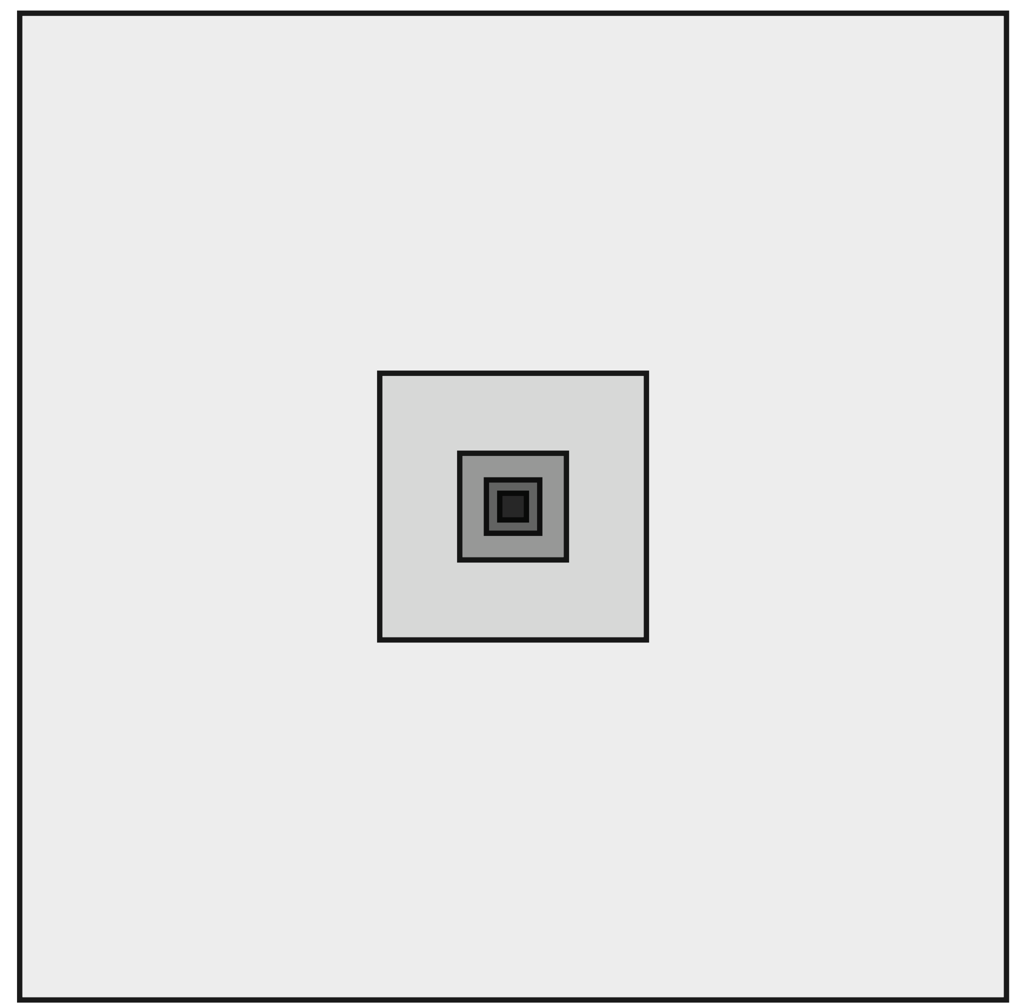
Need to distribute points according to singularities

Importance sampling (VEGAS)

Start from uniform distribution of points

Rapid changes in weights ⇒ divide into smaller areas



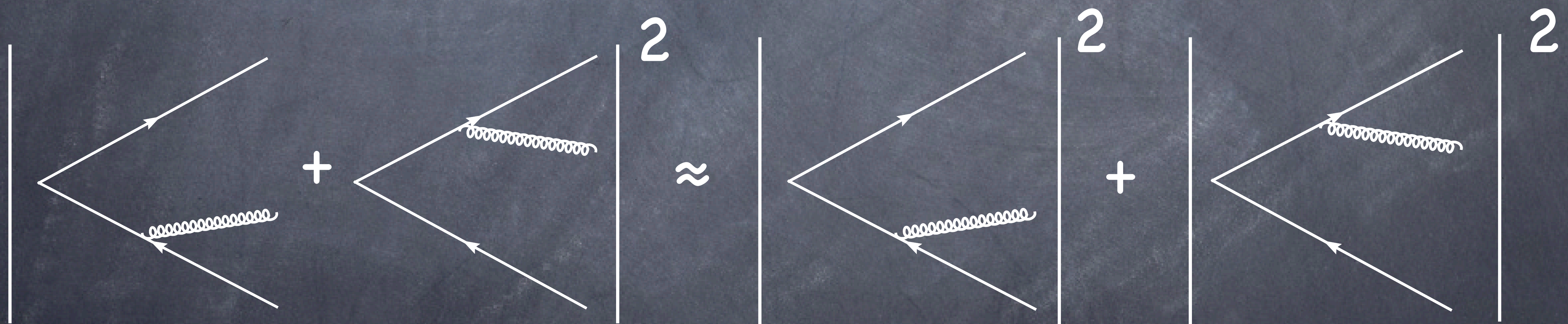


Better results, still inefficient for high multiplicity

Multi-channel (MadEvent)

$$\sum_i \frac{|M|^2}{\sum_i |M_i|^2} |M_i|^2$$

Singularities as propagators go on-shell



Integrate each term using singularity structure of $|M_i|^2$

Use Parton Shower (GenEvA)

Parton showers generate events with given kinematics

Kinematics correspond to point in phase space Φ_n

If one knows probability $P(\Phi_n)$ to generate phase space, can assign weight

$$w(\Phi_n) = |M|^2(\Phi_n) \;/\; P(\Phi_n)$$

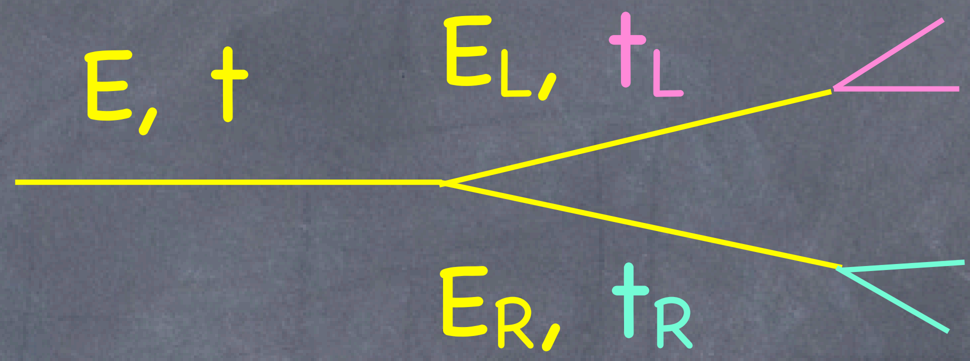
Since parton shower reproduces singularity structure of QCD

$$w(\Phi_n) = O(1)$$

What are the difficulties?

Some issues with a PS implementation

- Momentum is not conserved at leading order in parton shower
- Need to make somewhat ad-hoc corrections
- Can change the shape of distribution functions



-Start with single branch

-Add branching of left daughter

-will change values of E_L and E_R

-Add branching of right daughter

-will change values of E_L and E_R

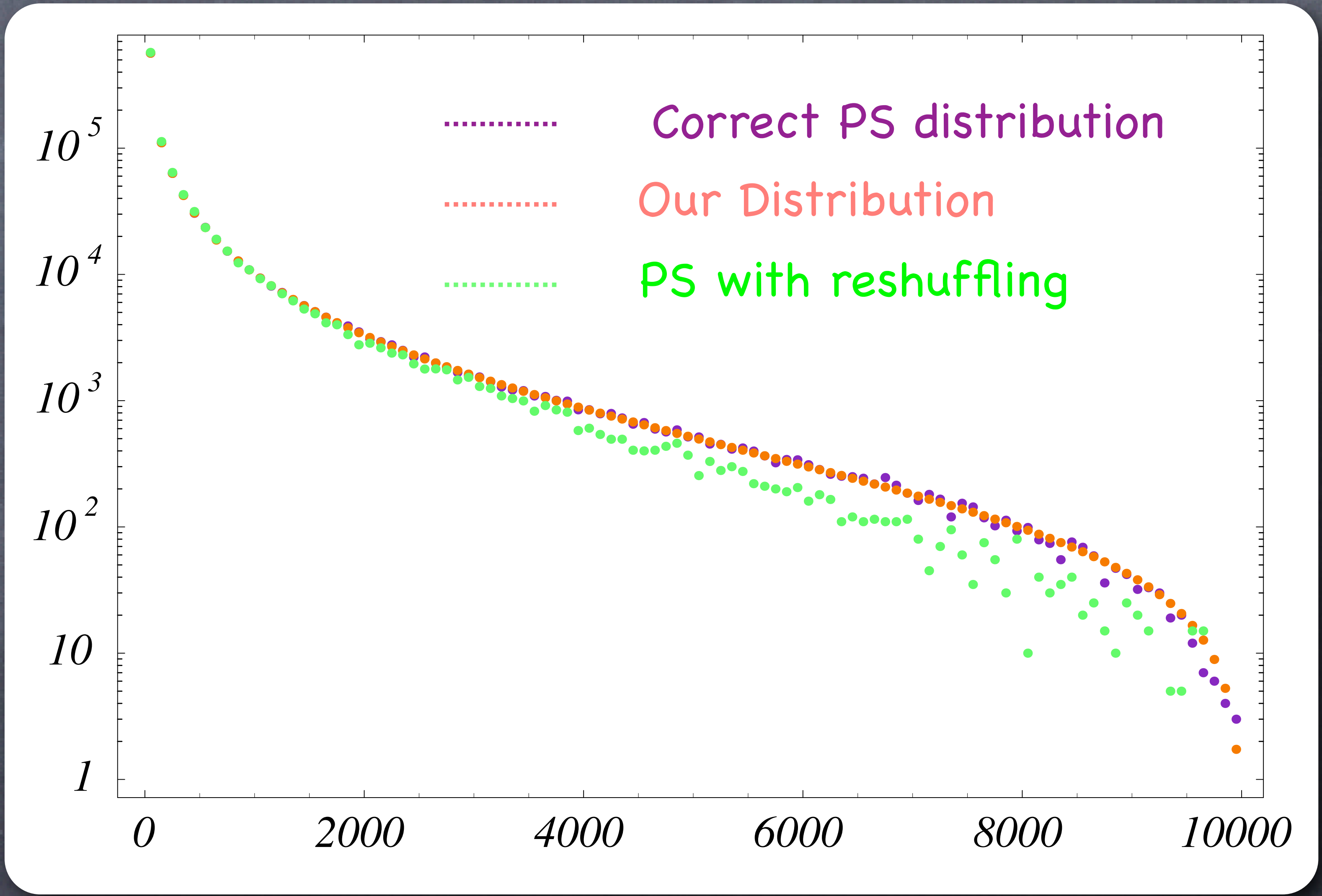
-might not be allowed given t_L

What are the difficulties?

Some issues with PS implementation

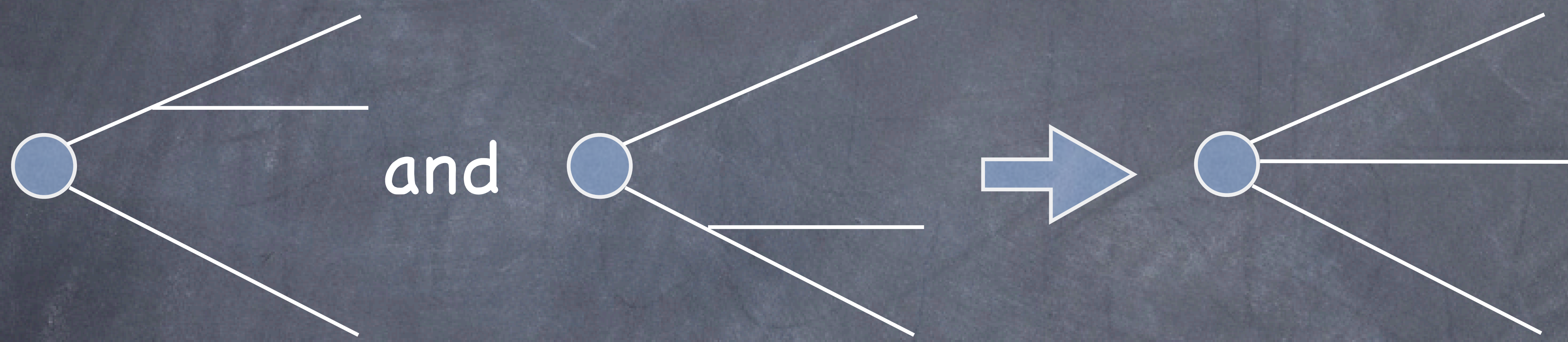
- Momentum is not conserved at leading order in parton shower
- Need to make somewhat ad-hoc corrections
- Can change the shape of distribution functions

Changes distribution



What are the difficulties?

Different parton shower histories can result in same point in phase space



Need to give different parton shower histories weights w_i , such that $\sum w_i = 1$

Have to make sure that these weights don't spoil efficiency

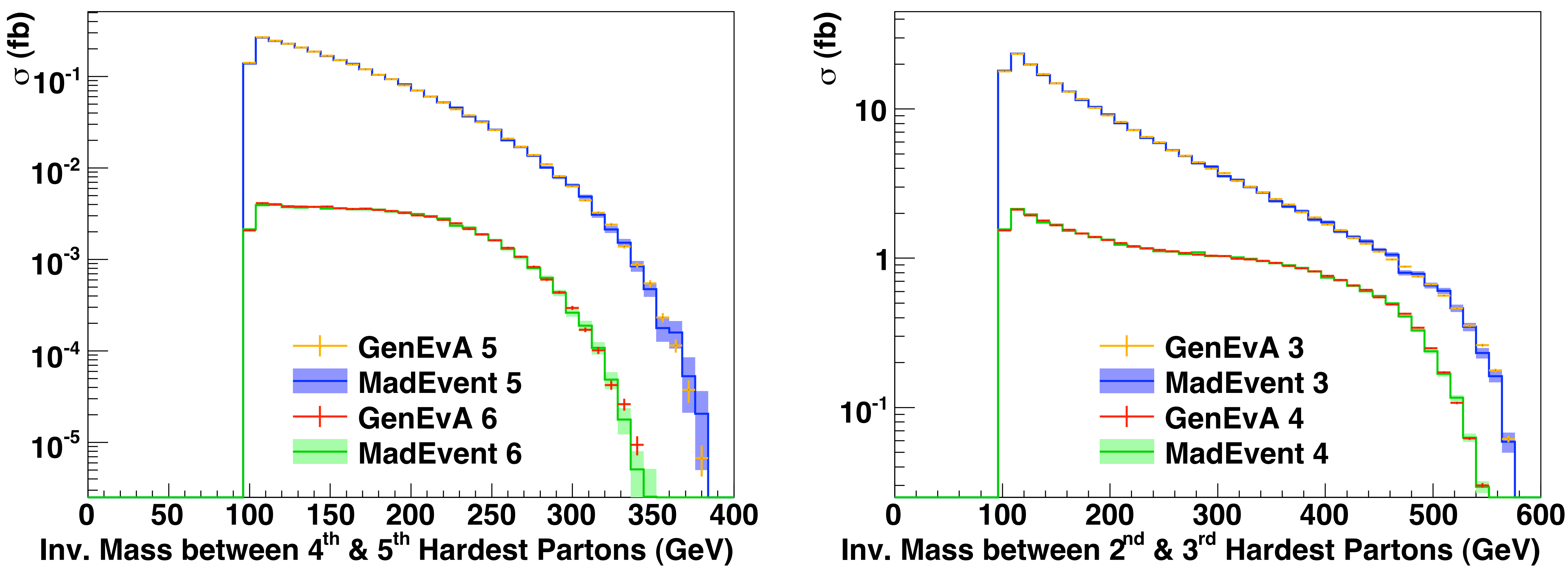
Verification of Phase Space

Use LO tree level matrix elements and compare output to other event generator (MadEvent)

process	MadEvent	GenEvA	process	MadEvent	GenEvA
LO 3 (fb)	216.71 ± 0.21	216.77 ± 0.22	LO 5 (ab)	2542 ± 3	2543 ± 3
$u\bar{u}g$	86.62 ± 0.13	86.60 ± 0.18	$u\bar{u}ggg$	912 ± 2	912 ± 2
$d\bar{d}g$	21.75 ± 0.07	21.55 ± 0.10	$d\bar{d}ggg$	227.5 ± 0.9	228.3 ± 0.8
$s\bar{s}g$	21.63 ± 0.06	21.73 ± 0.10	$u\bar{u}d\bar{d}g$	33.8 ± 0.2	34.3 ± 0.4
$c\bar{c}g$	86.71 ± 0.13	86.70 ± 0.18	$u\bar{u}u\bar{u}g$	25.6 ± 0.2	25.7 ± 0.3
LO 4 (fb)	36.44 ± 0.04	36.49 ± 0.04	LO 6 (ab)	67.9 ± 0.3	68.0 ± 0.2
$u\bar{u}gg$	14.00 ± 0.03	14.00 ± 0.02	$u\bar{u}gggg$	22.41 ± 0.09	22.29 ± 0.12
$d\bar{d}gg$	3.504 ± 0.013	3.511 ± 0.011	$u\bar{u}u\bar{u}gg$	1.117 ± 0.006	1.14 ± 0.03
$u\bar{u}d\bar{d}$	0.175 ± 0.001	0.180 ± 0.003	$u\bar{u}u\bar{u}u\bar{u}$	$0.005 \pm 0.001^{-}$	0.005 ± 0.001
$u\bar{u}u\bar{u}$	0.132 ± 0.001	0.132 ± 0.002	$u\bar{u}d\bar{d}s\bar{s}$	$0.019 \pm 0.001^{-}$	0.020 ± 0.005

Verification of Phase Space

Use LO tree level matrix elements and compare output to other event generator (MadEvent)



Comparison to MadEvent

	$\eta_{\rm eff}$	$T_{\rm eff}$ (msec)	$T_{0.9}$ (msec)
GenEvA LO 3	0.789	0.57	0.62
GenEvA LO/LL inc. 3	0.965	0.47	< 0.47
MadEvent 3	0.982	2.6	< 2.6
MadEvent $u\bar{u}g$	0.994	3.0	< 3.0
GenEvA LO 4	0.525	1.7	2.2
GenEvA LO/LL inc. 4	0.713	1.3	1.5
MadEvent 4	0.809	11.1	11.4
MadEvent $u\bar{u}gg$	0.752	5.4	5.7
GenEvA LO 5	0.390	10.0	15
GenEvA LO/LL inc. 5	0.557	8.6	10.8
MadEvent 5	0.843	62	64
MadEvent $u\bar{u}ggg$	0.833	27	27
GenEvA LO 6	0.298	160	250
GenEvA LO/LL inc. 6	0.396	150	230
MadEvent 6	0.809	1900	2300
MadEvent $u\bar{u}gggg$	0.784	330	350

GenEvA very competitive with MadEvent, even without tuning shower for efficiency

MadEvent is worse for LL improved results, while GenEvA is better

GenEva beats MadEvent dramatically for LL improved results

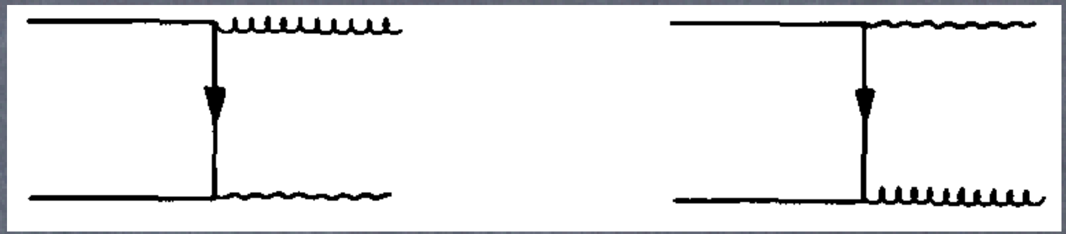
Calculation of $|M|^2$

Peturbative calculations

Calculations proceed by calculating Feynman diagrams

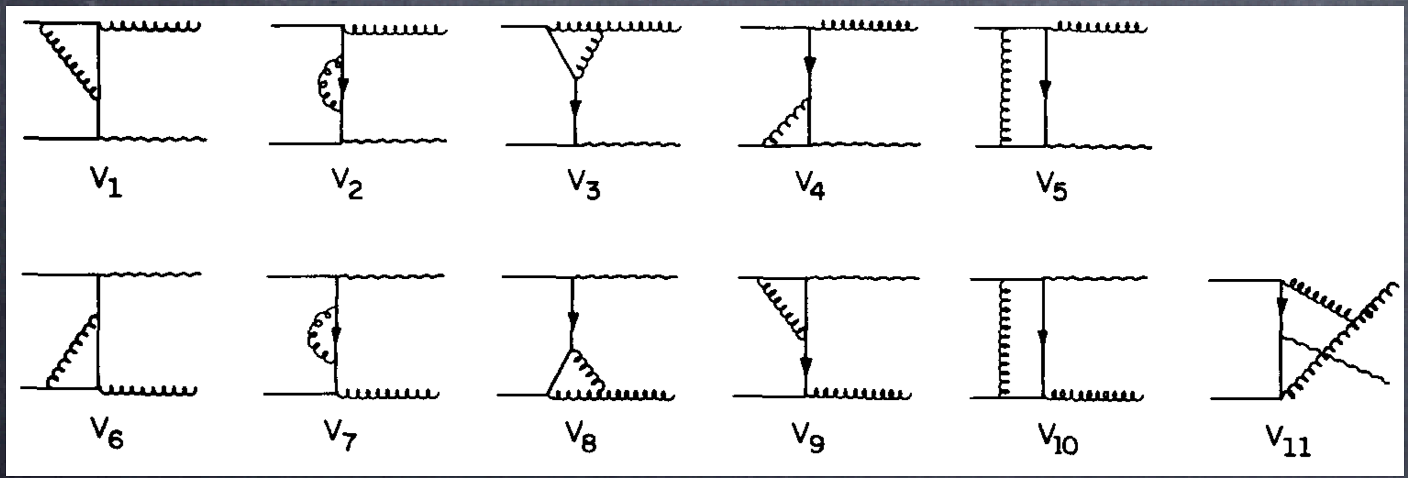
Example: pp → W j

Leading order:

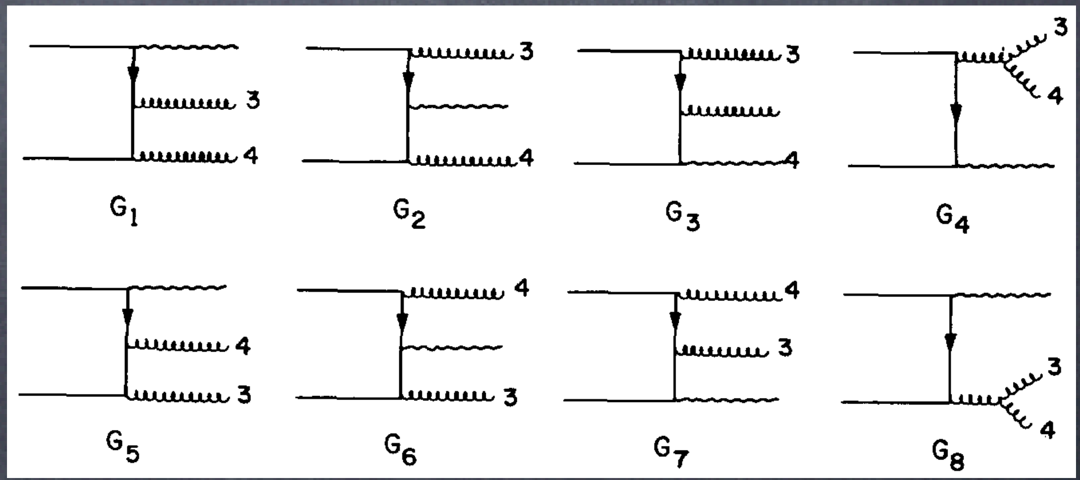


At NLO need:

virtual



real



Logarithmic resummation

Perturbative expressions at higher order always contain logarithms of ratios of scales in the problem

Take example of $pp \to W$+jets at small p_T

$$d\sigma/dp_T = N\times [\, 1 + \alpha_s L^2 + \alpha_s L + \alpha_s + \alpha_s^2 L^4 + \alpha_s^2 L^3 + \alpha_s^2 L^2 + \alpha_s^2 L + \alpha_s^2 + \ldots \,] \qquad L = \mathrm{Ln}(p_T/Q)$$

Presence of logarithms can clearly spoil perturbative expansion for large ratio p_T/Q

Need to sum all large logarithmic terms

Exclusive event generators

Most useful theoretical result for experimentalists is in the form of completely exclusive events

Need to generate events with many particles in the final state

Need to merge calculations with parton showers

Requiring such general results make calculations much more difficult

How precise can we make calculations in this form?

Minimum accuracy required

NLO accuracy

1. Required to have any idea of scale dependence
2. NLO corrections can be factor of 2 over LO

LL resummation

1. Required to be able to merge with parton shower
2. Unresummed Logarithms can completely destroy convergence of perturbation theory

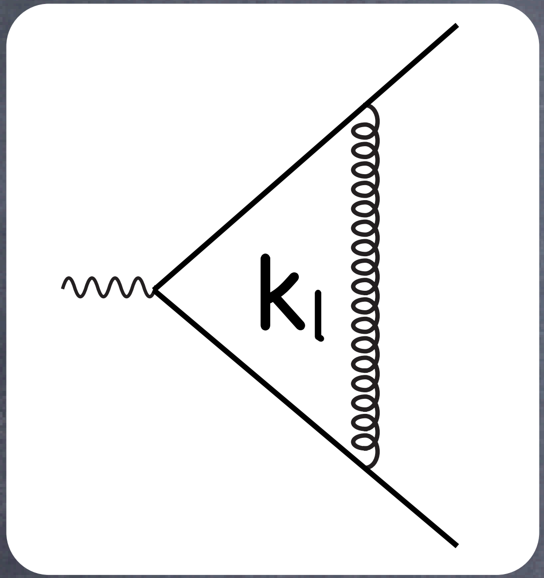
Combination of these two is the goal

Implementing NLO Calculations

Infrared divergences

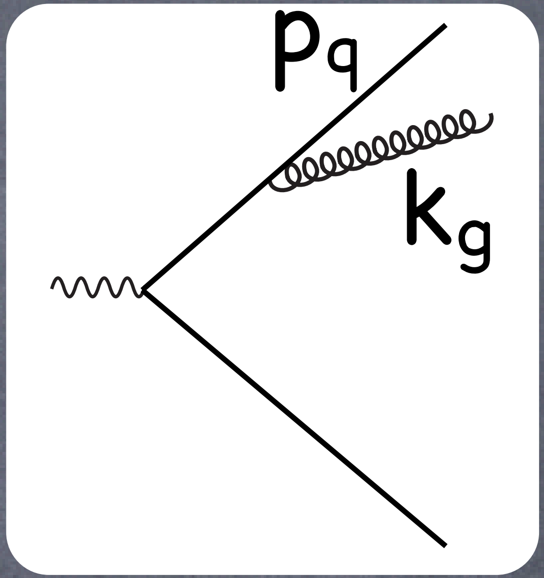
It is well known that both virtual and real contribution is IR divergent

virtual



IR div from $k_l \rightarrow 0$

real



IR div from $k_g \cdot p_q \rightarrow 0$

Note that real emission has different final state multiplicity than virtual graph

However, difference not detectable for $k_g \cdot p_q \rightarrow 0$ due to finite resolution of any detector

Analytical results

Virtual contribution

$$\left(\frac{s\,\mathrm{d}\sigma}{\mathrm{d}t}\right)_{q\bar{q}} = e_f^2 \frac{K_{qq}}{s} \alpha_s(M^2)\, T_0(Q^2,u,t) \left\{ 1 + \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \right.$$
$$\left. \times \left[-\frac{(2C_F+N_C)}{\varepsilon^2} - \frac{1}{\varepsilon}\left(3C_F - 2C_F \ln\left|\frac{s}{Q^2}\right| + \tfrac{11}{6}N_C + N_C \ln\frac{sQ^2}{ut} - \tfrac{2}{3}T_R \right)\right] + \ldots \right\}$$

Real contribution

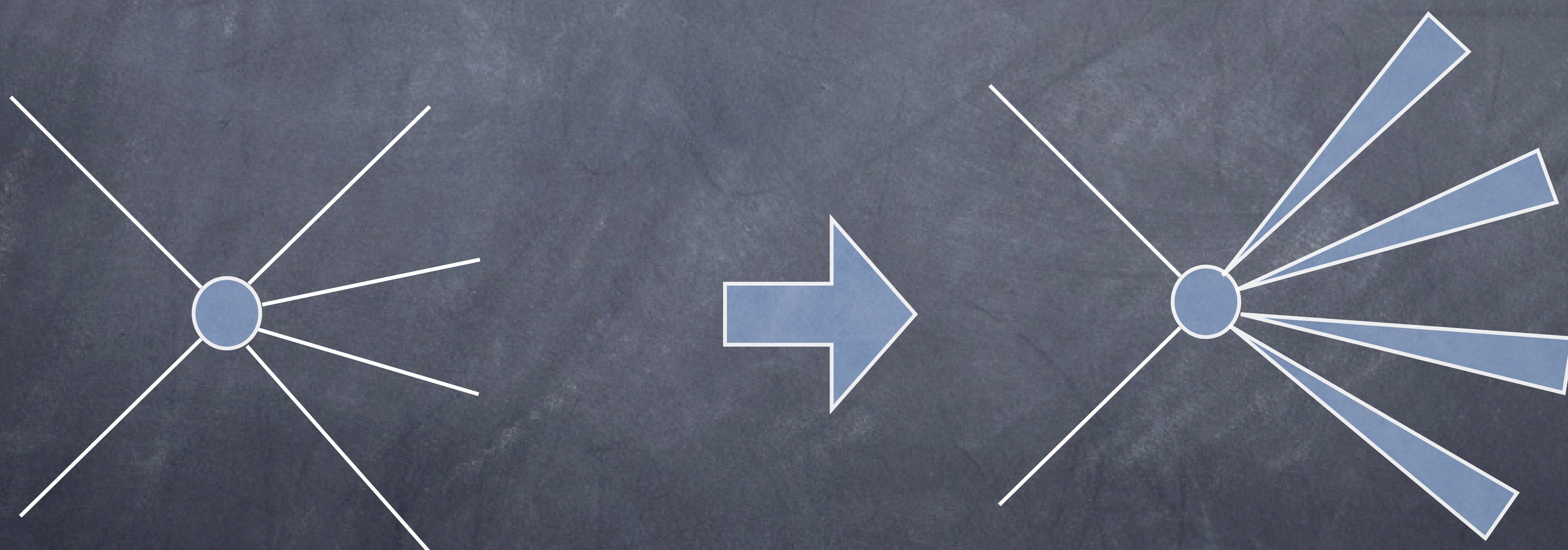
$$\frac{s\,\mathrm{d}\sigma}{\mathrm{d}t\,\mathrm{d}u} = e_f^2 \frac{K_{qq}}{s} \alpha_s \left\{ T_0(Q^2,u,t)\, \delta(s_2) \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)} \left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \right. \right.$$
$$\left. \left. \times \left[\frac{(2C_F+N_C)}{\varepsilon^2} + \frac{1}{\varepsilon}\left(3C_F + 2C_F \ln\frac{Q^2}{s} + N_C \ln\frac{Q^2 s}{ut} + \tfrac{11}{6}N_C - \tfrac{2}{3}T_R \right)\right] + \ldots \right\}$$

Divergences cancel, finite pieces left over

Infrared divergences

Problem is that IR divergences only cancel after phase space integration over real emission

Therefore, can not have fully exclusive events



Need to define exclusive cross section precisely

Definition of exclusive x-section

$$\frac{d\sigma_n^{\rm excl}}{d\Phi_n} = \left[\frac{d\sigma_n^{\rm parton}}{d\Phi_n} + \sum_{m\geq n+1}\int d\Phi_m \frac{d\sigma_m^{\rm parton}}{d\Phi_m} J_{\rm excl}(\Phi_m, \Phi_n; \mu)\right] R_n(\Phi_n)$$

"Jet" defintion, ensuring that we have no more than n final state "particles"

Restriction, making sure that we don't have less than n "particles"

In the end, have exactly n final states

Calculate @ NLO

Include both Born and Virtual term

$$\frac{d\sigma_n^{\rm parton}}{d\Phi_n} = B_n(\Phi_n) + V_n(\Phi_n) + \cdots$$

This gives

$$\left.\frac{d\sigma_n^{\rm excl}}{d\Phi_n}\right|_{\rm NLO} = \left[B_n(\Phi_n) + V_n(\Phi_n) + \int d\Phi_{n+1}\, B_{n+1}(\Phi_{n+1}) J_{\rm excl}(\Phi_{n+1}, \Phi_n, \mu_n)\right] R_n(\Phi_n)$$

$$\pm \int d\Phi_{n+1}\, S_{n+1}(\Phi_{n+1}) J_{\rm excl}(\Phi_{n+1}, \Phi_n, \mu_n) R_n(\Phi_n)$$

Define

$V^S = V + \int S$ $\qquad$ $B^S = B - S$

Calculate @ NLO

Include both Born and Virtual term

$$\frac{d\sigma_n^{\text{parton}}}{d\Phi_n} = B_n(\Phi_n) + V_n(\Phi_n) + \cdots$$

This gives

$$\left.\frac{d\sigma_n^{\text{excl}}}{d\Phi_n}\right|_{\text{NLO}} = \left[B_n(\Phi_n) + V_n^S(\Phi_n) + \int d\Phi_{n+1}\, B_{n+1}^S(\Phi_{n+1}) J_{\text{excl}}(\Phi_{n+1}, \Phi_n, \mu_n)\right] R_n(\Phi_n)$$

B^S has no more singularities, and J_{excl} only integrates over very small region

$\Rightarrow \int B^S J_{excl} \to 0$

Calculate @ NLO

Include both Born and Virtual term

$$\frac{\mathrm{d}\sigma_n^{\mathrm{parton}}}{\mathrm{d}\Phi_n} = B_n(\Phi_n) + V_n(\Phi_n) + \cdots$$

This gives

$$\left.\frac{\mathrm{d}\sigma_n^{\mathrm{excl}}}{\mathrm{d}\Phi_n}\right|_{\mathrm{NLO}} = \left[B_n(\Phi_n) + V_n^S(\Phi_n, \mu)\right] R_n(\Phi_n) + \mathcal{O}(\mu)$$

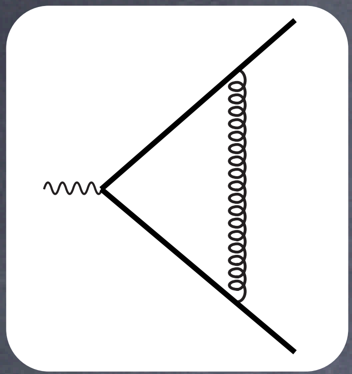
Can extract this from programs such as Blackhat etc

Note that σ_n^{excl} depends on scale μ

Resummation of Logs

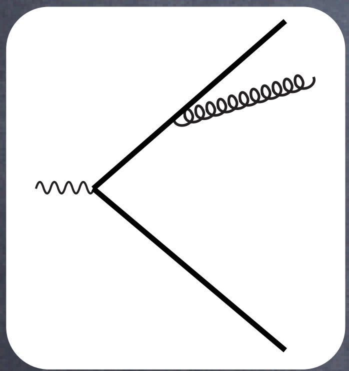
Logarithmic resummation

Logarithms related to IR divergences in theory



Divergences from loop integrations

$$\int_0^\infty dk_l\,(k_l - Q)^{-1-\varepsilon} = 1/\varepsilon + \ldots$$



Divergences from phase space integrations

$$\int_0^1 dk_g\,(k_g)^{-1-\varepsilon} = -\,1/\varepsilon + \ldots$$

Restrictions on phase space give rise to logarithmic remainders $0<k_g<\mu \Rightarrow -1/\varepsilon + \log(\mu)$

$$\sigma_V + \sigma_R = \log(\mu) + \ldots$$

Relating IR and UV divergences

Effective theories are defined to reproduce the IR physics of underlying theory

If EFT only contains no intrinsic mass scales, it only depends on Λ_{IR} and Λ_{UV}

In pure dim-reg $\Lambda_{IR} \to 0$ and $\Lambda_{UV} \to \infty$

No scale in problem $\Rightarrow$ result in EFT is 0

$\Rightarrow$ $\text{Log}(\Lambda_{IR}) = - \text{Log}(\Lambda_{UV})$

IR dependence of full theory can be extracted from UV dependence of EFT

Sum logs using RG Equations

Derive RG Equation, by taking μ d/dμ

$$\mu \frac{d}{d\mu} \frac{d\sigma_n^{\rm LL}(\mu)}{d\Phi_n} = \gamma_n(\mu) \frac{d\sigma_n^{\rm LL}(\mu)}{d\Phi_n}$$

Solve to find

$$\frac{d\sigma_n^{\rm LL}(\mu)}{d\Phi_n} = \frac{d\sigma_n^{\rm LL}(\mu_0)}{d\Phi_n} \Delta_n(\mu_0, \mu)$$

Δ_n = Sudakov factor

Resums all logarithms of μ

Resumming kinematic logs

Many scales in problem, such as different values of p_T

For universal result, want to resum ratios of all scales

For very particular choice of J_{excl}, possible to write the result as

$$\sigma_n(\mu_1,\mu_2,\ldots) = \sigma_n(Q)\ \Delta_n(Q,\mu_1)\ \Delta_n(Q,\mu_1)\ \ldots$$

Possible to sum all logarithms in the exclusive cross sections

Combining NLO and LL results

Schematically, write

$$\sigma_n^{excl} = \sigma_n^{LL} + M_n$$

Determine matching coefficient M_n by requiring correct NLO expression when expanded

$$M_n^{i_n,(0)}(\Phi_n) = S_n^{i_n}(\Phi_n)\left(\frac{B_n(\Phi_n)}{S_n(\Phi_n)} - 1\right),$$

$$M_n^{i_n,(1)}(\Phi_n) = S_n^{i_n}\left(\frac{V_n^S(\Phi_n,\mu_n)}{S_n(\Phi_n)} - \frac{V_{n-1}^S(\Phi_{n-1}^{i_n}, t_n^{i_n})}{B_{n-1}(\Phi_{n-1}^{i_n})} - \Delta_n^{(1)}(t_n^{i_n},\mu_n)\right)$$

Everything can be calculated analytically

Status of the work?

Have all the analytical results worked out in detail for e^+e^-

Finished implementation in C++ code

Working on the implementation with Pythia parton shower

Working on the extension to allow for hadron colliders

Will be interesting to see how well it works!

Conclusions

- SM predictions need to include many effects to give adequate predictions for experimental observables
- In order to compare with data, need exclusive events, distributed in phase space according to SM predictions
- New ideas in phase space generation can remove major bottleneck in efficiency of calculations
- Combination of NLO calculations with generic LL resummation is becoming reality

Look forward to testing these theoretical ideas against real data from the LHC