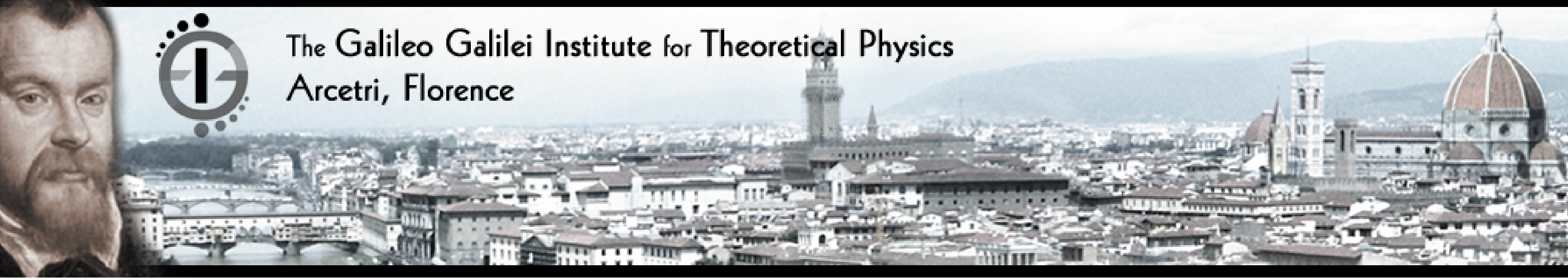

The ground state of finite density QCD

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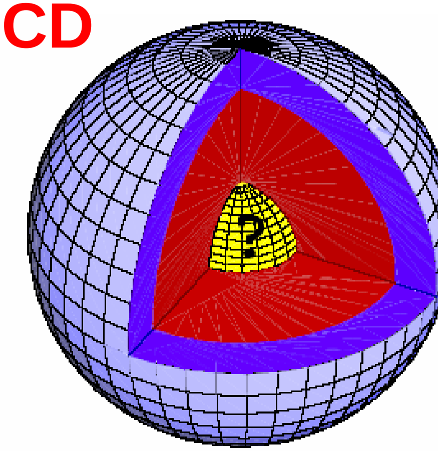
Summary

- Introduction
- Color Superconductivity: CFL and 2SC phases
- The case of pairing at different Fermi surfaces
- LOFF (or crystalline) phase
- Conclusions

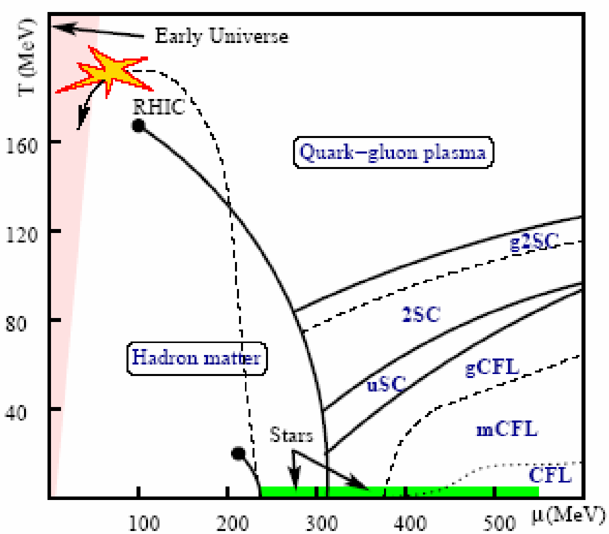
Introduction

Motivations for the study of high-density QCD



- Study of the QCD phase in a region difficult in lattice calculations
- Understanding the interior of CSO's

Asymptotic region in μ fairly well understood: existence of a CS phase.
Real question: does this type of phase persist at relevant densities ($\sim$5-6 ρ_0)?



CFL and S2C phases

- Ideas about CS back in 1975 (Collins & Perry-1975, Barrois-1977, Frautschi-1978).

- Only in 1998 (Alford, Rajagopal & Wilczek; Rapp, Schafer, Schuryak & Velkovsky) a real progress.

- Why CS? For an arbitrary attractive interaction it is energetically convenient the formation of Cooper pairs (difermions)

- Due to asymptotic freedom quarks are almost free at high density and we expect diquark condensation in the color attractive channel 3^* (the channel 6 is repulsive).

In matter SC only under particular conditions (phonon interaction should overcome the Coulomb force)

$$\frac{T_c(\text{electr.})}{E(\text{electr.})} \approx \frac{1 \div 10\ {}^0\text{K}}{10^4 \div 10^5\ {}^0\text{K}} \approx 10^{-3} \div 10^{-4}$$

In QCD attractive interaction (antitriplet channel)

$$\frac{T_c(\text{quarks})}{E(\text{quarks})} \approx \frac{50\ \text{MeV}}{100\ \text{MeV}} \approx 1$$

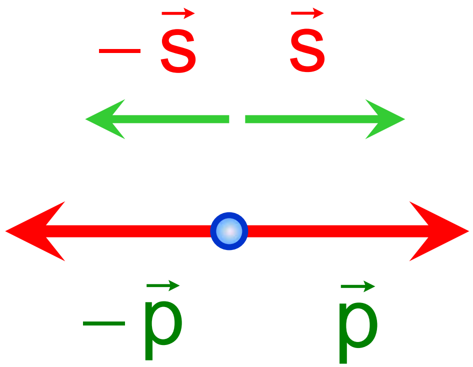
SC much more efficient in QCD

The symmetry breaking pattern of QCD at high-density depends on the number of flavors with mass $m < \mu$. Two most interesting cases: $N_f = 2, 3$. Consider the possible pairings at very high density

$$\langle 0 | \psi^{\alpha}_{ia} \psi^{\beta}_{jb} | 0 \rangle$$

α, β color; i, j flavor; a,b spin

- **Antisymmetry in spin (a,b) for better use of the Fermi surface**
- **Antisymmetry in color (a, b) for attraction**
- **Antisymmetry in flavor (i,j) for Pauli principle**



Only possible pairings LL and RR

For $\mu >> m_u, m_d, m_s$

Favorite state for $N_f = 3$, CFL (color-flavor locking) (Alford, Rajagopal & Wilczek 1999)

$$\langle 0 | \psi^{\alpha}_{iL} \psi^{\beta}_{jL} | 0 \rangle = -\langle 0 | \psi^{\alpha}_{iR} \psi^{\beta}_{jR} | 0 \rangle \propto \Delta \varepsilon^{\alpha\beta C} \varepsilon_{ijC}$$

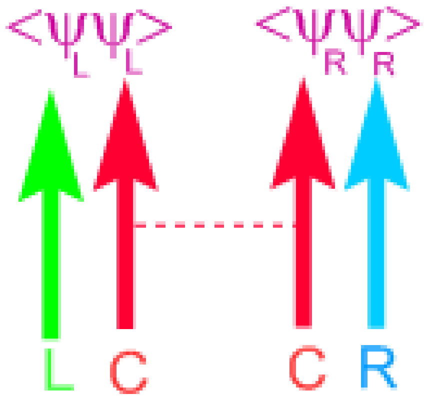
Symmetry breaking pattern

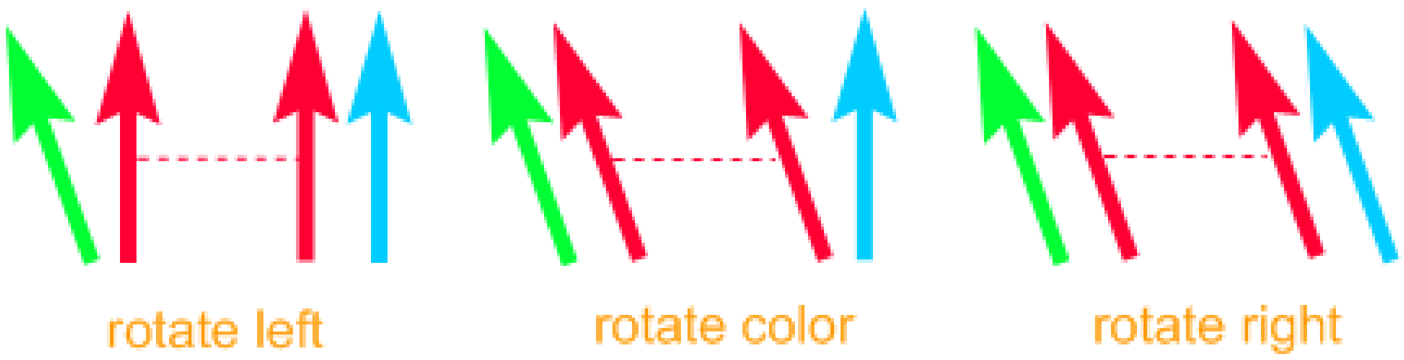
$$SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \Rightarrow SU(3)_{c+L+R}$$

Why CFL?

$$\langle 0 | \psi^{\alpha}_{iL} \psi^{\beta}_{jL} | 0 \rangle \propto \Delta \varepsilon^{\alpha\beta C} \varepsilon_{ijC}$$

$$\langle 0 | \psi^{\alpha}_{iR} \psi^{\beta}_{jR} | 0 \rangle \propto -\Delta \varepsilon^{\alpha\beta C} \varepsilon_{ijC}$$





What happens going down with μ? If $\mu \ll m_s$, we get 3 colors and 2 flavors (2SC)

$$\left\langle 0 \left| \psi^{\alpha}_{iL} \psi^{\beta}_{jL} \right| 0 \right\rangle = \Delta \varepsilon^{\alpha\beta 3} \varepsilon_{ij}$$

$$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \Rightarrow SU(2)_c \otimes SU(2)_L \otimes SU(2)_R$$

However, if μ is in the intermediate region we face a situation with quarks belonging to different Fermi surfaces (see later). Then other phases could be important (LOFF, etc.)

Pairing fermions with different Fermi momenta

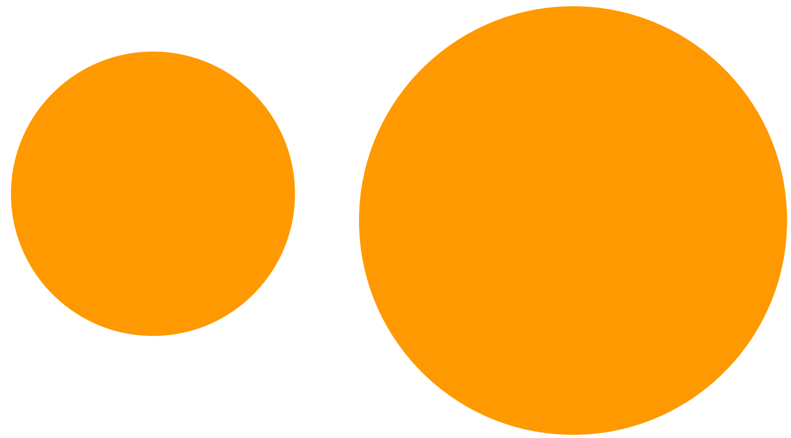
- M_s not zero
- Neutrality with respect to em and color
- Weak equilibrium

no free energy cost in neutral $\longrightarrow$ singlet, (Amore et al. 2003)

All these effects make Fermi momenta of different fermions unequal causing problems to the BCS pairing mechanism

Consider 2 fermions with $m_1 = M$, $m_2 = 0$ at the same chemical potential μ. The Fermi momenta are

$$p_{F_1} = \sqrt{\mu^2 - M^2}$$



$$p_{F_2} = \mu$$

Effective chemical potential for the massive quark

$$\mu_{\text{eff}} = \sqrt{\mu^2 - M^2} \approx \mu - \frac{M^2}{2\mu}$$

Mismatch: $$\boxed{\delta\mu \approx \frac{M^2}{2\mu}}$$

$M^2/2\mu$ effective chemical potential

Neutrality and β equilibrium

- **Weak equilibrium makes chemical potentials of quarks of different charges unequal:**

$$d \to u e \bar{\nu} \Rightarrow \mu_d - \mu_u = \mu_e$$

- **From this:** $\mu_i = \mu + Q_i \mu_Q$ **and**

$$\boxed{\mu_e = -\mu_Q}$$

- **N.B. μ_e is not a free parameter, it is fixed by the neutrality condition:**

$$Q = -\frac{\partial V}{\partial \mu_e} = 0$$

The case of non interacting quarks

$$\mu_u = \bar{\mu} - \frac{2}{3}\mu_e\,, \quad \mu_d = \mu_s = \bar{\mu} + \frac{1}{3}\mu_e$$

$$\boxed{\mu_{d,s} = \mu_u + \mu_e}$$

$$N_{u,d} = \int_0^{\mu_{u,d}} p^2 dp = \frac{\mu_{u,d}^3}{\pi^2}$$

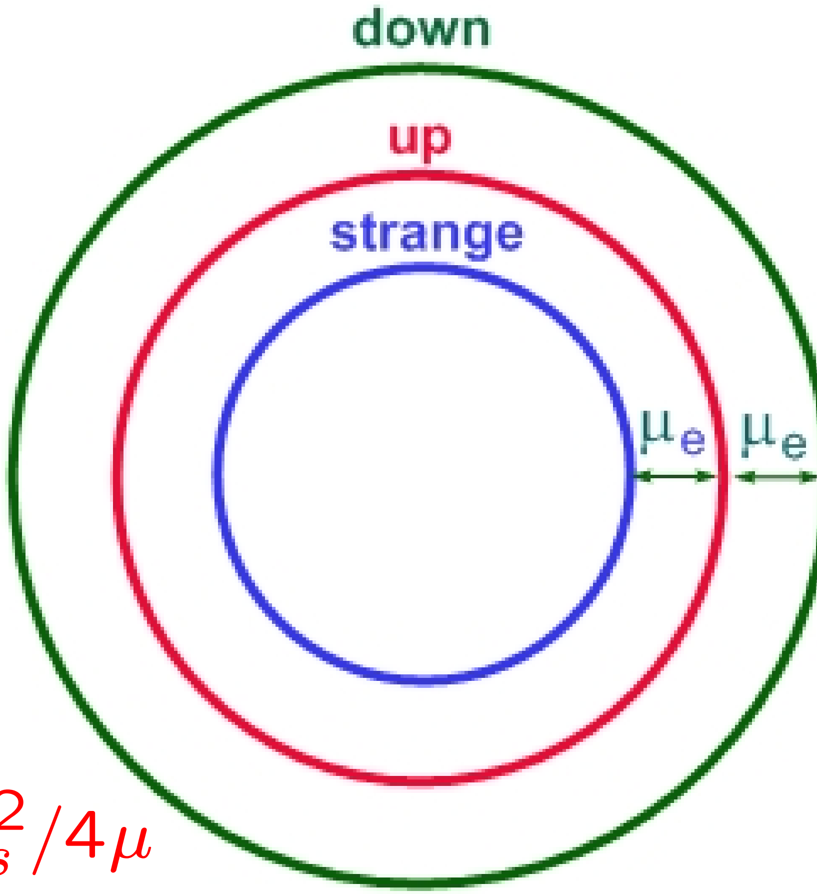
$$N_s = \frac{(\mu_s^2 - m_s^2)^{3/2}}{\pi^2}\,, \quad N_e = \frac{\mu_e^3}{3\pi^2}$$

Electric neutrality requires

$$\frac{2}{3}N_u - \frac{1}{3}N_d - \frac{1}{3}N_s - N_e = 0$$

If the strange quark is massless this equation has solution
$N_u = N_d = N_s$, $N_e = 0$; quark matter electrically neutral with no electrons

- **Fermi surfaces for neutral and color singlet unpaired quark matter at the β equilibrium and M_s not zero.**
- **In the normal phase $\mu_3 = \mu_8 = 0$.**



By taking into account M_s

$$\mu_e \approx p_F^d - p_F^u \approx p_F^u - p_F^s \approx M_s^2/4\mu$$

$$p_F^d - p_F^s \approx 2\mu_e, \qquad \mu_e = \frac{M_s^2}{4\mu}$$

As long as $\delta\mu$ is small no effects on BCS pairing, but when increased the BCS pairing is lost and two possibilities arise:

- **The system goes back to the normal phase**
- **Other phases can be formed**

- **Notice that there are also color neutrality conditions**

$$\boxed{\frac{\partial V}{\partial \mu_3} = T_3 = 0, \qquad \frac{\partial V}{\partial \mu_8} = T_8 = 0}$$

The problem of two fermions with different chemical potentials:

$$\mu_u = \mu + \delta\mu, \quad \mu_d = \mu - \delta\mu$$

$$\mu = \frac{\mu_u + \mu_d}{2}, \quad \delta\mu = \frac{\mu_u - \mu_d}{2}$$

can be described by an interaction hamiltonian

$$\boxed{H_I = -\delta\mu\,\psi^\dagger\sigma_3\psi}$$

In normal SC:

Gap equation:

$$1=\frac{g}{2}\int\frac{d^3p}{(2\pi)^3}\frac{1}{\varepsilon(\vec{p},\Delta)}(1-n_u-n_d)$$

$$n_{u,d}=\frac{1}{e^{(\varepsilon(\vec{p},\Delta)\pm\delta\mu)/T}+1}$$

$$\varepsilon(\vec{p},\Delta)=\sqrt{(|\vec{p}|-\mu)^2+\Delta^2}$$

For $T\to 0$

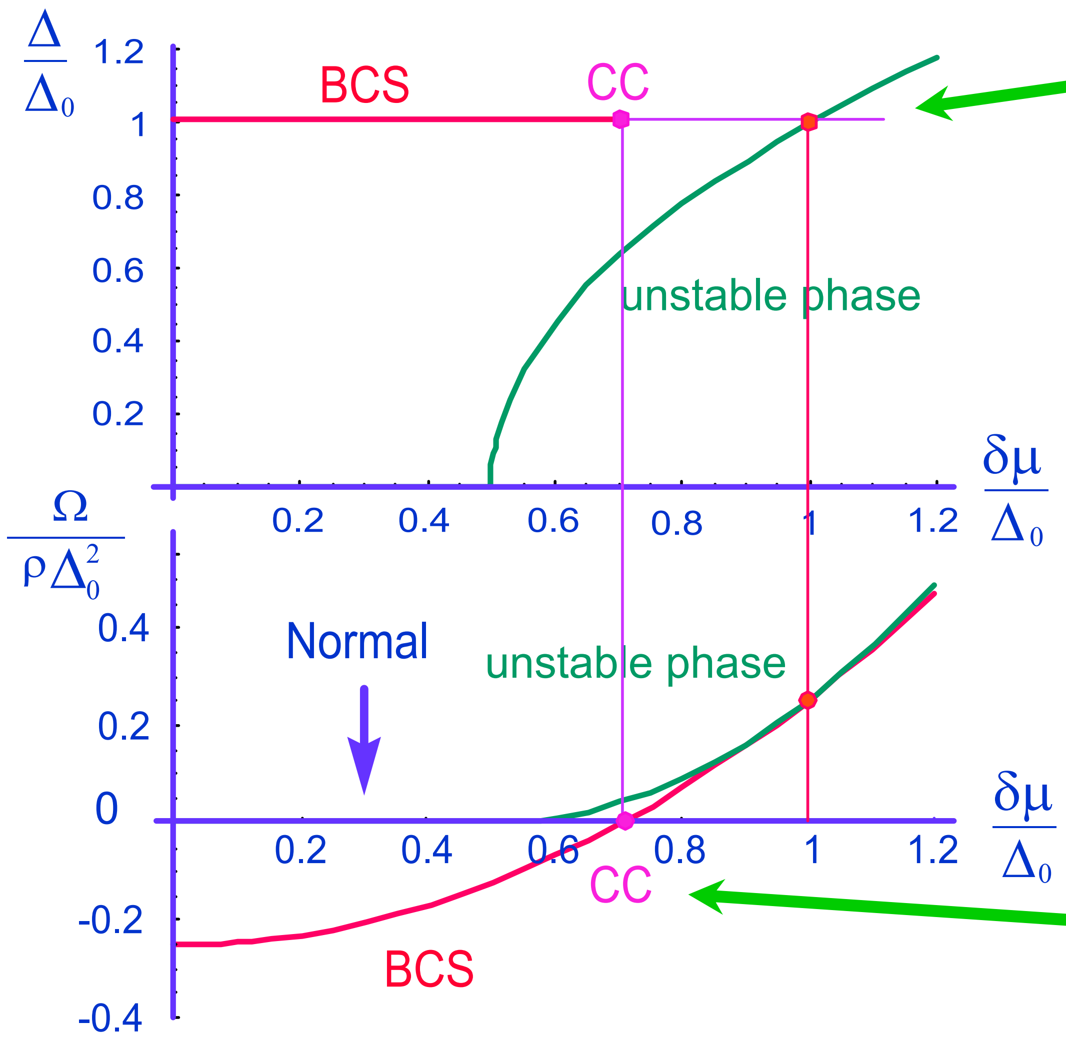
blocking region $\varepsilon<|\delta\mu|$

$$1=\frac{g}{2}\int\frac{d^3p}{(2\pi)^3}\frac{1}{\varepsilon(\vec{p},\Delta)}(1-\theta(-\varepsilon-\delta\mu)-\theta(-\varepsilon+\delta\mu))$$

The blocking region reduces the gap:

$$\boxed{\Delta<<\Delta_{BCS}}$$

In a simple model without supplementary conditions, with two fermions at chemical potentials μ_1 and μ_2, the system becomes normal at the Chandrasekhar - Clogston point. Another unstable phase exists.

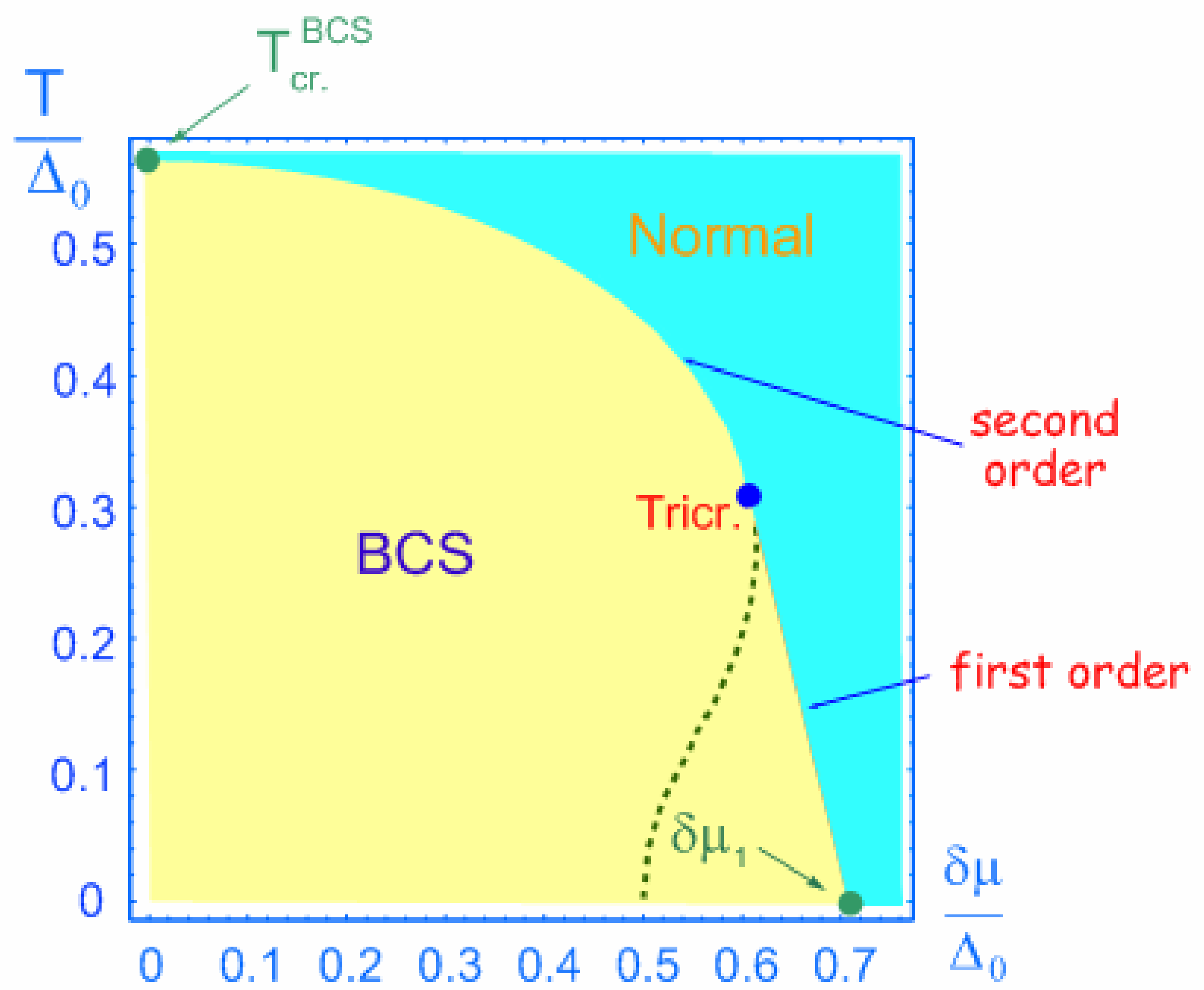


$$\delta\mu = \Delta_{BCS}$$

Sarma phase, tangential to the BCS phase at $\delta\mu=\Delta$

$$\delta\mu_1 = \frac{\Delta_{BCS}}{\sqrt{2}}$$

Phase diagram



g2SC

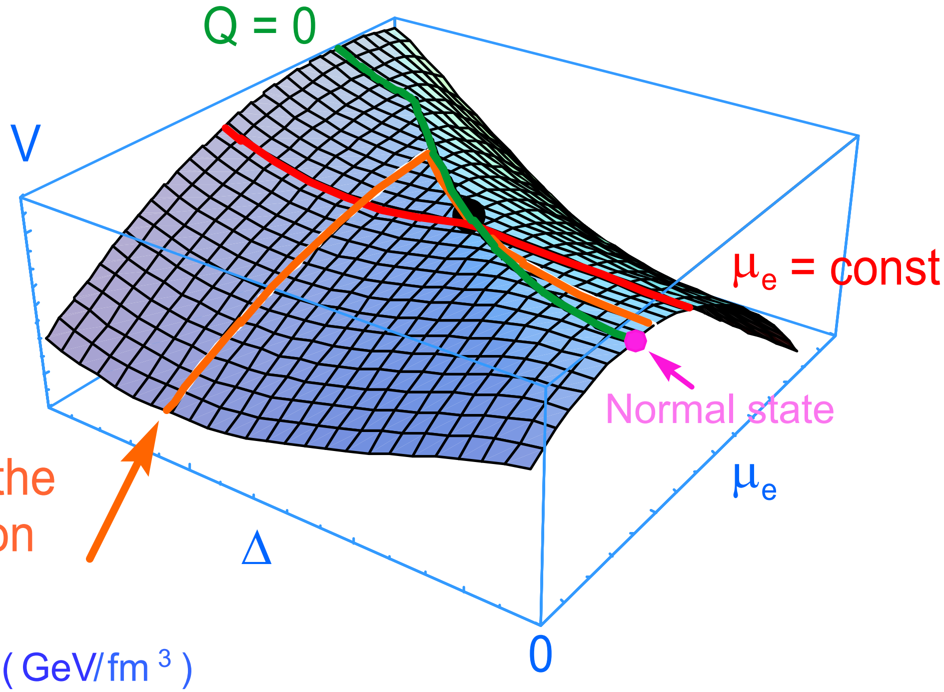
(Huang & Shovkovy, 2003)

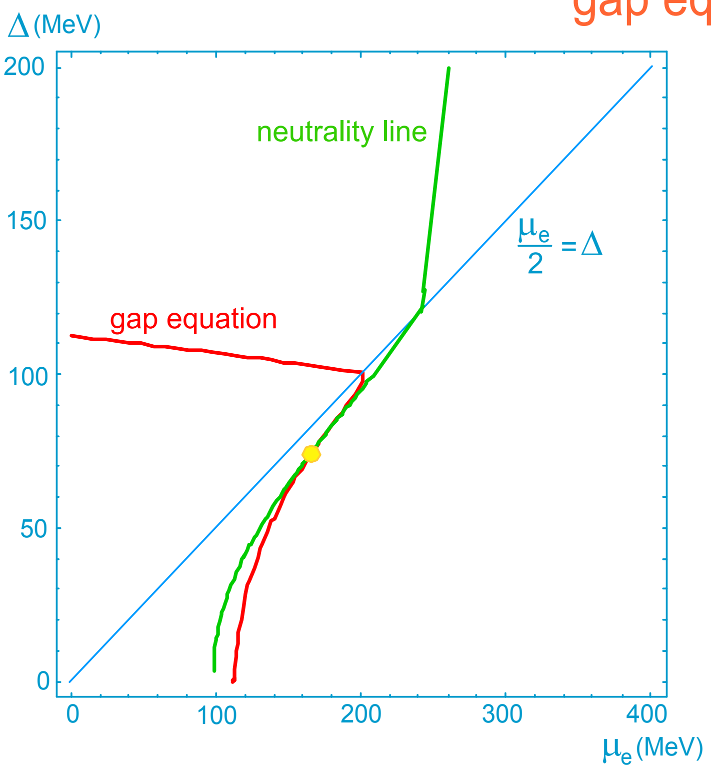
- If neutrality constraints are present the situation is different.

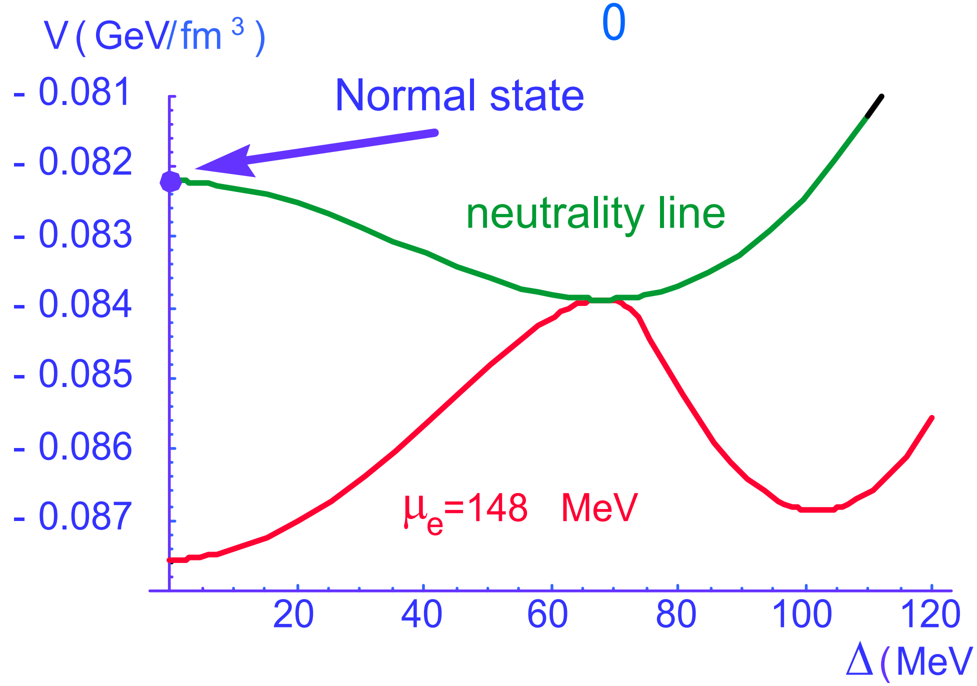
- For $|\delta\mu| > \Delta$ ($\delta\mu = \mu_e/2$) 2 gapped quarks become gapless. The gapped quarks begin to unpair destroying the BCS solution. But a new stable phase exists, the gapless 2SC (g2SC) phase.

- It is the unstable phase (Sarma phase) which becomes stable in this case (and in gCFL, see later) when charge neutrality is required.

$$\delta\mu = \frac{\mu_e}{2}$$

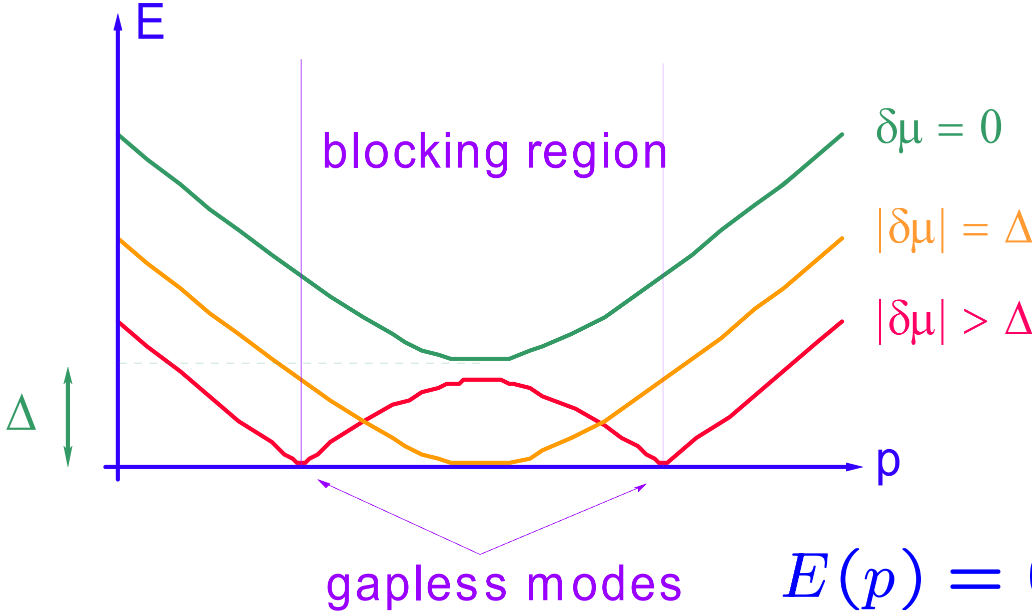






The point $|\delta\mu| = \Delta$ is special. In the presence of a mismatch new features are present. The spectrum of quasiparticles is

$$E(p) = |\pm\delta\mu + \sqrt{(p-\mu)^2 + \Delta^2}|$$



For $|\delta\mu| < \Delta$, the gaps are $\Delta - \delta\mu$ and $\Delta + \delta\mu$

For $|\delta\mu| = \Delta$, an unpairing (blocking) region opens up and gapless modes are present

$$E(p) = 0 \quad \Leftrightarrow \quad p = \mu \pm \sqrt{\delta\mu^2 - \Delta^2}$$

$2\delta\mu$ Energy cost for pairing

2Δ Energy gained in pairing

begins to unpair

$$\boxed{2\delta\mu > 2\Delta}$$

The case of 3 flavors

gCFL

(Alford, Kouvaris & Rajagopal, 2005)

$$\langle 0|\psi^{\alpha}_{aL}\psi^{\beta}_{bL}|0\rangle = \Delta_1\epsilon^{\alpha\beta 1}\epsilon_{ab1}+\Delta_2\epsilon^{\alpha\beta 2}\epsilon_{ab2}+\Delta_3\epsilon^{\alpha\beta 3}\epsilon_{ab3}$$

Different phases are characterized by different values for the gaps. For instance (but many other possibilities exist)

$$\text{CFL}: \quad \Delta_1 = \Delta_2 = \Delta_3 = \Delta$$

$$\text{g2SC}: \quad \Delta_3 \neq 0, \Delta_1 = \Delta_2 = 0$$

$$\text{gCFL}: \quad \Delta_3 > \Delta_2 > \Delta_1$$

$\tilde{Q}$	0	0	0	-1	+1	-1	+1	0	0
	ru	gd	bs	rd	gu	rs	bu	gs	bd
ru		Δ_3	Δ_2						
gd	Δ_3		Δ_1						
bs	Δ_2	Δ_1							
rd					$-\Delta_3$				
gu				$-\Delta_3$					
rs							$-\Delta_2$		
bu						$-\Delta_2$			
gs									$-\Delta_1$
bd								$-\Delta_1$	

Gaps in gCFL

Δ_1 : ds – pairing

Δ_2 : us – pairing

Δ_3 : ud – pairing

Strange quark mass effects:

- Shift of the chemical potential for the strange quarks:

$$\mu_{\alpha s} \Rightarrow \mu_{\alpha s} - \frac{M_s^2}{2\mu}$$

- Color and electric neutrality in CFL requires

$$\mu_8 = -\frac{M_s^2}{2\mu}, \quad \mu_3 = \mu_e = 0$$

- The transition CFL to gCFL starts with the unpairing of the pair **ds** with (close to the transition)

$$\delta\mu_{ds} = \frac{M_s^2}{2\mu}$$

It follows:

$\frac{M_s^2}{\mu}$ **Energy cost for pairing**

2Δ **Energy gained in pairing**

$\Big\}$ $\xrightarrow{\text{begins to unpair}}$ $\boxed{\frac{M_s^2}{\mu} > 2\Delta}$

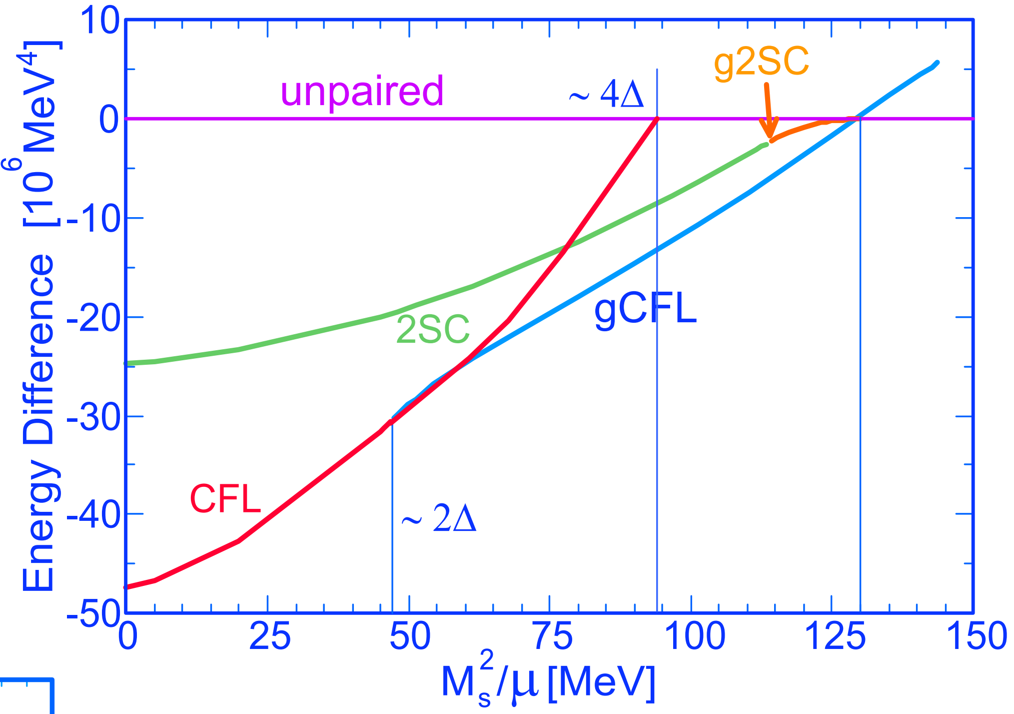
Calculations within a NJL model (modelled on one-gluon exchange):

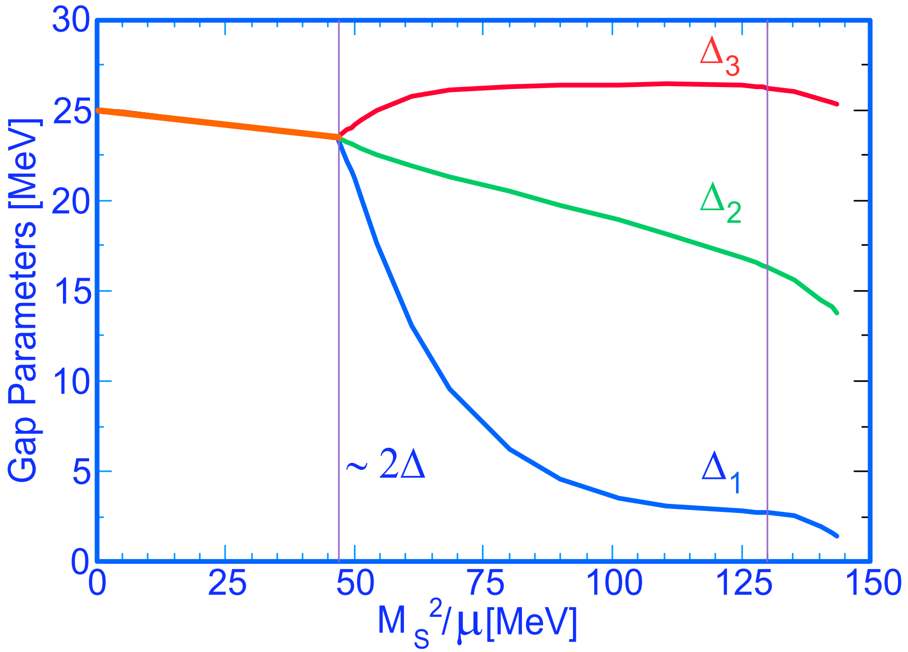
- **Write the free energy:** $V(\mu, \mu_3, \mu_8, \mu_e, \Delta_i)$
- **Solve:**

Neutrality $$\frac{\partial V}{\partial \mu_e} = \frac{\partial V}{\partial \mu_3} = \frac{\partial V}{\partial \mu_8} = 0$$

Gap equations $$\frac{\partial V}{\partial \Delta_i} = 0$$ $\Longrightarrow$

● **CFL $\mapsto$ gCFL 2nd order transition at $M_s^2/\mu \sim 2\Delta$, when the pairing ds starts breaking**

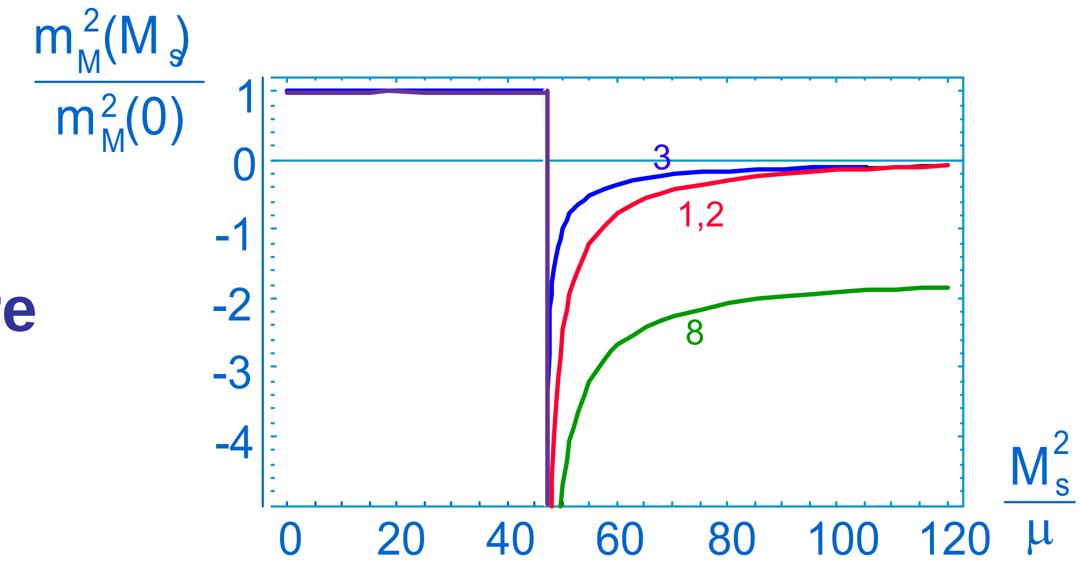




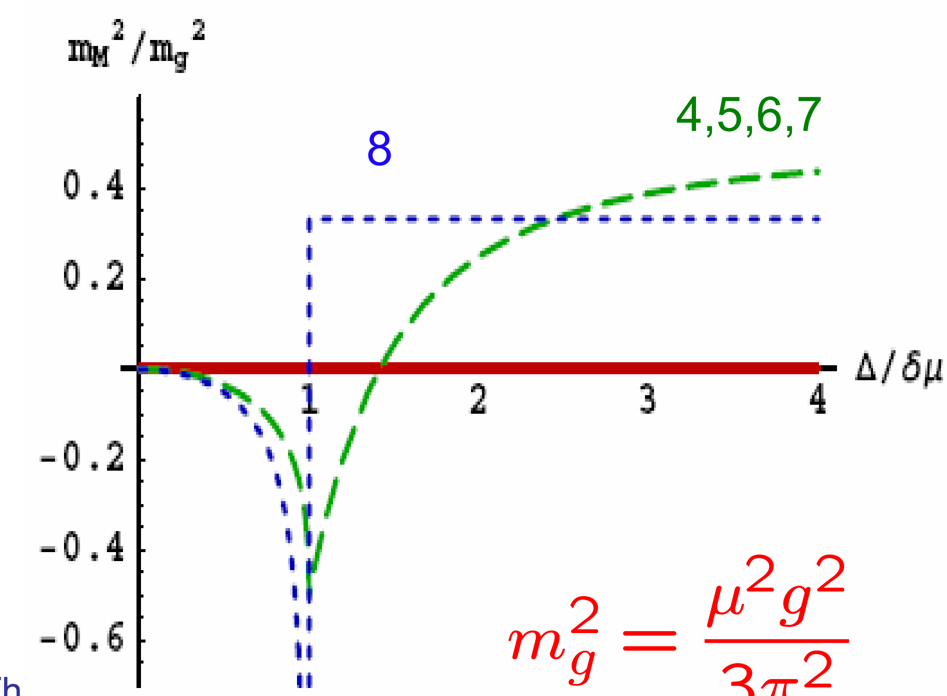
(Alford, Kouvaris & Rajagopal, 2005)

(Δ_0 = 25 MeV, μ = 500 MeV)

- **gCFL has gapless quasiparticles, and there are gluon imaginary masses** **(RC et al. 2004, Fukushima 2005).**



- **Instability present also in g2SC** **(Huang & Shovkovy 2004; Alford & Wang 2005)**



$$m_g^2 = \frac{\mu^2 g^2}{3\pi^2}$$

Proposals for solving the chromomagnetic instability

- **Gluon condensation. Assuming artificially $\langle A_{\mu\,3}\rangle$ or $\langle A_{\mu\,8}\rangle$ not zero (of order 10 MeV) this can be done (RC et al. 2004) . In g2SC the chromomagnetic instability can be cured by a chromo-magnetic condensate (Gorbar, Hashimoto, Miransky, 2005 & 2006; Kiriyama, Rischke, Shovkovy, 2006). Rotational symmetry is broken and this makes a connection with the inhomogeneous LOFF phase (see later). At the moment no extension to the three flavor case.**

• **CFL-K^0 phase.** When the stress is not too large (high density) the CFL pattern might be modified by a flavor rotation of the condensate equivalent to a condensate of K^0 mesons (Bedaque, Schafer 2002). This occurs for $m_s > m^{1/3}\,\Delta^{2/3}$. Also in this phase gapless modes are present and the gluonic instability arises (Kryjevski, Schafer 2005, Kryjevski, Yamada 2005). With a space dependent condensate a current can be generated which resolves the instability. Again some relations with the LOFF phase.

- **Single flavor pairing.** If the stress is too big single flavor pairing could occur but the gap is generally too small. It could be important at low μ before the nuclear phase (see for instance Alford 2006)

- **Secondary pairing.** The gapless modes could pair forming a secondary gap, but the gap is far too small (Huang, Shovkovy, 2003; Hong 2005; Alford, Wang, 2005)

- **Mixed phases** of nuclear and quark matter (Alford, Rajagopal, Reddy, Wilczek, 2001) as well as mixed phases between different CS phases, have been found either unstable or energetically disfavored (Neumann, Buballa, Oertel, 2002; Alford, Kouvaris, Rajagopal, 2004).

● **Chromomagnetic instability of g2SC makes the crystalline phase (LOFF) with two flavors energetically favored (Giannakis & Ren 2004), also there are no chromomagnetic instability although it has gapless modes (Giannakis & Ren 2005).**

This makes the LOFF phase very interesting

LOFF phase

- **LOFF (Larkin, Ovchinnikov; Fulde & Ferrel, 1964): ferromagnetic alloy with paramagnetic impurities.**

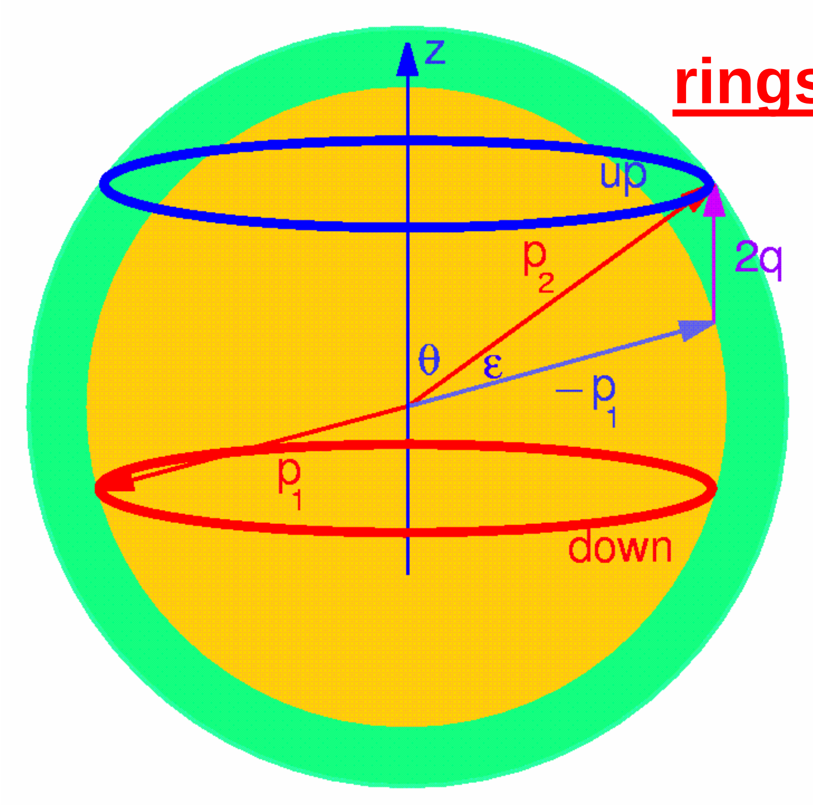
- **The impurities produce a constant exchange field acting upon the electron spin giving rise to an effective difference in the chemical potentials of the electrons producing a mismatch of the Fermi momenta**

- **Studied also in the QCD context (Alford, Bowers & Rajagopal, 2000, for a review R.C. & Nardulli, 2003)**

According to LOFF, close to first order point (CC point), possible condensation with non zero total momentum

$$\vec{p}_1 = \vec{k} + \vec{q},\ \ \vec{p}_2 = -\vec{k} + \vec{q} \ \longrightarrow\ \langle\psi(x)\psi(x)\rangle = \Delta e^{2i\vec{q}\cdot\vec{x}}$$

More generally $\longrightarrow$ $\langle\psi(x)\psi(x)\rangle = \sum_m \Delta_m e^{2i\vec{q}_m\cdot\vec{x}}$



rings

$$\boxed{\vec{p}_1 + \vec{p}_2 = 2\vec{q}}$$

$|\vec{q}\,|$ **fixed variationally**

$\vec{q}\,/|\vec{q}\,|$ **chosen spontaneously**

Single plane wave:

$$E(\vec{k})-\mu \Rightarrow E(\pm\vec{k}+\vec{q})-\mu\mp\delta\mu \approx \sqrt{(|\vec{k}|-\mu)^2+\Delta^2}\mp\bar{\mu}$$

$$\bar{\mu} = \delta\mu - \vec{v}_F \cdot \vec{q}$$

Also in this case, for $\bar{\mu} = \delta\mu - \vec{v}_F \cdot \vec{q} > \Delta$

an unpairing (blocking) region opens up and gapless modes are present

More general possibilities include a crystalline structure (Larkin & Ovchinnikov 1964, Bowers & Rajagopal 2002)

$$\langle\psi(x)\psi(x)\rangle = \Delta \sum_{\vec{q}_i} e^{2i\vec{q}_i\cdot\vec{x}}$$

The q_i's define the crystal pointing at its vertices.

The LOFF phase has been studied via a Ginzburg-Landau expansion of the grand potential

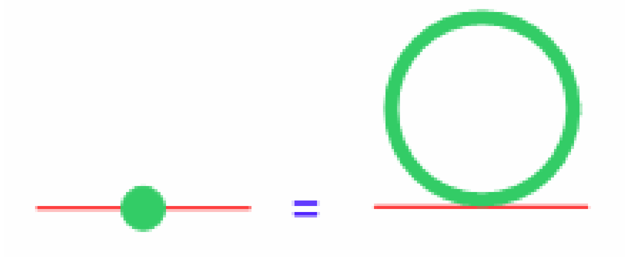
$$\Omega = \alpha\Delta^2 + \frac{\beta}{2}\Delta^4 + \frac{\gamma}{3}\Delta^6 + \cdots$$

(for regular crystalline structures all the Δ_q are equal)

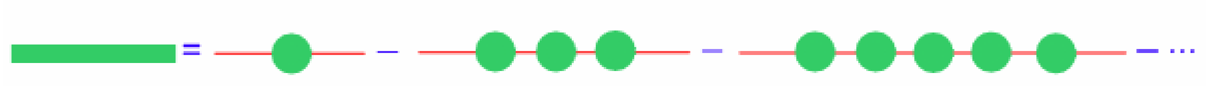
The coefficients can be determined microscopically for the different structures (Bowers and Rajagopal (2002))

General strategy

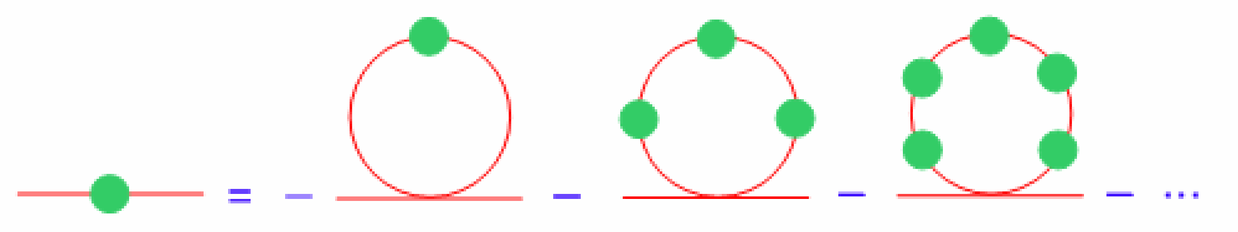
- **Gap equation**



- **Propagator expansion**



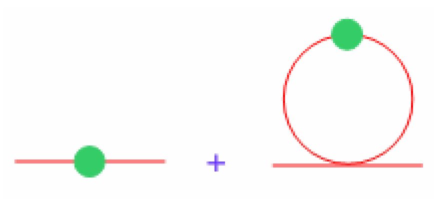
- **Insert in the gap equation**

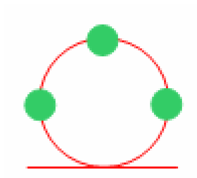


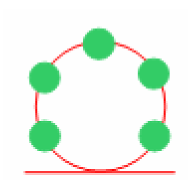
We get the equation

$$\alpha\Delta + \beta\Delta^3 + \gamma\Delta^5 + \cdots = 0$$

Which is the same as $\dfrac{\partial\Omega}{\partial\Delta} = 0$ **with**

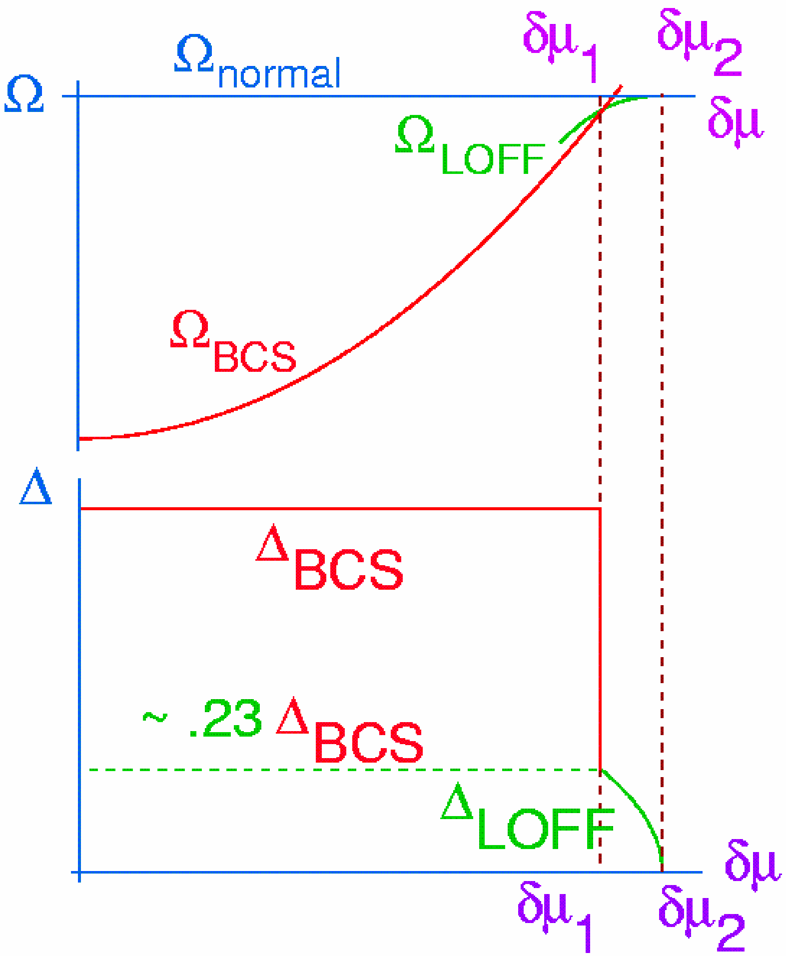
$\alpha\Delta =$ 

$\beta\Delta^3 =$ 

$\gamma\Delta^5 =$ 

The first coefficient has universal structure, independent on the crystal. From its analysis one draws the following results

LOFF and BCS



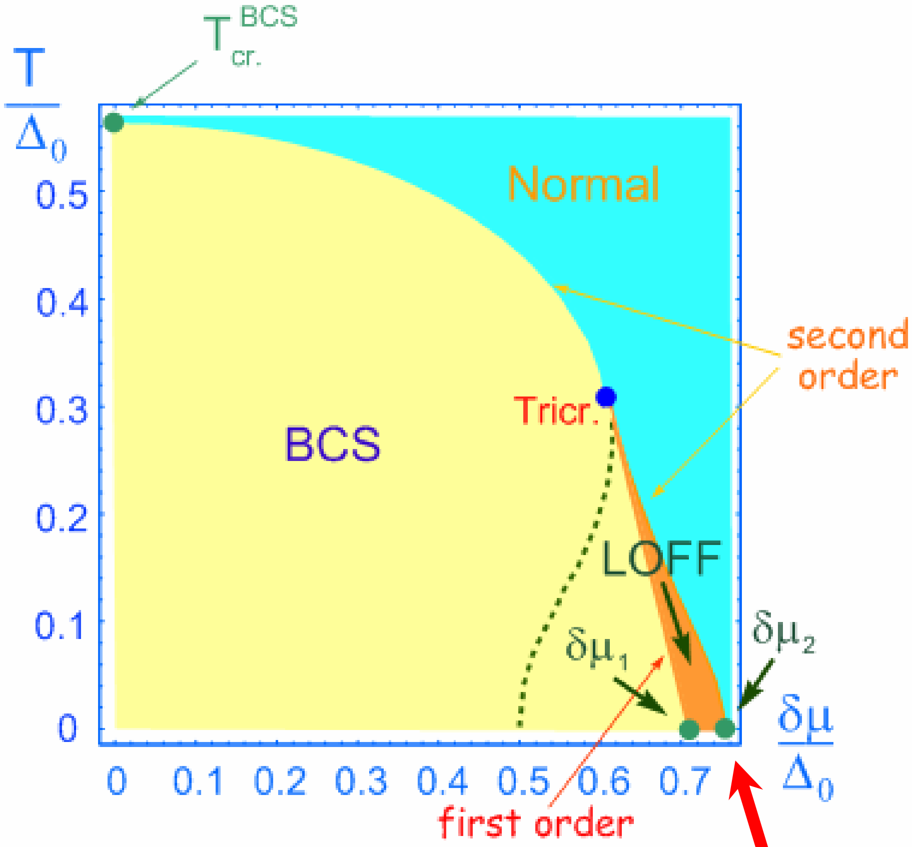
$$\Omega_{BCS} - \Omega_{normal} = -\frac{\rho}{4}(\Delta^2{}_{BCS} - 2\delta\mu^2)$$

$$\Omega_{LOFF} - \Omega_{normal} = -0.44\rho(\delta\mu - \delta\mu_2)^2$$

$$\Delta_{LOFF} \approx 1.15\sqrt{(\delta\mu_2 - \delta\mu)}$$

$$\boxed{\delta\mu_1 = \Delta_{BCS}/\sqrt{2}} \qquad \boxed{\delta\mu_2 \approx 0.754\Delta_{BCS}}$$

Small window. Opens up in QCD
(Leibovich, Rajagopal & Shuster 2001)



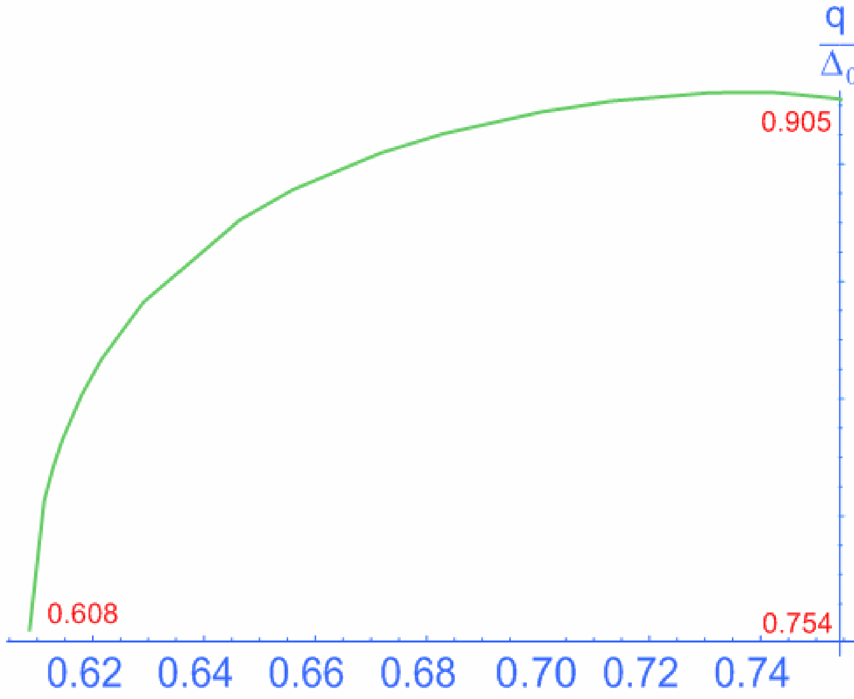
Single plane wave

Critical line from

$$\frac{\partial\Omega}{\partial\Delta} = 0, \qquad \frac{\partial\Omega}{\partial q} = 0$$

Along the critical line

$$(\text{at } T = 0,\ q = 1.2\delta\mu_2)$$



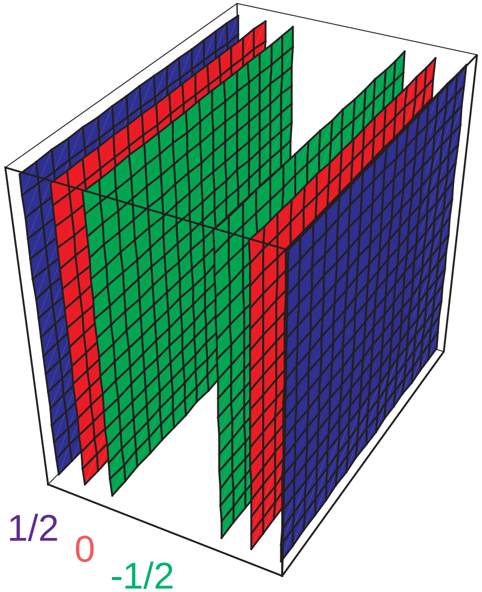
Structure	P	$\mathcal{G}$(Föppl)	$\bar{\beta}$	$\bar{\gamma}$	$\bar{\Omega}_{min}$	$\delta\mu_*/\Delta_0$
point	1	$C_{\infty v}(1)$	0.569	1.637	0	0.754
antipodal pair	2	$D_{\infty v}(11)$	0.138	1.952	0	0.754
triangle	3	$D_{3h}(3)$	-1.976	1.687	-0.452	0.872
tetrahedron	4	$T_d(13)$	-5.727	4.350	-1.655	1.074
square	4	$D_{4h}(4)$	-10.350	-1.538	–	–
pentagon	5	$D_{5h}(5)$	-13.004	8.386	-5.211	1.607
trigonal bipyramid	5	$D_{3h}(131)$	-11.613	13.913	-1.348	1.085
square pyramid	5	$C_{4v}(14)$	-22.014	-70.442	–	–
octahedron	6	$O_h(141)$	-31.466	19.711	-13.365	3.625
trigonal prism	6	$D_{3h}(33)$	-35.018	-35.202	–	–
hexagon	6	$D_{6h}(6)$	23.669	6009.225	0	0.754
pentagonal bipyramid	7	$D_{5h}(151)$	-29.158	54.822	-1.375	1.143
capped trigonal antiprism	7	$C_{3v}(13\bar{3})$	-65.112	-195.592	–	–
cube	8	$O_h(44)$	-110.757	-459.242	–	–
square antiprism	8	$D_{4d}(4\bar{4})$	-57.363	-6.866	–	–
hexagonal bipyramid	8	$D_{6h}(161)$	-8.074	5595.528	-2.8×10^{-6}	0.755
augmented trigonal prism	9	$D_{3h}(3\bar{3}\bar{3})$	-69.857	129.259	-3.401	1.656
capped square prism	9	$C_{4v}(144)$	-95.529	7771.152	-0.0024	0.773
capped square antiprism	9	$C_{4v}(14\bar{4})$	-68.025	106.362	-4.637	1.867
bicapped square antiprism	10	$D_{4d}(14\bar{4}1)$	-14.298	7318.885	-9.1×10^{-6}	0.755
icosahedron	12	$I_h(15\bar{5}1)$	204.873	145076.754	0	0.754
cuboctahedron	12	$O_h(4\bar{4}\bar{4})$	-5.296	97086.514	-2.6×10^{-9}	0.754
dodecahedron	20	$I_h(5555)$	-527.357	114166.566	-0.0019	0.772

Bowers and Rajagopal (2002)

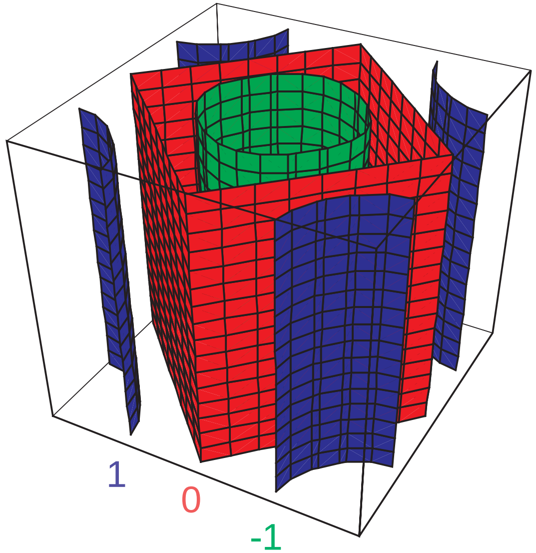
Preferred structure: face-centered cube

Crystalline structures in LOFF

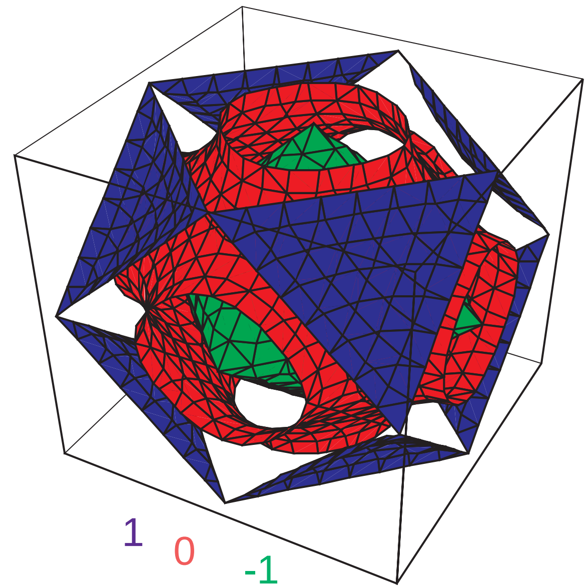
P = 2



P = 4

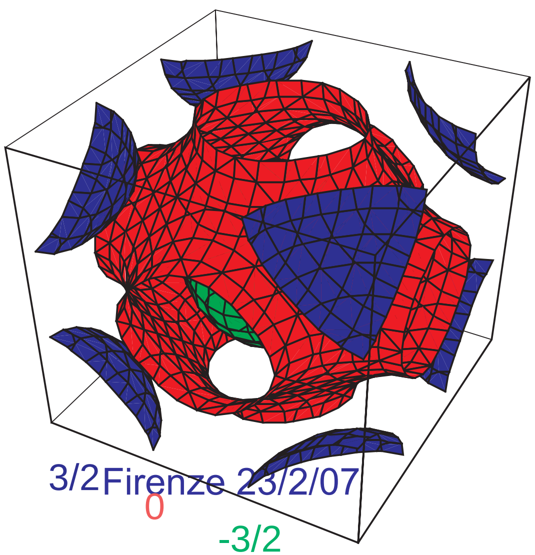


P = 6

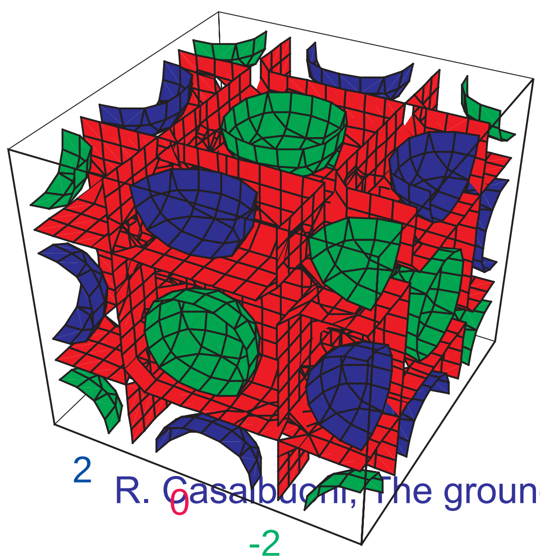


x 2 Δ

P = 6



P = 8



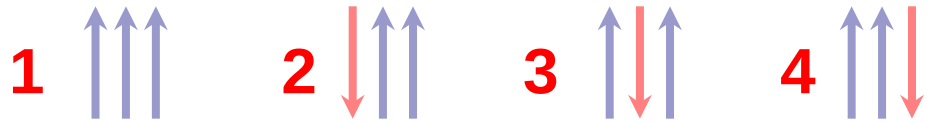
Preliminary results about LOFF with three flavors

Recent study of LOFF with 3 flavors within the following simplifying hypothesis (RC, Gatto, Ippolito, Nardulli & Ruggieri, 2005)

- **Study within the Landau-Ginzburg approximation.**
- **Only electrical neutrality imposed (chemical potentials μ_3 and μ_8 taken equal to zero).**
- **M_s treated as in gCFL. Pairing similar to gCFL with inhomogeneity in terms of simple plane waves, as for the simplest LOFF phase.**

$$\boxed{\langle \psi_{aL}^{\alpha} \psi_{bL}^{\beta} \rangle = \sum_{I=1}^{3} \Delta_I(\vec{x}) \epsilon^{\alpha\beta I} \epsilon_{abI}, \quad \Delta_I(\vec{x}) = \Delta_I e^{2i\vec{q}_I \cdot \vec{x}}}$$

- A further simplifications is to assume only the following geometrical configurations for the vectors q_I, i = 1,2,3



- The free energy, in the GL expansion, has the form

$$\boxed{\Omega-\Omega_{normal}=\sum_{I=1}^{3}\left(\frac{\alpha_I}{2}\Delta_I^2+\frac{\beta_I}{4}\Delta_I^4+\sum_{I\neq J}\frac{\beta_{IJ}}{4}\Delta_I^2\Delta_J^2\right)+O(\Delta^6)}$$

$$\Omega_{normal}=-\frac{3}{12}\pi^2(\mu_u^4+\mu_d^4+\mu_s^4)-\frac{1}{12}\pi^2\mu_e^4$$

- with coefficients α_I, β_I and β_{IJ} calculable from an effective NJL four-fermi interaction simulating one-gluon exchange

$$\Delta_0 \equiv \Delta_{BCS}, \quad \mu_u = \mu - \frac{2}{3}\mu_e, \quad \mu_d = \mu + \frac{1}{3}\mu_e, \quad \mu_s = \mu + \frac{1}{3}\mu_e - \frac{M_s^2}{2\mu}$$

$$\alpha_I(q_I, \delta\mu_I) = -\frac{4\mu^2}{\pi^2}\left(1 - \frac{\delta\mu_I}{2q_I}\log\left|\frac{q_I + \delta\mu_I}{q_I - \delta\mu_I}\right| - \frac{1}{2}\log\left|\frac{4(q_I^2 - \delta\mu_I^2)}{\Delta_0^2}\right|\right)$$

$$\beta_I(q_I, \delta\mu_I) = \frac{\mu^2}{\pi^2}\frac{1}{q_I^2 - \delta\mu_I^2}$$

$$\beta_{12} = -\frac{3\mu^2}{\pi^2}\int\frac{d\mathbf{n}}{4\pi}\frac{1}{(2\mathbf{q_1}\cdot\mathbf{n} + \mu_s - \mu_d)(2\mathbf{q_2}\cdot\mathbf{n} + \mu_s - \mu_u)}$$

Others by the exchange:

$$12 \to 23, \mu_s \leftrightarrow \mu_d$$

$$12 \to 13, \mu_s \leftrightarrow \mu_u$$

We require:

$$\boxed{\frac{\partial \Omega}{\partial \Delta_I} = \frac{\partial \Omega}{\partial q_I} = \frac{\partial \Omega}{\partial \mu_e} = 0}$$

At the lowest order in Δ_I

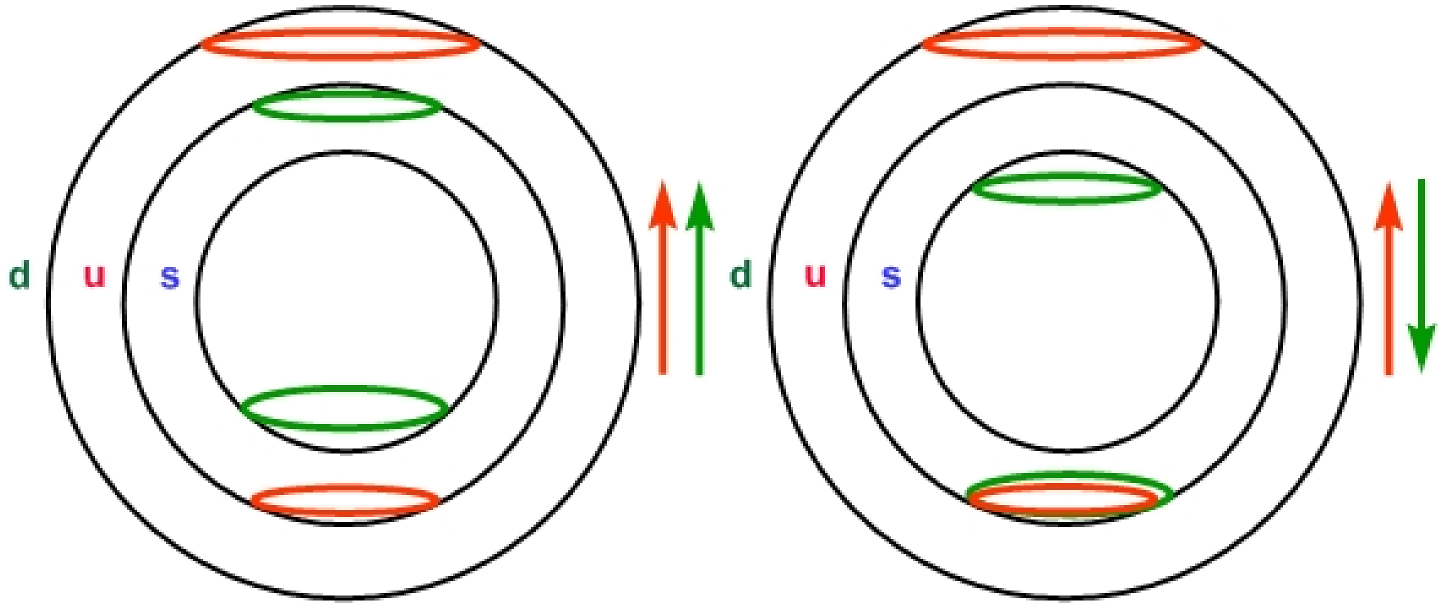
$$\frac{\partial \Omega}{\partial q_I} = 0 \Rightarrow \frac{\partial \alpha_I}{\partial q_I} = 0$$

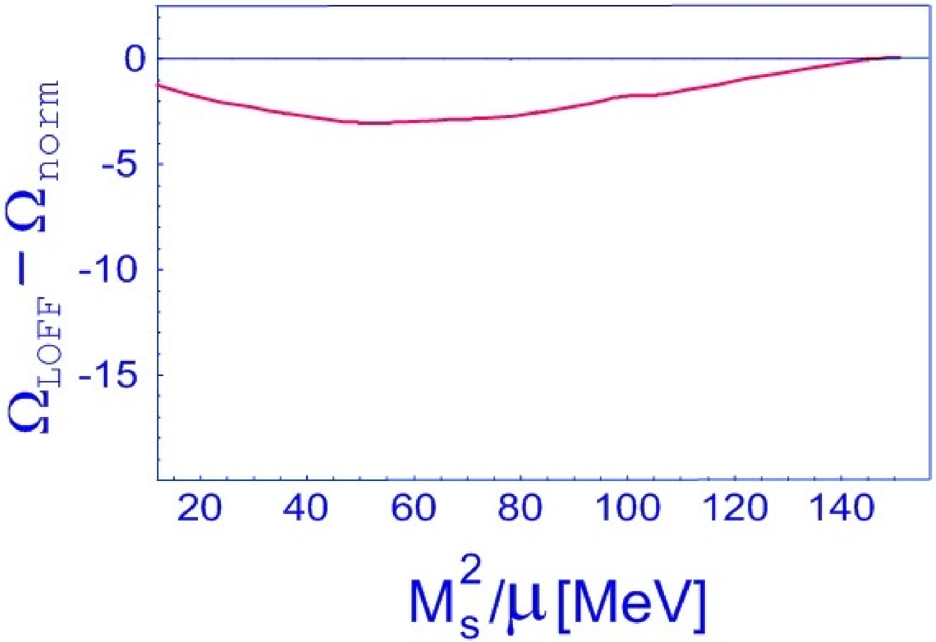
since α_I depends only on q_I and $\delta\mu_i$ we get the same result as in the simplest LOFF case:

$$|\vec{q}_I| = 1.2\delta\mu_I$$

In the GL approximation we expect to be pretty close to the normal phase, therefore we will assume $\mu_3 = \mu_8 = 0$. At the same order we expect $\Delta_2 = \Delta_3$ (equal mismatch) and $\Delta_1 = 0$ (ds mismatch is twice the ud and us).

Once assumed $\Delta_1 = 0$, only two configurations for q_2 and q_3, parallel or antiparallel. The antiparallel is disfavored due to the lack of configurations space for the up fermions.





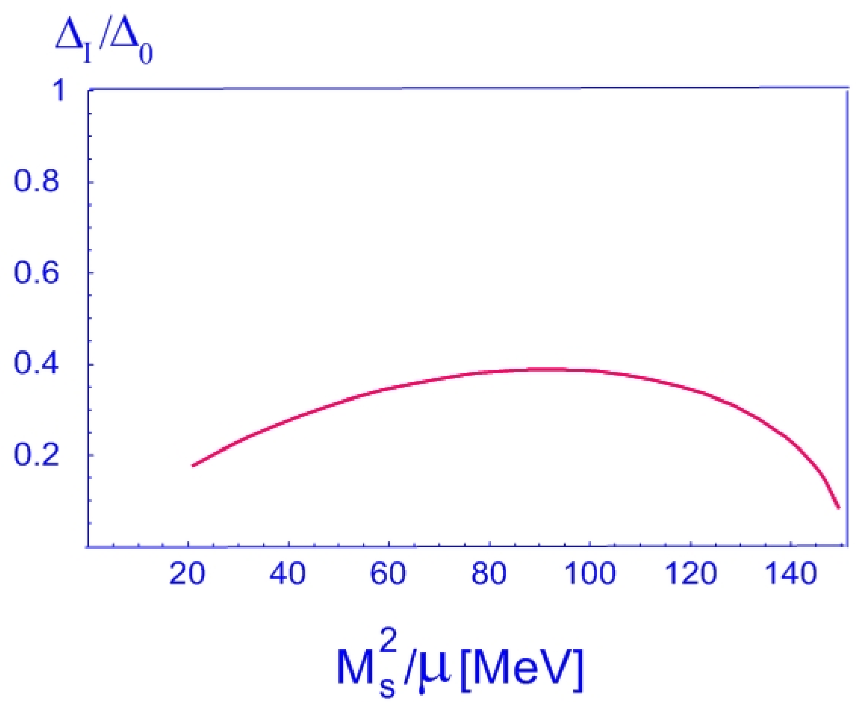
(we have assumed the same parameters as in Alford et al. in gCFL, Δ_0 = 25 MeV, μ = 500 MeV)

Δ_1 : ds – pairing

Δ_2 : us – pairing

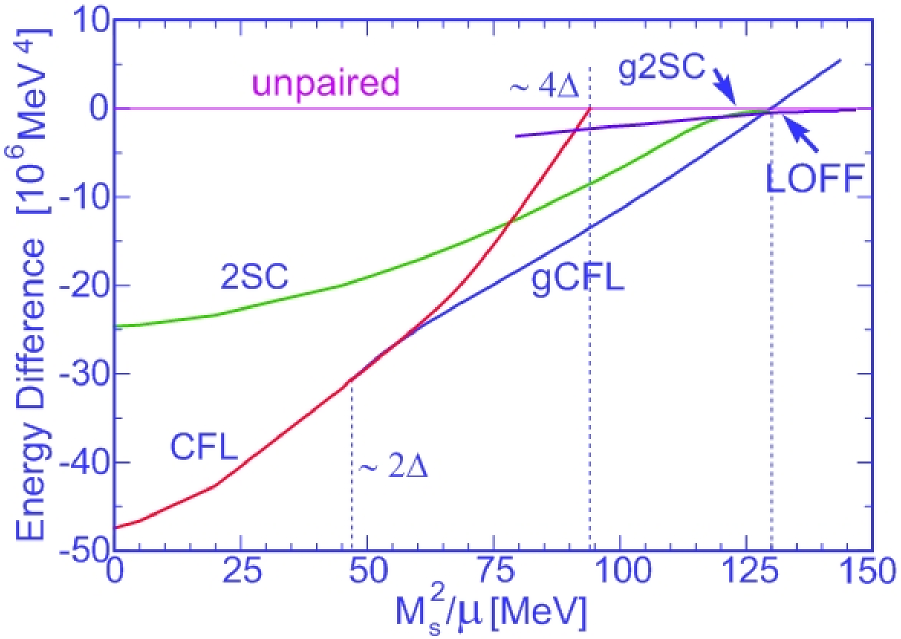
Δ_3 : ud – pairing

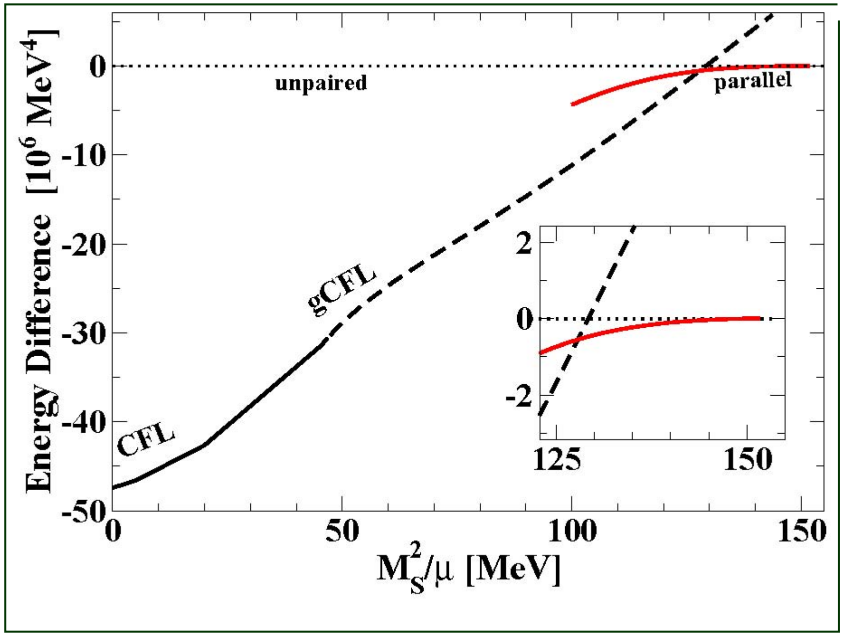
$$\boxed{\Delta_1 = 0, \quad \Delta_2 = \Delta_3}$$



Comparison with other phases

- LOFF phase takes over gCFL at about 128 MeV and goes over to the normal phase at about 150 MeV

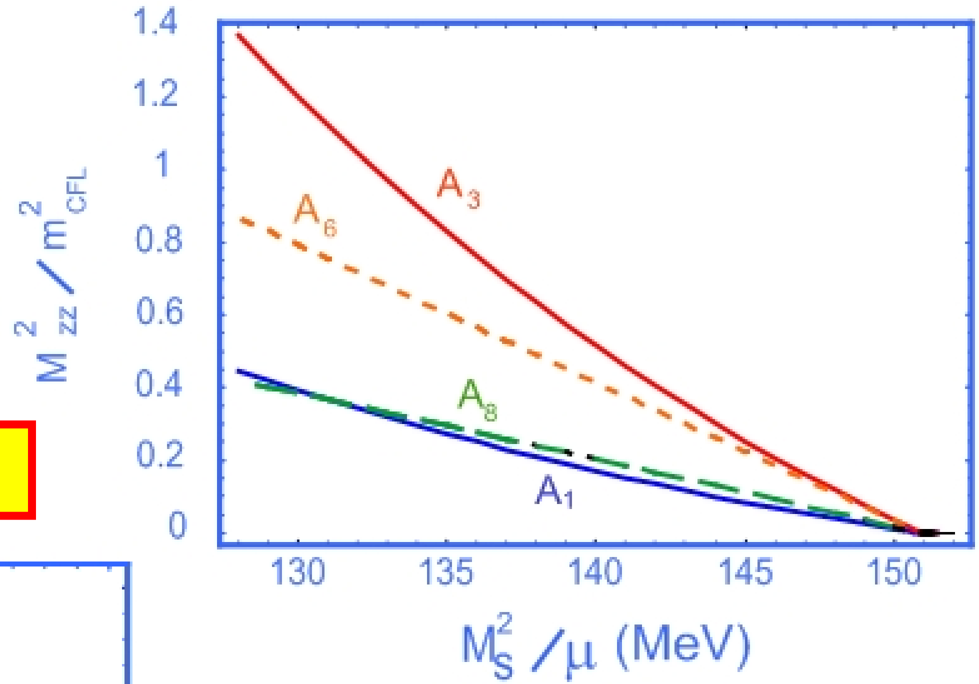




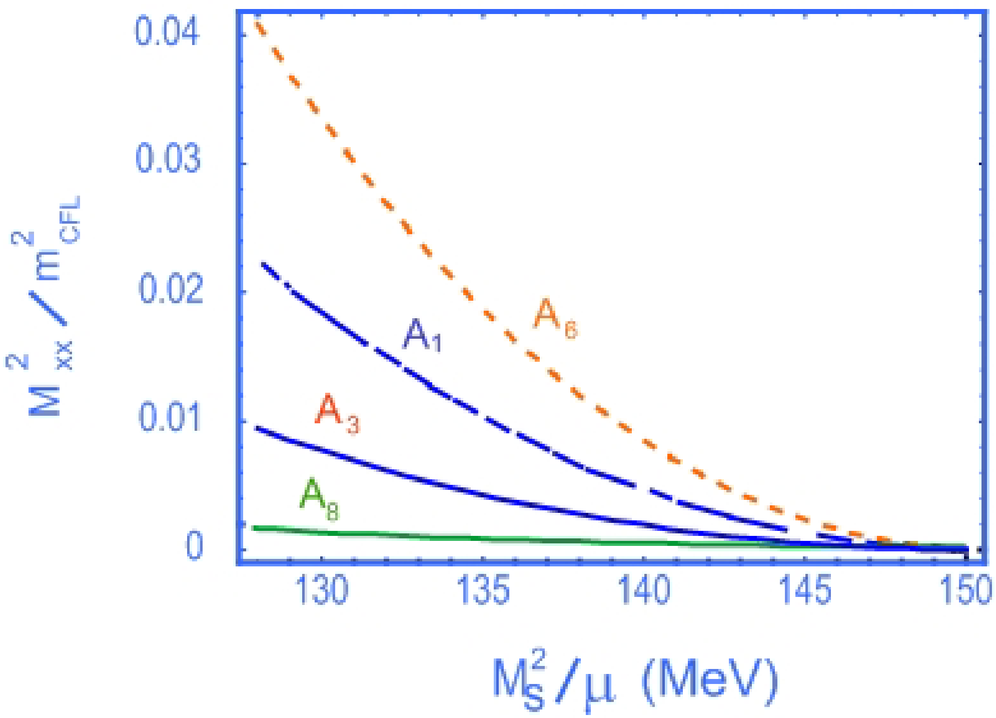
(RC, Gatto, Ippolito, Nardulli, Ruggieri, 2005)

Confirmed by an exact solution of the gap equation (Mannarelli, Rajagopal, Sharma, 2006)

No chromo-magnetic instability in the LOFF phase with three flavors (Ciminale, Gatto, Nardulli, Ruggieri, 2006)



Transverse masses



Longitudinal masses

$$M_1 = M_2 = M_4 = M_5$$

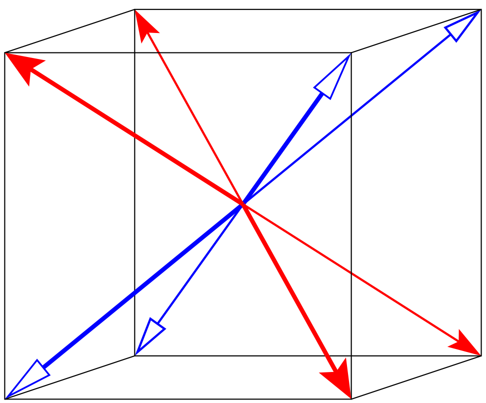
$$M_6 = M_7$$

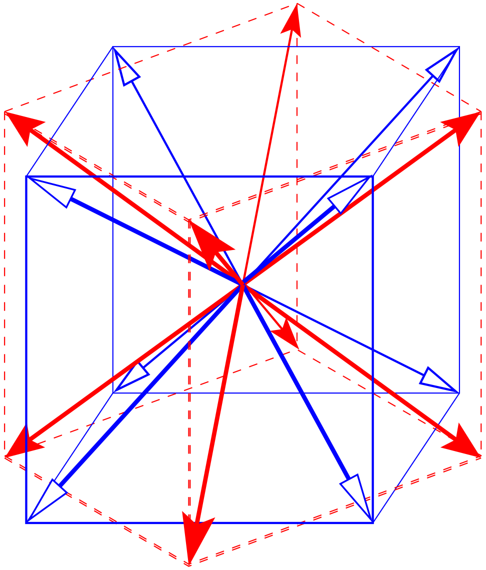
Extension to a crystalline structure (Rajagopal, Sharma 2006), always within the simplifying assumption $\Delta_1 = 0$ and $\Delta_2 = \Delta_3$

$$\langle ud\rangle \approx \Delta_3 \sum_a \exp(2i\vec{q}_3^a\cdot\vec{r}), \quad \langle us\rangle \approx \Delta_2 \sum_a \exp(2i\vec{q}_2^a\cdot\vec{r})$$

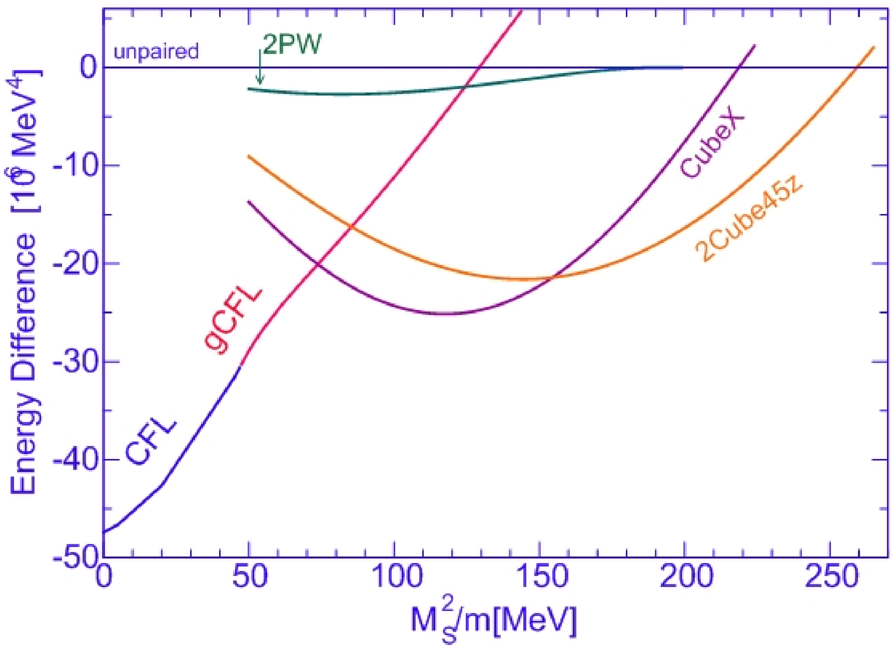
The sum over the index a goes up to 8 q_i^a. Assuming also $\Delta_2 = \Delta_3$ the favored structures (always in the GL approximation up to Δ^6) among 11 structures analyzed are

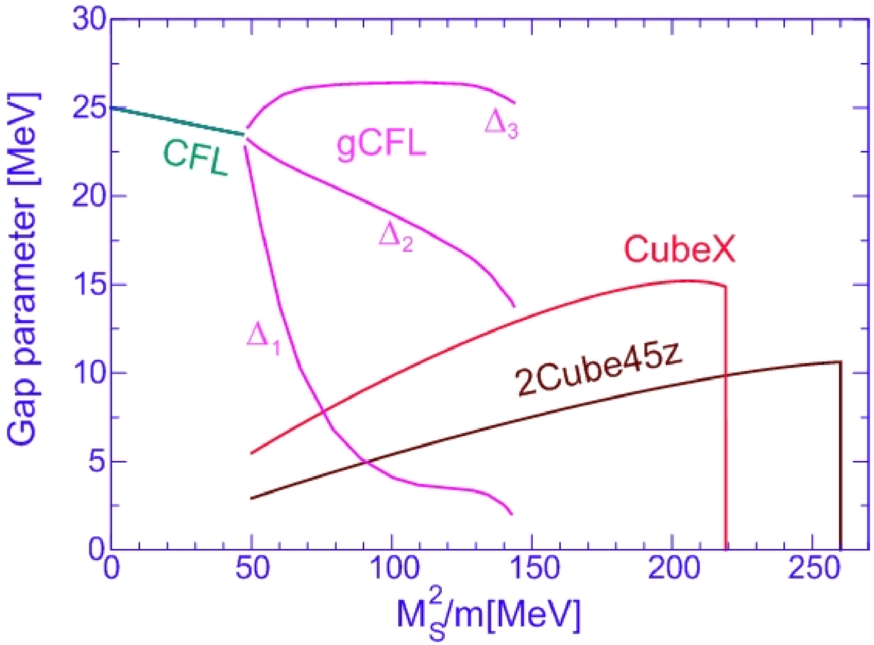
CubeX





2Cube45z


unpaired
2PW
Energy Difference [10^6 MeV^4]
0
-10
-20
-30
-40
-50
0
50
100
150
200
250
M_S^2/m[MeV]
CFL
gCFL
CubeX
2Cube45z


Gap parameter [MeV]
30
25
20
15
10
5
0
0
50
100
150
200
250
M_S^2/m[MeV]
CFL
gCFL
Δ3
Δ2
Δ1
CubeX
2Cube45z

Conclusions

- **Various phases are competing, many of them having gapless modes. However, when such modes are present a chromomagnetic instability arises.**
- **Also the LOFF phase is gapless but the gluon instability does not seem to appear.**
- **Recent studies of the LOFF phase with three flavors seem to suggest that this should be the favored phase after CFL, although this study is very much simplified and more careful investigations should be performed.**
- **The problem of the QCD phases at moderate densities and low temperature is still open.**