

Early 'Thermalization' in the CGC and a Couple of Other Crazy Ideas

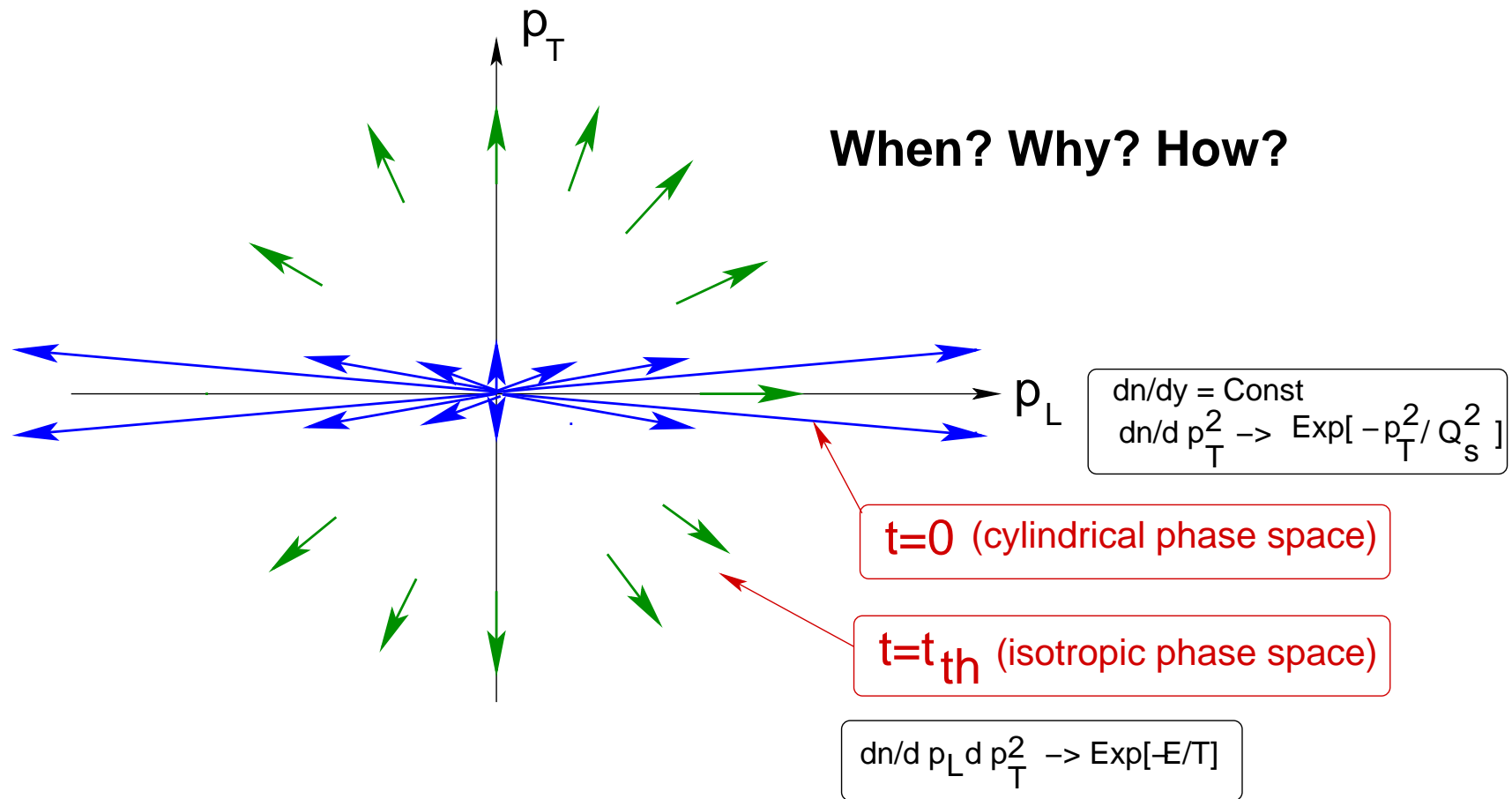
Eugene Levin, Tel Aviv University



WS: "High Density QCD"
GGI, Jan 15, 2007 - Mar 9, 2007

D. Kharzeev and E.L: (*in preparation*)
D. Kharzeev, E.L. and K. Tuchin: [hep-ph/0602063](https://arxiv.org/abs/hep-ph/0602063)
D. Kharzeev and K. Tuchin: [hep-ph/0501234](https://arxiv.org/abs/hep-ph/0501234)

Thermalization:

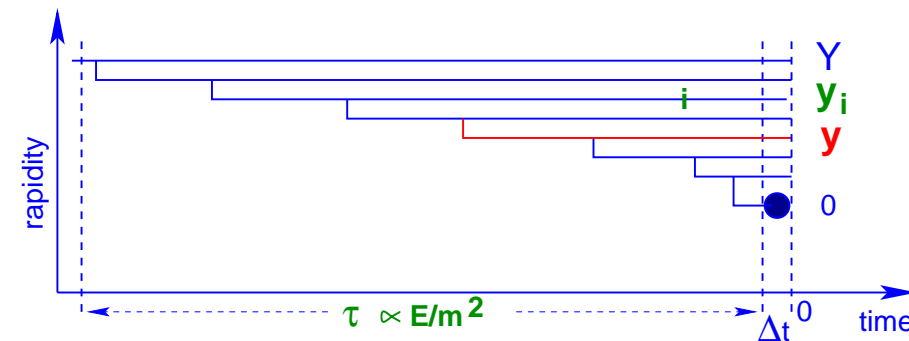


Colour Glass Condensate:



Structure of parton cascade in CGC

Space -time picture:



Low, Nussinov (1975); BFKL(1976–1986);
GLR (1981); Mueller & Qiu (1986); Mueller (1994);
McLerran & Venugopalan (1994); Balitsky (1995)
Kovchegov(1999); Bartels(1992 – 2000); Braun(2000)

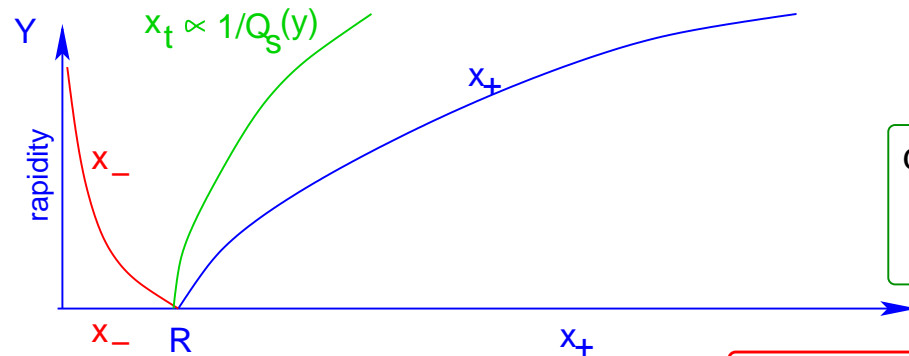
$$x_+ k_- \propto 1$$

$$x_- k_+ \propto 1$$

$$x_t Q_s \propto 1$$

LLA of pQCD:

$$\alpha_s (y_{i+1} - y_i) \propto 1$$



$$d\rho/dy = K \otimes (\rho - \rho^2)$$

$$\rho(y, Q_s) = 1$$

$$Q_s^2 \propto x G(x, Q_s) / \pi R^2 \propto (1/x)^\lambda$$

Classical fields:

Since all partons with rapidity larger than y live longer than parton with rapidity y , for a dense system as a nucleus they can be considered as the source of the classical field that emits a gluon with rapidity y .

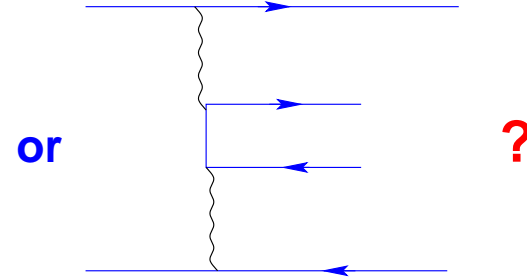
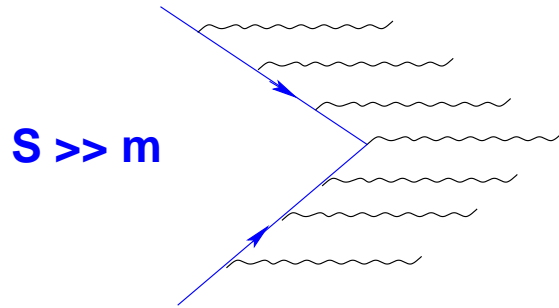
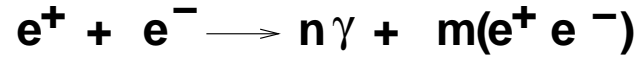
(L. McLerran & R. Venugopalan, 1994)

Duality principle:

Quantum emission in each stage of the process should give the same result as the emission of the classical field

*JIMWLK= J.Jalilian-Marian, E. Iancu, L. McLerran,
H. Weigert, A. Leonidov and A. Kovner 1999*

A question:



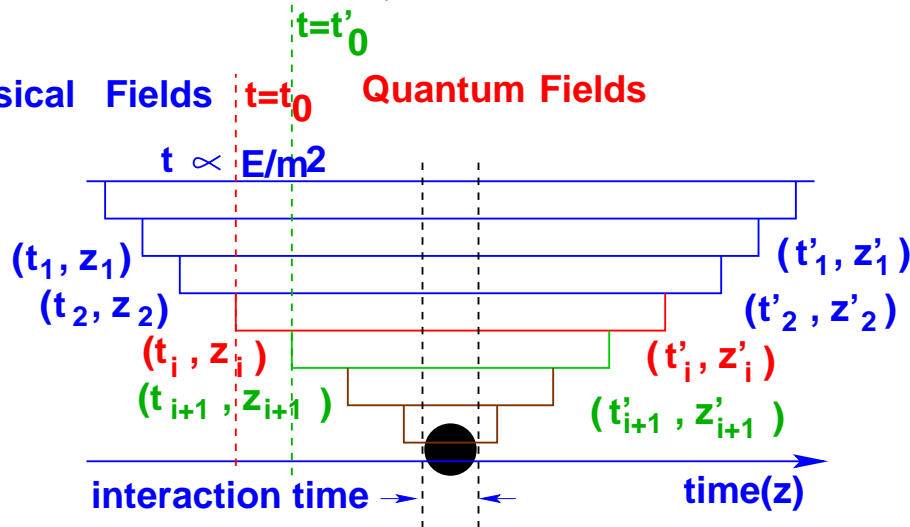
JIMWLK approach:

Classical Fields

Quantum Fields

Classical Fields

Quantum Fields



$$\text{At } t=t_0 : L(\rho) + j_\mu \cdot A_\mu + L(A)$$

$$\text{At } t=t'_0 : L(\rho) + j_\mu \cdot A_\mu + L(A)$$

$$t_1 - t'_1 \gg t_2 - t'_2 \gg \dots \gg t_i - t'_i \gg t_{i+1} - t'_{i+1}$$

Fields of CGC

Lienard-Weichart potential for a charge moving with $v \equiv v_z$

$$A_- = \frac{g\sqrt{2}}{4\pi} \frac{1}{\sqrt{2x_-^2 + (1-v^2)x_\perp^2}}; \quad A_+ = 0; \quad \vec{A}_\perp = 0;$$

Fields:

$$E_z = 0; \quad \vec{E}_\perp = \frac{g}{4\pi} \frac{(1-v^2)\vec{x}_\perp}{(2x_-^2 + (1-v^2)x_\perp^2)^{3/2}}; \quad \vec{H} = \vec{v} \times \vec{E};$$

Lienard-Weichart potential for a parton in the parton cascade

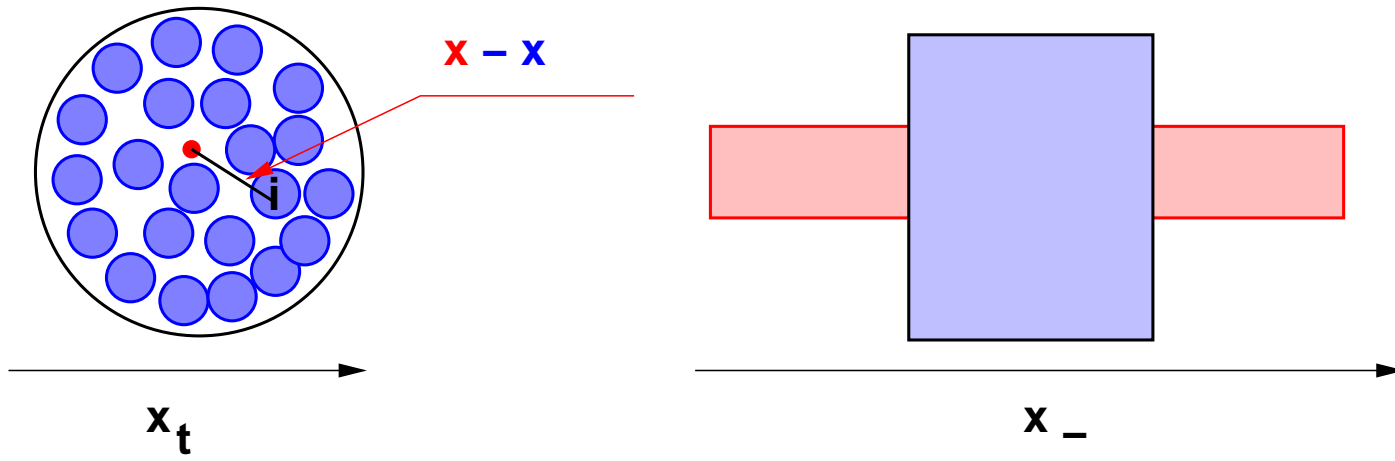
$$A_- = \frac{g\sqrt{2}}{4\pi} \frac{\Theta(x_+ - x_-)}{\sqrt{2x_-^2 + (1-v^2)x_\perp^2}}; \quad A_+ = 0; \quad \vec{A}_\perp = 0;$$

Fields:

$$E_z \propto \delta(x_+ - x_-);$$

$$\vec{E}_\perp = \frac{g}{4\pi} \frac{(1-v^2)\vec{x}_\perp}{(2x_-^2 + (1-v^2)x_\perp^2)^{3/2}} \Theta(x_+ - x_-); \quad \vec{H} = \vec{v} \times \vec{E};$$

$$|\vec{E}_\perp| = |\vec{H}_\perp| \ll E_z$$



A_+ exists only for $z \geq 0$ and the initial conditions depend on x_- and x_+ .

$$A = \int \frac{d^3x}{4\pi} \frac{\rho(\vec{x}, t - |\vec{x} - \vec{x}'|)}{|\vec{x} - \vec{x}'|} = \int \frac{dx_- d^2x}{4\pi} \frac{\sqrt{2}\rho\left(x_-, x_\perp, \mathbf{x}_+ + \mathbf{x}_- - \frac{(x_\perp - x'_\perp)^2}{2(x_- - x'_-)}\right)}{\mathbf{x}_- - \mathbf{x}'_- + \frac{(x_\perp - x'_\perp)^2}{2(x_- - x'_-)}}$$

$$\longrightarrow \int \frac{dx_- d^2x}{4\pi} \frac{\sqrt{2}\rho\left(x_-, x_\perp, \mathbf{x}_+ - \frac{(x_\perp)^2}{2x_-}\right)}{\mathbf{x}_- + \frac{x_\perp^2}{2x_-}}$$

At high energies

$$|\vec{E}_\perp| = |\vec{H}_\perp| \ll E_z$$

Main idea:

To replace complicated process of particle production by the production in the background fields $A_- = A_-(x_-)$

- At $t=0$:

$$E_z = E_0 = \text{Const} \text{ and } A_-(x_-) = -E_0 x_-$$

- At large t :

$$\text{In pQCD: } A_-(x_-) \xrightarrow{x_- \gg R} 1/x_-$$

$$(\text{Say } A_-(x_-) = -E_0 \frac{x_-}{1 + \omega^2 x_-^2});$$

$$\text{In n-pQCD: } A_-(x_-) \xrightarrow{x_- \gg R} e^{-\omega x_-};$$

Model for $A_+(x_-)$

- $A_+(x_-) = \frac{E_0}{\omega} (1 - e^{-\omega x_-}) \implies \sum_{\sigma} \text{Im } S = \frac{\pi p_{\perp}^2}{g E_0 (1 + \gamma)}$

- $\rho \left(x'_-, x'_{\perp}, x_+ + x'_- - \frac{(x'_{\perp})^2}{2x_-} \right) =$
 $c \int d^2 k_{\perp} e^{i \vec{k}_{\perp} \cdot \vec{x}'_{\perp}} \delta \left(x'_- - \omega^{-1} \right) \delta \left(k_{\perp}^2 - Q_s^2 \right),$

- $A_+(x_-) = c' \int \frac{d^2 x'_{\perp} d x'_-}{4 \pi} \frac{2 x_- \delta(x'_- - \omega^{-1}) J_0(x'_{\perp} Q_s)}{2x_-^2 + x_{\perp}'^2} = c' x_- K_0(x_- Q_s)$

Particle production in the background field

Educated guess: k_{\perp} factorization, for inclusive cross section:

(Catani, Ciafaloni & Hautman; E.L., Ryskin, Shabelski & Shuvaev; Collins & Ellis ; 1991)

$$\epsilon \frac{d\sigma}{d^3p} = \frac{4\pi N_c}{N_c^2 - 1} \frac{1}{p_{\perp}^2} \int dk_{\perp}^2 \alpha_S \varphi_P(Y - y, k_{\perp}^2) \varphi_T(y, (p - k)_{\perp}^2)$$

Proven in CGC by Kovchegov & Tuchin (2002) only for DIS !?

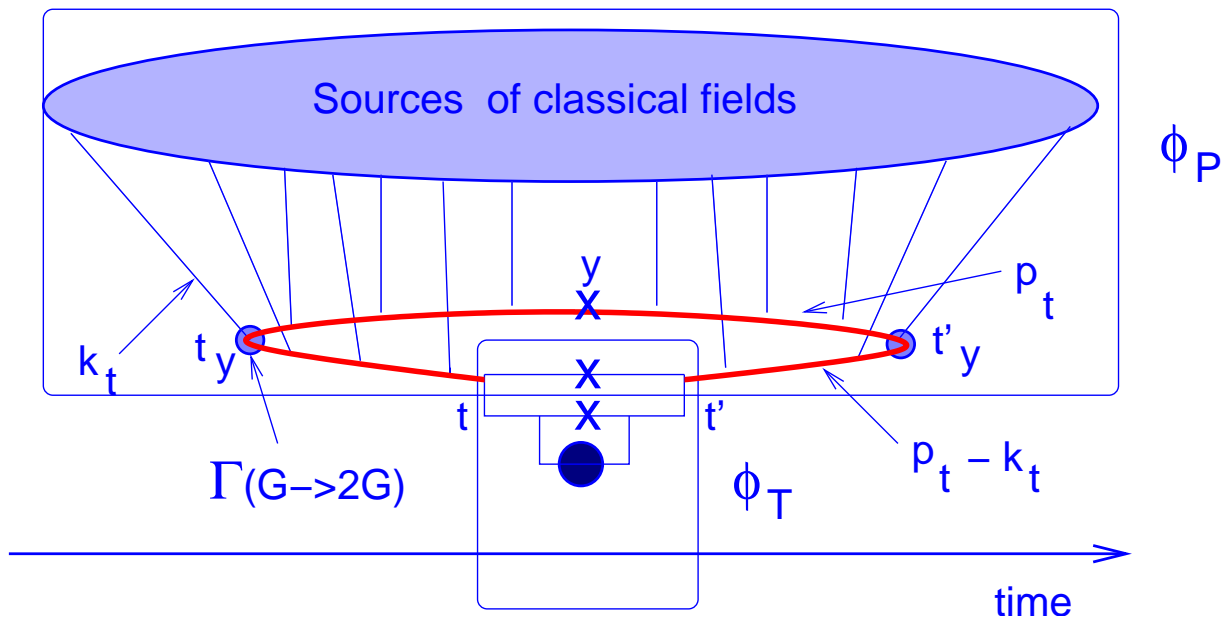
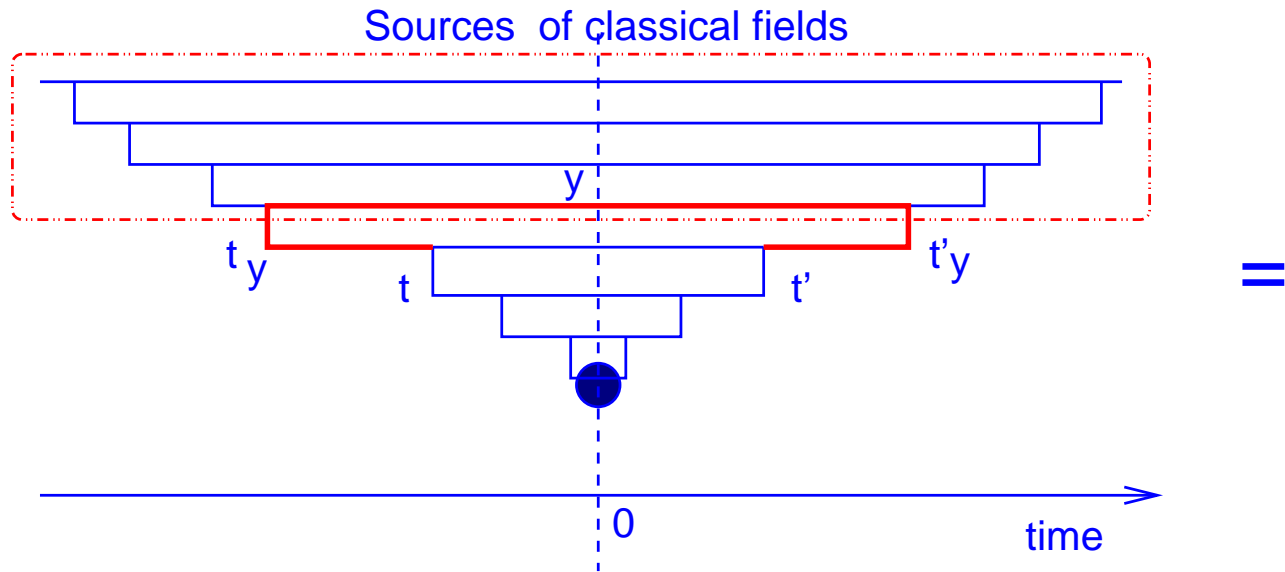
Our formula:

$$\frac{d\sigma}{dyd^2p_\perp} = \varphi_P(Y - y, p_\perp) \varphi_T(y, p_\perp)$$

with

$$\varphi_P(Y - y, p_\perp) \propto$$

$$\int d^2 k_\perp \text{Im} D(Y - y, \vec{p}_\perp - \vec{k}_\perp) \text{Im} D(Y - y, \vec{p}_\perp)$$



Equation of motion in the background field

- $G_\mu = A_\mu + W_\mu$, A_μ is a classical background field;

- $\mathcal{L}[A + W] = \mathcal{L}[A] + \frac{\partial \mathcal{L}[A]}{\partial A_\mu} W_\mu + \frac{1}{2} \frac{\partial^2 \mathcal{L}[A]}{\partial A_\mu \partial A_\nu} W_\mu W_\nu$;

- $(-(\partial_\lambda - igA_\lambda)^2 \delta_{\mu\nu} + 2ig F_{\mu\nu}[A]) W_\mu = 0$;

- $F_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & E_z \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -E_z & 0 & 0 & 0 \end{pmatrix}$,

- Introducing $W_\pm = W_0 \pm iW_3$ we obtain:

$$-((\partial_\lambda - igA_\lambda)^2 \pm 2gE_z) W_\pm = 0$$

Gluon propagator in the background field

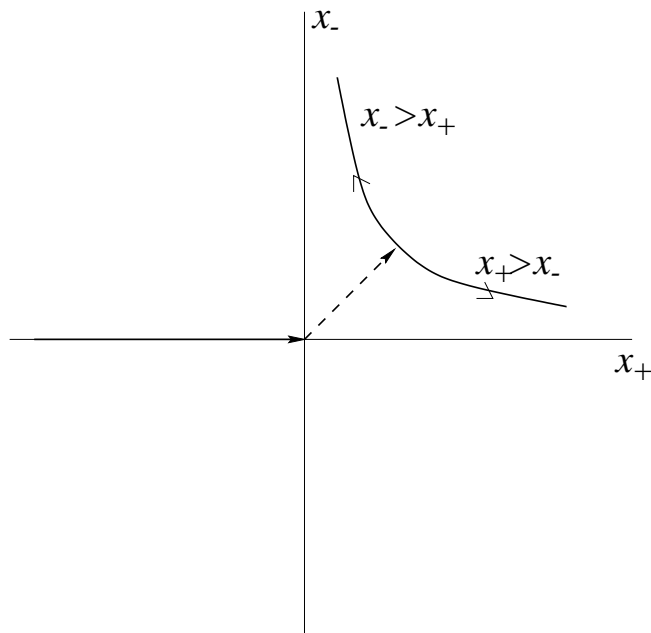
Schwinger (1951); Marinov & Popov (1972-1974); Kluger, Eisenberg, Svetitsky & ... (1991-1994); Dunne (2004)

Semiclassical solution:

- Solution in the form $W_\sigma = e^{-iS_\sigma - ip_\perp \cdot x_\perp}$ with $\sigma = \pm 1$ assuming $\partial_+ S \partial_- S \gg \partial_+ \partial_- S$;
- Introducing $p_-(x_-, x_+) = \partial S / \partial x_+$ and $p_+(x_-, x_+) = \partial S / \partial x_-$, we obtain $-2 p_+(p_- - gA_-(x_-)) + p_\perp^2 + 2g \sigma E_z = 0$;
- $\frac{dx_-}{dt} = -2(p_- - gA_-(x_-))$;
- $\frac{dx_+}{dt} = -2 p_+$;
- $\frac{dS}{dt} = -2p_+ p_- - 2p_+(p_- - gA_-(x_-))$;
- $\frac{dp_+}{dt} = -2p_- gA'_-(x_-) - 2g\sigma E'$;

Solution:

- $$S_K = - \int x_- \frac{d p_{\perp}^2 + 2 g \sigma E_z(x_-)}{p_-^0 - g A_-(x_-)} = - \frac{1}{g E_0} \int d \tau \frac{p_{\perp}^2 + 2 g \sigma E_0 f'(\tau)}{\gamma + f(\tau)}$$



- $$A_-(x_-) = - \frac{E_0}{\omega} f(\tau = \omega x_-);$$

- $$\text{Adiabaticity parameter: } \gamma = \frac{p_+^0 \omega}{g E_0} \approx \frac{p_+}{k_{i,+}};$$

- $$t=0: \gamma \ll 1; \quad \text{Im } S_{\sigma} = - \frac{\pi p_{\perp}^2}{2 g E_0};$$

- $$t=0: \text{Im } D(Y - y, p_{\perp}) \propto e^{-2 \sum_{\sigma} \text{Im } S_{\sigma}}$$

$$= \frac{S_P}{\alpha_S} e^{-\frac{2 \pi p_{\perp}^2}{g E_0}}; \quad Q_s^2(y) = \frac{g E_0}{2 \pi};$$

Thermalization by a pulse of the chromoelectric field

(Kluger, Eisenberg, Svetitsky & ... (1991 -1994)

- ★ $\frac{dk_{iz}}{dt} = g E_z \sim Q_s^2; \quad \frac{d\varepsilon}{dt} = 0;$
- ★ $\Delta k_i^+ \simeq \Delta k_i^- \sim Q_s;$
- ★ $\omega \implies Q_s; \quad \gamma \gg 1;$

For $A_- = \frac{E_0}{\omega} e^{-\tau}$ and $\gamma > 1$

- $Im S_\sigma = \frac{\pi p_\perp^2}{2 g E_0 \gamma} = \frac{4 \pi p_+}{\omega}$
- $\frac{d\sigma}{d^3p} \propto e^{-\frac{\epsilon}{T}}; \quad T = \frac{\omega \sqrt{2}}{4\pi};$

Thermalization time

- $\Delta p_- = p_-(x_-) - p_+^0 = -\omega; \quad \tau = \omega x_- \simeq \ln \gamma;$

- $\Delta p_+ = -\frac{gE_0}{\omega};$

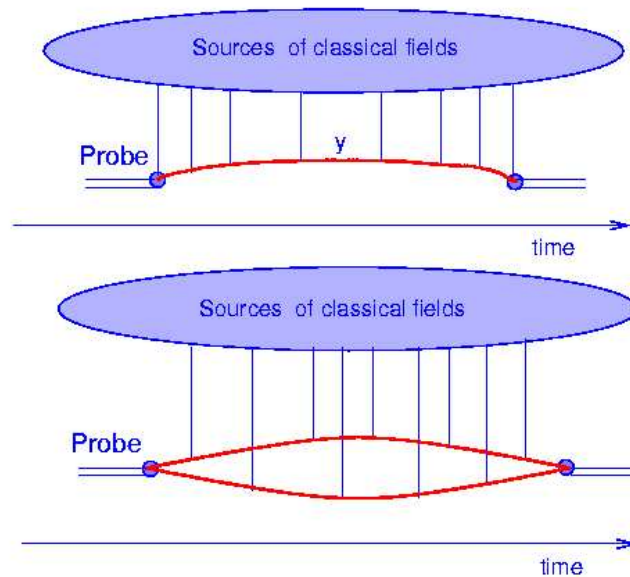
Therefore

- $\omega = \sqrt{g E_0}; \quad x_- \simeq \frac{\ln \gamma}{\sqrt{g E_0}} = \frac{\ln \gamma}{\sqrt{2 \pi} Q_s} \approx 0.6 \frac{1}{Q_s}$

- • • $T = \frac{Q_s}{\sqrt{4 \pi}} \approx 0.3 Q_s; \quad \bullet \bullet \bullet$

Nuclear gluon distributions

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McLerran-Venugopalan

formula:

- $\frac{dN^{MV}}{d^2x_{\perp} dy} \propto \frac{1}{\bar{\alpha}_S} \left(1 - e^{-\frac{1}{4}x_{\perp}^2 Q_s^2(x) \ln(x_{\perp}^2 Q_s^2(x))} \right)$

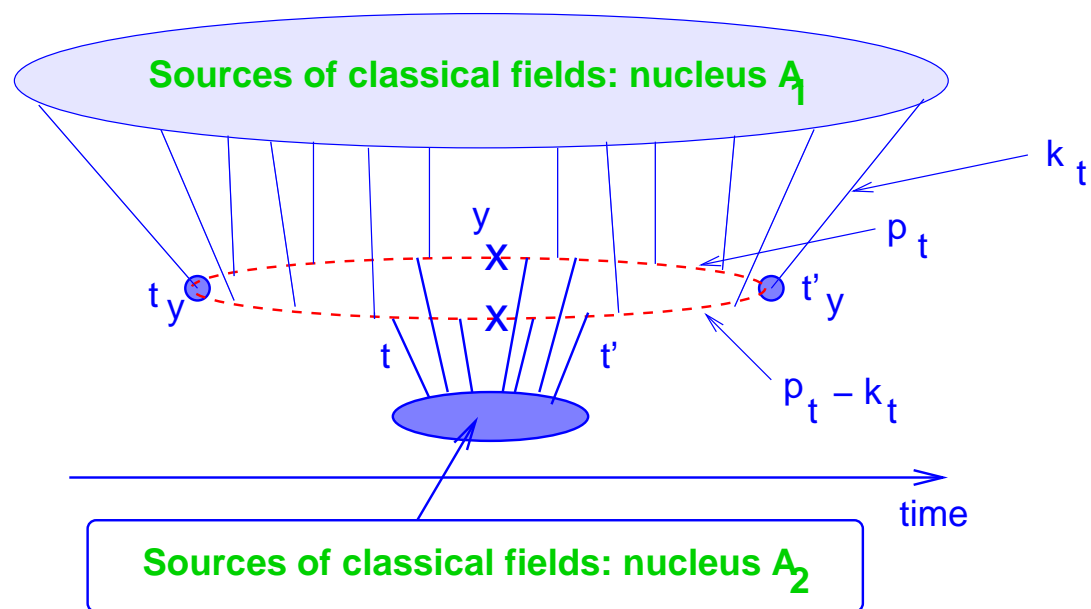
Our approach:

- $\frac{dN^{LLA}}{d^2x_{\perp} dy} \propto \ln(1/x) \left(1 - e^{-\frac{1}{4}x_{\perp}^2 Q_s^2(x) \ln(x_{\perp}^2 Q_s^2(x))} \right)$

- $\frac{dN^{LLA}}{d^2x_{\perp} dy} \propto \frac{1}{\bar{\alpha}_S} \ln(x_{\perp}^2 Q_s^2(x)) \left(1 - e^{-\frac{1}{4}x_{\perp}^2 Q_s^2(x) \ln(x_{\perp}^2 Q_s^2(x))} \right)$

Ion-Ion collisions

$$\text{Im } D(t - t') = \text{Im } D_{A_1}(t - t') \text{Im } D_{A_2}(t - t')$$



- $\frac{d\sigma}{dy d^2p_\perp} = \frac{S_{A_1} S_{A_2}}{\alpha_S} \frac{\pi^2 N_C}{2(N_C^2 - 1)} e^{-\frac{\epsilon}{T}}$

- $\frac{1}{T} = \frac{1}{T_{A_1}} + \frac{1}{T_{A_2}}$

Thermodynamics of heavy-ion collision

Statistical interpretation of gluon production

- Probability to produce n pairs is equal:

$$P_n = w_1^n(\sigma, \vec{p}) w_0(\sigma, \vec{p})$$

where

- w_1 is the probability to produce one pair;
- w_0 is the probability to produce no pair;

- Probability conservation yields

$$w_0(\sigma, \vec{p}) \sum_{n=0}^{\infty} w_1^n(\sigma, \vec{p}) = \frac{w_0(\sigma, \vec{p})}{1 - w_1(\sigma, \vec{p})} = 1;$$

- The total probability that the vacuum remain unchanged

$$W_0 = |\exp i\mathcal{L}V \Delta t|^2 = e^{-\frac{\Omega}{T}}$$

where $F\Omega$ is the thermodynamic potential;

- $$W_0 = \prod_{\sigma, \vec{p}} w_0(\sigma, \vec{p}) = e^{\sum_{\sigma, \vec{p}} \ln(1 - w_1(\sigma, \vec{p}))}$$

- $$\Omega = \mathcal{G} \frac{TV}{(2\pi)^3} \int d^3p \ln(1 - w_1(\sigma, \vec{p}))$$

where $\mathcal{G} = (2\sigma + 1)(N_c^2 - 1)$ - degeneration factor;

Bose-Einstein condensation

- Introducing

$$\mathcal{M}/T = 2 \operatorname{Im} S = \begin{cases} p_{\perp}^2/Q_s^2, & t \ll t_{\text{therm}} \\ p_{-}/T, & t \gg t_{\text{therm}} \end{cases}$$

- Number of produced pairs

$$\begin{aligned} N &= - \left. \frac{\partial \Omega(\mu)}{\partial \mu} \right|_{\mu=0} = \frac{\partial}{\partial \mu} \frac{VT}{(2\pi)^3} \mathcal{G} \int d^3p \ln(1 - e^{(\mu - \mathcal{M}/T)}) \Big|_{\mu=0} \\ &= \frac{\mathcal{G} V}{(2\pi)^2} \int dp_{\perp}^2 dp_z \frac{1}{e^{\mathcal{M}/T} - 1}. \end{aligned}$$

- At $t=0$: $N \propto \ln(Q_s^2/\Lambda^2)$ but the total emitted energy

$$\mathcal{E} = \int \varepsilon dN_{\varepsilon} = \frac{V \mathcal{G}}{(2\pi)^2} \Delta p_z Q_s^3 g E_0 \frac{\sqrt{\pi}}{2} \zeta(3/2)$$

- $\Delta p_z \simeq g E_0 t$ and $V \simeq t$, therefore, $\mathcal{E} \simeq t^2$.

- At $t = t_{therm}$:

$$N = \frac{V\mathcal{G}}{(2\pi)^2} g E t (\pi^2/6) T^2$$

We expect Bose-Einstein condensation of gluons at $T < T_0$ with

$$T_0 = \left(\frac{3(2\pi)^2 N}{V\mathcal{G} g E t \pi^2} \right)^{1/2}$$

$$N(p_{\perp} = 0) = N \left[1 - \left(\frac{T}{T_0} \right)^2 \right]$$

Equation of state

- Helmholtz free energy at $\mu = 0$ is equal to

$$F = \Omega = PV$$

- So long as $\Delta p_z \simeq gEt \ll p_\perp$

$$\mathcal{E}/V = 2P$$

- at $t > t_{therm}$ $p_z \sim p_\perp$

$$\mathcal{E}/V = 3P$$

Viscosity of the parton system

For $t > t_{therm} \implies$ equilibrium with

$$n \sim 1/\alpha_S \text{ and } \langle p_t \rangle \approx T \propto Q_s$$

Shear viscosity can be estimated:

- $\frac{\eta}{n} = \langle p_t \rangle \lambda \sim \frac{Q_s}{n \sigma}$
- $\sigma \sim \alpha_S / Q_s^2$
- The number of particle/unit volume $n \sim \frac{xG}{S+A L_z}$ where $L_z \sim 1/Q_s$ is the longitudinal extent of the system;

$$\frac{\eta}{n} \sim 1$$

When? Why? How?

1. **Why?** The origin of the initial ($t = 0$) distribution and of the process of thermalization ($t \approx t_{therm}$) are the same: **the clasasical fields that originated by all faster partons;**
2. **When?** $t_{therm} \simeq 0.8 \frac{1}{Q_s} \approx 0.1 - 0.2 fm;$
3. **How?** Creating the thermalized system of gluons with $T \simeq 0.3 Q_s(y) \approx 450 MeV$ at $y=0;$

What is bad?

- We failed to prove that the emitted gluons are in the thermodynamic equilibrium;
- We showed that the emitted gluons create the good initial conditions for **hydro**;
- We hope that **hydro** or something else will lead a system to the thermodynamic equilibrium;

What is good ?

- We gave a much simpler explanation for the GGC in which we are not losing any of essential ingredients of this approach;
- This explanation is not empty since it leads to a number of crazy ideas (see below).

The crazy idea: saturation of the saturation scale

- $$\frac{dN_{gluon}}{dt d^3 x d^2 p_t} = \frac{1}{4 \pi^3} \sum_{i=1}^3 |g \lambda_i| \ln \left(1 + e^{-\frac{\pi p_t^2}{|g \lambda_i|}} \right)$$
- $$\frac{dN_{gluon}}{dt d^3 x} = \int d^2 p_t \frac{dN_{gluon}}{dt d^3 x d^2 p_t} = \frac{\alpha_S}{8} C_1 = \frac{\alpha_S}{8} E^a E^a$$
- $$Q_s^2(Y) \equiv \langle |p_t^2| \rangle = \frac{\int p_t^2 d^2 p_t \frac{dN_{gluon}}{dt d^3 x d^2 p_t}}{\frac{dN_{gluon}}{dt d^3 x}} = \frac{9 \zeta(3)}{\pi^3} g \sqrt{C_1} \kappa_1$$
- $$Q_s^4(Y) = \frac{324 \zeta^2(3)}{\pi^5} \alpha_S \kappa_1^2 C_1$$
 •

The energy density $w(Y) = E^a E^a / 4\pi = C_1 / 4\pi$ and

$$\bullet w(Y + \Delta Y) - w(Y) = \int_Y^{Y+\Delta Y} dY' \frac{dN_{gluon}(Y')}{dt d^3x}$$

$$\xrightarrow{\Delta Y \rightarrow 0} \frac{dN_{gluon}(Y)}{dt d^3x} \Delta Y$$

$$\bullet \frac{E^a(Y) E^a(Y)}{4\pi} \neq \int_Y^{Y_b} dY' \frac{dN_{gluon}(Y')}{dt d^3x}$$

$$\bullet \frac{E^a(Y_b - Y) E^a(Y_b - Y)}{4\pi} = \int_Y^{Y_b} dY' \frac{dN_{gluon}(Y')}{dt d^3x}$$

$$\times \exp\left(-\int_Y^{Y'} dY'' \int dt d^3x \frac{dN_{gluon}(Y'')}{dt d^3x}\right)$$

The differential form is

$$\frac{1}{4\pi} \frac{d\{E^a(Y_b - Y) E^a(Y_b - Y)\}}{dY} = \frac{dN_{gluon}(Y)}{dt d^3x} - \int dt d^3x \frac{dN_{gluon}(Y)}{dt d^3x} \times$$

$$\times \int_Y^{Y_b} dY' \frac{dN_{gluon}(Y')}{dt d^3x} \exp\left(-\int_Y^{Y'} dY'' \int dt d^3x \frac{dN_{gluon}(Y'')}{dt d^3x}\right)$$

which can be rewritten as

$$\frac{2}{\pi\alpha_S} \frac{dQ_s^4(Y)}{dY} = Q_s^4(Y) - \frac{C}{4\pi\alpha_S} Q_s^8(Y) \times V \quad \text{where } V = \pi R_A^2 \times x_+ x_-$$

$$S \propto \int dx_+ \frac{p_\perp^2}{p_+^0 + g A_+(x_+)} \implies x_+ = \frac{p_+^0}{gE}$$

$$x_- = \frac{p_\perp^2}{2} \int \frac{dx_+}{(p_+^0 + g A_+(x_+))^2} = \frac{p_\perp^2}{2gE p_+^0}$$

$$\bullet \quad x_- x_+ = \frac{1}{2\pi g E} = \frac{81 \zeta^2(3) \kappa_1^2}{2\pi^6} \frac{1}{Q_s^2(Y)} \quad \bullet$$

Finally

$$\bullet \quad \frac{4}{\alpha_S \pi} \frac{d Q_s^2(Y)}{d Y} = Q_s^2(Y) - B Q_s^4(Y) \quad \bullet$$

where

$$\bullet \quad B = \frac{1}{32 \pi^2} \frac{\pi R_A^2}{\alpha_S} \quad \bullet$$

Solution:

$$\clubsuit \quad Q_s^2(Y) = \frac{Q_s^2(Y=Y_0) \exp\left(\frac{2\alpha_S}{\pi} (Y-Y_0)\right)}{1 + B Q_s^2(Y=Y_0) \left(\exp\left(\frac{2\alpha_S}{\pi} (Y-Y_0)\right) - 1\right)} \quad \clubsuit$$

