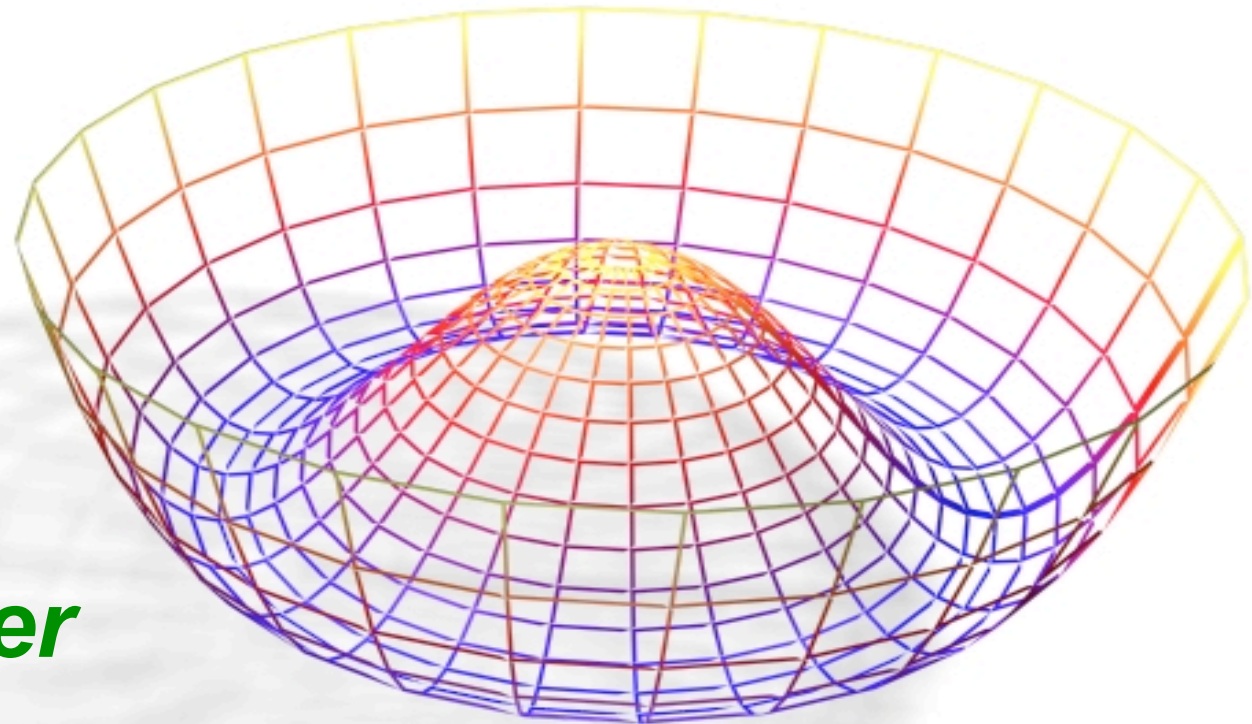




# *Searching for Exotic Higgs decays in Archived LEP Data*



***Kyle Cranmer***

***New York University***

***Center for Cosmology and Particle Physics***



Thank you to the Galileo Galilei Institute for the invitation

- apologies for arriving late, the program looks very interesting and I wish I could have been here for all of it

I joined ALEPH in '99, during its last year of data taking, and was active in the LEP Higgs searches

- In '05, I worked together with Marcello Maggi and Bruce Knuteson in the context of an ALEPH data archival project and to try Bruce's Quaero algorithm at LEP
- now possible to publish under ALEPH archival policy

I'd like to thank Neal Weiner, Spencer Chang, Tilman Plehn, and Bob McElrath in particular for pointing out this great opportunity.

- after a few failed attempts in the last few years to investigate these exotic scenarios, 3 things came together

1. the LHC "incident"

2. James Beacham, a graduate student at NYU was looking for a research project

3. Itay Yavin came to NYU and offered help (including learning to use ROOT)

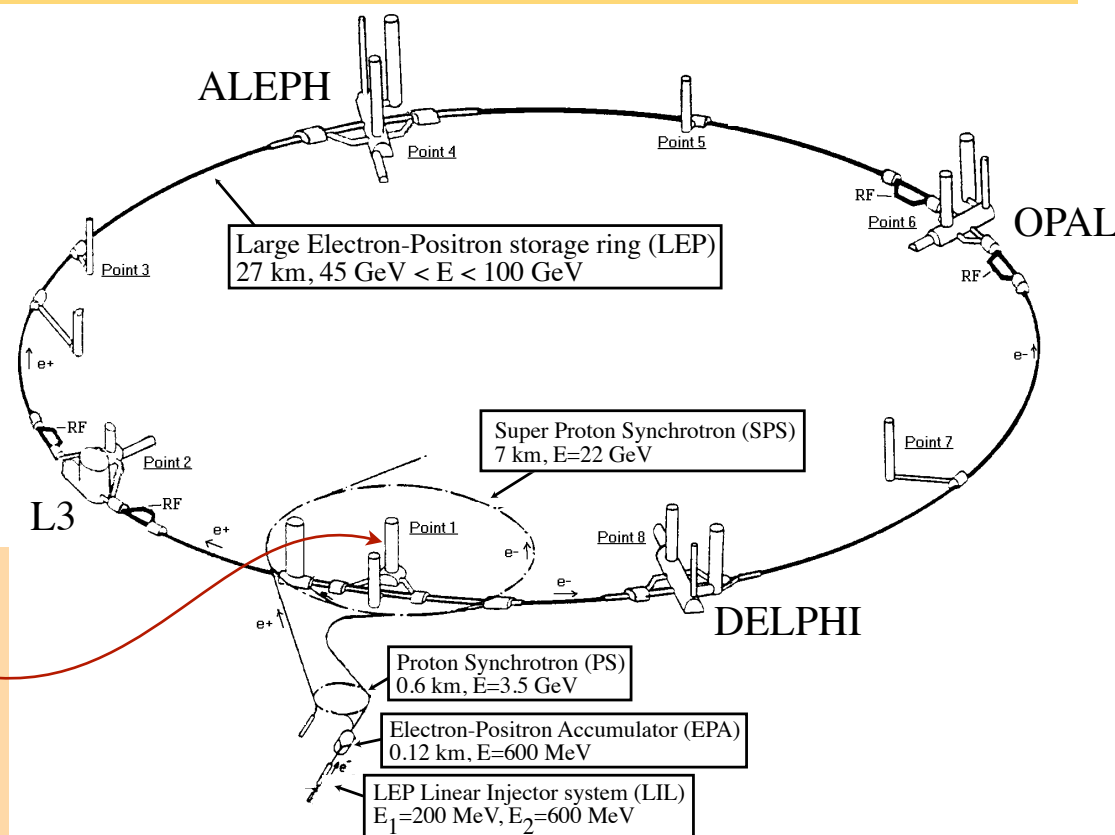
In addition Paolo Spagnolo @ INFN in Pisa was working on this independently.

we are merging our analyses into what will likely be the last ALEPH paper

## LEP operated from 1989–2000

- ▶ LEP1 running at the Z resonance (<1996)
- ▶ LEP2 running from  $\sqrt{s} = 183 - 207$  GeV

$E_{CM}$ (GeV)	183	189	192	196	200	202	205	207
$\int \mathcal{L} dt$ (pb <sup>-1</sup> )	56.82	174.21	28.93	79.83	86.30	41.90	81.41	133.21



I got to see the excavation of the ATLAS cavern directly above the LEP tunnel in the last days of running

Searches for the Standard Model Higgs put a limit at  $M_H > 114.4$  GeV

- searches dominated by  $H \rightarrow bb, \tau\tau$
- decay independent limit (from Z recoil) at 82 GeV
- searches in the (CP conserving) MSSM also quite stringent
  - $m_h, m_A < 93$  for  $0.5 < \tan \beta < 2.5$  in “ $m_h$ -max” scenario
- excesses seen at 97 and 115 GeV, but not consistent with SM or MSSM

Electroweak fits prefer a Higgs significantly lighter than this bound

- introduces fine tuning problems for Standard Model and MSSM
- LEP paradox:
  - no indication of new physics  $\Rightarrow$  scale of new physics  $> 1$ TeV
  - hard maintain naturalness if  $m_H > 114$  and scale of new physics is  $> 1$ TeV

This has motivated theories with extended Higgs sectors or next-to-minimal supersymmetric extensions to the Standard Model

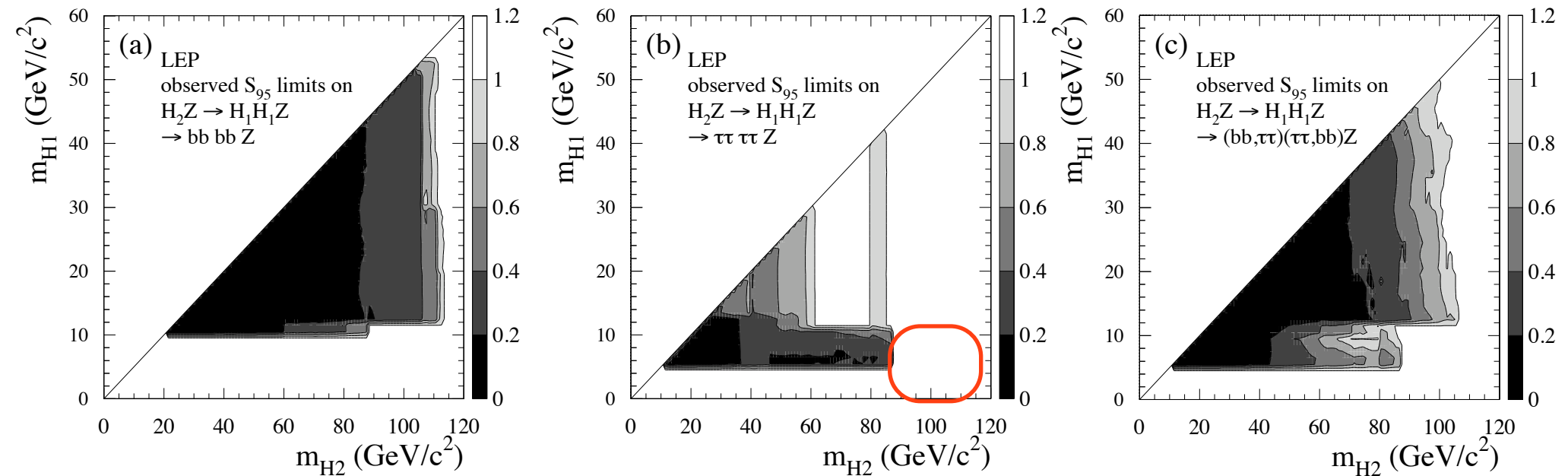
If the Higgs exists and is light, how could we have missed it at LEP?

- ▶ if the production cross-section were smaller than expected
  - this has direct implications on how the Higgs couples to the Z and its role in EWSB
- ▶ or maybe it decayed into something exotic that the standard analysis missed
  - Is that difficult to achieve? No, the  $Hbb$  coupling is quite small. It doesn't take much for a new decay mode to dominate the  $bb$  mode.
- ▶ would the existing analyses have seen it?
  - that depends, in some cases the existing searches may still be quite efficient.

# LEP Higgs limits in $H_1, H_2$ plane

## Search for Neutral MSSM Higgs Bosons at LEP

ALEPH, DELPHI, L3 and OPAL Collaborations  
The LEP Working Group for Higgs Boson Searches<sup>1</sup>



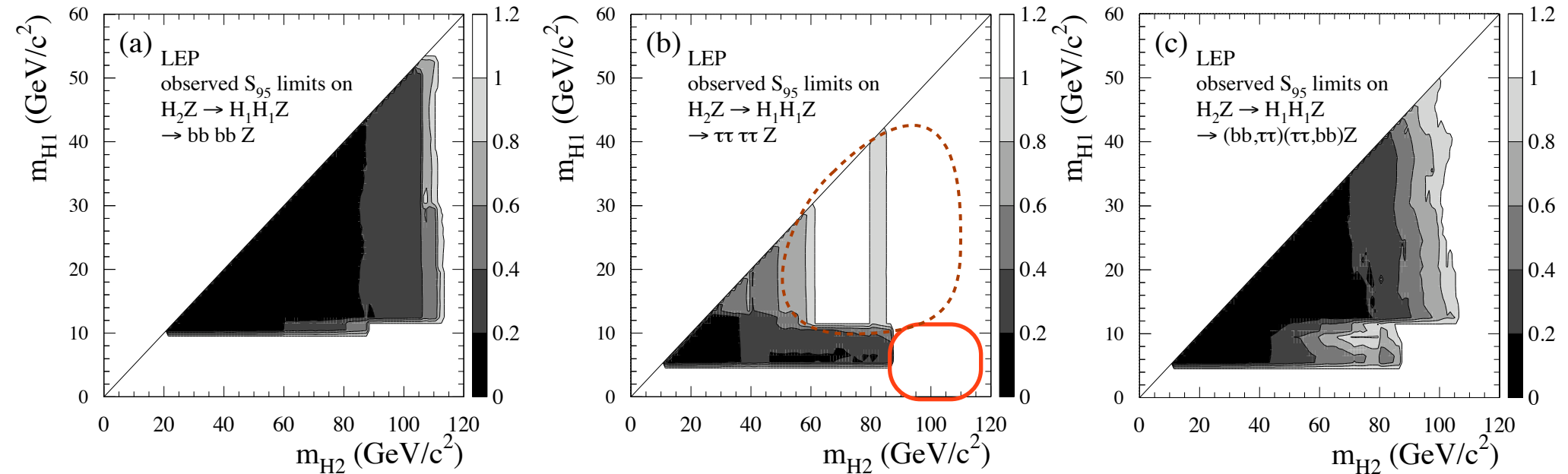
(factor x SM cross section that corresponds to 95% exclusion)

Here we see that Higgs bosons produced via Higgsstrahlung decaying to  $4b$  are highly constrained

- $4\tau$  are less constrained with a notable hole for  $m_h > 85$  &  $2m_\tau < m_a < 10$  GeV

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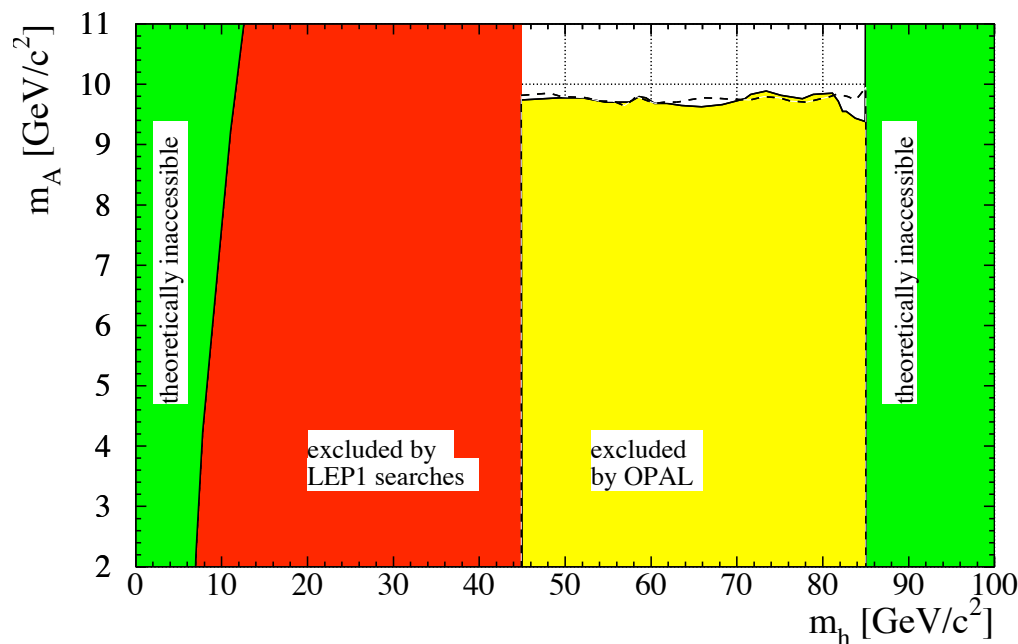
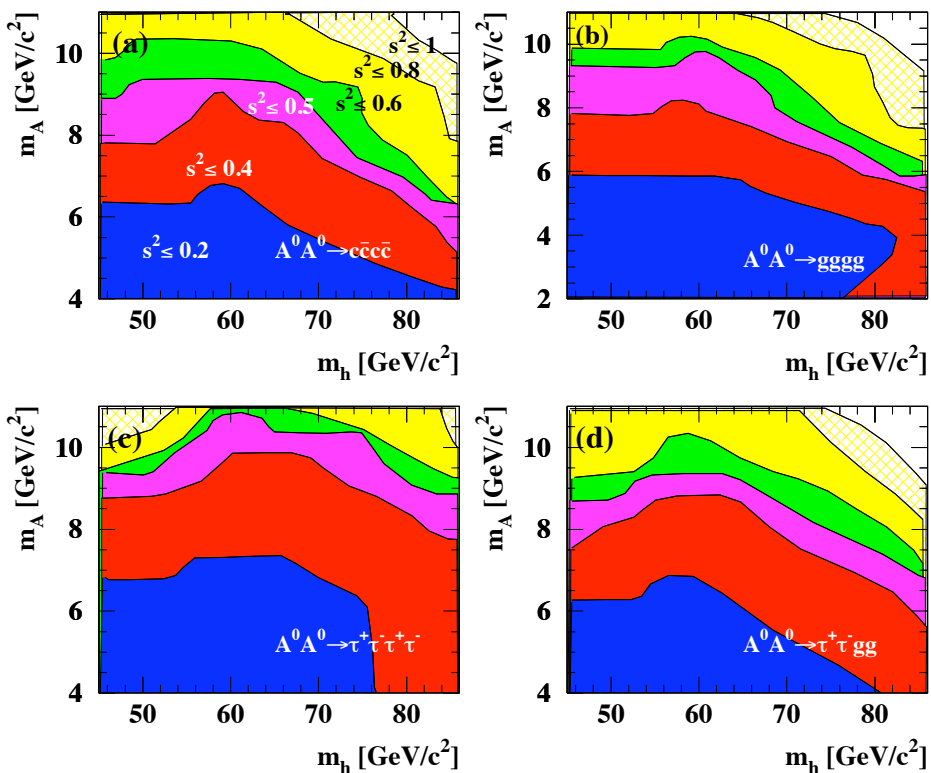
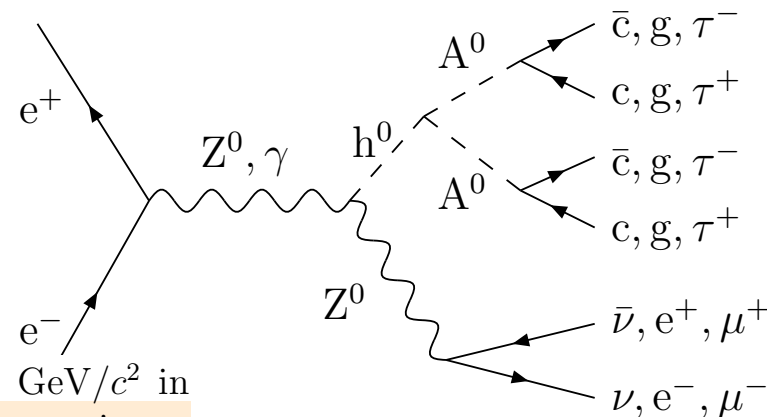
- $4\tau$  are less constrained with a notable hole for  $m_h > 85$  &  $2m_\tau < m_a < 10$  GeV

OPAL also carried out a searches in the region  $2m_\tau < m_a < 10 \text{ GeV}$

Search for a low mass CP-odd Higgs boson in  $e^+e^-$  collisions with the OPAL detector at LEP2

## 6.2 MSSM no-mixing scenario interpretation

We scan the region with  $2 \leq m_A \leq 11 \text{ GeV}/c^2$  and  $45 \text{ GeV}/c^2 \leq m_h \leq 85 \text{ GeV}/c^2$  in the  $m_A$  versus  $m_h$  plane for the MSSM benchmark parameter scenario. The maximum theoretically allowed value for  $m_h$  in this scenario is  $85 \text{ GeV}/c^2$  [6]. The scan procedure



95% CL in the  $m_A$  versus  $m_h$  plane for the MSSM no-mixing benchmark



The searches above were done with a 2 higgs doublet model in mind

- the same search is also sensitive to a wide range of theories with extended Higgs sectors
  - probably the most useful prototype is the next-to-minimal SSM, in which the MSSM is extended with an additional singlet superfield  $\hat{S}$ 
    - the scalar part naturally acquires a vev. and can provide a dynamical explanation for the size of the  $\mu$  term.
    - this gives rise to a (mostly singlet) CP-odd scalar boson  $a$
    - approximate accidental symmetries (à la Peccei-Quinn or when trilinear couplings vanish) can give a mechanism to make the  $a$  light
  - in addition, Hooper and Tait have considered similar scenarios in the context of the PAMELA excess

Here we are taking a very model independent attitude, and just look for all the uncovered  $h \rightarrow aa \rightarrow X$  scenarios that are not already ruled out and which are kinematically feasible

- in particular, we are also interested in looking for mixed decays that may not be expected if the  $a$  is a pseudo-scalar.

# Planned searches

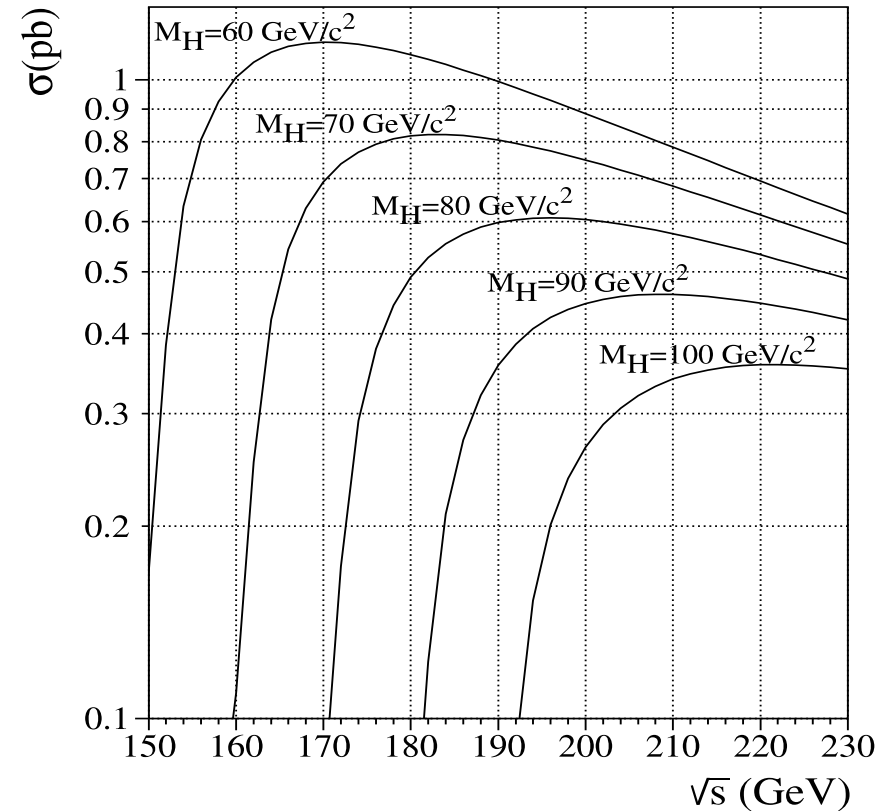
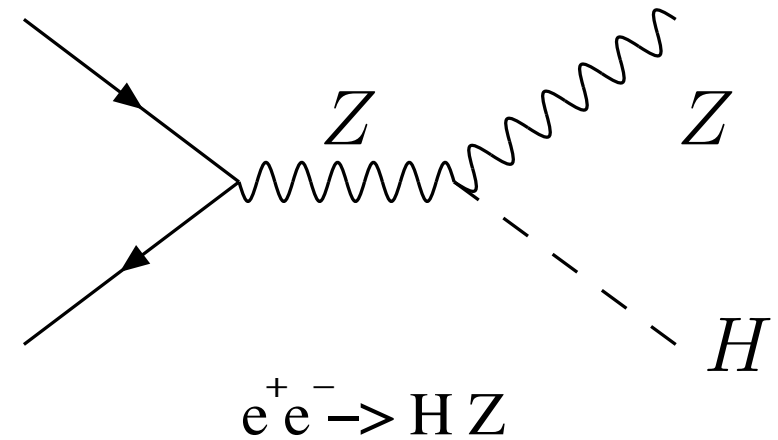
We're mainly interested in looking for **standard production** and **exotic decays**, thus expect to present result as 90/95% confidence limit on:

$$\xi^2 = \frac{\sigma \text{BR}(h \rightarrow aa) \text{BR}(a \rightarrow XX) \text{BR}(a \rightarrow YY) (2 - \delta_{XY})}{\sigma_{SM}}$$

particularly for:

- $e^+e^- \rightarrow Zh \rightarrow Z + 4\tau$  **focus today**
- $e^+e^- \rightarrow Zh \rightarrow Z + 2\mu 2\tau$
- $e^+e^- \rightarrow Zh \rightarrow Z + 4\mu$  **some progress**
- $e^+e^- \rightarrow Zh \rightarrow Z + (\mu\mu/\tau\tau)q\bar{q}$
- $e^+e^- \rightarrow Zh \rightarrow Z + (\mu\mu/\tau\tau)gg$
- $e^+e^- \rightarrow ah \rightarrow 6\mu$
- $e^+e^- \rightarrow ah \rightarrow 6\tau$  **suggested here at GGI**

(need to check on limits for electron modes)



# $H \rightarrow aa \rightarrow 2\mu 2\tau$ at the Tevatron

FERMILAB-PUB-09-257-E

Search for NMSSM Higgs bosons in the  $h \rightarrow aa \rightarrow \mu\mu \mu\mu, \mu\mu \tau\tau$  channels using  $p\bar{p}$  collisions at  $\sqrt{s} = 1.96$  TeV

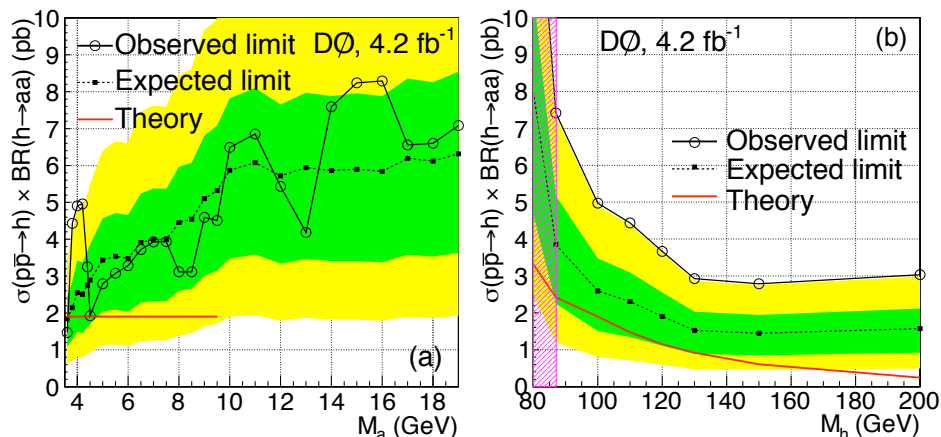


FIG. 3: The expected and observed limits and  $\pm 1$  s.d. and  $\pm 2$  s.d. expected limit bands for  $\sigma(p\bar{p} \rightarrow h + X) \times \text{BR}(h \rightarrow aa)$ , for (a)  $M_h = 100$  GeV and (b)  $M_a = 4$  GeV. The signal for  $\text{BR}(h \rightarrow aa) = 1$  is shown by the solid line. The region  $M_h < 86$  GeV is excluded by LEP.

$M_a$ (GeV)	$\sigma \times \text{BR}$ [exp] obs (fb)
0.2143	[10.0] 10.0
0.3	[9.5] 9.5
0.5	[7.3] 7.3
1	[6.1] 6.1
3	[5.6] 5.6

Sample	$\sigma \times 2 \times \text{BR}$
Data	
$M_a = 3.6$ GeV	[23.8] 19.1 fb
$M_a = 4$ GeV	[23.9] 45.9 fb
$M_a = 7$ GeV	[25.0] 24.6 fb
$M_a = 10$ GeV	[24.7] 27.3 fb
$M_a = 19$ GeV	[30.0] 33.7 fb

Andy Haas and company collaborated with Wacker and Lisanti to look for these signatures at the Tevatron

Discovering the Higgs with Low Mass Muon Pairs

Mariangela Lisanti and Jay G. Wacker<sup>1</sup>

<sup>1</sup> SLAC, Stanford University, Menlo Park, CA 94025  
Physics Department, Stanford University, Stanford, CA 94305

(Dated: March 8, 2009)

These searches are probing  $\sim 1\%$  of the expected production cross-section.

- there are not enough signal events at LEP to compete

However, the  $4\tau$  signature is significantly more difficult at hadron colliders than at

Tracking: silicon + large time projection chamber (~31 hits)



$$\frac{\Delta 1/p_T}{1/p_T} = (6 \cdot 10^{-4} \oplus 5 \cdot 10^{-3}/p_T)$$

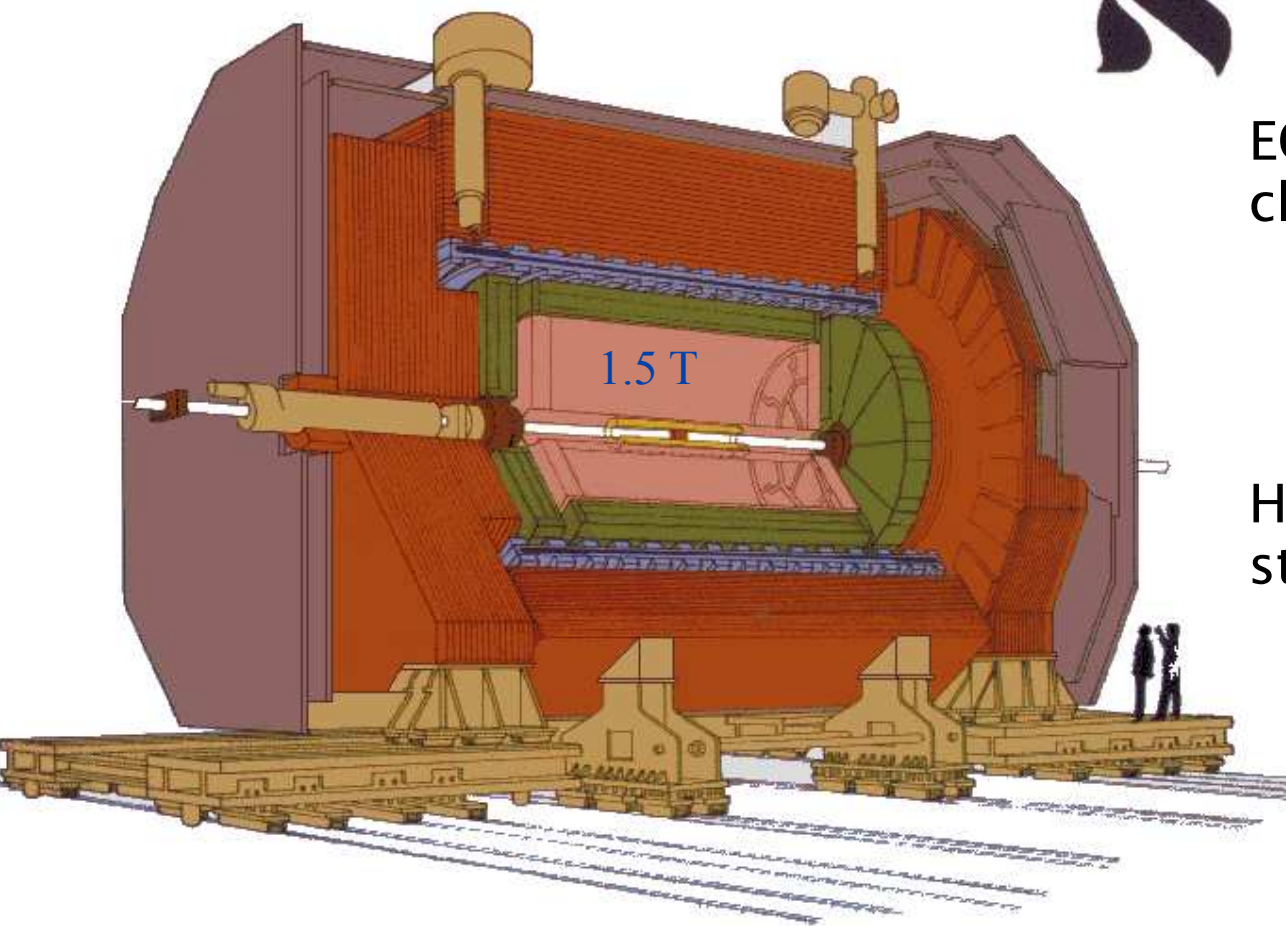
ECAL: lead + proportional wire chambers,  $22X_0$

$$\frac{\Delta E}{E} = 0.18/\sqrt{E}$$

HCAL: 23 layers of iron yolk + streamer tubes

$$\frac{\Delta E}{E} = 0.85/\sqrt{E}$$

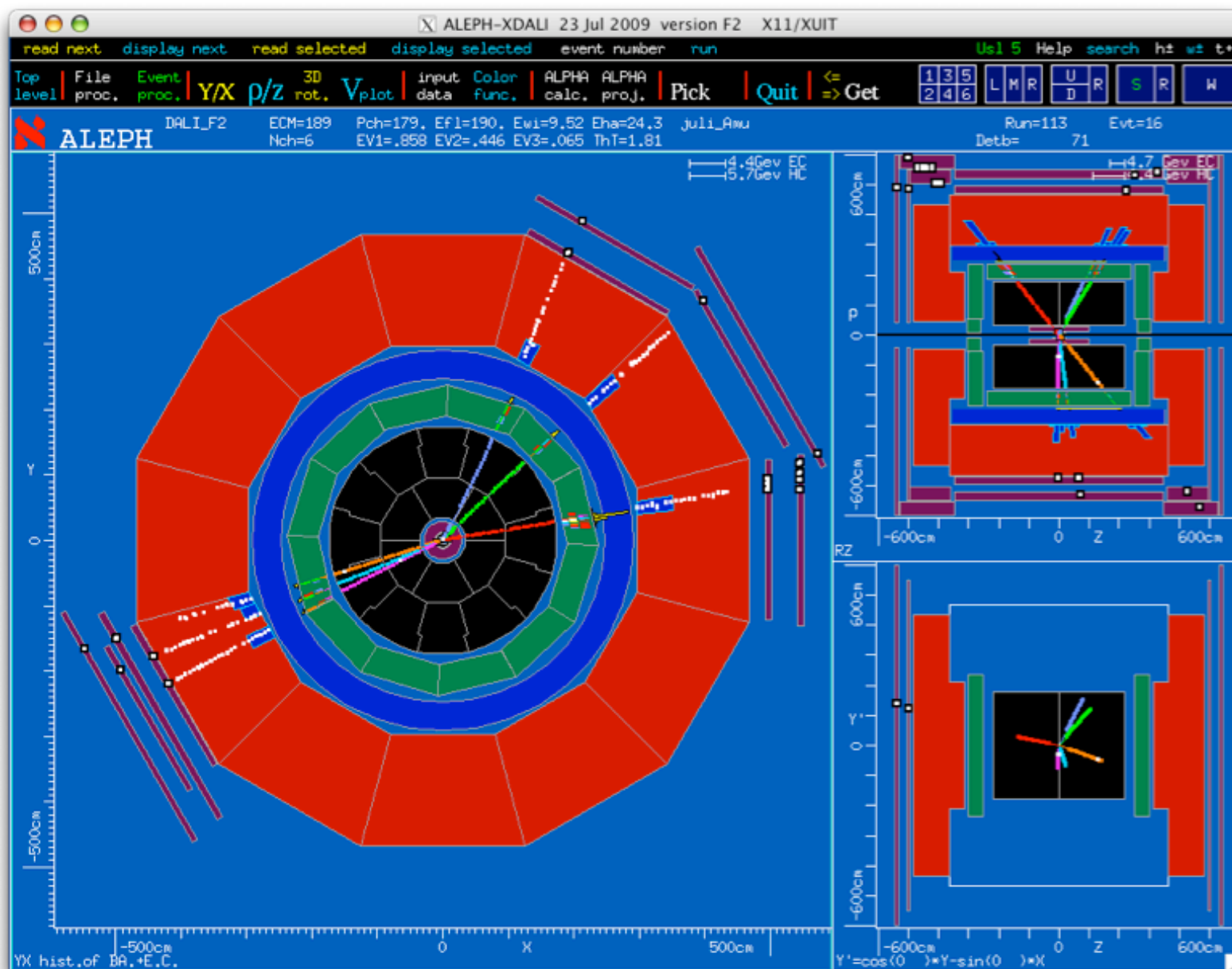
muons identified via HCAL + 2 muon chambers



Detector simulation based on Geant 3, analysis based on 10 year old fortran framework

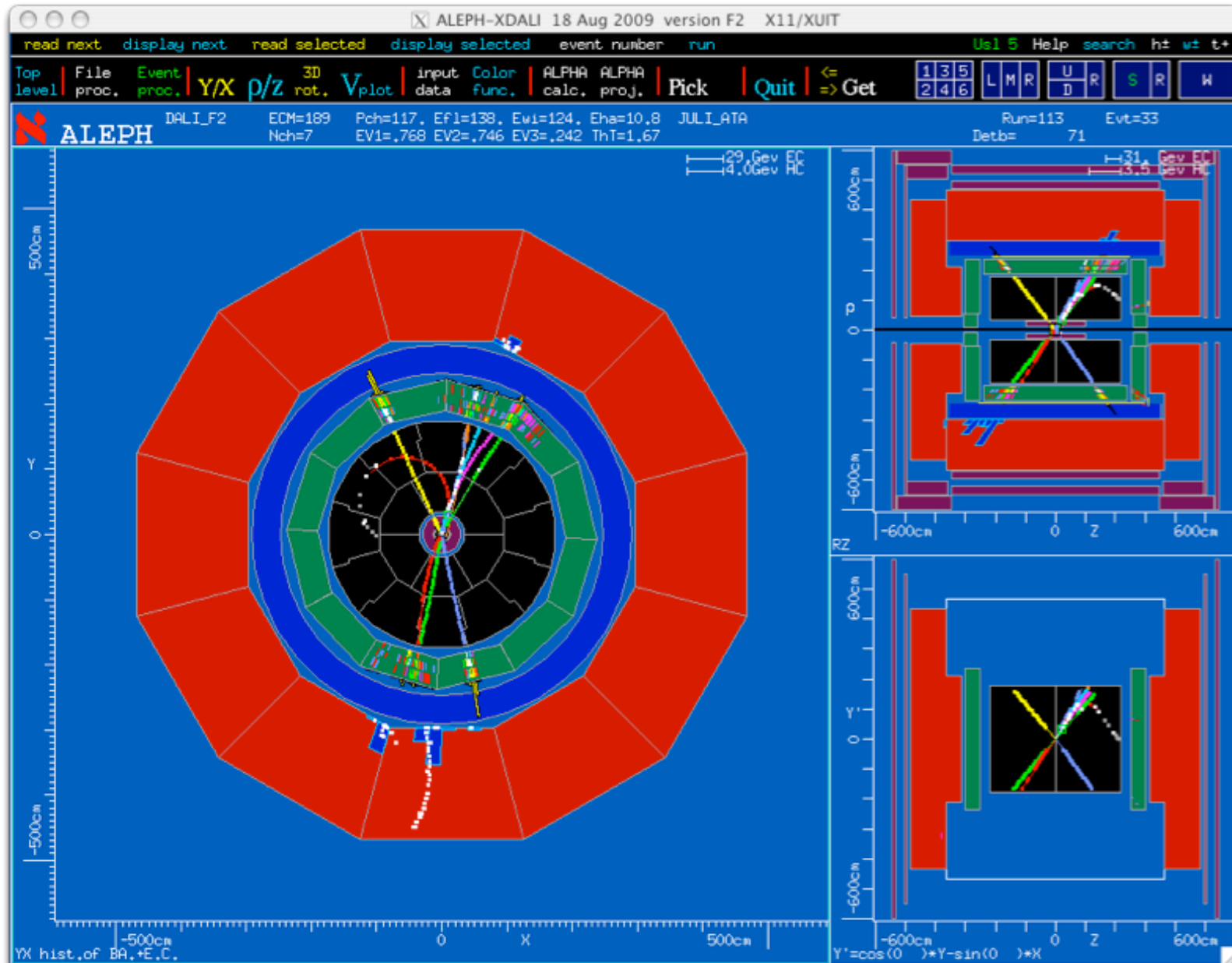
# Simulated signal event $e^+e^- \rightarrow ZH \rightarrow 6\mu$

Signal's generated with HZHA03 (using generic 2HDM) and run through full GEANT3 simulation, ALEPH reconstruction, and analysis chain (it's so clean! I love  $e^+e^-$ )



# Simulated signal event $e^+e^- \rightarrow ZH \rightarrow 2e4\tau$

2 back-to-back electrons clearly distinguished from 2 back-to-back jets.  
not much else in the event (about 50 GeV of missing energy)



# A Tevatron event, for comparison

Clearly the hadron colliders are more challenging

- lots of tracks, lots of hadronic energy deposits

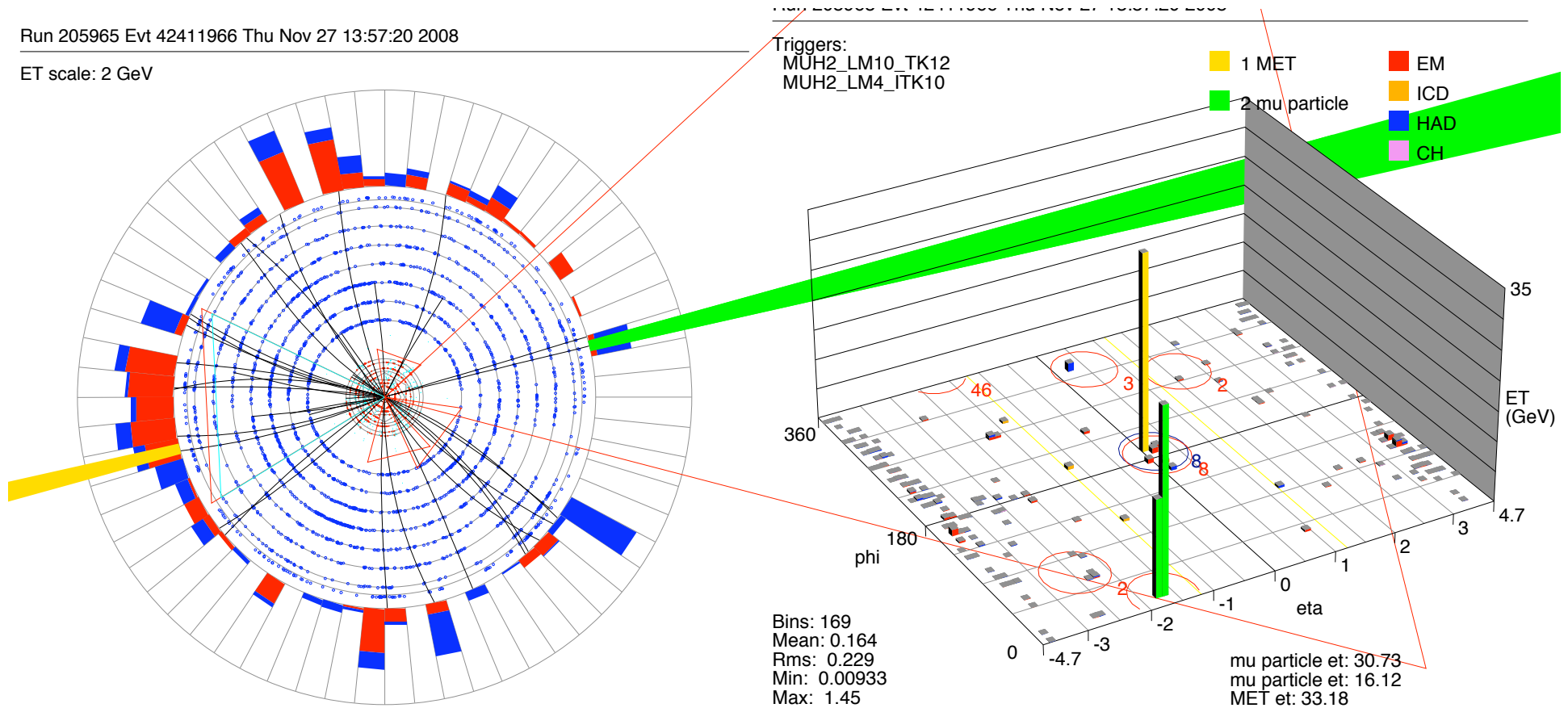


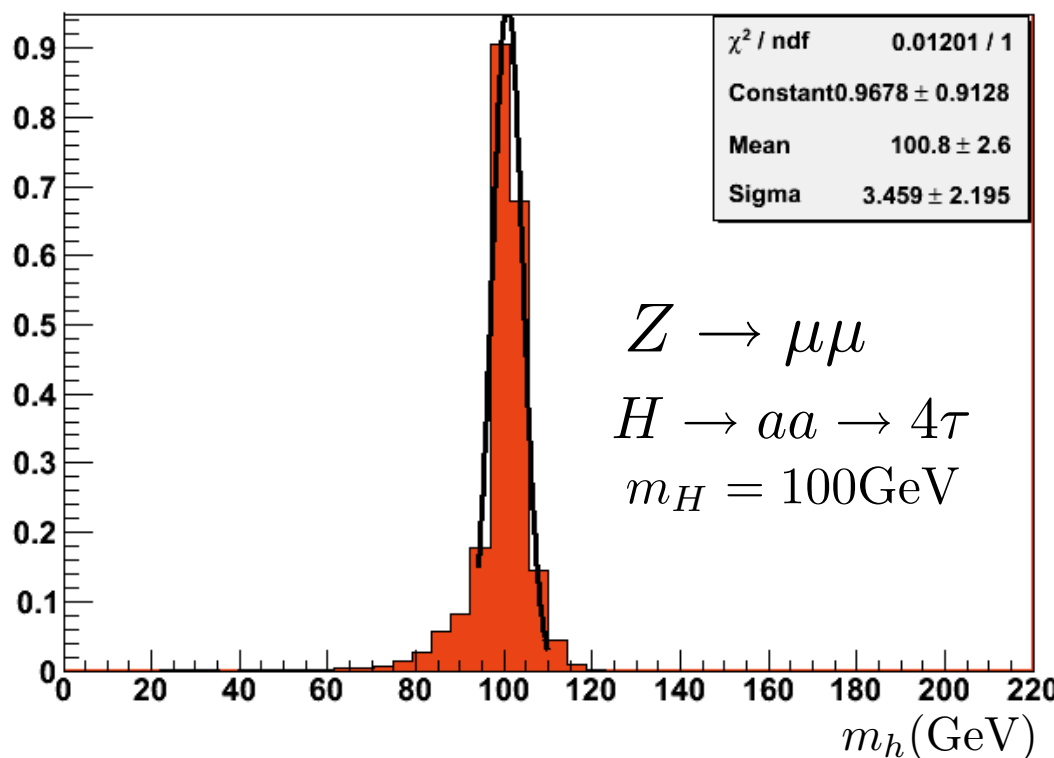
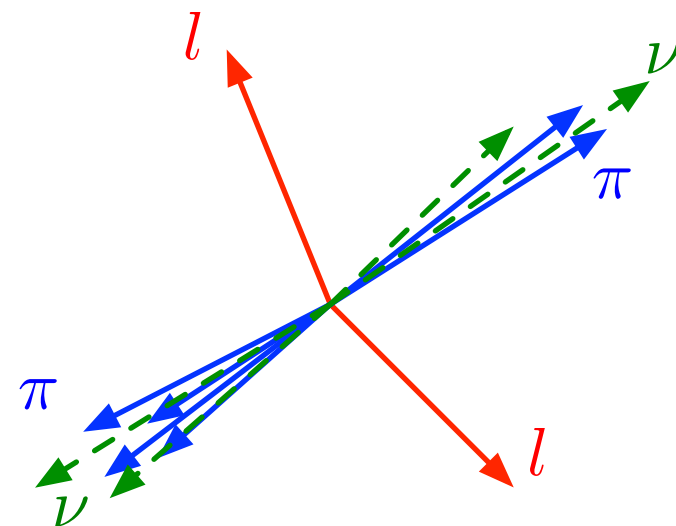
FIG. 6: Views of an event in data passing all the final “ $\cancel{E}_T$ ” selections for the  $2\mu 2\tau$  channel.

Even without resorting to the collinear approximation used for  $H \rightarrow \tau\tau$  at the LHC, it is possible to reconstruct the Higgs mass

- ▶ because it's  $e^+e^-$  have the full 4-vector for the neutrino system

In the  $Z \rightarrow \nu\nu$  channel, we do not have enough constraints to reconstruct the Higgs

- ▶ though several variables are sensitive to  $m_h$





After decades of running in a very clean environment, and tuning Monte Carlo to data the description of standard model processes in ALEPH is excellent.

$q\bar{q}$  The process  $e^+e^- \rightarrow Z/\gamma^* \rightarrow q\bar{q}(\gamma)$  is modeled using KK 4.14 [67], with initial state radiation from KK and final state radiation from PYTHIA.

$e^+e^-$  Bhabha scattering and  $e^+e^- \rightarrow Z/\gamma^* \rightarrow e^+e^-(\gamma)$  is modeled using BHWIDE 1.01 [68].

$\mu^+\mu^-$  Pair production of muons,  $e^+e^- \rightarrow Z/\gamma^* \rightarrow \mu^+\mu^-(\gamma)$ , is calculated using KK 4.14 [67], including initial and final state radiative corrections and their interference.

$\tau^+\tau^-$  Pair production of taus,  $e^+e^- \rightarrow Z/\gamma^* \rightarrow \tau^+\tau^-(\gamma)$ , is calculated using KK 4.14 [67], including initial and final state radiative corrections and their interference.

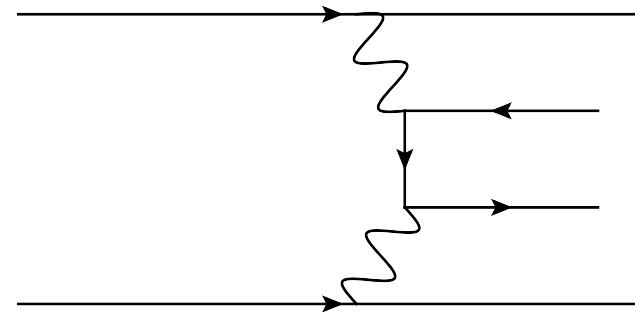
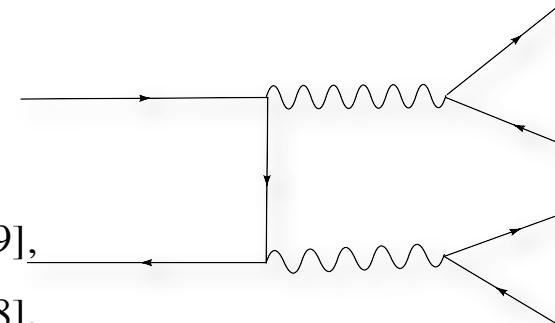
$1ph$  Single photon production,  $e^+e^- \rightarrow Z/\gamma^* \rightarrow \nu\bar{\nu}(\gamma)$ , is included in the background estimate.

$Nph$  Multiphoton production,  $e^+e^- \rightarrow n\gamma$ , with  $n \geq 2$ , is included in the background estimate.

## Two particularly important processes for these searches are 4 fermion and 2 photon processes

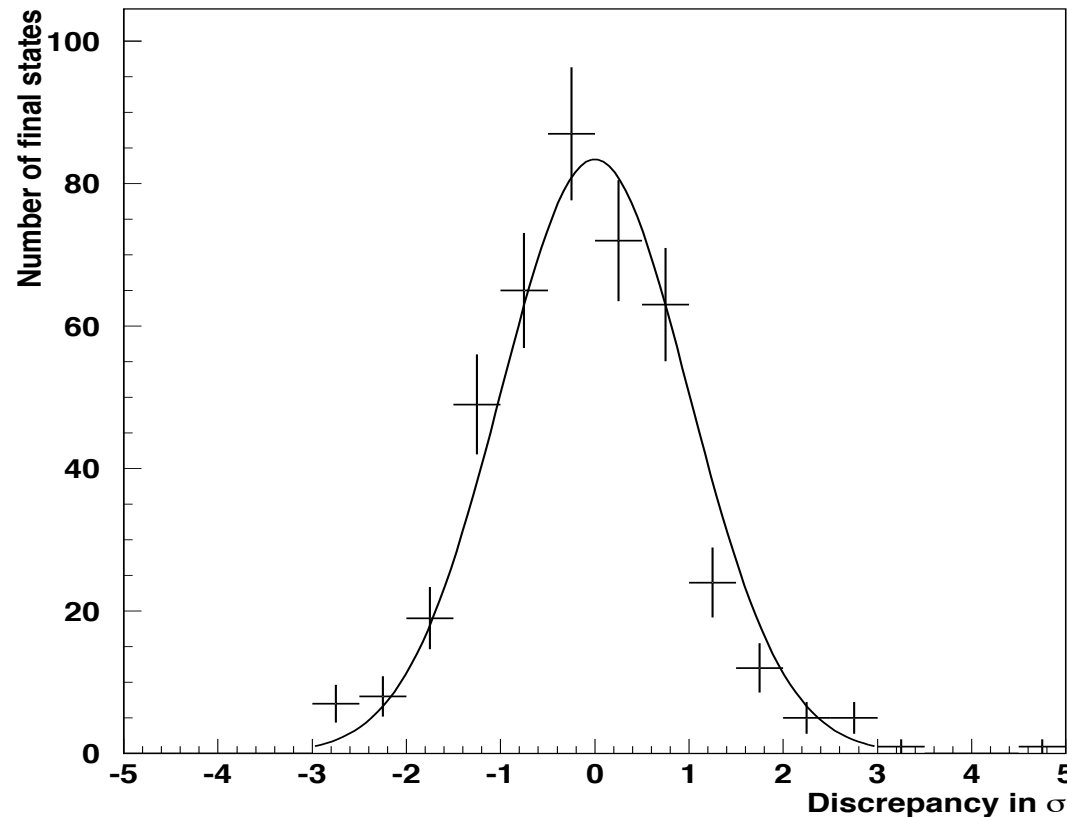
*4f* Four fermion events compatible with  $WW$  final states are generated using KoralW 1.51 [69], with quarks fragmented into parton showers and hadronized using either PYTHIA 6.1 [38]. Events with final states incompatible with  $WW$  production but compatible with  $ZZ$  production are generated with KoralZ

*2ph* Two-photon interaction processes,  $e^+e^- \rightarrow e^+e^-X$ , are generated with the PHOT02 generator [70]. When  $X$  is a pair of leptons, a QED calculation is used with preselection cuts to preferentially generate events that mimic  $WW$  production. When  $X$  is a multi-hadronic state, a modified version of PYTHIA is used to generate events with the incident beam electron and positron scattered at  $\theta < 12^\circ$  and  $168^\circ < \theta$ , respectively. Events in which the beam electron or positron is scattered through an angle of more than  $12^\circ$  are generated using HERWIG 6.2 [39].



In 2005, I worked together with Bruce Knuteson to try his Quaero algorithm on ALEPH's LEP2 data

- ▶ we used these same Monte Carlo samples and compared predictions to several hundred final states.
- ▶ That analysis did NOT use full simulation of ALEPH detector, but still saw excellent agreement with SM.



Because the LEP data is old and it is not possible to confirm anything with “next year’s data”, we had to be quite careful

- remember, we’re shooting for a discovery!
- no one would believe a signal if we adjusted our cuts looking at data
  - Also, we don’t want to spoil the other analyses that we might be interested in:  $a \rightarrow \text{jets}, \mu, ..$

But we do need to verify that our Monte Carlo is describing the data well.

- So we did a blind blind analysis and defined 5 control samples
  1. exclude  $m_{ll}$  around  $M_Z$ , that kills our signal, but otherwise similar
  2. Select events if  $\# \text{tracks} < 2$  for each jet (kills  $\tau\tau, \mu\mu, q\bar{q}, gg$  )
  3. in  $Z \rightarrow ll$  exclude events with  $M(j_1, j_2, \text{invisible}) > 60 \text{ GeV}$
  4. in  $Z \rightarrow \nu\nu$  exclude events with missing mass  $> 80 \text{ GeV}$
  5. exclude events with  $\# \text{track} > 6$  in both jets (to remove taus) AND if di-jet mass  $> 60$  (to avoid seeing  $h \rightarrow aa \rightarrow q\bar{q}, gg$  if it exists)

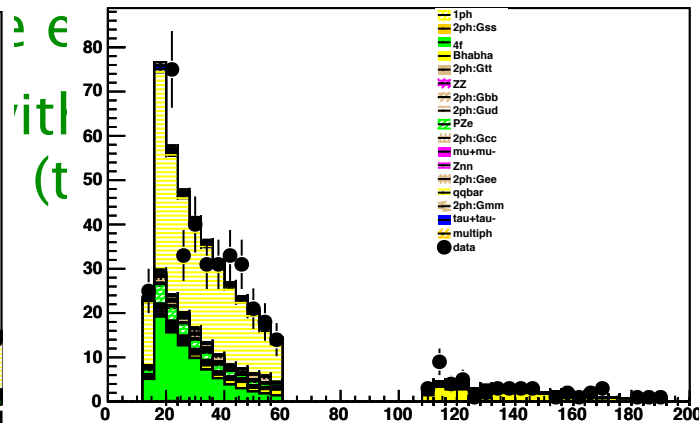
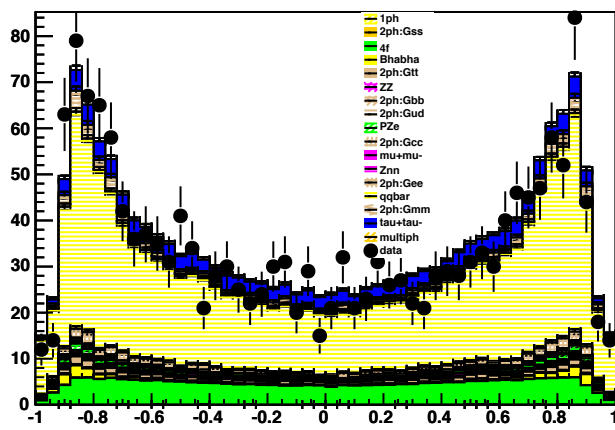
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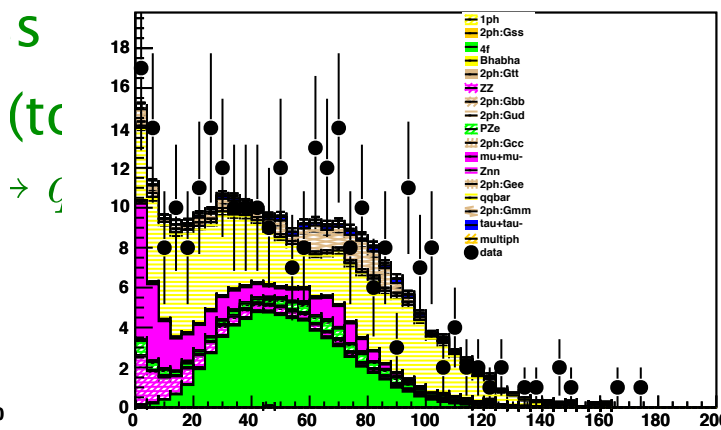
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jetct, nu



misse, muons



At LEP, the dominant jet algorithms were DURHAM and JADE.

- ▶ both are iterative recombination type algorithms: merge if  $m_{ij}^2 / E_{tot}^2 < y_{cut}$ 
  - $y_{cut}$  is an adjustable parameter and  $E_{tot}$  was often chosen to be the visible energy in the event
  - Often (as in the case of the OPAL analysis), events were “forced into N jets”, eg. the algorithm scanned  $y_{cut}$  until the event had exactly N jets.
    - Then that value of  $y_{cut}$  would be used as a discriminating variable together with the jet’s mass.
- ▶ DURHAM defines  $m_{ij}^2$  in a way that is more robust to soft radiation, which is good if you are interested in bona fide hadronic showers.
  - But we are looking for a purely electroweak decay, so the straight invariant mass combination of JADE is more natural.
  - Furthermore, we know that we are interested in  $m_a < 10 \text{ GeV}$  which leads to an obvious choice for  $y_{cut}$  if we use a fixed  $E_{tot}$ .

By choosing this approach our s/b was significantly higher than forcing to two jets with DURHAM and cutting on the jet mass

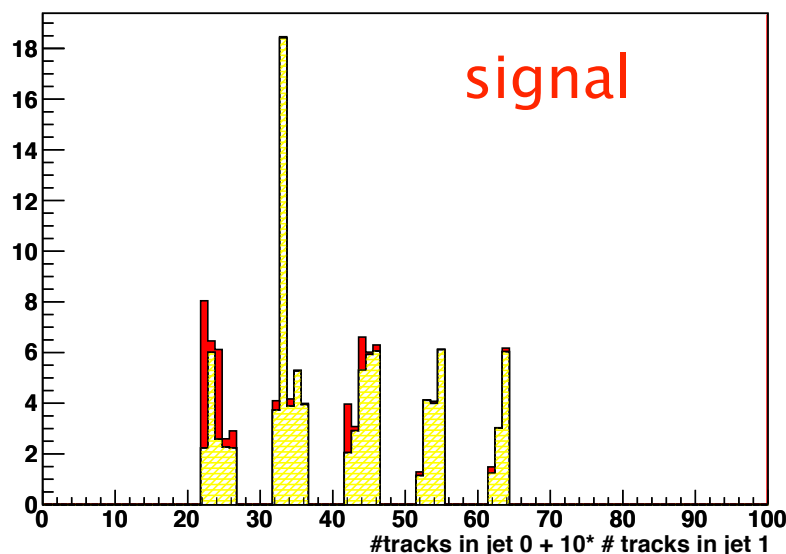
- Additionally we have track multiplicity in jets as a handle

# Thumbnail of $Zh \rightarrow \nu\nu 4\tau$

This channel drives the analysis because of the larger Z branching ratio

- it is also the most difficult, because you don't have a clean  $Z \rightarrow ll$
- initially ask for exactly 2 jets with at least 2 tracks with  $m_{jj} > 10\text{GeV}$
- to reject "2 photon" and beam bkg events  $\cos \theta_{miss} < 0.97$ ,  $E_{vis} > 5\% E_{CM}$
- require large missing energy, missing mass, that the jets aren't too forward, and remove events with very low aplanarity (unobserved initial state radiation in a 2 $\rightarrow$ 2 process with subsequent photon conversions or brehmsstrahlung)
- Finally, we have the track multiplicity distribution, which is very powerful at discriminating signal from background

Track Multiplicity in  $Z \rightarrow \nu\nu$  channel



After these cuts expect  
about 11 events from  
background

mainly 2ph and 4f

Total bg	11.01
1ph	0.13
2ph-Gss	0.49
4f	0.76
Bhabha	0.00
2ph-Gtt	0.24
ZZ	0.34
2ph-Gbb	0.06
2ph-Gud	4.32
PZe	0.10
2ph-Gcc	1.30
mu+mu-	0.00
Znn	0.01
2ph-Gee	0.00
qqbar	1.80
2ph-Gmm	0.00
tau+tau-	1.45
multiph	0.00

# Thumbnail of $Zh \rightarrow ee 4\tau$ & $Zh \rightarrow \mu\mu 4\tau$

These channels are significantly cleaner due to the clear Z peak, but the signal rate is very low and signal efficiency is precious

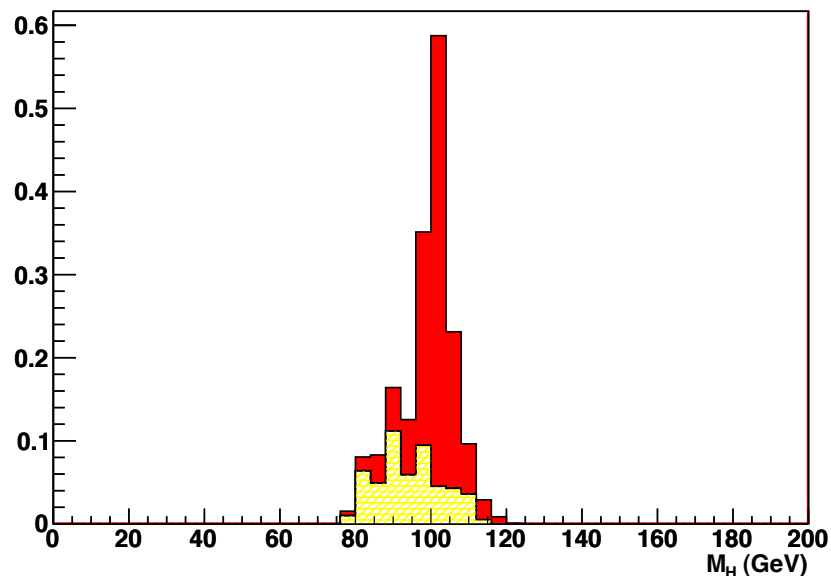
- use standard ALEPH lepton ID
  - worked hard to improve Z mass reconstruction by adding appropriate photons to Z (more severe for electron channel)
  - electron channel suffers from Bhabha background, where we have 2 good electrons which produce brehmsstrahlung photons that convert to give 2 track “jets”
  - note, in OPAL analysis, they had a requirement on  $E_{vis}$ . Makes sense for jet channels, but it is not efficient for the tau channel, so we dropped it.
- again, we make no attempt to reconstruct taus, we just remove leptons and photons from the event, and run our JADE jet algorithm on remainder
  - again we use track multiplicity to focus the analysis on taus
  - we can also use the reconstructed Higgs mass to cut down on background



# Expectations for a 100 GeV Higgs



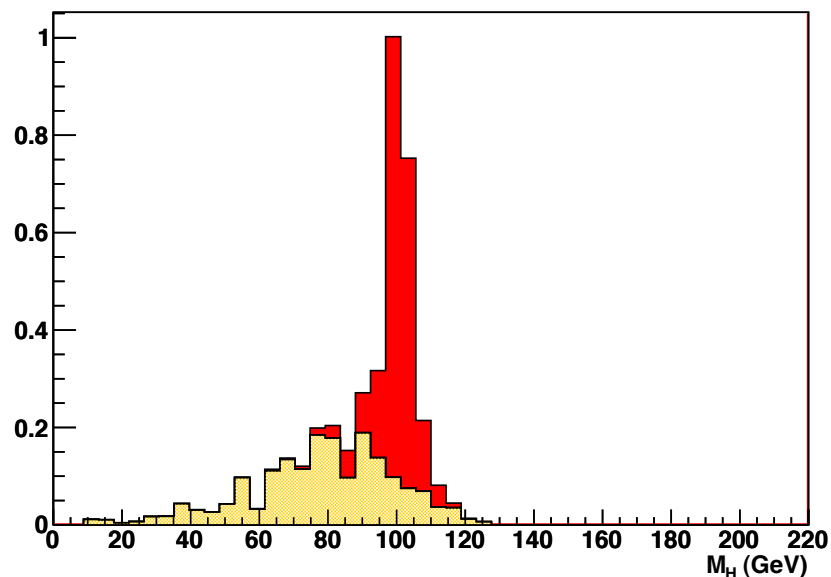
Reconstructed  $M_H$  in  $Z \rightarrow ee$  channel



## Background contributions for ee channel

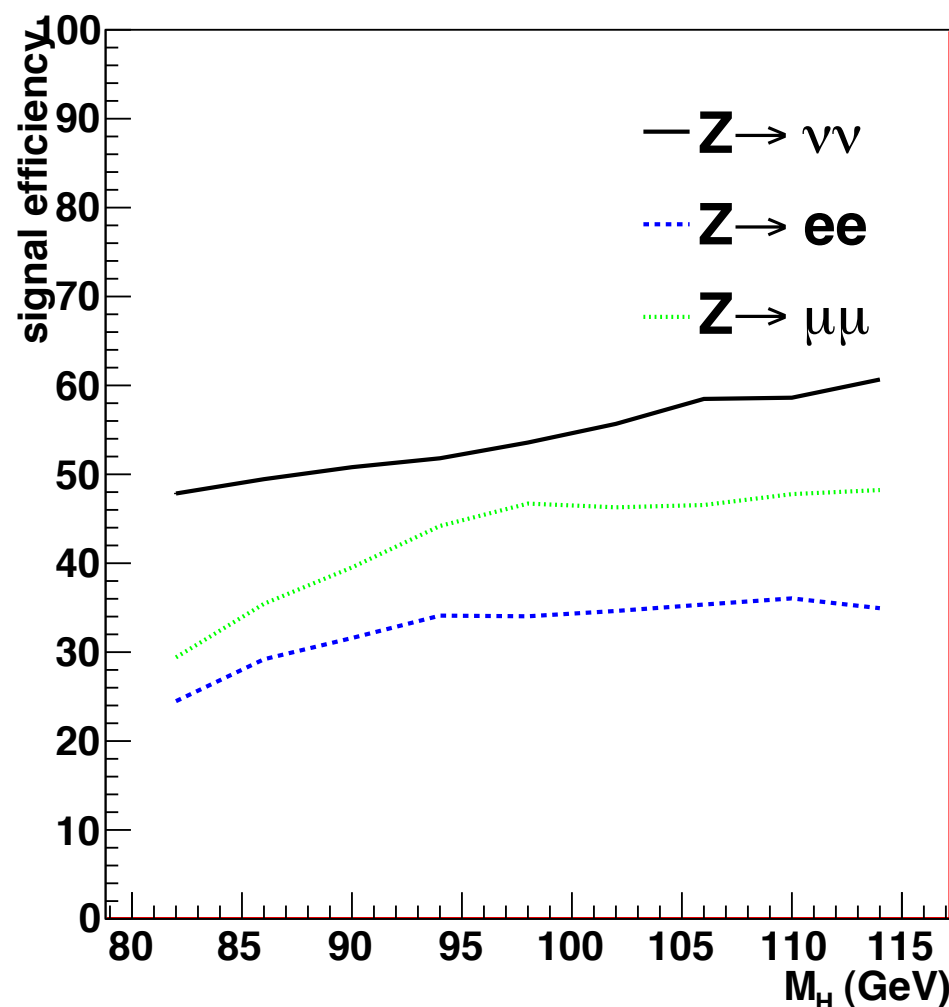
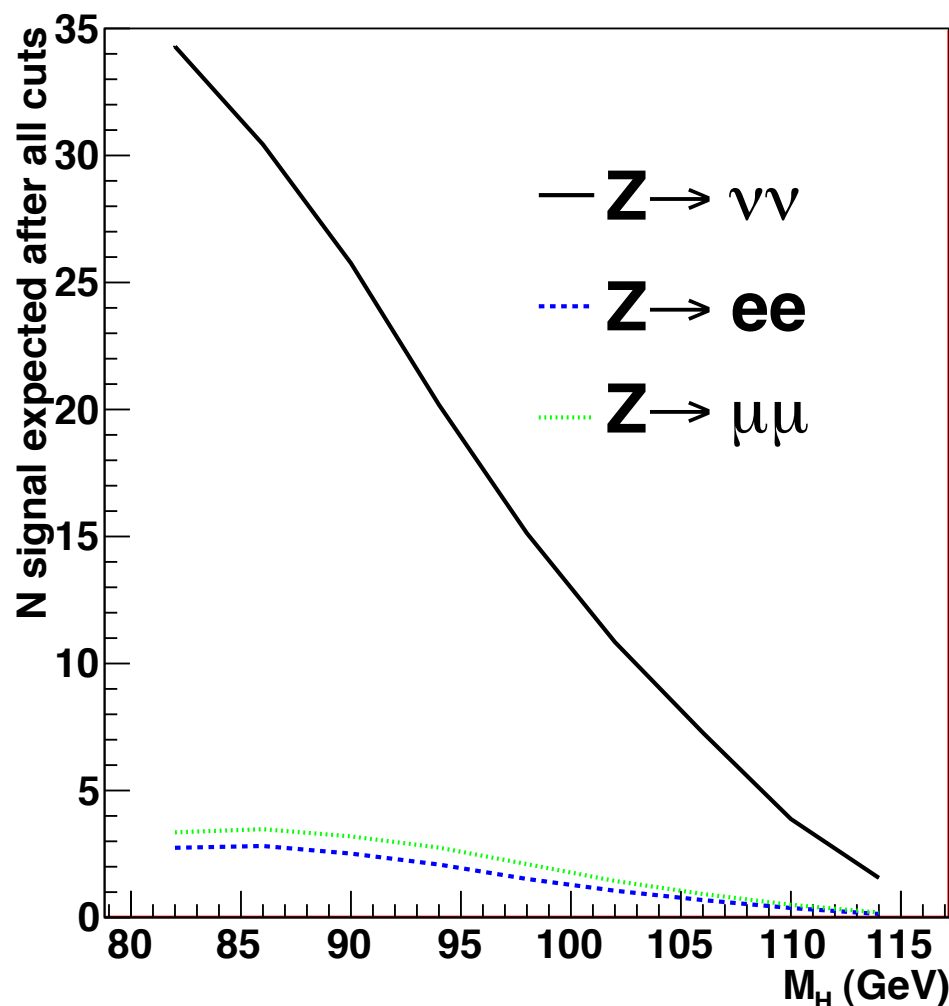
Total bg	0.52
1ph	0.00
2ph-Gss	0.00
4f	0.05
Bhabha	0.23
2ph-Gtt	0.00
PZZ	0.04
KZZ	0.07
2ph-Gbb	0.00
2ph-Gud	0.00
PZe	0.01
2ph-Gcc	0.00
mu+mu-	0.00
Znn	0.00
2ph-Gee	0.00
qqbar	0.07
2ph-Gmm	0.00
tau+tau-	0.05
multiph	0.00

Reconstructed  $M_H$  in  $Z \rightarrow \mu\mu$  channel



Our signal efficiency is pretty good, but clearly we have very few events in lepton channels

- but we also have almost no background in lepton channels

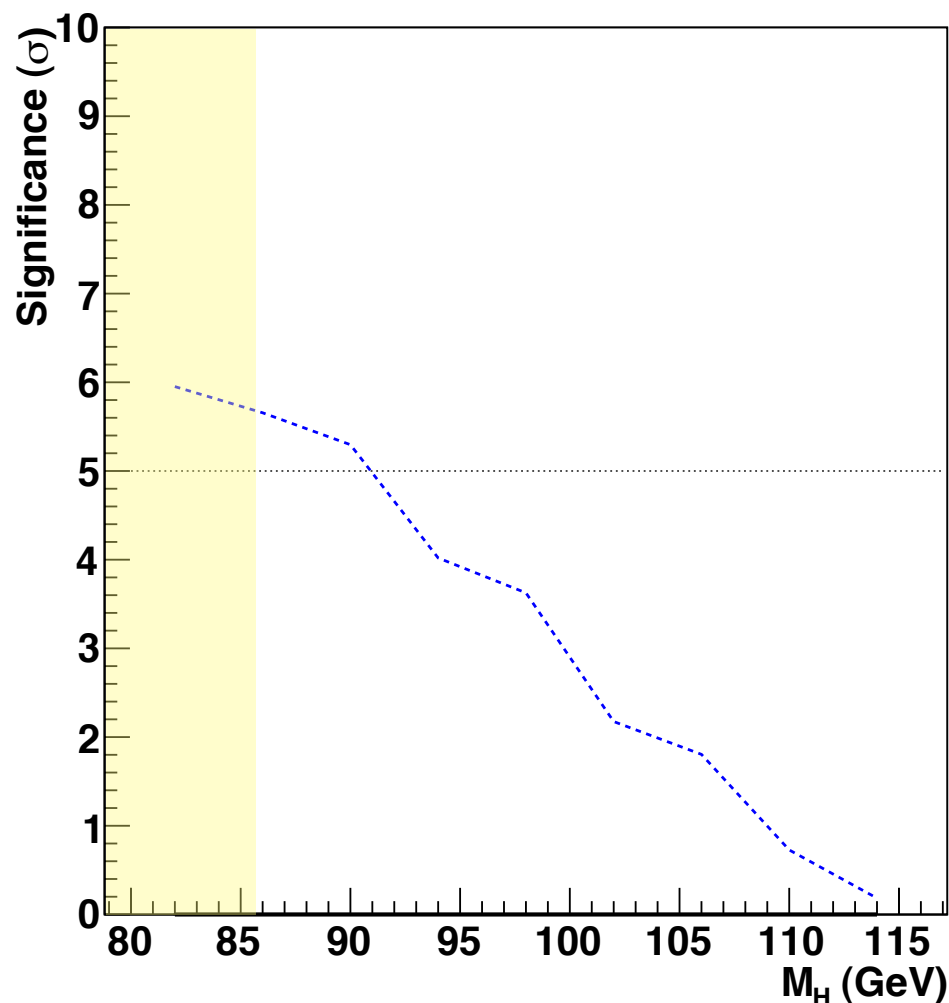


# Expected significance @ $m_a = 4, 10 \text{ GeV}$

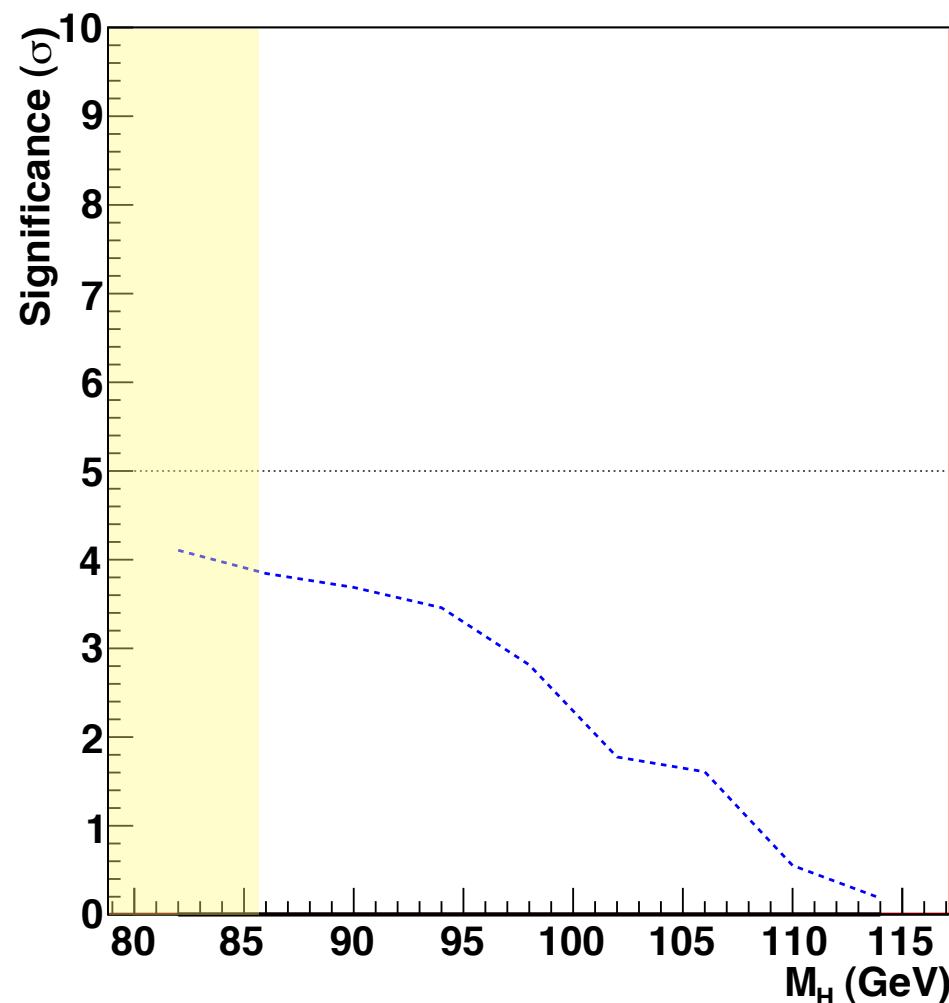
The final results are being considered as an ALEPH publication, so unfortunately I can't show them to you, but here are the expected limits

- ALEPH has it's 20th anniversary on Tuesday, results will be presented then, hopefully published soon after

expected discovery significance for  $m_a = 4 \text{ GeV}$



expected discovery significance for  $m_a = 10 \text{ GeV}$



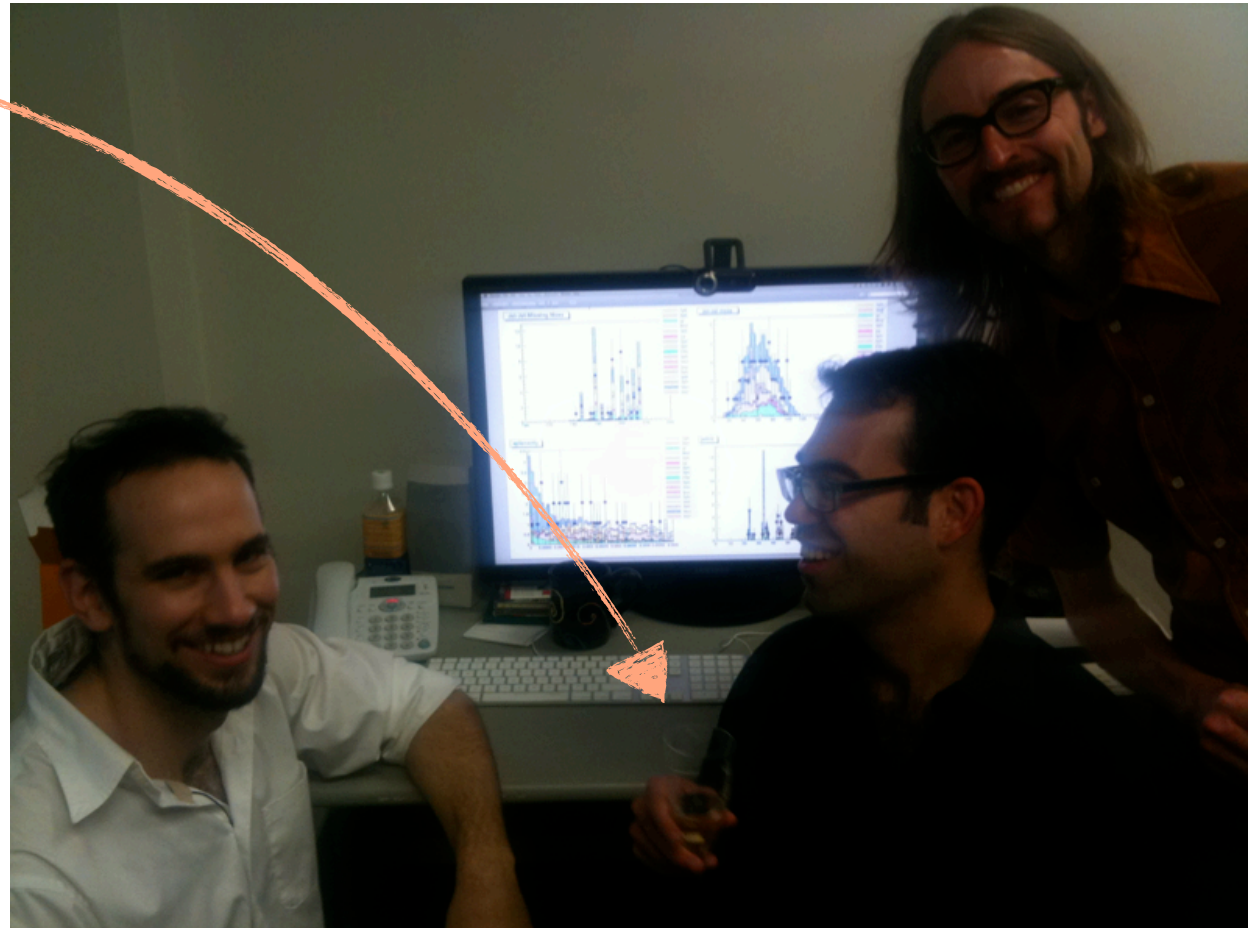
# “Unboxing” celebration

For what it's worth: Our goal was not to just set a limit... certainly not a mediocre one. We saw we had discovery sensitivity early on, so we really went for a discovery.

- since the analysis was blind, we really didn't know

Champaign

(to be consumed regardless of result)

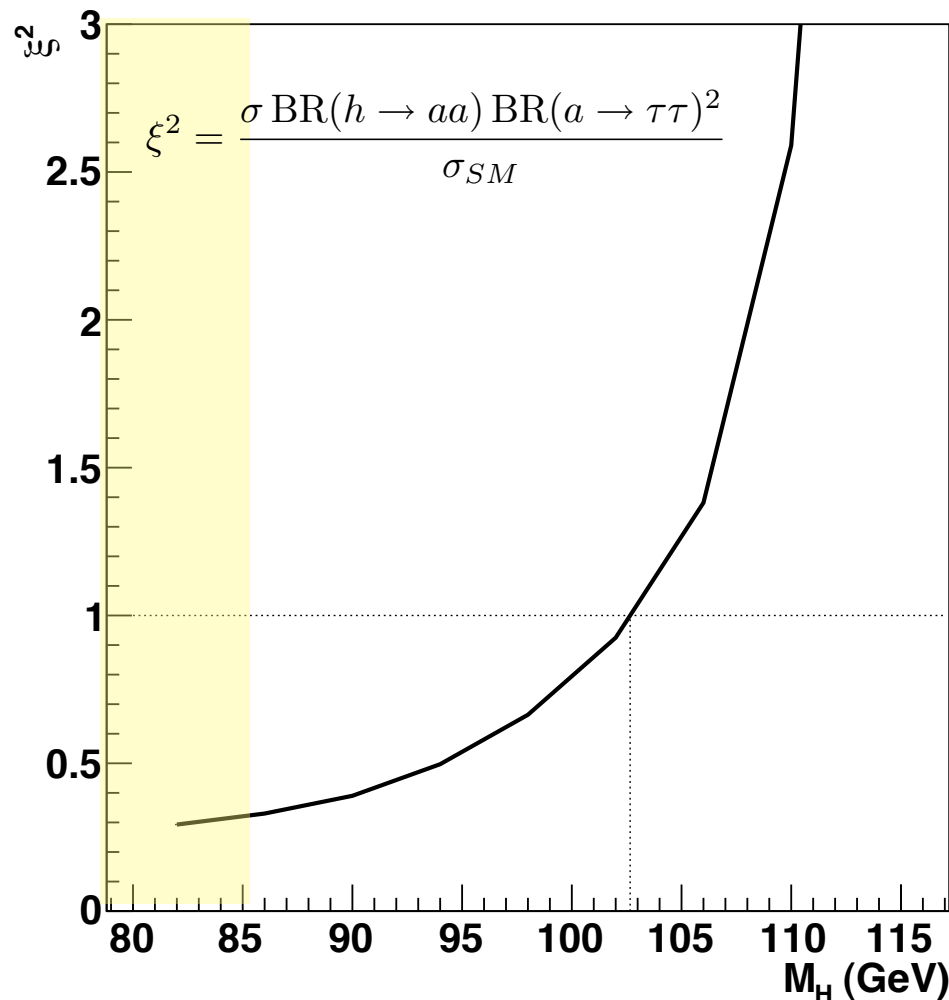


# Expected limits @ $m_a = 4, 10$ GeV

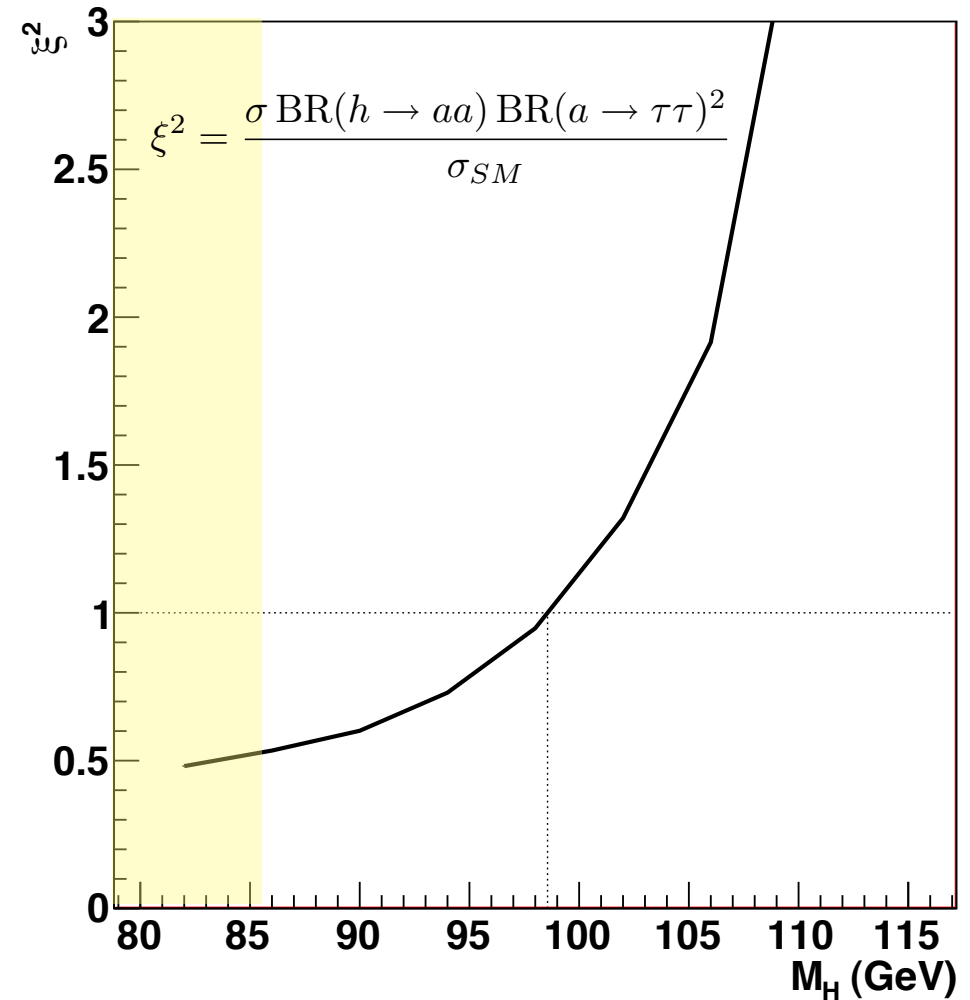
The final results are being considered as an ALEPH publication, so unfortunately I can't show them to you, but here are the expected limits

- ALEPH has it's 20th anniversary on Tuesday, results will be presented then, hopefully published soon after

expected limit for  $m_a = 4$  GeV



expected limit for  $m_a = 10$  GeV





After quite a bit of struggling, we have resurrected the ALEPH analysis engine (including the ability to produce Monte Carlo signal and simulate events in the ALEPH detector)

Our first analysis of  $e^+e^- \rightarrow Zh \rightarrow (ee, \mu\mu, \nu\nu) 4\tau$  is essentially complete, and will extend the reach of the OPAL analysis

- we have sensitivity for a  $5\sigma$  discovery up to  $\sim 90$  GeV
- expected limits ( $\xi^2 = 1$ ) are 99–103 GeV depending on  $m_a$

We plan to continue to look at other exotic decays, and your input is welcome (though we have finite time)

- I'd like to thank Itay and James again for helping this project gain critical mass
- Hopefully there will be a new ALEPH paper soon!

# Summary of similar LEP searches

$e^+e^- \rightarrow \mathcal{H}_2 Z \rightarrow (\mathcal{H}_1 \mathcal{H}_1) Z \rightarrow (\dots)(\dots)$			$m_{\mathcal{H}_2}$	$m_{\mathcal{H}_1}$	
(any)( $q\bar{q}$ )	91	16.2	12 – 70	< 0.21	[46]
( $V^0 V^0$ )(any but $\tau^+ \tau^-$ )	91	9.7	0.5 – 55	< 0.21	[46]
( $\gamma\gamma$ )(any)	91	12.5	0.5 – 60	< 0.21	[46]
(4 prongs)(any)	91	12.9	0.5 – 60	0.21 – 10	[46]
(hadrons)( $\nu\bar{\nu}$ )	91	15.1	1 – 60	0.21 – 30	[46]
( $\tau^+ \tau^- \tau^+ \tau^-$ )( $\nu\bar{\nu}$ )	91	15.1	9 – 73	3.5 – 12	[46]
(any)( $q\bar{q}, \nu\bar{\nu}$ )	161,172	20.0	40 – 70	20 – 35	[40]
( $b\bar{b}b\bar{b}$ )( $q\bar{q}$ )	183	54.0	45 – 85	12 – 40	[41]
( $b\bar{b}b\bar{b}, b\bar{b}c\bar{c}, c\bar{c}c\bar{c}$ )( $q\bar{q}$ )	192-208	452.4	30 – 105	12 – 50	[43, 44]
( $c\bar{c}c\bar{c}$ )( $q\bar{q}$ )	192-208	452.4	10 – 105	4 – 12	[47]
( $\mathcal{H}_1 \rightarrow b\bar{b}, c\bar{c}, g\bar{g}$ )( $q\bar{q}$ )	189 – 209	626.9	30 – 85	10 – 42	[56]
( $q\bar{q}q\bar{q}$ )( $\nu\bar{\nu}$ )	91	46.3	10 – 75	0 – 35	[64, 65]
( $b\bar{b}b\bar{b}$ )( $q\bar{q}$ )	183	54.1	40 – 80	10.5 – 38	[61]
( $b\bar{b}b\bar{b}$ )( $q\bar{q}$ )	189	172.1	40 – 100	10.5 – 48	[62]
( $b\bar{b}b\bar{b}$ )( $q\bar{q}$ )	192-209	421.2	80 – 120	12 – $m_{\mathcal{H}_2}/2$	[10]
( $b\bar{b}b\bar{b}$ )( $\nu\bar{\nu}$ )	183	53.9	50 – 95	10.5 – $m_{\mathcal{H}_2}/2$	[61]
( $q\bar{q}q\bar{q}$ )( $\nu\bar{\nu}$ )	189	171.4	50 – 100	10.5 – $m_{\mathcal{H}_2}/2$	[62]
( $b\bar{b}b\bar{b}$ )( $\nu\bar{\nu}$ )	199-209	207.2	100 – 110	12 – $m_{\mathcal{H}_2}/2$	[10]
( $b\bar{b}b\bar{b}$ )( $\tau^+ \tau^-$ )	183	53.7	30 – 100	10.5 – $m_{\mathcal{H}_2}/2$	[61]
( $b\bar{b}b\bar{b}$ )( $\tau^+ \tau^-$ )	189	168.7	30 – 100	10.5 – $m_{\mathcal{H}_2}/2$	[62]
( $b\bar{b}b\bar{b}, b\bar{b}\tau^+\tau^-, \tau^+\tau^-\tau^+\tau^-$ )( $\nu\bar{\nu}, e^+e^-, \mu^+\mu^-$ )	189-209	598.5	45 – 90	2 – 10.5	[68]

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[68] OPAL Collaboration, G. Abbiendi et al., Eur. Phys. J. **C27** (2003) 483.

$m_{\mathcal{H}_2}(\text{GeV}/c^2)$	$m_{\mathcal{H}_1}(\text{GeV}/c^2)$									
	10	15	20	25	30	35	40	45	50	55
20	0.020									
25	0.026									
30	0.037	0.046								
35	0.048	0.042								
40	0.053	0.056	0.051							
45	0.066	0.059	0.046							
50	0.087	0.058	0.048	0.049						
55	0.11	0.055	0.050	0.050						
60	0.29	0.103	0.094	0.094	0.053					
65	0.30	0.099	0.091	0.088	0.084					
70	0.25	0.098	0.097	0.095	0.083	0.059				
75	0.34	0.11	0.10	0.11	0.10	0.096				
80	0.39	0.13	0.14	0.14	0.13	0.12	0.13			
85	0.52	0.20	0.20	0.20	0.21	0.19	0.18			
90	$\geq 1$	0.23	0.23	0.23	0.27	0.26	0.24	0.28		
95	$\geq 1$	0.29	0.27	0.29	0.31	0.29	0.28	0.30		
100	$\geq 1$	0.30	0.29	0.31	0.30	0.27	0.28	0.29	0.29	
105	$\geq 1$	0.27	0.32	0.36	0.40	0.36	0.31	0.35	0.35	
110	$\geq 1$	0.44	0.54	0.55	0.96	0.97	$\geq 1$	$\geq 1$	0.89	$\geq 1$

Table 15: The 95% CL upper bound,  $S_{95}$ , obtained for the normalised cross-section (see text) of the Higgsstrahlung cascade process  $e^+e^- \rightarrow (\mathcal{H}_2 \rightarrow \mathcal{H}_1\mathcal{H}_1)Z \rightarrow (b\bar{b}b\bar{b})Z$ , as a function of the Higgs boson masses  $m_{\mathcal{H}_1}$  and  $m_{\mathcal{H}_2}$ . The numbers correspond to the contours shown in Figure 3 (a).



	5	10	15	20	25	30	35	40	45
10	0.26								
15	0.033								
20	0.048	0.32							
25	0.070	0.076							
30	0.10	0.11	0.38						
35	0.18	0.19	0.51						
40	0.22	0.22	0.40	0.39					
45	0.30	0.31	0.49	0.49					
50	0.18	0.38	0.66	0.66	0.63				
55	0.18	0.37	0.68	0.69	0.68				
60	0.20	0.38	0.95	0.96	0.96	0.94			
65	0.20	0.38	$\geq 1$	$\geq 1$	$\geq 1$	$\geq 1$			
70	0.21	0.43	$\geq 1$	$\geq 1$	$\geq 1$	$\geq 1$	$\geq 1$		
75	0.19	0.46	$\geq 1$	$\geq 1$	$\geq 1$	$\geq 1$	$\geq 1$	$\geq 1$	
80	0.20	0.44	0.83	0.83	0.83	0.83	0.84	0.84	
85	0.25	0.56	$\geq 1$	$\geq 1$	$\geq 1$	$\geq 1$	$\geq 1$	$\geq 1$	$\geq 1$

Table 16: The 95% CL upper bound,  $S_{95}$ , obtained for the normalised cross-section (see text) of the Higgsstrahlung cascade process  $e^+e^- \rightarrow (\mathcal{H}_2 \rightarrow \mathcal{H}_1\mathcal{H}_1)Z \rightarrow (\tau^+\tau^-\tau^+\tau^-)Z$ , as a function of the Higgs boson masses  $m_{\mathcal{H}_1}$  and  $m_{\mathcal{H}_2}$ . The numbers correspond to the contours shown in Figure 3 (b).



Benchmark parameters						
	(1)	(2)	(3)	(4)	(5)	(6)
	$m_h$ -max	no-mixing	large- $\mu$	gluophobic	small- $\alpha_{eff}$	CPX
Parameters varied in the scan						
$\tan \beta$	0.4–40	0.4–40	0.7–50	0.4–40	0.4–40	0.6–40
$m_A$ (GeV/ $c^2$ )	0.1–1000	0.1–1000	0.1–400	0.1–1000	0.1–1000	–
$m_{H^\pm}$ (GeV/ $c^2$ )	–	–	–	–	–	4–1000
Fixed parameters						
$M_{SUSY}$ (GeV)	1000	1000	400	350	800	500
$M_2$ (GeV)	200	200	400	300	500	200
$\mu$ (GeV)	–200	–200	1000	300	2000	2000
$m_{\tilde{g}}$ (GeV/ $c^2$ )	800	800	200	500	500	1000
$X_t$ (GeV)	$2 M_{SUSY}$	0	–300	–750	–1100	$A - \mu \cot \beta$
$A$ (GeV)	$X_t + \mu \cot \beta$	$X_t + \mu \cot \beta$	$X_t + \mu \cot \beta$	$X_t + \mu \cot \beta$	$X_t + \mu \cot \beta$	1000
$\arg(A) = \arg(m_{\tilde{g}})$	–	–	–	–	–	90°

