Gluon determination from $F_2$ and $F_2^c$

H. Jung (DESY)

- Towards precision determination of uPDFs ....
- why unintegrated parton density functions (uPDFs) ?

- Determination of uPDFs using $F_2$, $F_2^c$

- Is it all consistent ?

- What tells collinear approach ?
Need for uPDFs: transverse momenta

heavy quarks at HERA

heavy quarks at pp

Higgs at pp

NLO corrections will be very large for these LO processes .....
Need for uPDFs: transverse momenta


heavy quarks at HERA

heavy quarks at pp

Higgs at pp

⇒ doing kinematics correct at LO, reduces NLO corrections ... NEED uPDFs !!!!
Applications: beauty at HERA and LHC

Cross sections at parton level in central region

MNR (massive NLO) – FO NLO (matched NLL) – CASCADE (uPDF)

“Perfect” agreement of NLO(FMNR) calculation with CASCADE using uPDFs !!!
Evolution of uPDFs and x-section

- unintegrated PDFs (uPDFs): keep full $k_t$ dependence during perturbative evolution

  - using Dokshitzer, Gribov, Lipatov, Altarelli, Parisi, Balitski, Fadin, Kuraev, Lipatov or Ciafaloni, Catani, Fiorani, Marchesini evolution equations

  - CCFM treats explicitly real gluon emissions

    - according to color coherence ... angular ordering

    - angular ordering includes DGLAP and BFKL as limits...

- $k_t$ dependence in PDFs: from collinear to $k_t$ factorization

- cross section (in $k_t$ factorization):

  \[
  \frac{d\sigma^{jet}s}{dE_T d\eta} = \sum \int \int \int dx_g \ dQ^2 \ d \ldots \ [dk^2 x_g A_i(x_g, k^2, \bar{q})] \hat{\sigma}_i(x_g, k^2)\]

  - can be reduced to the collinear limit:

  \[
  \frac{d\sigma^{jet}s}{dE_T d\eta} = \sum \int \int \int dx \ dQ^2 \ d \ldots x f_i(x, Q^2) \hat{\sigma}_i(x, Q^2, \ldots)
  \]
only gluons densities are considered here !!!
evolve with CCFM using

\[ \times \] full gluon splitting function and \( \alpha_s(M_Z) = 0.118 \)
\[ \times \] starting scale for evolution \( Q_0 = 1.2 \text{ GeV} \)

**Fitting uPDFs:**

- using **FitPDF** (E. Perez [Saclay])
  - applicable also for collinear DGLAP evolution
  - allowing different treatment of correlated systematic uncertainties

- **uPDF** is a convolution of starting distribution \( A_0(x_0) \) with perturbative evolution:

\[
x A(x, k_{\perp}, \bar{q}) = \int dx_0 A_0(x_0) \cdot \frac{x}{x_0} \tilde{A} \left( \frac{x}{x_0}, k_{\perp}, \bar{q} \right)
\]

- Calculate x-section for \( x, Q^2 \) for inclusive quantities

  - Optionally use full event simulation including parton showering and hadronization of **CASCADE MC generator** for final state predictions

- optimize parameters in starting distribution \( A_0(x_0) \) with \( \chi^2 \)

- **General procedure**, applicable also for DGLAP fits
Fit to $F_2$ data

- $\chi^2 = \sum_i \left( \frac{(T - D)^2}{\sigma_{i \text{stat}}^2 + \sigma_{i \text{uncorr}}^2} \right)$

- fit parameters of starting distribution

- $x g(x, \mu_0^2) = N x^{-B_g} \cdot (1 - x)^4$

- using $F_2$ data H1
  

  $x < 0.05 \quad Q^2 > 5 \text{ GeV}^2$

- parameters: $\mu_r^2 = p_t^2 + m_{q, Q}^2$

  $m_q = 250 \text{ MeV}, m_c = 1.5 \text{ GeV}$

- Fit (only stat+uncorr):

  $\frac{\chi^2}{\text{ndf}} = \frac{111.8}{61} = 1.83$

  $B_g = 0.018 \pm 0.003$

  ➔ similar to DGLAP fits (~1.5)
Check sensitivity to parameterization

\[ xg(x, \mu_0^2) = N x^{-B_g} \cdot (1 - x)^{C_g} (1 - D_g x) \]

Strong sensitivity to small x part \( B_g \)

Clear preference for large x parameters.

\[ \Rightarrow \text{keep } C_g \text{ and } D_g \text{ fixed in fit} \ldots \]
Check sensitivity to alphas

sensitive to:

\[ \alpha_s(\mu) \cdot x A(x, k_\perp, q) \]

here use (1-loop):

\[ \alpha_s(\mu) \sim \frac{1}{\log \frac{\mu}{\Lambda_{qcd}}} \]

\[ \Lambda_{qcd} \sim 0.13 \quad \text{gives:} \]

\[ \alpha_s(M_Z) = 0.118 \]
Fit to $F_2^c$ data

$\chi^2 = \sum_i \left( \frac{(T - D)^2}{\sigma_{stat}^2 + \sigma_{syst}^2} \right)$

- fit parameters of starting distribution

$x g(x, \mu_0^2) = N x^{-B_g} \cdot (1 - x)^4$

- using $F_2^c$ data H1


$Q^2 > 1 \text{ GeV}^2$

- fit result: $\frac{\chi^2}{\text{ndf}} = \frac{18.8}{20} = 0.94$

- with $B_g = 0.286 \pm 0.002$
- higher than for $F_2$ !?!?!?!?
- compare to $\frac{\chi^2}{\text{ndf}} = \frac{190.4}{50} = 3.81$

for gluon from $F_2$ fit
Fit to $F_2^c$ data

\[ \chi^2 = \sum_i \left( \frac{(T - D)^2}{\sigma^2_{i \text{ stat}} + \sigma^2_{i \text{ syst}}} \right) \]

fit parameters of starting distribution

\[ xg(x, \mu_0^2) = N x^{-B_g} \cdot (1 - x)^4 \]

using $F_2^c$ data H1


$Q^2 > 1 \text{ GeV}^2$

fit result: $\frac{\chi^2}{\text{ndf}} = \frac{18.8}{20} = 0.94$

with $B_g = 0.286 \pm 0.002$

uncertainty obtained with CTEQ (eigenvector) method, using

$\Delta \chi^2 = 1$ but, CTEQ uses tolerance $T^2 = 100$ for global fits....
**Fit to $F_2^c$ data**

- $\chi^2 = \sum_i \left( \frac{(T - D)^2}{\sigma_{\text{stat}}^2 + \sigma_{\text{syst}}^2} \right)$

- fit parameters of starting distribution

$$xg(x, \mu_0^2) = N x^{-B_g} \cdot (1 - x)^4$$

- using $F_2^c$ data H1


- $Q^2 > 1 \text{ GeV}^2$

- fit result: $\frac{\chi^2}{\text{ndf}} = \frac{18.8}{20} = 0.94$

  - with $B_g = 0.286 \pm 0.002$

  - higher than for $F_2$ !?!?!?!

  - significant change of uPDF
Fit to $F_2^c$ data: uPDF

- $\chi^2 = \sum_i \left( \frac{(T-D)^2}{\sigma^2_{i\,\text{stat}} + \sigma^2_{i\,\text{uncor}}} \right)$

- fit parameters of starting distribution
  
  $$xg(x, \mu_0^2) = N x^{-B_g} \cdot (1 - x)^4$$

- using $F_2^c$ data H1


  $$Q^2 > 1 \text{ GeV}^2$$

- fit result: $\frac{\chi^2}{\text{ndf}} = \frac{18.8}{20} = 0.94$

  with $B_g = 0.286 \pm 0.002$

  $\Rightarrow$ higher than for $F_2$ !?!?!?!
Calculating $F_L$: sensitive to gluon

$\sigma_L(\gamma g \rightarrow q\bar{q}) \rightarrow F_L$

- calculate contribution to $F_L$ in $k_t$-factorization
- similar level of agreement for CCFM uPDF as obtained in collinear factorization with best parametrization

A. Kotikov, A. Lipatov, N. Zotov

$W = 276$ GeV

$F_L(Q^2)$ vs $Q^2 (GeV^2)$
Geometric scaling: $F_2$ and $F_2^c$

- do we expect geometric scaling also for $F_2^c$?
Geometric scaling: $F_2$ and $F_2^c$

- do we expect geometric scaling also for $F_2^c$?

  ➔ even not when changing scale to $Q^2 + m_c^2$?
Fit of intrinsic $k_t$ distribution

- Fit parameters of intrinsic $k_t$ distribution
  \[ \sim \exp\left(-\frac{(\mathbf{k}_\perp - \mathbf{k}_\perp^0)^2}{\sigma^2}\right) \]

- Fit results from $F_2$ fit
  \[ \mathbf{k}_\perp^0 \sim 0.8 \]
  \[ \sigma \sim 0.5 \]

- Small change in $\chi^2$
- Essentially no sensitivity from $F_2^c$
$k_t$ in $F_2$

- investigate $k_t$ from Monte Carlo using uPDF

$F_2$ dominated by small $kt$ ..... large size
$k_t$ in $F_2$ and $F_2^c$

- investigate $k_t$ from Monte Carlo using uPDF

- $F_2^c$ ..... has larger $k_t$ (~1 - 2 GeV) ..... smaller size dipole .....
Gluon from \( F_2 \) and \( F_2^c \)

- investigations:
  - suppress small \( k_t \) region for \( F_2 \) with different ansaetze:
    - linear suppression
    - GBW \( k_t \) distribution
  - no significant change in steepness of gluon
  - fit parameters for GBW agree with original formulation
  - fit parameters for gaussian \( k_t \) are reasonable ....
  - what makes \( F_2^c \) so different from \( F_2 \)?
What tells collinear factorization?

- Are similar effects seen in collinear factorization using DGLAP?

- How to constrain parton densities:
  - in collinear factorization is much more tricky ...
  - How to tell the difference of gluon and sea?
  - How to find appropriate parameterization?
Fit to $F_2^c$ in collinear approach

- $F_2^c$ from $F_2$ fit

- $F_2^c$ from $F_2^c$ fit
Significantly steeper gluon required from $F_2^c$
using FitPDF from E. Perez
compare gluon from $F_2$ (H1 published) and $F_2^c$ fits.....

- gluon comes out very different... consistent picture?
  ➔ fix gluon with $F_2^c$
  ➔ fit quarks with $F_2$
  ➔ consistent fit?
F_{2c} in NLO, NNLO

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R. Thorne, hep-ph/0511351

Gluon LO, NLO and NNLO

Hannes Jung, GGI – high density workshop, Firenze 2007
Conclusion

- Full treatment of kinematics in calculations is necessary – **NEED uPDFs**
  - NLO corrections are MUCH smaller then ....
    - **uPDFs** are needed for precision calculations at LHC:
      - see heavy quarks, Higgs etc ...
  - first **real** uPDF fits to data from HERA presented !!!!

- \( F_2 \) data suggest flat gluon distribution

- \( F_2^c \) data suggest steeply rising gluon distribution
  - very different from \( F_2 \) gluon !!!!
    - even for collinear case !!!
    - \( \Rightarrow \) do we see new effects, saturation etc ?

\( \Rightarrow \) **Heavy quark measurements could be the tool for saturation ... !!!!!!!! ????????????????**
Backup slides
Fit to $F_{2}^{c}$ data: checking results

- Check sensitivity to parameterization

$$xg(x, \mu_{0}^{2}) = N x^{-B_{g}} \cdot (1 - x)^{C_{g}} (1 - D_{g} x)$$

- Strong sensitivity to small x rise
- Sensitivity to large x parts...
**Fit to $F_2$ data**

\[ \chi^2 = \sum_i \left( \frac{(T - D)^2}{\sigma_{i \text{ stat}}^2 + \sigma_{i \text{ uncor}}^2} \right) \]

- fit parameters of starting distribution
  \[ x g(x, \mu_0^2) = N x^{-B_g} \cdot (1 - x)^4 \]

- using $F_2$ data H1

\[ x < 0.01 \quad Q^2 > 5 \text{ GeV}^2 \]

- Fit (only stat+uncorr):
  \[ B_g = 0.018 \pm 0.003 \text{ from } F_2 \]
  \[ B_g = 0.286 \pm 0.002 \text{ from } F_2^c \]
**$F_2$ with GLR effects (EHKQS)**

- use $F_2$ HERA data
- fit with DGLAP+GLRMQ

K. Eskola, H. Honkanen, V. Kolhinen, L. Qiu, C. Salgado


\[
\frac{dx g}{d \log Q^2} \sim \left| \frac{dx g}{d \log Q^2} \right|_{DGLAP} - \frac{1}{R^2} K \otimes [xg]^2
\]

\[Q_0^2 = 1.4 \text{ GeV}^2\]
Advantage of uPDFs

\[ A(x, k^\perp, \bar{q}) = A_0(x, k^\perp) \Delta_s(\bar{q}, Q_0) + \int \frac{dz}{z} \frac{d^2q}{q^2} \Delta_s(\bar{q}, zq) \tilde{P}(z, \ldots) A \left( \frac{x}{z}, k'^\perp, q \right) \]

Advantage of uPDF:
- possibility to separate no-branching piece from resolvable branching contributions

What is actually fitted?
- Fit adjusts mainly the no-branching contribution
- large kt region fixed by evolution
- influence on final state predictions

Hannes Jung, GGI – high density workshop, Firenze 2007
Application to $D^*$ + jets in $\gamma p$

- Geros analysis

\[ d\sigma_{ep} \to D^* + \gamma + X \ (\text{nb/GeV}) \]

\[ p_t(D^*) \ (\text{GeV}) \]

\[ d\sigma_{ep} \to D^* + X + \gamma \ (\text{nb}) \]

\[ \eta(D^*) \]

\[ d\sigma_{ep} \to D^* + \gamma + X \ (\text{nb}) \]

\[ 2 \lesssim p_t \lesssim 3 \]

\[ 3 \lesssim p_t \lesssim 4.5 \]

\[ 4.5 \lesssim p_t \lesssim 8.5 \]
chi**2 for inclusive D* (pt and eta)

$F_2$ gluon:

$\chi^2 = 22.32/13 = 1.72$

new $F_2^c$ gluon:

$\chi^2 = 24.33/13 = 1.87$

no big effect from new fit

although gluon is very different

$\gamma p \rightarrow D^* + X$ probes different $xg$ range: $<x_g> = -2.5$ and $<k_T> = 3.7$ compared to $F2c$

is still consistent

use sensitivity to $k_T$ from jets
F2c in dipole model:

- K.Golec-Biernat and S.Sapeta
  hep-ph/0607276

Figure 4: Predictions for the charm structure function \( F_2^{c \bar{c}} \) in the BGK model with heavy quarks (solid lines). The predictions in the GBW model [1] are shown for reference (dashed lines).
F2c in dipole model:

H. Kowalski and D. Teaney
hep-ph/0304189

**FIG. 11:** A comparison of the measured $F_2^c$ [32] to the results of the model.

The dipole model makes a direct prediction for the inclusive charm contribution to $F_2$. In the dipole approach the charm quark distribution is calculated from the gluon distribution. Figure 11 shows a comparison between the measured and predicted values of the charm structure function $F_2^c$. The results depend weakly on the charm mass. The good agreement with data for both ZEUS and H1 experiments [32, 33] confirms the consistency of the model and supports the dipole picture.