Radiative and rare semileptonic $B$ decays
(news 2009/2010)

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# 1. New, more precise determination of $B(B \rightarrow X_s l^+l^-)$ by Belle. Slide from T. Ijima at Lepton-Photon 2009:

For entire MXs region

$Br(B \rightarrow X_s ee) = (4.56 \pm 1.15^{+0.33}_{-0.40}) \times 10^{-6}$

$Br(B \rightarrow X_s \mu\mu) = (1.91 \pm 1.02^{+0.16}_{-0.18}) \times 10^{-6}$

$Br(B \rightarrow X_s \ell\ell) = (3.33 \pm 0.80^{+0.19}_{-0.24}) \times 10^{-6}$

Note: Measured $Br(M(XS):0.2-2.0 \text{ GeV/c}^2) \times [1.10 \pm 0.002]$ --- based on signal MC
Dilepton mass spectrum in $\bar{B} \rightarrow X_s l^+ l^-$. 

$$m_b \frac{d\mathcal{B}(\bar{B} \rightarrow X_s l^+ l^-)}{dm_{l^+ l^-}} \times 10^5$$

with perturbative $c\bar{c}$ using "naive" factorization

[F. Krüger, L.M. Sehgal
hep-ex/9603237]
New HFAG average (2009): \( \mathcal{B}(X_s \rightarrow l^+l^-) = (3.66^{+0.76}_{-0.77}) \times 10^{-6} \)

\[ \Rightarrow \text{Non-SM sign of } C_7 \text{ is excluded at more than } 4\sigma \]

(as compared to 3\(\sigma\) that we’ve had so far)

provided \(C_{9,10}\) remain unchanged.

\[ \mathcal{L} = \mathcal{L}_{QCD \times QED}(q, l) + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu)O_i \]

\[
O_i = \begin{cases} 
(\bar{s}\Gamma_i c)(\bar{c}\Gamma_i'b), & i = 1, 2, \quad |C_i(m_b)| \sim 1 \\
(\bar{s}\Gamma_i b)\Sigma_q(\bar{q}\Gamma_i'q), & i = 3, 4, 5, 6, \quad |C_i(m_b)| < 0.07 \\
\frac{e m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}, & i = 7, \quad C_7(m_b) \sim -0.3 \\
\frac{g m_b}{16\pi^2} \bar{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a, & i = 8, \quad C_8(m_b) \sim -0.15 \\
\frac{e^2}{16\pi^2} (\bar{s}_L \gamma_{\mu} b_L)(\bar{l}\gamma^\mu \gamma_5 l), & i = 9, 10, \quad |C_i(m_b)| \sim 4
\end{cases}
\]

\[ \text{[P. Gambino, U. Haisch, MM, PRL 94 (2005) 061803]} \]

using \((4.5 \pm 1.0) \times 10^{-6}\).
Inclusive decay rates and the sign of $C_7$

\[
\frac{d\Gamma(\bar{B} \to X_sl^+l^-)}{d\hat{s}} = \frac{G_F^2 m_{b,\text{pole}}^5 |V_{ts}V_{tb}|^2}{48\pi^3} \left(\frac{\alpha_{\text{em}}}{4\pi}\right)^2 (1 - \hat{s})^2 \times \\
\times \left\{ (1 + 2\hat{s}) \left( |C_9^{\text{eff}}(\hat{s})|^2 + |C_{10}^{\text{eff}}(\hat{s})|^2 \right) + \left( 4 + \frac{8}{\hat{s}} \right) |C_7^{\text{eff}}(\hat{s})|^2 + 12 \text{Re} \left( C_7^{\text{eff}}(\hat{s}) C_{10}^{\text{eff}}(\hat{s}) \right) \right\} + R_1,
\]

\[
\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \frac{G_F^2 m_{b,\text{pole}}^5 |V_{ts}V_{tb}|^2}{8\pi^3} \frac{\alpha_{\text{em}}}{4\pi} |C_7^{\text{eff}}(\hat{s} = 0)|^2 + R_2
\]

are conveniently expressed in terms of the so-called effective coefficients

\[C_i^{\text{eff}}(\hat{s}) = C_i(\mu_b) + (\text{loop corrections})(\hat{s}).\]

The quantities $R_i$ stand for small bremsstrahlung contributions and for the non-perturbative corrections.

\[\text{sgn } C_7(\mu_b) = \text{("sign of the } b \to s\gamma \text{ amplitude")}.\]

This sign matters for the $\bar{B} \to X_s l^+ l^-$ rate and (even more) for the forward-backward asymmetry:

\[A_{FB} = \int_{-1}^{1} dy \frac{d^2\Gamma(\bar{B} \to X_sl^+l^-)}{d\hat{s} dy} \text{sgn } y \sim (1 - \hat{s})^2 \text{Re} \left[ C_{10}^{\text{eff}}(\hat{s}) (\hat{s} C_9^{\text{eff}}(\hat{s}) + 2 C_7^{\text{eff}}(\hat{s})) \right] + R_3,
\]

where $y = \cos \theta_l$ and $\theta_l$ is the angle between the momenta of $\bar{B}$ and $l^+$ in the dilepton rest frame. Forward-backward asymmetries for the exclusive $\bar{B} \to K^{(*)} l^+ l^-$ modes are defined analogously.
AD 2005  model-independent constraints on additive new physics contributions to $C_{9,10}$ at 90% C.L.

The three lines correspond to three different values of $B(\bar{B} \to X_s \gamma) \times 10^4$: the experimental central value and borders of the 90% C.L. domain for this branching ratio.

The dot at the origin indicates the SM case for $C_{9,10}$.

The SM values have been assumed for $C_1, \ldots, C_6$ and for $C_8$. New physics in $C_8$ would have little effect provided one accepts the bound $B(b \to \text{charmless})_{NP} = 3.7\%$ @ 95% C.L. [DELPHI, PLB 426 (1998) 193].

In the rightmost plot, the maximal MFV MSSM ranges for $C_{9,\text{NP}}$ and $C_{10,\text{NP}}$ are indicated by the dashed cross. They were obtained in hep-ph/0112300 by A. Ali, E. Lunghi, C. Greub and G. Hiller who scanned over the following parameter ranges:

\[ 2.3 < \tan \beta < 50, \quad 0 < M_2 < 1 \text{ TeV}, \quad -1 \text{ TeV} < \mu < 1 \text{ TeV}, \]
\[ 78.6 \text{ GeV} < M_{H^\pm} < 1 \text{ TeV}, \quad 90 \text{ GeV} < M_{\tilde{t}_{1,2}} < 1 \text{ TeV}, \]
\[ -\frac{\pi}{2} < \theta_{\tilde{t}} < \frac{\pi}{2}, \quad M_{\tilde{\nu}} \geq 50 \text{ GeV}. \]
# 2. Updated forward-backward asymmetries in $B(B \to K^{*}l^{+}l^{-})$.

Slide from T. Ijima at Lepton-Photon 2009:

\[ A_{FB} \text{ extracted from fits to} \]

\[ \frac{3}{4} F_{L}(1 - \cos^{2} \theta_{BL}) + \frac{3}{8}(1 - F_{L})(1 + \cos^{2} \theta_{BL}) + A_{FB} \cos \theta_{BL} \]


384M BB, PRD79, 031102(R) (2009)

$A_{FB}$ exceeds SM?
3. Updated $\mathcal{B}(B \rightarrow X_s \gamma)$ measurement by Belle.


The displayed measurements are only the fully-inclusive, no-hadronic-tag ones. Other methods (included in the HFAG average):

- Semi-inclusive (systematics-limited),
- With hadronic tags of the recoiling $B$ meson (not necessarily fully reconstructed).

Low systematic errors, but statistics-limited at present.
4. Evaluation of $\mathcal{O}(\alpha_s \Lambda^2/m_b^2)$ corrections to $\Gamma_{77}(\bar{B} \rightarrow X_s \gamma)$ and moments of the photon spectrum.


$$\Gamma_{77}|_{E_\gamma>E_0} = \Gamma_{77}^{(0)} \left[ 1 + \frac{\lambda_1-9\lambda_2(\mu)}{2m_b^2} + \frac{\alpha_s(\mu)}{\pi} f_{\text{pert.NLO}}(\delta) + \frac{\alpha_s^2(\mu)}{\pi^2} f_{\text{pert.NNLO}}(\delta) + \frac{\lambda_1\alpha_s(\mu)}{3m_b^2\pi} \left( \frac{3+4\ln\delta}{6\delta^2} + g_1(\delta) \right) + \frac{\lambda_2(\mu)\alpha_s(\mu)}{m_b^2\pi} g_2(\delta) + \ldots \right]$$

$$\delta = 1 - \frac{2E_0}{m_b}$$

$g_{1,2}$ contain $\frac{\ln\delta}{\delta}$, $\frac{1}{\delta}$, $\ln^2\delta$, $\ln\delta$, and non-singular terms.
Clarification of quark-hadron duality issues in \( \bar{B} \to X_s l^+ l^- \)


If the intermediate \( J/\psi \) and \( \psi' \) resonances are included, \( \Gamma(\bar{B} \to X_s l^+ l^-) \) exceeds the perturbative \( \Gamma(b \to X_s l^+ l^-) \) by around two orders of magnitude.

Is the quark-hadron duality violated here?

G.B. 2000: No, because we need to resum Coulomb-like interactions in the \( c\bar{c} \) state.

BBNS 2009: Yes, because we need to resum Coulomb-like interactions in the \( c\bar{c} \) state.

Both answers are satisfactory, because they differ only linguistically, while the physics remains the same.
Technically: Coulomb resummation effects get washed out after smearing over $q^2$ in the correlator (as in $b \rightarrow sc\bar{c}$), but not in the squared correlator (as in $b \rightarrow se^+e^-$).

Pedagogical toy model: consider fictitious leptons (heavy $l_1$ instead of $b$, and massless $l_2$ instead of $s$) to single out bound-state effects in the $c\bar{c}$ system only.

The decays $l_1 \rightarrow l_2c\bar{c}$ and $l_1 \rightarrow l_2e^+e^−$ are described by:

In the case (b), we integrate imaginary part of the correlator $\Pi(q^2)$ of two $c\bar{c}$ currents. In the case (a), we get $|\Pi(q^2)|^2$.

In the acknowledgments, thanks to Tobias Hurth for persistent encouragement.
# 6. Many BSM studies... Let's have a look at the past 2 weeks.

# 6a. G. Degrassi and P. Slavich, arXiv:1002:1071 (Feb 4th)

Evaluation of the NLO QCD corrections to $R_b$ and $b \to s\gamma$ in generic MVF two-Higgs-doublet models.

$$\mathcal{L}_{H^+} = -\frac{g}{\sqrt{2m_W}} \sum_{i,j=1}^{3} \bar{u}_i T^{(a)}_R \left( A^{i}_{u} m^{i}_{u} \frac{1-\gamma_5}{2} - A^{i}_{d} m^{d}_{j} \frac{1+\gamma_5}{2} \right) V_{ij} d^{j}_{a} H^{+}_{(a)} + h.c.$$ 

Question: Do the two-loop $b \to s\gamma$ matching results agree analytically with those from hep-ph/9904413 (C. Bobeth, J. Urban, MM)?
### 6b. Fourth generation (congratulations to George Hou!)


Scans over the SM4 parameter space (Fig. 16 from the latter paper):

**LO $b \to s\gamma$ matching for 4th gen.**

Would the left plot remain qualitatively the same for $q^2 \in [1, 6]$ GeV$^2$ and/or with the updated HFAG result for the full $q^2$ range?
To conclude, the following topics have been missed in my list of 2009/2010 news:

- Isospin asymmetries in $B \to K^*\gamma$ and $B \to K^{(*)}l^+l^-$,
- CP asymmetries in those decays,
- Theory upgrades in the full angular analyses of $B \to K^*l^+l^-$,
- Many other new BSM studies, some of them even more recent.

(see e.g. arXiv:1002.2758 (Feb 14th), Q. Chang, X.-Q. Li, Y.-D. Yang,
“$B \to K^*l^+l^-$, $Kl^+l^-$ decays in a family non-universal $Z'$ model.”)

- ....
Energetic photon production in charmless decays of the $\bar{B}$-meson
$(E_\gamma \gtrsim \frac{m_b}{3} \simeq 1.6 \text{ GeV})$
[see MM, arXiv:0911.1651]

A. Without long-distance charm loops:

1. Hard
   ![Diag1](image1.png)
   Dominant, well-controlled.

2. Conversion
   ![Diag2](image2.png)
   $\mathcal{O}(\alpha_s \Lambda/m_b)$, $(-1.5 \pm 1.5)\%$.
   [Lee, Neubert, Paz, 2006]

3. Collinear
   ![Diag3](image3.png)
   Pert. $< 1\%$, nonp. $\sim -0.2\%$.
   [Kapustin,Ligeti,Politzer, 1995]

4. Annihilation
   $(q\bar{q} \neq c\bar{c})$
   ![Diag4](image4.png)
   Exp. $\pi^0, \eta, \eta', \omega$ subtracted.
   Perturbatively $\sim 0.1\%$.

B. With long-distance charm loops:

5. Soft
   ![Diag5](image5.png)
   $\mathcal{O}(\Lambda^2/m_c^2)$, $\sim +3.1\%$.
   [Voloshin, 1996], [...],

6. Boosted light $c\bar{c}$
   ![Diag6](image6.png)
   state annihilation (e.g. $\eta_c, J/\psi, \psi'$)
   Exp. $J/\psi$ subtracted ($< 1\%$).
   Perturbatively (including hard): $\sim +3.6\%$.
   $\phi^{(1)}(\delta), \phi^{(2)}(\delta), i,j = 1,2$

7. Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state
   ![Diag7](image7.png)
   $\mathcal{O}(\alpha_s(\Lambda/M)^2)$
   $M \sim 2m_c, 2E_\gamma, m_b$.
   $\mathcal{O}(\alpha_s \Lambda/M)$
Gluon-to-photon conversion in the QCD medium

This is hard gluon scattering on the valence quark or a “sea” quark that produces an energetic photon. The quark that undergoes this Compton-like scattering is assumed to remain soft in the $\bar{B}$-meson rest frame to ensure effective interference with the leading “hard” amplitude. Without interference the contribution would be negligible ($\mathcal{O}(\alpha_s^2\Lambda^2/m_b^2)$).

Suppression by $\Lambda$ can be understood as originating from dilution of the target (size of the $\bar{B}$-meson $\sim \Lambda^{-1}$).

A rough estimate using vacuum insertion approximation gives

$$\frac{\Delta \Gamma}{\Gamma} \in [-3\%, -0.3\%] \quad (\mathcal{O}(\alpha_s \Lambda/m_b)).$$


However:

1. Contribution to the interference from scattering on the ”sea” quarks vanishes in the $SU(3)_{\text{flavour}}$ limit because $Q_u + Q_d + Q_s = 0$.

2. If the valence quark dominates, then the isospin-averaged $\Delta \Gamma/\Gamma$ is given by:

$$\frac{\Delta \Gamma}{\Gamma} \approx \frac{Q_d + Q_u}{Q_d - Q_u} \Delta_{0-} = -\frac{1}{3} \Delta_{0-} = (+0.2 \pm 1.9_{\text{stat}} \pm 0.3_{\text{sys}} \pm 0.8_{\text{ident}}) \%,$$

using the BABAR measurement (hep-ex/0508004) of the isospin asymmetry

$$\Delta_{0-} = \left[ \Gamma(\bar{B}^0 \rightarrow X_s\gamma) - \Gamma(B^- \rightarrow X_s\gamma) \right] / \left[ \Gamma(\bar{B}^0 \rightarrow X_s\gamma) + \Gamma(B^- \rightarrow X_s\gamma) \right],$$

for $E_\gamma > 1.9$ GeV.

Quark-to-photon conversion gives a soft $s$-quark and poorly interferes with the ”hard” $b \rightarrow s\gamma g$ amplitude.
Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state

Heavy $\Leftrightarrow$ Above the $D\bar{D}$ production threshold

Long-distance $\Rightarrow$ Annihilation amplitude is suppressed with respect to the open-charm decay due to the order $\Lambda^{-1}$ distance between $c$ and $\bar{c}$. By analogy to the $B$-meson decay constant $f_B \sim \Lambda(\Lambda/m_b)^{1/2}$, we may expect that the suppression factor scales like $(\Lambda/M)^{3/2}$, where $M \sim 2m_c, 2E_\gamma, m_b$.

Hard gluon $\Leftrightarrow$ Suppression by $\alpha_s$ of the interference with (non-soft)

Altogether: $O(\alpha_s(\Lambda/M)^{3/2})$. To stay on the safe side, assume $O(\alpha_s\Lambda/m_b)$ for numerical error estimates.

This type of amplitude interferes with the leading term but receives an additional $\Lambda/M$ suppression (at least) due to participation of the $s$-quark in the hard annihilation.
The inclusive branching ratio in the SM:

\[ \mathcal{B}(\bar{B} \rightarrow X_s \gamma)_{\text{NNLO}}^{E_\gamma>1.6 \text{ GeV}} = \begin{cases} 
(3.15 \pm 0.23) \times 10^{-4}, & \text{hep-ph/0609232, using the 1S scheme}, \\
(3.26 \pm 0.24) \times 10^{-4}, & \text{following the kin scheme analysis of arXiv:0805.0271, but } \overline{m}_c(\overline{m}_c)^{2\text{loop}} \\
& \text{rather than } \overline{m}_c(\overline{m}_c)^{1\text{loop}}.
\end{cases} \]

Contributions to the total uncertainty:

- **5%** non-perturbative, mainly \( \mathcal{O}(\alpha_s \, \frac{\Lambda}{m_b}) \) \( \rightarrow \) Improved measurements of \( \Delta_0^- \) should help.

- **3%** parametric \( (\alpha_s(M_Z), \mathcal{B}_{\text{semileptonic}}^\text{exp}, m_c & C, \ldots) \)
  - 2.0% 1.6% 1.1% (1S) 2.5% (kin)

- **3%** \( m_c \)-interpolation ambiguity \( \rightarrow \) The calculation of \( G_{17} \) and \( G_{27} \) for \( m_c = 0 \) should help a lot.

- **3%** higher order \( \mathcal{O}(\alpha_s^3) \) \( \rightarrow \) This uncertainty will stay with us.
Missing ingredients in the perturbative NNLO matrix elements

$$\Gamma(b \rightarrow X_{\text{parton}} \gamma)_{E\gamma > E_0} = \frac{G_F^2 m_b^5 \alpha_{\text{em}}}{32\pi^4} |V_{ts}^* V_{tb}|^2 \sum_{i,j=1}^{8} C_i(\mu_b) C_j(\mu_b) G_{ij}(E_0, \mu_b)$$

**LO:** \( G_{ij} = \delta_i^7 \delta_j^7 \)

**NLO:** The most important \( G_{ij} \) \((i, j = 1, 2, 7, 8)\) are known since 1996.

The remaining \( G_{ij} \) are known since 2002.

**NNLO:** Only \( i, j = 1, 2, 7, 8 \) have been considered so far.

Only \( G_{77} \) is fully known:

\[ G_{27}: \]

(and analogous \( G_{17} \))

Two-particle cuts:~
\[ \sim 160 \text{ four-loop master integrals} \ (m_c = 0) \]
recently completed by T. Schutzmeier.

Three- and four-particle cuts:
\[ \text{R. Boughezal,} \quad \text{M. Czakon,} \quad \text{T. Schutzmeier,} \quad \text{in progress...} \]


Diagrams with quark loops on gluon lines for \( m_c \neq 0 \): arXiv:0707.3090.
Two-particle cuts:
finished in 2007
(unpublished)

H.M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub, G. Ossola.

Three- and four-particle cuts:
in progress...

Two-particle cuts
are known (just $\vert$NLO$^2$).

Three- and four-particle cuts
vanish at the endpoint $E_\gamma = m_b/2$.

Analogous NLO corrections are not big ($+3.6\%$).

The current phenomenological analysis at the NNLO relies on using the BLM approximation together
with the large-$m_c$ asymptotics of the non-BLM correction. The latter correction is interpolated
in $m_c$ under the assumption that it vanishes at $m_c = 0$.

Large-$m_c$ asymptotics
of $G_{ij}^{NNLO}$ ($m_c \gg m_b/2$):

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The BLM approximation
for $G_{ij}^{NNLO}$ (arbitrary $m_c$):

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The BLM corrections to $G_{78}$, $G_{88}$ are small.
$G_{18}$ and $G_{28}$ are small at the NLO.

[MM, Steinhauser, 2006]
[Bieri, Greub, Steinhauser, 2003]
[Ferroglia, Haisch, 2007]
The operators $Q_i$ that matter for $b \rightarrow s\gamma$ read:

$$O_{1,2} = \begin{array}{c}
\begin{array}{c}
\text{c} \\
\text{b} \\
\text{c}
\end{array}
\end{array} = \begin{array}{c}
\begin{array}{c}
\bar{s}\Gamma_i c \\
\bar{c}\Gamma'_i b
\end{array}
\end{array}, \quad \text{from} \quad \begin{array}{c}
\begin{array}{c}
\text{c} \\
\text{b} \\
\text{c}
\end{array}
\end{array} \quad \Rightarrow \quad C_i(m_b) \sim 1$$

$$O_{3,4,5,6} = \begin{array}{c}
\begin{array}{c}
\text{q} \\
\text{b} \\
\text{q}
\end{array}
\end{array} = \begin{array}{c}
\begin{array}{c}
\bar{s}\Gamma_i b \sum_q (\bar{q}\Gamma'_i q)
\end{array}
\end{array}, \quad \Rightarrow \quad |C_i(m_b)| < 0.07$$

$$O_7 = \begin{array}{c}
\begin{array}{c}
\text{b} \\
\text{s}
\end{array}
\end{array} = \frac{em_b}{16\pi^2} \begin{array}{c}
\begin{array}{c}
\bar{s}\sigma^{\mu\nu}b_R F_{\mu\nu}
\end{array}
\end{array}, \quad C^\text{SM}_7(m_b) \simeq -0.3$$

$$O'_7 = \begin{array}{c}
\begin{array}{c}
\text{b} \\
\text{s}
\end{array}
\end{array} = \frac{em_b}{16\pi^2} \begin{array}{c}
\begin{array}{c}
\bar{s}\sigma^{\mu\nu}b_L F_{\mu\nu}
\end{array}
\end{array}, \quad C'^\text{SM}_7(m_b) = \frac{m_s}{m_b} C^\text{SM}_7$$

$$O_8 = \begin{array}{c}
\begin{array}{c}
\text{g} \\
\text{b} \\
\text{g}
\end{array}
\end{array} = \frac{gm_b}{16\pi^2} \begin{array}{c}
\begin{array}{c}
\bar{s}\sigma^{\mu\nu}T^a b_R G^a_{\mu\nu}
\end{array}
\end{array}, \quad C^\text{SM}_8(m_b) \simeq -0.15$$

$$O'_8 = \begin{array}{c}
\begin{array}{c}
\text{g} \\
\text{b} \\
\text{g}
\end{array}
\end{array} = \frac{gm_b}{16\pi^2} \begin{array}{c}
\begin{array}{c}
\bar{s}\sigma^{\mu\nu}T^a b_L G^a_{\mu\nu}
\end{array}
\end{array}, \quad C'^\text{SM}_8(m_b) = \frac{m_s}{m_b} C^\text{SM}_8$$

Their SM Wilson coefficients are known up to $O(\alpha_s^2)$ (NNLO).

Assumption: no relevant NP effects in the 4-quark operators.
$\Gamma(\bar{B}^0 \to K^{*0}\gamma)_{\text{exp}} = (4.01 \pm 0.20) \times 10^{-5}$ [HFAG],

$\Gamma(\bar{B}_s \to \phi\gamma)_{\text{exp}} = \left(5.7^{+1.8}_{-1.5}(\text{stat})^{+1.2}_{-1.1}(\text{syst})\right) \times 10^{-5}$ [BELLE, PRL 100 (2008) 121801].

The decay rates $\Gamma(\bar{B} \to \bar{K}^{*}\gamma)$ and $\Gamma(\bar{B}_s \to \phi\gamma)$ are proportional to (practically) the same combinations of the Wilson coefficients as the inclusive rate $\Gamma(\bar{B} \to X_s\gamma)$.

Errors in the inclusive rate are $O(7\%)$, both EXP and TH. Theory uncertainties in the exclusive rates are $O(30\%)$ due to non-perturbative form-factors.

A promising exclusive observable for constraining the Wilson coefficients:

The mixing-induced CP asymmetry

$$A_{CP}(t) = \frac{\Gamma[\bar{B}^0(t) \to \bar{K}^{*0}\gamma] - \Gamma[B^0(t) \to K^{*0}\gamma]}{\Gamma[B^0(t) \to K^{*0}\gamma] + \Gamma[B^0(t) \to K^{*0}\gamma]} = C_{K^*\gamma} \cos(\Delta m_B t) + S_{K^*\gamma} \sin(\Delta m_B t).$$

$S_{K^*\gamma}^{\text{th}} = -\frac{2|z|}{1+|z|^2} \sin[2\beta - \arg(C_7 C_7^t)] + ... \overset{\text{SM}}{\sim} -0.03, \quad z = \frac{C_7^t}{C_7} \overset{\text{SM}}{\sim} \frac{m_s}{m_b}.$

$S_{K^*\gamma}^{\text{exp}} = -0.19 \pm 0.23$ [BaBar,Belle $\to$ HFAG].
Constraints in the \((C_7^{\text{NP}} \equiv C_7 - C_7^{\text{SM}}, C_7')\) plane from C. Bobeth, G. Hiller and G. Piranishvili, arXiv:0805.2525

**Fig. 2a**

**Green:** \(\bar{B} \rightarrow X_s \gamma\),

**Blue:** \(\bar{B} \rightarrow X_s l^+ l^-\) \(q_{\text{dilept}}^2 \in [1, 6] \text{ GeV}^2\),

**Red:** \(S_{K^*\gamma}\)

Black dotted lines: Effect of enlarging the uncertainty in the SM prediction for \(S_{K^*\gamma}\) due to the \(\mathcal{O}(\Lambda/m_b)\) fraction of right-handed photons originating from:

Assumptions for the above plot:

(i) \(C_7^{\text{NP}}\) and \(C_7'\) are real.

(ii) All the other Wilson coefficients are fixed at their SM values.

The operators $Q_i$ that matter for $\bar{B} \to \bar{K}^* \mu^+ \mu^-$ and $\bar{B}_s \to \phi \mu^+ \mu^-$ are the same as those for $\bar{B} \to \bar{K}^* \gamma$ and $\bar{B}_s \to \phi \gamma$, plus:

\[
O_9 = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s}_L \gamma^\nu b_L) (\bar{\mu} \gamma_\nu \mu), \quad O'_9 = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s}_R \gamma^\nu b_R) (\bar{\mu} \gamma_\nu \mu),
\]

\[
O_{10} = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s}_L \gamma^\nu b_L) (\bar{\mu} \gamma_\nu \gamma_5 \mu), \quad O'_{10} = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s}_R \gamma^\nu b_R) (\bar{\mu} \gamma_\nu \gamma_5 \mu),
\]

and, in principle, also the four chirality-violating operators that do not contribute to $\bar{B}_s \to \mu^+ \mu^-$:

\[
O'_S = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s} b) (\bar{\mu} \mu), \quad O'_P = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s} b) (\bar{\mu} \gamma_5 \mu),
\]

\[
O_T = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s} \sigma^{\nu \lambda} b) (\bar{\mu} \sigma_\nu \lambda \mu), \quad O'_T = \frac{\alpha_{\text{em}}}{4\pi} (\bar{s} \sigma^{\nu \lambda} b) (\bar{\mu} \sigma_\nu \lambda \gamma_5 \mu).
\]
The full angular distribution of $\bar{B} \rightarrow \bar{K}^* (\rightarrow \bar{K}\pi)\mu^+\mu^-$:

[e.g.: C. Bobeth, G. Hiller and G. Piranishvili, arXiv:0805.2525]

$$\frac{d^4\Gamma}{dq^2\,d\cos\theta_l\,d\cos\theta_{K^*}\,d\phi} = \frac{3}{8\pi} J(q^2, \theta_l, \theta_{K^*}, \phi),$$

$$J(q^2, \theta_l, \theta_{K^*}, \phi) = J_1^s \sin^2 \theta_{K^*} + J_1^c \cos^2 \theta_{K^*} + (J_2^s \sin^2 \theta_{K^*} + J_2^c \cos^2 \theta_{K^*}) \cos 2\theta_l + J_3 \sin^2 \theta_{K^*} \sin^2 \theta_l \cos 2\phi + J_4 \sin 2\theta_{K^*} \sin 2\theta_l \cos \phi + J_5 \sin 2\theta_{K^*} \sin \theta_l \cos \phi + J_6 \sin^2 \theta_{K^*} \cos \theta_l + J_7 \sin 2\theta_{K^*} \sin \theta_l \sin \phi + J_8 \sin 2\theta_{K^*} \sin 2\theta_l \sin \phi + J_9 \sin^2 \theta_{K^*} \sin^2 \theta_l \sin 2\phi.$$

$q^2 = $ dilepton invariant mass squared,

$\theta_l = $ angle between the $\mu^-$ and $\bar{B}$ momenta in the dilepton c.m.s.,

$\theta_{K^*} = $ angle between the $\bar{K}$ and $\bar{B}$ momenta in the $\bar{K}\pi$ c.m.s.,

$\phi = $ angle between the normals to the $\bar{K}\pi$ and $\mu^+\mu^-$ planes in the $\bar{B}$-meson rest frame.

The forward-backward asymmetry:

$$A_{FB}(q^2) = \left(\frac{d\Gamma}{dq^2}\right)^{-1} \left[I_1^1 - I_0^0\right] d\cos\theta_l \frac{d^2\Gamma}{dq^2\,d\cos\theta_l} = \left(\frac{d\Gamma}{dq^2}\right)^{-1} J_6(q^2)$$
Quantities similar to $A_{FB}(q^2)$ can be obtained by integrating the full distribution with various angular weighting functions. Such quantities are functions of ratios of the Wilson coefficients $C_i/C_j$ and ratios of $q^2$-dependent form-factors.

**In general:** 7 independent form-factors
[see e.g. F. Krüger, J. Matias, Phys. Rev. D71 (2005) 094009].

In the large $E_{K^*}$ limit ($m_{K^*}/E_{K^*} \sim \Lambda/m_b \ll 1$): only $\xi_\perp(q^2)$ and $\xi_\parallel(q^2)$, up to $O(\alpha_s, \Lambda/m_b)$.

**Two strategies:**

1. **Determine** $\xi_\perp/\xi_\parallel$ together with $C_i/C_j$ from experiment.

2. **Search** for quantities in which the form-factors cancel out.  
**Example:** see next slide