$V_{cb}$ and $V_{ub}$

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GGI, Feb. 2010

Some Transparencies borrowed from
A. Khodjamirian, F. Tackman, E. Gardi, R. van der Water
Contents

1. Inclusive $V_{cb}$
2. Exclusive $V_{cb}$
3. Exclusive $V_{ub}$
4. Inclusive $V_{ub}$
Structure of the expansion (@ tree):

\[ d\Gamma = d\Gamma_0 + \left( \frac{\Lambda_{\text{QCD}}}{m_b} \right)^2 d\Gamma_2 + \left( \frac{\Lambda_{\text{QCD}}}{m_b} \right)^3 d\Gamma_3 + \left( \frac{\Lambda_{\text{QCD}}}{m_b} \right)^4 d\Gamma_4 \]

\[ + d\Gamma_5 \left( a_0 \left( \frac{\Lambda_{\text{QCD}}}{m_b} \right)^5 + a_2 \left( \frac{\Lambda_{\text{QCD}}}{m_b} \right)^3 \left( \frac{\Lambda_{\text{QCD}}}{m_c} \right)^2 \right) \]

\[ + \ldots + d\Gamma_7 \left( \frac{\Lambda_{\text{QCD}}}{m_b} \right)^3 \left( \frac{\Lambda_{\text{QCD}}}{m_c} \right)^4 \]

Power counting \( m_c^2 \sim \Lambda_{\text{QCD}} m_b \)
Present state of the $b \to c$ semileptonic Calculations

- Tree level terms up to and including $1/m_b^5$ known
- $O(\alpha_s)$ and full $O(\alpha_s^2)$ for the partonic rate known
- $O(\alpha_s)$ for the $\mu_\pi^2/m_b^2$ is known
- In the pipeline:
  - Complete $\alpha_s/m_b^2$, including the $\mu_G$ terms
- The upshot:

\[
V_{cb} = (41.54 \pm 0.44 \pm 0.58_{HQE}) \times 10^{-3}
\]

(PDG 2010)

Relative uncertainty of 1.7% !!
Higher Orders in the 1/m Expansion

Beyond $\mu_\pi^2$, $\mu_G^2$, $\rho_D$ and $\rho_{LS}$: At $\mathcal{O}(1/m^4)$
Dimension 7 matrix elements = four derivatives

Spin-independent

\[ 2M_B m_1 = \langle (\vec{p})^2 \rangle^2 \]
\[ 2M_B m_2 = g^2 \langle \vec{E}^2 \rangle \]
\[ 2M_B m_3 = g^2 \langle \vec{B}^2 \rangle \]
\[ 2M_B m_4 = g \langle \vec{p} \cdot \text{rot} \vec{B} \rangle \]
### Spin-dependent

\[
2 M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle \\
2 M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle \\
2 M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle \\
2 M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle \\
2 M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle
\]

- In the published paper three of the matrix elements were ommitted!
- An erratum and a more sophisticated estimate of the matrix elements will be published soon
Quantitative Results

- Estimate by “Ground State Saturation”: (Spatial Components only)

\[
\frac{1}{2M_B} \langle B(v) | b(D_{\mu_1})(D_{\mu_2})b | B(v) \rangle \langle B(v) | \bar{b}(iD_{\mu_3})(iD_{\mu_4})b | B(v) \rangle \\
+ \frac{1}{2M_B} \sum_{\text{Pol}} \langle B(v) | b(D_{\mu_1})(D_{\mu_2})b | B^*(v) \rangle \langle B^*(v) | \bar{b}(iD_{\mu_3})(iD_{\mu_4})b | B(v) \rangle
\]

- Calculate via “Trace Formulae”
  → reduce them to $\mu_\pi$ and $\mu_G$ (Bigi, Zwicky, Uraltsev)
With two time derivatives:

\[
\langle B(v) | \bar{b} (iD_{\mu_1})(iD_0)(iD_0)(iD_{\mu_4})b | B(v) \rangle \\
\approx \bar{\epsilon}^2 \langle B(v) | \bar{b} (iD_{\mu_1})(iD_{\mu_4})b | B(v) \rangle
\]

\(\bar{\epsilon}\): Excitation energy to the first excited state

Numerical values

(\(\mu_\pi^2 = 0.45\,\text{GeV}^2, \mu_G^2 = 0.35\,\text{GeV}^2, \bar{\epsilon} = 400\,\text{MeV}\))

\[
\begin{array}{cccccccccc}
  m_1 & m_2 & m_3 & m_4 & m_5 & m_6 & m_7 & m_8 & m_9 \\
  0.11 & -0.07 & -0.08 & 0.39 & -0.06 & -0.16 & 0.42 & 1.26 & 0.40 \\
\end{array}
\]

(all values in GeV^4)
Effect has been studied in detail on the moments
→ small effects of expected size!
Effect on the total rate:
\[(\delta \Gamma |_{1/m_b^i} = (\Gamma |_{1/m_i} - \Gamma |_{1/m_{i-1}}) / \Gamma_{\text{parton}})\]
\[\delta \Gamma |_{1/m_b^4} \approx +0.29\%\]
\[\delta \Gamma |_{1/m_b^3} \approx -2.84\%\]
\[\delta \Gamma |_{1/m_b^2} \approx -4.29\%\]
Impact on \( V_{cb} \): Slight improvement of the uncertainly related to the application of the HQE
Total improvement small, \( \mathcal{O}(0.25\%) \)
\( \mathcal{O}(\alpha_s \mu_\pi^2 / m_b^2) \) corrections

- One-Loop \( \alpha_s \) corrections known since a long time
- Corrections to the leading (partonic) rate
- Make use of Reparametrization invariance:

\[
v \to v' = v + \frac{k}{m_b}
\]

- Relates different orders of the \( 1/m_b \) expansion
- Valid to all orders in \( \alpha_s \)
- \( \to \) Compute \( \mathcal{O}(\alpha_s) \)-Correction with \( p_b = m_b v + k \)
  and expand in \( k \)

\[
k_\mu k_\nu \to (g_{\mu\nu} - v_\mu v_\nu) \frac{\mu_\pi^2}{3}
\]
For the complete $\alpha_s/m_b^2$ also the $O(\alpha_s \mu_G^2/m_b^2)$
Corrections need to be computed

Significantly more complicated

→ Needs the one gluon matrix elements at one loop

Doable, is in the pipeline

The knowledge of the partonic $\alpha_s^2$ corrections also
give us the $\alpha_s^2 \mu^2_{\pi}/m_b^2$ by RPI

.... and the $\alpha_s m_1/m_b^4$, and the $\alpha_s^2 m_1/m_b^4$
Technically challenging

- Partially numerical calculation
- Analytic Results for limiting cases
- \( \rightarrow \) allows for an interpolation
- Recently: Complete differential distributions available

\( O(\alpha_s^2) \) corrections Czarnecki, Pak; Melnikov

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\( V_{cb} \) and \( V_{ub} \)
• Contributions to the Moments (\(d\Gamma_0\): Partonic rate)

\[
L_n(E_{\text{cut}}) = \frac{\langle (E_i/m_b)^n \theta(E_i - E_{\text{cut}}) d\Gamma \rangle}{\langle d\Gamma_0 \rangle}
\]

\[
H_n(E_{\text{cut}}) = \frac{\langle (E_h/m_b)^n \theta(E_i - E_{\text{cut}}) d\Gamma \rangle}{\langle d\Gamma_0 \rangle}
\]

• Expansion:

\[
L_n(E_{\text{cut}}) = L_n^{(0)} + \frac{\alpha_s}{\pi} L_n^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 \left[ \beta_0 L_n^{2,\text{BLM}} + L_n^{(2)} \right]
\]

\[
H_n(E_{\text{cut}}) = H_n^{(0)} + \frac{\alpha_s}{\pi} H_n^{(1)} + \left( \frac{\alpha_s}{\pi} \right)^2 \left[ \beta_0 H_n^{2,\text{BLM}} + H_n^{(2)} \right]
\]

• \(\beta_0 = 11 - 2N_f/3\) and \(\alpha_s = \alpha_s^{\text{MS},N_f=5}(m_b)\)
### TABLE I: Lepton energy moments.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$E_{\text{cut}}$, GeV</th>
<th>$L_n^{(0)}$</th>
<th>$L_n^{(1)}$</th>
<th>$L_n^{(2,\text{BLM})}$</th>
<th>$L_n^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>-1.77759</td>
<td>-1.9170</td>
<td>3.40</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0.307202</td>
<td>-0.55126</td>
<td>-0.6179</td>
<td>1.11</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.10299</td>
<td>-0.1877</td>
<td>-0.2175</td>
<td>0.394</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0.81483</td>
<td>-1.4394</td>
<td>-1.5999</td>
<td>2.63</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0.27763</td>
<td>-0.49755</td>
<td>-0.5667</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.09793</td>
<td>-0.17846</td>
<td>-0.20875</td>
<td>0.382</td>
</tr>
</tbody>
</table>

### TABLE II: Hadronic energy moments.

<table>
<thead>
<tr>
<th>$n$</th>
<th>$E_{\text{cut}}$, GeV</th>
<th>$H_n^{(0)}$</th>
<th>$H_n^{(1)}$</th>
<th>$H_n^{(2,\text{BLM})}$</th>
<th>$H_n^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.334</td>
<td>-0.57728</td>
<td>-0.6118</td>
<td>1.02</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.14111</td>
<td>-0.23456</td>
<td>-0.2343</td>
<td>0.362</td>
</tr>
</tbody>
</table>
Even higher orders: $\mathcal{O}(1/m_b^n)$, $n > 4$ Corrections

- $1/m_b^5$ has been studied in the context of “intrinsic charm”
  Numerical estimates of the $1/m_b^5$ are available
- General Structure of the higher order terms have been studied
- Proliferation of new parameters
Estimates of $1/m_b^5$

- In total 18 parameters (Singlet and Triplet)
- Estimates of the parameters by “Ground State Saturation”
- Full expressions for doubly differential rates are available
- Numerical estimates

\[
\frac{\Gamma|^{1/m_b^5}_{\text{complete}}}{\Gamma_0} \approx 0.36\%
\]
\[
\frac{\Gamma|^{1/m_b^5}_{1/m_c^2}}{\Gamma_0} \approx 0.46\%
\]
Beyond $1/m_b^5$

**Proliferation of parameters in high orders $1/m_b$:**

<table>
<thead>
<tr>
<th></th>
<th>Dim 5</th>
<th>Dim 6</th>
<th>Dim 7</th>
<th>Dim 8</th>
<th>Dim 9</th>
<th>Dim 10</th>
<th>Dim 11</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>24</td>
<td>60</td>
<td>216</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>11</td>
<td>48</td>
<td>150</td>
<td>624</td>
</tr>
<tr>
<td>tot</td>
<td>2</td>
<td>2</td>
<td>9</td>
<td>18</td>
<td>72</td>
<td>210</td>
<td>840</td>
</tr>
</tbody>
</table>

At high orders: (n = Dim - 3)

$$N_1(n) \approx \frac{1}{2} \sum_{n_g=1}^{\left[\frac{n}{2}\right]} (2n_g - 1)!! \binom{n-2}{n-2n_g}$$

$$N_\sigma(n) \approx \frac{1}{2} \sum_{n_g=1}^{\left[\frac{n}{2}\right]-1} (2n_g - 1)!! \binom{n-2}{n-2n_g-2} \binom{2+2n_g}{2}$$

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$V_{cb}$ and $V_{ub}$
Exclusive $V_{cb}$

- Kinematic variable for a heavy quark: Four Velocity $v$
- Differential Rates

\[
\frac{d\Gamma}{d\omega}(B \rightarrow D^* \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 m_D^3 (\omega^2 - 1)^{1/2} P(\omega) (\mathcal{F}(\omega))^2
\]

\[
\frac{d\Gamma}{d\omega}(B \rightarrow D \ell \bar{\nu}_\ell) = \frac{G_F^2}{48\pi^3} |V_{cb}|^2 (m_B + m_D)^2 m_D^3 (\omega^2 - 1)^{3/2} (\mathcal{G}(\omega))^2
\]

- with $\omega = vv'$ and
- $P(\omega)$: Calculable Phase space factor
- $\mathcal{F}$ and $\mathcal{G}$: Form Factors
Heavy Quark Symmetries

- Normalization of the Form Factors is known at $\nu\nu' = 1$: (both initial and final meson at rest)
- Corrections can be calculated / estimated

\[
\mathcal{F}(\omega) = \eta_{\text{QED}}\eta_A \left[1 + \delta_1/\mu^2 + \cdots\right] (\omega - 1)\rho^2 + \mathcal{O}\left((\omega - 1)^2\right)
\]

\[
\mathcal{G}(1) = \eta_{\text{QED}}\eta_V \left[1 + \mathcal{O}\left(\frac{m_B - m_D}{m_B + m_D}\right)\right]
\]

- Parameter of HQS breaking: \( \frac{1}{\mu} = \frac{1}{m_c} - \frac{1}{m_b} \)
- \( \eta_A = 0.960 \pm 0.007, \eta_V = 1.022 \pm 0.004, \delta_1/\mu^2 = -(8 \pm 4)\% , \eta_{\text{QED}} = 1.007 \)
$B \rightarrow D^{(*)}$ Form Factors from the Lattice

- Unquenched Calculations become available!
- Heavy Mass Limit is not used
- Lattice Calculations of the deviation from unity

\[ \mathcal{F}(1) = 0.927 \pm 0.024 \]

\[ \mathcal{G}(1) = 1.074 \pm 0.018 \pm 0.016 \]


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$V_{cb}$ and $V_{ub}$
$B \rightarrow D^* \ell \bar{\nu}_\ell$

$\Delta \chi^2 = 1$

- OPAL (excl.)
- DELPHI (excl.)
- OPAL (part. reco.)
- DELPHI (part. reco.)
- BABAR (Global Fit)
- BABAR (D*0)
- BELLE
- BABAR (excl.)

AVERAGE

$\chi^2$/dof = 39.6/21

$F(1) \times |V_{cb}| [10^{-3}]$

$\rho^2$

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$V_{cb}$ and $V_{ub}$
$B \rightarrow D^{\ell}\bar{\nu}_\ell$

The graph shows the correlation between $G(1) \times |V_{cb}| [10^{-3}]$ and $\rho^2$ with contours for different experiments and fits. The BABAR global fit is indicated by a green contour, BABAR tagged by a blue contour, and an AVERAGE fit by a red contour. The CLEO result is shown by a blue point. The $\chi^2$/dof for the global fit is 1.3/8.

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$V_{cb}$ and $V_{ub}$
$V_{cb, excl} = (38.7 \pm 1.1) \times 10^{-3}$

- from Zero Recoil Sum Rules: Hints that $\mathcal{F}(1) \leq 0.9$
  Ural'tsev, Gambino, TM: Work to be completed at GGI

- Likewise, old estimate by Kolya for $\mathcal{G}(1)$
  $$\mathcal{G}(1) = 1.04 \pm 0.02$$
Ist there a problem $V_{incl}^{cb}$ vs. $V_{excl}^{cb}$?

- Take the $V_{cb}$ value from the (very mature!) inclusive determination
  ... and compute $F(1)$ and $G(1)$!

  $$F(1) = 0.86 \pm 0.03$$
  $$G(1) = 1.02 \pm 0.04$$

- Both lattice results are on the high side.
Exclusive $V_{ub}$: $B \rightarrow \pi \ell \bar{\nu}_\ell$

- Problem: Calculation of the form factor:

$$\frac{d\Gamma(\bar{B}^0 \rightarrow \pi^+ l^- \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \rho_\pi^3 |f_{B\pi}^+(q^2)|^2 + O(m_l^2)$$

- Lattice QCD
- QCD (Light Cone) Sum Rules
Tools: Form Factor Parametrizations

- **Becirevic Kaidalov Parametrization**

\[
f_+(q^2) = \frac{f_+(0)}{(1 - q^2/m_{B^*}^2)(1 - \alpha q^2/m_{B^*}^2)}
\]

- **z Parametrization**

\[
P(t) \phi(t, t_0) f_+(t) = \sum_{k=0}^{\infty} a_k(t_0) z^k(t, t_0)
\]

with

\[
z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}, \quad t_+ = (m_B + m_\pi)^2
\]
\[ |V_{ub}| \times 10^3 = 3.38 \pm 0.36 \]

\[ a_0 = 0.0218 \pm 0.0021 \]
\[ a_1 = -0.0301 \pm 0.0063 \]
\[ a_2 = -0.059 \pm 0.032 \]
\[ a_3 = 0.079 \pm 0.068 \]
Status of LCSR calculation

- last update: $f_{B\pi}^+(q^2)$ at $0 \leq q^2 \leq 12 \text{ GeV}^2$
- with $f_B$ from two-point QCD sum rules (exp. $f_B$ still with a larger error)
- fitting the shape to the BABAR data to constrain LCSR input (twist-2 DA)
- in this region fitted to BK parameterization with $\alpha_{BK} = 0.53 \pm 0.06$
  (equally well to Boyd-Grinstein-Lebed series-parameterization)

$$f_{B\pi}^+(q^2) / f_{B\pi}^+(0)$$

the result: $f_{B\pi}^+(0) = 0.26^{+0.04}_{-0.03}$
Combining LCSR⊕BaBar shape ⊕ lattice QCD

[ Bourrely, Caprini, Lellouch, 0807.222 hep-ph ]

- Modified series-parameterization used:
  - LCSR
  - FNAL-MILC
  - HPQCD

$q^2 = 0$: LCSR [DKMNO], $q^2 > 15$ GeV$^2$: lattice QCD [FNAL-MILC, HPQCD]
| $|V_{ub}|$ from $B \rightarrow \pi l \nu_l$ |

A sample of most recent results

| [Ref.]          | $f_{B\pi}^+(q^2)$ calculation | $f_{B\pi}^+(q^2)$ input | $|V_{ub}| \times 10^3$ |
|-----------------|-------------------------------|--------------------------|-----------------------|
| FNAL-MILC '08   | lattice                       | -                        | 3.38 ± 0.35           |
| HPQCD '07       | lattice                       | -                        | 3.55 ± 0.25 ± 0.50    |
| Ball, Zwicky '04| LCSR                          | -                        | 3.5 ± 0.4 ± 0.1       |
| Flynn, Nieves '07| -                             | lattice ⊕ LCSR           | 3.47 ± 0.29 ± 0.03    |
| DKMMO '07       | LCSR                          | -                        | 3.5 ± 0.4 ± 0.2 ± 0.1 |
| Bourrely, Caprini, Lellouch '08 | - | lattice ⊕ LCSR | 3.54 ± 0.24 |

Non-lattice QCD calculations for semileptonic $B$ and $D$ decays
Exclusive $V_{ub}$: PDG Average 2010

$$V_{ub} = (3.38 \pm 0.36) \times 10^{-4}$$

(PDG 2010)
$|V_{ub}|$ from Inclusive $B \rightarrow X_u \ell \nu$

Removing huge charm background requires stringent phase space cuts

$$\mathcal{B}(B \rightarrow X_c \ell \nu) / \mathcal{B}(B \rightarrow X_u \ell \nu) \approx 50$$

- Cuts can drastically enhance perturbative and nonperturbative corrections

Rates become sensitive to $b$-quark PDFs in $B$ meson

- Determine shape of spectra
- Leading order: Universal shape function (SF) [Neubert (1993); Bigi et al. (1993)]
- $\mathcal{O}(\Lambda_{QCD} / m_b)$: Several more subleading shape functions [Bauer, Luke, Mannel (2001)]
- Need to be extracted from data (like any PDF)
Regions of Phase Space

Kinematic variables: \( p_{X}^{\pm} = E_X \mp |\vec{p}_X| \)

Shape function region (SCET region): \( p_{X}^{\pm} \ll p_{X}^{-} \)

- Leading order in \( 1/m_b \) requires nonperturbative shape function \( S(\omega) \)
  
  \[ d\Gamma = H(E_\ell, p_{X}^{\pm}) \int d\omega \ J[p_{X}^{-} (p_{X}^{\pm} - \omega)] S(\omega) \]
  
  \[ \mathcal{O}(\alpha_s^2) \] corrections recently completed

Local OPE region: \( p_{X}^{\pm} \sim p_{X}^{-} \) (\( q^2 \) spectrum, small \( E_\ell \))

- Leading order in \( 1/m_b \) given by quark decay (as in \( B \rightarrow X_c \ell \nu \))
  known to \( \mathcal{O}(\alpha_s, \alpha_s^2/\beta_0) \) [De Fazio, Neubert (1999); Gardi, Ridolfi, Gambino (2006)]

Cut on \( m_X < m_D \) does not imply \( p_{X}^{\pm} \ll p_{X}^{-} \Rightarrow \) depends on both regions
Non-Experimental Uncertainties

Theoretical uncertainties
- Unknown higher orders in $\alpha_s$, $1/m_b$ expansions
- Weak annihilation (open question $\Rightarrow$ separate data into $B^+$ and $B^0$)

Dominant uncertainties on $|V_{ub}|$ come from input parameters
- $m_b$: Total rate $\sim |V_{ub}|^2 m_b^5$, partial rates with cuts $\sim |V_{ub}|^2 m_b^{10}$
  - Need as precise as possible $m_b$ to get precise $|V_{ub}|$
- Shape function(s): Sensitivity depends on phase space region
  (a) SCET region: small $p_X^+$, very large $E_\ell$
    $\Rightarrow$ Need the full shape (i.e. all moments)
  (b) Local OPE region: total rate, $q^2$ spectrum, small $E_\ell$
    $\Rightarrow$ Only need 1st moments (i.e. $m_b, \mu_\pi^2$)
  (c) something in between: $m_X$, moderately large $E_\ell$
- $m_b$ and SF uncertainties are separate but correlated
The OPE hard–cutoff approach

Gambino, Giordano, Ossola & Uraltsev propose to write each structure function as a convolution

\[ W_i = \int dk^+ F_i(k^+, q^2; \mu) W_i^{\text{pert}}(\mu), \]

A hard cutoff \( \mu = 1 \) GeV is implemented in the ‘kinetic scheme’. \( F_i(k^+, q^2; \mu) \) are non-perturbative functions, parametrized subject to constraints on the moments of \( W_i \) computed by OPE.

Advantages: simple and prudent! Perturbation theory is used in a safe regime above 1 GeV; the infrared is parametrized.

Limitations:
- Extensive parametrization: the unknown functions \( F_i(k^+, q^2; \mu) \) depends on two kinematic variables!
- Known structure of infrared singularities not used!
The elaborate shape function approach

For jet kinematics $P^+ \ll P^- \simeq m_b$ one has

$$\frac{d\Gamma}{dP^- dP^+ dE_l} = H J \otimes S(k^+, \mu) + \frac{\sum H_n J_n \otimes S_n(k^+, \mu)}{m_b} + \ldots \tag{1}$$

The shape function approach by Bosch, Lange, Neubert & Paz combines (1), valid for jet kinematics, with the local OPE.

Advantages: elaborate use of theoretical tools. Sudakov resummation of jet logs.

Limitations:

- starting at $O(1/m_b)$ more unknowns than observables
- Even the first $S(k^+, \mu)$ cannot be computed non-perturbatively. It is parametrized based on known center ($m_b$) and width ($\mu^2$) alone.
Dressed Gluon Exponentiation (DGE)

- Resummed perturbation theory (on-shell heavy quark) yields:

\[
\frac{1}{\Gamma_{\text{tot}}} \frac{d\Gamma}{dP^+dP^-dE_l} = \int_{-\infty}^{\infty} \frac{dN}{2\pi i} \left(1 - \frac{P^+ - \Lambda}{P^- - \Lambda}\right)^{-N} H(N, P^-, E_l) \overline{\text{Sud}}(P^-, N)
\]

- Soft and collinear radiation is summed into a Sudakov factor

\[
\overline{\text{Sud}}(p^-, N) = \exp \left\{ \frac{C_F}{\beta_0} \int_0^\infty \frac{du}{u} T(u) \left(\frac{\Lambda}{p^-}\right)^{2u} \left[ B_J(u)\Gamma(-u)(1 - N^u) - B_S(u)\Gamma(-2u)(1 - N^{2u}) \right] \right\}
\]

- Renormalon resummation indicates the presence of specific power corrections \((N\Lambda/p^-)^k\) in the exponent!

  - \(u = 1/2\) ambiguity cancels with the pole mass renormalon.
  - \(u = 1\) renormalon is missing \((B_S(1) = 0)\).
  - \(u \geq 3/2\) ambiguities are present in the on-shell spectrum.
Dressed Gluon Exponentiation (DGE)

- Resummed on-shell calculation in moment space, with no cutoff!
  - Resummation includes:
    - Sudakov logs of both jet and quark–distribution — both currently at NNLL accuracy!
    - Renormalon resummation in the exponent.
- Parametrization of power corrections in moment space
- Advantages: Ultimate use of resummed perturbation theory; minimal parametrization.
- Limitations: difficult to relate the magnitude of power corrections to conventional cutoff based definitions.
HFAG Ave. (BLNP)  
4.06 ± 0.15 + 0.25 - 0.27

HFAG Ave. (DGE)  
4.25 ± 0.15 + 0.21 - 0.17

HFAG Ave. (GGOU)  
4.03 ± 0.15 + 0.20 - 0.25

HFAG Ave. (ADFR)  
3.84 ± 0.13 + 0.23 - 0.20

HFAG Ave. (BLL)  
4.87 ± 0.24 ± 0.38

BABAR (LLR)  
4.43 ± 0.45 ± 0.29

BABAR endpoint (LLR)  
4.28 ± 0.29 ± 0.48

BABAR endpoint (LNP)  
4.40 ± 0.30 ± 0.47

\[
|V_{ub}| \times 10^{-3}
\]
Inclusive $V_{ub}$: PDG Average 2010

$$V_{ub} = (4.27 \pm 0.24) \times 10^{-4}$$

(PDG 2010)

Recent BELLE Analysis of inclusive $V_{ub}$:

Table II. Values for $|V_{ub}|$ with relative errors (in %).

| Theory   | $|V_{ub}| \times 10^3$ | Stat | Syst | $m_b$  | Th.    |
|----------|------------------------|------|------|--------|--------|
| BLNP [5] | 4.37                   | 4.3  | 4.0  | +3.1   | +4.3   |
|          |                        |      |      | -2.7   | -4.0   |
| DGE [6]  | 4.46                   | 4.3  | 4.0  | +3.2   | +1.0   |
|          |                        |      |      | -3.3   | -1.5   |
| GGOU [7] | 4.41                   | 4.3  | 4.0  | 1.9    | +2.1   |
|          |                        |      |      |        | -4.5   |
Instead of conclusions ...

- Exclusive $V_{cb}$ vs Inclusive $V_{cb}$: My point of view: The Problem lies in the Lattice calculation of the Form Factors
- Exclusive $V_{cb}$ vs Inclusive $V_{cb}$: More severe ...
Inclusive $V_{cb}$

Exclusive $V_{cb}$

Exclusive $V_{ub}$

Inclusive $V_{ub}$

Thomas Mannel, Uni. Siegen

$V_{cb}$ and $V_{ub}$
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$V_{cb}$ and $V_{ub}$