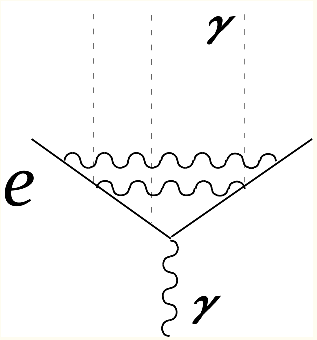
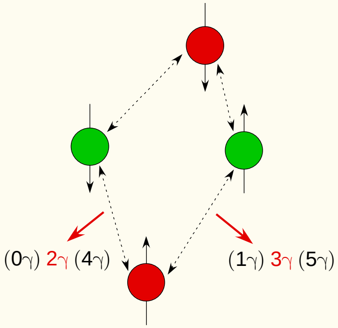
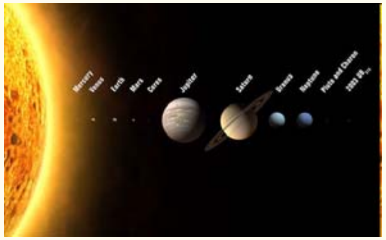
Low-energy QED tests
(and what we can learn from them)







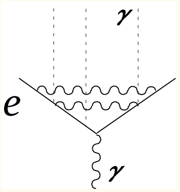
Indirect Searches for New Physics at the time of LHC

Florence, March 2010

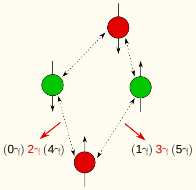
Andrzej Czarnecki U. of Alberta & CERN

Outline

Gyromagnetic factors and the determination of fundamental constants (α, m_e)



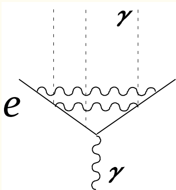
Polyelectrons and tests of few-body QED



Muonic atoms and new physics searches (lepton flavor violation, new weak-scale forces)

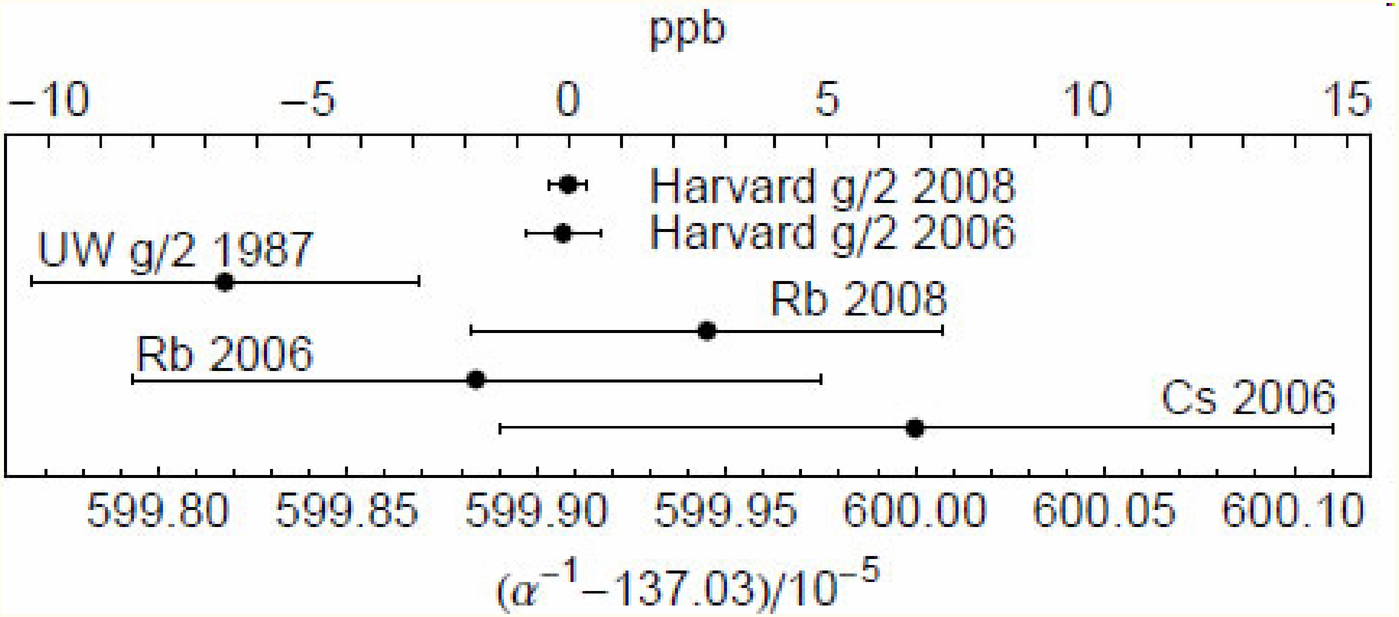
Gyromagnetic factors and the determination of

- the electron mass
- the fine structure constant



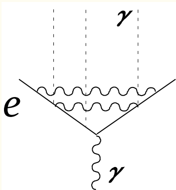
Free-electron g-factor

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 + \ldots + a_{\mu\tau} + a_{\mathrm{hadronic}} + a_{\mathrm{weak}},$$



$$\alpha = 1/137.035999084(51) \quad [0.37\,\mathrm{ppb}]$$

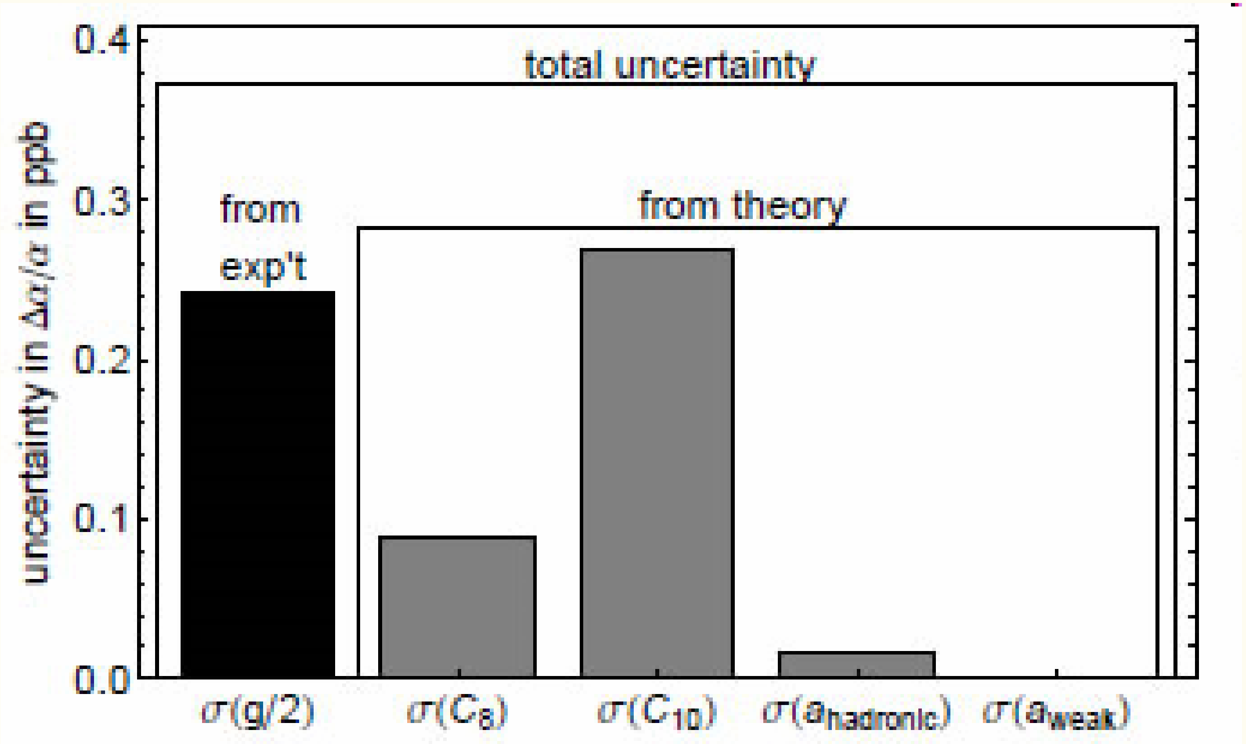
If you remember three decimal places, 137.036, you get another three free!

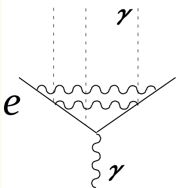


Free-electron g-factor

Experimental and theoretical uncertainties:

$$\alpha = 1/137.035999084(33)(39) \quad [0.37\,\text{ppb}]$$

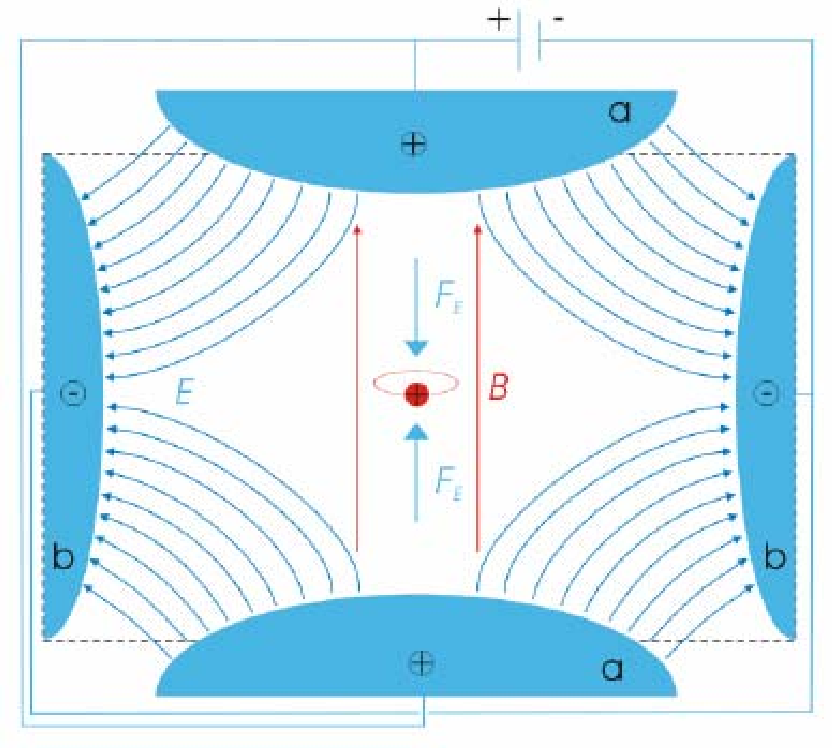




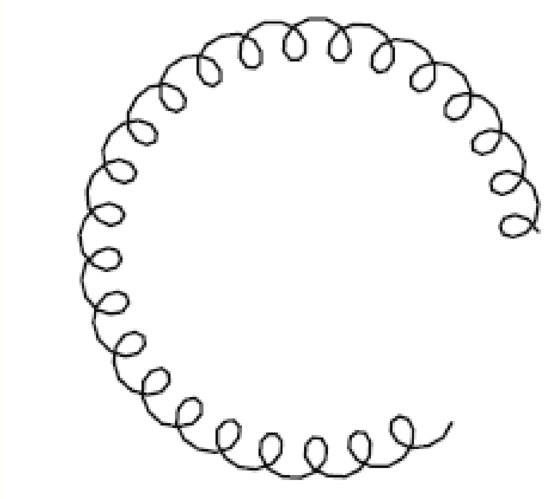
Motion in the Penning trap

$$\ddot{x} + \omega_c \dot{y} - \frac{\omega_z^2}{2} x = 0 \qquad \ddot{y} - \omega_c \dot{x} - \frac{\omega_z^2}{2} y = 0 \qquad \ddot{z} + \omega_z^2 z = 0$$

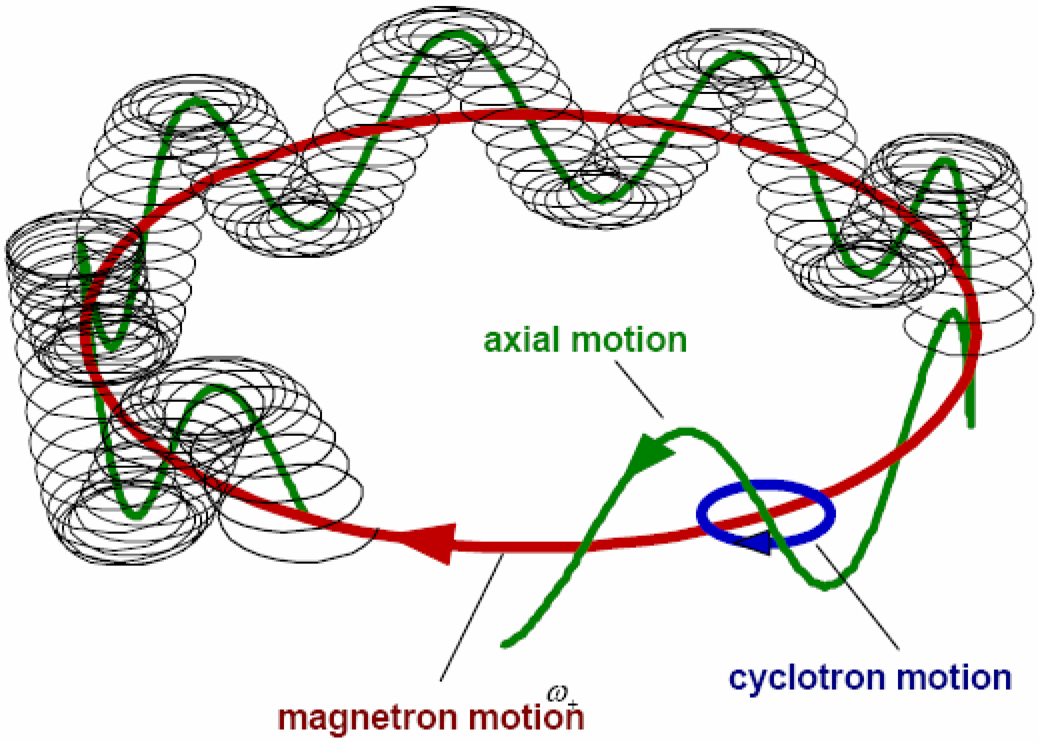
$$\omega_z \ll \omega_c$$



Motion in the xy plane:



Bound-electron g-2: measurement



From Werth

Spin precession (Larmor) frequency

$$h\nu_L = g\cdot\mu_B\cdot B$$

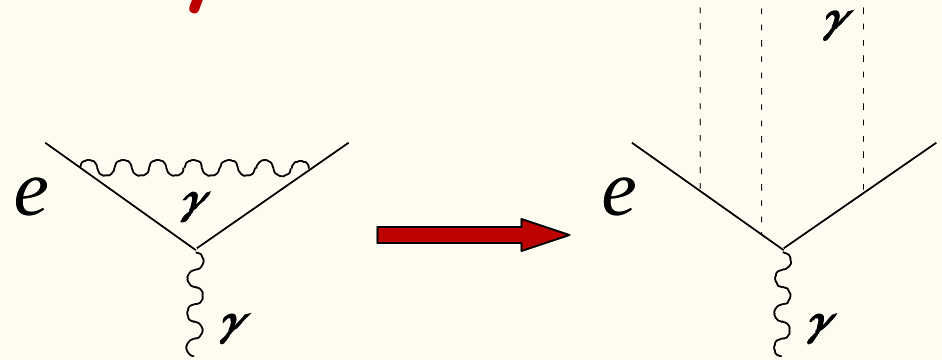
Cyclotron frequency:

$$h\nu_C = \frac{q}{M}B$$

$$g = 2\frac{\nu_L}{\nu_C}\frac{q}{e}\frac{m}{M}$$

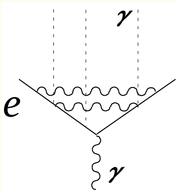
M and *m* have different origins!

Bound-electron g-2: theory

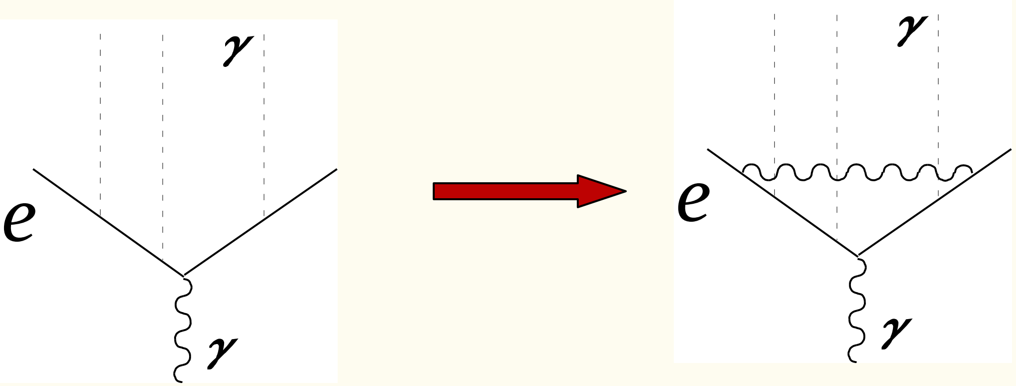


$$g = 2 - \frac{2(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + O(Z\alpha)^6 = \underbrace{\frac{2}{3}\left[1 + 2\sqrt{1-(Z\alpha)^2}\,\right]}_{\text{Breit 1928 – Dirac theory}}$$

Note: Breit's calculation predates Schwinger's by 20 years



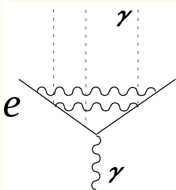
Bound-electron g-2: theory



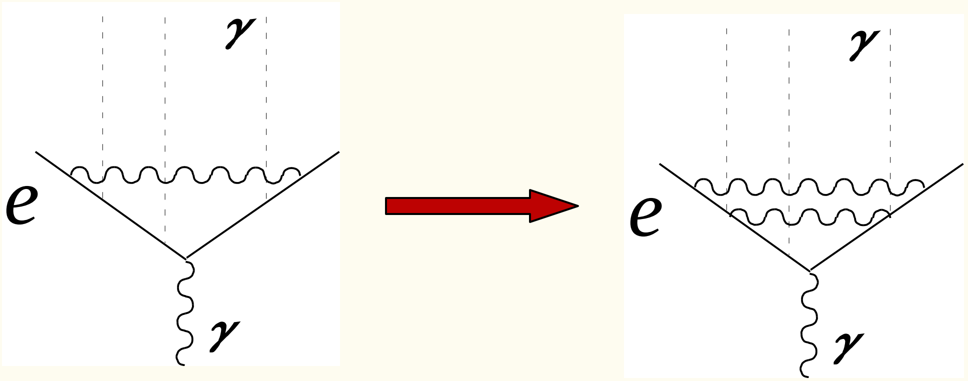
$$g = 2 - \frac{2(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + O(Z\alpha)^6$$

$$\underbrace{+\frac{\alpha}{\pi}\left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\left(a_{41}\ln\frac{1}{(Z\alpha)^2} + a_{40}\right) + O(Z\alpha)^5\right]}_{\text{one-loop corrections}}$$

Pachucki, Jentschura, Yerokhin
2004



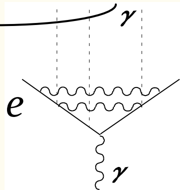
Bound-electron g-2: theory



$$g = 2 - \frac{2(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + O(Z\alpha)^6$$

$$+\frac{\alpha}{\pi}\left[1+\frac{(Z\alpha)^2}{6}+(Z\alpha)^4\left(a_{41}\ln\frac{1}{(Z\alpha)^2}+a_{40}\right)+O(Z\alpha)^5\right]$$

$$\underbrace{+\left(\frac{\alpha}{\pi}\right)^2\left[-0.65..\left(1+\frac{(Z\alpha)^2}{6}\right)+(Z\alpha)^4\left(b_{41}\ln\frac{1}{(Z\alpha)^2}+b_{40}\right)+..\right]}_{\text{two-loop corrections}}$$



The two-loop bound-state effect

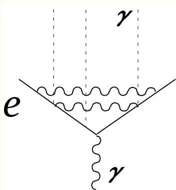
$$b_{41} = \frac{28}{9}$$

$$b_{40} = -16.4$$

Pachucki, AC, Jentschura, Yerokhin
2005

$$\boxed{m_e\left({}^{12}C^{5+}\right) = 0.000\,548\,579\,909\,31\left(29\right)_{\mathrm{exp}}\left(1\right)_{\mathrm{th}}\ u}$$

Theoretical error: negligible



2010: new measurement with oxygen, $^{16}O^{7+}$

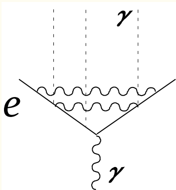
Theoretical prediction:

$$g^{\text{th}}(Z=8) = 2.000\,047\,020\,32(11)$$

Measured value:

$$g^{\text{exp}}(Z=8) = 2.000\,047\,0201(25)$$

J. Verdú,[1] H. Häffner,[2] W. Quint,[3] T. Valenzuela,[4] and G. Werth[5] (preliminary)



The "kinematic" method of finding alpha

Rydberg constant is extremely well measured,

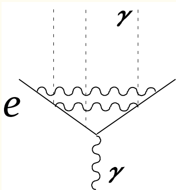
$$\mathrm{R}_\infty = \frac{m_e c^2 \alpha^2}{2} \frac{1}{hc} \simeq \frac{13.6\,\mathrm{eV}}{hc}$$

We can find alpha if we measure the quotient of the Planck constant and the "electron" mass,

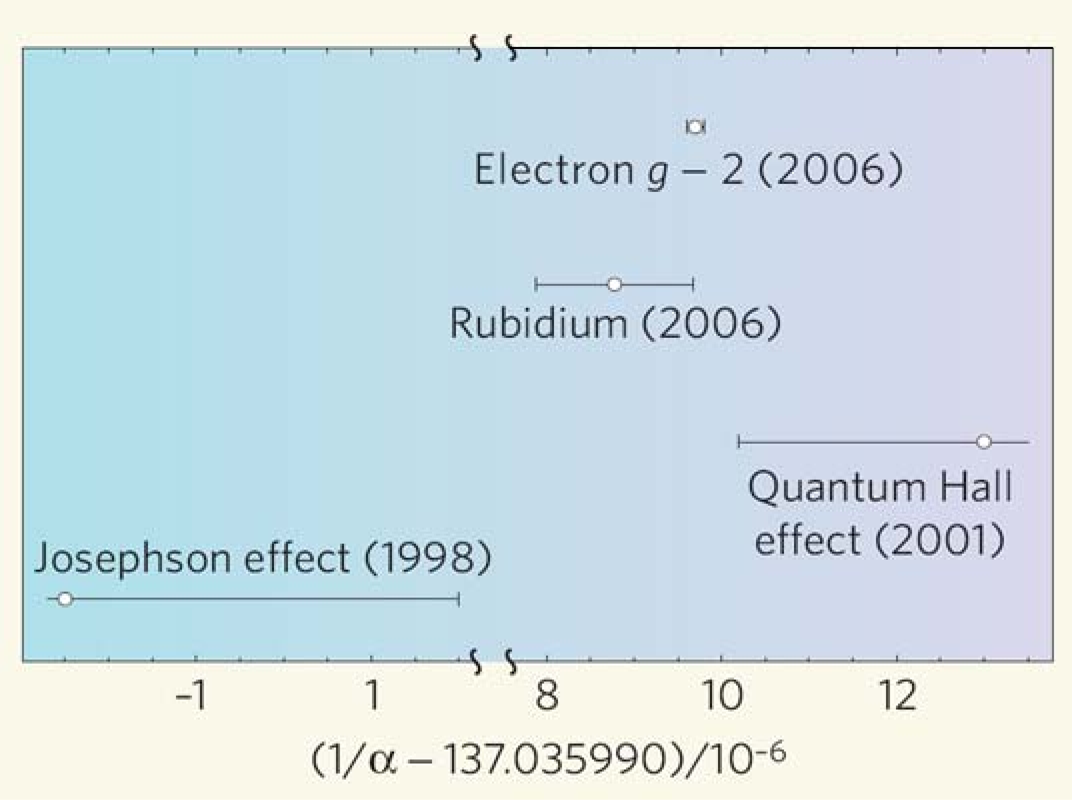
$$\alpha^2 = \frac{2h\mathrm{R}_\infty}{m_e c}$$

$$= \frac{2\mathrm{R}_\infty}{c} \frac{m_p}{m_e} \frac{m_{\mathrm{Cs}}}{m_p} \frac{h}{m_{\mathrm{Cs}}}$$

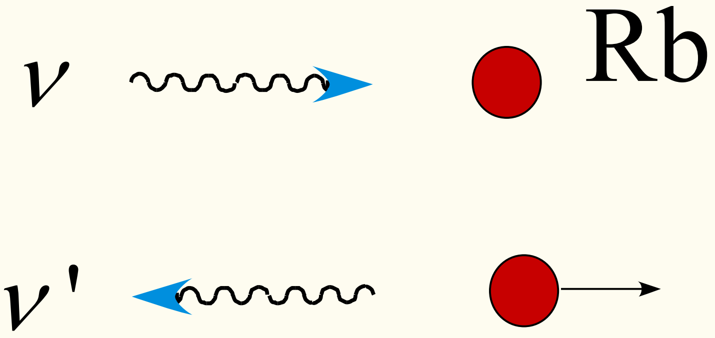
In practice heavier particles are better: neutrons or atoms.



Fine structure constant: other methods



Nature 442 (2006) 516.

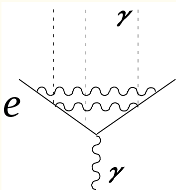


$$\alpha^2 = \frac{2h\,\mathrm{Ry}}{m_e c}$$

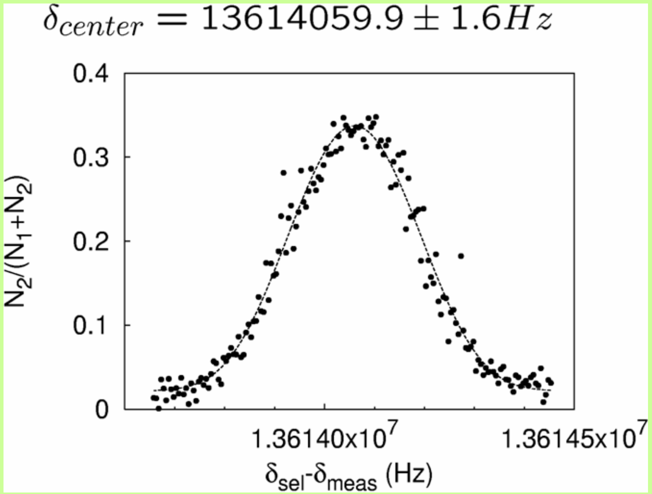
$$= \frac{2\mathrm{Ry}}{c}\,\frac{m_p}{m_e}\,\frac{m_{\mathrm{Rb}}}{m_p}\,\frac{h}{m_{\mathrm{Rb}}}$$

$$h\cdot(\nu-\nu') \simeq \frac{2h^2\nu^2}{m_{\mathrm{Rb}}c^2}$$

$$\frac{h}{m_{\mathrm{Rb}}} = \frac{\Delta\nu}{2\nu^2}c^2$$



α from Paris



about 450 Bloch oscillations
in each direction : 1800 recoils

from F. Nez

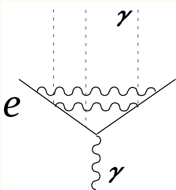
Statistical uncertainty on α = 4.4×10^{-9}

α^{-1} = 137.035 998 78 (91)

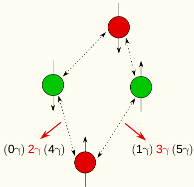
uncertainty 6.7×10^{-9}

Cladé et al, PRL **96**, 033001 (2006)

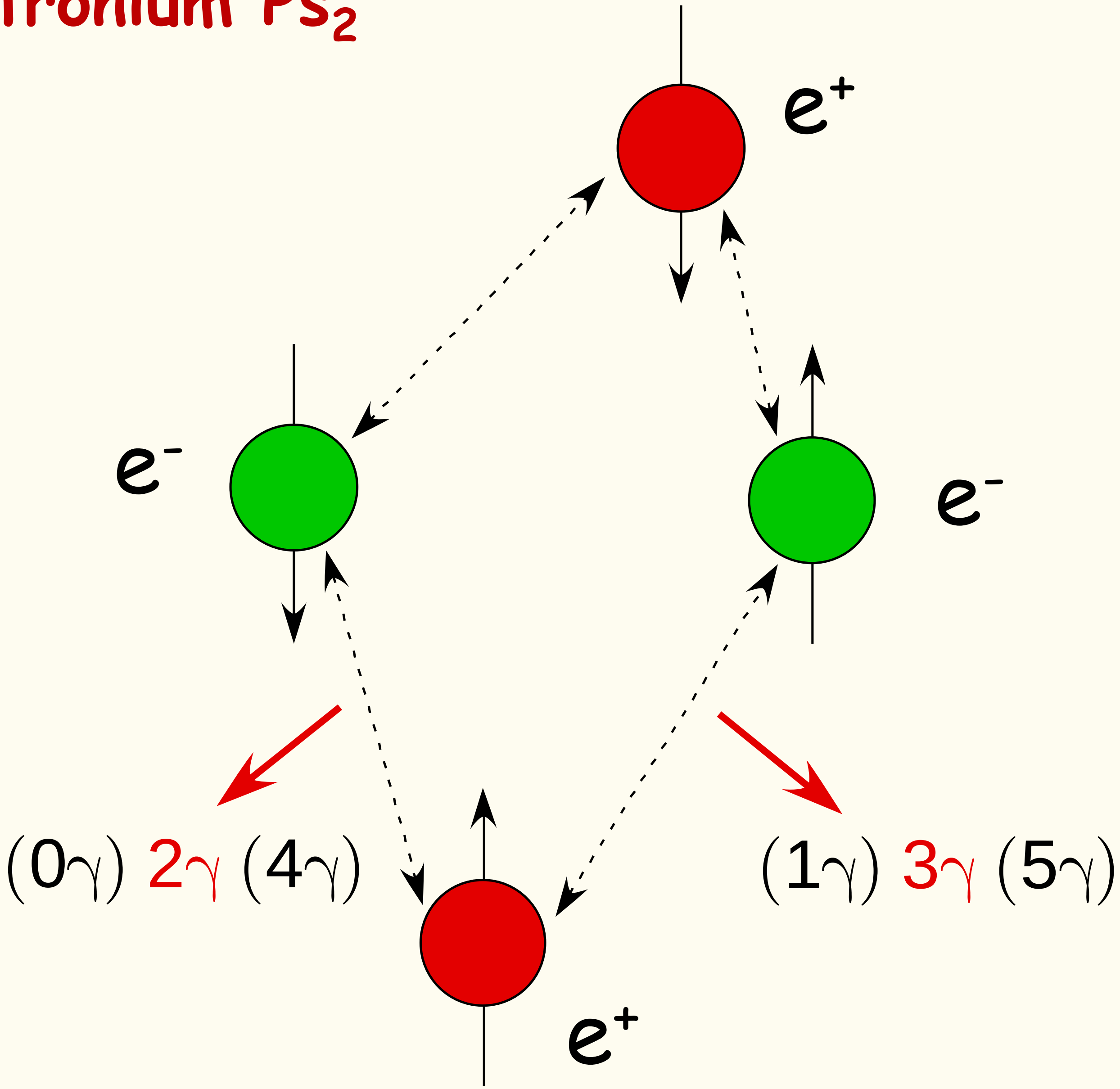
Future goal: 1ppb



Polyelectrons



Dipositronium Ps_2



e^+

e^-

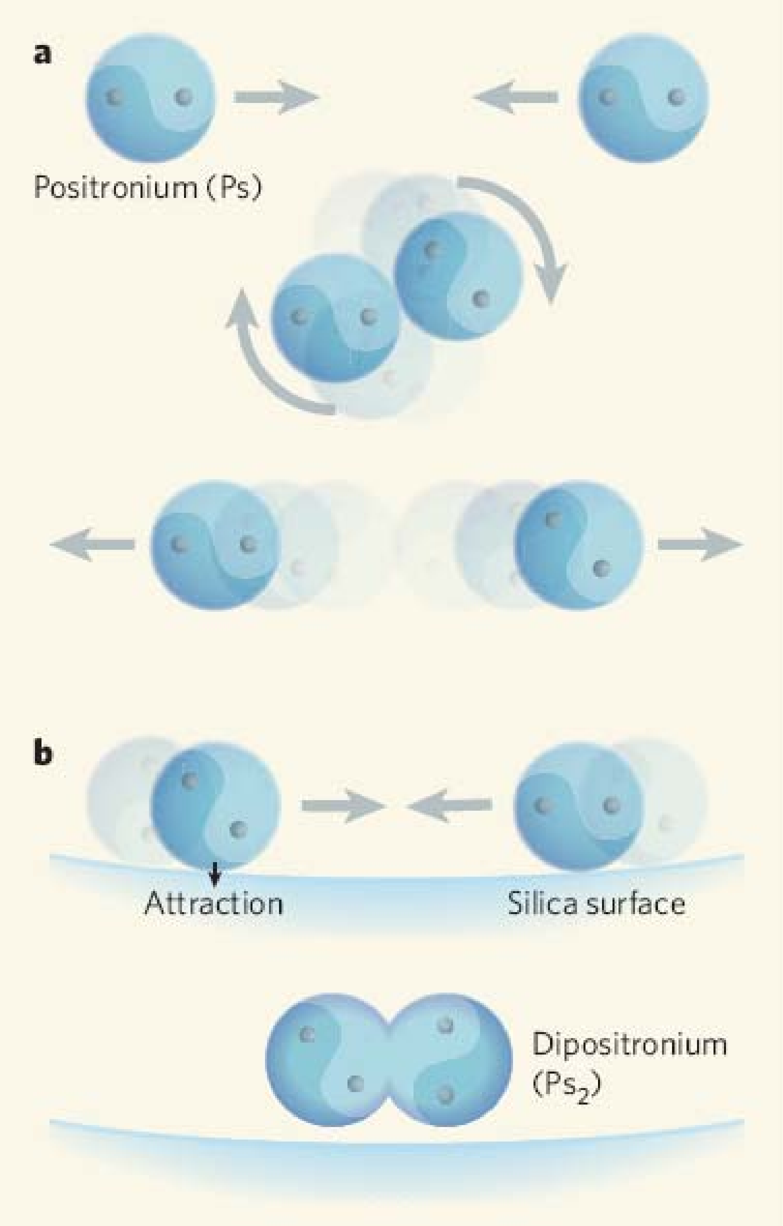
e^-

$(0\gamma)\ 2\gamma\ (4\gamma)$

$(1\gamma)\ 3\gamma\ (5\gamma)$

e^+

Discovery of dipositronium 2007

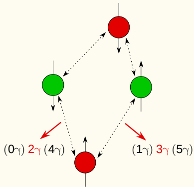


Molecule formation kills long-lived positronia.

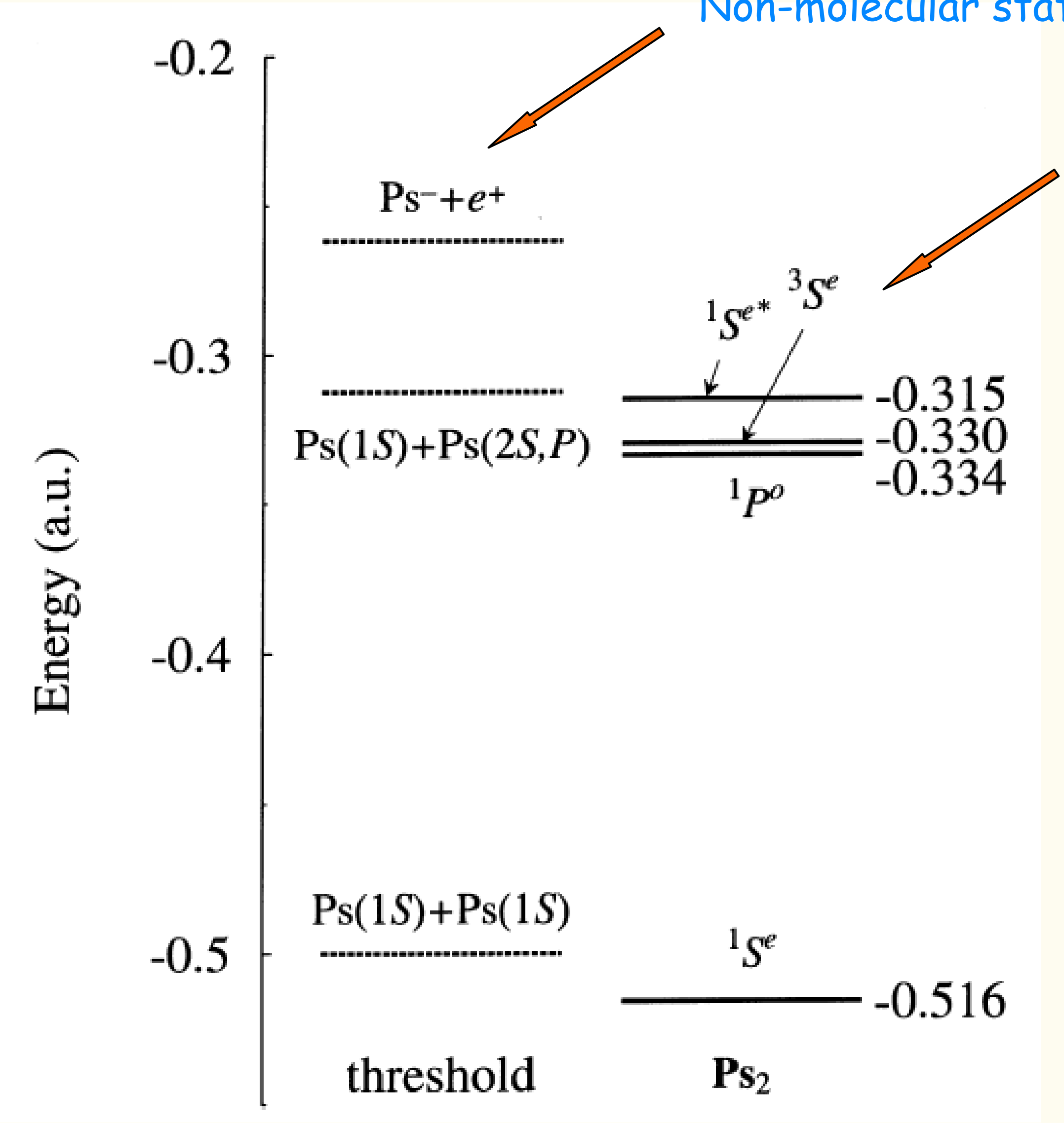
At higher temperature,
fewer atoms on the surface,
fewer molecules formed.

Indeed: at high-T, more long-lived positronia observed.

Cassidy & Mills, Nature 2007



Spectrum of the molecule Ps_2

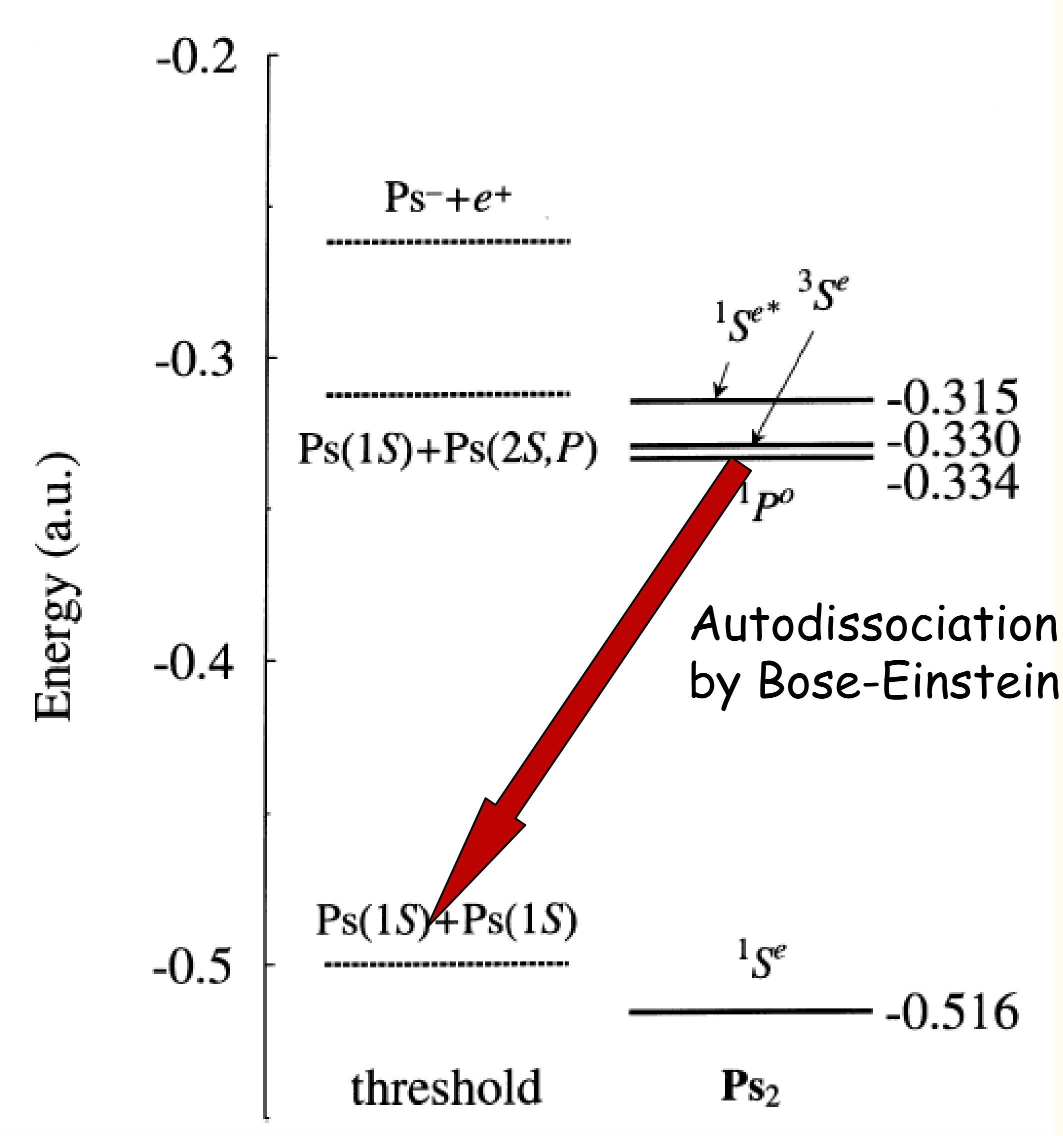


Non-molecular states of $e^- e^- e^+ e^+$

Molecular states

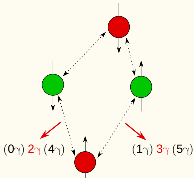
From Suzuki & Usukura, 2000

A direct signal of the molecule: transition line.

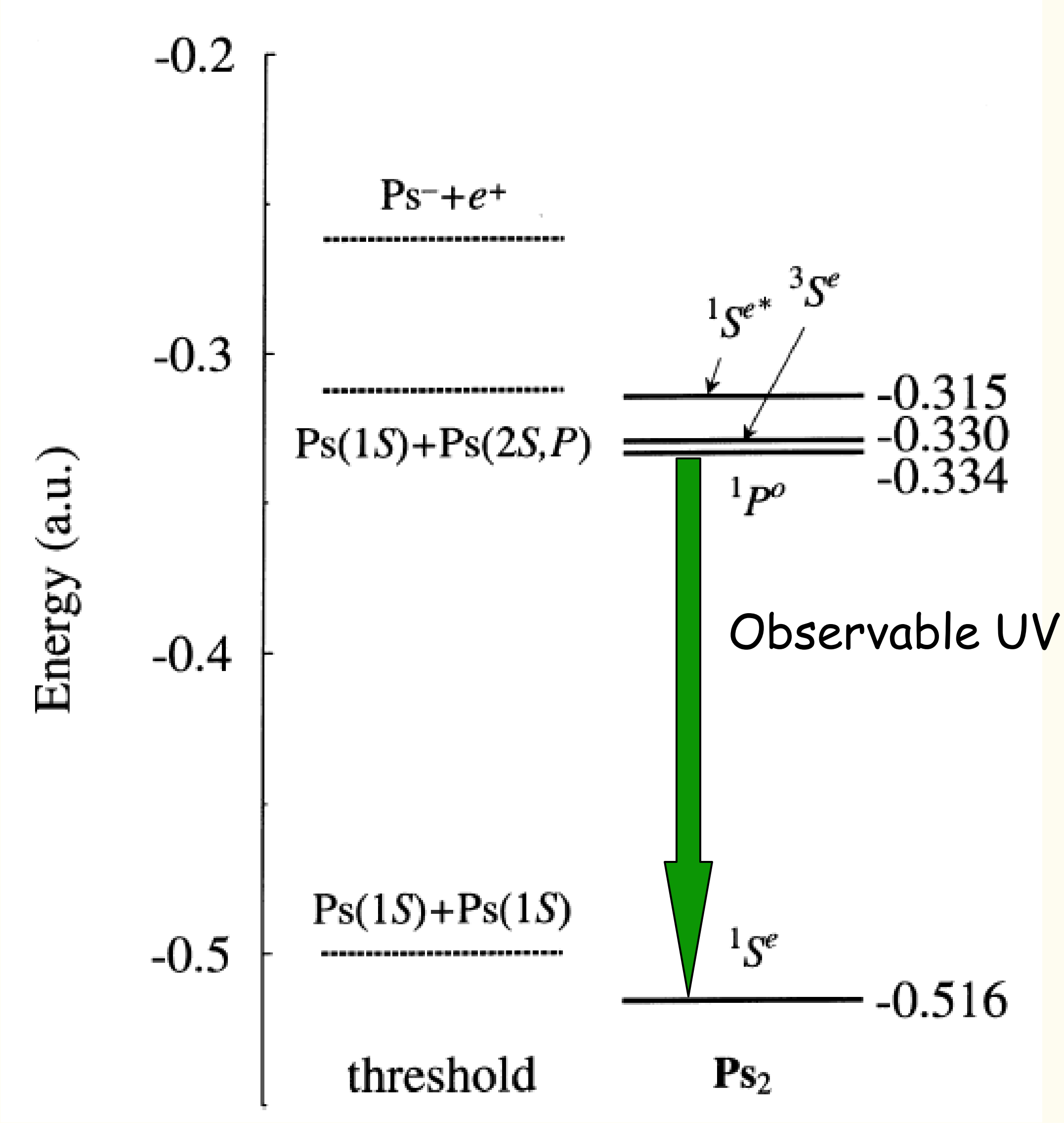


Autodissociation forbidden
by Bose-Einstein statistics

From Suzuki & Usukura, 2000



A direct signal of the molecule: transition line.



Observable UV transition

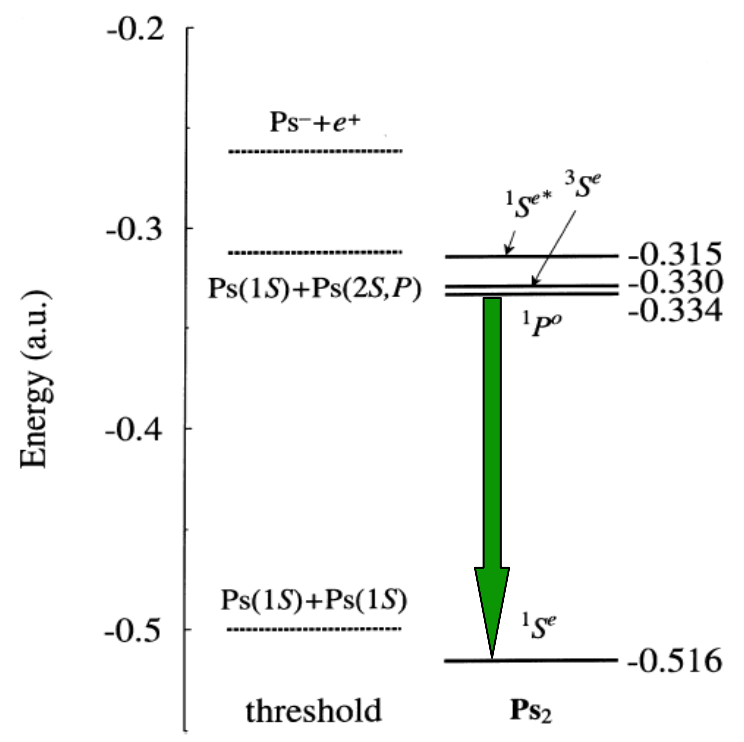
From Suzuki & Usukura, 2000

with Puchalski,
PRL 101, 183001 (2008)

Questions about this transition:

What is its accurate energy?

$$\Delta E = E_P - E_S = 0.1815867(8)\,\text{a.u.}$$
$$\simeq 4.9\,\text{eV}$$



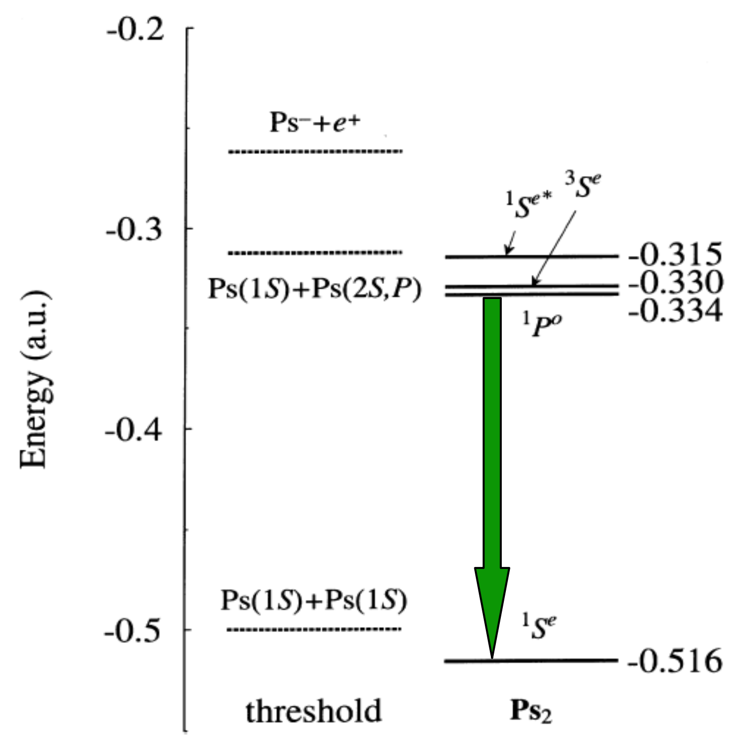
Similar to atomic positronium,
but softer (dielectric effect?):

$$E_P - E_S = \frac{3}{4}\times\frac{1}{4}\,\text{a.u.}=0.1875\,\text{a.u.}$$

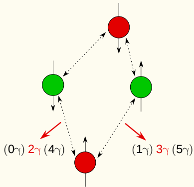
with Puchalski,
PRL 101, 183001 (2008)

Questions about this transition:

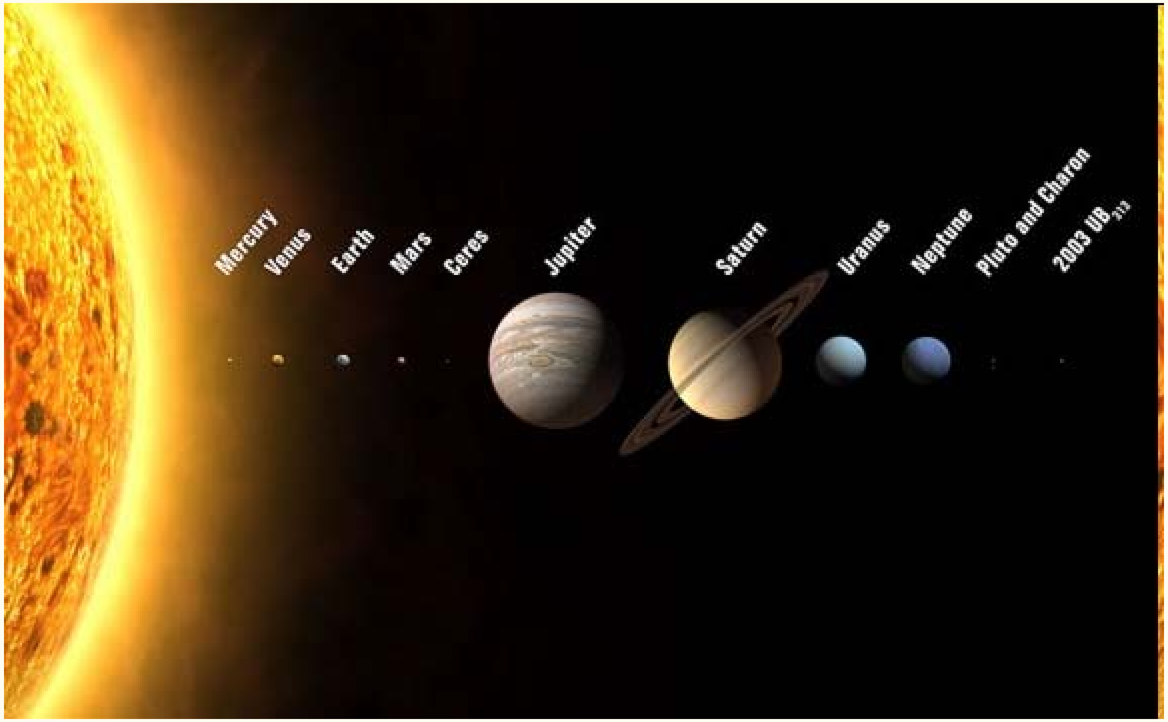
How often does radiative transition appear (before annihilation)?



$$\mathrm{BR}\left(P \to S\right) = \frac{\Gamma_{\mathrm{dip}}\left(P \to S\right)}{\Gamma_{\mathrm{annih}}\left(P\right) + \Gamma_{\mathrm{dip}}\left(P \to S\right)} = 0.191(2)$$



Muonic atoms



Muonic hydrogen Lamb shift and the proton radius

Slides on this topic are not included in this version; please contact Randolf Pohl randolf.pohl@mpq.mpg.de for detailed information about the recent PSI results.

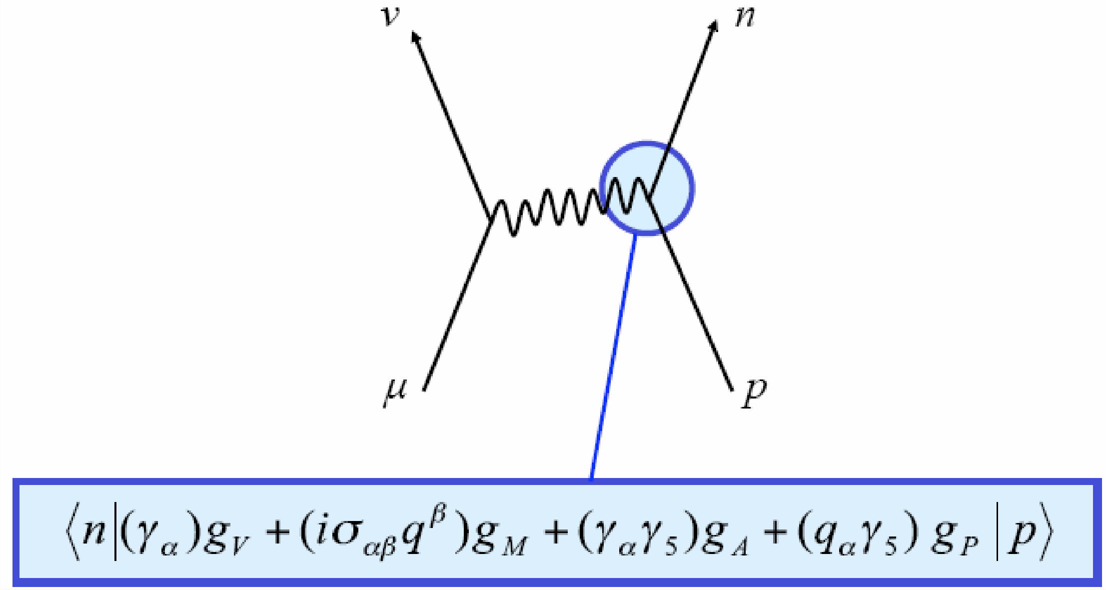
Searches for Lepton Flavor Violation

We heard yesterday about MEG: now 10^{-11}, goal ~10^{-13}

Muon-electron conversion: Fermilab proposal Mu2E: 10^{-16}

New idea: $\mu^- e^- \rightarrow e^- e^-$ Koike et al, 1003.1578,
in a large-Z atom. Competition with muon capture.

Muon capture



$$\langle n|(\gamma_\alpha)g_V + (i\sigma_{\alpha\beta}q^\beta)g_M + (\gamma_\alpha\gamma_5)g_A + (q_\alpha\gamma_5)\,g_P\,|p\rangle$$

Theory and experiment agree, after years of confusion.

Average of HBChPT calculations of Λ_S:

$$(687.4\ \mathrm{s}^{-1} + 695\ \mathrm{s}^{-1})/2 = 691.2\ \mathrm{s}^{-1}$$

Apply new rad. correction (2.8%):

$$(1 + 0.028)691.2\ \mathrm{s}^{-1} = 710.6\ \mathrm{s}^{-1}$$

→

further sub percent theory required

$\Lambda_S^{\mathrm{Theory}} = 710.6\ \mathrm{s}^{-1}$

PRL **99**, 032001 (2007)

$\Lambda_S^{\mathrm{MuCap}} = 725.0 \pm 13.7_{\mathrm{stat}} \pm 10.7_{\mathrm{sys}}\ \mathrm{s}^{-1}$

Summary

Continuing progress in determination of α and m_e.

New opportunities to test QED with three- and four-body bound states.

An open problem: organization of the perturbative series; origin of dominant corrections.