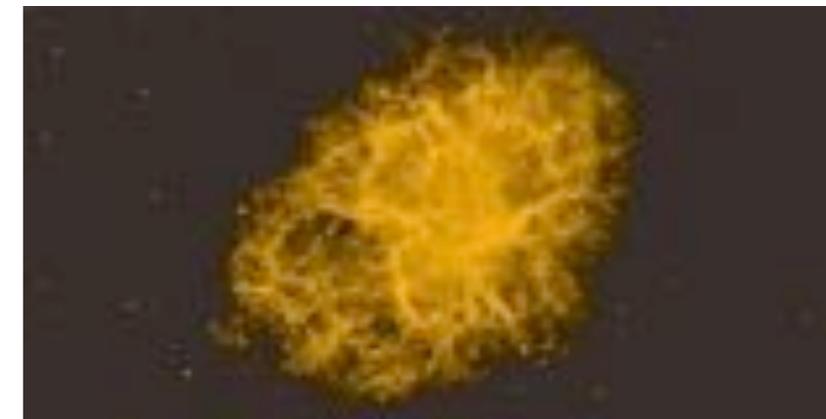


# Kaon Physics

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# Indirect searches for new physics

Search of remnants of new physics:

In decays induced by

higher-dimensional operators ← new particles

Yet the standard model will also induce

higher-dimensional operators ← W, Z & top

NP competes with SM  $\frac{M_{SM}}{M_{NP}}$  need high statistics

If SM contribution is suppressed: NP sensitivity enhanced

# Interesting Topics (incomplete)

very clean

- (Semi-)leptonic Kaon Decays

Lepton Universality &  
CKM Unitarity

clean and suppressed by  $V_{ts}^* V_{td}$

- Rare Kaon Decays

SM Prediction  
MSSM & new light particles

- CP violation in  $\epsilon_K$

new NNLO results

# Leptonic and Semileptonic

$$K(\pi) \rightarrow l \bar{\nu}_l \quad \& \quad K \rightarrow \pi l \bar{\nu}_l$$

## Observables

$$\Gamma(K_{l3}) \quad |V_{us}| f_+(0) = 0.21661(47)$$

$$\frac{\Gamma(K_{l2})}{\Gamma(\pi_{l2})} \quad \frac{|V_{us}| f_K}{|V_{ud}| f_\pi} = 0.27599(59)$$

[FlavianetKaon '08]

and nuclear  $\beta$  decay  $V_{ud} = 0.97425(22)$

[Hardy, Towner '08]

$$\begin{aligned} \Delta_{CKM} &= |V_{ud}^2| + |V_{us}^2| + |V_{ub}^2| - 1 \\ &= (0.1 \pm 0.6) \times 10^{-3} \end{aligned}$$

# CKM Unitarity (Model Independent)

[Cirigliano et. al. '09]

$\Lambda_{NP} \gg M_W$  Neglect  $\mathcal{O}\left(\frac{M_W}{\Lambda_{NP}}\right)$  corrections

Use  $SU(2) \otimes U(1)$  invariant operators [Buchmüller-Wyler '06]  
(plus  $U(3)^5$  flavour symmetry)

$$O_{lq}^{(3)} = (\bar{l}\gamma^\mu \sigma^a l)(\bar{q}\gamma_\mu \sigma^a q) \quad O_{ll}^{(3)} = \frac{1}{2}(\bar{l}\gamma^\mu \sigma^a l)(\bar{l}\gamma_\mu \sigma^a l)$$

Constrained from EW precision data [Han, Skiba '05]

Redefine

$$G_F(\mu \rightarrow e \nu \bar{\nu}) \rightarrow G_F(1 - 2\bar{\alpha}_{ll}^{(3)}) \longrightarrow G_F^\mu$$
$$G_F(d \rightarrow u e \bar{\nu}) \rightarrow G_F(1 - 2\bar{\alpha}_{lq}^{(3)}) \longrightarrow G_F^{SL}$$

# CKM Unitarity (Model Independent)

$$V_{udi}^{\text{PDG}} = \frac{G_F^{\text{SL}}}{G_F^\mu} V_{udi} \longrightarrow \Delta_{\text{CKM}} = 4 \left( \bar{\alpha}_{ll}^{(3)} - \bar{\alpha}_{lq}^{(3)} + \dots \right)$$

[Cirigliano et. al. '09]

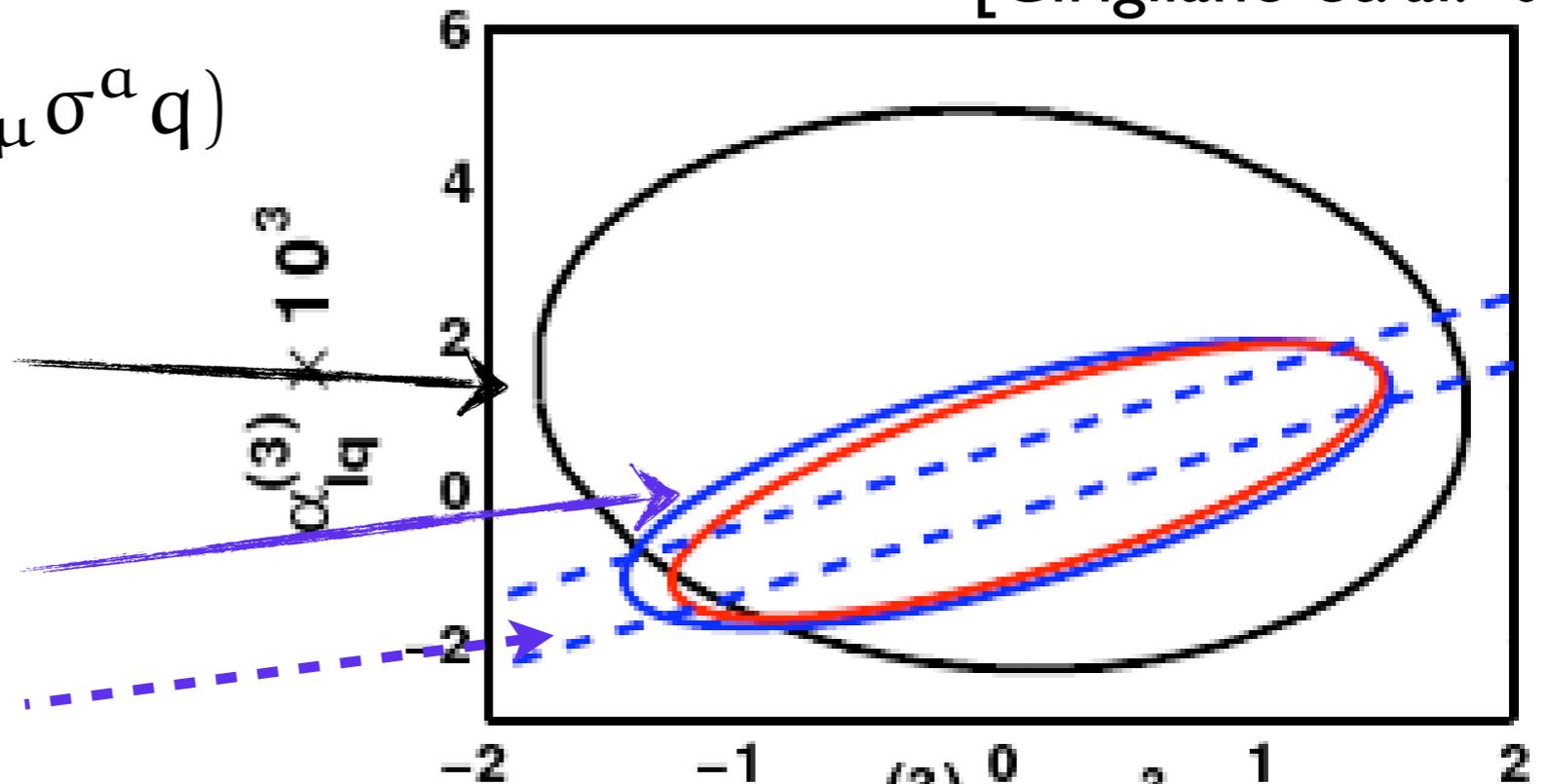
$$O_{lq}^{(3)} = (\bar{l}\gamma^\mu \sigma^a l)(\bar{q}\gamma_\mu \sigma^a q)$$

from HEP

HEP + CKM

CKM

$$\Lambda_{\text{NP}} > 10\text{TeV}$$



$$O_{ll}^{(3)} = \frac{1}{2} (\bar{l}\gamma^\mu \sigma^a l)(\bar{l}\gamma_\mu \sigma^a l)$$

# Leptonic and Semileptonic

$$K(\pi) \rightarrow l \bar{\nu}_l \quad \& \quad K \rightarrow \pi l \bar{\nu}_l$$

## Observables

$$R_K = \frac{\Gamma(K \rightarrow e \bar{\nu})}{\Gamma(K \rightarrow \mu \bar{\nu})} \quad R_K^{SM} = 2.477(1) \times 10^{-5}$$

[Cirigliano, Rosell '07]  
See also [Marciano, Sirlin '93]

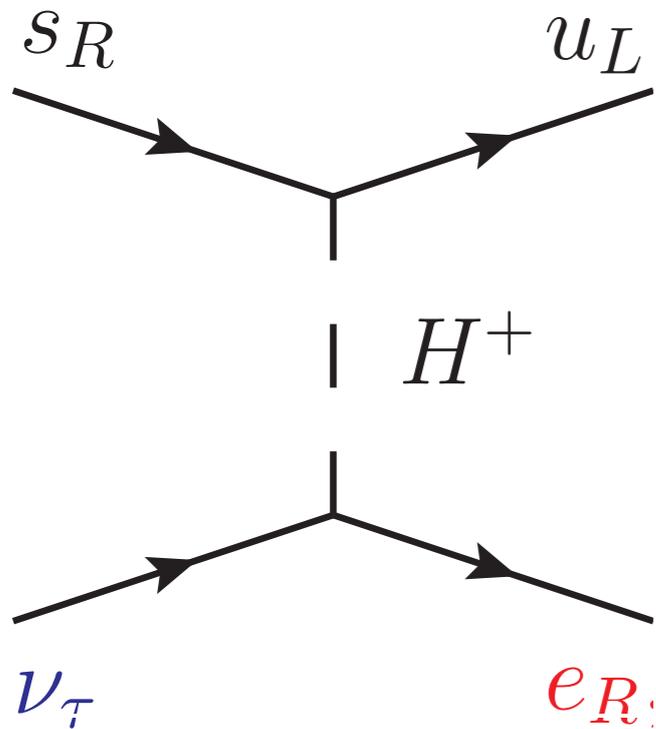
$$R_K^{NA62} = 2.500(16) \times 10^{-5}$$

$$R_K^{KLOE} = 2.493(25)(19) \times 10^{-5}$$

[numbers from KAON09]

Test of lepton universality violation  
driven by experimental precision

# Lepton Universality in the MSSM



LF Conserving:  $\sim$  lepton mass

Lepton Flavour Violation:  $\Delta_R^{31} \sim \frac{g_2^2}{16\pi^2} \delta_{RR}^{31}$   
 [Masiero, Paradisi, Petronzio '08]

$$R_K^{\text{LFV}} = \frac{\Gamma_{SM}(K \rightarrow e \nu_e) + \Gamma_{SM}(K \rightarrow e \nu_\tau)}{\Gamma_{SM}(K \rightarrow \mu \nu_\mu)}$$

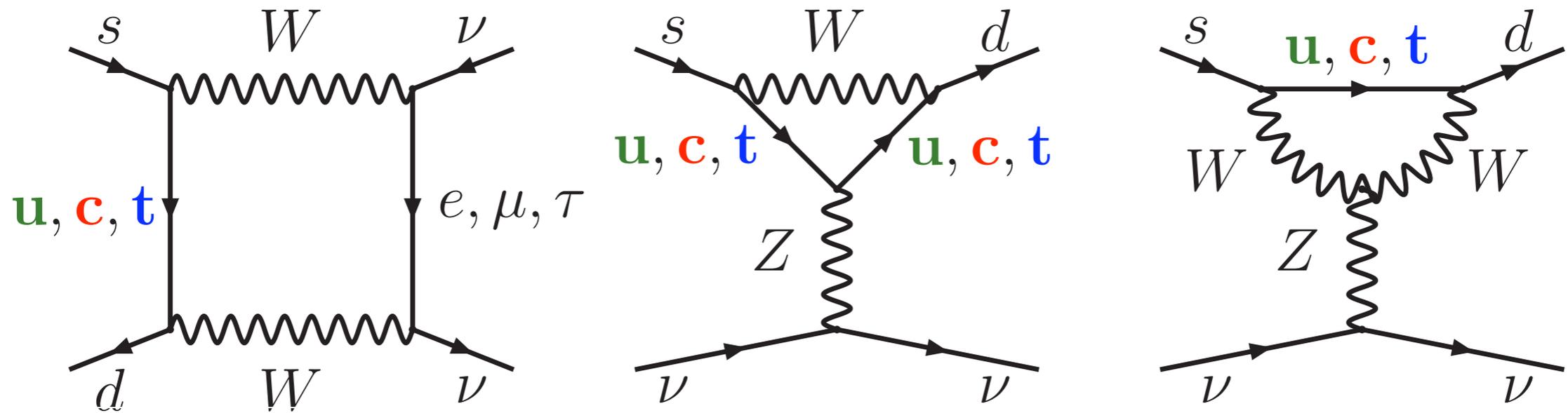
$$\Delta r_K \sim \frac{m_K^4}{m_{H^+}^4} \frac{m_\tau}{m_e} |\Delta_R^{31}|^2 \tan^6 \beta \longrightarrow \text{can reach } 10^{-2}$$

But: finetuning of  $m_e$  necessary [Girrbach et. al. '09]

Modelindependent MLFV and GUT analysis

[Isidori et. al. '09]

# Introduction: $K \rightarrow \pi \nu \bar{\nu}$



- Dominant Operator:  $Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td} (F(x_t) - F(x_u)) + V_{cs}^* V_{cd} (F(x_c) - F(x_u))$$

Quadratic GIM:

$$\lambda^5 \frac{m_t^2}{M_W^2}$$

$$\lambda \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c}$$

$$\lambda \frac{\Lambda^2}{M_W^2}$$

Use isospin symmetry and normalise to:  $K^+ \rightarrow \pi^0 e^+ \nu$

# $s \rightarrow d$ and New Physics (NP)

$$\begin{array}{lll} b \rightarrow s: & b \rightarrow d: & s \rightarrow d: \\ |V_{tb}^* V_{ts}| \propto \lambda^2 & |V_{tb}^* V_{td}| \propto \lambda^3 & |V_{ts}^* V_{td}| \propto \lambda^5 \end{array}$$

Rare K Decays: Additional Cabibbo suppression  $\lambda^5$

$$\mathcal{L}_{\text{eff}} = \frac{C(b \rightarrow s)}{\Lambda_{\text{NP}}^2} (\bar{b}\Gamma s)(\bar{\nu}\Gamma\nu) + \frac{C(b \rightarrow d)}{\Lambda_{\text{NP}}^2} (\bar{b}\Gamma d)(\bar{\nu}\Gamma\nu) + \frac{C(s \rightarrow d)}{\Lambda_{\text{NP}}^2} (\bar{s}\Gamma d)(\bar{\nu}\Gamma\nu)$$

Low NP scale  $\Lambda_{\text{NP}} \simeq 1 \text{ TeV}$

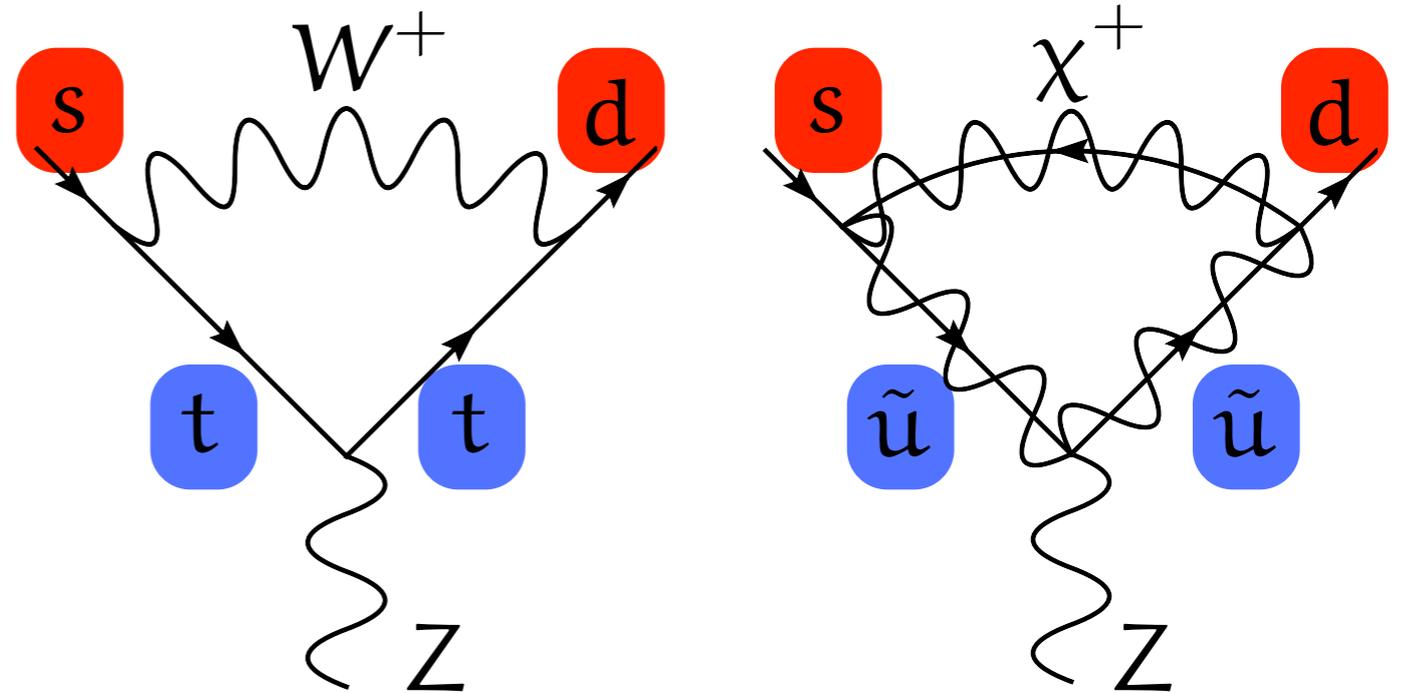
NP Flavour Sector  $C(s \rightarrow d) < \lambda^5$

For Generic NP  $C(s \rightarrow d) \simeq 1$

New Physics scale  $\Lambda_{\text{NP}} > 75 \text{ TeV}$

# Rare K decays and New Physics:

- Test deviation of flavour alignment (Minimal Flavour Violation MFV)



- Precise theory prediction
- Sensitive to small deviations from MFV

$$K_L \rightarrow \pi^0 \bar{\nu} \nu$$

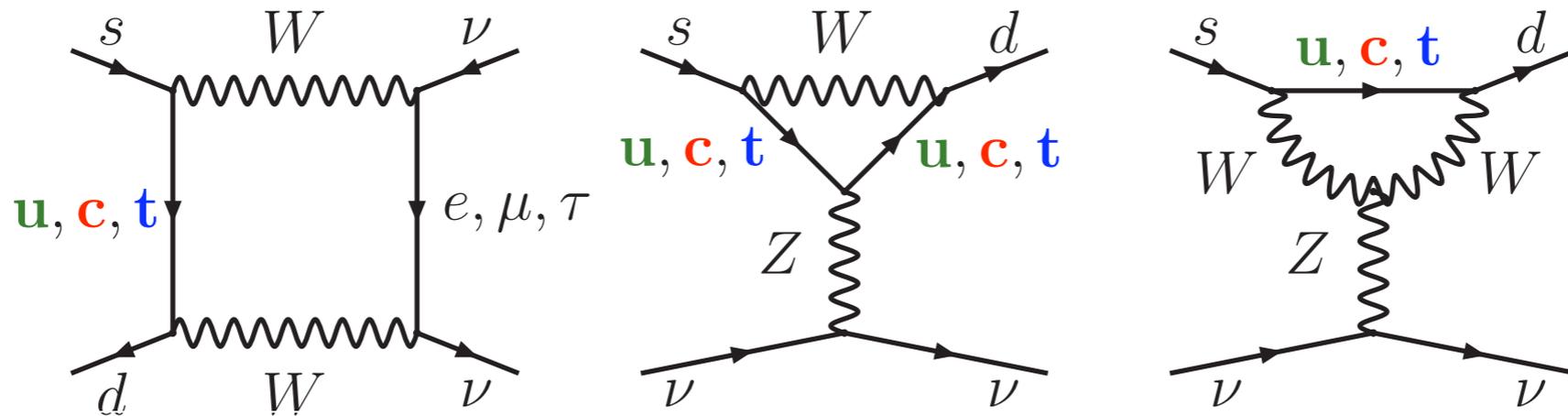
$$K^+ \rightarrow \pi^+ \bar{\nu} \nu$$

also:

$$K_L \rightarrow \pi^0 \mu^+ \mu^-$$

$$K_L \rightarrow \pi^0 e^+ e^-$$

# $K_L \rightarrow \pi^0 \bar{\nu} \nu$ : Effective Hamiltonian



CP violating

Only top quark contributes:  $H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha V_{ts}^* V_{td}}{2\pi \sin^2 \Theta_W} X(x_t) Q_\nu$

Use isospin symmetry and normalise to:  $K^+ \rightarrow \pi^0 e^+ \nu$

$$\text{Br}(K_L \rightarrow \pi^0 \bar{\nu} \nu) = \kappa_L \left( \frac{\text{Im}(V_{ts}^* V_{td})}{\lambda^5} X(x_t) \right)^2$$

# $K_L \rightarrow \pi^0 \bar{\nu} \nu$ : short distance

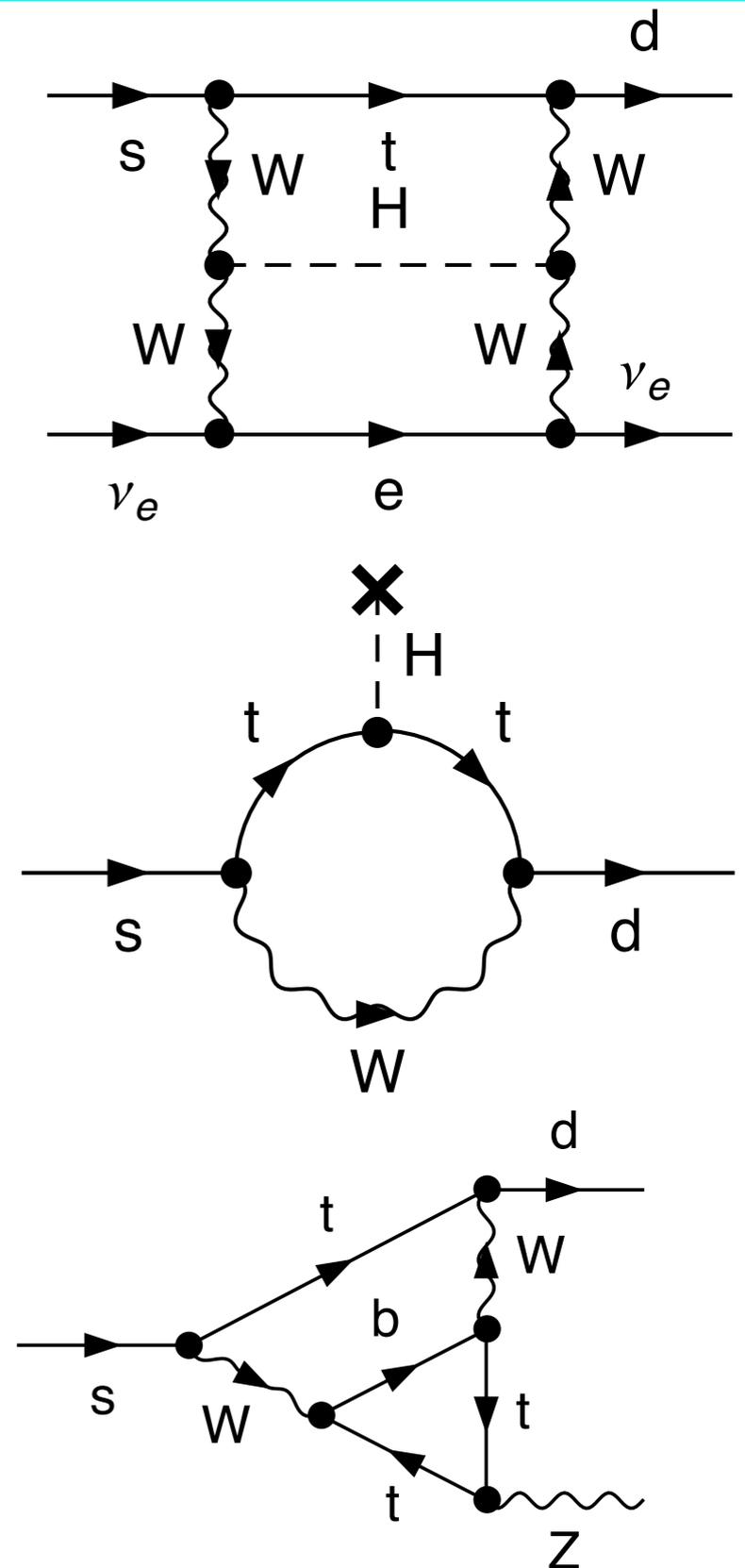
$X_t$  Is purely short distance

- NLO QCD:  $\pm 1\%$  (theory) [Misiak et.al., Buchalla et. al. '99]
- EW corrections large  $m_t$ :  
 $\pm 2\%$  uncertainty [Buchalla, Buras '99]
- $X(x_t)$ : Dominant theory uncertainty for  $K_L \rightarrow \pi^0 \bar{\nu} \nu$
- For example a change  $\sin^{\text{OS}} \theta_W \leftrightarrow \sin^{\overline{\text{MS}}} \theta_W$   
results in 5% uncertainty  $H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \frac{\alpha V_{ts}^* V_{td}}{2\pi \sin^2 \Theta_W} X(x_t) Q_\nu$

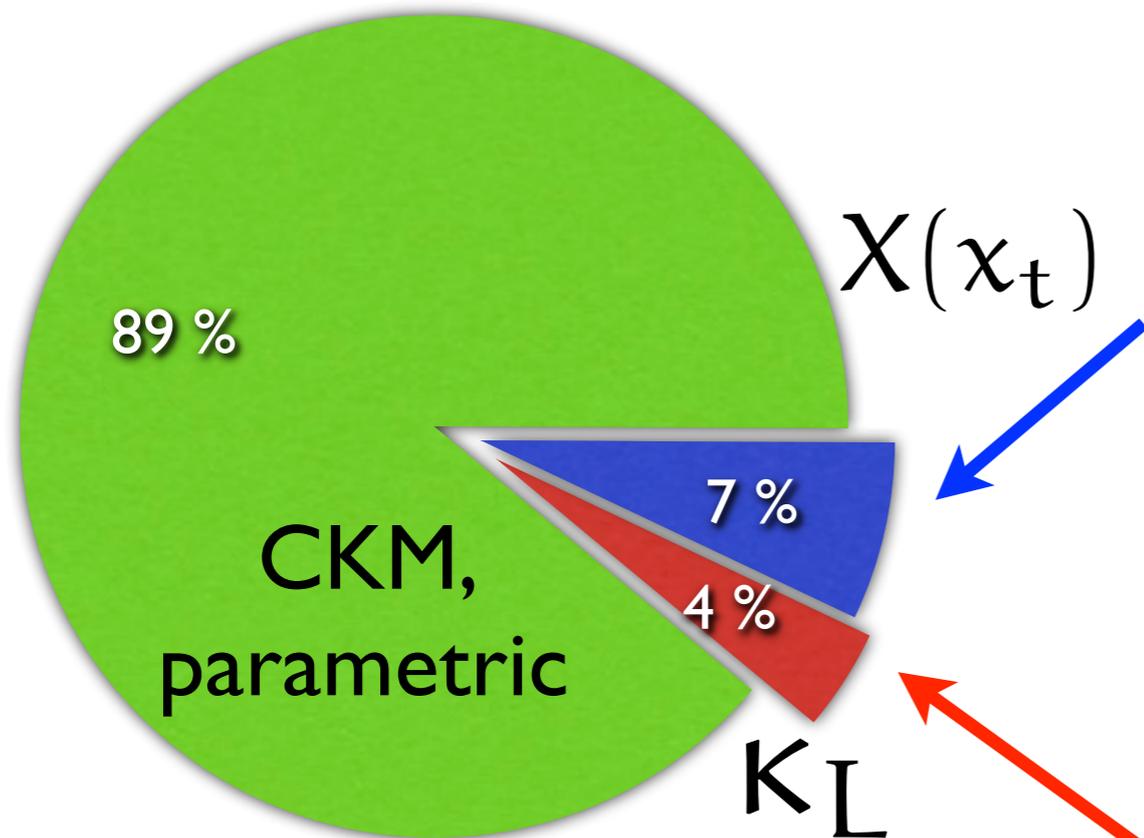
# $X(x_t)$ : Electroweak Corrections

- Use the  $\overline{\text{MS}}$  scheme
- Normalise to  $G_F$
- VEV minimises renormalised potential: include tadpoles
- Traces with  $\gamma_5$ : use HV scheme
- NLO EW: +0.5% shift

[Brod, Gorbahn, Stamou '10]



# $K_L \rightarrow \pi^0 \nu \bar{\nu}$ : Theoretical Status



$X(x_t)$ : Full NLO

electroweak corrections

[Brod, MG, Stamou '10]

Reduce theory uncertainty  
by factor of 2

Matrix element extracted

from  $K_{l3}$  decays.  $N^{\frac{3}{2}}$  LO  $\chi$ PT

[Mescia, Smith '07; Bijmans, Ghorbani '07]

No further long distance  
uncertainty

$$\mathcal{B}r_{K_L} = (2.6 \pm 0.4) \times 10^{-11}$$

$$< 6.7 \times 10^{-8} \text{ [E391a '08]}$$

$$K^+ \rightarrow \pi^+ \bar{\nu} \nu \text{ and } K_L \rightarrow \pi^0 \bar{\nu} \nu$$

Different from  $K_L \rightarrow \pi^0 \bar{\nu} \nu$

- CP conserving: **Top** & **charm** contribute

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}(\gamma)) = \kappa_+ (1 + \Delta_{\text{EM}})$$

$$\times \left| \frac{V_{ts}^* V_{td} \chi_t(m_t^2) + \lambda^4 \text{Re} V_{cs}^* V_{cd} (P_c(m_c^2) + \delta P_{c,u})}{\lambda^5} \right|^2$$

$$\frac{m_c^2}{M_W^2} \text{ suppression lifted by } \log\left(\frac{m_c}{M_W}\right) \frac{1}{\lambda^4}$$

Like in  $K_L \rightarrow \pi^0 \bar{\nu} \nu$

- Only  $Q_\nu$ : Quadratic GIM & Isospin symmetry
- Top quark contribution like in  $K_L \rightarrow \pi^0 \bar{\nu} \nu$

# $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ Long distance

- Matrix element extracted from  $K_{l3}$  decays  
[Mescia, Smith '07]

- $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  is  $K^+ \rightarrow \pi^+ \nu \bar{\nu}(\gamma)$

QED radiative corrections included:

$$\Delta_{EM}(E_\gamma < 20\text{MeV}) = -0.003$$

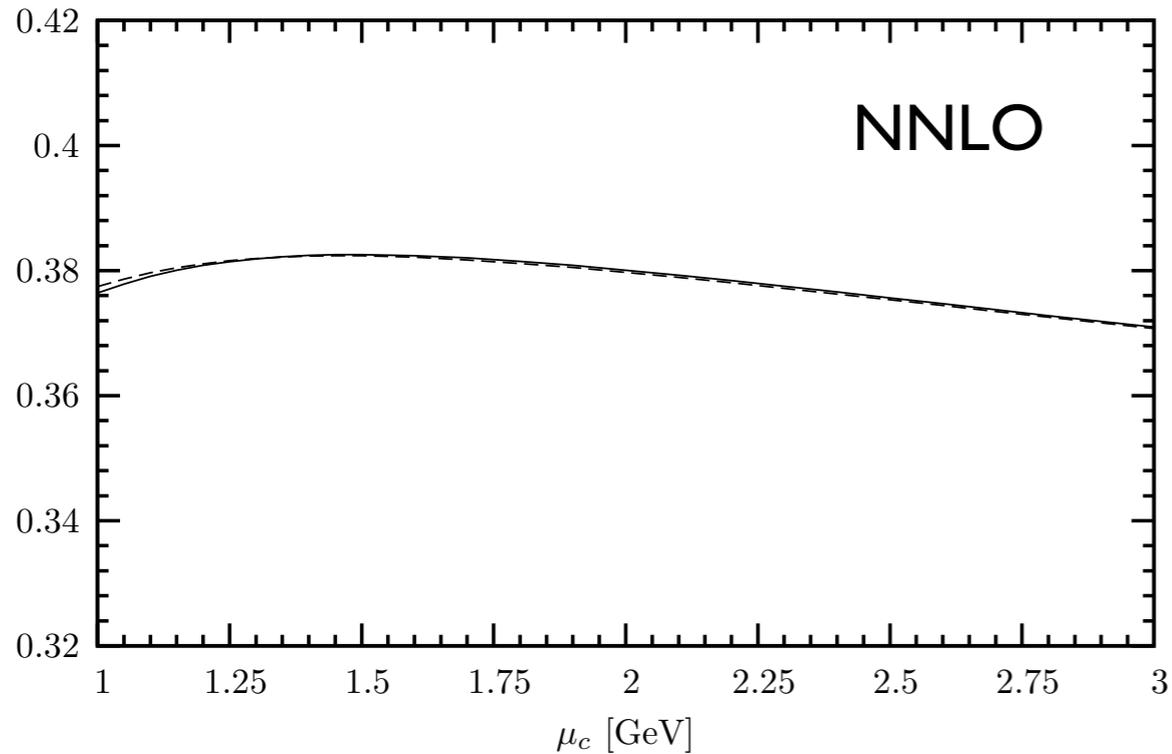
- Uncertainty in  $\kappa_+(1 - \Delta_{EM})$  reduced by  $\frac{1}{7}$

- Below charm scale: Dimension 8 operators  
[Falk et. al. '01]

- Together with light quarks:  $\delta P_{c,u} = 0.04 \pm 0.02$   
[Isidori, Mescia, Smith '05]

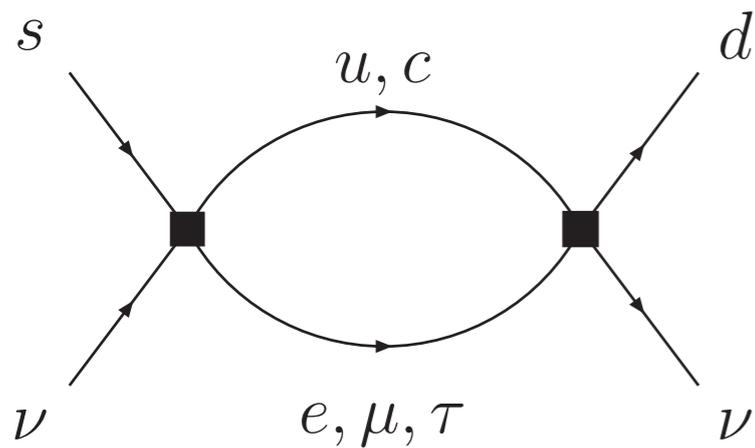
- Could be Improved by Lattice [Isidori et. al. '05]

# $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ charm contribution



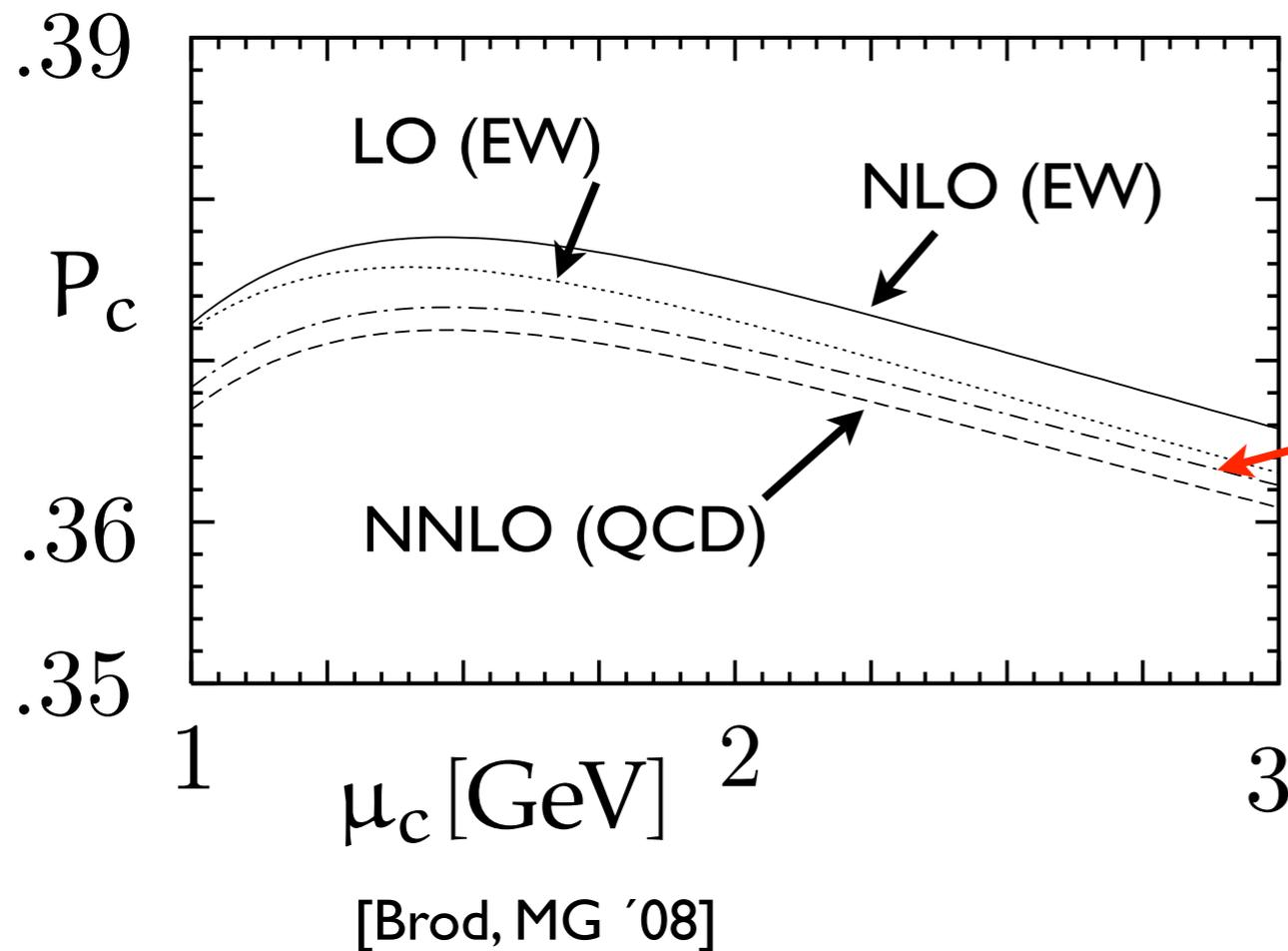
- Resum  $\log \frac{m_c}{M_W}$  in  $P_c$

$P_c$  at NNLO:  $\pm 2.5\%$  (theory)  
[Buras, MG, Haisch, Nierste '06]



- Bilocal mixing is  $\mathcal{O}(G_F^2)$
- What is the parameter  $x_c = \frac{m_c^2}{M_W^2}$
- EW corrections define  $M_W$

# $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ charm contr. (EW)



- Use  $\overline{MS}$  scheme

- Normalise to  $G_F$

- use

$$x_c = \sqrt{2} \frac{\sin^2 \theta_W}{\pi \alpha} G_F m_c^2(\mu_c)$$

- instead of

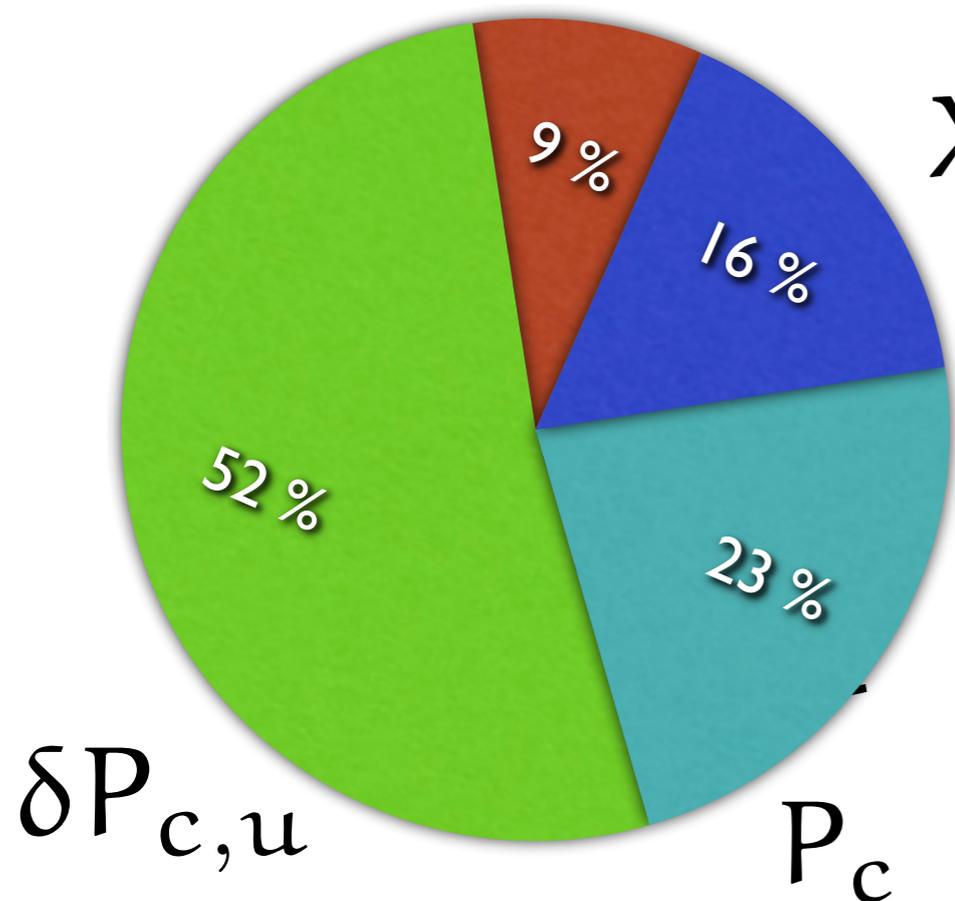
$$x_c = \frac{m_c(\mu)^2}{M_W^2}$$

- $P_c$  enhanced by up to 2% for all EW

# $K^+ \rightarrow \pi^+ \bar{\nu} \nu$ Error budget

## Theory error budget

$K_+$

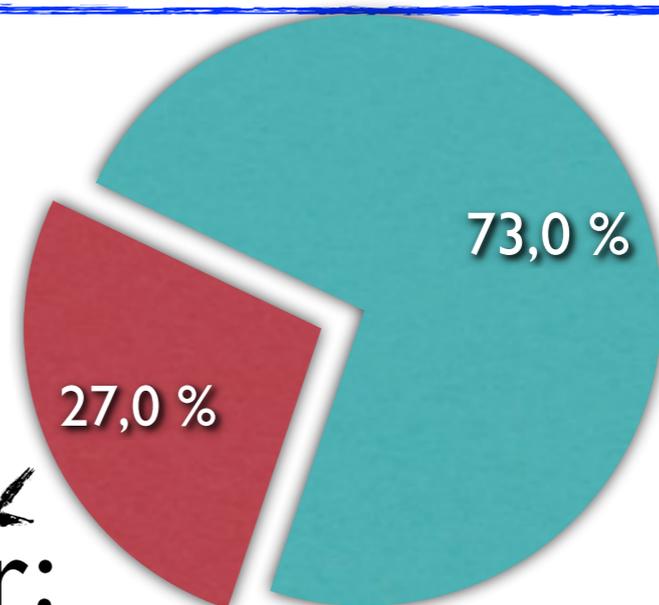


for  $m_c(m_c) = (1286 \pm 13) \text{ MeV}$

[Kühn et. al. '07]

$$\text{Br}_{K^+} = (0.85 \pm 0.07) \times 10^{-10}$$

$X(x_t)$



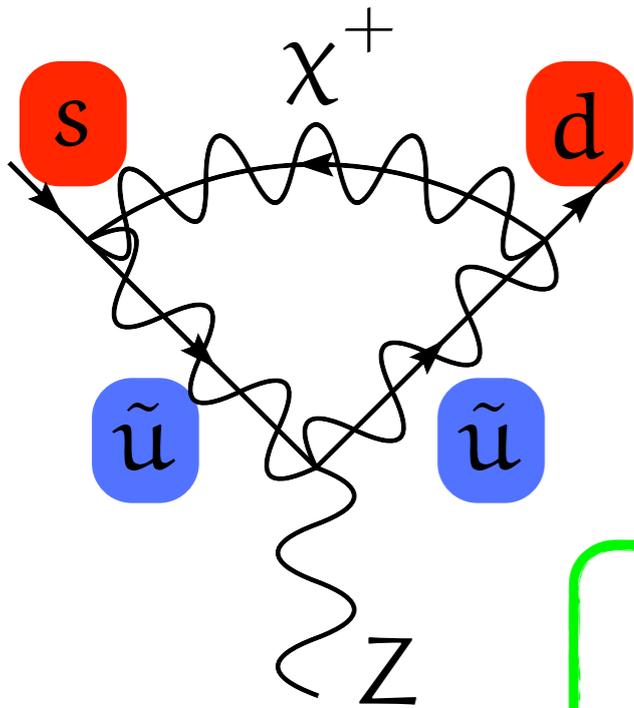
Theory error:  
 $10\% \times 30\% = 3\%$

Parametric  
uncertainty

Experiment [E787, E949 '08]

$$\text{Br}_{K^+} = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$$

# $K \rightarrow \pi \bar{\nu} \nu$ in the MSSM



New physics in:  $X(x_t) \rightarrow X(x_t, \tilde{m}, \tilde{M})$

MSSM is a 2HDM of Type II:

$$\mathcal{L} = -Y_{ij}^d H_d \bar{d}_R^i q^j - Y_{ij}^u H_u \bar{u}_R^i q^j + \text{h.c.}$$

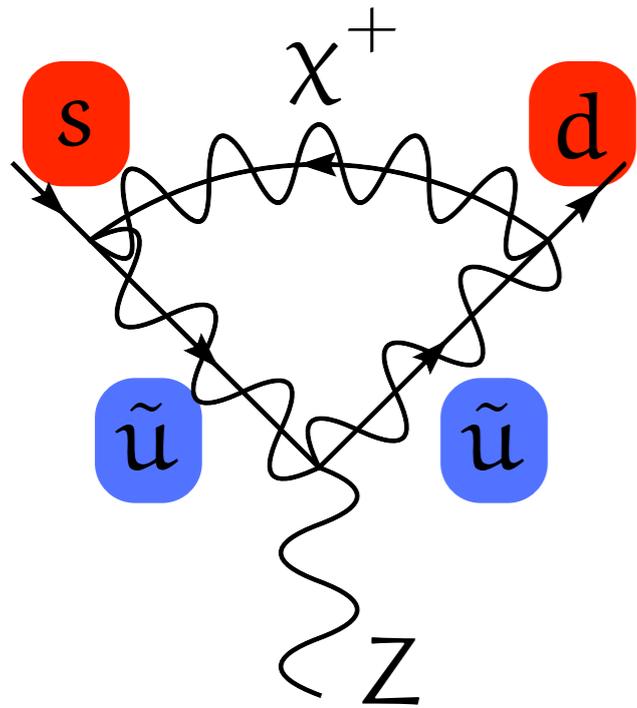
for small  $\tan \beta = v_u / v_d = \mathcal{O}(1)$

Yukawa and mass-matrix aligned

Flavour Violation in squark mass matrix

$$\hat{\mathcal{M}}_{\tilde{u}}^2 = \begin{pmatrix} \hat{M}_{\tilde{u}_L}^2 & v_u \hat{A}_u^\dagger - v_d \mu \hat{Y}_u^\dagger \\ v_u \hat{A}_u - v_d \mu^* \hat{Y}_u & \hat{M}_{\tilde{u}_R}^2 \end{pmatrix}$$

# $K \rightarrow \pi \bar{\nu} \nu$ in the MSSM: MFV



Minimal Flavour Violation:  
Aligned squarks and quarks

No strong enhancement possible.  
Interesting correlations with other  
observable

[Buras, Gambino, MG, Jäger, Silvestrini '00; Isidori, Mescia, Paradisi, Smith, Trine '06]

Flavour Violation in  
squark mass matrix

$$\hat{\mathcal{M}}_{\tilde{u}}^2 = \begin{pmatrix} \hat{M}_{\tilde{u}_L}^2 & v_u \hat{A}_u^\dagger - v_d \mu \hat{Y}_u^\dagger \\ v_u \hat{A}_u - v_d \mu^* \hat{Y}_u & \hat{M}_{\tilde{u}_R}^2 \end{pmatrix}$$

Diagonal

# $K \rightarrow \pi \bar{\nu} \nu$ and non MFV

Offdiagonal squark mass-matrix: Extra Flavour Violation

Diagonalisation:  
Mass insertions

$$\delta_{LR}^u{}_{ij} = \frac{\hat{M}_{\tilde{u}}^2{}_{i_R j_L}}{\hat{M}_{\tilde{u}}^2}$$

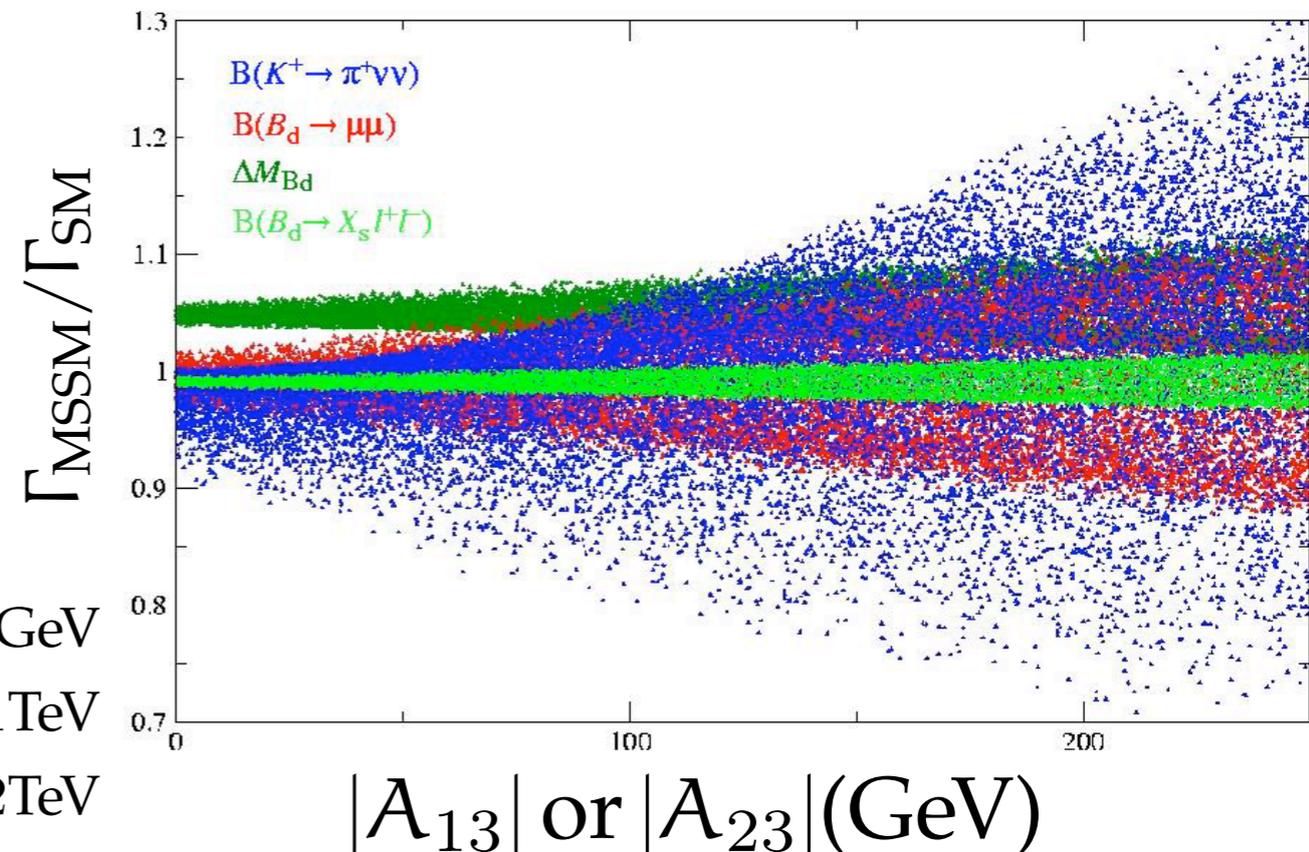
2 LR insertions dominant

[Colangelo, Isidori '98]

$$\chi_{LL}^2 \propto \delta_{LR_{ts}}^{u*} \delta_{LR_{td}}^u$$

No CKM suppression

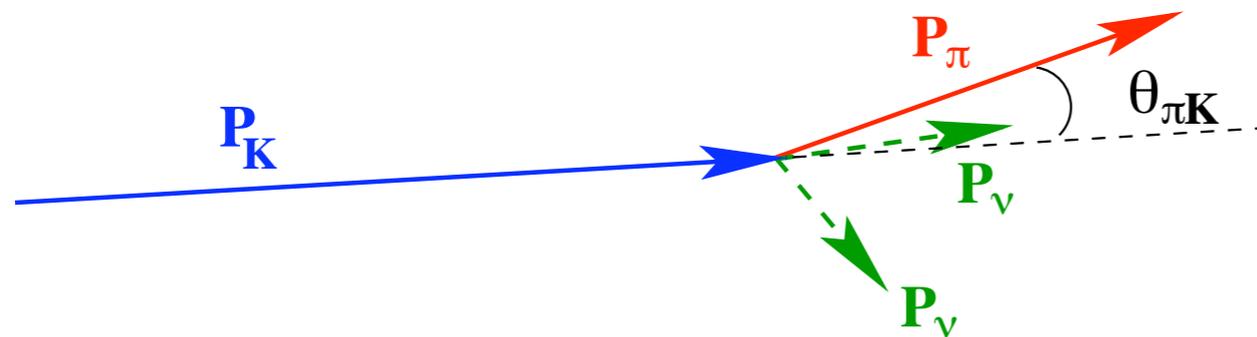
sensitive to  $A_u$  [Isidori, Mescia, Paradisi, Smith, Trine '06]



$$\begin{aligned} \tan \beta &= 2 - 4 & \mu &= 500 \pm 10 \text{ GeV} & M_2 &= 300 \pm 10 \text{ GeV} \\ M_{\tilde{u}_R} &= 600 \pm 20 \text{ GeV} & M_{\tilde{q}_L} &= 800 \pm 20 \text{ GeV} & A_0 &= 1 \text{ TeV} \\ M_1 &= 500 \text{ GeV} & M_{\tilde{d}_R} &= M_{\tilde{l}} = M_3 = M_{\tilde{H}^+} & &= 2 \text{ TeV} \end{aligned}$$

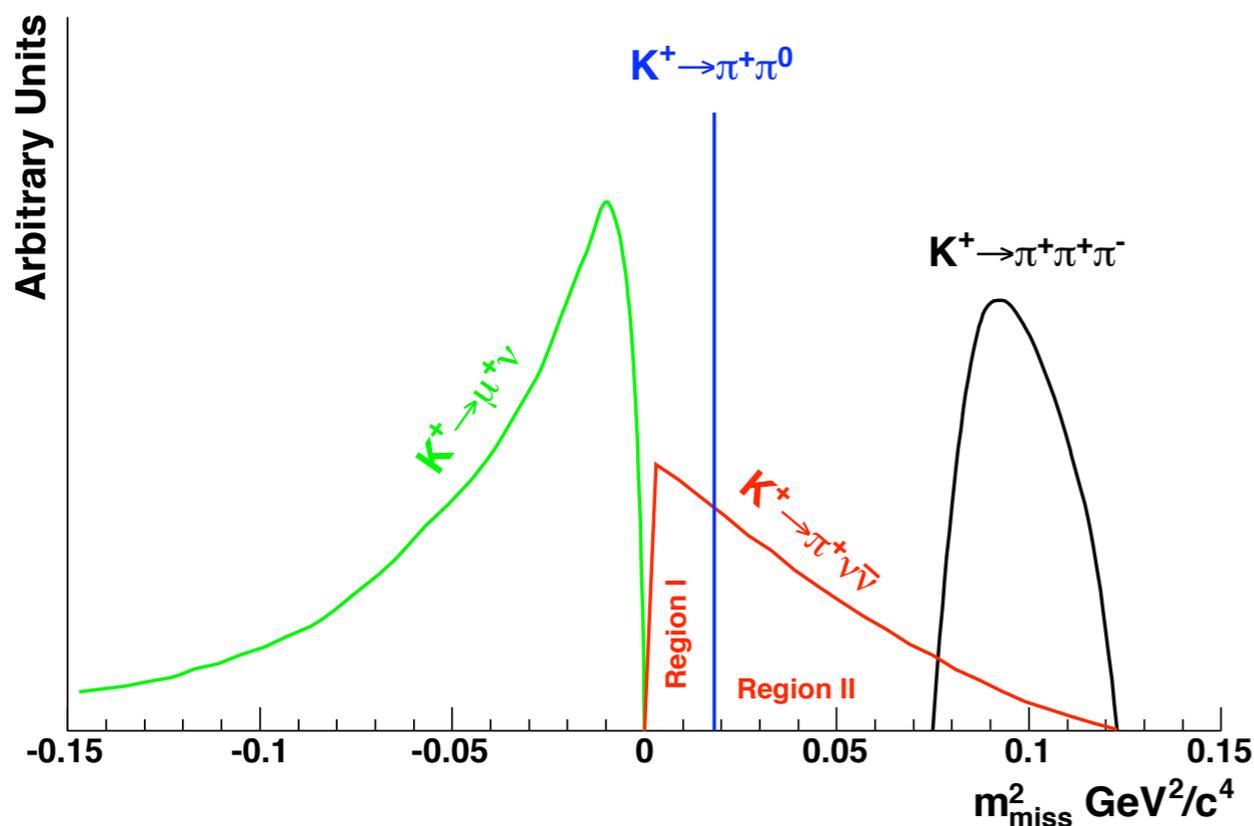
# Beyond the Z-Penguin

Experiment: Background from frequent  $K^+$ -Decays



Measure  $p_\pi$  &  $\theta_{\pi K}$

cut on: 
$$m_{miss}^2 \simeq m_K^2 \left( 1 - \frac{|P_\pi|}{|P_K|} \right) + m_\pi^2 \left( 1 - \frac{|P_K|}{|P_\pi|} \right) - |P_K| |P_\pi| \theta_{\pi K}^2$$



- Can new physics change the shape?

- In all models there is only one operator for

$$K \rightarrow \pi \bar{\nu} \nu$$

# Missing Mass Distribution (NP)

Can new physics change the missing mass distribution?

$K^+ \rightarrow \pi^+ +$  and new light particles:

Sensitivity to the mass

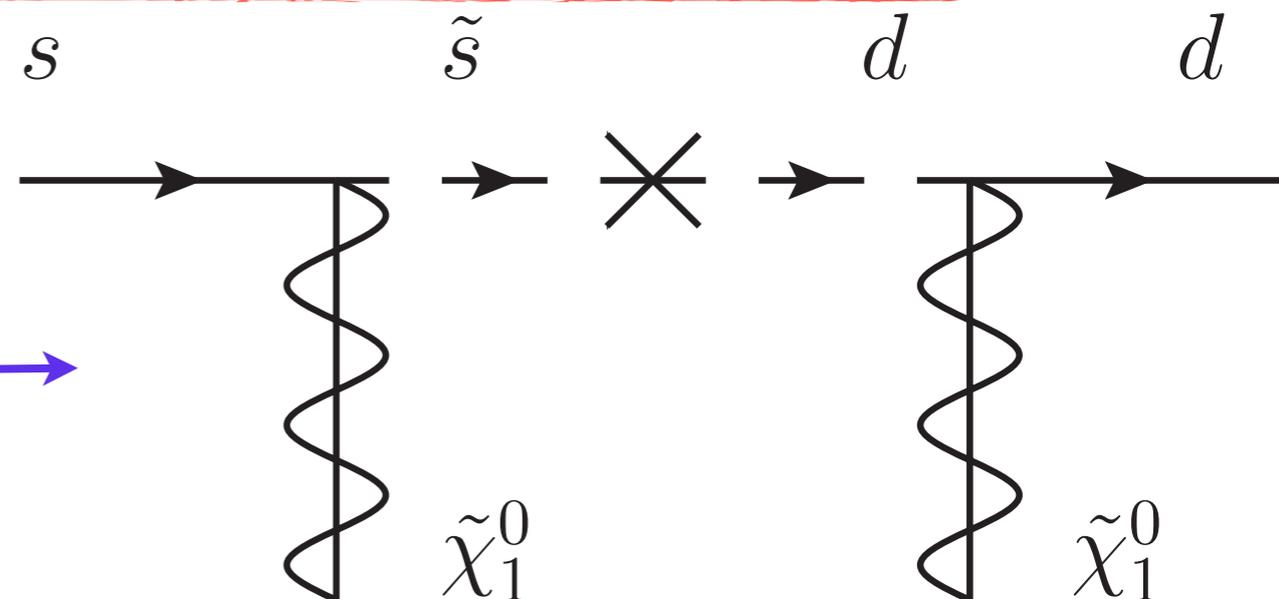
New Operators

Toy example recently studied [Dreiner et. al. '09]:  
light neutralinos:

$$K^+ \rightarrow \pi^+ \tilde{\chi}_1^0 \tilde{\chi}_1^0$$

MFV: one loop

Non-MFV: tree level



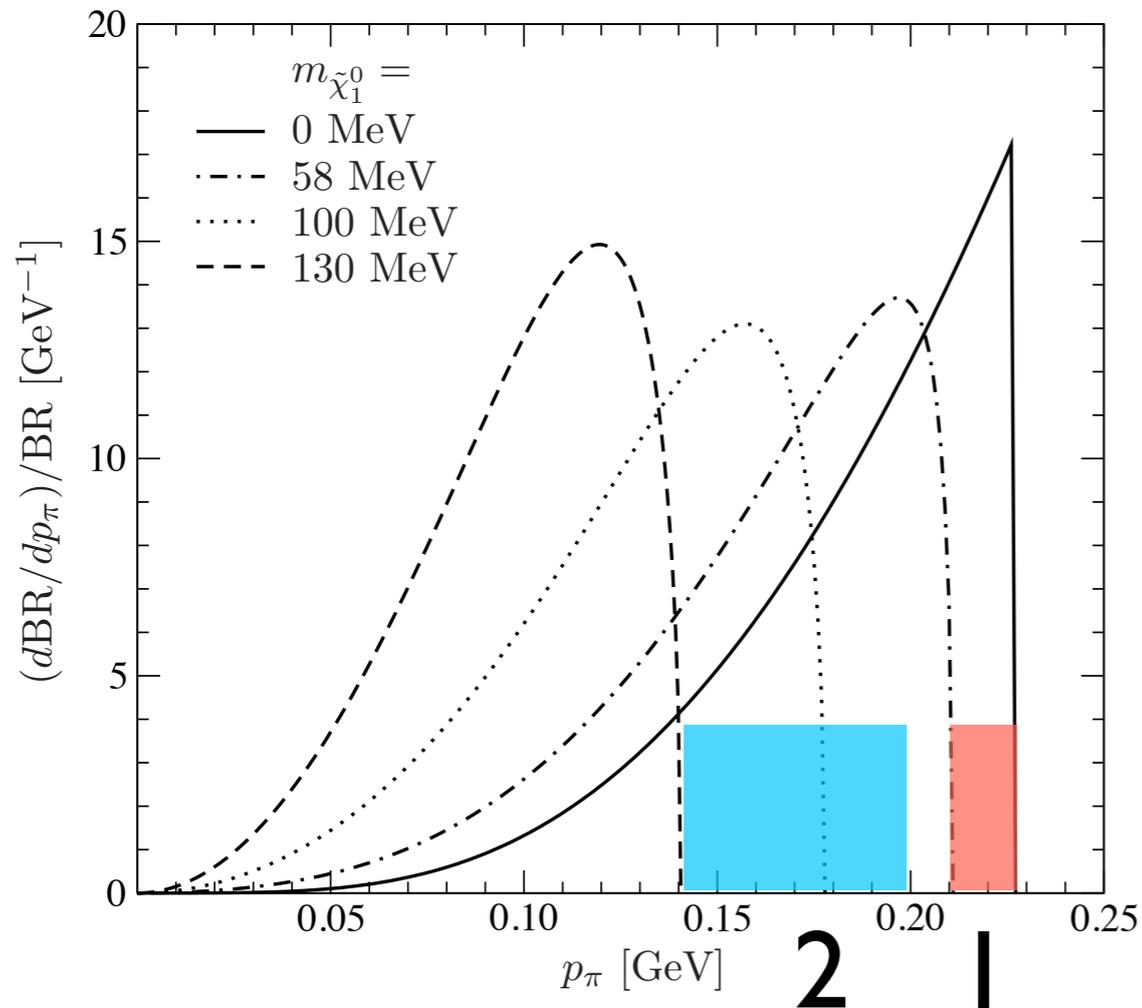
# Light Neutralinos

Decay in very light neutralinos:  $K^+ \rightarrow \pi^+ \tilde{\chi}_1^0 \tilde{\chi}_1^0$  [Dreiner et. al. '09]

in Non-MFV scenarios:

$m_{\tilde{\chi}_1^0} = 0$  :  $\propto$  to  $K^+ \rightarrow \pi^+ \bar{\nu} \nu$

$m_{\tilde{\chi}_1^0} \neq 0$  :  $\not\propto$  to  $K^+ \rightarrow \pi^+ \bar{\nu} \nu$



in MFV scenarios:

effects are smaller but  
new operators like

$$(\bar{s}d)(\tilde{\chi}_1^0 P_L \chi_1^0)$$

appear, yet the SM like

$$(\bar{s}\gamma_\mu d)(\tilde{\chi}_1^0 \gamma^\mu \gamma_5 \tilde{\chi}_1^0)$$

Operator dominates

- Estimate the missing mass distribution for new Ops

# Decay in One Light Boson $\mathcal{P}$

- Peak in the missing mass distribution
- Already constrained by Experiment:  $\mathcal{B}_{K \rightarrow \pi + \mathcal{P}} \lesssim \mathcal{B}_{K \rightarrow \pi + \bar{\nu}\nu}$
- e.g.: Meta-stable SUSY Violation [Banks, Haber '09]

$$\mathcal{L}_{\text{eff}} = \frac{\alpha_2^3 m_t^2}{\Lambda_{\text{ISS}}^3} V_{td} V_{ts}^* \bar{d}(1 - \gamma_5) \gamma^\mu s \partial_\mu \mathcal{P}$$

coupling to light pseudo-Nambu Goldstone boson

SM like Flavour suppression

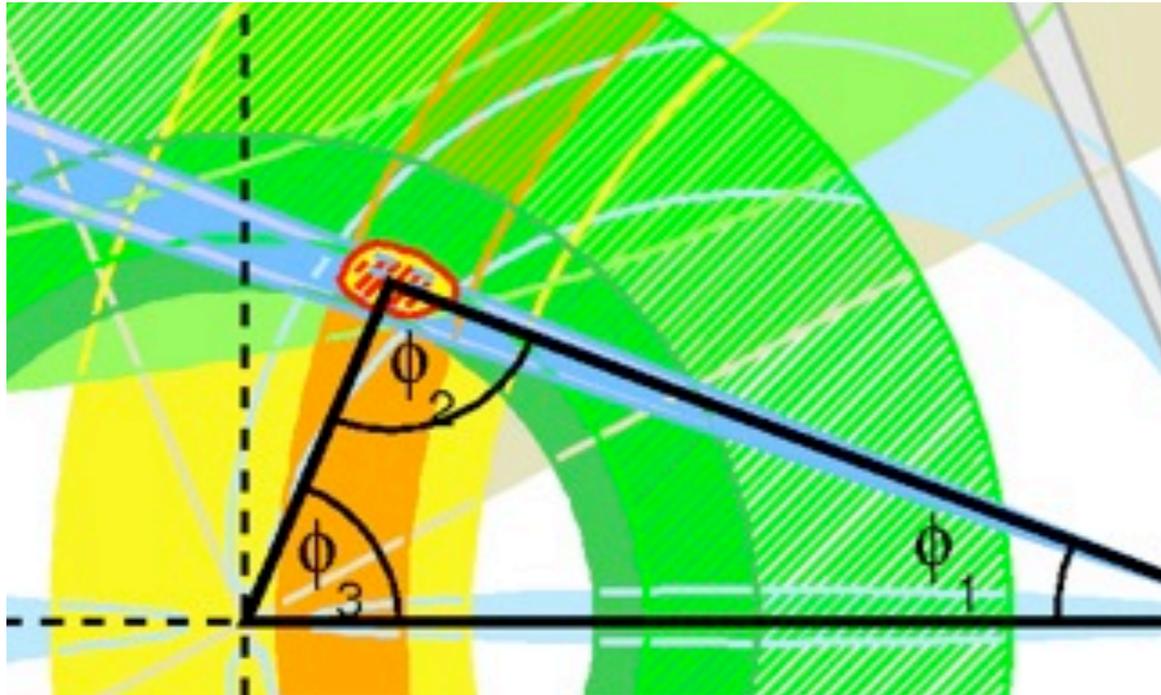
$$\mathcal{B}_{K^+ \rightarrow \pi^+ \mathcal{P}} \sim 5 \cdot 10^{-15} \left( \frac{2\text{TeV}}{\Lambda_{\text{ISS}}} \right)^6 \quad \text{contributes only for small scale } \Lambda_{\text{ISS}}$$

In this model there is also a coupling to electrons:

For light  $m_{\mathcal{P}}$  red star cooling gives a much stronger bound

MFV like & electron coupling: Model dependent

# $\epsilon_K$ : Indirect CP violation



$$\epsilon_K \simeq \frac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle}$$

$$\epsilon_K = e^{i\phi_\epsilon} \sin \phi_\epsilon \left( \frac{\text{Im}(M_{12}^K)}{\Delta M_K} + \xi \right)$$

- In almost all old analysis:  $\phi_\epsilon = 45^\circ$  and  $\xi = 0$
- In reality:  $\xi \neq 0$   $\phi_\epsilon \neq 45^\circ$  [Andriyash et. al.'04]

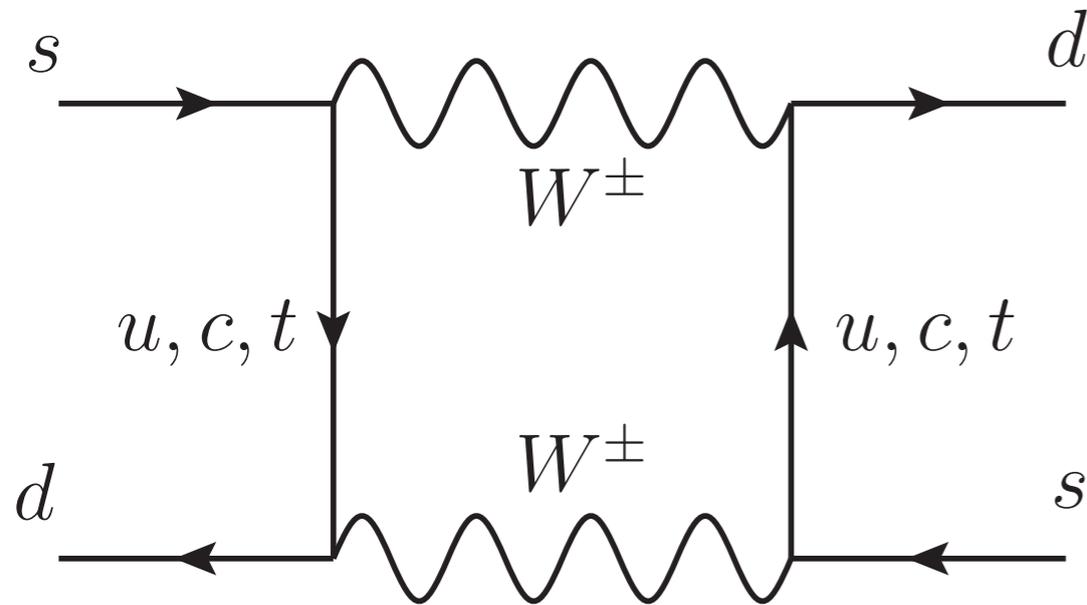
$$|\epsilon_K^{SM}| = \kappa_\epsilon |\epsilon_K| (\phi_\epsilon = 45^\circ, \xi = 0)$$

also similar effect as  $\delta P_{c,u}$  in  $\epsilon_K$

$$\kappa_\epsilon = 0.94 \pm 0.02 \quad [\text{Buras, Guadagnoli, Isidori '10}]$$

# Calculation of $M_{12}^K = \langle K^0 | \mathcal{H}_{eff}^{\Delta S=2} | \bar{K}^0 \rangle$

Box diagram  
with internal **u, c, t**



$$\lambda_i \lambda_j A(x_i, x_j)$$

$$\lambda_i = V_{is}^* V_{id}$$

plus GIM:

$$\lambda_c + \lambda_t = -\lambda_u$$

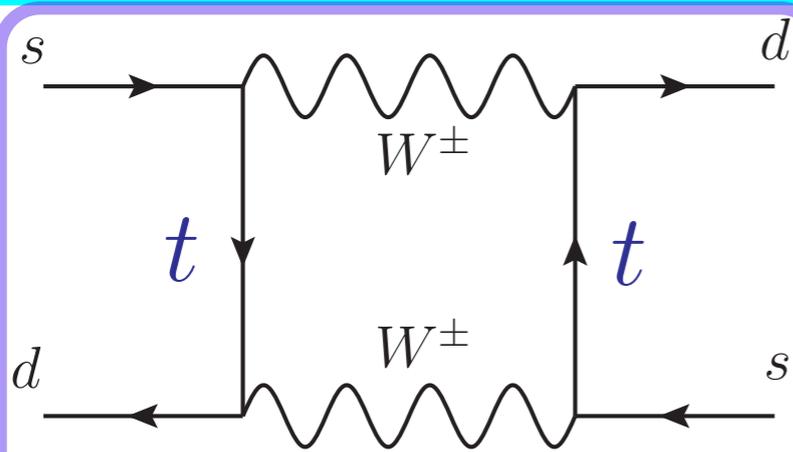
Gives three different  
contributions for

$$M_{12}^K = \langle K^0 | \mathcal{H}_{eff}^{\Delta S=2} | \bar{K}^0 \rangle$$

$$\begin{aligned} \mathcal{H} \propto & \left[ \lambda_t^2 \eta_t S(x_t) \quad \text{top} \right. \\ & + 2\lambda_c \lambda_t \eta_{ct} S(x_c, x_t) \quad \text{charm top} \\ & \left. + \lambda_c^2 \eta_c S(x_c) \right] b(\mu) \tilde{Q} \quad \text{charm} \end{aligned}$$

$$\tilde{Q} = (\bar{s}_L \gamma_\mu d_L) (\bar{s}_L \gamma^\mu d_L)$$

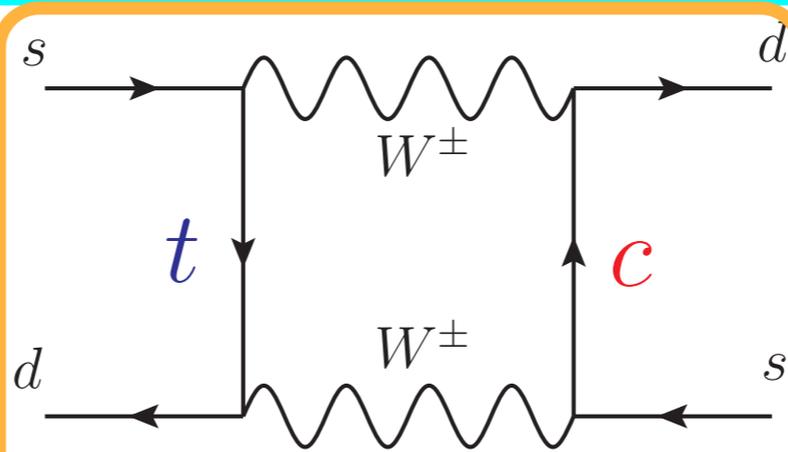
# Calculation of $M_{12}^K = \langle K^0 | \mathcal{H}_{eff}^{\Delta S=2} | \bar{K}^0 \rangle$



**top**  
 $\log \chi_t$

**LO**  $(\alpha_s \log \chi_c)^n$   
**NLO**  $\alpha_s (\alpha_s \log \chi_c)^n$

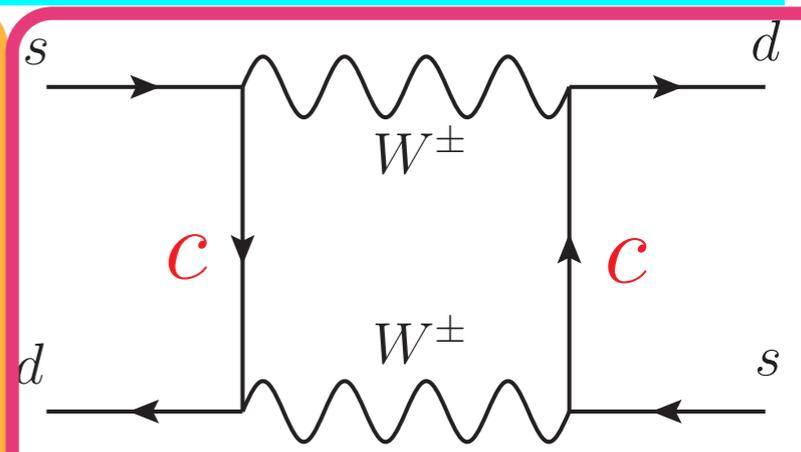
$\epsilon_K$  **75%**  
 scale **1.8%**



**charm top**  
 $\log \chi_c$

$(\alpha_s \log \chi_c)^n \log \chi_c$   
 $(\alpha_s \log \chi_c)^n$

**37%**  
**7.5%**



**charm**  
 $(\log \chi_c)^0$    
**hard GIM**

$(\alpha_s \log \chi_c)^n$   
 $\alpha_s (\alpha_s \log \chi_c)^n$

**-12%**  
**17.7%**

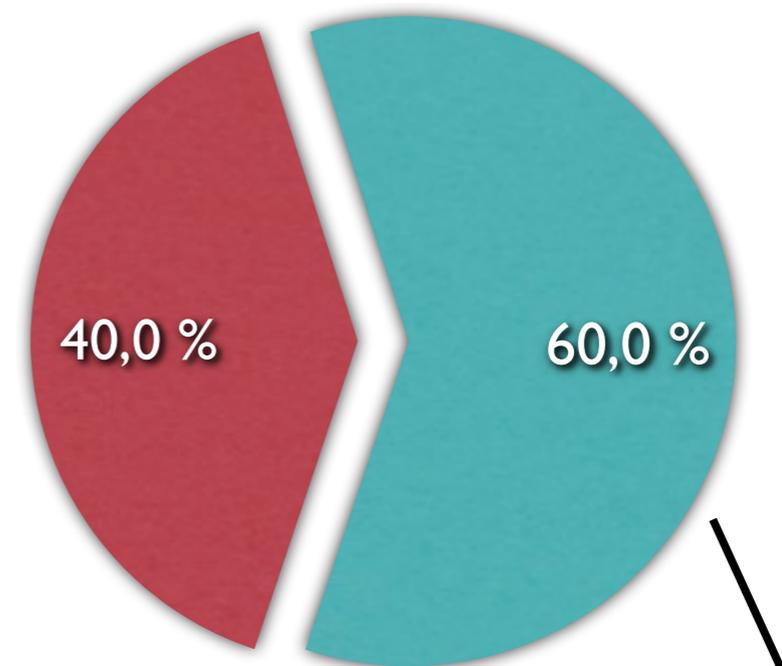
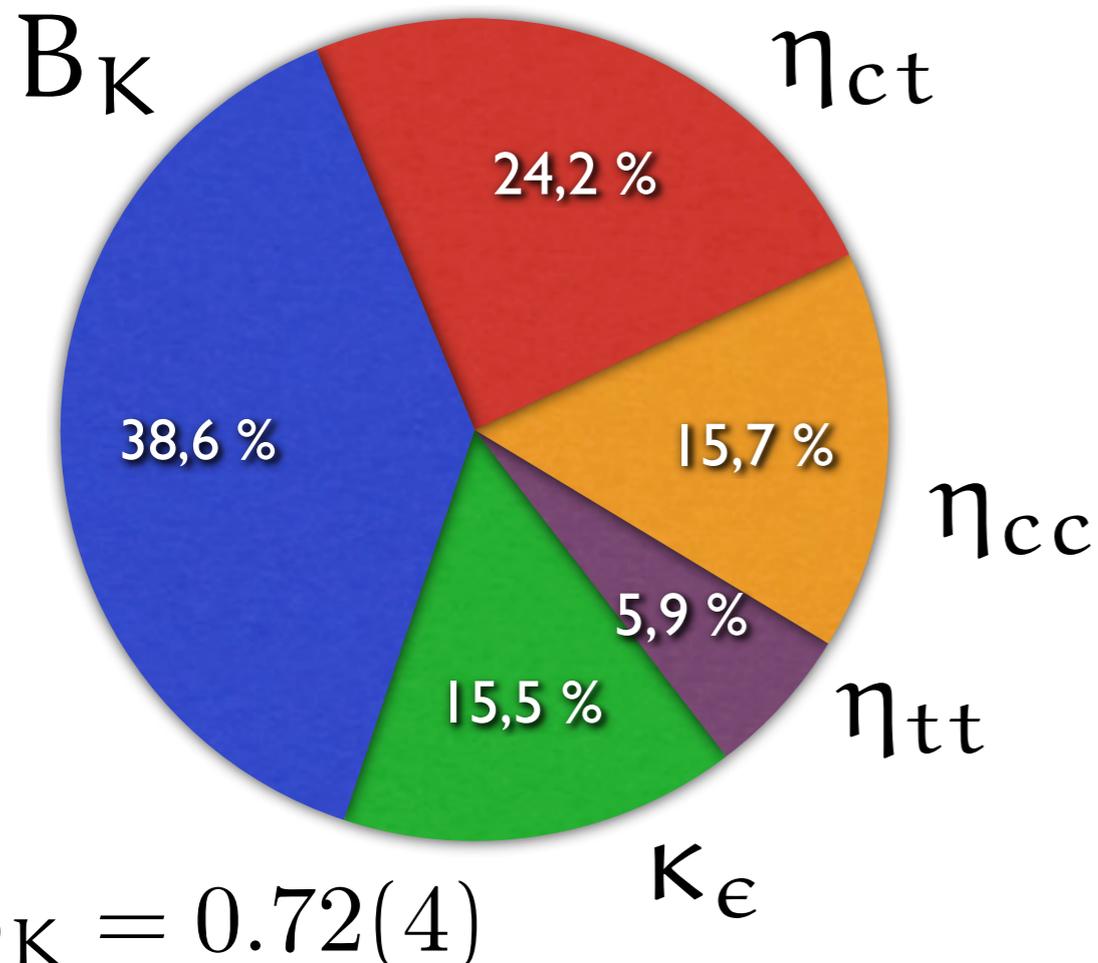
# Error Budget for $\epsilon_K$ @ NLO

$$\epsilon_K = \kappa_\epsilon C_\epsilon B_K |V_{cb}|^2 \lambda^2 \bar{\eta} (|V_{cb}|^2 (1 - \bar{\rho}) \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_c, x_t) - \eta_{cc} x_c)$$

$$\epsilon_K = (1.78 \pm 0.25) \cdot 10^{-3}$$

[Buras, Guadagnoli'09]

Theory uncertainty



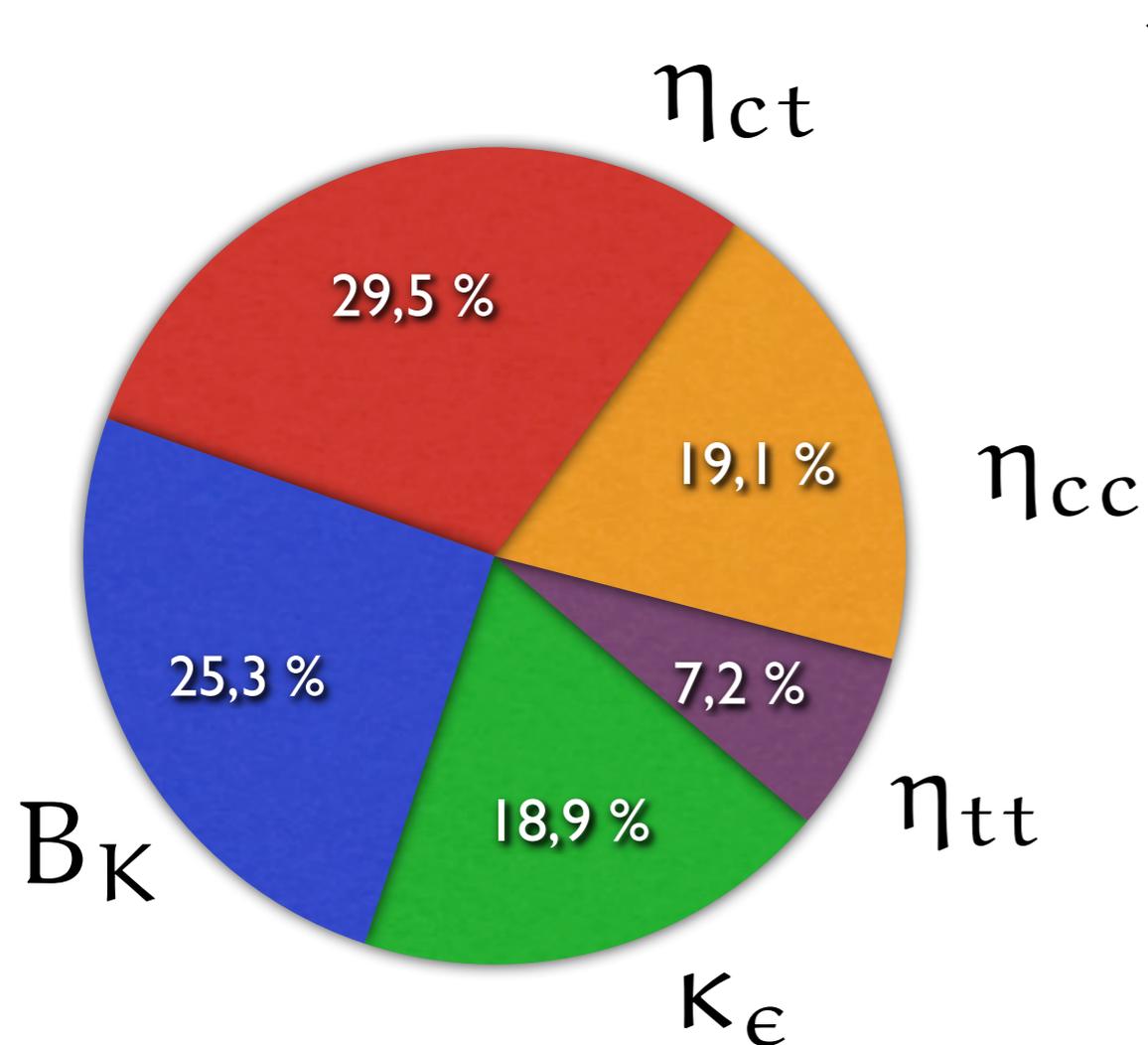
Parametric uncertainty

$$|V_{cb}| = 41.2(1.1) \cdot 10^{-3}$$

$\bar{\eta}, \bar{\rho}, \dots$

# Error Budget for $\epsilon_K$ @ NLO

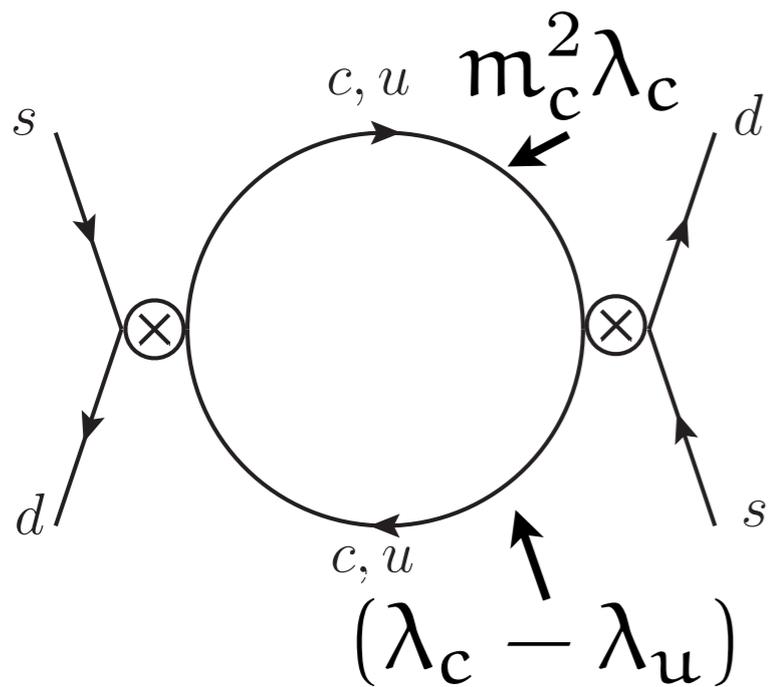
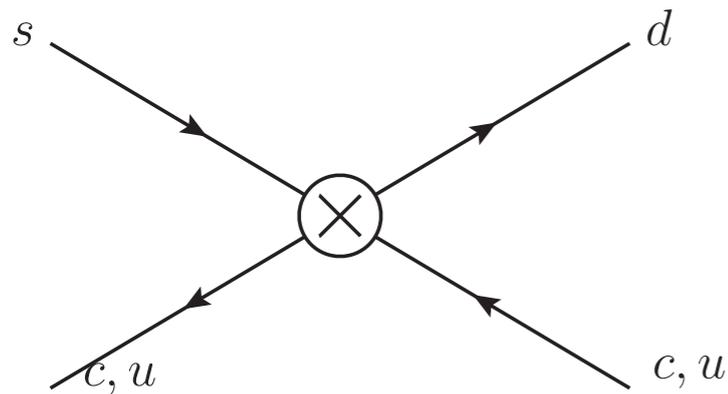
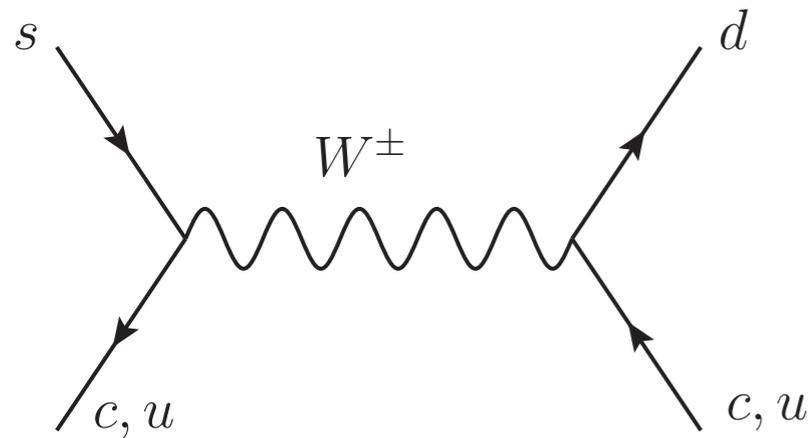
For a 3% uncertainty in  $B_K$  the perturbative uncertainties become dominant



$\eta_{ct}$  : largest uncertainty  
needs a 3 loop RGE analysis

$\eta_{cc}$  : second largest  
perturbative uncertainty  
needs a 3 loop matching  
calculation

# $\eta_{ct}$ : Charm Top at LO

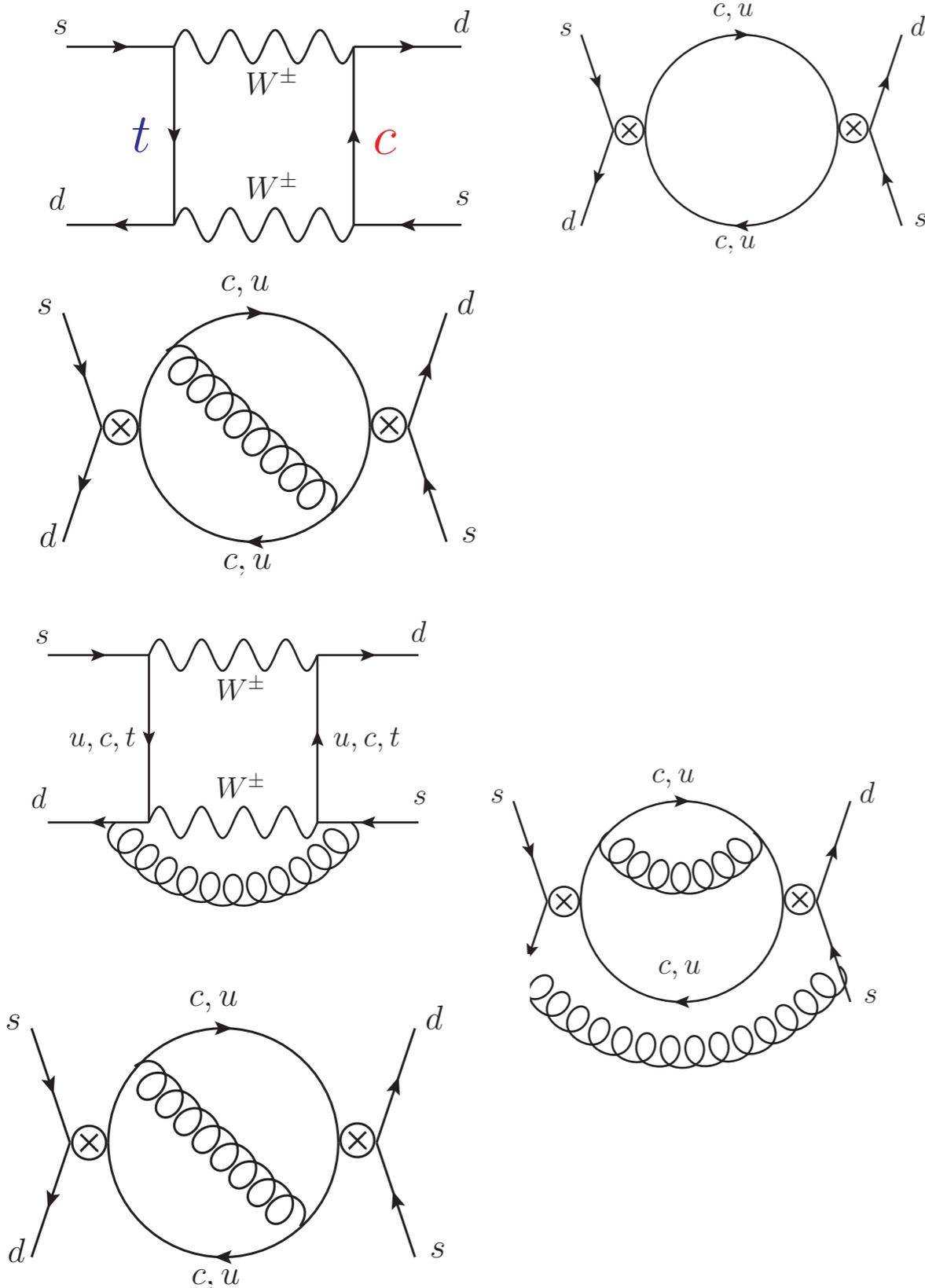


- The Leading Order result  
 $(\alpha_s \log x_c)^n \log x_c$   
 starts with a  $\log x_c$
- Tree level matching
- One-loop Renormalisation Group Equation

$$m_c^2 \lambda_c (\lambda_c - \lambda_u) \log \frac{m_c}{M_W}$$

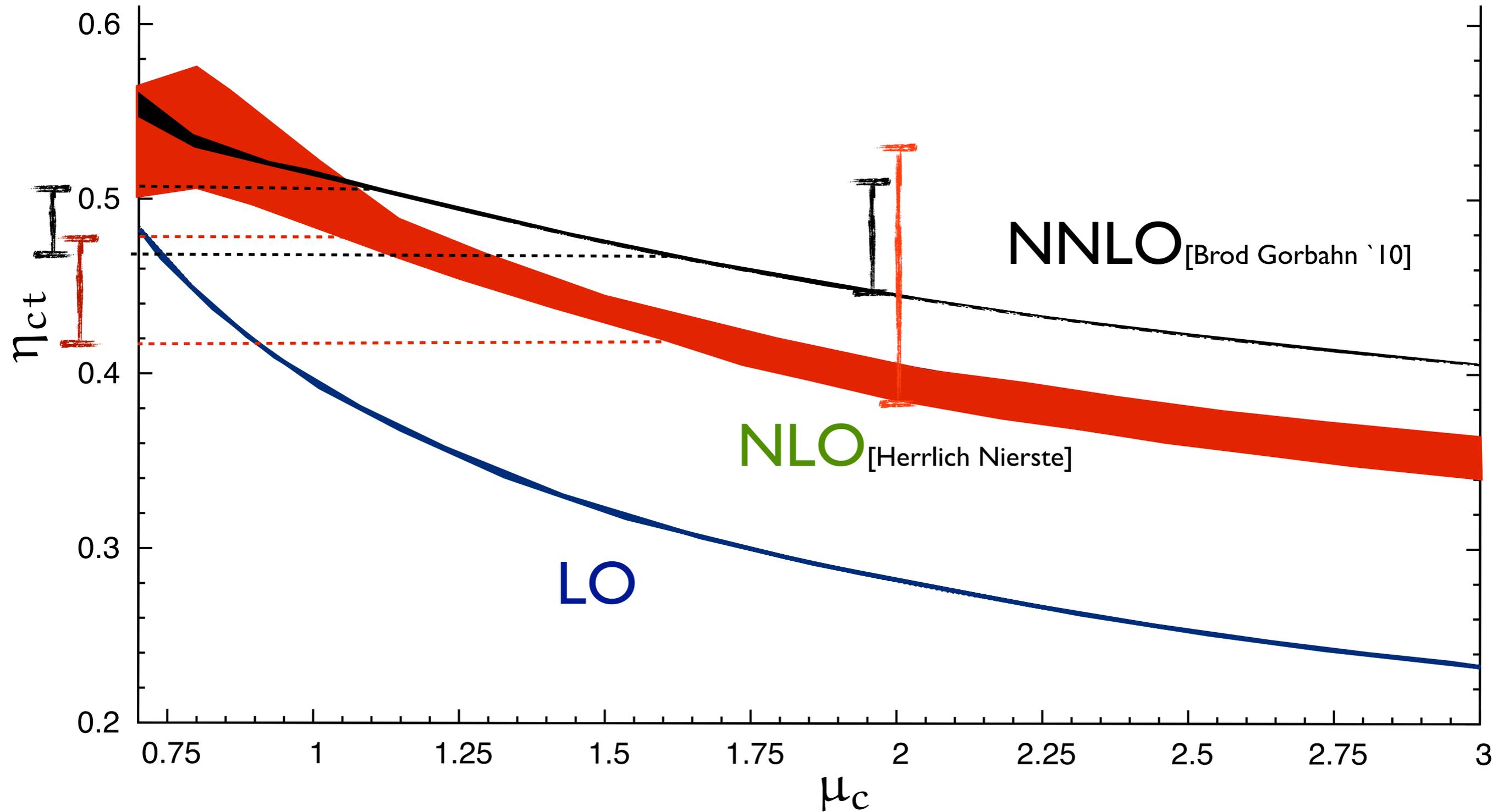
$$\rightarrow m_c^2 \lambda_c \lambda_t \tilde{Q} \log x_c$$

# $\eta_{ct}$ : Charm Top beyond LO



- One-loop matching at  $\mu_t$
- One-loop matching at  $\mu_c$
- Two-loop RG running
- Plus  $d=6$  operators NLO  
[Herrlich, Nierste]  
 $\eta_{ct} = 0.47 \pm 0.04$
- NNLO: RGE and matching for  $d=6$  operators RGE: [MG, Haisch '04], Matching: [Bobeth, et. al. '00]
- Still  $O(10000)$  diagrams were calculated

# $\eta_{ct}$ at NNLO



# Conclusions

$K \rightarrow \pi \bar{\nu} \nu$  very clean and suppressed

probe of the new physics flavour structure

$\epsilon_K$  Improvements  $B_K \rightarrow$  NNLO calculation

Strong NP sensitivity: Talks by Blanke & Haisch

very clean semileptonic and leptonic modes  
are also interesting