Status of the Muon g-2

Michel Davier  (LAL – Orsay)

• the muon magnetic anomaly
• revisited \( \tau \) spectral functions: Belle + updated corrections
• ee spectral functions after KLOE and BaBar
• combination of all ee data
• discussion and perspectives
Lepton Magnetic Anomaly: from Dirac to QED

\[ \vec{\mu} = g \frac{e}{2m} \vec{s}, \quad a = \frac{(g - 2)}{2} \]

Dirac (1928) \( g_e = 2 \quad a_e = 0 \)

anomaly discovered:
Kusch-Foley (1948) \( a_e = (1.19 \pm 0.05) \ 10^{-3} \)

and explained by O(\( \alpha \)) QED contribution:
Schwinger (1948) \( a_e = \alpha / 2\pi = 1.16 \ 10^{-3} \)

first triumph of QED

\( \Rightarrow a_e \) sensitive to quantum fluctuations of fields
More Quantum Fluctuations

\[ \alpha = \alpha_{\text{QED}} + \alpha_{\text{had}} + \alpha_{\text{weak}} + \text{? a new physics ?} \]

QED up to \(O(\alpha^4)\), \(\alpha^5\) in progress (Kinoshita et al.)

Hadrons vacuum polarization light-by-light (models)

Electroweak

\[ \Rightarrow a_\mu \text{ much more sensitive to high scales} \]

\[ \delta a_\ell \propto \frac{m_\ell^2}{M^2} \]
Dominant uncertainty from lowest-order HVP piece
Cannot be calculated from QCD (low mass scale), but one can use experimental
data on $e^+e^-$→hadrons cross section (Bouchiat-Michel 1961)

\[ a_{\mu}^{\text{had}} = \frac{\alpha^2}{3\pi^2} \int_{4m_e^2}^{\infty} ds \frac{K(s)}{s} R(s) \]

Dispersion relation
The E-821 $a_\mu$ Measurement at BNL Updated

$a_\mu$ measured from a ratio of frequencies

$$\omega_a = \omega_{\text{precession}} - \omega_{\text{cyclotron}}$$

$$\omega_{\text{precession}} = \omega_L + \omega_T$$

$$\omega_a = a_\mu \frac{eB}{m_\mu}$$

$$a_\mu = \frac{\omega_a}{\omega_L - \omega_a} = \frac{\omega_a/\tilde{\omega}_p}{\omega_L/\tilde{\omega}_p - \omega_a/\tilde{\omega}_p} = \frac{\mathcal{R}}{\lambda - \mathcal{R}}$$

$$\lambda = \omega_L / \omega_p = \mu_\mu / \mu_p$$  from muonium hyperfine splitting

value used by E-821 3.18334539(10)
new value 3.183345137(85) Mohr et al., RMP 80 (2008) 633

$\Rightarrow$ change in $a_\mu$ (+0.92 $10^{-10}$)
(review in RPP2009 (Höcker-Marciano))

$$a_\mu^{\text{exp}} = (11\ 659\ 208.9 \pm 5.4 \pm 3.3)\ 10^{-10} \text{ updated} \ (\pm 6.3) \ (0.54 \text{ ppm})$$
The Role of $\tau$ Data through CVC – SU(2)

Hadronic physics factorizes (spectral Functions)

$$\sigma^{(l=1)}[e^+e^- \to \pi^+\pi^-] = \frac{4\pi\alpha^2}{s} \nu[\tau^- \to \pi^-\pi^0\nu_{\tau}]$$

$$\nu[\tau^- \to \pi^-\pi^0\nu_{\tau}] \propto \frac{BR[\tau^- \to \pi^-\pi^0\nu_{\tau}]}{BR[\tau^- \to e^-\nu_e\nu_{\tau}]} \frac{1}{N_{\pi^0}} \frac{dN_{\pi^0}}{ds} \frac{m_{\tau}^2}{(1-s/m_{\tau}^2)(1+s/m_{\tau}^2)}$$

branching fractions  mass spectrum  kinematic factor (PS)

R. Alemany, MD, A. Höcker, EPJC 1998
Corrections for SU(2) breaking applied to $\tau$ data for dominant $\pi^-\pi^+$ contrib.: 

- **Electroweak radiative corrections:**
  - dominant contribution from short distance correction $S_{EW}$
  - subleading corrections (small)
  - long distance radiative correction $G_{EM}(s)$

- **Charged/neutral mass splitting:**
  - $m_{\pi^-} \neq m_{\pi^0}$ leads to phase space (cross sec.) and width (FF) corrections
  - $\rho - \omega$ mixing (EM $\omega \rightarrow \pi^-\pi^+$ decay) corrected using FF model
  - $m_{\rho^-} \neq m_{\rho^0}$ *** and $\Gamma_{\rho^-} \neq \Gamma_{\rho^0}$ ***

- **Electromagnetic decays:** $\rho \rightarrow \pi \pi \gamma$ ***, $\rho \rightarrow \pi \gamma$, $\rho \rightarrow \eta \gamma$, $\rho \rightarrow l^+l^-$

- **Quark mass difference** $m_u \neq m_d$ (negligible)
Situation at ICHEP’06 / 08

\[ a_\mu^{\text{had}}[ee] = (690.9 \pm 4.4) \times 10^{-10} \]

\[ a_\mu[ee] = (11 \,659 \,180.5 \pm 4.4_{\text{had}} \pm 3.5_{\text{LBL}} \pm 0.2_{\text{QED+EW}}) \times 10^{-10} \]

Hadronic HO  
\(- (9.8 \pm 0.1) \times 10^{-10}\)

Hadronic LBL  
\(+ (12.0 \pm 3.5) \times 10^{-10}\)

Electroweak  
\((15.4 \pm 0.2) \times 10^{-10}\)

QED  
\((11 \,658 \,471.9 \pm 0.1) \times 10^{-10}\)

Kinoshita-Nio (2006)

Observed Difference with BNL using \(e^+e^-\):

\[ a_\mu[\text{exp}] - a_\mu[\text{SM}] = (27.5 \pm 8.4) \times 10^{-10} \]

\(\Rightarrow 3.3\) „standard deviations“

But estimate using \(\tau\) data consistent with E-821!
Most recent data and analyses

- $\tau \rightarrow \pi \pi^0 \nu_{\tau}$ data from Belle
  PRD 78 (2008) 072006

- $e^+ e^- \rightarrow \pi^+ \pi^-$ data
  - KLOE
    PLB 670 (2009) 285
  - BaBar
    arXiv:0908.3589v1 (PRL)

- updated $\tau$-based analysis
  arXiv:0906.5443v3 (EPJC)
  MD, A. Hoecker, G. Lopez Castro, B. Malaescu, X.H.Mo, G. Toledo Sanchez,
  P. Wang, C.Z. Yuan, Z. Zhang

- updated ee-based analysis
  arXiv:0908.4300v2 (EPJC)
  MD, A. Hoecker, B. Malaescu, C.Z. Yuan, Z. Zhang
Revisited Analysis using $\tau$ Data: including Belle

Test of the spectral function shapes from different experiments: WA BR used

M.Davier  HVP/g-2  Physics LHC Era  21/3/2010
Revisited Analysis $\tau$ Data: new IB corrections

\[
\begin{array}{ll}
\text{Source} & \Delta a_{\mu}^{\text{had},\text{LO}}[\pi\pi, \tau] \ (10^{-10}) \\
& \begin{array}{ll}
\text{GS model} & \text{KS model} \\
\hline
S_{\text{EW}} & -12.21 \pm 0.15 \\
G_{\text{EM}} & -1.92 \pm 0.90 \\
\text{FSR} & +4.67 \pm 0.47 \\
\rho-\omega \text{ interference} & +2.80 \pm 0.19 \hspace{1cm} +2.80 \pm 0.15 \\
\text{m}_{\pi}^{\pm} - \text{m}_{\pi}^{0} \text{ effect on } \sigma & -7.88 \\
\text{m}_{\pi}^{\pm} - \text{m}_{\pi}^{0} \text{ effect on } \Gamma_{\rho} & +4.09 \hspace{1cm} +4.02 \\
\text{m}_{\rho}^{\pm} - \text{m}_{\rho}^{0} \text{ bare} & 0.20_{-0.19}^{+0.27} \hspace{1cm} 0.11_{-0.11}^{+0.19} \\
\pi\pi\gamma, \text{ electrom. decays} & -5.91 \pm 0.59 \hspace{1cm} -6.39 \pm 0.64 \\
\text{Total} & -16.07 \pm 1.22 \hspace{1cm} -16.70 \pm 1.23 \\
& \hspace{1cm} -16.07 \pm 1.85
\end{array}
\end{array}
\]

disagreement with Maltman-Wolfe
arXiv:0908.2391

\[
\begin{array}{ll}
\ll & \text{large change} \\
& \text{since DEHZ (2003)}
\end{array}
\]
Consistency of $\tau$ Data: Dispersion Integrals

- using BR from each experiment makes results independent from each other
- consistent results (disagreement with Benayoun et al. arXiv:09075603v1)

- using WA BR checks consistency for the spectral function shapes

- WA BR + combined spectral function $\Rightarrow$

$$a_\mu^{2\pi,LO} = (515.2 \pm 2.0_{\text{exp}} \pm 0.9_{\text{Be}} \pm 2.1_{\text{B_{\pi\pi}}} \pm 1.6_{\text{IB}}) \times 10^{-10}$$

- 0.7% precision
Relative comparison of IB-corrected $\tau$ and ee spectral functions  \( (\tau\ \text{green band}) \)

\[ \Rightarrow \text{better agreement than before with CMD2-SND} \]
\[ \Rightarrow \text{strong disagreement with KLOE : slope…} \]
Data on $e^+e^- \rightarrow$ hadrons


CMD-2 (2006)

SND (2006)

KLOE (2009)
The BaBar Analysis

\[ \text{e}^+ \text{e}^- \rightarrow \mu^+ \mu^- \gamma (\gamma) \text{ and } \pi^+ \pi^- \gamma (\gamma) \text{ measured simultaneously} \]

\[ x = \frac{2E_\gamma^*}{\sqrt{s}} \]

\[ s' = s(1 - x) \]

\[ \text{ISR} \quad \text{FSR} \]

\[ \text{ISR + add. ISR} \quad \text{ISR + add. FSR} \]

LO FSR negligible for \( \pi\pi \) at \( s \sim (10.6 \text{ GeV})^2 \)
QED Test with $\mu\mu\gamma$ sample

- absolute comparison of $\mu\mu$ mass spectra in data and in simulation
- simulation corrected for data/MC efficiencies
- AfkQed corrected for incomplete NLO using Phokhara
- strong test (ISR probability drops out for $\pi\pi$)

\[
\frac{\sigma_{\mu\mu\gamma(\gamma)}^{\text{data}}}{\sigma_{\mu\mu\gamma(\gamma)}^{\text{NLO QED}}} = 1 + (4.0 \pm 1.9 \pm 5.5 \pm 9.4) \times 10^{-3}
\]

$0.2 \text{ – } 3 \text{ GeV}$

BaBar ee luminosity

ISR $\gamma$ efficiency 3.4 syst.
trig/track/PID 4.0
Data on $e^+e^- \rightarrow$ hadrons (2)

BaBar (PRL Dec 2009)
VDM Fit of the BaBar Pion Form Factor

\[ F_\pi(s) = \frac{BW^{GS}_\rho(s, m_\rho, \Gamma_\rho)^{1+\alpha BW^{KS}_\omega(s, m_\omega, \Gamma_\omega)}}{1+\alpha} + \beta BW^{GS}_\rho(s, m_\rho', \Gamma_\rho') + \gamma BW^{GS}_\rho(s, m_\rho'', \Gamma_\rho'') }{1 + \beta + \gamma} \]

\[ |F_\pi|^2(s') = \frac{3s'}{\pi \alpha^2(0) \beta^3_\pi} \sigma_{\pi\pi}(s') \]

\[ \sigma_{\pi\pi}(s') = \frac{\sigma^0_{\pi\pi}(\gamma)(s')}{1 + \frac{\alpha}{\pi} \eta(s')} \left( \frac{\alpha(s')}{\alpha(0)} \right)^2 \]

add. FSR   \( \alpha \) Running (VP)
BaBar vs. other ee data (0.5-1.0 GeV)

direct relative comparison of cross sections with BaBar fit (stat + syst errors included)
(green band)
BaBar vs. IB-corrected $\tau$ data (0.5-1.0 GeV)

relative comparison w.r.t. BaBar of isospin-breaking corrected $\tau$ spectral functions

IB corrections: radiative corr., $\pi$ masses, $\rho$-$\omega$ interference, $\rho$ masses/widths

each $\tau$ data normalized to its own BR
Combination of all $e^+e^-$ Data

arXiv: 0908.4300 (EPJC)
MD-Höcker-Malaescu-Yuan-Zhang

Improved procedure and software (HVPTools) for combining cross section data with arbitrary point spacing/binning
Obtaining the average cross section

- local weighted average performed
- full covariance matrices
- local $\chi^2$ used for error rescaling
- average dominated by BaBar and KLOE, BaBar covering full range
## Computing $a_\mu^{\pi\pi} [2m_\pi, 1.8 \text{ GeV}]$

<table>
<thead>
<tr>
<th>Energy range (GeV)</th>
<th>Experiment</th>
<th>$a_\mu^{\text{had, LO}[\pi\pi]} (10^{-10})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2m_\pi \pm 0.3$</td>
<td>Combined $e^+e^-$ (fit)</td>
<td>0.55 ± 0.01</td>
</tr>
<tr>
<td>0.30 – 0.63</td>
<td>Combined $e^+e^-$</td>
<td>132.6 ± 0.8 ± 1.0 (1.3$_{\text{tot}}$)</td>
</tr>
<tr>
<td>0.63 – 0.958</td>
<td>CMD2 03</td>
<td>361.8 ± 2.4 ± 2.1 (3.2$_{\text{tot}}$)</td>
</tr>
<tr>
<td></td>
<td>CMD2 06</td>
<td>360.2 ± 1.8 ± 2.8 (3.3$_{\text{tot}}$)</td>
</tr>
<tr>
<td></td>
<td>SND 06</td>
<td>360.7 ± 1.4 ± 4.7 (4.9$_{\text{tot}}$)</td>
</tr>
<tr>
<td></td>
<td>KLOE 08</td>
<td>356.8 ± 0.4 ± 3.1 (3.1$_{\text{tot}}$)</td>
</tr>
<tr>
<td></td>
<td>BABAR 09</td>
<td>365.2 ± 1.9 ± 1.9 (2.7$_{\text{tot}}$)</td>
</tr>
<tr>
<td></td>
<td>Combined $e^+e^-$</td>
<td>360.8 ± 0.9 ± 1.8 (2.0$_{\text{tot}}$)</td>
</tr>
<tr>
<td>0.958 – 1.8</td>
<td>Combined $e^+e^-$</td>
<td>14.4 ± 0.1 ± 0.1 (0.2$_{\text{tot}}$)</td>
</tr>
<tr>
<td>Total</td>
<td>Combined $e^+e^-$</td>
<td>508.4 ± 1.3 ± 2.6 (2.9$_{\text{tot}}$)</td>
</tr>
<tr>
<td>Total</td>
<td>Combined $\tau$ [1]</td>
<td>515.2 ± 2.0$<em>{\text{exp}}$ ± 2.2$</em>{\text{B}}$ ± 1.6$<em>{\text{IB}}$ (3.4$</em>{\text{tot}}$)</td>
</tr>
</tbody>
</table>

- Pre-BaBar combined ee: $503.5 \pm 3.5 \times 10^{-10}$
- BaBar: $514.1 \pm 3.8 \times 10^{-10}$
- Combined ee: $508.4 \pm 2.9 \times 10^{-10}$
- Combined $\tau$: $515.2 \pm 3.0 \pm 1.6 (3.4) \times 10^{-10}$

<table>
<thead>
<tr>
<th>Modes</th>
<th>Energy [GeV]</th>
<th>$e^+e^-$</th>
<th>$\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+\pi^−2\pi^0$</td>
<td>$2m_\pi−1.8$</td>
<td>$16.8\pm1.3\pm0.2_{\text{rad}}$</td>
<td>$21.4\pm1.3\pm0.6_{\text{SU(2)}}$</td>
</tr>
<tr>
<td>$2\pi^+2\pi^−(+\text{BaBar})$</td>
<td>$2m_\pi−1.8$</td>
<td>$13.1\pm0.4\pm0.0_{\text{rad}}$</td>
<td>$12.3\pm1.0\pm0.4_{\text{SU(2)}}$</td>
</tr>
<tr>
<td>$\omega$ (782)</td>
<td>0.3 – 0.81</td>
<td>$38.0\pm1.0\pm0.3_{\text{rad}}$</td>
<td>–</td>
</tr>
<tr>
<td>$\phi$ (1020)</td>
<td>1.0 – 1.055</td>
<td>$35.7\pm0.8\pm0.2_{\text{rad}}$</td>
<td>–</td>
</tr>
<tr>
<td>Other excl. (+BaBar)</td>
<td>$2m_\pi−1.8$</td>
<td>$24.3\pm1.3\pm0.2_{\text{rad}}$</td>
<td>–</td>
</tr>
<tr>
<td>$J/\psi, \psi(2S)$</td>
<td>3.08 – 3.11</td>
<td>$7.4\pm0.4\pm0.0_{\text{rad}}$</td>
<td>–</td>
</tr>
<tr>
<td>$R$ [QCD]</td>
<td>1.8 – 3.7</td>
<td>$33.9\pm0.5_{\text{theo}}$</td>
<td>–</td>
</tr>
<tr>
<td>$R$ [data]</td>
<td>3.7 – 5.0</td>
<td>$7.2\pm0.3\pm0.0_{\text{rad}}$</td>
<td>–</td>
</tr>
<tr>
<td>$R$ [QCD]</td>
<td>5.0 – $\infty$</td>
<td>$9.9\pm0.2_{\text{theo}}$</td>
<td>–</td>
</tr>
</tbody>
</table>

⇒ another large long-standing discrepancy in the $\pi^+\pi^-2\pi^0$ channel!
The Problematic $2\pi 2\pi^0$ Contribution

preliminary BaBar data:
A. Petzold, EPS-HEP (2007)

only statistical errors

ee data used now (CMD2 discarded)

old contribution         $16.8 \pm 1.3$
update                    $17.6 \pm 1.7$ probably still underestimated (BaBar prelim.)
$\tau$                     $21.4 \pm 1.4$
BaBar Multi-hadronic Results

Still more channels under analysis: $K^+K^-$, $KK\pi\pi$ with $K^0$

only statistical errors
syst. 5-10%
Where are we?

- including BaBar $2\pi$ results in the $e^+e^-$ combination + estimate of hadronic LBL contribution (Prades-de Rafael-Vainhstein, 2009) yields
  \[
a_{\mu}^{\text{SM}}[e^+e^-] = (11 659 183.4 \pm 4.1 \pm 2.6 \pm 0.2) \times 10^{-10}
  \]

- E-821 updated result
  \[
  11 659 208.9 \pm 6.3
  \]

- deviation (ee) 25.5 ± 8.0 (3.2 $\sigma$)

- updated $\tau$ analysis + Belle + revisited IB corrections

- deviation ($\tau$) 15.7 ± 8.2 (1.9 $\sigma$)
Discussion

• BaBar $2\pi$ data complete and the most accurate, but expected gain in precision not fully realized because of discrepancy with KLOE
• however, previous $\tau/ee$ disagreement strongly reduced (resolved?)
  \[2.9\sigma \text{ (2006)} \rightarrow 2.4\sigma \text{ (}\tau\text{ update)} \rightarrow 1.5\sigma \text{ (including BaBar)}\]
• a range of values for the deviation from the SM can be obtained, depending on the $2\pi$ data used:

<p>| | |</p>
<table>
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</thead>
<tbody>
<tr>
<td>BaBar</td>
<td>2.4$\sigma$</td>
</tr>
<tr>
<td>all ee</td>
<td>3.2$\sigma$</td>
</tr>
<tr>
<td>all ee −BaBar</td>
<td>3.7$\sigma$</td>
</tr>
<tr>
<td>all ee −KLOE</td>
<td>2.9$\sigma$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>1.9$\sigma$</td>
</tr>
</tbody>
</table>

• all approaches yield a deviation, but SM test limited by systematic effects not accounted for in the experimental analyses (ee) and/or the corrections to $\tau$ data

• at the moment some evidence for a deviation ($\sim 3\sigma$), but not sufficient to establish a contribution from new physics (NP)
• however if NP is found at LHC, this deviation will constraint the NP phenomenology
Impact on new physics: ex. SUSY

LHC: direct search for SUSY partners
difficult to measure couplings and disentangle between models (ILC)
g-2 measurement + theory prediction: sensitivity to couplings

taking one MSSM parameter point
(D. Stöckinger)
Perspectives

• first priority is a clarification of the BaBar-τ / KLOE discrepancy:
  - origin of the ‘slope’ (was very pronounced with the 2004 KLOE results, reduced now with the 2008 results)
  - normalization difference on ρ peak (most direct effect on $a_\mu$)
  - Novosibirsk results in-between and not precise enough

• further checks of the KLOE results are possible: as method is based on MC simulation for ISR and additional ISR/ISR probabilities ⇒ long-awaited test with $\mu\mu\gamma$ analysis

• contribution from multi-hadronic channels will continue to be updated with more results forthcoming from BaBar, particularly $2\pi$ $2\pi^0$

• new precise data expected from VEPP-2000 in Novosibirsk

• experimental error of E-821 direct $a_\mu$ measurement is a limitation, already now ⇒ new projects necessary
  proposal accepted in Fermilab (financed?): improve accuracy by a factor 4 project at JPARC
Comparison of ee and $\tau$ Data Revisited (2)

Global test of spectral functions:
prediction of $\tau$ BR using ee data

⇒ apply to $\pi\pi^0$ channel

IB corrections applied to ee data this time

• data from CMD2-SND
  overconsistent?

• fair agreement CMD2-SND with $\tau$

• larger disagreement with KLOE

\[
B_X^{\text{CVC}} = \frac{3}{2} B_e |V_{ud}|^2 \int_{s_{\text{min}}}^{m_{\tau}^2} ds s \sigma_I \left(1 - \frac{s}{m_{\tau}^2}\right)^2 \left(1 + \frac{2s}{m_{\tau}^2}\right)
\]
Obtaining the $\pi\pi(\gamma)$ cross section

$$\frac{dN_{\pi\pi\gamma(\gamma)}}{d\sqrt{s'}} = \frac{dL_{ISR}^{eff}}{d\sqrt{s'}} \varepsilon_{\pi\pi\gamma(\gamma)}(\sqrt{s'}) \sigma_{\pi\pi}(\sqrt{s'})$$

Unfolded spectrum

Acceptance from MC + data/MC corrections

Effective ISR luminosity from $\mu\mu\gamma(\gamma)$ analysis (similar equation + QED)

$\pi\pi$ mass spectrum unfolded (Malaescu arXiv:0907-3791) for detector response

Additional ISR almost cancels in the procedure ($\pi\pi\gamma(\gamma) / \mu\mu\gamma(\gamma)$ ratio)
Correction (2.5 ±1.0) $10^{-3}$ $\Rightarrow$ $\pi\pi$ cross section does not rely on accurate description of NLO in the MC generator

ISR luminosity from $\mu\mu\gamma\gamma$ in 50-MeV energy intervals (small compared to variation of efficiency corrections)
BaBar results (arXiv:0908.3589)

\[ e^+ e^- \rightarrow \pi^+ \pi^- (\gamma) \]  

bare (no VP) cross section  

diagonal errors stat+syst
BaBar results in $\rho$ region

2-MeV energy intervals

[Graph showing the cross section as a function of $\sqrt{s'}$ (GeV), with data points labeled BABAR]
Additional ISR

Angular distribution of add. ISR /beams!

Energy cut-off for add. ISR in AelfQed
Additional FSR

Large-angle add.ISR in data ≠ AfkQed

Evidence for FSR data ~ AfkQed

\[
\text{data/MC} \\
\mu\mu & 0.96 \pm 0.06 \\
\pi\pi & 1.21 \pm 0.05
\]

Angle between add $\gamma$ and closest track

M.Davier  HVP/g-2  Physics LHC Era  21/3/2010
Consistency of Experiments with Average
Computing $a_{\mu}^{\pi\pi}$

$$a_{\mu}^{\pi\pi}(\gamma),LO = \frac{1}{4\pi^3} \int ds K(s) \sigma_{\pi\pi}(\gamma)(s),$$

where $K(s)$ is the QED kernel,

$$K(s) = x^2 \left(1 - \frac{x^2}{2}\right) + (1 + x)^2 \left(1 + \frac{1}{x^2}\right) \left[\ln(1+x) - x + \frac{x^2}{2}\right] + x^2 \frac{1+x}{1-x} \ln x,$$

with $x = (1 - \beta_{\mu})/(1 + \beta_{\mu})$ and $\beta_{\mu} = (1 - 4m_{\mu}^2/s)^{1/2}$.

<table>
<thead>
<tr>
<th>$m_{\pi\pi}$ range (GeV)</th>
<th>$a_{\mu}^{\pi\pi}(\gamma),LO$ (GeV)</th>
<th>BABAR</th>
<th>0.28–0.30</th>
<th>$0.55 \pm 0.01 \pm 0.01$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30–0.50</td>
<td></td>
<td></td>
<td>0.55</td>
<td>$57.62 \pm 0.63 \pm 0.55$</td>
</tr>
<tr>
<td>0.50–1.00</td>
<td></td>
<td></td>
<td>0.55</td>
<td>$445.94 \pm 2.10 \pm 2.51$</td>
</tr>
<tr>
<td>1.00–1.80</td>
<td></td>
<td></td>
<td>0.55</td>
<td>$9.97 \pm 0.10 \pm 0.09$</td>
</tr>
<tr>
<td>0.28–1.80</td>
<td></td>
<td></td>
<td>0.55</td>
<td>$514.09 \pm 2.22 \pm 3.11$</td>
</tr>
</tbody>
</table>

0.28–1.8 (GeV)

| BABAR            | 514.1 ± 3.8 |
| previous $e^+e^-$ combined | 503.5 ± 3.5 * |
| $\tau$ combined  | 515.2 ± 3.5 * |

* $0.7\%$ precision

0.630–0.958 GeV range

CMD-2 96

CMD-2 98

SND

KLOE 08

BABAR

$\tau$ average

* $arXiv:0906-5443$ MD et al.