A Lee-Wick Extension of the Standard Model

Benjamin Grinstein

Indirect Searches for New Physics at the time of LHC - Conference

GGI Florence, March 23, 2010

Work mostly with Donal O'Connell and Mark Wise

Incomplete list of references

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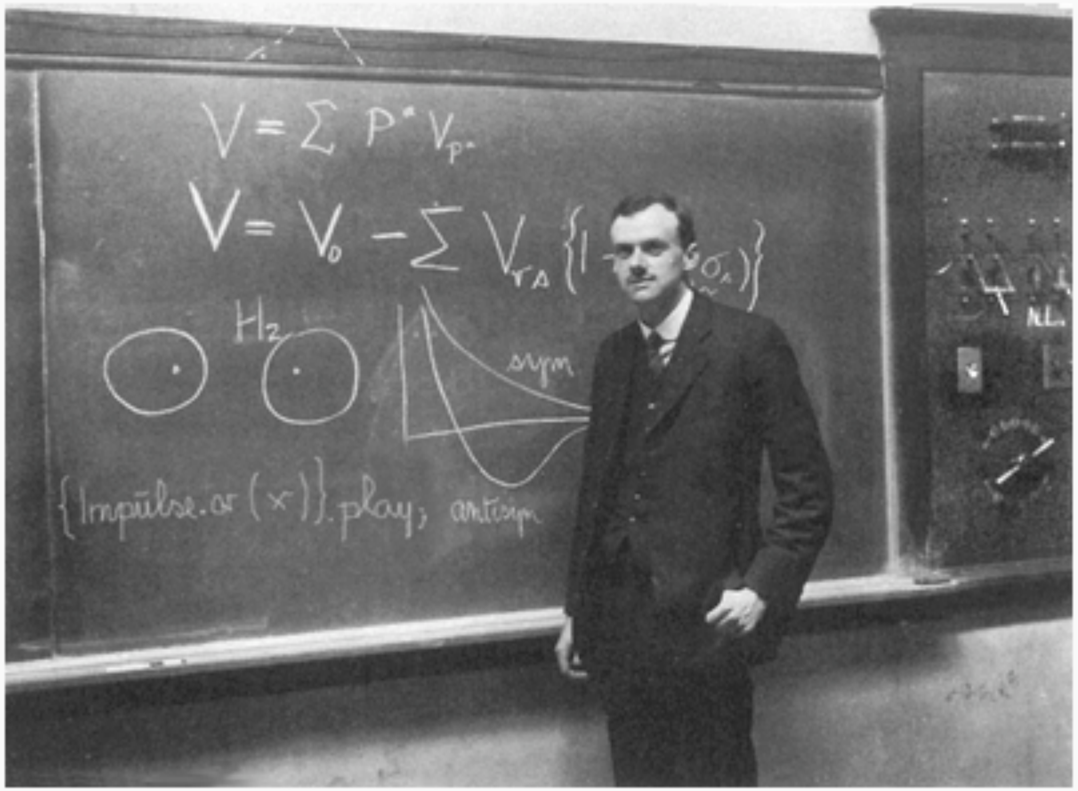
Quantum Mechanics with Indefinite Metric



Paul Dirac at a SuperCollider workshop in the early 1930s.

There is a sign error in the Maxwell equations.

Pauli

Indefinite Metric Quantization

$$\langle i|j\rangle = \eta_{ij}$$

- Hamiltonian is self-adjoint but not hermitian

$$\bar{H} = H \qquad\qquad \bar{H} = \eta H^\dagger \eta$$

- H eigenvalues are either of
  - real with non-zero norm

$$E_r^* = E_r \qquad \langle r|r\rangle \neq 0$$

  - complex, in c.c. pairs, with zero norm

$$E_\pm = E_R \pm iE_I \qquad \langle +|+\rangle = \langle -|-\rangle = 0 \qquad \langle +|-\rangle = 1$$

- H self-adjoint implies S-matrix is pseudo-unitary

$$S^\dagger \eta S = \eta$$

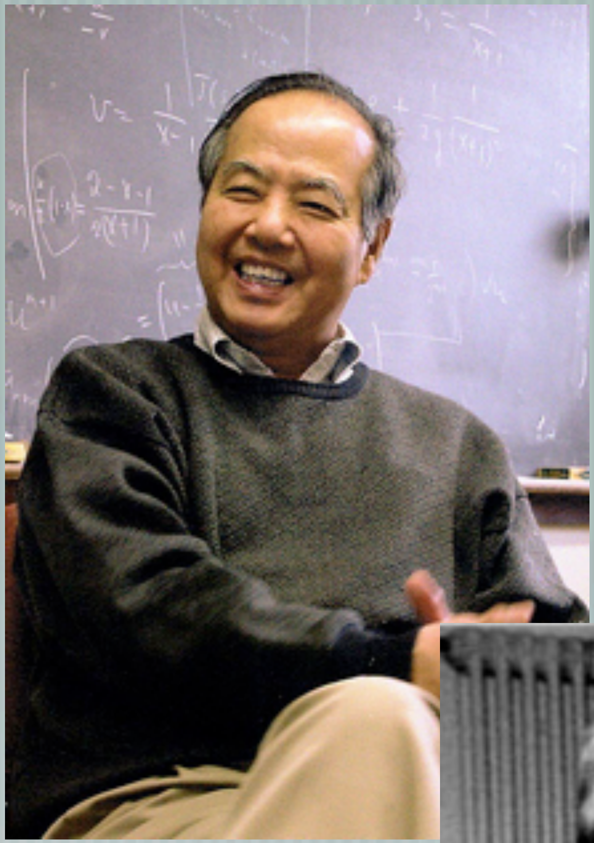
- LW condition: all eigenstates with real eigenvalues have positive norm
  - restriction of S-matrix to states with real eigenvalues gives a unitary S-matrix

$$S^\dagger S = 1 \qquad \langle r|r\rangle > 0$$

Don’t be afraid of indefinite metric:

- Lorentz metric is indefinite
- Gauge fields have a negative metric component
  - Combined with the longitudinal mode give pairs of zero norm states
  - S-matrix is unitary because they are not allowed as external asymptotic states (and current conservation)
- Likewise in string theory (X^0 component has negative norm)

TD Lee
and
Giancarlo Wick



Basic idea: unitary S-matrix possible if negative metric states are unstable

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- Strategy (arranging for real eigenvalue states to have positive norm automatically):
  - In absence of interactions have "heavy" (n) negative metric states and "light" (p) positive metric states
  - Turn on interactions; a pp state is degenerate with an n state; n unstable
  - n and pp states mix; complex eigen-energy (c.c. pair), zero norm

$$|\pm\rangle = \frac{|pp\rangle \pm |n\rangle}{\sqrt{2}}$$

  - all negative metric states have disappeared

Consider an example

Three equivalent Lagrangians:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \hat{\phi})^2 - \frac{1}{2M^2}(\partial^2 \hat{\phi})^2 - V(\hat{\phi})$$

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$$\phi = \hat{\phi} + \chi$$

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$$\phi = \hat{\phi} + \chi$$

- Indefinite metric problem explicit

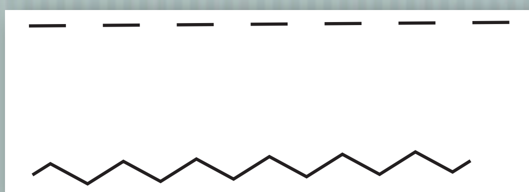
To explain basic ideas consider toy model for simplicity: $g\phi^3$

Recall, equivalent lagrangians

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \hat{\phi})^2 - \frac{1}{2M^2}(\partial^2 \hat{\phi})^2 - V(\hat{\phi})$$

$$\mathcal{L}'' = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}(\partial_\mu \chi)^2 + \frac{1}{2}M^2\chi^2 - V(\phi - \chi)$$

$$g\phi^3 \to g(\phi - \chi)^3$$



$$\frac{i}{p^2 - m^2}$$

$$\frac{-i}{p^2 - M^2}$$

Scattering:

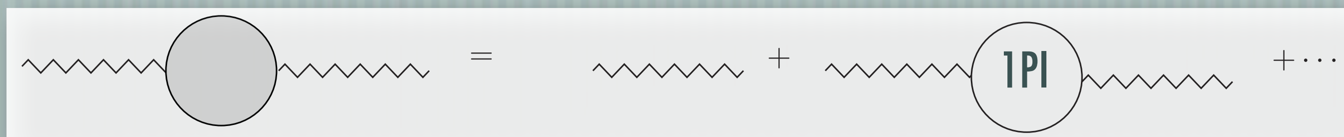
$$= -ig^2 \left(\frac{1}{p^2 - m^2} - \frac{1}{p^2 - M^2} \right)$$

$$\Rightarrow \quad \mathrm{Im}\ \mathcal{A}_{\mathrm{fwd}} = \pi g^2 \left[\delta(p^2 - m^2) - \delta(p^2 - M^2) \right]$$

This is a disaster: optical theorem is violated

$$\mathrm{Im}\ \mathcal{A}_{\mathrm{fwd}} = \pi \sqrt{s(s - 4m^2)}\sigma_T > 0$$

Reorganize perturbation theory (old school, resonances, think W/Z):
(i) Replace all propagators by dressed propagators (old well known way to deal with resonances)
(ii) Define amplitude by analytic continuation from positive and large $\mathrm{Im}(p^2)$



$iG^{(2)}$ $\quad i\Delta \quad i\Delta \quad i\Pi \quad i\Delta$

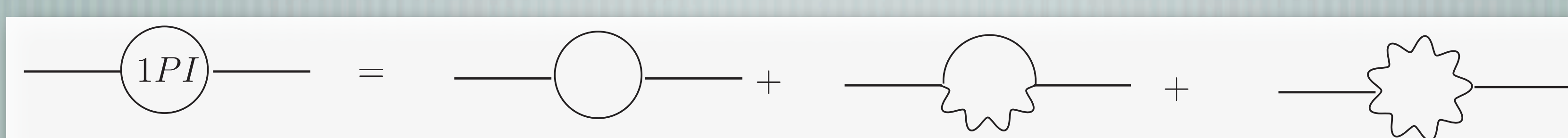
$$\Rightarrow iG^{(2)} = \frac{i}{\Delta^{-1} + \Pi}$$

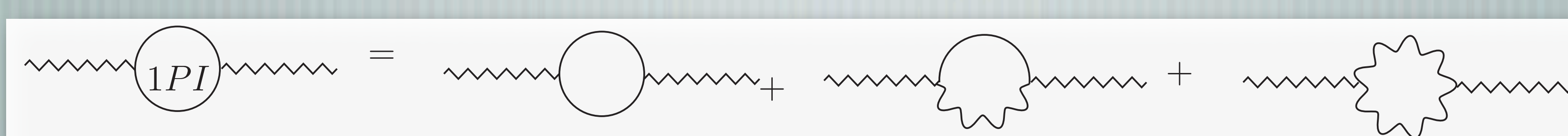
very familiar, but now use $\quad i\Delta = \dfrac{-i}{p^2 - M^2} \quad$ to get the surprising

$$iG^{(2)} = \frac{-i}{p^2 - M^2 - \Pi}$$

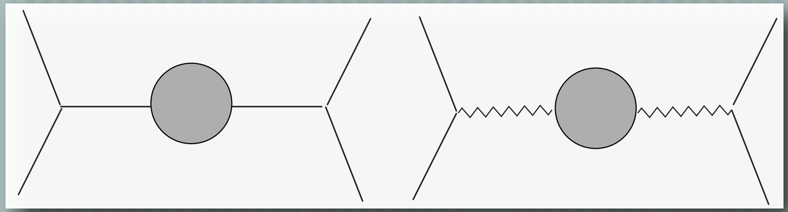
Compare this with normal case: $\quad iG^{(2)} = \dfrac{i}{p^2 - m^2 + \Pi}$

Π itself is very "normal," it is the same for normal and LW fields:

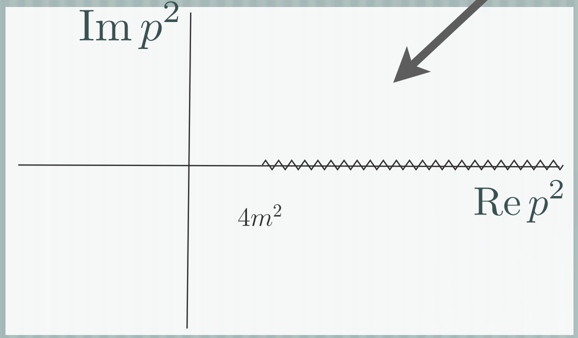


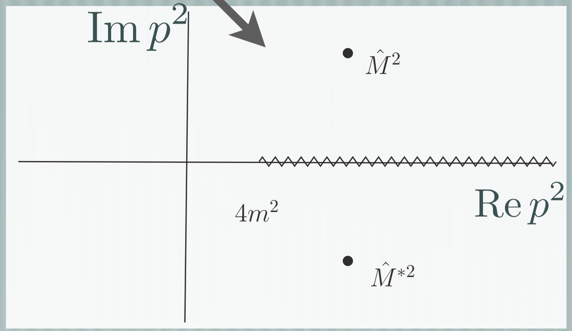


Pole in the scattering amplitude!



$$i\mathcal{A} = -ig^2\left[\frac{1}{p^2 - m^2 + \Pi} - \frac{1}{p^2 - M^2 - \Pi}\right]$$





so in fact, the LW propagator is

$$G^{(2)} = -\frac{A}{p^2 - \hat{M}^2} - \frac{A^*}{p^2 - \hat{M}^{*2}} + \int_{4m^2}^{\infty} d\mu^2\, \frac{\rho(\mu^2)}{p^2 - \mu^2}$$

properties: $\rho(\mu^2) \geq 0 \qquad -A - A^* + \int d\mu^2 \rho(\mu^2) = -1$

Imaginary part of forward amplitude: complex dipole cancels out

$$\mathrm{Im}\, \mathcal{A}_{\mathrm{fwd}} = \pi g^2 \left[\rho_{\mathrm{normal}}(\mu^2) + \rho_{\mathrm{LW}}(\mu^2)\right]$$

This is a positive discontinuity.
You can see it is precisely the total cross section (to the order we have carried this out)

Above calculation ok because single LW-resonance in intermediate state can never go "on-shell" when energies of incoming particles are real

Subtleties first encountered in 1-loop amplitude:
with real energy may still produce two LW-resonances with masses M and M^*

$$I = \int \frac{d^4p}{(2\pi)^4} \frac{-i}{(p+q)^2 - M_1^2} \frac{-i}{p^2 - M_2^2},$$

has poles at $p^0 = \pm\sqrt{\mathbf{p}^2 + M_2^2}$ and $p^0 = -q^0 \pm \sqrt{\mathbf{p}^2 + M_1^2}$

Lee & Wick:

Start from $g = 0$, masses real, take usual Feynman contour.
Turn on interaction. As M develops imaginary part deform contour to avoid crossing poles

CLOP:

Issue when contour is pinched, which can only happen when ${M_1}^* = M_2$
Take M_1 and M_2 independent, $M_2 - M_1 = i\delta$
After integration complete take $\delta \to 0$

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- We have solved the $\mathrm{O}(N)$ model in large N limit. The width or LW resonance is $O(1/N)$, so positivity of spectral function easily shown. Hence example exists for which

  i) used LW-CLOP prescription

  ii) unitary shown explicitly (directly checked optical theorem)

Lee, Wick, Coleman, Gross.... not everyone who has worked on this is a crackpot

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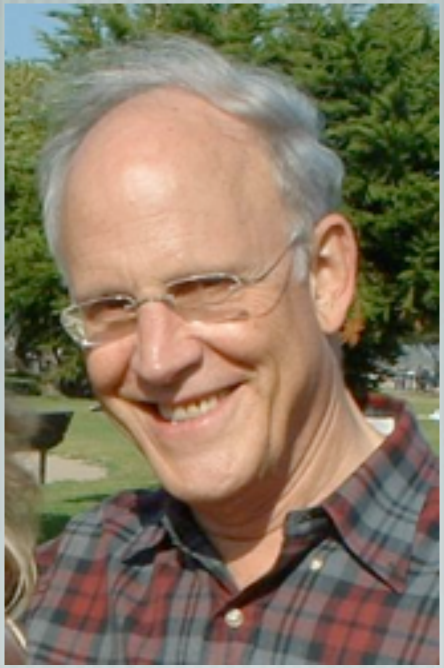

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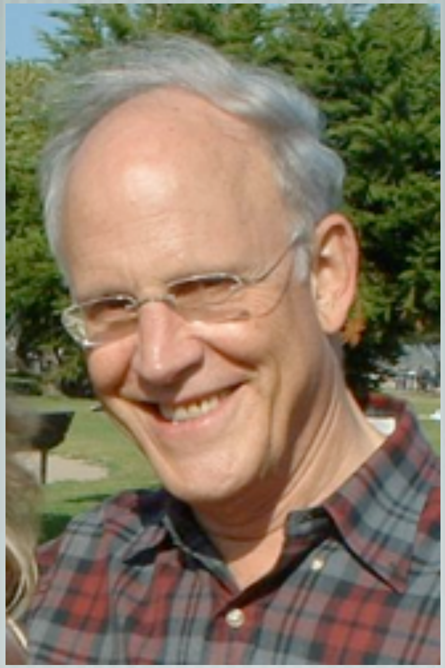
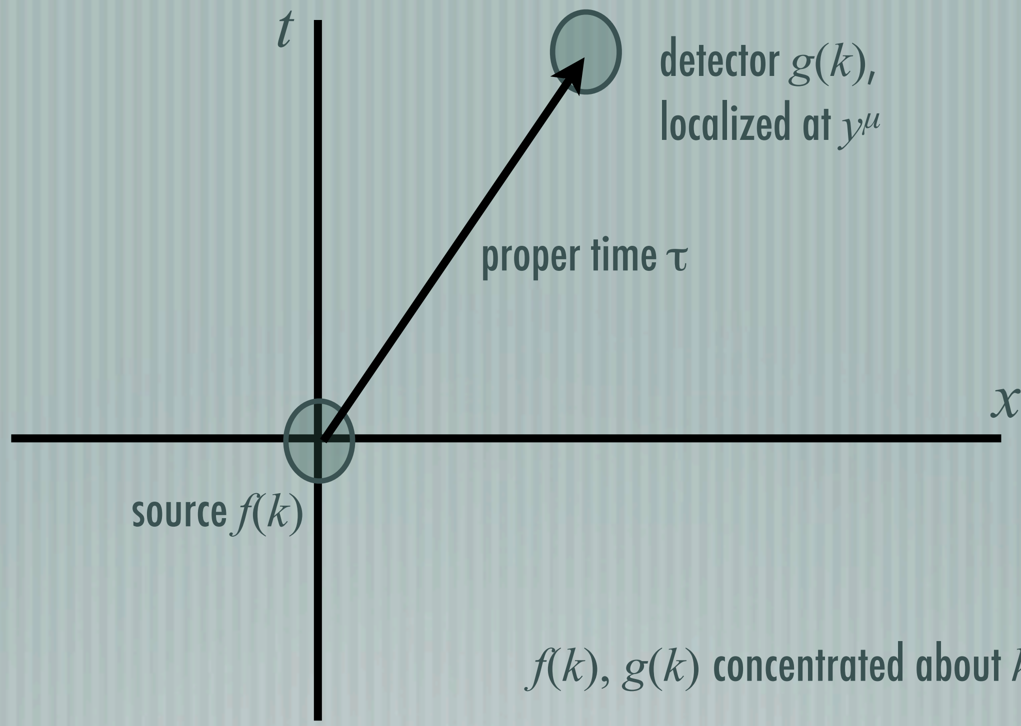


Indefinite metric quantization: Dirac, Pauli, ...

Peculiar effects: Non-locality?

Recall "response theory"



$f(k)$, $g(k)$ concentrated about $k = k_0$

stable particle

$$\langle \text{detector} | \text{source} \rangle \propto g^*(my/\tau) f(my/\tau) \frac{1}{\tau^{3/2}} e^{-im\tau} \theta(y^0)$$

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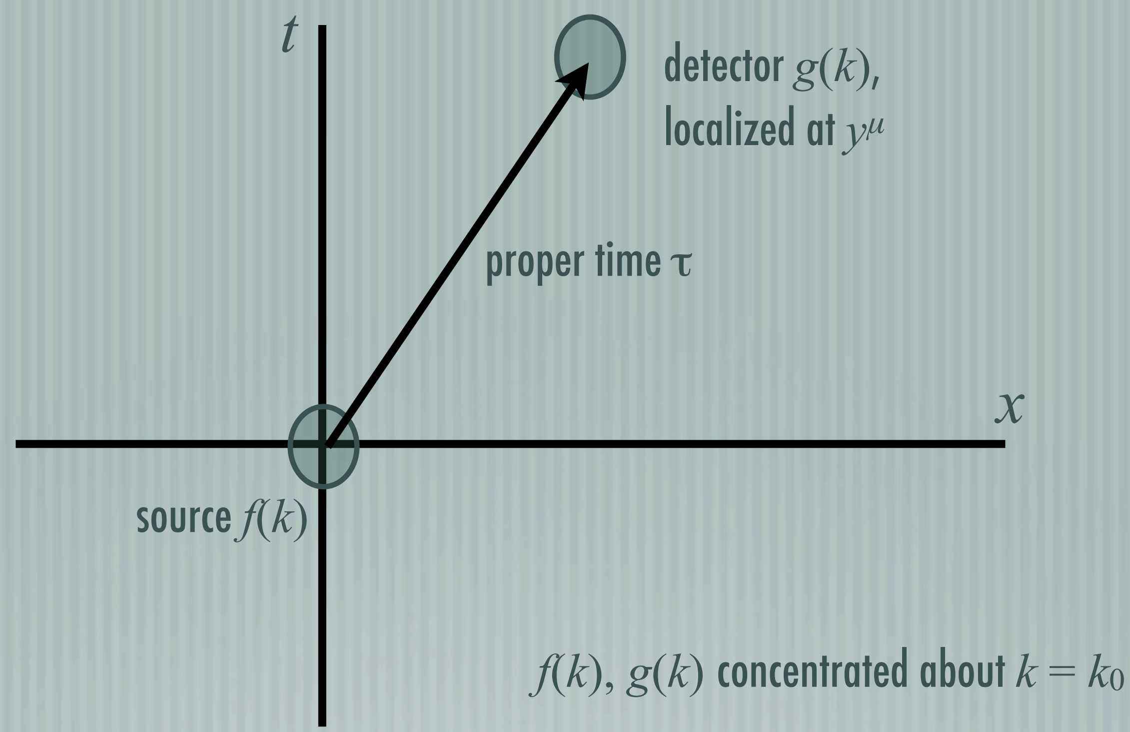

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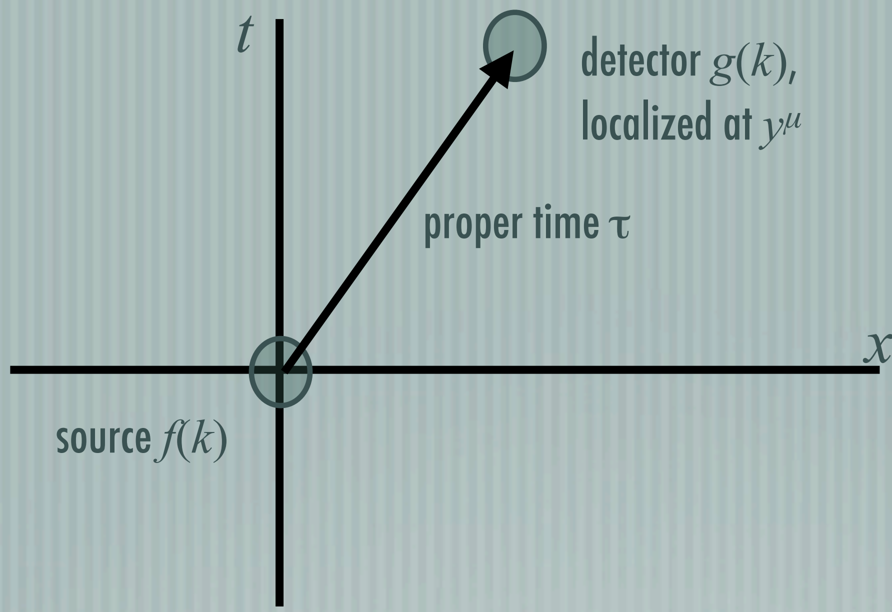
and for narrow resonance, production and decay, (pole in second sheet)

$$\langle \text{detector} | \text{source} \rangle \propto g^*(my/\tau) f(my/\tau) \frac{1}{\tau^{3/2}} e^{-im\tau} e^{-\Gamma\tau/2} \theta(y^0)$$

“source” can be from collision of two normal (non-LW) particles
“detector” from decay into normal particles

Now for LW resonance



$f(k)$, $g(k)$ concentrated about $k = k_0$

$$\langle \text{detector}|\text{source}\rangle \propto g^*(-my/\tau)f(-my/\tau)\frac{1}{\tau^{3/2}}e^{im\tau}e^{-\Gamma\tau/2}\theta(-y^0)$$

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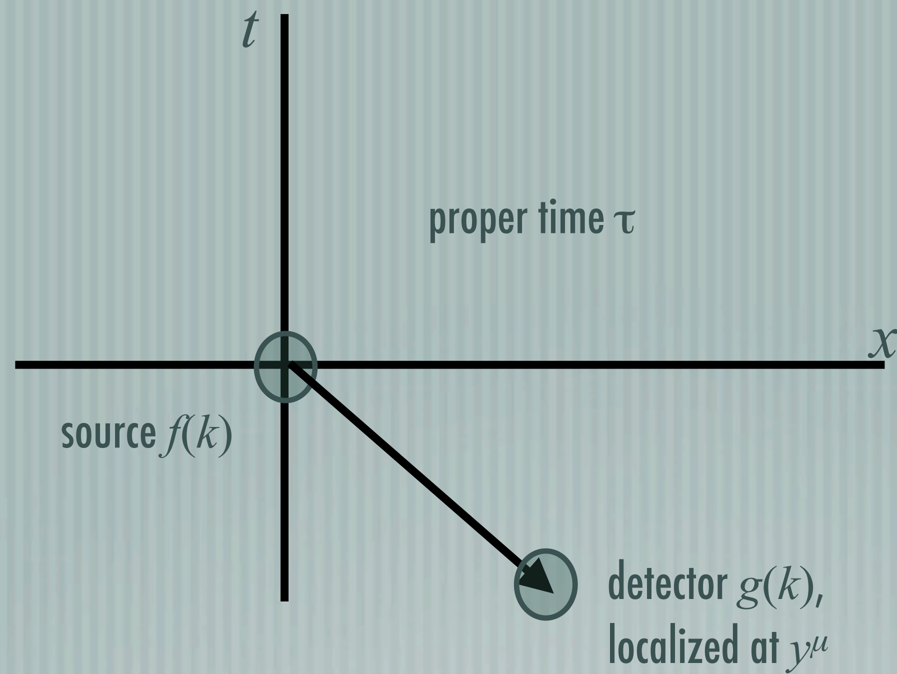

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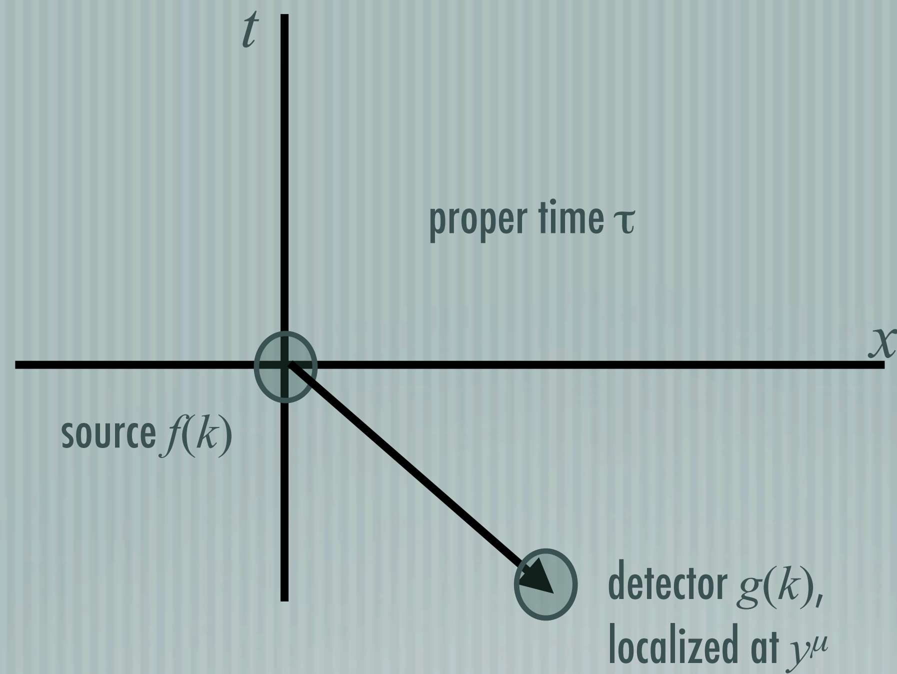

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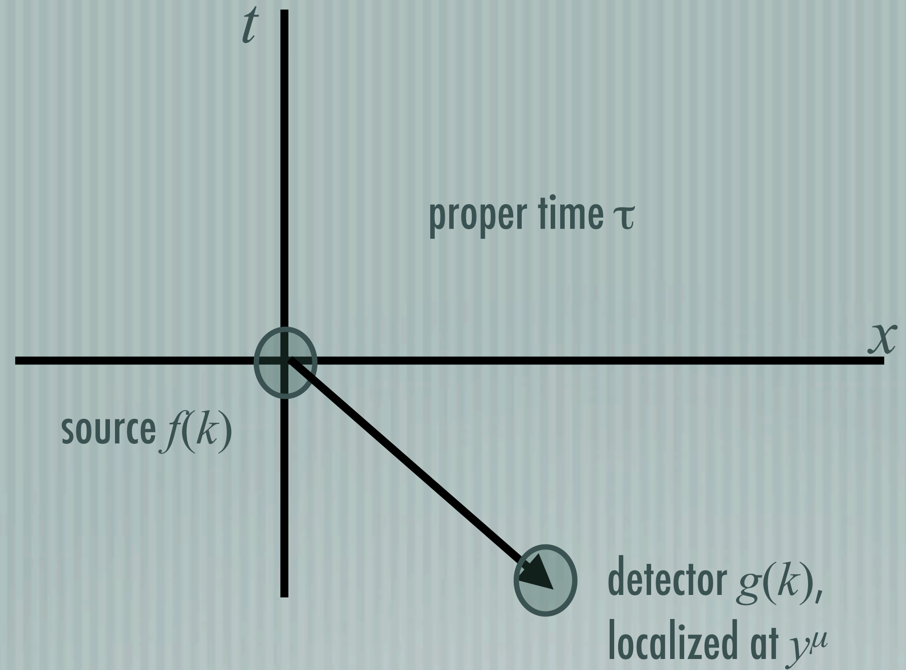

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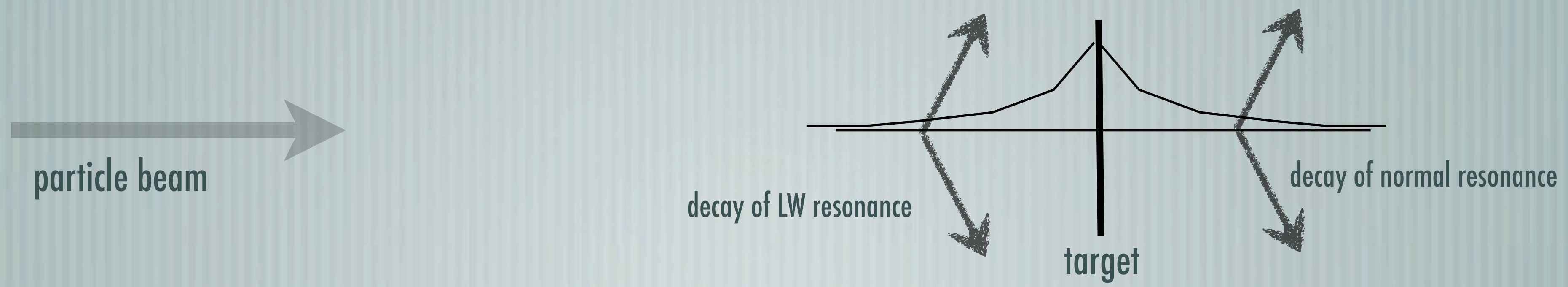

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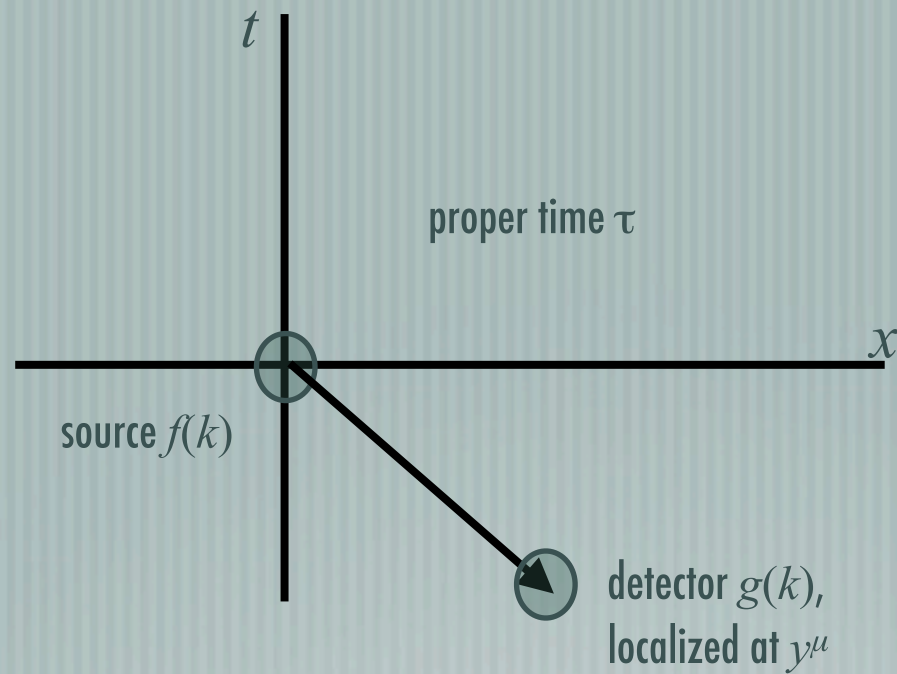

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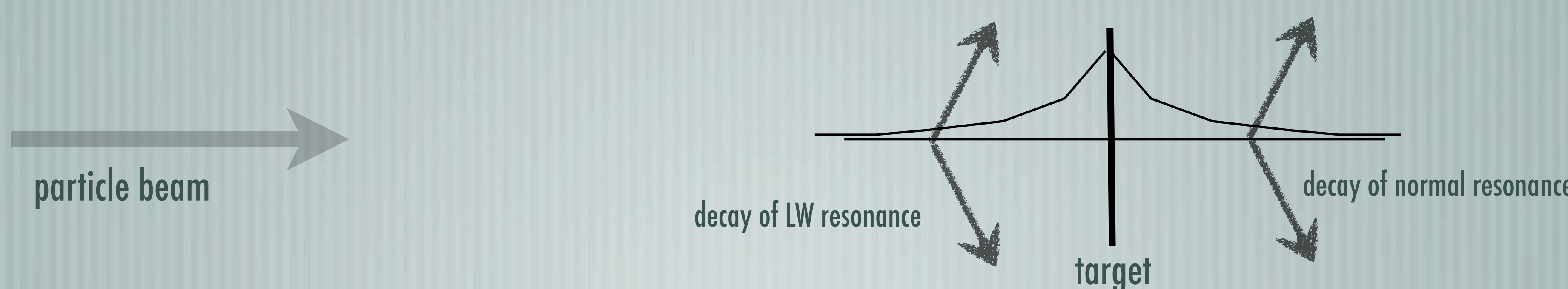

(And for LW virtual "dipole") $\langle \text{detector}|\text{source}\rangle \sim g^*(-my/\tau) f(-my/\tau) \frac{1}{\tau^3} e^{-2i\mathrm{Re}(M)\tau}$

LW-SM: Introduction

- Lore:
  Symmetry+Field Content+Renormalizability+Unitarity = SM
- Higher Derivative (HD) terms:
  - can be made of same fields and preserve symmetries
  - renormalizability preserved
  - unitarity?? Lee-Wick says yes
- Should be explored

Outline

- Minimalistic presentation of six results:
  - No "big" fine-tuning problem
  - No flavor problem
  - EW precision OK, if mass of new resonances few TeV
  - Renormalization and GUTs
  - High energy vector-vector scattering: the special operators
  - LHC examples

The LW SM (or HD SM)

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\rm HD}$$

$$\mathcal{L}_{\rm HD} = \frac{1}{2M_1^2}\left(D^\mu F_{\mu\nu}\right)^a \left(D^\lambda F_\lambda{}^\nu\right)^a - \frac{1}{2M_2^2}\left(D_\mu D^\mu H\right)^\dagger \left(D_\nu D^\nu H\right) - \frac{1}{M_3^2}\bar{\psi}_L (i\not{D})^3 \psi_L$$

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(one for each gauge group factor)

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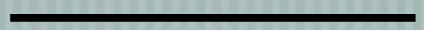
Gauge fixing can be as usual $\mathcal{L}_{\rm GF} = \frac{1}{2\xi}(\partial \cdot A)^2$

or can include HD's, eg,
(convenient for power counting) $\mathcal{L}_{\rm GF} = \frac{1}{2\xi}(\partial \cdot A)(1 + \frac{\partial^2}{M_3^2})(\partial \cdot A)$

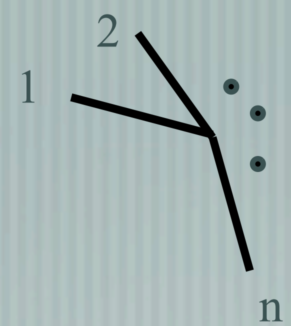
Naive degree of divergence, naively done (but correct!)

propagators



$$\sim \frac{i}{p^2 - p^4/M^2}$$

vertices



$$\sim p^{6-n} \quad \text{(leading)}$$

L = # of loops

V_n = # of vertices with n lines

I = # of internal propagators

E = # of external lines

naive degree of divergence:

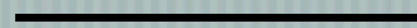
$$D = 4L + \sum_n (6-n)V_n - 4I$$

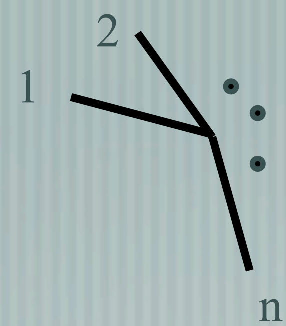
topological identities

$$L = I - \sum_n V_n + 1 \qquad \sum_n nV_n = 2I + E$$

$$\Rightarrow \quad \boxed{D = 6 - 2L - E}$$

Naive degree of divergence, naively done (but correct!)

propagators  $\sim \dfrac{i}{p^2 - p^4/M^2}$

vertices 

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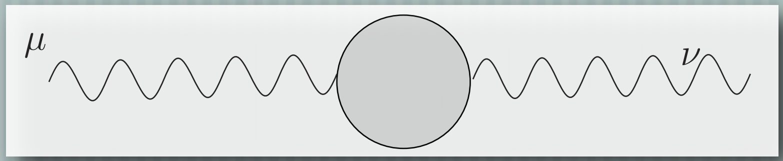
possible divergences:

$$D = \begin{cases} 4 - E & L = 1 \\ 2 - E & L = 2 \end{cases}$$ quadratic only for L=1, E=2

Note: renormalizability straightforward

1. Quadratic divergences?

(i) Gauge fields: gauge invariance decreases divergence to $D = 0$



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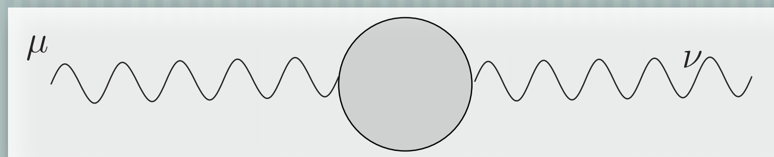
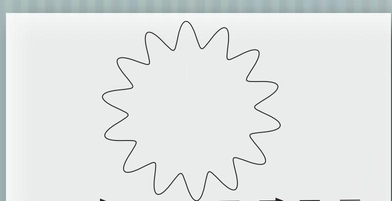
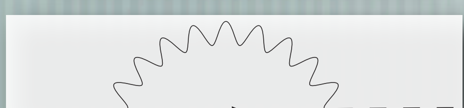


$$= i(p_\mu p_\nu - g_{\mu\nu} p^2)\Pi(p^2)$$

(ii) Higgs field: quadratic divergence from vertex with 2/3 derivatives



$$(D^2 H)^\dagger (D^2 H) \qquad D^2 H = [\partial^2 + 2igA\cdot\partial + ig(\partial\cdot A)]H$$

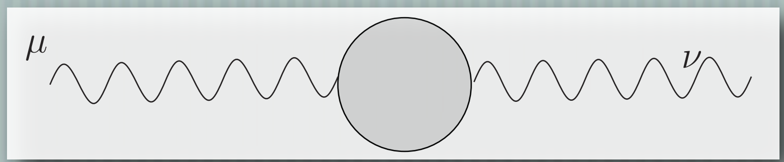


Choose gauge $\partial \cdot A = 0$ and integrate by parts: there are at least two derivatives on external field

$$\Rightarrow \boxed{\delta m_H^2 \sim M^2 \ln \Lambda^2}$$

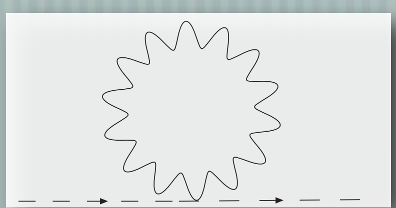
1. Quadratic divergences?

(i) Gauge fields: gauge invariance decreases divergence to $D = 0$

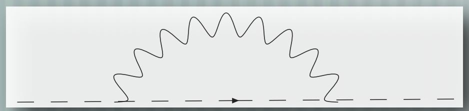


$$= i(p_\mu p_\nu - g_{\mu\nu} p^2)\Pi(p^2)$$

(ii) Higgs field: quadratic divergence from vertex with 2/3 derivatives



$$(D^2 H)^\dagger (D^2 H) \qquad D^2 H = [\partial^2 + 2igA\cdot\partial + ig(\partial\cdot A)]H$$



Choose gauge $\partial \cdot A = 0$ and integrate by parts:
there are at least two derivatives on external field

$$\Rightarrow \boxed{\delta m_H^2 \sim M^2 \ln \Lambda^2}$$

Notes:

1. Physical mass is gauge independent. Quadratic divergences found in unphysical quantities
2. Result checked by explicit calculation (arbitrary ξ-gauge)

2. FCNC's

This is of particular interest at this meeting on "Flavor Physics"

What is interesting is that there is no need for additional restrictions artificially imposed (eg, MFV couplings for the HDs) nor an additional huge superstructure to deal with this (like in SUSY with gauge mediation).

I think this merits more study.

Notation: SM Yukawas:

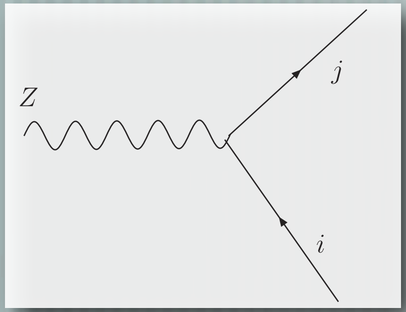
$$\mathcal{L}_{\rm SM} \supset \lambda_U H \bar{q}_L u_R + \lambda_D H^* \bar{q}_L d_R + \lambda_E H^* \bar{\ell}_L e_R$$

For low energy FCNCs treat HDs as small.
Use EOM on HD terms:

$$\frac{1}{M^2} r_{ij} \bar{q}^i_L (i\not{D})^3 q^j_L = \frac{1}{M^2} (\lambda_U^\dagger r \lambda_U)_{ij}\, \bar{u}^i_R H^* i\not{D}(H u^j_R)$$

completely arbitrary matrix (order(1))

:: There are off-diagonal tree level Z couplings, but suppressed



$$\sim \delta_{ij} + \Delta_{ij} \qquad \Delta_{ij} \sim \frac{m_i m_j r_{ij}}{M^2}$$

So, for example, with M = 1 TeV

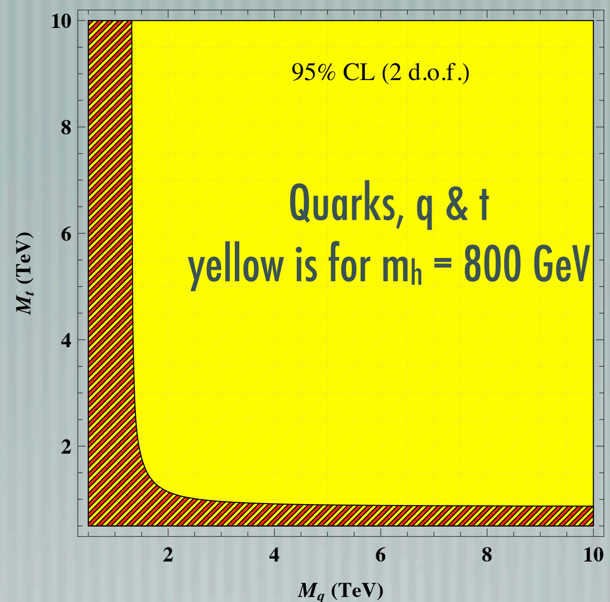
$$\Delta_{bs} \sim \frac{m_b m_s r_{bs}}{M^2} \sim 10^{-6}$$

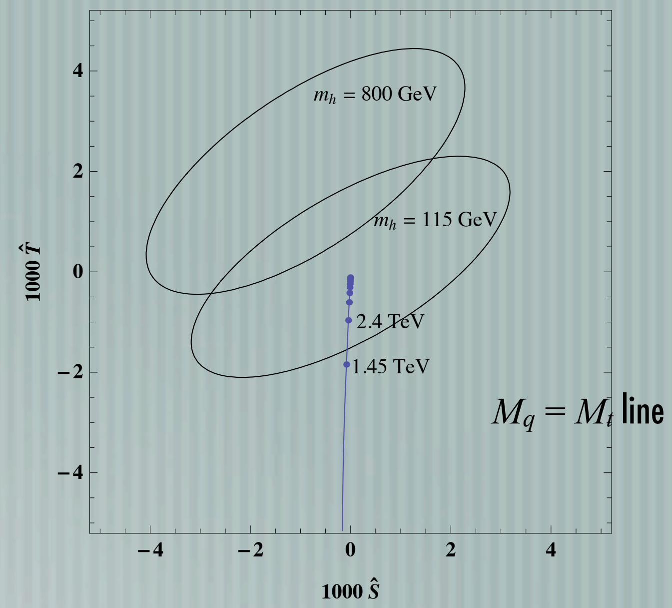
Even for LFV, this mass suppression is sufficient

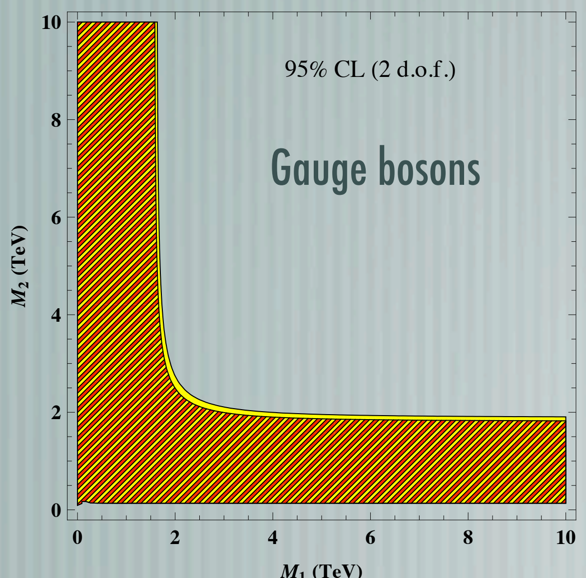
(HD-2HDM at large tan β ? not done)

3. EW precision

Alvarez, Da Rold, Schat & Szynkman, JHEP 0804:026,2008
Underwood & Zwicky, Phys. Rev. D79:035016,2009
Carone & Lebed, Phys. Lett.B668: 221-225,2008
S. Chivukula et al, arXiv:1002.0343 (this reported below)







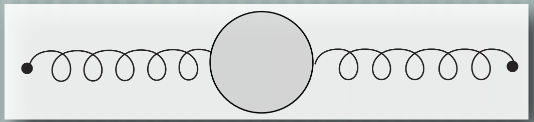
Bounds, quark or gauge bosons, largely decouple: enter into (S,T), or (W, Y)

Light higgs favored

back on plan:

4. YM-beta function

Background-Field Gauge



1-loop, normal $$\beta = -\frac{g^3}{16\pi^2} C_2 \left(\frac{10}{3} + \frac{1}{3}\right)$$

1-loop, HD^2 theory $$\beta = -\frac{g^3}{16\pi^2} C_2 \left(2 \times \frac{10}{3} + \frac{1}{3} + \frac{1}{6}\right)$$

1/6 is easy to understand: doubling obvious only when longitudinal and transverse modes all have same power counting. Need HD GF. But then get determinant from exponentiation trick:

$$\sqrt{\det(1 + D^2/M^2)} \int [d\alpha] e^{\frac{i}{2\xi} \int d^4x\, \alpha \left(1 + \frac{D^2}{M^2}\right)\alpha} \delta(\partial \cdot A - \alpha)$$

This det is, for UV, same as usual ghosts in BFG. The sqrt gives an additional 1/2

1-loop, HD^3 theory $$\beta = -\frac{g^3}{16\pi^2} C_2 \left(2 \times \frac{10}{3} + \frac{1}{3} + \frac{1}{6} + 1\right)$$

C. Carone, arXiv:0904.2359

More generally, in HD[2] $\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\psi + \mathcal{L}_\phi,$

$$\mathcal{L}_A = -\frac{1}{2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + \frac{1}{m^2}\mathrm{Tr}(D^\mu F_{\mu\nu})^2 - \frac{i\gamma g}{m^2}\mathrm{Tr}(F^{\mu\nu}[F_{\mu\lambda}, F_\nu{}^\lambda])$$

$$\mathcal{L}_\psi = \bar\psi_L i\not{D}\psi_L + \frac{i}{m^2}\bar\psi_L\left[\sigma_1\not{D}\not{D}\not{D} + \sigma_2\not{D}D^2 + ig\sigma_3F^{\mu\nu}\gamma_\nu D_\mu + ig\sigma_4(D_\mu F^{\mu\nu})\gamma_\nu\right]\psi_L$$

$$\mathcal{L}_\phi = -\phi^* D^2\phi - \frac{1}{m^2}\phi^*\left[\delta_1(D^2)^2 + ig\delta_2(D_\mu F^{\mu\nu})D_\nu + g^2\delta_3F^{\mu\nu}F_{\mu\nu}\right]\phi$$

$$\beta(g) = -\frac{g^3}{16\pi^2}\left[\left(\frac{43}{6} - 18\gamma + \frac{9}{2}\gamma^2\right)C_2 - n_\psi\left(\frac{\sigma_1^2 - \sigma_2\sigma_3 + \frac{1}{2}\sigma_3^2}{(\sigma_1+\sigma_2)^2}\right) - n_\phi\left(\frac{\delta_1 + 6\delta_3}{3\delta_1}\right)\right]$$

$$\gamma_\psi(g) = -\frac{g^2}{16\pi^2}\frac{3}{4}C_1\left(\frac{2\sigma_1(2\sigma_2+\sigma_3-2\sigma_4) + \sigma_2(2\sigma_2+2\sigma_3-\sigma_4) - \sigma_3^2 - \sigma_4^2 + \sigma_3\sigma_4}{\sigma_1+\sigma_2}\right)$$

$$\gamma_\phi(g) = -\frac{g^2}{16\pi^2}\frac{3}{8}C_1\left(\frac{8\delta_1^2 - \delta_2^2 - 4\delta_1\delta_2}{\delta_1}\right)$$

$$\mu\frac{\partial\gamma}{\partial\mu} = 0 \qquad \mu\frac{\partial(g^2\sigma_i)}{\partial\mu} = 2(g^2\sigma_i)\gamma_\psi(g) \quad \text{and} \quad \mu\frac{\partial(g^2\delta_i)}{\partial\mu} = 2(g^2\delta_i)\gamma_\phi(g).$$

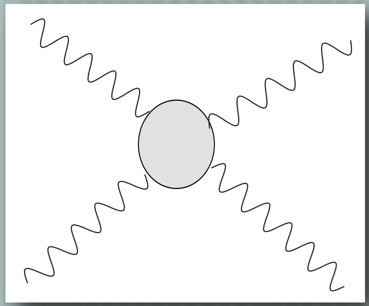
This is for general HD terms, but not all have good high energy behavior (next section)

GUT (Carone): some fields have HD^2, others HD^3

model	$N=3$ fields	(b_3, b_2, b_1)	$\alpha_3^{-1}(m_Z)$	error
SM	-	$(-7, -19/6, 41/10)$	14.04	$+50.8\sigma$
MSSM	-	$(-3, 1, 33/5)$	8.55	$+2.9\sigma$
$N=2$ 1H LWSM	none	$(-19/2, -2, 61/5)$	14.03	$+50.6\sigma$
$N=3$ 1H LWSM	all	$(-9/2, 25/6, 203/10)$	13.76	$+48.3\sigma$
$N=2$ 8H LWSM	none	$(-19/2, 1/3, 68/5)$	7.76	-4.01σ
$N=3$ 6H LWSM	all	$(-9/2, 20/3, 109/5)$	7.85	-3.16σ
$N=2$ 1H LWSM,	gluons	$(-25/2, -2, 61/5)$	7.81	-3.55σ
$N=2$ 1H LWSM	gluons, 1 gen. quarks	$(-59/6, 0, 41/3)$	8.40	$+1.55\sigma$
$N=2$ 1H LWSM	1 gen. LH fields	$(-49/6, 2/3, 191/15)$	8.03	-1.66σ
$N=2$ 2H LWSM	LH leptons	$(-19/2, 1/3, 68/5)$	7.76	-4.01σ
$N=2$ 2H LWSM	gluons, quarks, 1H	$(-9/2, 9/2, 169/10)$	8.21	-0.06σ

TABLE I: Predictions for $\alpha_3^{-1}(m_Z)$ assuming one-loop unification. The experimental value is 8.2169 ± 0.1148 [10]. The abbreviations used are as follows: H=Higgs doublets, gen.=generation, LH=left handed.

but M_{GUT} low, proton decay a problem. Fermions at orbifold fixed points in Higher-dim's where wave-function vanishes?



5. Massive V V-scattering: Special HD terms

Consider VV-scattering, first in non-HD case:

- if described by massive vector boson lagrangian, $\mathcal{A} \sim E^2 \quad E >> m$

unitarity violated (perturbatively)

- growth could be E^4, $\epsilon_L^\mu(p) = 1/M(p,0,0,E)$

but approximate GI at large E reduces growth by E^2, since $\epsilon_L^\mu(p) = p^\mu/M + (M/2E)n^\mu$

$(n^2 = 0)$

- HD:
  - + Gauge Invariance (GI) is maintained, exact ward identities
  - + Use LW-form (2-fields): amplitude has no inverse powers of M

$$\Rightarrow \quad \mathcal{A} \sim E^0$$

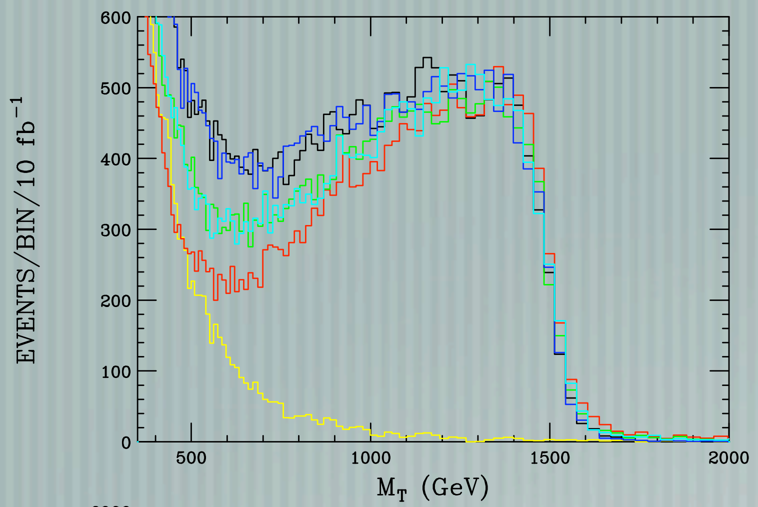
Unacceptable growth is controlled by GI and absence of $1/M$ terms in lagrangian.

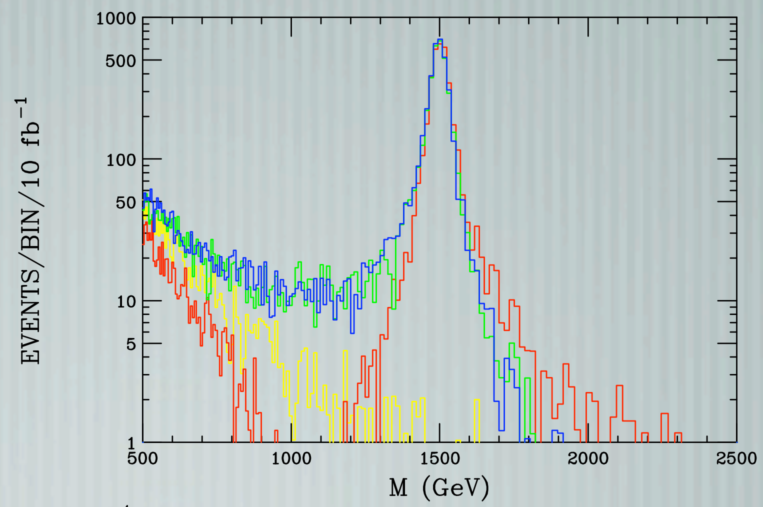
- HD with no LW-form, like F^3, does have E^2 growth at tree level (verified by explicit calculation)

6. LHC examples

T. Rizzo, JHEP 06:070(2007)

LW-Wboson
M=1.5TeV
ATLAS-like cuts
10 fb^{-1} (14TeV)
(LW=black)





LW-Zboson
M=1.5TeV
ATLAS-like cuts
10 fb^{-1} (14TeV)
(LW=green)

The End

- There exist unitary HD theories (at least large N to all orders g)
- HDSM Solves big fine tuning, flavor OK, EWP fine (M > 3 TeV)
- GUT trouble... open questions on completion and gravity
- Acausal (non-local?) at short distances, but does not build macroscopic acausality (at least not in thermal equilibrium)
- Other applications? Cosmology?

fin

Extra slides

3. EW precision, very rough

Use perturbation theory in HD operators, again because E << M

Then from operator analysis (eff theory; eg, Han and Skiba) know that T and S are, respectively

$$(H^\dagger D_\mu H)^2 \qquad \text{and} \qquad H^\dagger \tau^a W^a_{\mu\nu} H B_{\mu\nu}$$

Neither of these are HD ops, but we generate them using EOM.

$$(DF)_\mu = g(H^\dagger \overleftrightarrow{\partial}_\mu H) \qquad \Rightarrow \qquad \frac{g^2}{M^2}(H^\dagger D_\mu H)^2$$

Bound on boundary of total naturalness:

$$T = -\pi \frac{g_1^2 + g_2^2}{g_2^2}\frac{v^2}{M^2} \qquad \Rightarrow M \gtrsim 3 \text{ TeV}$$

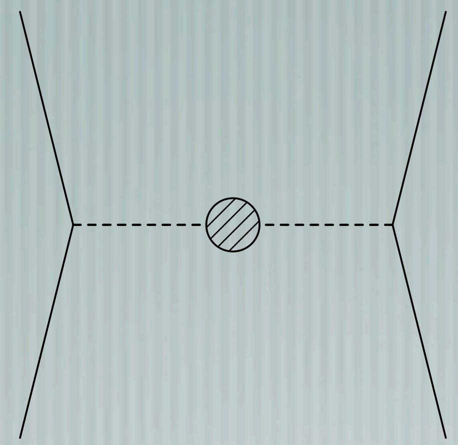
while $\delta m_H^2 \sim \dfrac{g^2}{16\pi^2} M^2 \lesssim m_H^2 \Rightarrow M \lesssim 3 \text{ TeV}$

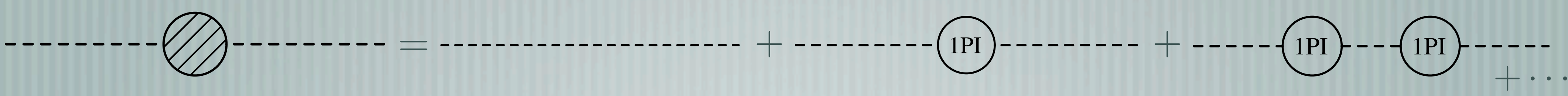
Global analysis constraints M to 3 TeV'ish. Alvarez, Da Rold, Schat & Szynkman, JHEP 0804:026,2008

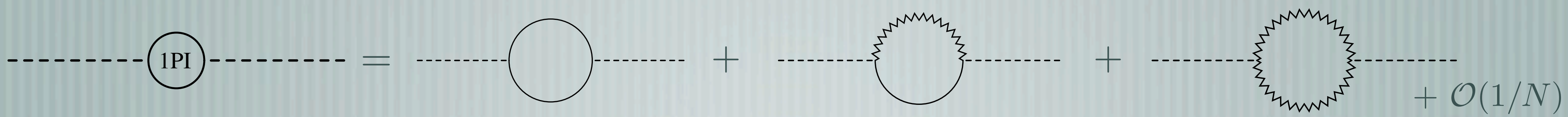
(ii) $O(N)$ model

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^a)^2 - \frac{1}{2}m^2(\phi^a)^2 - \frac{1}{2}(\partial_\mu \Phi^a)^2 + \frac{1}{2}M^2(\Phi^a)^2 - \frac{1}{8}\lambda[(\phi^a - \Phi^a)^2]^2$$

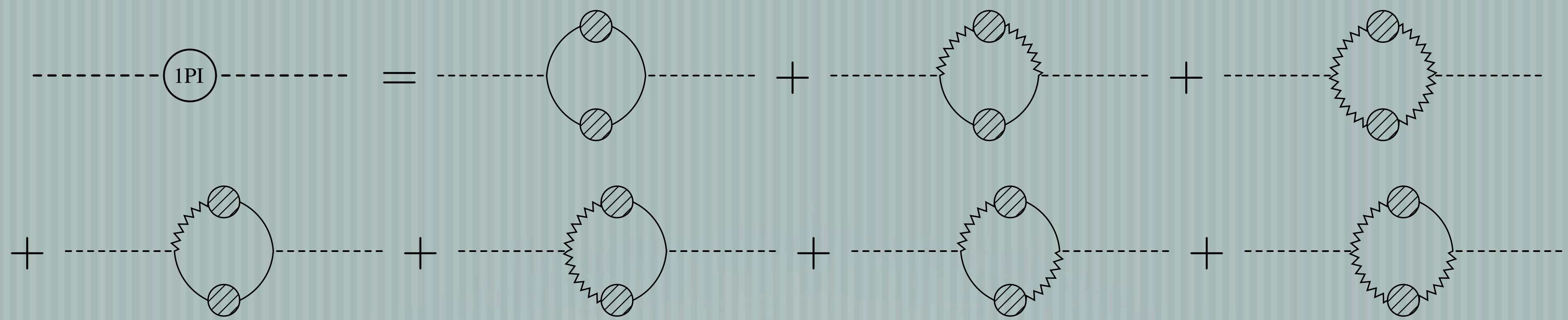
use auxiliary field σ, $\quad \mathcal{L}_{\text{int}} = \frac{1}{2}\sigma^2 + \frac{1}{2}g\sigma(\phi^a - \Phi^a)^2, \quad g^2 N = g_0^2$ fixed

$i\mathcal{A} =$ 





same story as above, this does not satisfy optical theorem, need to dress propagators



but now only Im part of pole need to be kept, Re is a $1/N$ correction

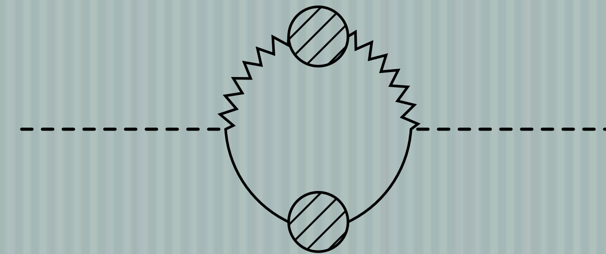
full LW propagator formally as before

$$G^{(2)} = -\frac{A}{p^2 - \hat{M}^2} - \frac{A^*}{p^2 - \hat{M}^{*2}} + \int_{9m^2}^{\infty} d\mu^2 \, \frac{\rho(\mu^2)}{p^2 - \mu^2}$$

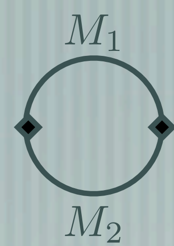
but now A=1+$O(1/N)$ and

$$\rho(\mu^2) \approx \frac{1}{\pi} \, \mathrm{Im} \frac{1}{\mu^2 - M^2 - iM\Gamma} \to \delta(\mu^2 - M^2)$$

We can see very explicitly how unitarity works; consider the contribution to the forward scattering amplitude from 1 normal and 1 LW



Let

$i\tilde{\mathcal{I}}(M_1, M_2) =$ 

defined with p^0 integral along the imaginary axis**

3 terms in LW propagator: $\mathcal{I} = -A\tilde{\mathcal{I}}(m, \hat{M}) - A^*\tilde{\mathcal{I}}(m, \hat{M}^*) + \int_{(3m)^2}^{\infty} d\mu^2\, \rho(\mu^2)\tilde{\mathcal{I}}(m, \mu)$

$$\mathrm{Im}(\mathcal{A}) = \frac{g^4 N}{16\pi} \frac{1}{|1 + \Pi_\sigma(s)|^2} \int_{(3m)^2}^{\infty} d\mu^2\, \rho(\mu^2) I(s, m, \mu)$$

where $\mathrm{Im}(\tilde{\mathcal{I}}(m, \mu)) = \pi I(m, \mu)$ is the usual phase space factor

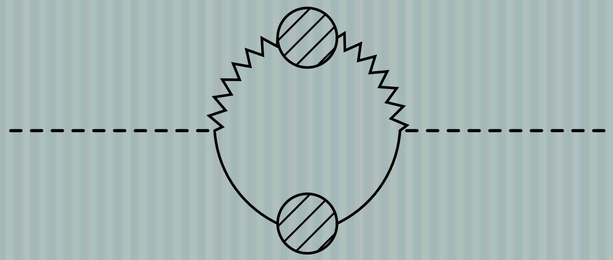
---

Replacing $\rho(\mu^2) \to \delta(\mu^2 - M^2)$ satisfies *exactly* the optical theorem

$$\sigma(\phi\phi \to \phi\Phi) = \frac{1}{\sqrt{s(s - 4m^2)}} \left(\frac{g^4 N}{16\pi} \frac{1}{|1 + \Pi_\sigma(s)|^2} \right) I(s, m, M) \qquad (\text{“}\Phi\text{”} = 3\phi)$$

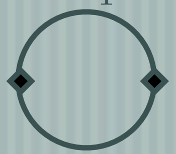
**subtleties @ dinner tonight after wine

Physically:



recall

$$i\tilde{\mathcal{I}}(M_1, M_2) =$$



is a function of $p^2 = 4E^2$ (in CM frame)

$$\mathcal{I} = -A\tilde{\mathcal{I}}(m, \hat{M}) - A^*\tilde{\mathcal{I}}(m, \hat{M}^*) + \int_{(3m)^2}^{\infty} d\mu^2\, \rho(\mu^2)\tilde{\mathcal{I}}(m, \mu)$$

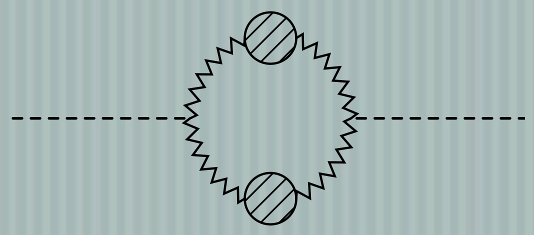
look for discontinuities in E in each of three terms

discontinuity only arises from internal propagators going on shell

for first two this can only happen for complex E

but E is external energy, always real (if external particles are the stable “normal” modes)

2 LW case is on the surface similar



3x3 terms:

$$\tilde{\mathcal{I}}(\hat{M},\hat{M}) + \tilde{\mathcal{I}}(\hat{M}^*,\hat{M}^*) + 2\tilde{\mathcal{I}}(\hat{M},\hat{M}^*) + \tilde{\mathcal{I}}(M,M) - 2\tilde{\mathcal{I}}(M,\hat{M}) - 2\tilde{\mathcal{I}}(M,\hat{M}^*)$$

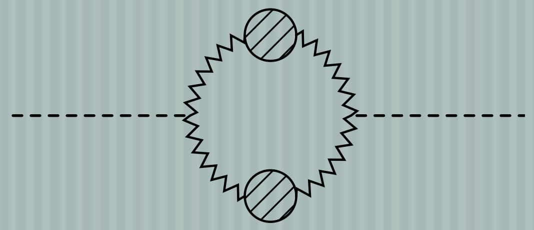
problem: both $\tilde{\mathcal{I}}(M,M)$ and $\tilde{\mathcal{I}}(\hat{M},\hat{M}^*)$ may give $\mathrm{disc}(A)$

and this one comes with wrong sign

more specifically

the integral $\tilde{\mathcal{I}}(M_1,M_2)$ as a function of E has a cut with branch point at $(M_1+M_2)^2$
this is for real E in both terms above

2 LW case is on the surface similar



3x3 terms:

$$\tilde{\mathcal{I}}(\hat{M},\hat{M})+\tilde{\mathcal{I}}(\hat{M}^*,\hat{M}^*)+2\tilde{\mathcal{I}}(\hat{M},\hat{M}^*)+\tilde{\mathcal{I}}(M,M)-2\tilde{\mathcal{I}}(M,\hat{M})-2\tilde{\mathcal{I}}(M,\hat{M}^*)$$

problem: both $\tilde{\mathcal{I}}(M,M)$ and $\tilde{\mathcal{I}}(\hat{M},\hat{M}^*)$ may give $\mathrm{disc}(A)$

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more specifically

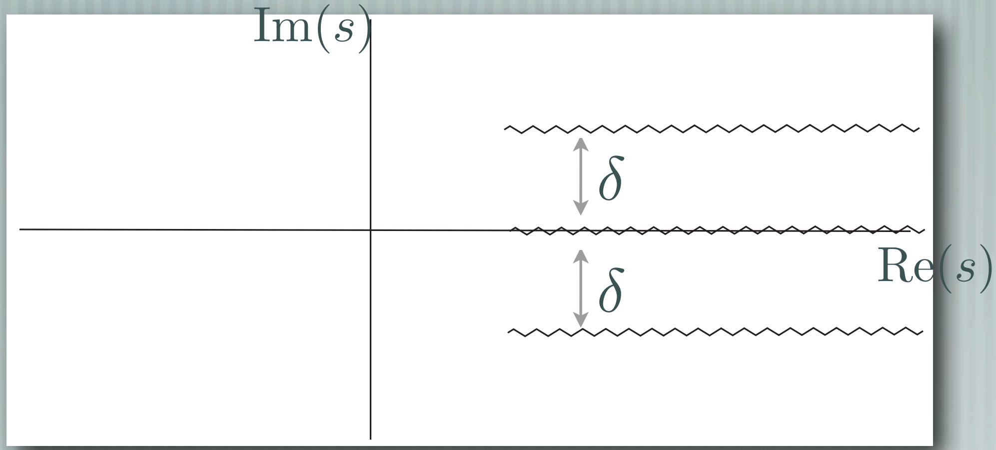
the integral $\tilde{\mathcal{I}}(M_1,M_2)$ as a function of E has a cut with branch point at $(M_1+M_2)^2$
this is for real E in both terms above

oopsie!

CLOP prescription:
result of integration depends on choice of contour
equivalent to taking different complex masses in the two propagators, with

$$M_2 - M_1 = i\delta$$

then letting, at the end, $\delta \to 0$
This prescription is explicitly Lorentz covariant.



"bad" cuts move off real axis,
discontinuity across real axis
is only from "good" cut

This distortion of the normal Feynman rules is what makes
the non-perturbative formulation elusive

-we have checked the optical theorem for this case
-easy to generalize argument to all scattering amplitudes