

GGI Workshop,
May 2010

Streams and caustics: the fine structure of Λ CDM halos and its implications for dark matter detection

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The four elements of Λ CDM halos

I Smooth background halo

- NFW-like cusped density profile
- near-ellipsoidal equidensity contours

II Bound subhalos

- most massive typically 1% of main halo mass
- total mass of all subhalos $\lesssim 10\%$
- less centrally concentrated than the smooth component

III Tidal streams

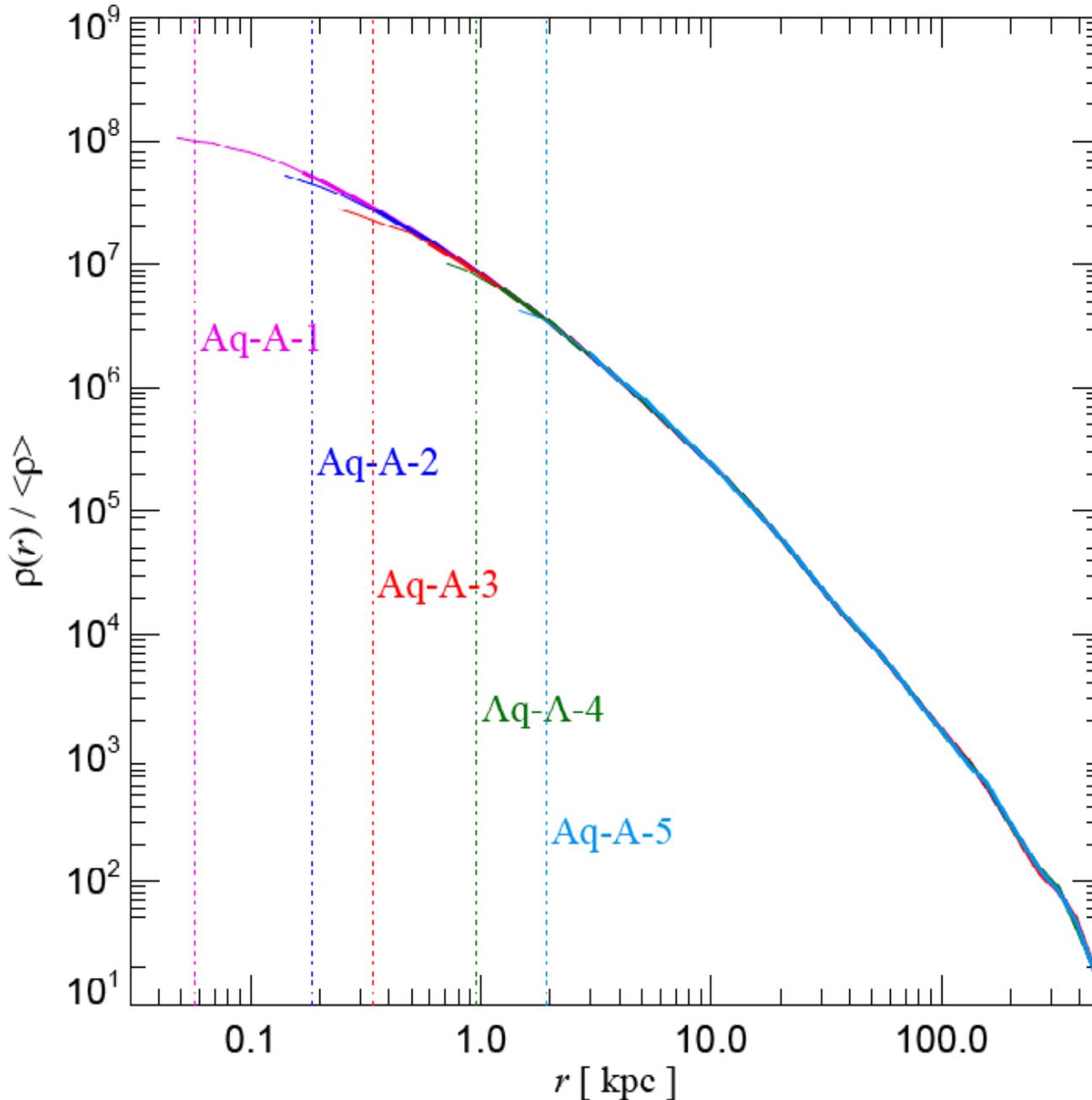
- remnants of tidally disrupted subhalos

IV Fundamental streams

- consequence of smooth and cold initial conditions
- very low internal velocity dispersions
- produce density caustics at projective catastrophes

I. Smooth background halo

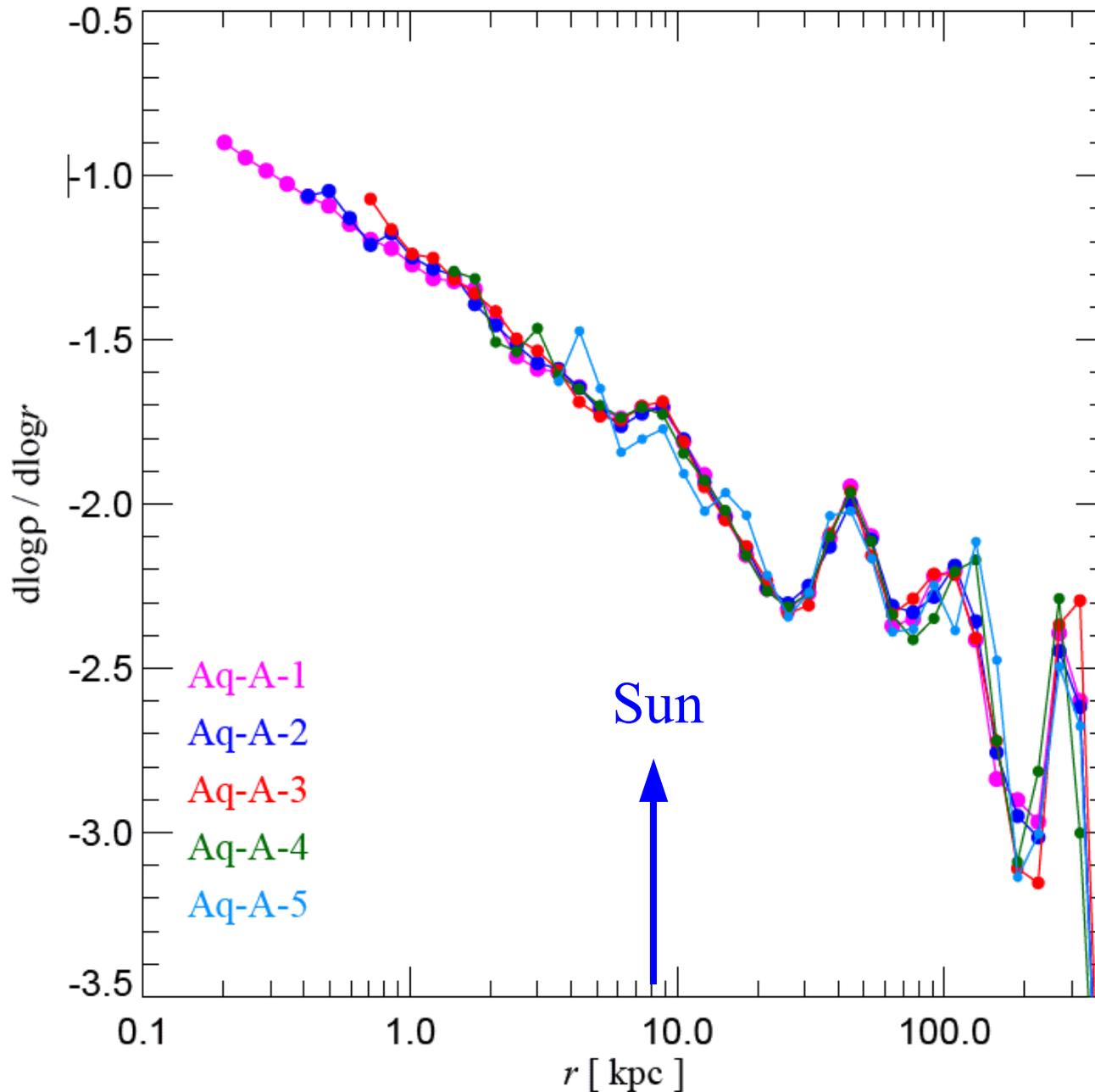
Aquarius Project: Springel et al 2008



- Density profiles of simulated DM-only Λ CDM halos are now very well determined

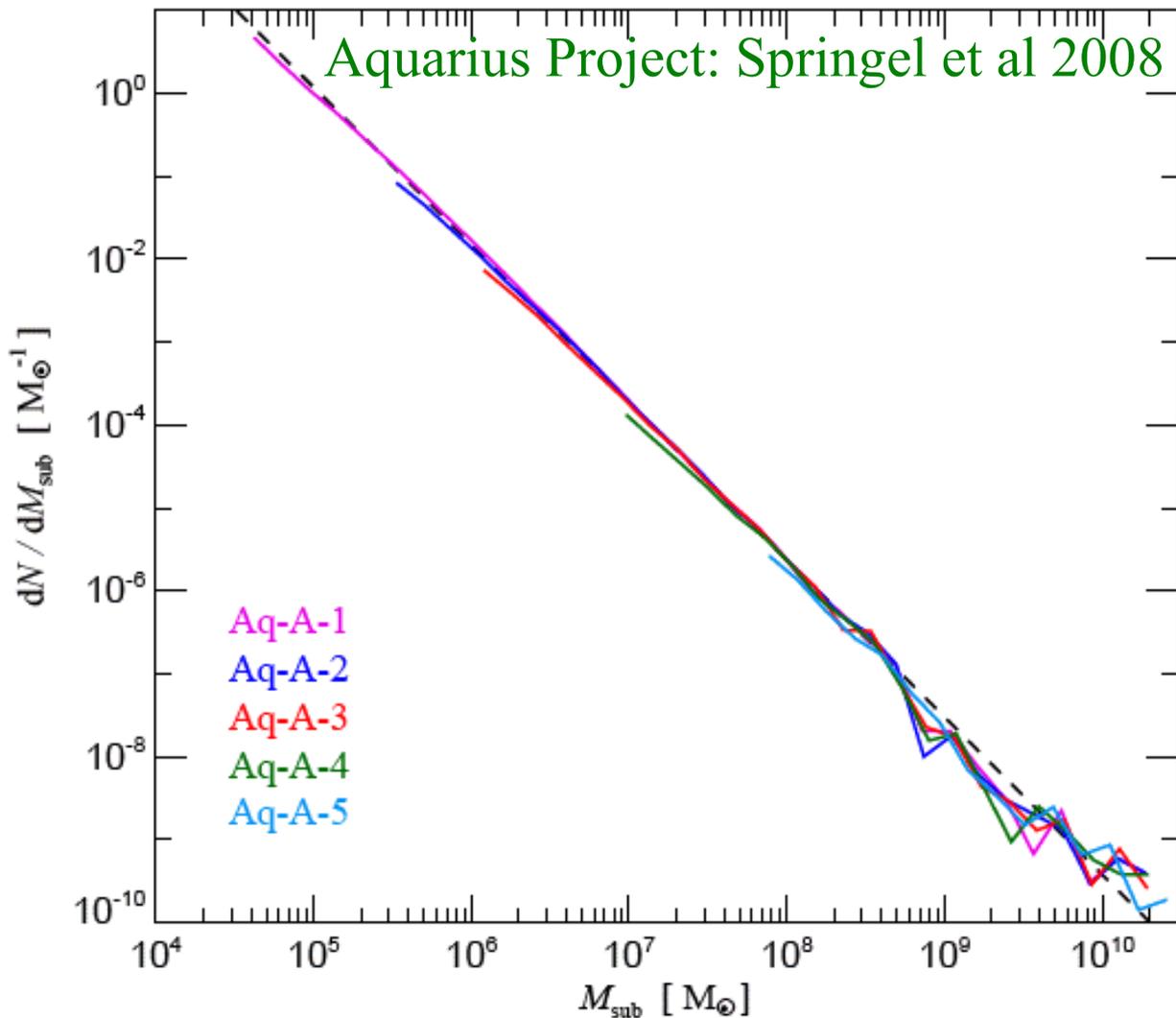
I. Smooth background halo

Aquarius Project: Springel et al 2008



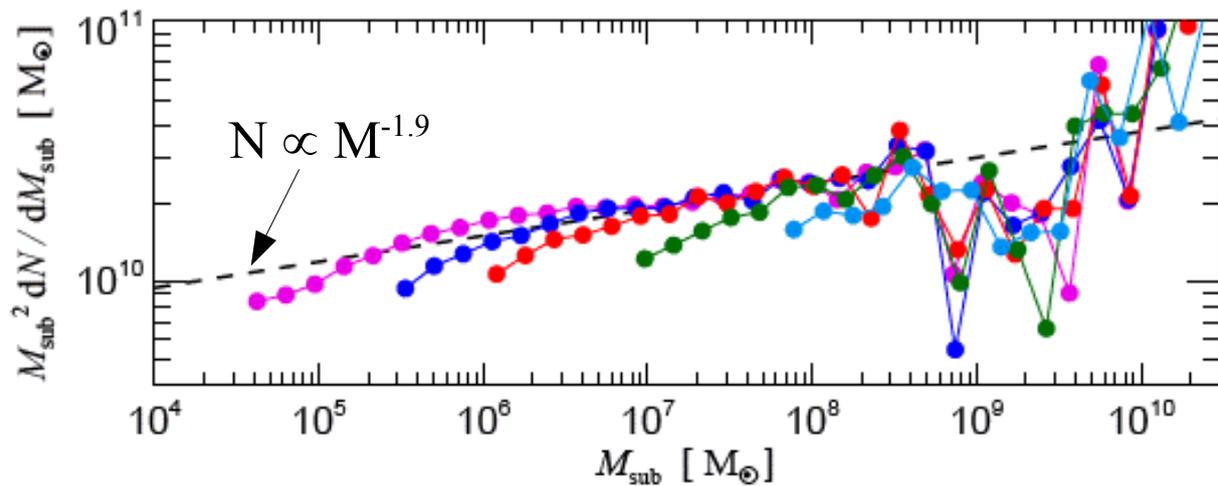
- Density profiles of simulated DM-only Λ CDM halos are now very well determined
- The inner cusp does not appear to have a well-defined power law slope
- Treating baryons more important than better DM simulations

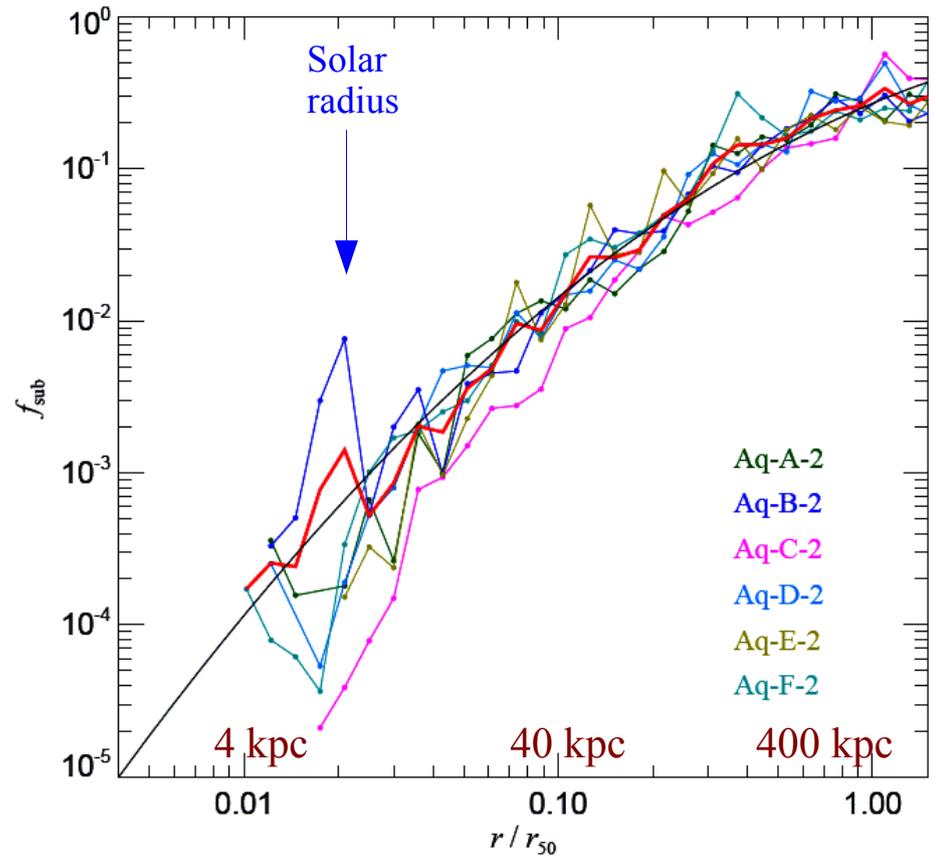
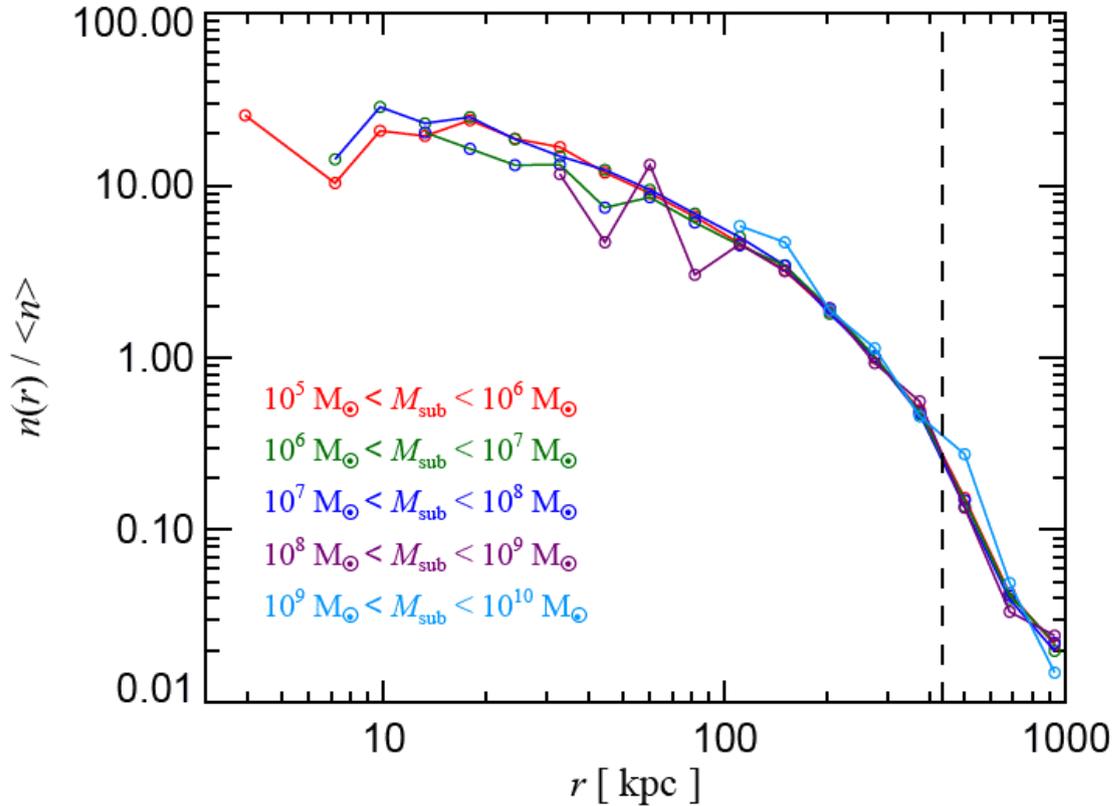
Aquarius Project: Springel et al 2008



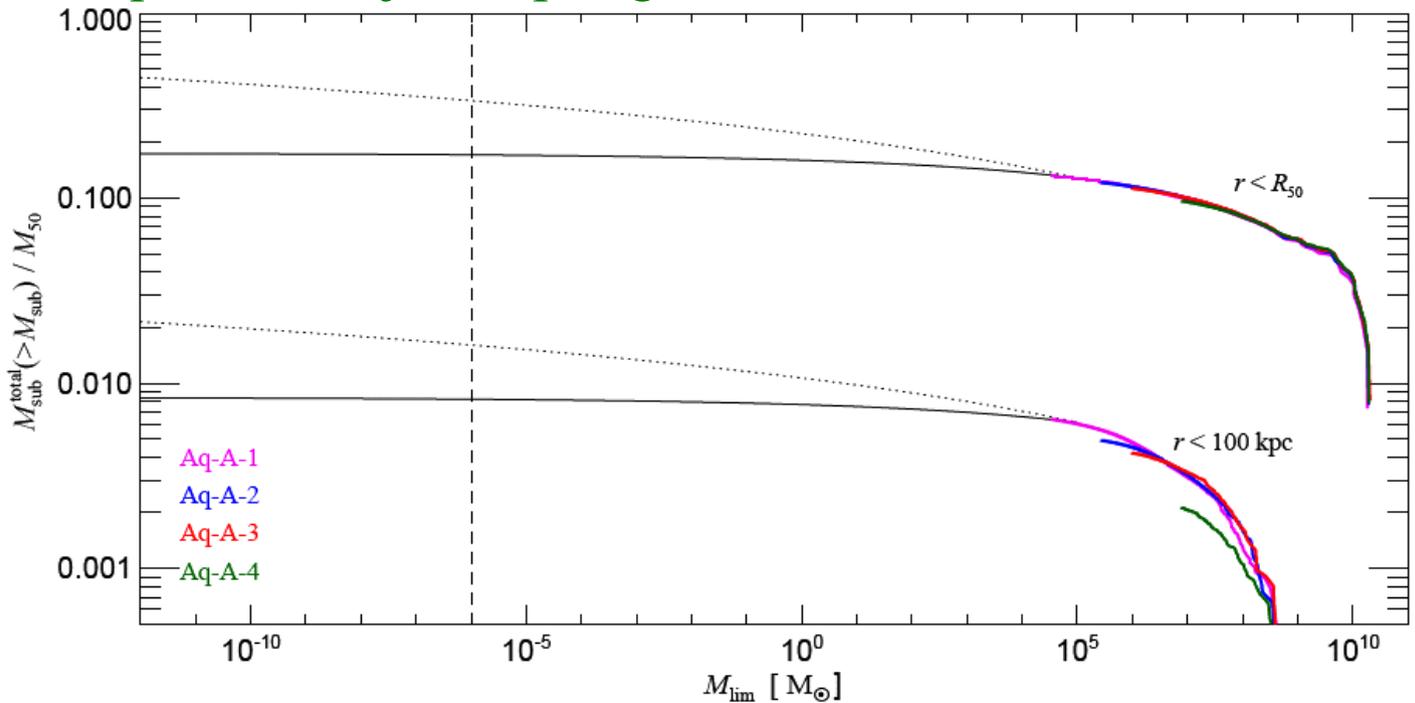
II. Bound subhalos

- Abundance of self-bound subhalos is measured to below $10^{-7} M_{\text{halo}}$
- Most subhalo mass is in the biggest objects (just)





Aquarius Project: Springel et al 2008

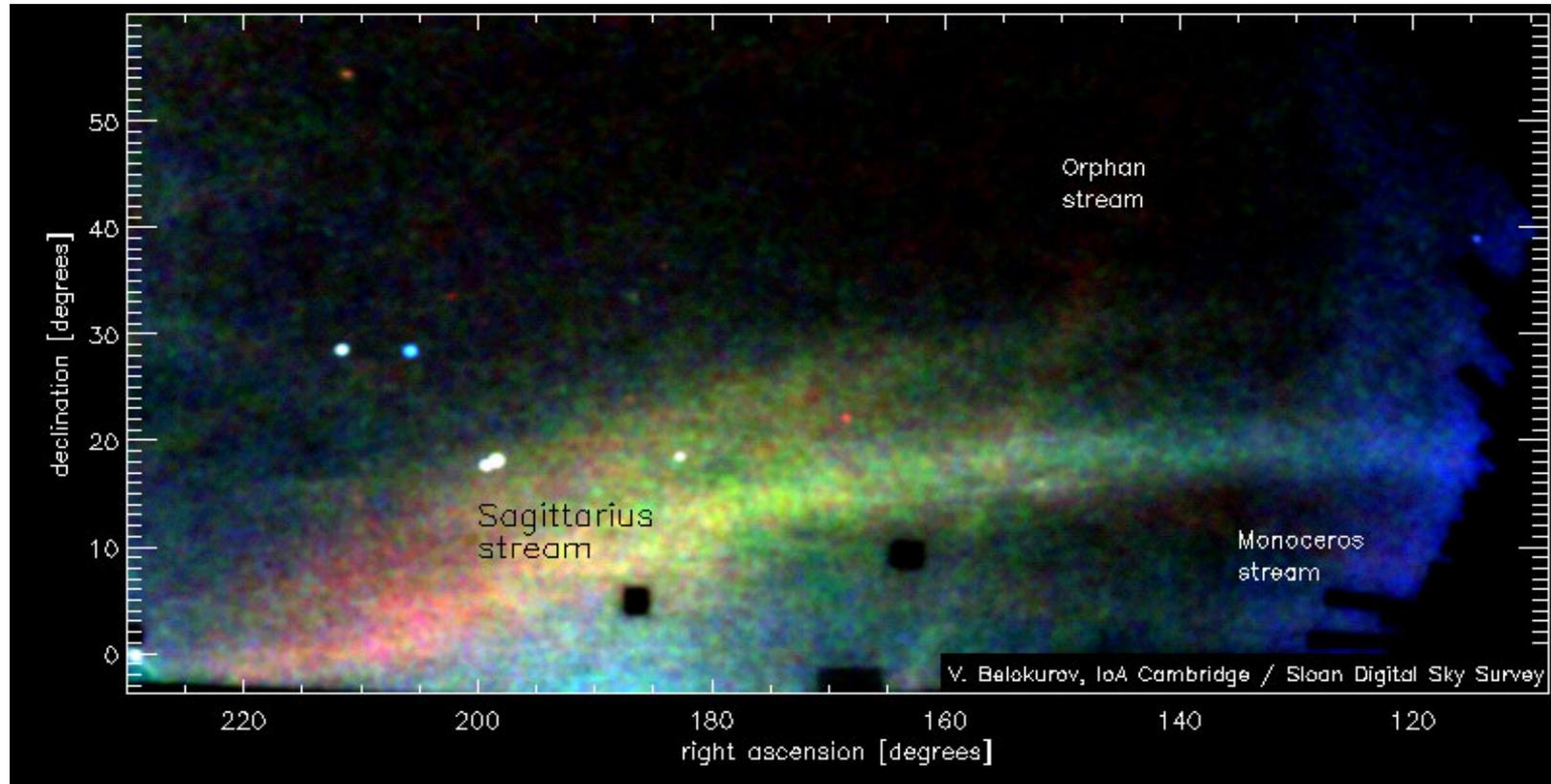


- All mass subhalos are similarly distributed
- A small fraction of the inner mass in subhalos
- $\ll 1\%$ of the mass near the Sun is in subhalos

Bound subhalos: conclusions

- Substructure is primarily in the outermost parts of halos
- The radial distribution of subhalos is almost mass-independent
- Subhalo populations scale (almost) with the mass of the host
- The total mass in subhalos converges only weakly at small m
- Subhalos contain a very small mass fraction in the inner halo

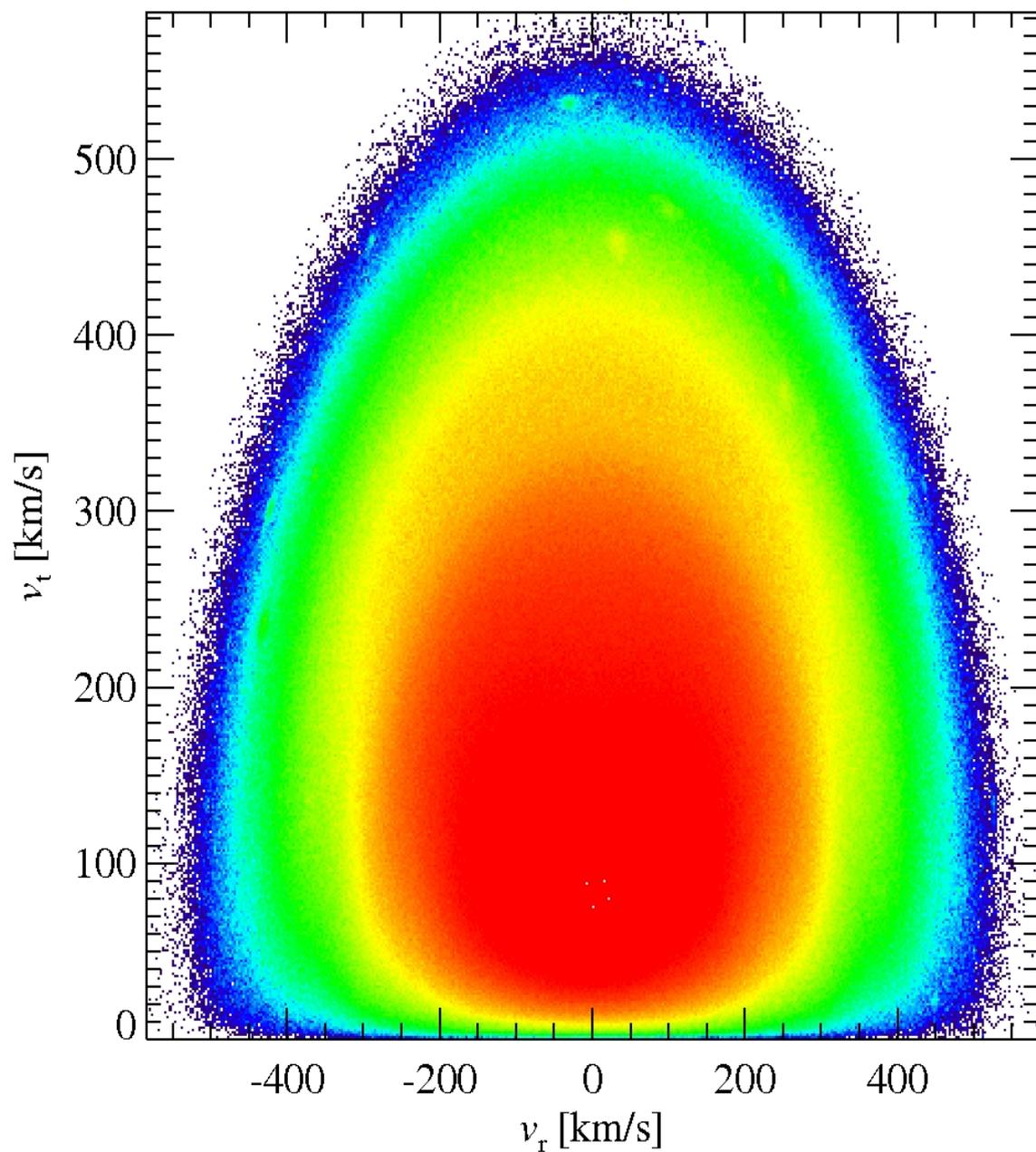
III. Tidal Streams



- Produced by partial or total tidal disruption of subhalos
- Analogous to observed stellar streams in the Galactic halo
- Distributed along/around orbit of subhalo (c.f. meteor streams)
- Localised in almost 1-D region of 6-D phase-space (\underline{x} , \underline{v})

Dark matter phase-space structure in the inner MW

M. Maciejewski



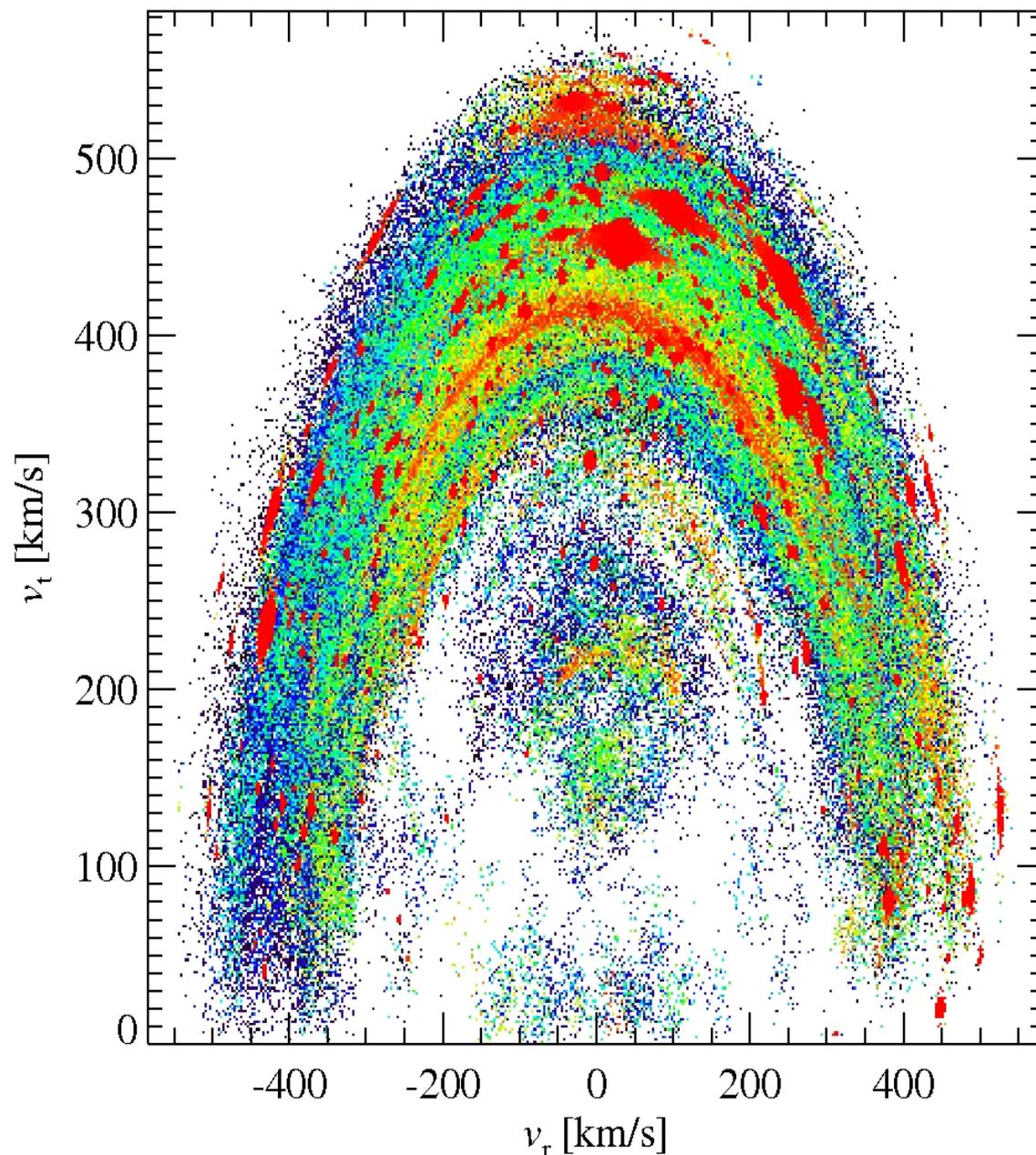
$6 \text{ kpc} < r < 12 \text{ kpc}$

All particles

$N = 3.8 \times 10^7$

Dark matter phase-space structure in the inner MW

M. Maciejewski



$6 \text{ kpc} < r < 12 \text{ kpc}$

Particles in detected
phase-space structure

$$N = 3.0 \times 10^5$$

$$N_{\text{subhalo}} = 3.9 \times 10^4$$

IV. Fundamental streams

After CDM particles become nonrelativistic, but *before* they dominate the density (e.g. $z \sim 10^5$) their distribution function is

$$f(\mathbf{x}, \mathbf{v}, t) = \rho(t) [1 + \delta(\mathbf{x}, t)] N [\{\mathbf{v} - \mathbf{V}(\mathbf{x}, t)\} / \sigma]$$

where $\rho(t)$ is the mean mass density of CDM,

$\delta(\mathbf{x}, t)$ is a Gaussian random field with finite variance $\ll 1$,

$\mathbf{V}(\mathbf{x}, t) = \nabla \psi(\mathbf{x}, t)$ where $\nabla^2 \psi \propto \delta$,

and N is normal with $\sigma^2 \ll \langle |\mathbf{V}|^2 \rangle$ (today $\sigma \sim 0.1$ cm/s)

CDM occupies a thin 3-D 'sheet' within the full 6-D phase-space and its projection onto \mathbf{x} -space is near-uniform.

$Df / Dt = 0$  only a 3-D subspace is occupied at *all* times.

Nonlinear evolution leads to multi-stream structure and caustics

IV. Fundamental streams

Consequences of $Df/Dt = 0$

- The 3-D phase sheet can be stretched and folded but not torn
- At least one sheet must pass through every point \mathbf{x}
- In nonlinear objects there are typically many sheets at each \mathbf{x}
- Stretching which reduces a sheet's density must also reduce its velocity dispersions to maintain $f = \text{const.}$ $\longrightarrow \sigma \sim \rho^{-1/3}$
- At a caustic, at least one velocity dispersion must $\longrightarrow \infty$
- All these processes can be followed in fully general simulations by tracking the phase-sheet local to each simulation particle

The geodesic deviation equation

Particle equation of motion: $\dot{X} = \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{v}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ -\nabla\phi \end{bmatrix}$

Offset to a neighbor: $\delta\dot{X} = \begin{bmatrix} \delta\mathbf{v} \\ \mathbf{T} \cdot \delta\mathbf{x} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{T} & 0 \end{bmatrix} \cdot \delta X$; $\mathbf{T} = -\nabla(\nabla\phi)$

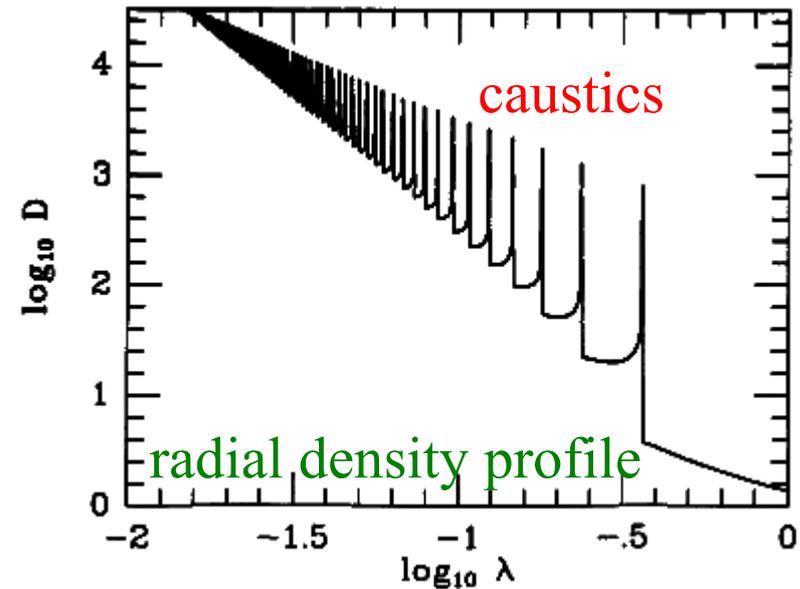
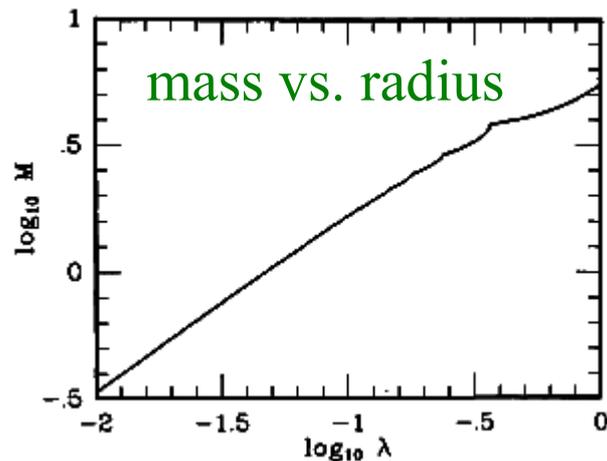
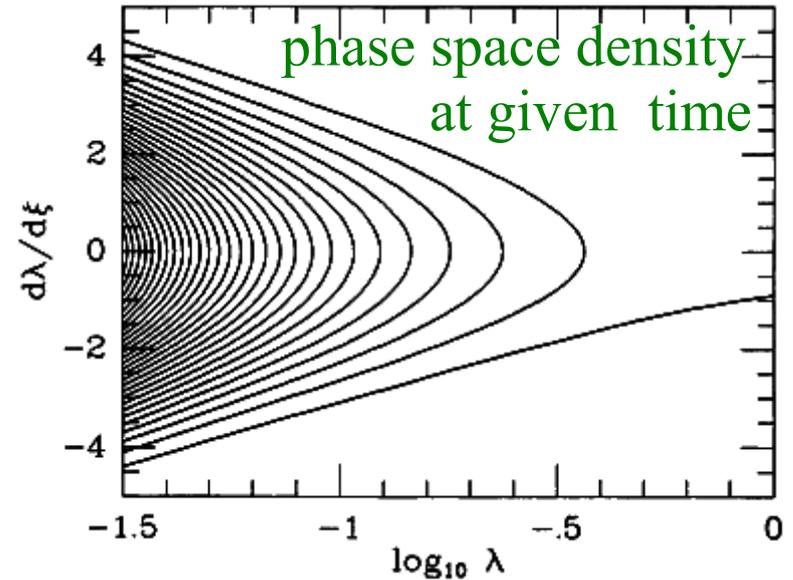
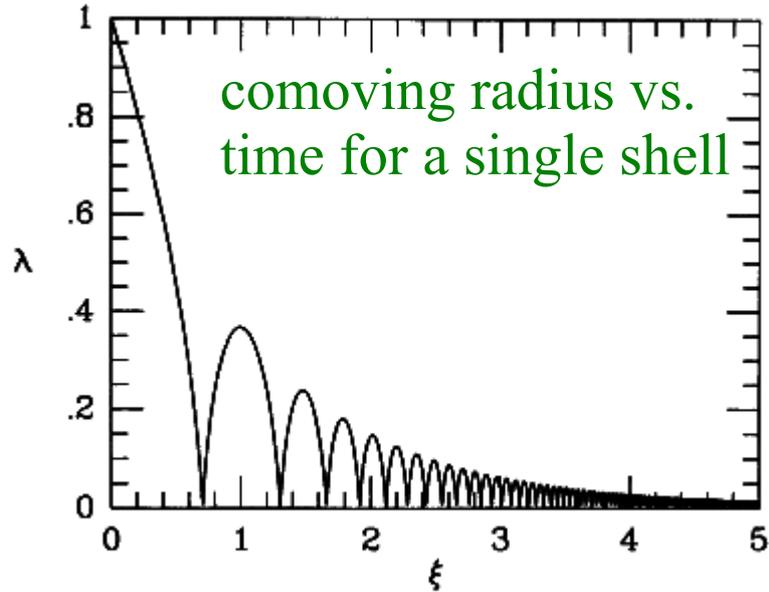
Write $\delta X(t) = D(X_0, t) \cdot \delta X_0$, then differentiating w.r.t. time gives,

$$\dot{D} = \begin{bmatrix} 0 & \mathbf{I} \\ \mathbf{T} & 0 \end{bmatrix} \cdot D \quad \text{with } D_0 = I$$

- Integrating this equation together with each particle's trajectory gives the evolution of its local phase-space distribution
- No symmetry or stationarity assumptions are required
- $\det(D) = 1$ at all times by Liouville's theorem
- For CDM, $1/|\det(D_{\mathbf{xx}})|$ gives the decrease in local 3D space density of each particle's phase sheet. Switches sign and is infinite at caustics.

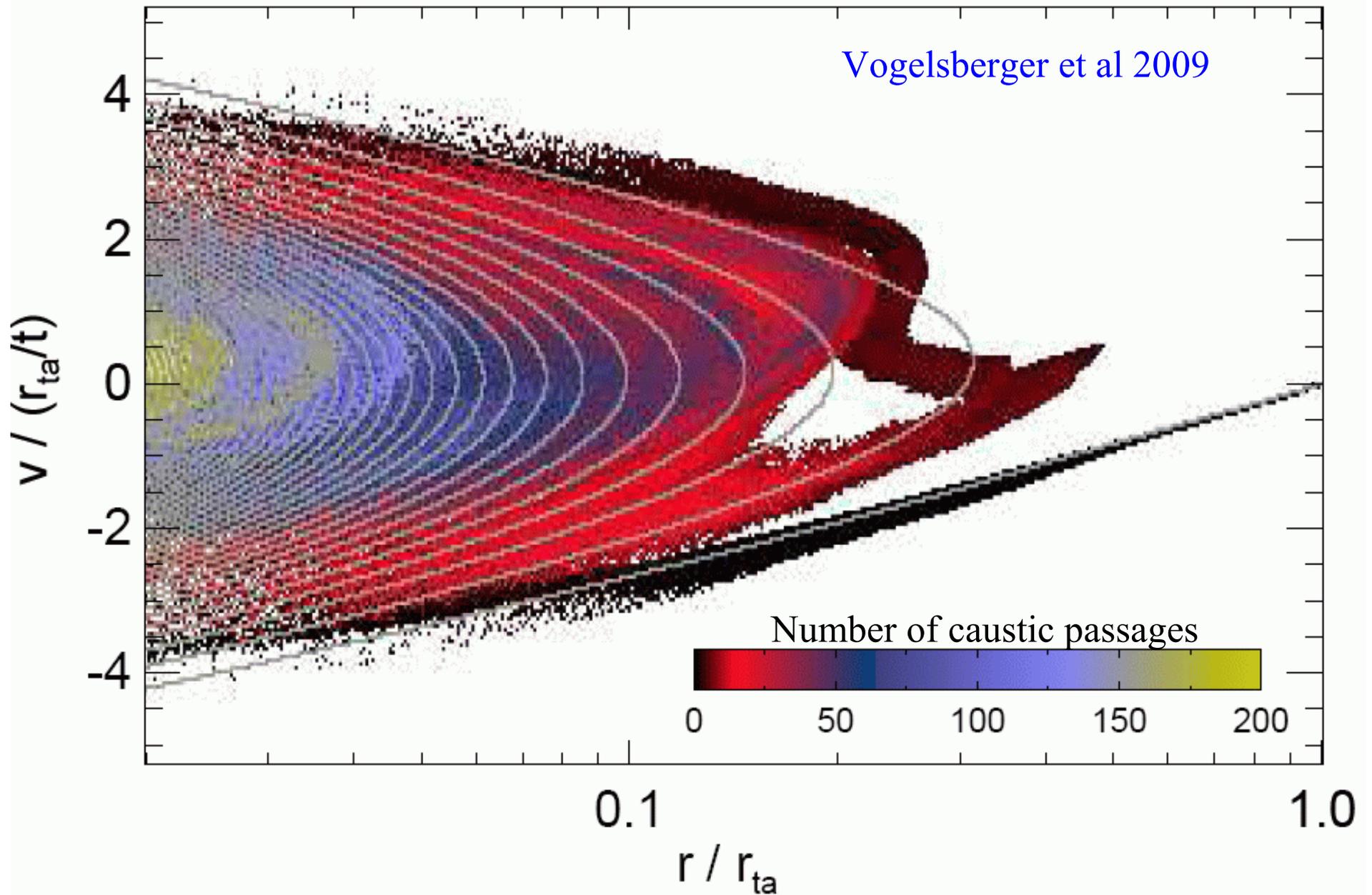
Similarity solution for spherical collapse in CDM

Bertschinger 1985

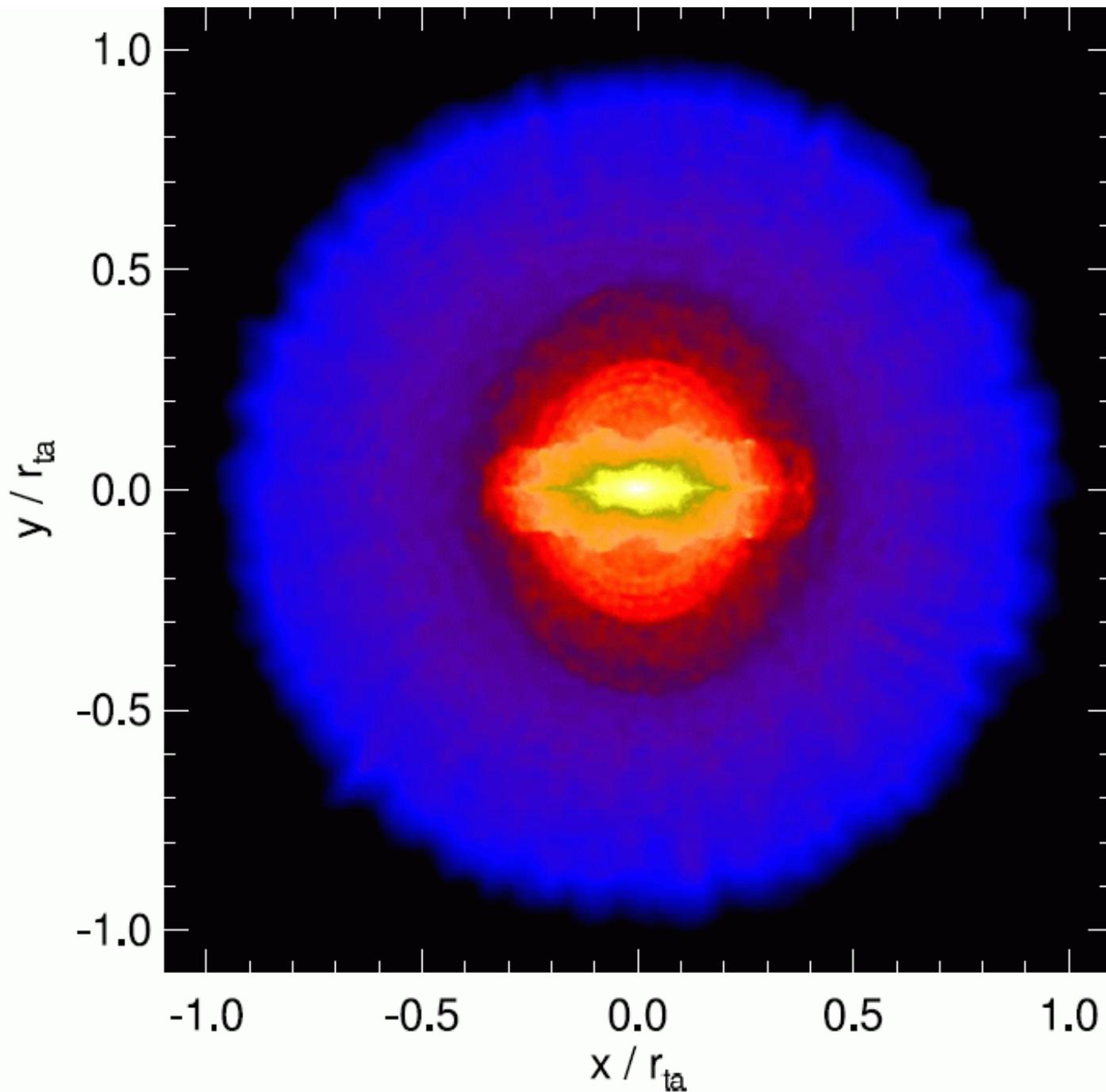


Simulation from self-similar spherical initial conditions

Geodesic deviation equation \longrightarrow phase-space structure local to each particle



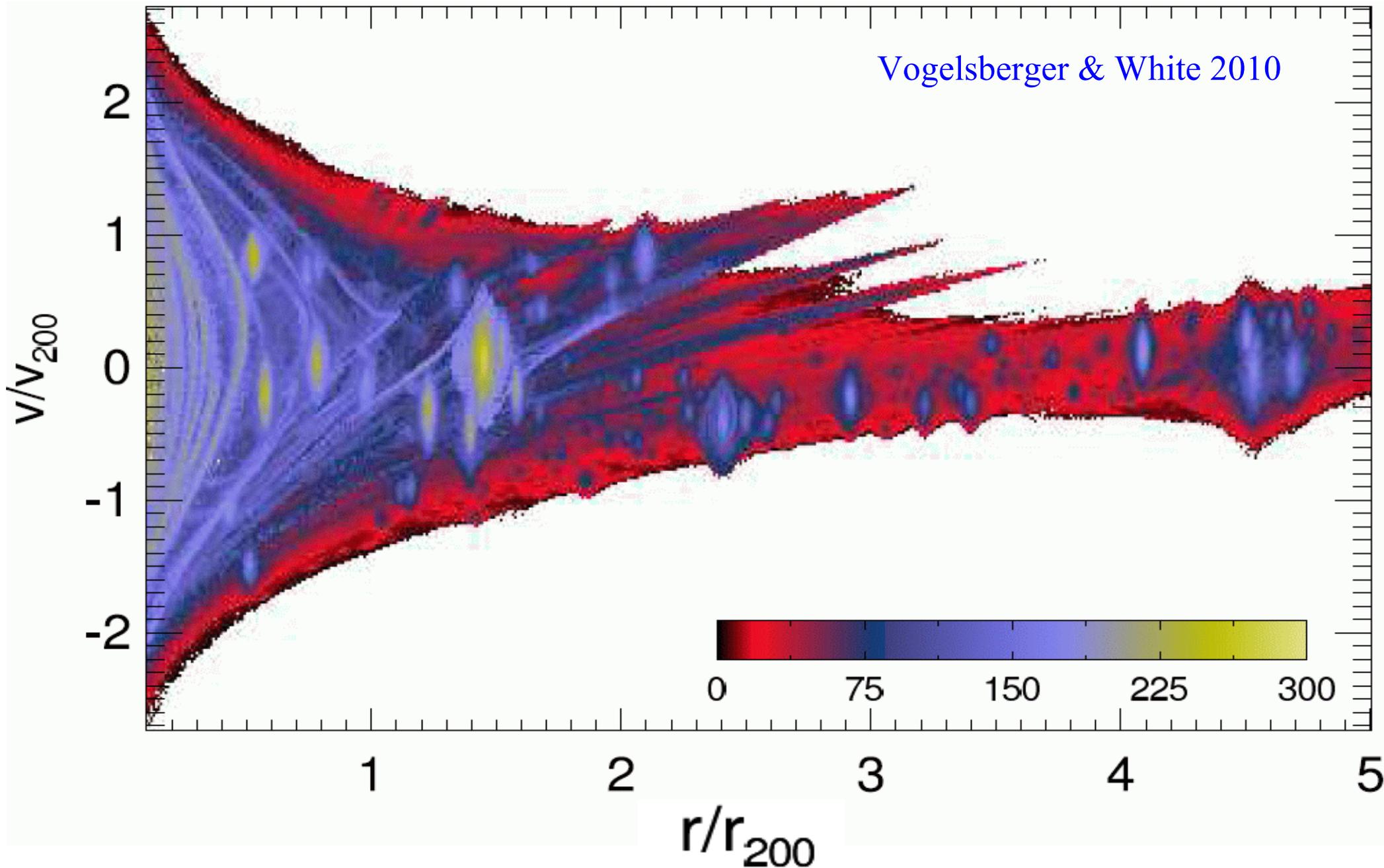
Simulation from self-similar spherical initial conditions



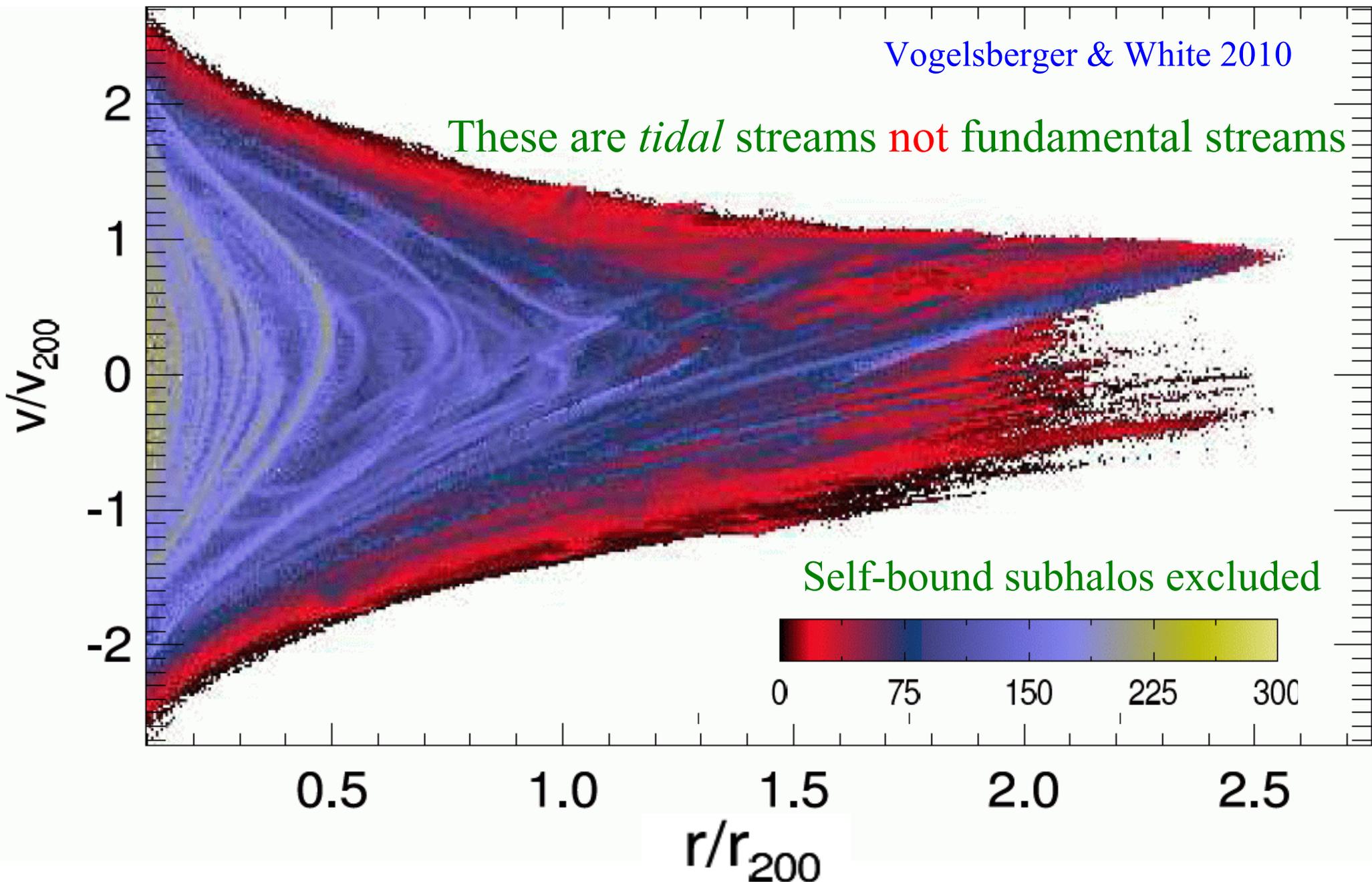
Vogelsberger et al 2009

The radial orbit instability leads to a system which is strongly prolate in the inner nonlinear regions

Caustic crossing counts in a Λ CDM Milky Way halo

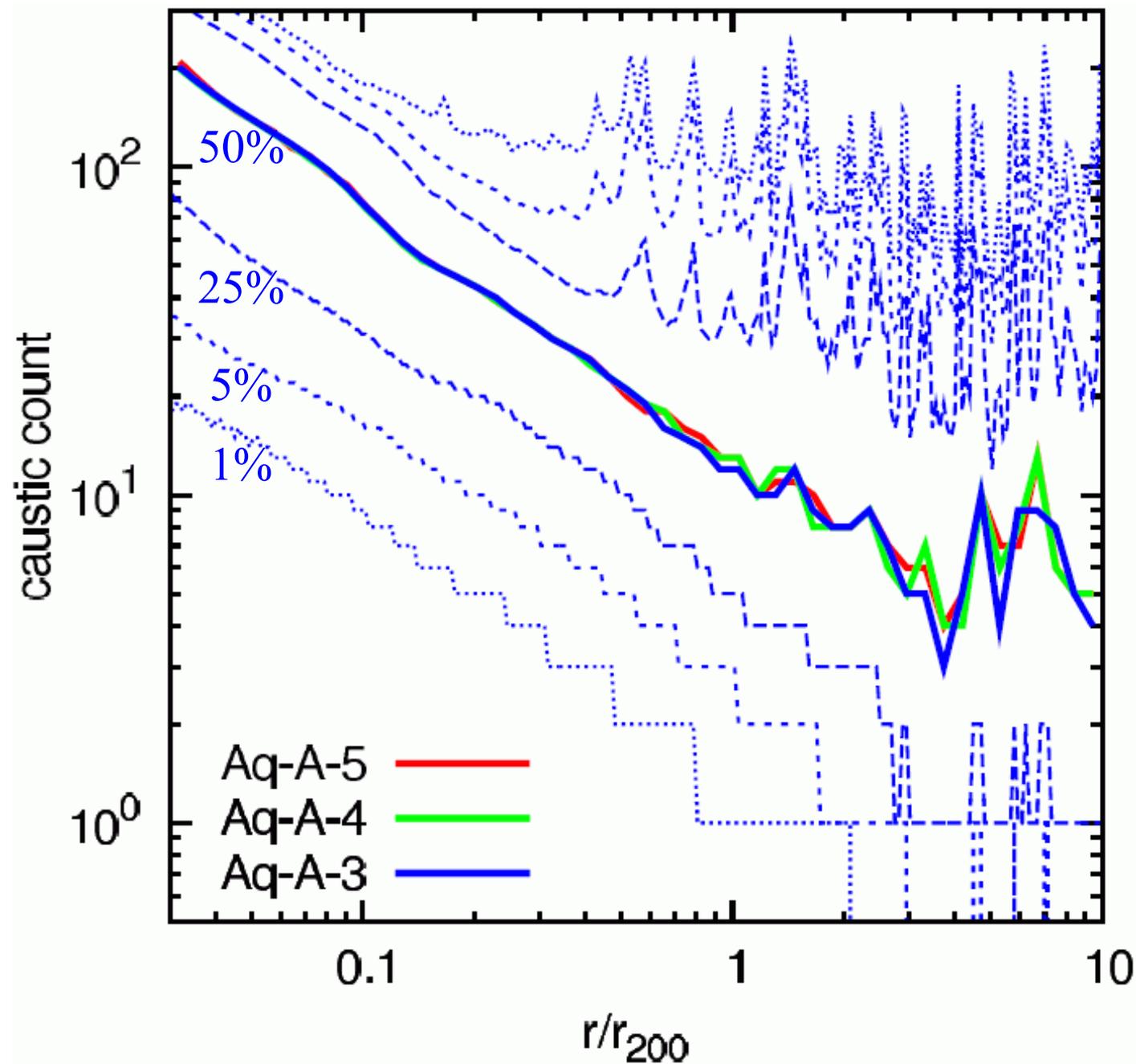


Caustic crossing counts in a Λ CDM Milky Way halo



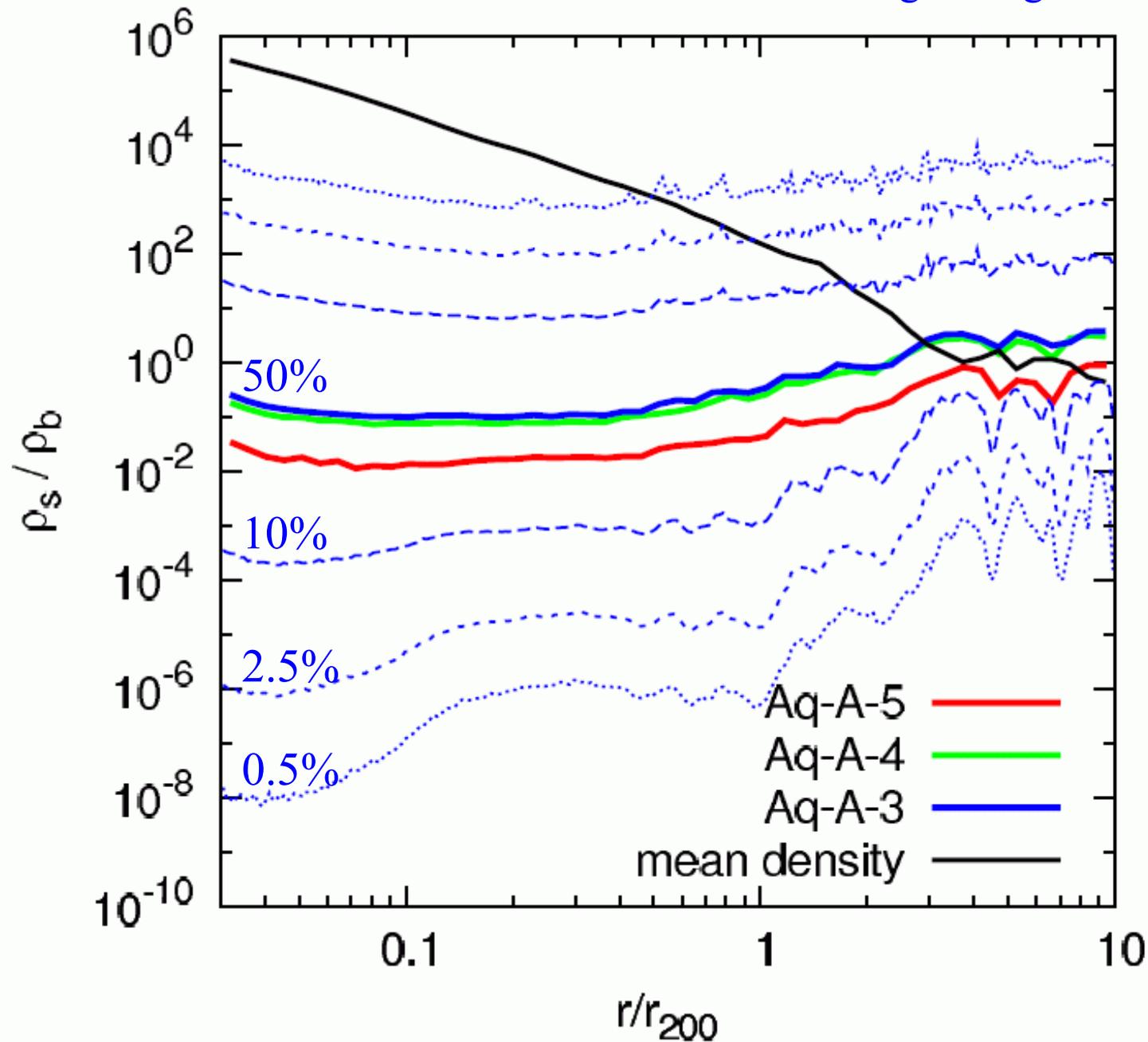
Caustic count profiles for Aquarius halos

Vogelsberger & White 2010



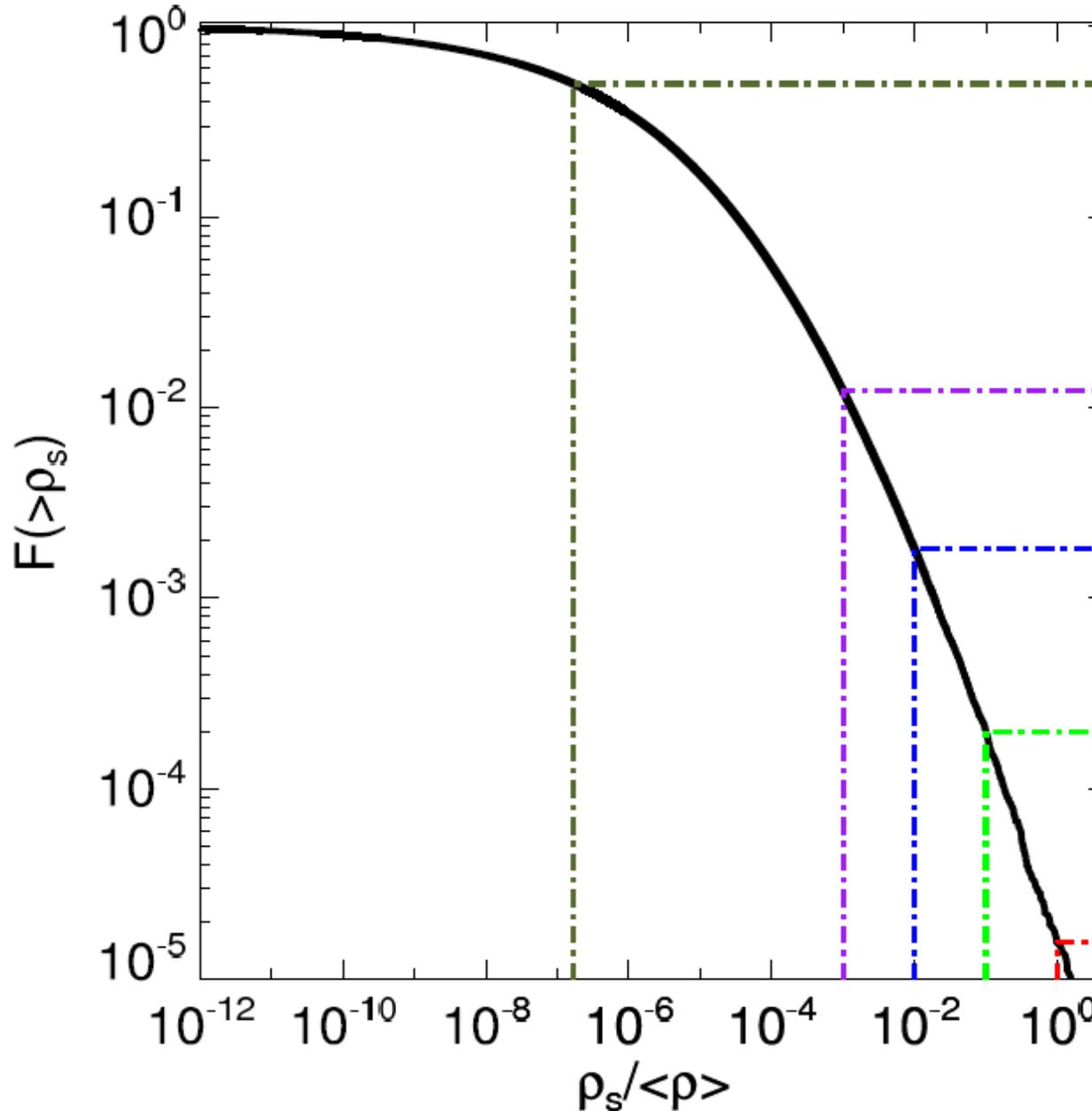
Stream density distribution in Aquarius halos

Vogelsberger & White 2010



Stream density distribution at the Sun

Vogelsberger & White 2010



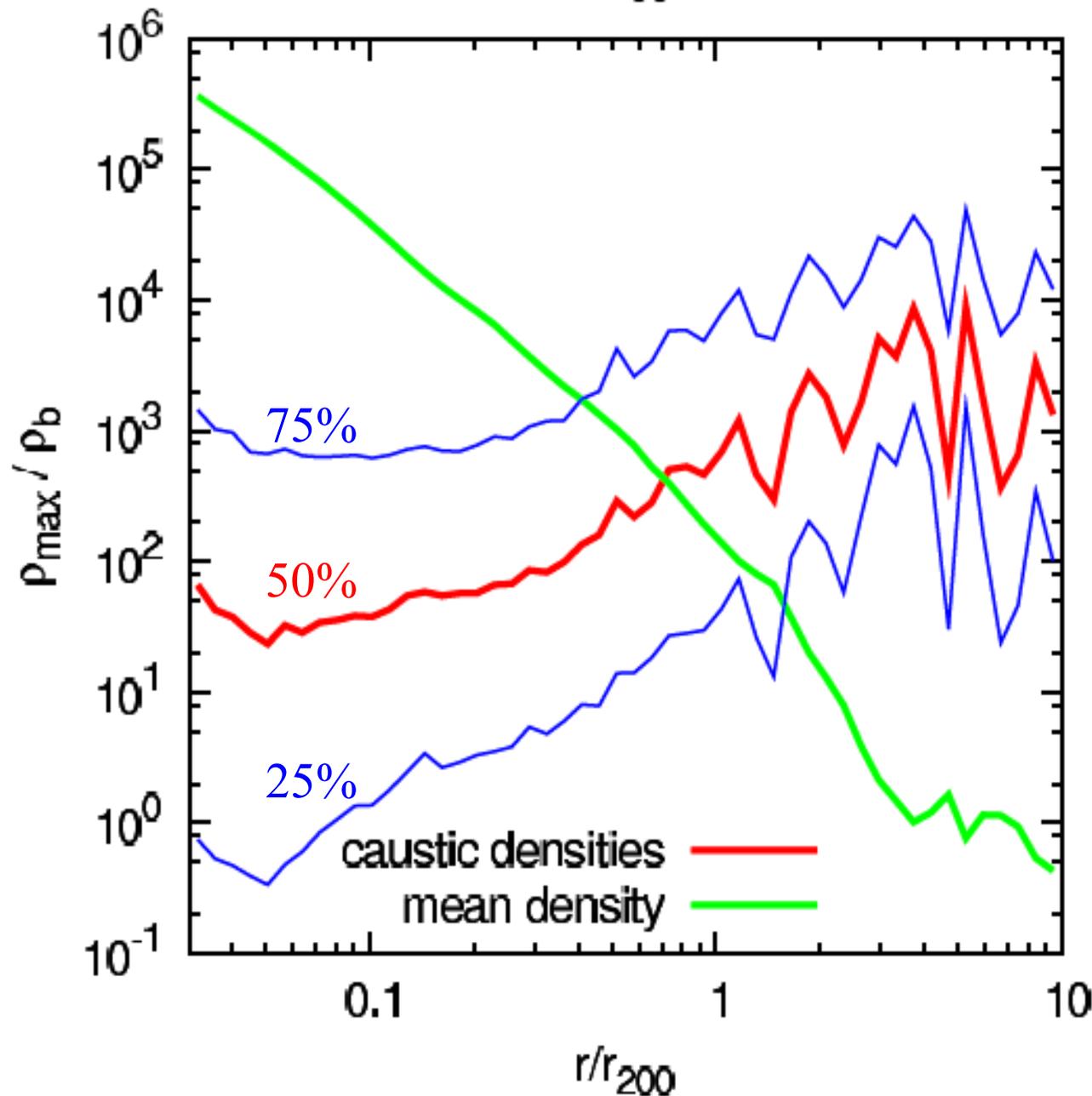
Cumulative stream density distribution for particles with $7 \text{ kpc} < r < 13 \text{ kpc}$

Probability that the Sun is in a stream with density $> X \langle\rho\rangle$ is P

X	P
1.0	0.00001
0.1	0.002
0.01	0.2
0.001	~ 1

Radial distribution of peak density at caustics

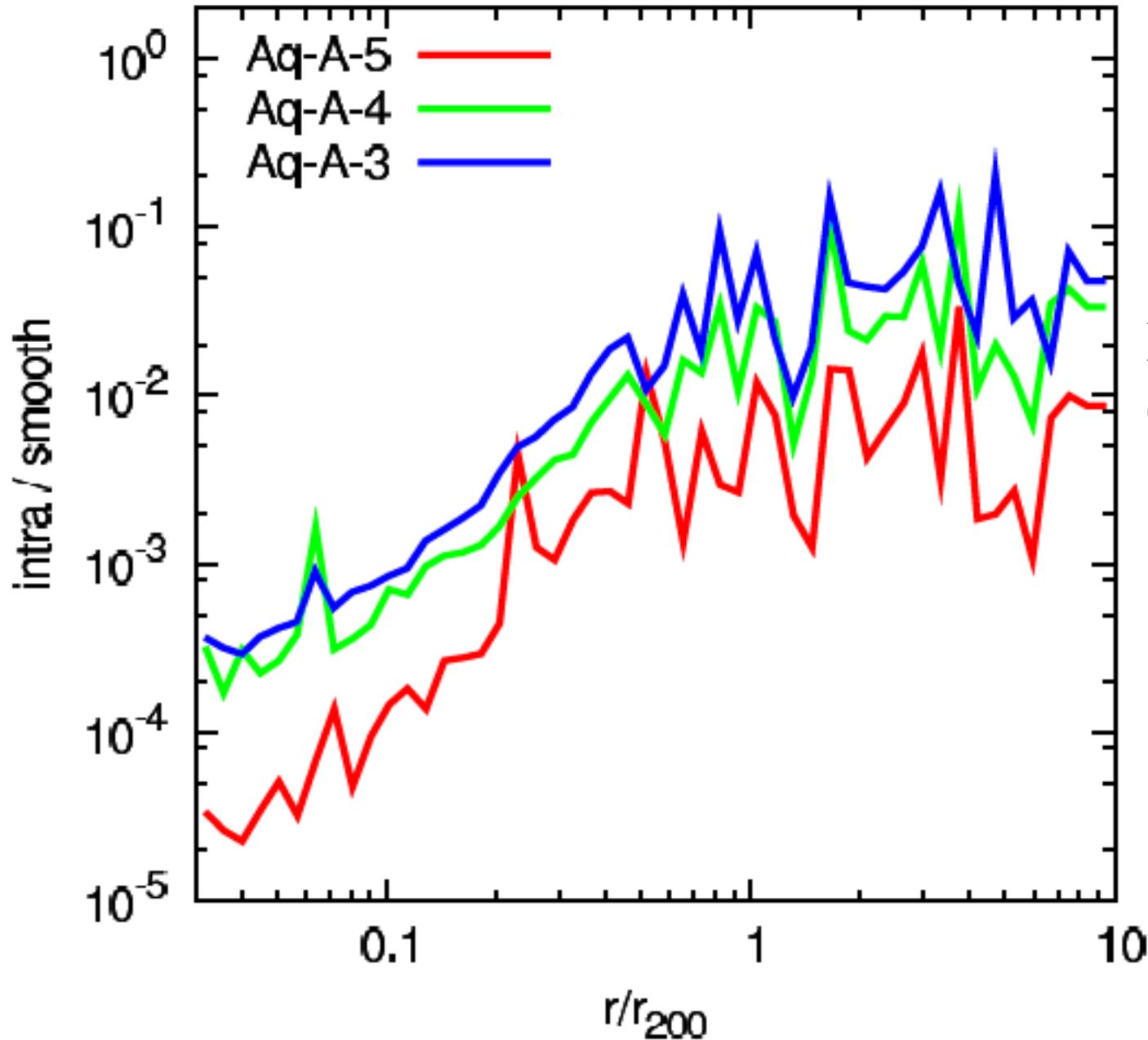
Vogelsberger & White 2010



Initial velocity dispersion assumes a standard WIMP with $m = 100 \text{ GeV}/c^2$

Fraction of annihilation luminosity from caustics

Vogelsberger & White 2010



Initial velocity dispersion assumes a standard WIMP with $m = 100 \text{ GeV}/c^2$

Conclusions: fundamental streams and caustics

- Integration of the GDE can augment the ability of Λ CDM simulations to resolve fine-grained structure by 15 to 20 orders of magnitude
- Fundamental streams and their associated caustics will have no significant effect on direct and indirect Dark Matter detection experiments
- The most massive stream at the Sun should contain roughly 0.001 of the local DM density and would have an energy spread $\Delta E/E < 10^{-10}$. It might be detectable in an axion experiment

