Extreme QCD at RHIC and LHC

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OUTLINE

* QCD at high temperature
  * Phase transition: hadrons to partons (QGP)

* QCD at high energy
  * Unitarity: small to large (CGC)

* RHIC and LHC
QCD at high T

Hadrons vs. partons: energy density

\[ \varepsilon = n \frac{\pi^2}{30} T^4 \]

Hadronic Matter: quarks and gluons confined up to \( T \sim 200 \text{ MeV} \), 3 pions with spin=0

\[ \varepsilon = 3 \frac{\pi^2}{30} T^4 \]

Quark Gluon Matter:
8 gluons; 2 quark flavors, antiquarks, 2 spins, 3 colors

\[ \varepsilon = \left\{ 2.8_g + \frac{7}{8} \cdot 2_s \cdot 2_a \cdot 2_f \cdot 3_c \right\} \frac{\pi^2}{30} T^4 \]

\( 37 \gg 3 \)
QGP vs. Hadron Gas

Transition values:

\[ T = 170 \text{ MeV} \]
\[ \varepsilon_c = 0.8 \text{ GeV}/\text{fm}^3 \]

Assumes thermal system

Lattice QCD

\[ \frac{\varepsilon}{T^4} \]

\[ \frac{\varepsilon_{\text{eq}}}{T^4} \]

\[ \text{hadrons} \Rightarrow \text{quark/gluon} \]

NEED TO CREATE \[ \varepsilon \gg \varepsilon_c \]
RHIC

Center of mass energy: 20, 60, 130, 200 GeV

Hot nuclear matter:
gold-gold, copper-copper

Cold nuclear matter:
deuteron-gold

Baseline:
proton-proton

Central:
maximum overlap

Peripheral:
“Almond” of overlap region

RHIC-II
Colliding heavy ions at high energies

Bjorken: high $p_t$ partons scatter from the medium and “lose energy” (radiate gluons)
QGP at RHIC

* Note deuteron-gold control experiment with no suppression
Probing the medium
QGP at RHIC

disappearance of back to back jets
From CGC to QGP: Space-Time History of a Heavy Ion Collision

Initial conditions

Freeze-out
(System falls apart)

Hadronic Gas

Thermalization Region
(Quark-Gluon Plasma)

nucleus #1

nucleus #2
**Degrees of Freedom in a Nucleus?**

It depends on the scales probed!

A point particle \( \lambda \gg 10 \, \text{fm} \)

A collection of protons and neutrons \( \lambda \sim 1 \, \text{fm} \)

A dense system of quarks and gluons \( \lambda \ll 1 \, \text{fm} \)
Deeply Inelastic Scattering (DIS)

The simplest way to study QCD in a hadron/nuucleus

$e p (A) \longrightarrow e X$

Kinematic Invariants:

Center of mass energy squared

$S \equiv (p + q)^2$

Momentum resolution squared

$Q^2 \equiv -q^2$

$X_{b,j} \equiv \frac{Q^2}{2 p \cdot q}$

QCD: Structure Functions $F_1, F_2$
The hadron at high energy

★Bjorken: \( Q^2, \nu \to \infty \) but \( \frac{Q^2}{\nu} \) fixed

structure functions \( F_1, F_2 \) depend only on \( x_{bj} \)

★Feynman:

Parton constituents of proton are “quasi-free” on interaction time scale \( 1/Q \ll 1/\Lambda \) (interaction time scale between partons)

\[ X_{bj} = \text{fraction of hadron momentum carried by a parton} = X_F \]
The hadron at high energy

Parton model

QCD - bound quarks
pQCD--RG evolution (radiation)

\[ \int_0^1 \frac{dx}{x} \left[ xq(x) - x\bar{q}(x) \right] = 3 \quad \text{# of valence quarks} \]

\[ \int_0^1 \frac{dx}{x} \left[ xq(x) + x\bar{q}(x) \right] \rightarrow \infty \quad \text{# of quarks ....} \]
\( pQCD--RG \) evolution (radiation)

\[
F_2(x, Q^2) = \sum e_q^2 \left[ x q(x, Q^2) + x \bar{q}(x, Q^2) \right]
\]
# of gluons grows rapidly at small $x$…

**pQCD--RG evolution (radiation)**
The number of gluons increases but the phase space density decreases: hadron becomes more dilute.

Radiated gluons have smaller and smaller sizes (~ $1/Q^2$) as $Q^2$ grows.
QCD in the Regge-Gribov limit

$Q^2$ fixed, $S \to \infty$

$X_{bj} \to 0$
BFKL evolution

- The infrared sensitivity of bremsstrahlung favors the emission of ‘soft’ (= small–$x$) gluons

\[ k_z = x P_z \quad \text{with} \quad x << 1 \]

\[ P(1) \propto \alpha_s \int_x^1 \frac{dx_1}{x_1} = \alpha_s \ln \frac{1}{x} \]

- The ‘price’ of an additional gluon:

\[ d\mathcal{P} \propto \alpha_s \frac{dk_z}{k_z} = \alpha_s \frac{dx}{x} \]
The ‘last’ gluon at small $x$ can be emitted off any of the ‘fast’ gluons with $x' > x$ radiated in the previous steps:

\[ \frac{\partial n}{\partial Y} \simeq \alpha_s n \quad \Longrightarrow \quad n(Y) \propto e^{\omega \alpha_s Y} \]

- Dipole scattering amplitude:
  \[ T \sim \alpha_s n \]

- Unitarity bound:
  \[ SS^\dagger = 1 \quad \Longrightarrow \quad T \leq 1 \quad — \quad \text{violated by BFKL!} \]
The hadron at high energy

QCD Bremsstrahlung

Non-linear evolution-Gluon recombination:

this is essential if proton is a dense object
Radiated gluons have the same size \( (1/Q^2) \) - the number of partons increase due to the increased longitudinal phase space.

How to achieve high gluon density

Or/and large nuclei

Increase the energy

![Diagram showing energy levels and gluon density growth]

- Low Energy 
  - Large x
- High Energy 
  - Small x

\[ R \sim A^{1/3} \]
Parton saturation

Competition between “attractive” bremsstrahlung and “repulsive” recombination effects

maximal phase space density

\[
\frac{1}{2(N_c^2 - 1)} \frac{x G(x, Q^2)}{\pi R^2 Q^2} = \frac{1}{\alpha_s(Q^2)}
\]

saturated for

\[
Q = Q_s(x) \gg \Lambda_{QCD} \approx 0.2 \text{ GeV}
\]
Consider a large nucleus in the IMF frame

One large component of the current-others suppressed by \( \frac{1}{P^+} \)

Wee partons see a large density of valence color charges at small transverse resolutions
**Born-Oppenheimer: separation of large x and small x modes**

Large X partons are static over small X parton life times

\[ \tau_{\text{wee}} \sim \frac{1}{k^-} = \frac{2k^+}{k^2} \equiv \frac{2xP^+}{k_\perp^2} \quad \tau_{\text{valence}} = \frac{2P^+}{k_\perp^2} \gg \tau_{\text{wee}} \text{ for } x \ll 1 \]
The effective action

\[ S = -\frac{1}{4} \int d^4 x \ G^2 \]

\[ + \frac{i}{N_c} \int d^2 x_t \ dx^- \ \delta(x^-) \ \text{tr} \ \rho(x_t) \ U_{-\infty, \infty} [A^{-}](x^-, x_t) \]

\[ + i \int d^2 x_t F[\rho^a(x_t)] \]

where

\[ U[A^-] \equiv \hat{P} e^{-ig \int dx^+ A_a^- T_a} \]

MV:

\[ F[\rho] \rightarrow \frac{\rho^2}{\mu^2} \quad \text{tr} \ \rho \ U \rightarrow \rho A^- \]
Hadron/nucleus at high energy is a Color Glass Condensate

- Gluons are colored
- Random sources evolving on time scales much larger than natural time scales - very similar to spin glasses
- Bosons with a large occupation number $n \sim \frac{1}{\alpha_s}$
- Typical momentum of gluons is $Q_s(x)$
QCD at High Energy: from classical to quantum \( (\alpha_s \log 1/x) \)

Integrate out small fluctuations => Increase color charge of

\[
\frac{\partial W_x[\rho]}{\partial \ln(1/x)} = \frac{1}{2} \int_{x_\perp, y_\perp} \frac{\delta}{\delta \rho_x^a(x_\perp)} \chi^{ab}(x_\perp, y_\perp)[\rho] \frac{\delta}{\delta \rho_x^b(y_\perp)} W_x[\rho] \]

B-JIMWLK
B-JIMWLK equations describe evolution of all \( N \)-point correlation functions with energy

the 2-point function: \( \text{Tr} \left[ 1 - U^+(x_t) U(y_t) \right] \)
(probability for scattering of a quark-anti-quark dipole on a target)

B-JIMWLK in two limits:
I) Strong field: exact scaling - \( f \left( \frac{Q^2}{Q_s^2} \right) \) for \( Q < Q_s \)
II) Weak field: BFKL
**BK: mean field + large $N_c$**

A closed form equation

\[
\partial_Y \langle T_{xy} \rangle = \frac{\tilde{\alpha}}{2\pi} \int d^2 z \frac{(x - y)^2}{(x - z)^2(z - y)^2} \left[ \langle T_{xz} \rangle + \langle T_{zy} \rangle - \langle T_{xy} \rangle - \langle T_{xz} \rangle \langle T_{zy} \rangle \right]
\]

The simplest equation to include unitarity: $T < 1$

Exhibits geometric scaling

\[
T(x, r_t) \rightarrow T[r_t Q_s(x)]
\]

for

\[
Q_s < Q < \frac{Q_s^2}{\Lambda_{QCD}}
\]
Geomtric scaling at HERA

$x < 0.01, \quad 0.045 < Q^2 < 450 \text{ GeV}^2$

Scaling variable $Q^2 R_0^2(x)$
A New Paradigm of QCD

Saturation region: dense system of gluons

Extended scaling region: dilute system - anomalous dimension

Double Log: BFKL meets DGLAP

DGLAP: collinearly factorized pQCD
Relation to statistical physics

\( \partial_y N = \bar{\alpha} \chi [ - \partial_L ] N - \bar{\alpha} N^2 \)

can be written as with

\( N \rightarrow u, \ y \rightarrow t, L \rightarrow x \)

\( \partial_t u = \partial_x^2 u + u - u^2 \)

traveling wave solution

F-KPP equation in statistical mechanics

with applications in biology, ....
Beyond B-JIMWLK (BK)

some undesirable features

merging vs. splitting
2 --> 1 vs. 1 --> 2

reaction-diffusion in statistical mechanics: sF-KPP

Pomeron loops

BFKL
saturation
fluctuation
The new phase diagram

The “phase–diagram” revisited

\[ Y = \ln \frac{1}{x} \]

\[ \text{Saturation} \]

\[ \ln \frac{Q^2}{\Lambda_{QCD}^2} = \lambda Y \]

\[ (\lambda + D)Y \]

\[ \text{Diffusive scaling} \]

\[ \text{Geometric scaling} \]

\[ \text{Low density} \]

Introduction
Outline
Gluon evolution
Mean–field approximation
Fluctuations and pomeron loops

!Fluctuations
!Pomeron loops.

We don't really know! (large theoretical uncertainties)

Compare theoretical expectations with the data!
Applications to RHIC and LHC
Colliding sheets of color glass

solve the classical eqs. of motion in the forward light cone: subject to initial conditions given by one nucleus solution

Initial energy and multiplicity of produced gluons depends on \( Q_s \)

\[
\frac{1}{\pi R^2} \frac{dE}{d\eta} = \frac{0.25}{g^2} Q_s^3
\]

\[
\frac{1}{\pi R^2} \frac{dN}{d\eta} = \frac{0.3}{g^2} Q_s^2
\]

Fermion production (Gelis et al.)
Colliding Sheets of Colored Glass

adding final state effects: hydro, energy loss
Colliding Sheets of Colored Glass

What happens to produced gluons?

- Is there thermalization of QCD matter?
- Can it be described by weak coupling?

Bottom up scenario (Baier, Mueller, Schiff, Son)

Production of “hard” gluons: $k \sim Q_s$
Radiation of “soft” gluons: $k \ll Q_s$

Soft gluons thermalize
Hard gluons thermalize

Thermalization time:

$$\tau \sim \frac{1}{\alpha_s^{13/5} Q_s} \quad T_{\text{max}} \sim \alpha_s^{2/5} Q_s$$

Instabilities?  Fast thermalization?
**Signatures of CGC at RHIC: pA**

- **Multiplicities (dominated by** $p_t < Q_s$): energy, rapidity, centrality dependence

- **Single particle production**: hadrons, EM rapidity, $p_t$, centrality dependence
  
  i) Fixed $p_t$: vary rapidity (evolution in $x$)
  
  ii) Fixed rapidity: vary $p_t$ (transition from dense to dilute)

- **Two particle production**: back to back correlations
Classical (multiple elastic scattering):

\[ p_t \gg Q_s : \text{enhancement (Cronin effect)} \]

\[ R_{pA} = 1 + \left( \frac{Q_s^2}{p_t^2} \right) \log \frac{p_t^2}{\Lambda^2} + \ldots \]

\[ R_{pA} (p_t \sim Q_s) \sim \log A \]

position and height of enhancement are increasing with centrality

Evolution in x:

can show analytically the peak disappears as energy/rapidity grows

and levels off at \( R_{pA} \sim A^{-1/6} < 1 \)

These expectations are confirmed at RHIC
What we see is a transition from DGLAP to BFKL to CGC kinematics. Centrality, flavor, species dependence.
The future is promising!

\[ x_{1,2} = (M/\sqrt{s}) e^{\pm y} \]

- **M (GeV)**
- **x**

- **1 TeV**
- **LHC**
- **RHIC**
- **PS (PS)**

- **y**
- **10 GeV**
- **y = 6**
- **y = 4**
- **y = 2**
- **y = 0**
Exciting time in high energy QCD again

- Frenetic pace of theoretical developments
- Hints for CGC from HERA
- Strong evidence for CGC from RHIC

Significant ramifications for strong interaction physics at LHC and eRHIC