Bayesian method of SUSY parameter reconstruction - a case study

Leszek Roszkowski

U. of Sheffield, England and SINS, Warsaw, Poland

with Roberto Ruiz de Austri and Roberto Trotta, arXiv:0907.0594

public tool: SuperBayes package, available from www.superbayes.org

Outline

Outline

- SUSY, Constrained MSSM (CMSSM)
- case study: ATLAS SU3 benchmark point

Outline

- SUSY, Constrained MSSM (CMSSM)
- case study: ATLAS SU3 benchmark point
- Bayesian parameter reconstruction for SU3

Outline

- SUSY, Constrained MSSM (CMSSM)
- case study: ATLAS SU3 benchmark point
- Bayesian parameter reconstruction for SU3
- impact of additional info on $\Omega_\chi h^2$
- prior dependence, profile likelihood
- summary

A conjecture

A conjecture

SUSY cannot be experimentally ruled out

A conjecture

SUSY cannot be experimentally ruled out

it can only be discovered...

A conjecture

SUSY cannot be experimentally ruled out

it can only be discovered...

...or abandoned

Parameter reconstruction

Parameter reconstruction

...once positive measurements are made...

Parameter reconstruction

...once positive measurements are made...

task: reconstruct underlying SUSY parameters

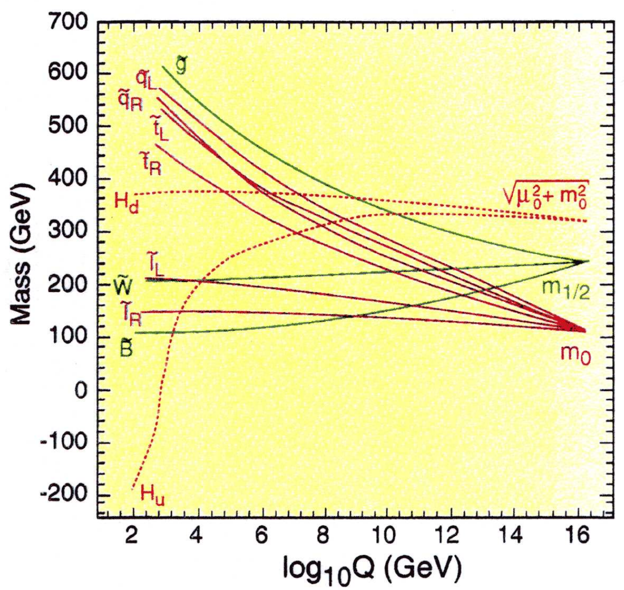
model dependent program

Constrained MSSM (CMSSM)

Kane, Kolda, LR, Wells (1993)

..."benchmark framework" for the LHC

(...e.g., mSUGRA)

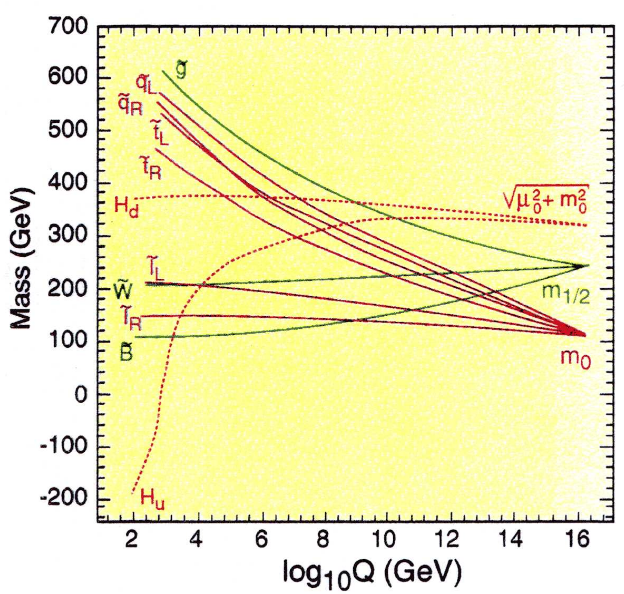


Constrained MSSM (CMSSM)

Kane, Kolda, LR, Wells (1993)

...“benchmark framework” for the LHC

(...e.g., mSUGRA)



At $M_{\mathrm{GUT}} \simeq 2 \times 10^{16}\,\mathrm{GeV}$:

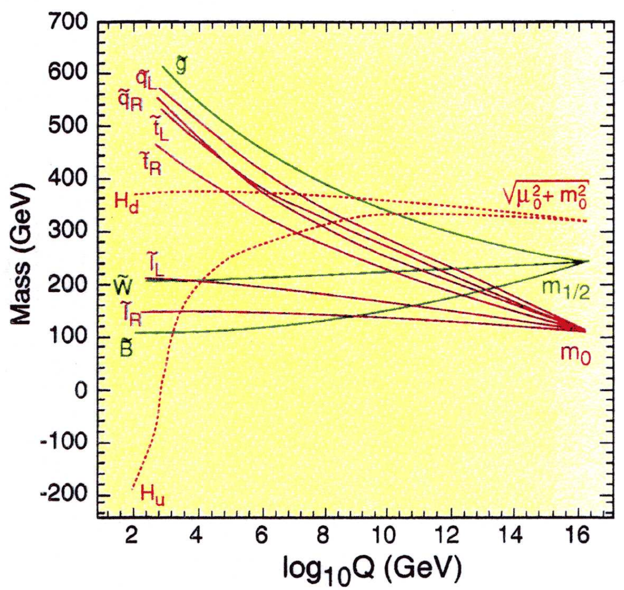
- gauginos $M_1 = M_2 = m_{\widetilde{g}} = m_{1/2}$
- scalars $m^2_{\widetilde{q}_i} = m^2_{\widetilde{l}_i} = m^2_{H_b} = m^2_{H_t} = m^2_0$
- 3–linear soft terms $A_b = A_t = A_0$

Constrained MSSM (CMSSM)

Kane, Kolda, LR, Wells (1993)

..."benchmark framework" for the LHC

(...e.g., mSUGRA)



At $M_{\rm GUT} \simeq 2 \times 10^{16}\,{\rm GeV}$:

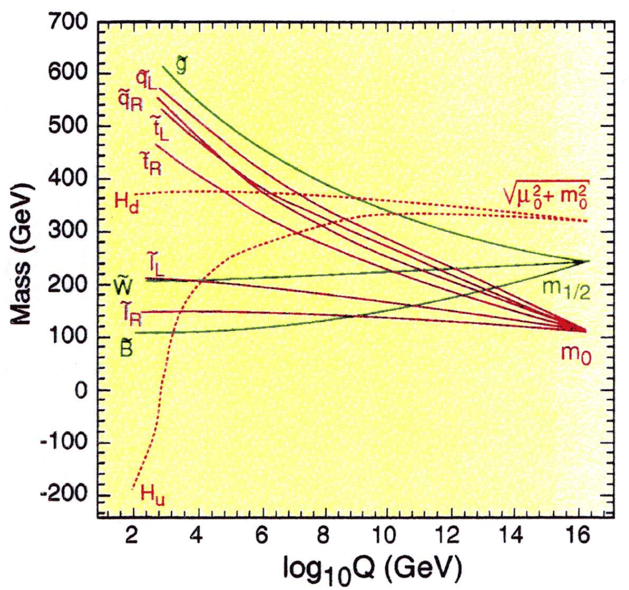
- gauginos $M_1 = M_2 = m_{\widetilde{g}} = m_{1/2}$
- scalars
  $m^2_{\widetilde{q}_i} = m^2_{\widetilde{l}_i} = m^2_{H_b} = m^2_{H_t} = m^2_0$
- 3–linear soft terms $A_b = A_t = A_0$
- radiative EWSB
  $\mu^2 = \frac{m^2_{H_b} - m^2_{H_t}\tan^2\beta}{\tan^2\beta - 1} - \frac{m_Z^2}{2}$

Constrained MSSM (CMSSM)

Kane, Kolda, LR, Wells (1993)

..."benchmark framework" for the LHC

(...e.g., mSUGRA)



At $M_{\rm GUT} \simeq 2 \times 10^{16}\,{\rm GeV}$:

- gauginos $M_1 = M_2 = m_{\widetilde g} = m_{1/2}$
- scalars $m^2_{\widetilde q_i} = m^2_{\widetilde l_i} = m^2_{H_b} = m^2_{H_t} = m^2_0$
- 3–linear soft terms $A_b = A_t = A_0$
- radiative EWSB $\mu^2 = \frac{m^2_{H_b} - m^2_{H_t}\tan^2\beta}{\tan^2\beta - 1} - \frac{m_Z^2}{2}$
- five independent parameters:

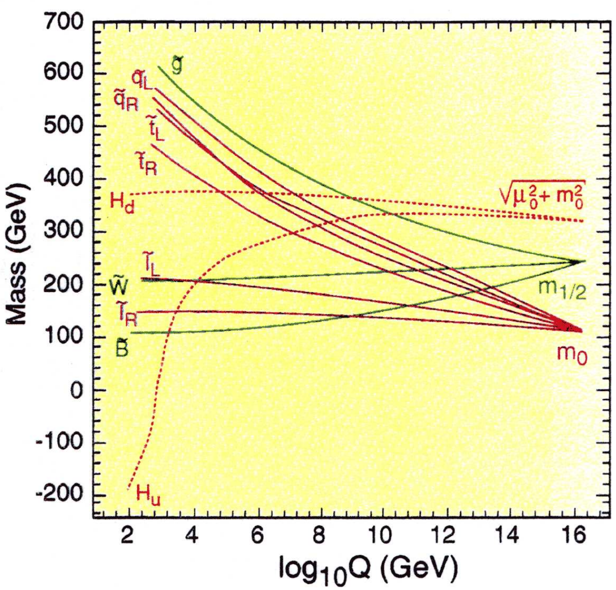
$$m_{1/2},\ m_0,\ A_0,\ \tan\beta,\ {\rm sgn}(\mu)$$

Constrained MSSM (CMSSM)

Kane, Kolda, LR, Wells (1993)

..."benchmark framework" for the LHC

(...e.g., mSUGRA)



At $M_{\rm GUT} \simeq 2 \times 10^{16}\,{\rm GeV}$:

- gauginos $M_1 = M_2 = m_{\tilde{g}} = m_{1/2}$
- scalars
  $m^2_{\tilde{q}_i} = m^2_{\tilde{l}_i} = m^2_{H_b} = m^2_{H_t} = m_0^2$
- 3–linear soft terms $A_b = A_t = A_0$
- radiative EWSB
  $\mu^2 = \frac{m^2_{H_b} - m^2_{H_t} \tan^2\beta}{\tan^2\beta - 1} - \frac{m_Z^2}{2}$
- five independent parameters:
  $m_{1/2},\ m_0,\ A_0,\ \tan\beta,\ {\rm sgn}(\mu)$
- well developed machinery to compute masses and couplings

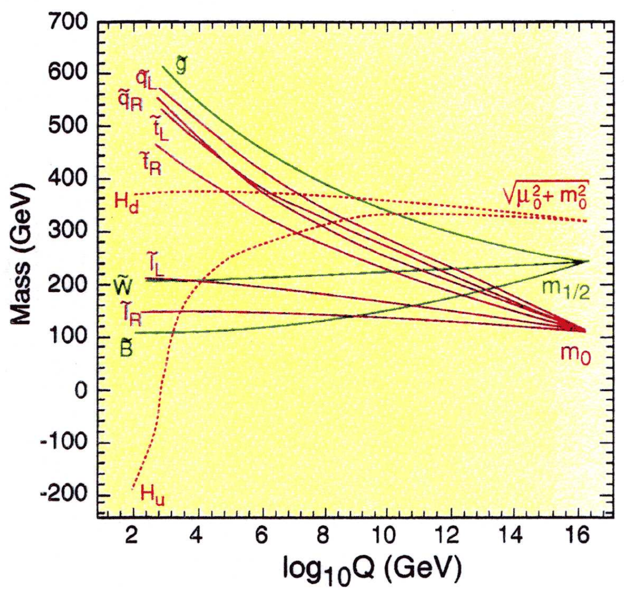
Constrained MSSM (CMSSM)

Kane, Kolda, LR, Wells (1993)

..."benchmark framework" for the LHC

(...e.g., mSUGRA)

At $M_{\rm GUT} \simeq 2 \times 10^{16}\,{\rm GeV}$:



- gauginos $M_1 = M_2 = m_{\tilde{g}} = m_{1/2}$
- scalars
  $$m^2_{\tilde{q}_i} = m^2_{\tilde{l}_i} = m^2_{H_b} = m^2_{H_t} = m_0^2$$
- 3–linear soft terms $A_b = A_t = A_0$
- radiative EWSB
  $$\mu^2 = \frac{m^2_{H_b} - m^2_{H_t}\tan^2\beta}{\tan^2\beta - 1} - \frac{m_Z^2}{2}$$
- five independent parameters:
  $$\boxed{m_{1/2},\ m_0,\ A_0,\ \tan\beta,\ {\rm sgn}(\mu)}$$
- well developed machinery to compute masses and couplings
- neutralino χ mostly bino

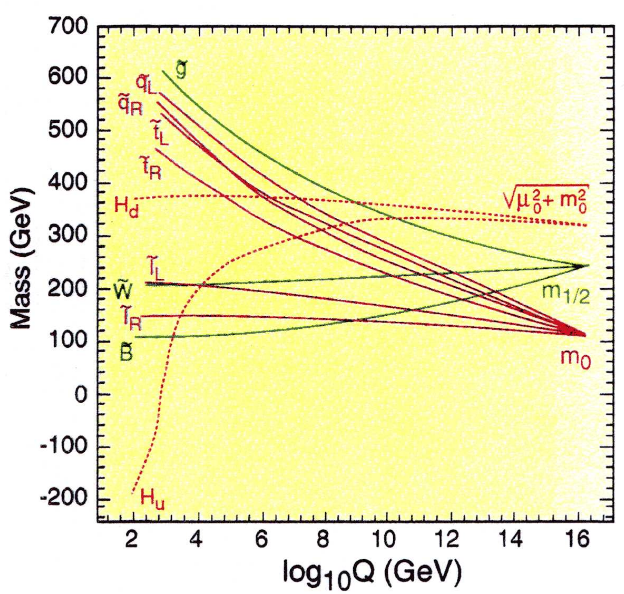
Constrained MSSM (CMSSM)

Kane, Kolda, LR, Wells (1993)

..."benchmark framework" for the LHC

(...e.g., mSUGRA)

At $M_{\rm GUT} \simeq 2 \times 10^{16}\,{\rm GeV}$:



- gauginos $M_1 = M_2 = m_{\tilde{g}} = m_{1/2}$
- scalars $m^2_{\tilde{q}_i} = m^2_{\tilde{l}_i} = m^2_{H_b} = m^2_{H_t} = m_0^2$
- 3–linear soft terms $A_b = A_t = A_0$
- radiative EWSB $\mu^2 = \frac{m^2_{H_b} - m^2_{H_t}\tan^2\beta}{\tan^2\beta - 1} - \frac{m_Z^2}{2}$
- five independent parameters:

$$m_{1/2},\ m_0,\ A_0,\ \tan\beta,\ {\rm sgn}(\mu)$$

- well developed machinery to compute masses and couplings
- neutralino χ mostly bino

some useful mass relations:

- bino: $m_\chi \simeq 0.4 m_{1/2}$
- gluino $\tilde{g}$: $m_{\tilde{g}} \simeq 2.7 m_{1/2}$
- supersymmetric tau (stau) $\tilde{\tau}_1$: $m_{\tilde{\tau}_1} \simeq \sqrt{0.15 m^2_{1/2} + m_0^2}$

Case study: ATLAS SU3 Point

ATLAS SU3 benchmark point, arXiv:0901.0512

Case study: ATLAS SU3 Point

ATLAS SU3 benchmark point, arXiv:0901.0512

Parameter	SU3 benchmark value
m_0	100 GeV
$m_{1/2}$	300 GeV
$\tan\beta$	6.0
A_0	-300 GeV
$\Omega_\chi h^2$	0.23319 $\Leftarrow$
SUSY mass spectrum	
$\chi = \chi_1^0$	117.9 GeV
χ_2^0	223.4 GeV
$\widetilde{m}_{\tilde{l}}$	152.2 GeV
$m_{\tilde{q}}$	652.4 GeV

- $\widetilde{m}_{\tilde{l}}$ - lightest slepton mass
- $m_{\tilde{q}}$ - average light squark mass

Case study: ATLAS SU3 Point

ATLAS SU3 benchmark point, arXiv:0901.0512

Parameter	SU3 benchmark value
m_0	100 GeV
$m_{1/2}$	300 GeV
$\tan\beta$	6.0
A_0	-300 GeV
$\Omega_\chi h^2$	0.23319 $\Leftarrow$
SUSY mass spectrum	
$\chi = \chi_1^0$	117.9 GeV
χ_2^0	223.4 GeV
$\widetilde{m}_{\widetilde{l}}$	152.2 GeV
$m_{\widetilde{q}}$	652.4 GeV

- study endpoint measurements
  - dileptons + lepton+jets analysis of the decay chain $\widetilde{q}_L \to \chi_2^0 (\to \widetilde{l}^\pm l^\mp) q \to \chi_1^0 l^+ l^- q$ and
  - the high-p_T and large missing energy analysis of the decay chain $\widetilde{q}_R \to \chi_1^0 q$
- χ^2 minimization
- int. lum. 1 fb^{-1}

- $\widetilde{m}_{\widetilde{l}}$ - lightest slepton mass
- $m_{\widetilde{q}}$ - average light squark mass

ATLAS SU3 measurements

Observable	SU3 m_{meas} [GeV]	SU3 m_{MC} [GeV]
$m_{\tilde{\chi}_1^0}$	$88 \pm 60 \mp 2$	118
$m_{\tilde{\chi}_2^0}$	$189 \pm 60 \mp 2$	219
$m_{\tilde{q}}$	$614 \pm 91 \pm 11$	634
$m_{\tilde{\ell}}$	$122 \pm 61 \mp 2$	155
Observable	**SU3 Δm_{meas} [GeV]**	**SU3 Δm_{MC} [GeV]**
$m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$	$100.6 \pm 1.9 \mp 0.0$	100.7
$m_{\tilde{q}} - m_{\tilde{\chi}_1^0}$	$526 \pm 34 \pm 13$	516.0
$m_{\tilde{\ell}} - m_{\tilde{\chi}_1^0}$	$34.2 \pm 3.8 \mp 0.1$	37.6

Aad, *et al.*, arXiv:0901.0512

- 1st errors: parabolic
- 2nd errors: jet energy scale

The covariance matrix (ATLAS):

	$m_{\chi_1^0}$	$m_{\chi_2^0} - m_{\chi_1^0}$	$\widetilde{m}_{\tilde{l}} - m_{\chi_1^0}$	$m_{\tilde{q}} - m_{\chi_1^0}$
$m_{\chi_1^0}$	3.72×10^3	53.40	1.92×10^3	10.75×10^2
$m_{\chi_2^0} - m_{\chi_1^0}$		3.6	29.0	-1.3
$\widetilde{m}_{\tilde{l}} - m_{\chi_1^0}$			1.12×10^3	4.65
$m_{\tilde{q}} - m_{\chi_1^0}$				14.1

SU3 parameter reconstruction by ATLAS

Aad, *et al.*, arXiv:0901.0512

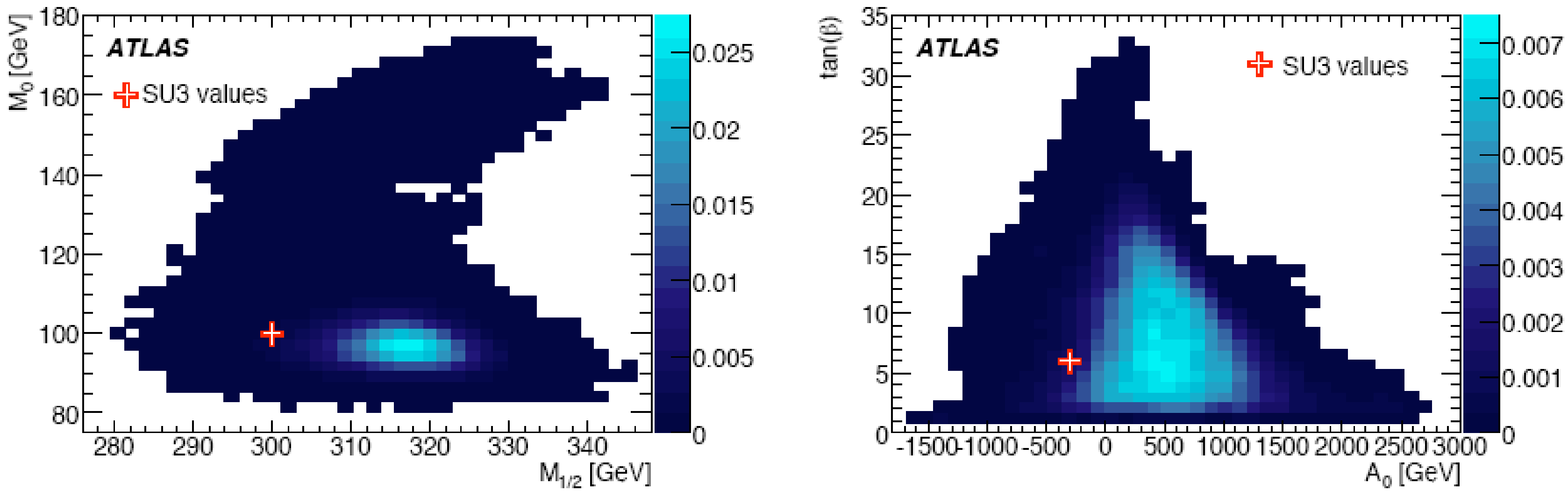


Figure 12: Two-dimensional Markov chain likelihood maps for mSUGRA parameters M_0 and $M_{1/2}$ (left) as well as $\tan\beta$ and A_0 (right) for sign $\mu = +1$, for benchmark point SU3, with integrated luminosity of $1\ \mathrm{fb}^{-1}$. The crosses indicate the actual values of the parameters for that benchmark point.

- 2D likelihood maps (int. lum. $1\ \mathrm{fb}^{-1}$)
- theory errors neglected
- neglect effect of SM parameters
- ranges around the true value found

Bayesian Analysis of the CMSSM

Apply to the CMSSM: recent development, led by 2 groups

Bayesian Analysis of the CMSSM

Apply to the CMSSM:

recent development, led by 2 groups

- $\boxed{m = (\theta, \psi)}$ – model's all relevant parameters

Bayesian Analysis of the CMSSM

Apply to the CMSSM: recent development, led by 2 groups

- $\boxed{m = (\theta, \psi)}$ – model's all relevant parameters
- CMSSM parameters $\boxed{\theta = m_{1/2}, m_0, A_0, \tan\beta}$
- relevant SM param's $\boxed{\psi = M_t, m_b(m_b)^{\overline{MS}}, \alpha_s^{\overline{MS}}, \alpha_{\rm em}(M_Z)^{\overline{MS}}}$

Bayesian Analysis of the CMSSM

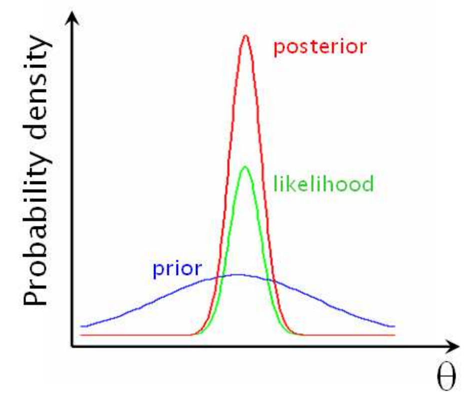
Apply to the CMSSM: recent development, led by 2 groups

- $m = (\theta, \psi)$ – model's all relevant parameters
- CMSSM parameters $\theta = m_{1/2}, m_0, A_0, \tan\beta$
- relevant SM param's $\psi = M_t, m_b(m_b)^{\overline{MS}}, \alpha_s^{\overline{MS}}, \alpha_{\rm em}(M_Z)^{\overline{MS}}$
- $\xi = (\xi_1, \xi_2, \ldots, \xi_m)$: set of derived variables (observables): $\xi(m)$

Bayesian Analysis of the CMSSM

Apply to the CMSSM: recent development, led by 2 groups

- $\boxed{m = (\theta, \psi)}$ – model's all relevant parameters
- CMSSM parameters $\boxed{\theta = m_{1/2}, m_0, A_0, \tan\beta}$
- relevant SM param's $\boxed{\psi = M_t, m_b(m_b)^{\overline{MS}}, \alpha_s^{\overline{MS}}, \alpha_{\rm em}(M_Z)^{\overline{MS}}}$
- $\xi = (\xi_1, \xi_2, \ldots, \xi_m)$: set of derived variables (observables): $\boxed{\xi(m)}$
- d: data ($\Omega_{\rm CDM}h^2$, $b \to s\gamma$, m_h, etc)



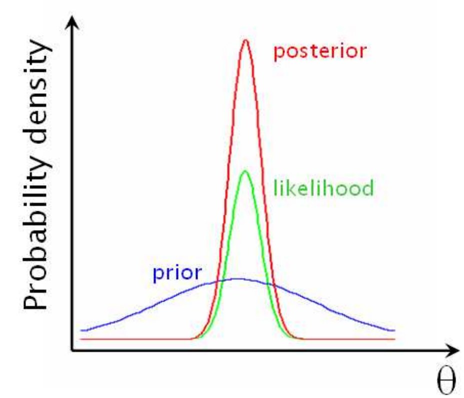
Bayesian Analysis of the CMSSM

Apply to the CMSSM: recent development, led by 2 groups

- $m = (\theta, \psi)$ – model's all relevant parameters
- CMSSM parameters $\theta = m_{1/2}, m_0, A_0, \tan\beta$
- relevant SM param's $\psi = M_t, m_b(m_b)^{\overline{MS}}, \alpha_s^{\overline{MS}}, \alpha_{\rm em}(M_Z)^{\overline{MS}}$
- $\xi = (\xi_1, \xi_2, \ldots, \xi_m)$: set of derived variables (observables): $\xi(m)$
- d: data $(\Omega_{\rm CDM}h^2, b \to s\gamma, m_h$, etc)
- Bayes' theorem: posterior pdf

$$p(\theta, \psi | d) = \frac{p(d|\xi)\pi(\theta,\psi)}{p(d)}$$

- $p(d|\xi) = \mathcal{L}$: likelihood
- $\pi(\theta, \psi)$: prior pdf
- $p(d)$: evidence (normalization factor)



$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{normalization factor}}$$

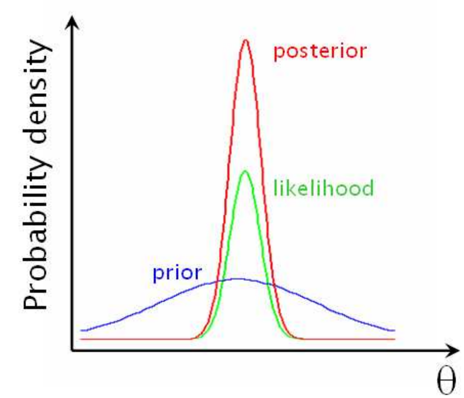
Bayesian Analysis of the CMSSM

Apply to the CMSSM:

recent development, led by 2 groups

- $m = (\theta, \psi)$ – model's all relevant parameters
- CMSSM parameters $\theta = m_{1/2}, m_0, A_0, \tan\beta$
- relevant SM param's $\psi = M_t, m_b(m_b)^{\overline{MS}}, \alpha_s^{\overline{MS}}, \alpha_{\rm em}(M_Z)^{\overline{MS}}$
- $\xi = (\xi_1, \xi_2, \ldots, \xi_m)$: set of derived variables (observables): $\xi(m)$
- d: data ($\Omega_{\rm CDM} h^2$, $b \to s\gamma$, m_h, etc)
- Bayes' theorem: posterior pdf

$$p(\theta, \psi | d) = \frac{p(d|\xi)\pi(\theta,\psi)}{p(d)}$$



$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{normalization factor}}$$

- $p(d|\xi) = \mathcal{L}$: likelihood
- $\pi(\theta, \psi)$: prior pdf
- $p(d)$: evidence (normalization factor)
- usually marginalize over SM (nuisance) parameters ψ $\Rightarrow$ $p(\theta|d)$

The likelihood: 1-dim case

Take a single observable $\xi(m)$ that has been measured

(e.g., M_W)

The likelihood: 1-dim case

Take a single observable $\xi(m)$ that has been measured

(e.g., M_W)

- c – central value, σ – standard exptal error

The likelihood: 1-dim case

Take a single observable $\xi(m)$ that has been measured

(e.g., M_W)

- c – central value, σ – standard exptal error
- define

$$\chi^2 = \frac{[\xi(m) - c]^2}{\sigma^2}$$

The likelihood: 1-dim case

Take a single observable $\xi(m)$ that has been measured

(e.g., M_W)

- c – central value, σ – standard exptal error
- define

$$\chi^2 = \frac{[\xi(m) - c]^2}{\sigma^2}$$

- assuming Gaussian distribution ($d \longrightarrow (c, \sigma)$):

$$\mathcal{L} = p(\sigma, c|\xi(m)) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\chi^2}{2}\right]$$

The likelihood: 1-dim case

Take a single observable $\xi(m)$ that has been measured

(e.g., M_W)

- c – central value, σ – standard exptal error
- define

$$\chi^2 = \frac{[\xi(m)-c]^2}{\sigma^2}$$

- assuming Gaussian distribution ($d \longrightarrow (c, \sigma)$):

$$\mathcal{L} = p(\sigma, c|\xi(m)) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\chi^2}{2}\right]$$

- when include theoretical error estimate τ (assumed Gaussian):

$$\sigma \rightarrow s = \sqrt{\sigma^2 + \tau^2}$$

TH error “smears out” the EXPTAL range

The likelihood: 1-dim case

Take a single observable $\xi(m)$ that has been measured

(e.g., M_W)

- c – central value, σ – standard exptal error
- define

$$\chi^2 = \frac{[\xi(m) - c]^2}{\sigma^2}$$

- assuming Gaussian distribution ($d \longrightarrow (c, \sigma)$):

$$\mathcal{L} = p(\sigma, c|\xi(m)) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\chi^2}{2}\right]$$

- when include theoretical error estimate τ (assumed Gaussian):

$$\sigma \longrightarrow s = \sqrt{\sigma^2 + \tau^2}$$

TH error "smears out" the EXPTAL range

- for several uncorrelated observables (assumed Gaussian):

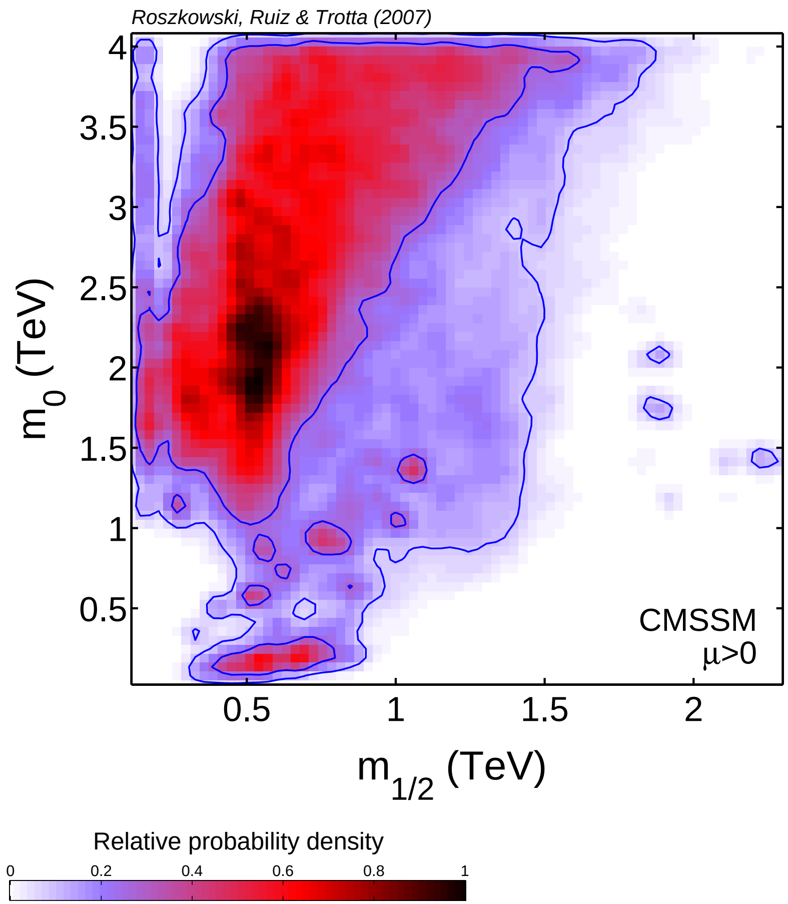
$$\mathcal{L} = \exp\left[-\sum_i \frac{\chi_i^2}{2}\right]$$

Probability maps of the CMSSM

Bayesian posterior pdf

Probability maps of the CMSSM

arXiv:0705.2012

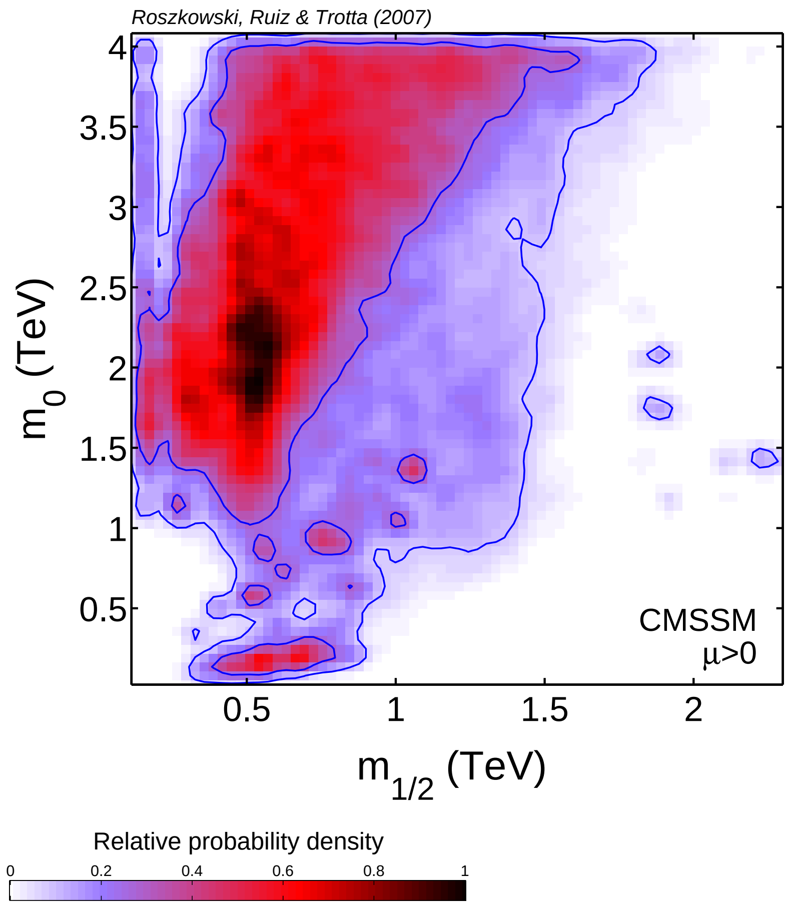


Bayesian posterior pdf

- MCMC scan (4 CMSSM + 4 SM param's)
- Bayesian analysis
- relative probability density fn (pdf)
- flat priors
- 68% total prob. – inner contours
- 95% total prob. – outer contours
- 2-dim pdf $p(m_0, m_{1/2}|d)$
- favored: $m_0 \gg m_{1/2}$ (FP region)

Probability maps of the CMSSM

arXiv:0705.2012



Bayesian posterior pdf

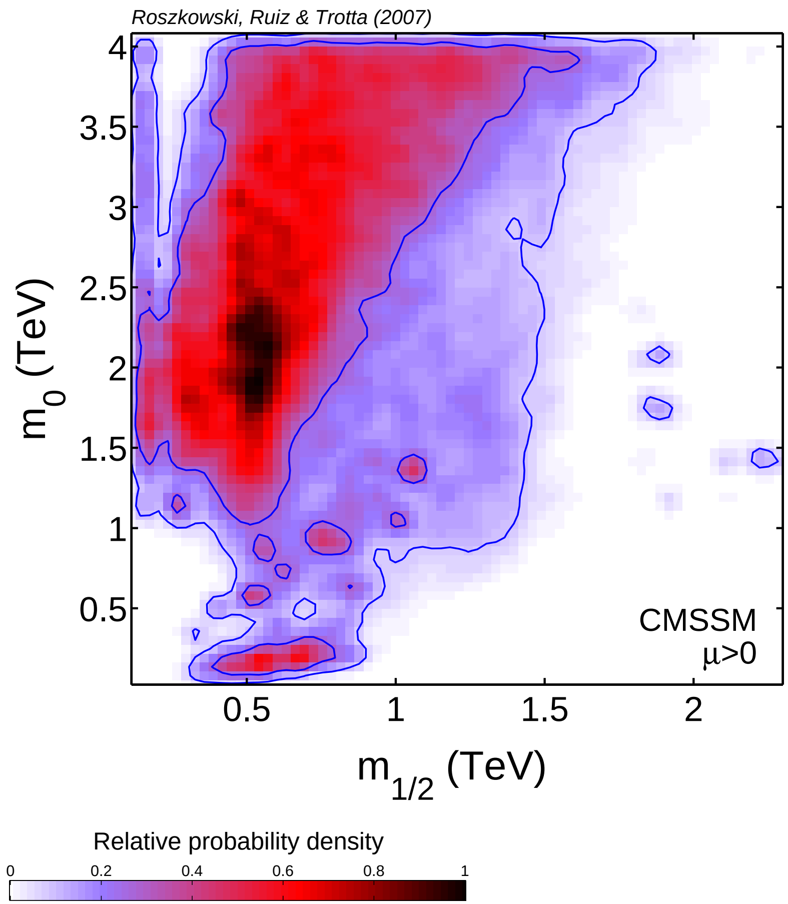
- MCMC scan (4 CMSSM + 4 SM param's)
- Bayesian analysis
- relative probability density fn (pdf)
- flat priors
- 68% total prob. – inner contours
- 95% total prob. – outer contours
- 2-dim pdf $p(m_0, m_{1/2}|d)$
- favored: $m_0 \gg m_{1/2}$ (FP region)

similar study by Allanach+Lester(+Weber)

see also, Ellis et al (EHOW, χ^2 approach, no MCMC, fixed SM parameters)

Probability maps of the CMSSM

arXiv:0705.2012



Bayesian posterior pdf

- MCMC scan (4 CMSSM + 4 SM param's)
- Bayesian analysis
- relative probability density fn (pdf)
- flat priors
- 68% total prob. – inner contours
- 95% total prob. – outer contours
- 2-dim pdf $p(m_0, m_{1/2}|d)$
- favored: $m_0 \gg m_{1/2}$ (FP region)

unlike others (except for A+L), we vary also SM parameters

Reconstruction of $m_{1/2}, m_0$ with SB

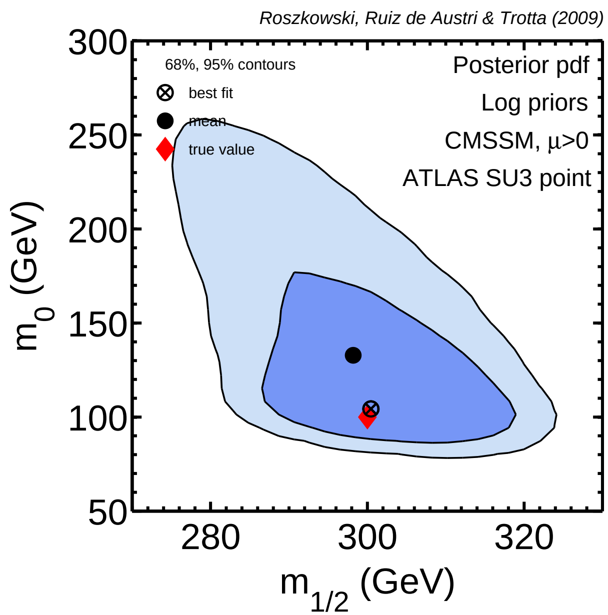
ATLAS SU3 benchmark point

Bayesian analysis, use Gaussian approx. with publicly available info

Reconstruction of $m_{1/2}, m_0$ with SB

ATLAS SU3 benchmark point

Bayesian analysis, use Gaussian approx. with publicly available info

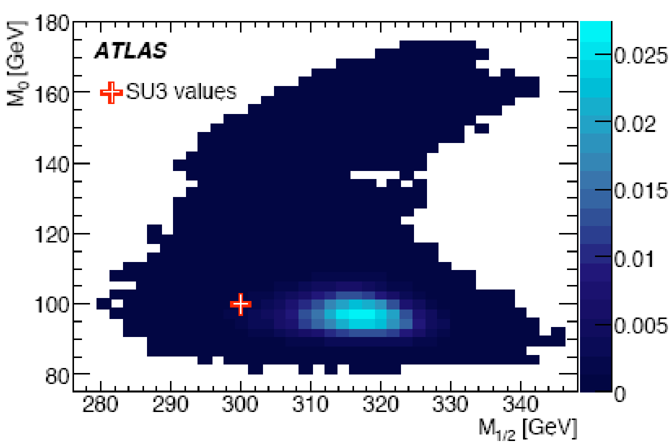


$$\theta = \{m_{\chi_1^0}, m_{\chi_2^0} - m_{\chi_1^0}, \widetilde{m}_{\tilde{l}} - m_{\chi_1^0}, m_{\tilde{q}} - m_{\chi_1^0}\}$$

$$-2\ln \mathcal{L}_{\mathrm{ATLAS}} = \chi^2_{\mathrm{ATLAS}} = (\theta - \theta_{\mathrm{ML}})^t C^{-1} (\theta - \theta_{\mathrm{ML}})$$

- red diamond: SU3 point
- green cross in circle: best-fit value
- big dot: posterior mean
- dark blue: 68% total prob. region
- light blue: 95% total prob. region

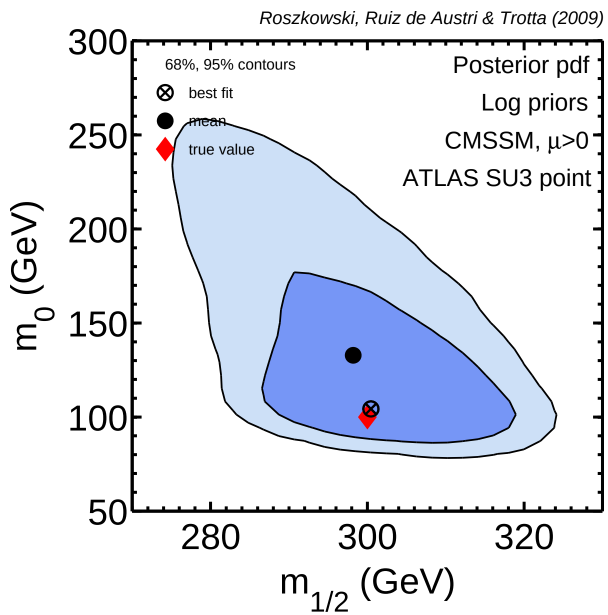
ATLAS analysis



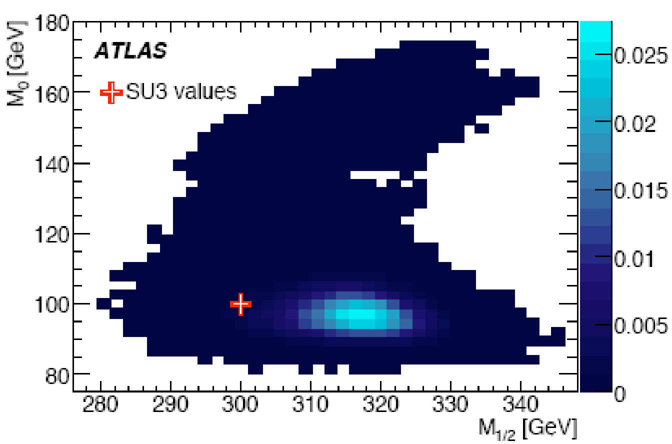
Reconstruction of $m_{1/2}, m_0$ with SB

ATLAS SU3 benchmark point

Bayesian analysis, use Gaussian approx. with publicly available info



ATLAS analysis



- Nested Sampling (NS) scan
- $50\,\mathrm{GeV} \leq m_{1/2}, m_0 \leq 500\,\mathrm{GeV}, \mu > 0$
- $-2\,\mathrm{TeV} \leq A_0 \leq 2\,\mathrm{TeV}, 2 \leq \tan\beta \leq 62$
- follow ATLAS input
- NO exptal constraints applied ($b \rightarrow s\gamma$, $\Omega_\chi h^2$, etc)
- similar for flat prior and profile likelihood (akin to χ^2)
- determination of m_0 a bit poorer than ATLAS

Reconstruction of $A_0, \tan\beta$ with SB

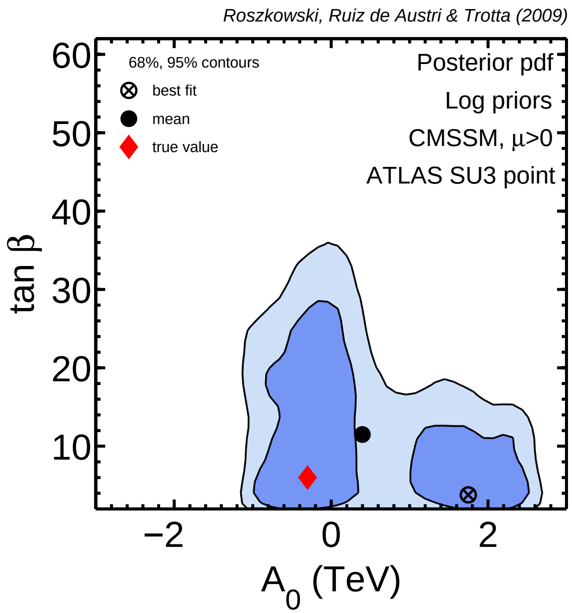
ATLAS SU3 benchmark point

Bayesian analysis, use Gaussian approx. with publicly available info

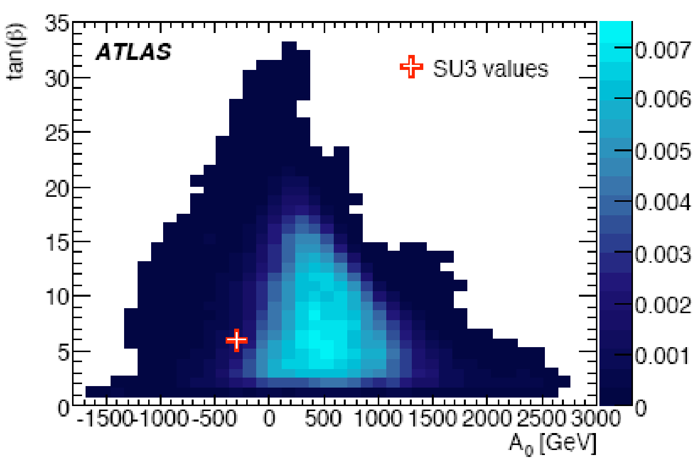
Reconstruction of A_0, $\tan\beta$ with SB

ATLAS SU3 benchmark point

Bayesian analysis, use Gaussian approx. with publicly available info



ATLAS analysis



$$\theta = \{m_{\chi_1^0}, m_{\chi_2^0} - m_{\chi_1^0}, \widetilde{m}_{\tilde{l}} - m_{\chi_1^0}, m_{\tilde{q}} - m_{\chi_1^0}\}$$

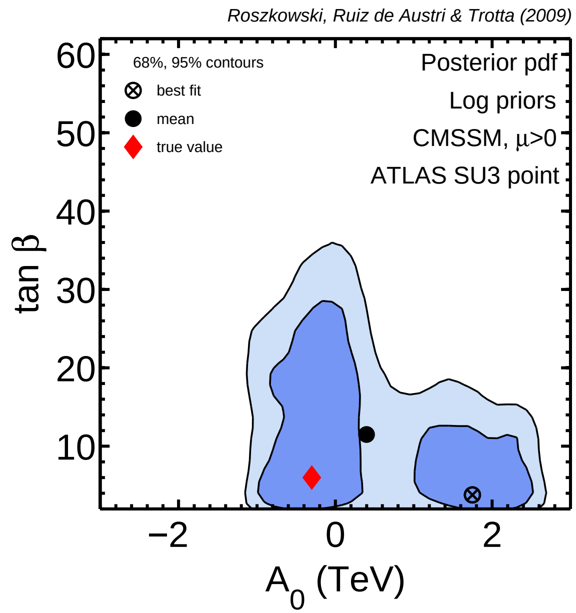
$$-2\ln\mathcal{L}_{\rm ATLAS} = \chi^2_{\rm ATLAS} = (\theta - \theta_{\rm ML})^t C^{-1} (\theta - \theta_{\rm ML})$$

- red diamond: SU3 point
- green cross in circle: best-fit value
- big dot: posterior mean
- dark blue: 68% total prob. region
- light blue: 95% total prob. region

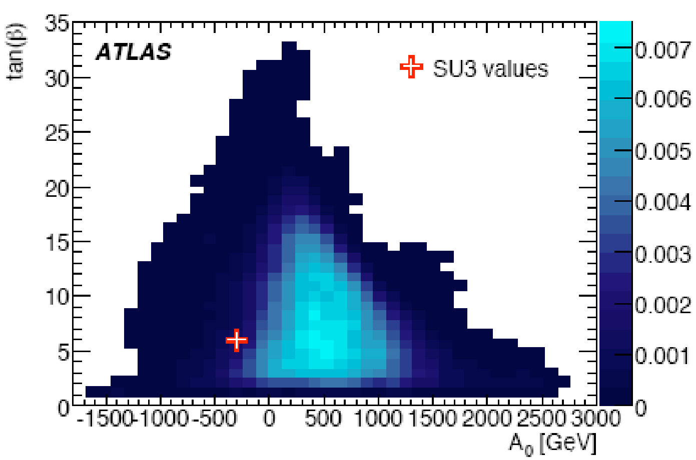
Reconstruction of A_0, $\tan\beta$ with SB

ATLAS SU3 benchmark point

Bayesian analysis, use Gaussian approx. with publicly available info



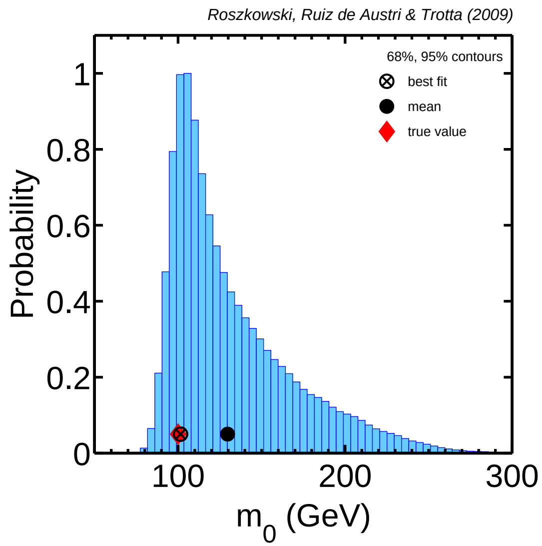
ATLAS analysis

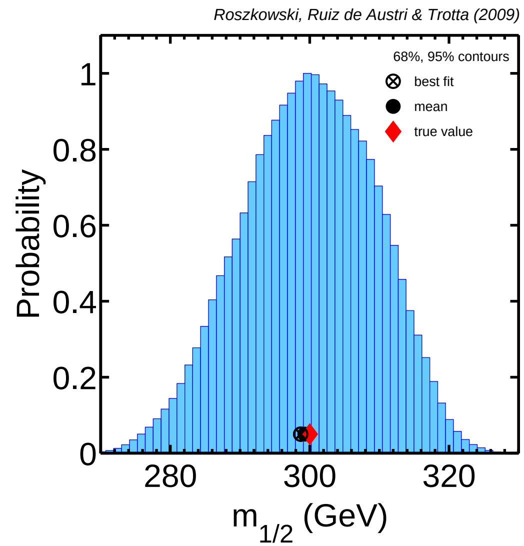


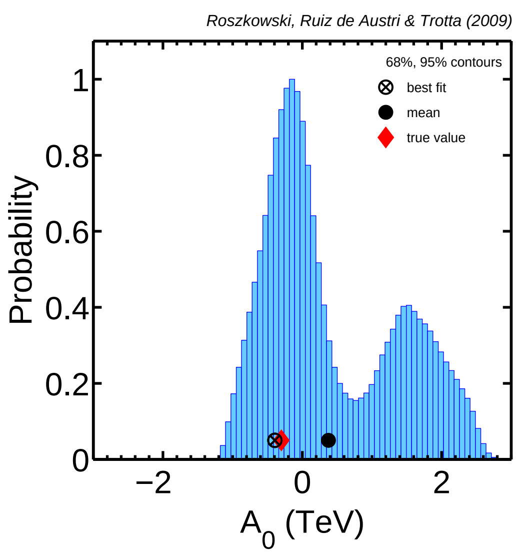
- NS scan
- $50\,\mathrm{GeV} \leq m_{1/2}, m_0 \leq 500\,\mathrm{GeV}$, $\mu > 0$
- $-2\,\mathrm{TeV} \leq A_0 \leq 2\,\mathrm{TeV}$, $2 \leq \tan\beta \leq 62$
- follow ATLAS input
- fix SM (nuisance) parameters
- NO exptal constraints applied ($b \rightarrow s\gamma$, $\Omega_\chi h^2$, etc)
- similar result for flat prior and profile likelihood (akin to χ^2)
- cannot resolve sign of A_0

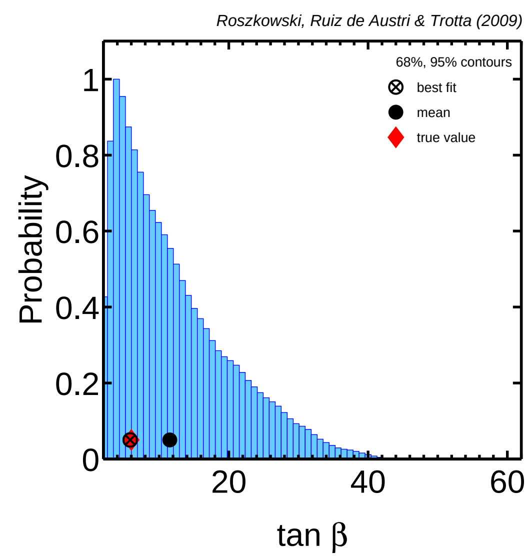
1dim posterior pdfs

ATLAS SU3 benchmark point



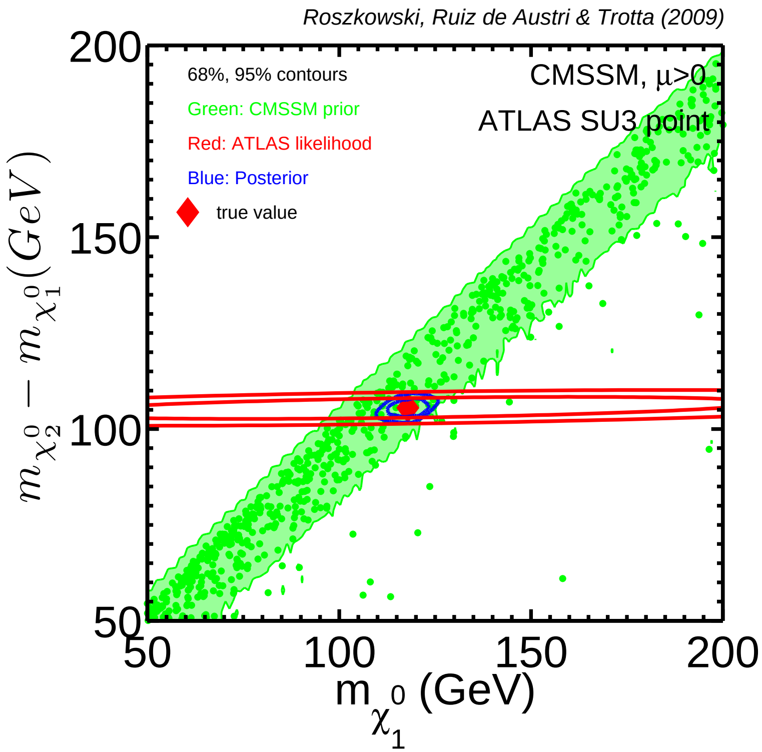




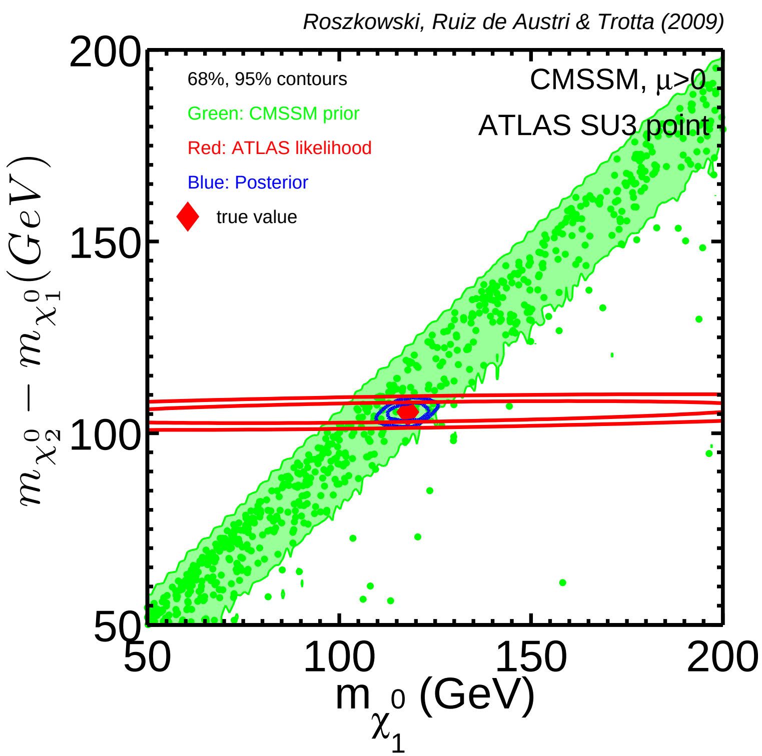


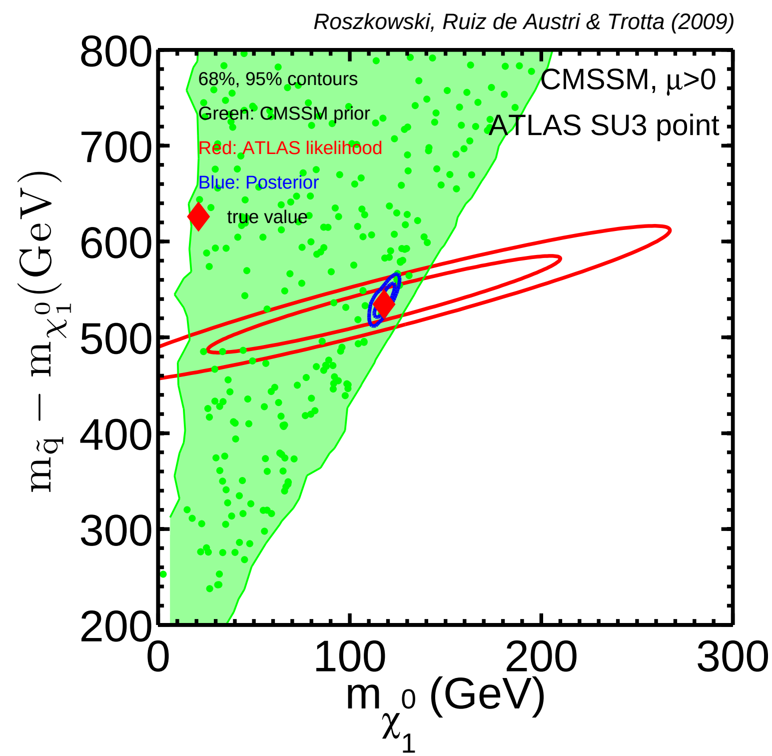
SU3: CMSSM vs MSSM

SU3: CMSSM vs MSSM



SU3: CMSSM vs MSSM





- green points: allowed by CMSSM
- red ellipses: ATLAS likelihood
- blue ellipses: posterior constraints

theory advantage:

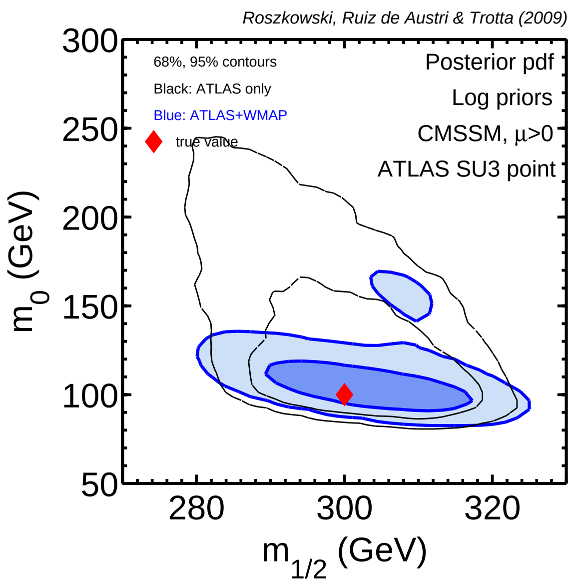
$\Rightarrow$ using posterior allows much better determination of $m_{\chi_1^0}$

Add info about $\Omega_\chi h^2$

take WMAP error on $\Omega_\chi h^2$: 0.0062

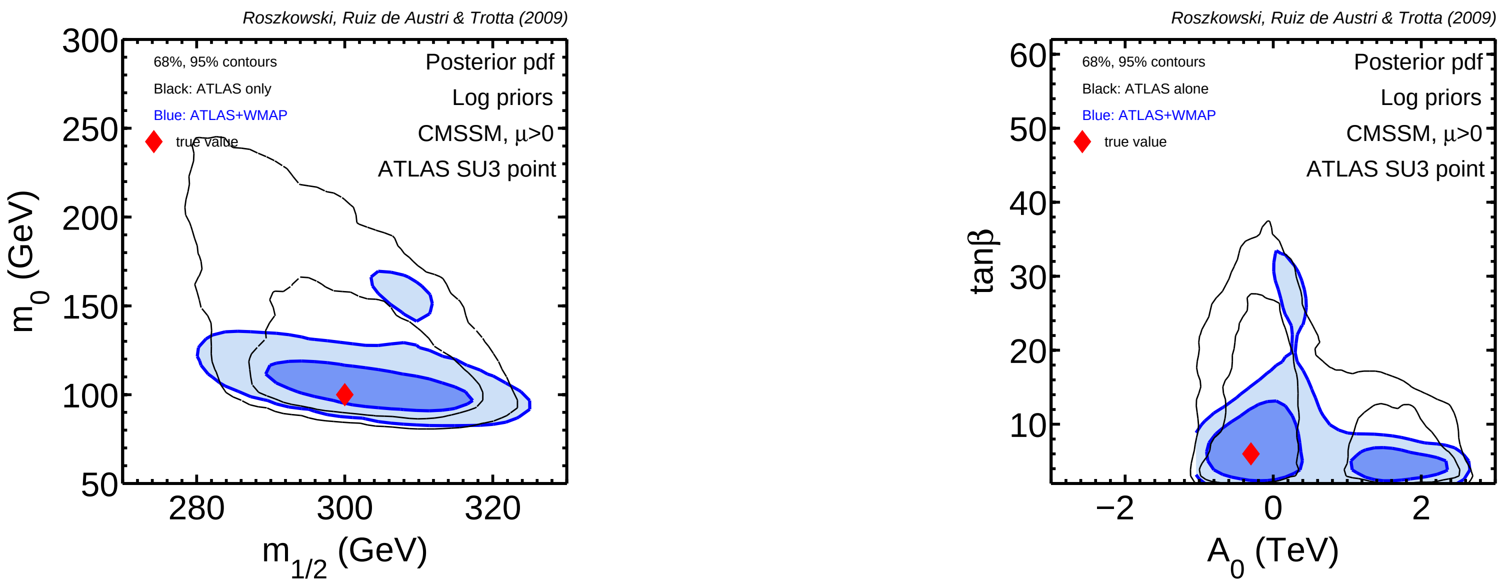
Add info about $\Omega_\chi h^2$

take WMAP error on $\Omega_\chi h^2$: 0.0062



Add info about $\Omega_\chi h^2$

take WMAP error on $\Omega_\chi h^2$: 0.0062



similar result for flat prior and profile likelihood

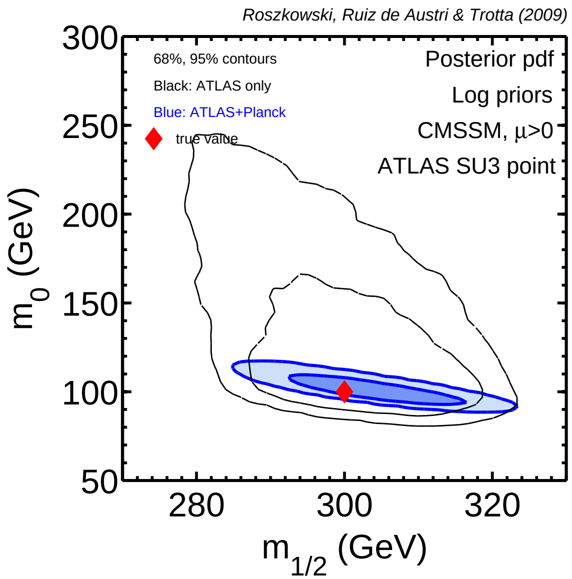
- determination of $m_{1/2}, m_0$ spot on!
- $\tan\beta$ resolved reasonably well
- determination of A_0 remains poor
- still cannot resolve sign of A_0

Add info about $\Omega_\chi h^2$ from Planck

assume Planck-like error on $\Omega_\chi h^2$ of $\lesssim 0.0016$ (WMAP error$/5$)

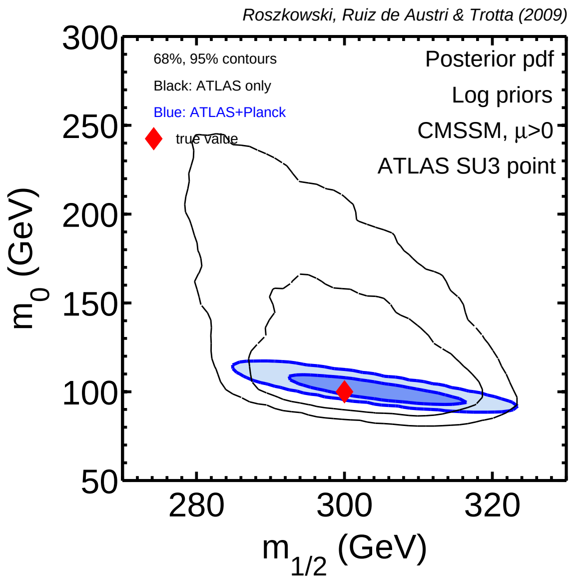
Add info about $\Omega_\chi h^2$ from Planck

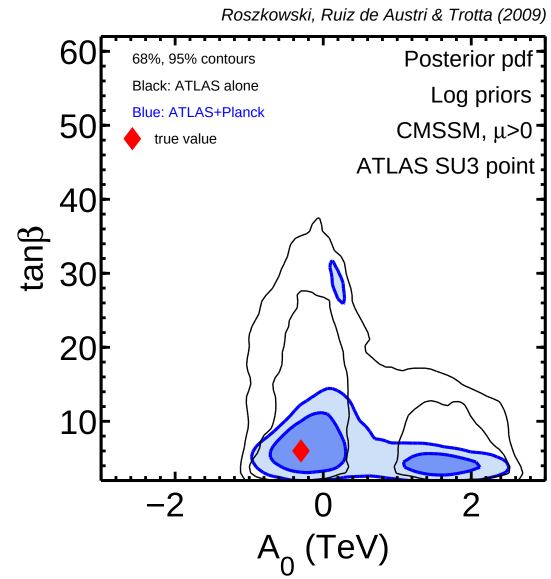
assume Planck-like error on $\boldsymbol{\Omega_\chi h^2}$ of $\lesssim \mathbf{0.0016}$ (WMAP error$/\mathbf{5}$)



Add info about $\Omega_\chi h^2$ from Planck

assume Planck-like error on $\Omega_\chi h^2$ of $\lesssim 0.0016$ (WMAP error$/5$)





similar result for flat prior and profile likelihood

- determination of $m_{1/2}, m_0$ spot on!
- $\tan\beta$ resolved reasonably well
- determination of A_0 remains poor
- still cannot resolve sign of A_0

Profile likelihood

different statistical measure, independent of priors

Regions of high posterior probability do not always give the best fits

Profile likelihood

different statistical measure, independent of priors

Regions of high posterior probability do not always give the best fits

- Take two regions: a ‘spike’ - tiny region with excellent fit to data and a large region with somewhat worse fit to data

Profile likelihood

different statistical measure, independent of priors

Regions of high posterior probability do not always give the best fits

- Take two regions: a ‘spike’ - tiny region with excellent fit to data and a large region with somewhat worse fit to data
- Bayesian statistics: pdf would peak at large region (‘volume’ effect)

Profile likelihood

different statistical measure, independent of priors

Regions of high posterior probability do not always give the best fits

- Take two regions: a ‘spike’ - tiny region with excellent fit to data and a large region with somewhat worse fit to data
- Bayesian statistics: pdf would peak at large region (‘volume’ effect)

define profile likelihood for, e.g., parameter m_1

$$\boxed{\mathfrak{L}(m_1) \equiv \max_{m_2,\ldots,m_N} \mathcal{L}(d|m)}$$

Profile likelihood

different statistical measure, independent of priors

Regions of high posterior probability do not always give the best fits

- Take two regions: a ‘spike’ - tiny region with excellent fit to data and a large region with somewhat worse fit to data
- Bayesian statistics: pdf would peak at large region (‘volume’ effect)

define profile likelihood for, e.g., parameter m_1

$$\mathfrak{L}(m_1) \equiv \max_{m_2,\ldots,m_N} \mathcal{L}(d|m)$$

- PL maximizes the likelihood along marginalized dimensions
- marginal posterior integrates them out

Profile likelihood

different statistical measure, independent of priors

Regions of high posterior probability do not always give the best fits

- Take two regions: a ‘spike’ - tiny region with excellent fit to data and a large region with somewhat worse fit to data
- Bayesian statistics: pdf would peak at large region (‘volume’ effect)

define profile likelihood for, e.g., parameter m_1

$$\mathfrak{L}(m_1) \equiv \max_{m_2,\ldots,m_N} \mathcal{L}(d|m)$$

- PL maximizes the likelihood along marginalized dimensions
- marginal posterior integrates them out
- any tension between Bayesian pdf and profile likelihood indicates that data is not constraining enough

Profile likelihood

different statistical measure, independent of priors

Regions of high posterior probability do not always give the best fits

- Take two regions: a 'spike' - tiny region with excellent fit to data and a large region with somewhat worse fit to data
- Bayesian statistics: pdf would peak at large region ('volume' effect)

define profile likelihood for, e.g., parameter m_1

$$\mathfrak{L}(m_1) \equiv \max_{m_2,\dots,m_N} \mathcal{L}(d|m)$$

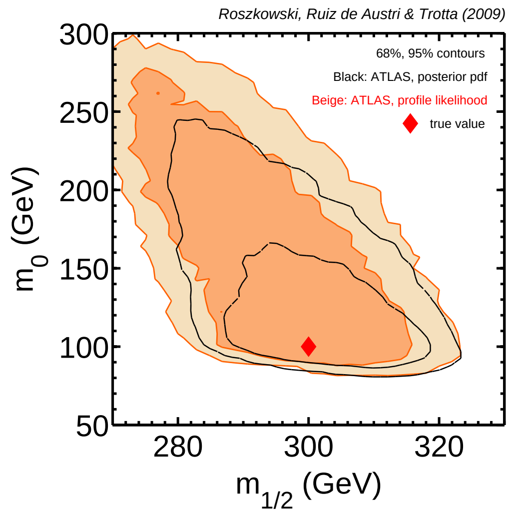
- PL maximizes the likelihood along marginalized dimensions
- marginal posterior integrates them out
- any tension between Bayesian pdf and profile likelihood indicates that data is not constraining enough

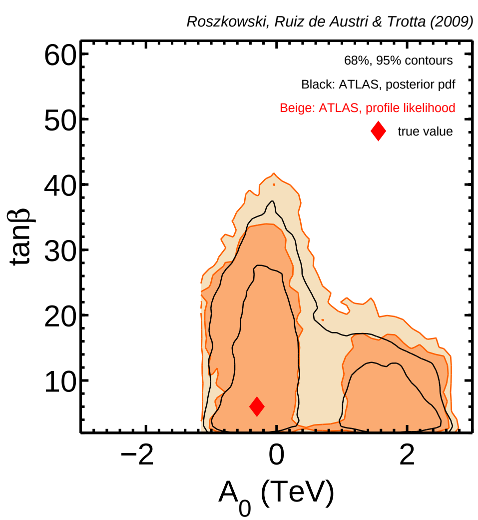
need to do both to see if that is the case

Posterior pdf vs. profile likelihood

Posterior pdf vs. profile likelihood

ATLAS data only

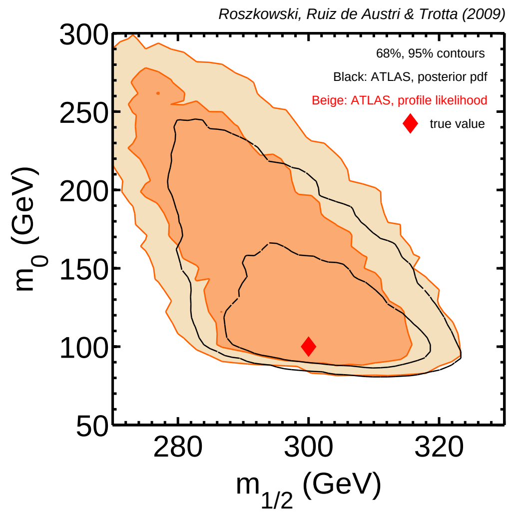


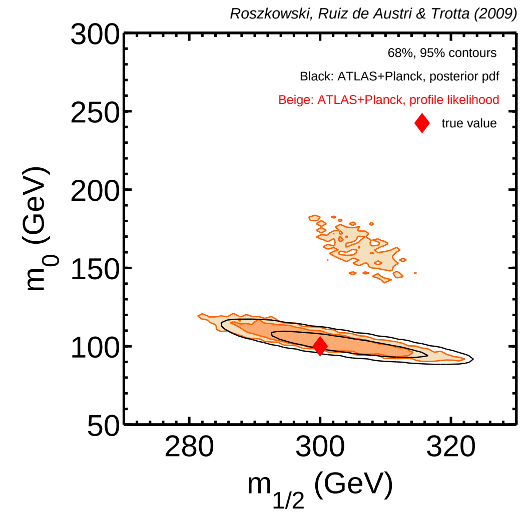


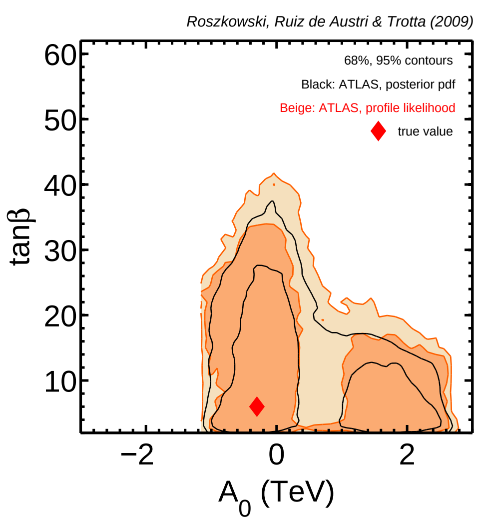
Posterior pdf vs. profile likelihood

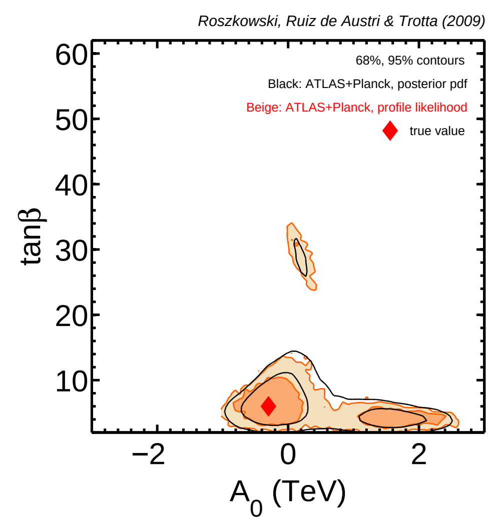
ATLAS data only

add $\Omega_\chi h^2$+ Planck-like error





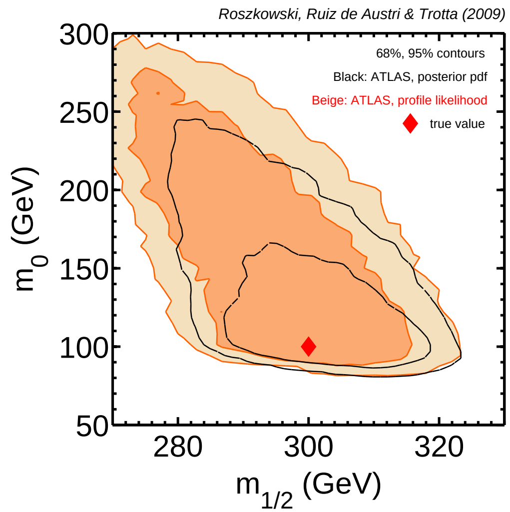


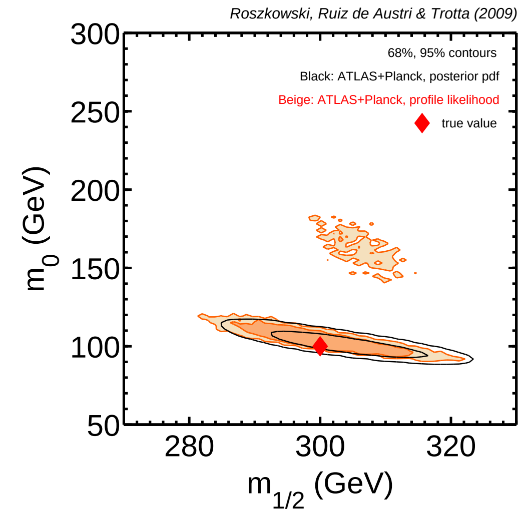


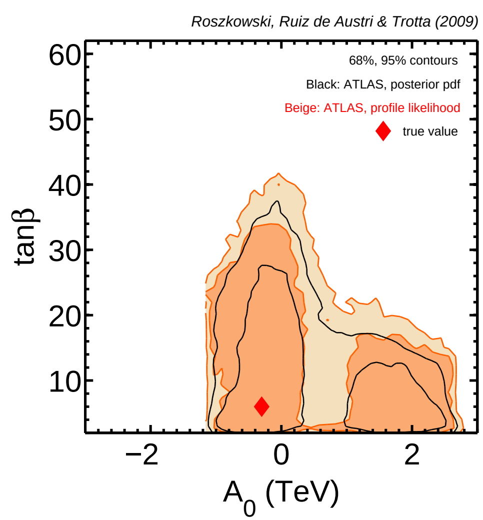
Posterior pdf vs. profile likelihood

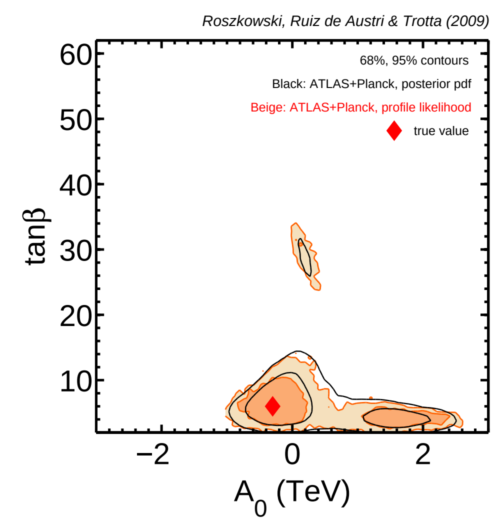
ATLAS data only

add $\Omega_\chi h^2$+ Planck-like error









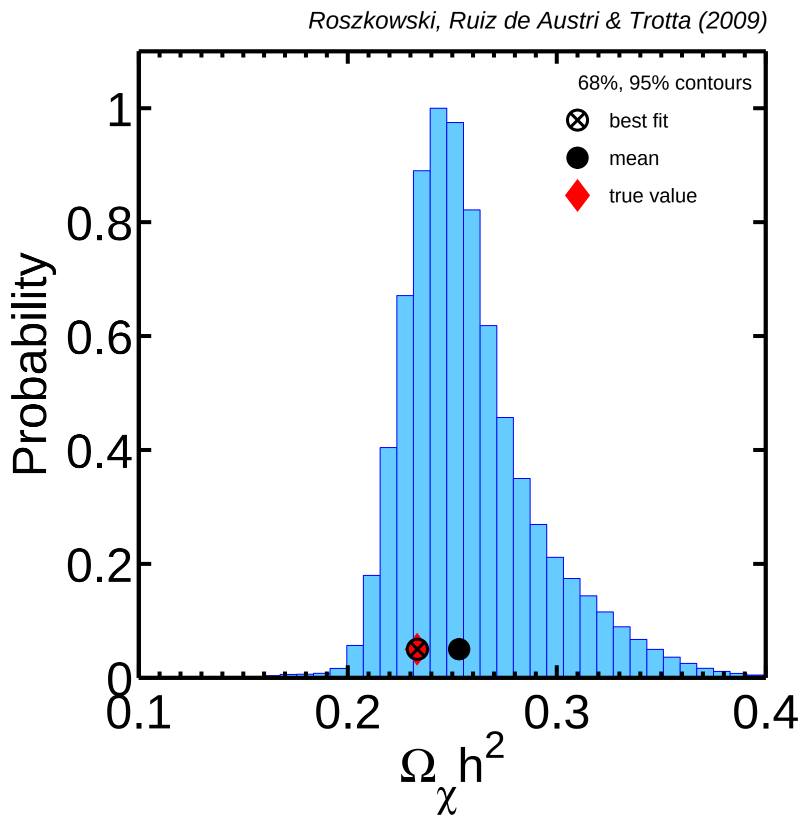
$\Rightarrow$ good agreement

Determination of $\Omega_\chi h^2$

ATLAS SU3 point

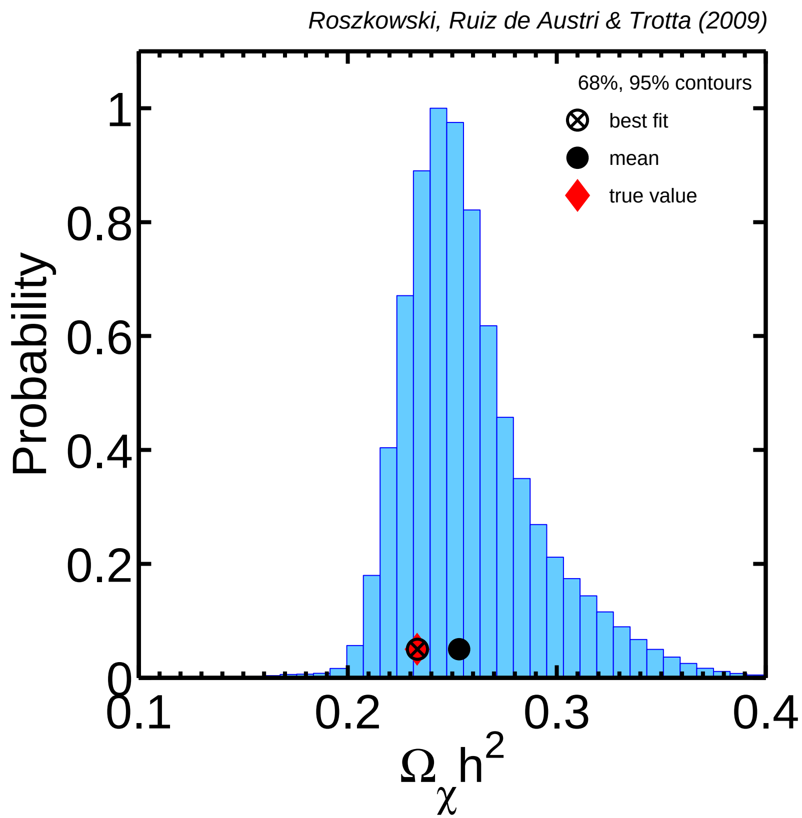
Determination of $\Omega_\chi h^2$

ATLAS SU3 point



Determination of $\Omega_\chi h^2$

ATLAS SU3 point

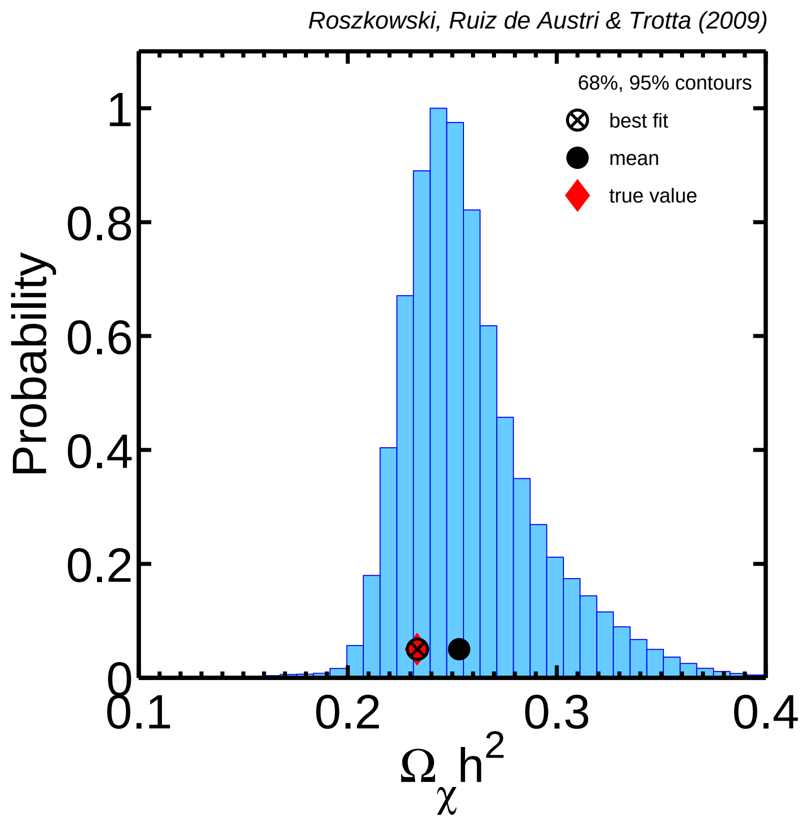


- use only ATLAS data
- similar result for log prior and profile likelihood

- red diamond: SU3 point
- green cross in circle: best-fit value
- dark blue dot: posterior mean

Determination of $\Omega_\chi h^2$

ATLAS SU3 point



- use only ATLAS data
- similar result for log prior and profile likelihood

- red diamond: SU3 point
- green cross in circle: best-fit value
- dark blue dot: posterior mean

$$\Rightarrow \; \Omega_\chi h^2 = 0.253 \pm 0.034$$

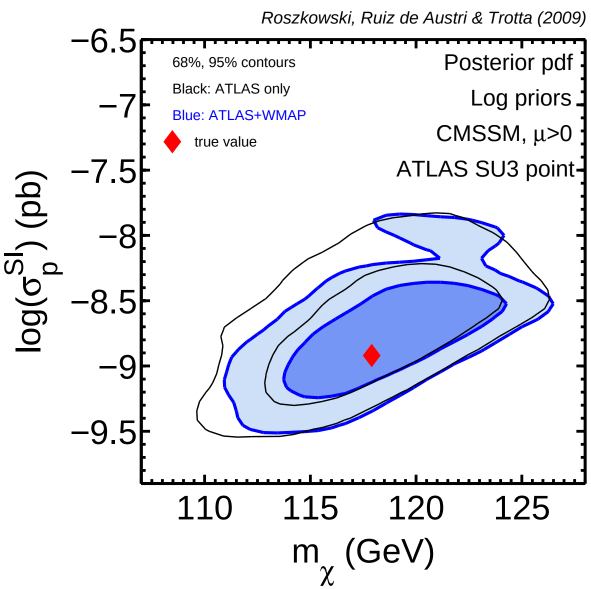
relative accuracy of $\sim 10\%$

Determination of σ_p^{SI}

assume Planck-like error: reduce WMAP error on $\Omega_\chi h^2$ by ~ 5 ($\lesssim 0.0016$)

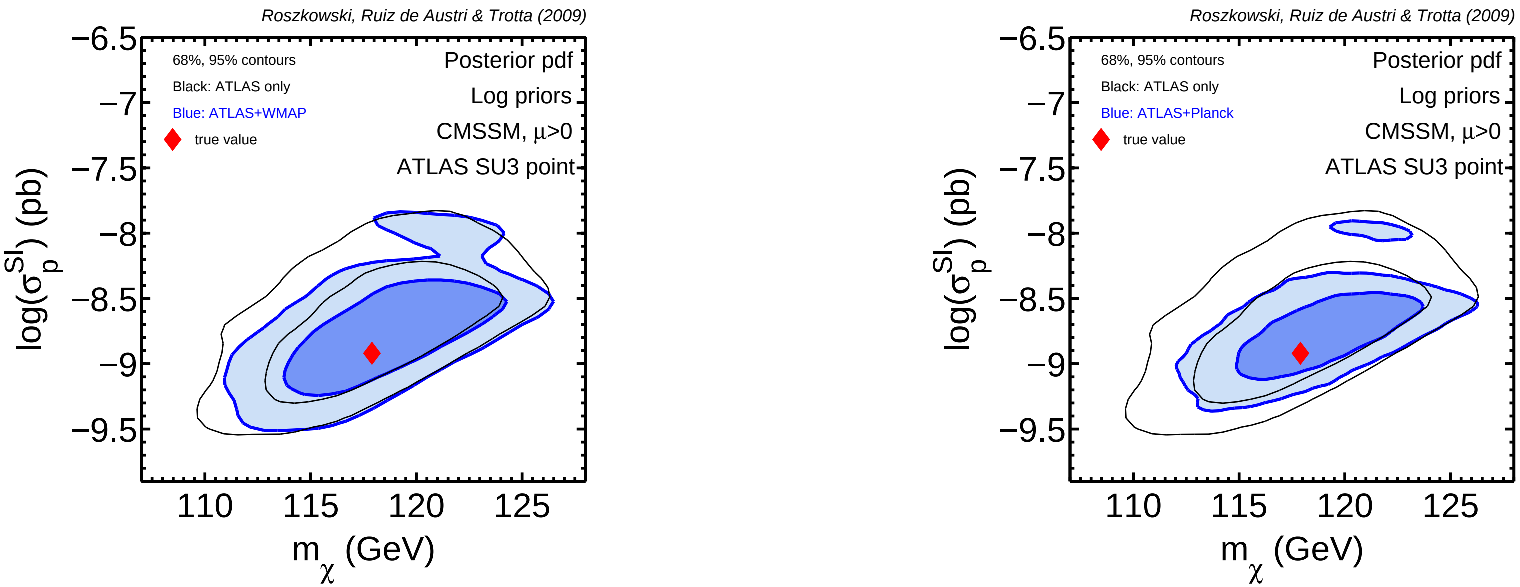
Determination of σ_p^{SI}

assume Planck-like error: reduce WMAP error on $\Omega_\chi h^2$ by ~ 5 ($\lesssim 0.0016$)



Determination of σ_p^{SI}

assume Planck-like error: reduce WMAP error on $\Omega_\chi h^2$ by ~ 5 ($\lesssim 0.0016$)

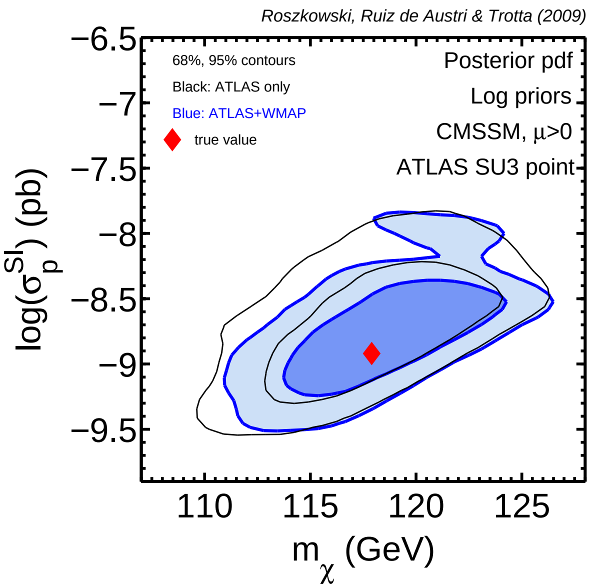


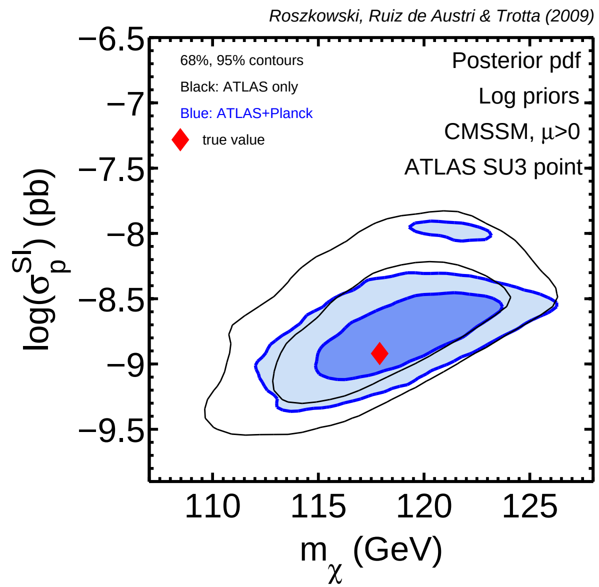
similar result for flat prior and profile likelihood

- determination of σ_p^{SI} much improved by adding WMAP error on $\Omega_\chi h^2$
- Planck: limited impact

Determination of $\sigma_p^{\rm SI}$

assume Planck-like error: reduce WMAP error on $\Omega_\chi h^2$ by ~ 5 ($\lesssim 0.0016$)





similar result for flat prior and profile likelihood

- determination of $\sigma_p^{\rm SI}$ much improved by adding WMAP error on $\Omega_\chi h^2$
- Planck: limited impact

Summary

Summary

- MCMC + Bayesian statistics: powerful tool for LHC/Planck era to properly analyze multi-dim. “new physics” models like SUSY
- tool: SuperBayes package, available from www.superbayes.org

Summary

- MCMC + Bayesian statistics: powerful tool for LHC/Planck era to properly analyze multi-dim. “new physics” models like SUSY
- tool: SuperBayes package, available from www.superbayes.org

Constrained MSSM – currently underconstrained

Summary

- MCMC + Bayesian statistics: powerful tool for LHC/Planck era to properly analyze multi-dim. “new physics” models like SUSY
- tool: SuperBayes package, available from www.superbayes.org

Constrained MSSM – currently underconstrained

CMSSM SU3 benchmark point with 1 fb^{-1}

- Bayesian reconstruction of CMSSM parameters with simple Gaussian approximation and public info (except covariance matrix) comparable to that claimed in ATLAS analysis

almost no prior dependence left

Summary

- MCMC + Bayesian statistics: powerful tool for LHC/Planck era to properly analyze multi-dim. "new physics" models like SUSY
- tool: SuperBayes package, available from www.superbayes.org

Constrained MSSM – currently underconstrained

CMSSM SU3 benchmark point with 1 fb^{-1}

- Bayesian reconstruction of CMSSM parameters with simple Gaussian approximation and public info (except covariance matrix) comparable to that claimed in ATLAS analysis

almost no prior dependence left

theory extras: adding WMAP or Planck error on $\Omega_\chi h^2$:

- determination of $m_{1/2}, m_0$ excellent (for SU3)
- $\tan\beta$ resolved reasonably well
- still cannot resolve A_0, not even its sign
- DM: $\sigma_p^{\rm SI}$ improved by a factor of ~ 2

Planck error on $\Omega_\chi h^2$: limited further impact

Summary

- MCMC + Bayesian statistics: powerful tool for LHC/Planck era to properly analyze multi-dim. "new physics" models like SUSY
- tool: SuperBayes package, available from www.superbayes.org

Constrained MSSM – currently underconstrained

CMSSM SU3 benchmark point with 1 fb^{-1}

- Bayesian reconstruction of CMSSM parameters with simple Gaussian approximation and public info (except covariance matrix) comparable to that claimed in ATLAS analysis

almost no prior dependence left

theory extras: adding WMAP or Planck error on $\Omega_\chi h^2$:

- determination of $m_{1/2}, m_0$ excellent (for SU3)
- $\tan\beta$ resolved reasonably well
- still cannot resolve A_0, not even its sign
- DM: σ_p^{SI} improved by a factor of ~ 2

Planck error on $\Omega_\chi h^2$: limited further impact

$\Rightarrow$ SUSY parameter reconstruction with open-access data (+ convariance matrix) seems doable