Defects in AdS_4/CFT_3

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Overview

Probe branes in AdS/CFT: Added flavour degrees of freedom

In $AdS_5 \times S^5$:

- D7 probe: Codimension zero defect
  Application example: Holographic Superconductor

  [Ammon, J.E., Kaminski, Kerner 0810.2316, 0903.1864]
- D5 (D3) probe: Codimension 1 (2) defect

In the ABJM geometry $AdS_4 \times \mathbb{C}P^3$:

- Four embeddings constructed, including dual field theories

  [Ammon, J. E., Meyer, O'Bannon, Wrase 0909.3845]
- Applications: Aspects of Quantum Hall effect

  [work in progress]

Outline

1. Motivation
2. AdS_5/CFT_4
   - Adding flavour
   - Holographic Superconductors from D7 brane probes
   - Defects
3. The ABJM model: AdS_4/CFT_3
   - Review
   - Adding flavour
   - Defects

Outline

Motivation

Embedding probe branes in $AdS_5 \times S^5$

- generates gravity duals of defect CFT's
- adds flavour degrees of freedom

$\Rightarrow$ many applications

AdS_4/CFT_3

- Recently established from M2 branes [Aharony, Bergman, Jafferis, Maldacena 2008]
- Chern-Simons theory with gauge group $SU(N)_k \times SU(N)_{-k}$

Goal:

Determine general recipe
for adding supersymmetric flavour and defects
to AdS_4/CFT_3 in field theory and gravity description

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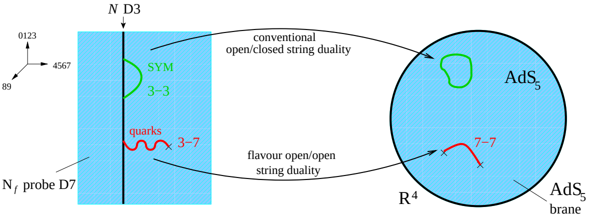
[Karch, Katz 2002]



$N \to \infty$ (standard Maldacena limit), N_f small (probe approximation)

Duality acts twice:

Field theory

$\mathcal{N} = 4$ SU(N) Super Yang-Mills theory

coupled to

$\mathcal{N} = 2$ fundamental hypermultiplet

$\Leftrightarrow$

Gravity

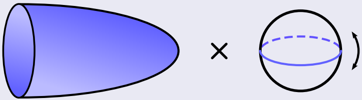
IIB supergravity on $AdS_5 \times S^5$

+

Probe brane action (DBI) on $AdS_5 \times S^3$

Phenomenological models for studying quarks and mesons in QCD-like theories

- Fluctuations of brane probes ⇒ Mesons



- Brane embeddings in confining 10d backgrounds ⇒ Chiral symmetry breaking
- Brane probes in AdS black hole geometry ⇒ Quarks added to finite temperature field theory
- Finite density ⇒ Phase diagram
- Hydrodynamics

Condensed matter:

- Superfluids/Superconductors, Quantum Hall Effect
- Quarks ⇒ 'electrons'

Holographic superconductor from D7 brane probes

Holographic superconductor with (3+1)-dimensional field theory in for which

- the dual field theory is explicitly known
- there is a ten-dimensional string theory picture of condensation

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p-wave superconductor

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p-wave superconductor

Holographic superconductor at finite isospin density

- Embed two coincident D7-branes into AdS-Schwarzschild

  gauge fields $A_\mu = A_\mu^a \sigma^a \in u(2) = u(1)_B \oplus su(2)_I$
- Finite isospin density: $A_0^3 \neq 0 \Rightarrow$ Explicit breaking to $u(1)_3$
- Dynamics of Flavour degrees is described by non-abelian DBI action

Field theory described:

$\mathcal{N} = 4$ Super Yang-Mills plus two flavors of fundamental matter at finite temperature and finite isospin density

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Holographic superconductor at finite isospin density

There is a new solution to the equations of motion
with non-zero vev for $A_3^1\sigma^1$ in addition to the non-zero $A_0^3\sigma^3$

$$A_0^3 = \mu - \frac{\tilde{d}_0^3}{2\pi\alpha'}\frac{\rho_H}{\rho^2} + \dots, \qquad A_3^1 = -\frac{\tilde{d}_1^3}{2\pi\alpha'}\frac{\rho_H}{\rho^2} + \dots$$

This new solution has lower free energy

Order parameter $\tilde{d}_3^1 \propto \langle \bar{\psi}_u\gamma_3\psi_d + \bar{\psi}_d\gamma_3\psi_u + \mathit{bosons} \rangle \neq 0$

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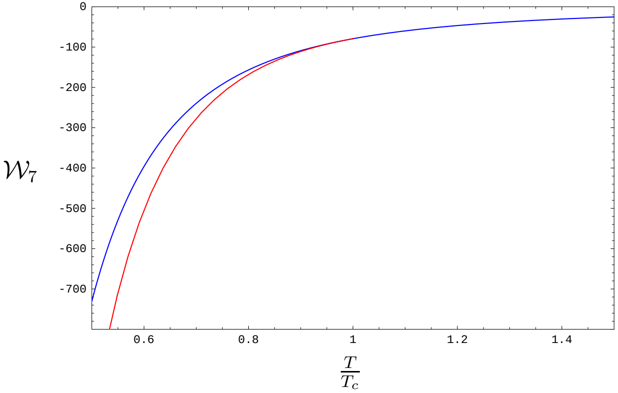
Free energy (Grand potential) vs. temperature



The new ground state has properties known from superconductors:

- infinite DC conductivity, gap in the AC conductivity
- second order phase transition, critical exponent of 1/2 (mean field)
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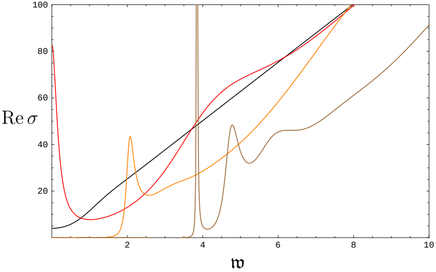
Frequency-dependent conductivity $\sigma(\omega) = \frac{i}{\omega} G^R(\omega)$

G^R retarded Green function for fluctuation a_2^3



$\mathfrak{w} = \omega/(2\pi T)$

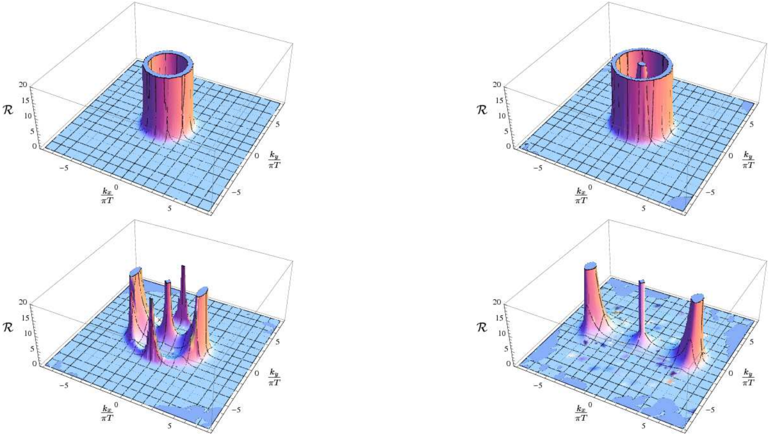
T/T_c: Black: ∞, Red: 1, Orange: 0.5, Brown: 0.28.

Interpretation: Frictionless motion of mesons through plasma

Fermions

Use fermionic part of D7 DBI action to study fermionic fluctuations

[Ammon, J.E., Kaminski, O'Bannon 1003.1134]



Defects

D3/D7:

Codimension zero defect theory

Even before:

- D3/D5 (codimension 1) [Karch, Randall 2001; Freedman, Ooguri, DeWolfe 2001]
- D3/D3 (codimension 2) [Constable, J.E., Guralnik, Kirsch 2002]
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Supersymmetric embeddings

[Skenderis, Taylor 2002]

Brane	$ND = 4$ intersections	Embedding
D1	$(0\|D1 \perp D3)$	AdS_2
D3	$(1\|D3 \perp D3)$	$AdS_3 \times S^1$
D5	$(2\|D5 \perp D3)$	$AdS_4 \times S^2$
D7	$(3\|D7 \perp D3)$	$AdS_5 \times S^3$
	$ND = 8$ intersections	
D5	$(0\|D5 \perp D3)$	$AdS_2 \times S^4$
D7	$(1\|D7 \perp D3)$	$AdS_3 \times S^5$

D3/D5

[J.E., Guralnik, Kirsch 2002]

Action in $\mathcal{N} = 2$, $d = 3$ superspace

$$S_{\mathrm{bulk}} = \frac{1}{g^2} \int \mathrm{d}z\mathrm{d}^3x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta} \left(\Sigma^2 - \frac{1}{2}(\sqrt{2}\partial_z V + \Phi + \bar{\Phi})^2 + \bar{Q}_i Q_i \right)$$

$$+ \int \mathrm{d}z\mathrm{d}^3x\mathrm{d}^2\theta \, \epsilon_{ij} Q_i \partial_z Q_j + \int \mathrm{d}z\mathrm{d}^3x\mathrm{d}^2\bar{\theta} \, \epsilon_{ij} \bar{Q}_i \partial_z \bar{Q}_j \, ,$$

$$S^{3\mathrm{d}}_{\mathrm{bdy}} = \int d^3x d^2\theta d^2\bar{\theta} \left(\bar{B}^+ e^{gV} B^+ + \bar{B}^- e^{-gV} B^- \right)$$

$$+ \frac{ig}{\sqrt{2}} \left[\int d^3x d^2\theta B^+ Q_2 B^- + c.c. \right]$$

D3/D5

[J.E., Guralnik, Kirsch 2002]

Non-renormalization

Write field theory action in $\mathcal{N} = 2$, $d = 3$ superspace

No contributions possible to divergence of supercurrent

$\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = D_\alpha S$, $\qquad S = 0$ here!
($V\Sigma$ is vector multiplet)

$SO(3,2)$ conformal symmetry preserved even in quantized theory

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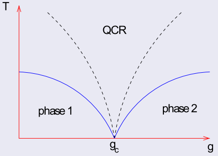
Applications

Defect systems described are useful for

- uncovering universal behaviour of systems at quantum critical points



- due to conformal invariance and strong coupling

Recent developments

- Conductivities, specific heat, speed of sound [Myers, Wapler; Karch, Parnachev, ..]
- BKT-Transitions [Karch, Son et al; Evans et al]
- Fractional Quantum Hall Effect [Keski-Vakkuri, Kraus et al '09] [Fujita, Li, Ryu, Takayanagi '09]

Outline

Low–Energy descriptions of M2–Branes

[Aharony, Bergman, Jafferis, Maldacena, '08]

	0	1	2	3	4	5	6	7	8	9	10
N_c *M2*	•	•	•	-	-	-	-	-	-	-	-

N_c M2–Branes on C^4/Z_k:
two different low–energy descriptions for $N_c \to \infty$ and $N_c \gg k$:

Gravity side

- For $N_c \gg k^5$: 11D Supergravity on asymptotically $AdS_4 \times S^7/Z_k$
- For $N_c \ll k^5$: 10D IIA Supergravity on asymptotically $AdS_4 \times CP^3$

Gauge theory side

- $U(N_c)_k \times U(N_c)_{-k}$ Chern-Simons Matter Theory (CSM)
- $\mathcal{N} = 6$ supersymmetric for general k
- conformal, invariant under parity, $SU(4)_{\mathcal{R}} \simeq SO(6)_{\mathcal{R}}$

Deriving AdS_4 / CFT_3 from type IIB setup

Four steps:

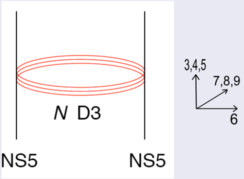
1. Write D3-brane theory, add 2 NS5-branes
2. Add k D5-branes, form $(1, k)$ 5-brane bound state ($\Rightarrow$ Chern-Simons theory)
3. Lift to M-theory
4. Low-energy limit

IIB Brane construction (Step 1)

Brane Setup

	0	1	2	3	4	5	6	7	8	9
N $D3$	•	•	•	-	-	-	•	-	-	-
2 $NS5$	•	•	•	•	•	•	-	-	-	-

IIB Picture



Low–Energy Field Theory

$\mathcal{N} = 4$, $3+1$ dim. $U(N_c) \times U(N_c)$ gauge theory + bifundamental fields, Vector Multiplet (A_μ^{0126}, 345789).

Dimensional reduction along 6 direction: Vector multiplet splits into

$\mathcal{N} = 4$, $2+1$ dim. Vector (A_μ^{012}, 345) and

$\mathcal{N} = 4$, $2+1$ dim. Hyper (A_6, 789)

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	0	1	2	3	4	5	6	7	8	9
N D3	•	•	•	-	-	-	•	-	-	-
2 *NS5*	•	•	•	•	•	•	-	-	-	-

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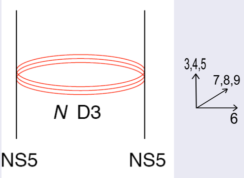

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Hyper removed by NS5 – Branes

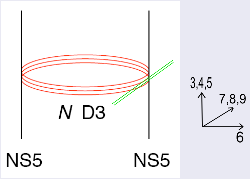
IIB Brane construction (Step 2)

Chern – Simons terms [Bergman, Hanany, Karch,Kol '99]

Brane Setup

	0	1	2	3	4	5	6	7	8	9
N $D3$	•	•	•	-	-	-	•	-	-	-
2 $NS5$	•	•	•	•	•	•	-	-	-	-
k $D5$	•	•	•	•	•	-	-	-	-	•

IIB Picture



Low – Energy Field Theory

- 3–5 strings ⇒ "Flavour" Fields
- Supersymmetry broken down to $\mathcal{N} = 2$.
- Give mass to "flavour" fields and integrate them out
- Via parity anomaly generate Chern–Simons terms.

IIB Brane construction (Step 2)

Maximally supersymmetric mass deformation: Bind k D5–Branes to NS5 forming $(1,k)$5–Brane and rotate in (37), (48), (59) plane.

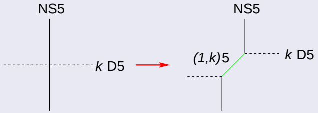
	0	1	2	3	4	5	6	7	8	9
N $D3$	•	•	•	-	-	-	•	-	-	-
1 $NS5$	•	•	•	•	•	•	-	-	-	-
$(1,k)5$	•	•	•	$[3,7]_\theta$	$[4,8]_\theta$	$[5,9]_\theta$	-	-	-	-

Low – Energy Field Theory

- Chern-Simons term generated
- $\mathcal{N}=3$ $U(N_c)_k \times U(N_c)_{-k}$ Yang-Mills theory with a Chern-Simons term
- 4 massless bifundamental matter fields (A_a, B_a)

IIB Picture

Web deformation



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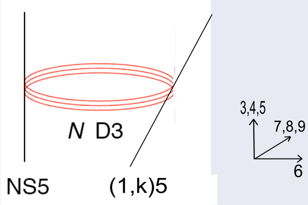

Uplift to M-theory (Step 3)

T–Dualize in x^6–direction

	0	1	2	3	4	5	6	7	8	9
N D2	•	•	•	-	-	-	-	-	-	-
KK	•	•	•	•	•	•	-	-	-	-
$KK + D6$ flux	•	•	•	$[3,7]_\theta$	$[4,8]_\theta$	$[5,9]_\theta$	-	-	-	-

Lift to M-theory

	0	1	2	3	4	5	6	7	8	9	10
N M2	•	•	•	-	-	-	-	-	-	-	-
X_8				•	•	•	•	•	•	•	•

where X_8 is the intersection of two KK monopoles.

Field Theory

Still $\mathcal{N} = 3$ SYM + CS + matter

Near–Horizon Limit (Step 4)

Enhancement to $\mathcal{N} = 6$ supersymmetry:

Gravity side

- X_8 has singularity; near singularity spacetime locally C^4/Z_k.
- take "near-horizon" limit

Field Theory side

- low–energy limit
- write effective theory at scales below $\sim g_{YM}^2 k$
  $\Rightarrow$ discard YM terms, only CS terms survive
- $\mathcal{N} = 6$ supersymmetric.

Chern-Simons Matter Theory

Field content

- Two $\mathcal{N}=2$ vector superfields V_i, one for each gauge group,
- Two $\mathcal{N}=2$ chiral superfields Φ_i in the adjoint representation,
- Four $\mathcal{N}=2$ chiral superfields, A_1, A_2, B_1 and B_2, where A_k in $(N_c, \overline{N_c})$ and B_k in $(\overline{N_c}, N_c)$ representation.

Action

$$S_{\text{ABJM}} = S_{\text{CS}} + S_{\text{bifund}} + S_{\text{pot}}$$

with

- $S_{\text{CS}} = kS_{\text{CS},1} - kS_{\text{CS},2}$,
- $S_{\text{bifund}} = \int d^3x d^4\theta \left[\bar{A}_a e^{-V_1} A_a e^{V_2} + \bar{B}_a e^{-V_2} B_a e^{V_1}\right]$,
- $S_{\text{pot}} = \int d^3x\, d^2\theta\, W + c.c.$,

and superpotential $W = -\frac{k}{8\pi}\text{Tr}\left(\Phi_1^2 - \Phi_2^2\right) + \text{Tr}\left(B_a\Phi_1 A_a\right) + \text{Tr}\left(A_a\Phi_2 B_a\right)$

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$$S_{\text{ABJM}} = S_{\text{CS}} + S_{\text{bifund}} + S_{\text{pot}}$$

with

- $S_{\text{CS}} = kS_{\text{CS},1} - kS_{\text{CS},2}, \qquad S_{\text{CS,k}} = -\frac{i}{4\pi}\int d^3x\, d^4\theta \int_0^1 dt\, \text{Tr}\, V_k \bar{D}^\alpha \left(e^{tV_k} D_\alpha e^{-tV_k}\right)$
- $S_{\text{bifund}} = \int d^3x d^4\theta \left[\bar{A}_a e^{-V_1} A_a e^{V_2} + \bar{B}_a e^{-V_2} B_a e^{V_1}\right],$
- $S_{\text{pot}} = \int d^3x\, d^2\theta\, W + c.c.,$

and superpotential $W = -\frac{k}{8\pi}\text{Tr}\left(\Phi_1^2 - \Phi_2^2\right) + \text{Tr}\left(B_a\Phi_1 A_a\right) + \text{Tr}\left(A_a\Phi_2 B_a\right)$

Chern-Simons Matter Theory

Field content

- Two $\mathcal{N}=2$ vector superfields V_i, one for each gauge group,
- Two $\mathcal{N}=2$ chiral superfields Φ_i in the adjoint representation,
- Four $\mathcal{N}=2$ chiral superfields, A_1, A_2, B_1 and B_2, where A_k in $(N_c, \overline{N_c})$ and B_k in $(\overline{N_c}, N_c)$ representation.

Action

$$S_{\text{ABJM}} = S_{\text{CS}} + S_{\text{bifund}} + S_{\text{pot}}$$

with

- $S_{\text{CS}} = kS_{\text{CS},1} - kS_{\text{CS},2}, \qquad S_{\text{CS,k}} = -\frac{i}{4\pi}\int d^3x\, d^4\theta \int_0^1 dt\, \text{Tr}\, V_k \bar{D}^\alpha \left(e^{tV_k} D_\alpha e^{-tV_k}\right)$
- $S_{\text{bifund}} = \int d^3x d^4\theta \left[\bar{A}_a e^{-V_1} A_a e^{V_2} + \bar{B}_a e^{-V_2} B_a e^{V_1}\right],$
- $S_{\text{pot}} = \int d^3x\, d^2\theta\, W + c.c.,$

and superpotential $W = \frac{2\pi}{k}\varepsilon^{ab}\,\varepsilon^{cd}\,\text{Tr}\,(A_a B_c A_b B_d)$ after integrating out Φ_1 and Φ_2

Add in type IIB flavour branes and follow the four steps:

Supersymmetric flavour branes in type IIB

Type IIB	Type IIA	M theory	codim	wrapping	SUSY	SUSY (anti)
D1	D2	M2	2	0(7)	2	2
D3	D2	M2	0	0126	6	0
D3	D4	M5	1	01(37)	3	3
D3	D4	M5	1	01(38)	2	2
D3	D2	M2	2	0(34)6	2	2
D3	D2	M2	2	06(78)	2	2
D5	D6	KK	0	012(347)	2	2
D5	D6	KK	0	012(349)	4	2
D5	D6	KK	0	012789	6	0
D5	D4	M5	1	013456	3	3
D5	D4	M5	1	01(378)6	2	2
D5	D4	M5	1	01(389)6	3	3
D5	D6	KK	2	0(34)789	2	2
D7	D6	KK	0	0126(3478)	2	4
D7	D6	KK	0	0126(3479)	2	2
D7	D8	M9	1	01345789	3	3

[Ammon, J.E., Meyer, O'Bannon, Wrase 2009]

Codimension zero Flavour, Step 1

Consider D5–Brane in 012789 direction.

[Hohenegger, Kirsch], [Hikida, Li, Takayanagi], [Gaiotto, Jafferis]

N_c D3–Branes and N_f D5–Branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	•	•	•	-	-	-	•	-	-	-
N_f D5	•	•	•	-	-	-	-	•	•	•

- 2+1 dimensional $\mathcal{N} = 4$ supersymmetry
- Action of Flavour degrees in $\mathcal{N} = 2$ superspace

$$S_{fl} = \int d^3x\, d^4\theta \left(\bar{Q}e^V Q + \tilde{Q}e^{-V}\bar{\tilde{Q}}\right) + \int d^3x\, d^2\theta \tilde{Q}\Phi Q\,,$$

where (V, Φ) is the $\mathcal{N} = 4$ Vector Multiplet

Codimension zero Flavour, Step 1

Add NS5–Branes

	0	1	2	3	4	5	6	7	8	9
N_c $D3$	•	•	•	-	-	-	•	-	-	-
N_f $D5$	•	•	•	-	-	-	-	•	•	•
1 $NS5$	•	•	•	•	•	•	-	-	-	-

- Dimensionally reduce on x_6, set hypermultiplet to zero

$\Rightarrow$ S_{fl} unchanged

Compactify x_6

Add N_f D5–Branes intersecting <u>each</u> stack of N_c $D3$–Branes

$\Rightarrow$ massless flavour in each gauge group.

$$S_{fl} = \int d^3x\, d^4\theta \left(\bar{Q}_k e^{V_k} Q_k + \tilde{Q}_k e^{-V_k} \bar{\tilde{Q}}_k\right) + \int d^3x\, d^2\theta \tilde{Q}_k (-1)^k \Phi_k Q_k \,,$$

Codimension zero Flavour, Step 2+3

$(1, k)$5–Brane

- Supersymmetry broken to $\mathcal{N} = 3$.
- Flavour action unchanged

T-duality along x_6 and Lift to M-theory

- type IIA configuration:

  N_f $D5 \rightarrow N_f$ $D6$–Branes.
- M-theory configuration:

  N_f $D6 \rightarrow KK$–Monopole associated with M–theory circle.
- action S_{fl} unchanged.

Codimension zero Flavour, Step 4

Gravity side

- zoom in on C^4/Z_k.
- For $N_c \gg k^5$: KK–Monopole wrapping a three cycle on S^7/Z_k.
- For $N_c \ll k^5$: $D6$–Brane wrapping $AdS_4 \times RP^3$.
- preserves 12 supercharges, i.e. $\mathcal{N} = 3$ in 2+1 dimensions, as well as $U(1)_b$ and $SU(2)_R \times SU(2)_D \simeq SO(4) \subset SO(6)_{\mathcal{R}}$

Field theory side

- Determine effective theory valid on scales $\ll g_{YM}^2 k$.
- In S_{fl}, write down all terms consistent with 2+1 dimensional $\mathcal{N} = 3$ supersymmetry and $SO(3)_R$.

  $\Rightarrow$ No such terms! $\Rightarrow$ S_{fl} unchanged. [Gaiotto, Yin '07]
- Integrate out Φ_1 and Φ_2.

Codimension zero Flavour, Action

Action

$$S = S_{\rm fl} + S_{\rm ABJM} = S_{\rm fl} + S_{\rm CS} + S_{\rm bifund} + S_{\rm pot}\,,$$

where

- $S_{\rm CS}$ and $S_{\rm bifund}$ unchanged, $S_{\rm pot} = \int d^3x\, d^2\theta\, W + c.c.$, with

$$W = \frac{2\pi}{k}\,\mathrm{Tr}\left[(A_aB_a + Q_1\tilde{Q}_1)^2 - (B_aA_a - Q_2\tilde{Q}_2)^2\right]\,.$$

- $S_{fl} = \int d^3x\, d^4\theta\, \left(\bar{Q}_k e^{V_k} Q_k + \tilde{Q}_k e^{-V_k} \bar{\tilde{Q}}_k\right)\,,$

Symmetries of the action

- preserves 12 supersymmetry charges, i.e. $\mathcal{N} = 3$ in 2+1 D
- $U(1)_b$ Symmetry as well as $SU(2)_D \times SU(2)_R = SO(4)_{\mathcal{R}}$
- Symmetries on gravity and field theory side match

Defects

Example: codimension one $\mathcal{N} = (0, 6)$ chiral flavour

D7 brane probe
Repeat the four steps given above

$\mathcal{N} = (0,6)$ codimension one flavour, Step 1

Consider D7–Brane in 01345789 direction. [Fujita, Li, Ryu, Takayanagi]

N_c D3–Branes and N_f D7–Branes

	0	1	2	3	4	5	6	7	8	9
N_c D3	•	•	•	-	-	-	•	-	-	-
N_f D7	•	•	-	•	•	•	-	•	•	•

8 supercharges, Flavour fields confined to 1+1 dimensional defect.

Flavour fields

- study spectrum of 3–7 strings $\rightarrow$ single 1+1 dim. Weyl fermion ψ.
- Fermions are *left–handed*, preserved supercharges *right–handed*, i.e. $\mathcal{N} = (0,8)$.
- $S_{def} = \int dx_+ dx_- \psi^\dagger (i\partial_- - A_-)\psi$. [Harvey, Royston '08]

Codimension one Flavour, Step 1

Add NS5–Branes

	0	1	2	3	4	5	6	7	8	9
N_c $D3$	•	•	•	-	-	-	•	-	-	-
N_f $D7$	•	•	-	•	•	•	-	•	•	•
1 $NS5$	•	•	•	•	•	•	-	-	-	-

- Dimensionally reduce on x_6, set $\mathcal{N} = 4$ Hypermultiplet to zero Supersymmetry broken down to $\mathcal{N} = (0, 4)$.

$\Rightarrow$ S_{def} unchanged

Compactify x_6

Add N_f D7–Branes intersecting each stack of N_c $D3$–Branes

$$S_{def} = \int dx_+ dx_- \psi^\dagger_{(k)} (i\partial_- - A_{(k)\,-})\psi_{(k)}.$$

Codimension one Flavour, Step 2+3

$(1,k)$5–Brane

- Bind k D5 and NS5 into $(1,k)$5–Brane

	0	1	2	3	4	5	6	7	8	9
N_c $D3$	•	•	•	-	-	-	•	-	-	-
N_f $D7$	•	•	-	•	•	•	-	•	•	•
1 $NS5$	•	•	•	•	•	•	-	-	-	-
$(1,k)5$	•	•	•	$[3,7]_\theta$	$[4,8]_\theta$	$[5,9]_\theta$	-	-	-	-

- Supersymmetry broken to $\mathcal{N}=(0,3)$.
- Flavour action S_{def} unchanged

T-duality along x_6 and Lift to M-theory

- type IIA configuration: N_f $D7 \rightarrow N_f$ $D8$–Branes.
- M-theory configuration: N_f $D8 \rightarrow$ "M9"–Branes.
- action S_{def} unchanged.

Codimension one Flavour, Step 4

Gravity side

- zoom in on C^4/Z_k.
- For $N_c \gg k^5$: "M9"–Branes wrapping $AdS_3 \times S^7/Z_k$.
- For $N_c \ll k^5$: D8–Branes wrapping $AdS_3 \times CP^3$.
- preserves 6 real supercharges, as well as $U(1)_b$ and $SU(4)_{\mathcal{R}}$.

Field theory side

- Determine effective theory valid on scales $\ll g_{YM}^2 k$.
- For S_{def}, write down all terms consistent with 1+1 dimensional $\mathcal{N} = (0, 3)$ supersymmetry, $SO(3)_{\mathcal{R}}$, 1+1 D Lorentz- and gauge invarance $\Rightarrow$ No such terms! $\Rightarrow$ S_{def} unchanged.
- Integrate out Φ_1 and Φ_2 (trivial) $\Rightarrow$ action $S = S_{\text{ABJM}} + S_{def}$.

Codimension one Flavour, Step 4

Gravity side

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- Integrate out Φ_1 and Φ_2 (trivial) $\Rightarrow$ action $S = S_{\text{ABJM}} + S_{def}$.
- $\mathcal{N} = (0,6)$ susy, $SU(4)_{\mathcal{R}} \times U(1)_b$

Codimension one Flavour, Step 4

Gravity side

- zoom in on C^4/Z_k.
- For $N_c \gg k^5$: "M9"–Branes wrapping $AdS_3 \times S^7/Z_k$.
- For $N_c \ll k^5$: D8–Branes wrapping $AdS_3 \times CP^3$.
- preserves 6 real supercharges, as well as $U(1)_b$ and $SU(4)_{\mathcal{R}}$.

Field theory side

- Determine effective theory valid on scales $\ll g^2_{YM} k$.
- For S_{def}, write down all terms consistent with 1+1 dimensional $\mathcal{N} = (0,3)$ supersymmetry, $SO(3)_{\mathcal{R}}$, 1+1 D Lorentz- and gauge invarance $\Rightarrow$ No such terms! $\Rightarrow$ S_{def} unchanged.
- Integrate out Φ_1 and Φ_2 (trivial) $\Rightarrow$ action $S = S_{\text{ABJM}} + S_{def}$.
- $\mathcal{N} = (0,6)$ susy, $SU(4)_{\mathcal{R}} \times U(1)_b \Rightarrow$ Symmetries match!

Generalizations

Further examples arXiv: 0909.3845.

- D3-brane in type IIB (D4-brane in type IIA, M5-brane in M-theory) $\rightarrow$ codimension one, non-chiral, $\mathcal{N} = (3, 3)$ flavour fields.
- D3-brane in type IIB (D2-brane in type IIA, M2-brane in M-theory) $\rightarrow$ codimension two, $\mathcal{N} = 4$ flavour fields.

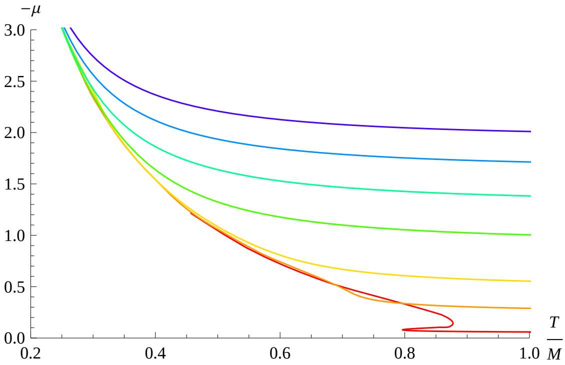
Applications of chiral codimension one theory

Chiral fermions on 1+1-dimensional defect coupled to Chern-Simons theory

$\Rightarrow$ compare to Quantum Hall theory with edge states

$\Rightarrow$ Calculate Phase diagram at finite temperature and density

D6 brane embedding at finite temperature and density



Lines of constant density

Conclusion

Summary

General *recipe* for adding flavour to AdS_4/CFT_3, in particular

- codimension zero $\mathcal{N} = 3$ flavour

Defect theories:

- codimension one $\mathcal{N} = (0, 6)$ chiral flavour
- codimension one $\mathcal{N} = (3, 3)$ non-chiral flavour
- codimension two $\mathcal{N} = 4$ flavour

Conclusion

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- codimension one $\mathcal{N} = (3, 3)$ non-chiral flavour
- codimension two $\mathcal{N} = 4$ flavour

Future Directions

- More examples, complete classification!
- Introduce mass, compute meson spectra.
- Study thermodynamics and hydrodynamics.

Conclusion

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General *recipe* for adding flavour to AdS_4/CFT_3, in particular

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