Defects in AdS$_4$/CFT$_3$

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Overview

Probe branes in AdS/CFT: Added flavour degrees of freedom

**In $AdS_5 \times S^5$:**
- D7 probe: Codimension zero defect
  Application example: Holographic Superconductor
  [Ammon, J.E., Kaminski, Kerner 0810.2316, 0903.1864]
- D5 (D3) probe: Codimension 1 (2) defect

**In the ABJM geometry $AdS_4 \times CP^3$:**
- Four embeddings constructed, including dual field theories
  [Ammon, J. E., Meyer, O’Bannon, Wrase 0909.3845]
- Applications: Aspects of Quantum Hall effect
  [ work in progress ]
Outline

1 Motivation

2 AdS$_5$/CFT$_4$
   - Adding flavour
   - Holographic Superconductors from D7 brane probes
   - Defects

3 The ABJM model: AdS$_4$/CFT$_3$
   - Review
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Embedding probe branes in AdS$_5 \times S^5$

- generates gravity duals of defect CFT’s
- adds flavour degrees of freedom

$\Rightarrow$ many applications

AdS$_4$/CFT$_3$

- Recently established from M2 branes [Aharony, Bergman, Jafferis, Maldacena 2008]
- Chern-Simons theory with gauge group $SU(N)_k \times SU(N)_{-k}$

Goal:

Determine general recipe for adding supersymmetric flavour and defects to AdS$_4$/CFT$_3$ in field theory and gravity description
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$N \to \infty$ (standard Maldacena limit), $N_f$ small (probe approximation)

Duality acts twice:

**Field theory**
\[ \mathcal{N} = 4 \text{ SU}(N) \text{ Super Yang-Mills theory} \]
\[ \text{coupled to} \]
\[ \mathcal{N} = 2 \text{ fundamental hypermultiplet} \]

**Gravity**
\[ \text{IIB supergravity on } AdS_5 \times S^5 \]
+ 
Probe brane action (DBI) on $AdS_5 \times S^3$
Phenomenological models for studying quarks and mesons in QCD-like theories

- Fluctuations of brane probes $\Rightarrow$ Mesons
- Brane embeddings in confining 10d backgrounds $\Rightarrow$ Chiral symmetry breaking
- Brane probes in AdS black hole geometry $\Rightarrow$ Quarks added to finite temperature field theory
- Finite density $\Rightarrow$ Phase diagram
- Hydrodynamics

Condensed matter:

- Superfluids/Superconductors, Quantum Hall Effect
- Quarks $\Rightarrow$ ‘electrons’
Holographic superconductor from D7 brane probes

Holographic superconductor with (3+1)-dimensional field theory in for which

- the dual field theory is explicitly known
- there is a ten-dimensional string theory picture of condensation

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Embed two coincident D7-branes into AdS-Schwarzschild gauge fields $A_\mu = A^a_\mu \sigma^a \in u(2) = u(1)_B \oplus su(2)_I$.

Finite isospin density: $A^3_0 \neq 0 \Rightarrow$ Explicit breaking to $u(1)_3$

Dynamics of Flavour degrees is described by non-abelian DBI action

Field theory described:

$\mathcal{N} = 4$ Super Yang-Mills plus two flavors of fundamental matter at finite temperature and finite isospin density
Holographic superconductor at finite isospin density

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There is a new solution to the equations of motion with non-zero vev for $A^1 \sigma^1$ in addition to the non-zero $A^3 \sigma^3$

$$A^3_0 = \mu - \frac{\tilde{d}^3_0}{2\pi \alpha'} \frac{\rho_H}{\rho^2} + \ldots, \quad A^1_3 = -\frac{\tilde{d}^3_1}{2\pi \alpha'} \frac{\rho_H}{\rho^2} + \ldots.$$  

This new solution has lower free energy

Order parameter $\tilde{d}^1_3 \propto \langle \bar{\psi}_u \gamma_3 \psi_d + \bar{\psi}_d \gamma_3 \psi_u + \text{bosons} \rangle \neq 0$
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Free energy (Grand potential) vs. temperature

\[ \mathcal{W}_7 \]

\[ \frac{T}{T_c} \]
The new ground state has properties known from superconductors:

- infinite DC conductivity, gap in the AC conductivity
- second order phase transition, critical exponent of 1/2 (mean field)
- a remnant of the Meissner–Ochsenfeld effect
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Frequency-dependent conductivity \( \sigma(\omega) = \frac{i}{\omega} G^R(\omega) \)

\( G^R \) retarded Green function for fluctuation \( a_2^3 \)

\[ \omega = \frac{\omega}{2\pi T} \]

\( T/T_c \): Black: \( \infty \), Red: 1, Orange: 0.5, Brown: 0.28.

Interpretation: Frictionless motion of mesons through plasma
Fermions

Use fermionic part of D7 DBI action to study fermionic fluctuations

[Ammon, J.E., Kaminski, O'Bannon 1003.1134]
D3/D7:
Codimension zero defect theory

Even before:
- D3/D5 (codimension 1) [Karch, Randall 2001; Freedman, Ooguri, DeWolfe 2001]
- D3/D3 (codimension 2) [Constable, J.E., Guralnik, Kirsch 2002]
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Supersymmetric embeddings

[Skenderis, Taylor 2002]

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<th>Embedding</th>
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<td>D7</td>
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Action in $\mathcal{N} = 2, d = 3$ superspace

$$
S_{\text{bulk}} = \frac{1}{g^2} \int dz d^3 x d^2 \theta d^2 \bar{\theta} \left( \Sigma^2 - \frac{1}{2} (\sqrt{2} \partial_z V + \Phi + \bar{\Phi})^2 + \bar{Q}_i Q_i \right)
+ \int dz d^3 x d^2 \theta \epsilon_{ij} Q_i \partial_z Q_j + \int dz d^3 x d^2 \bar{\theta} \epsilon_{ij} \bar{Q}_i \partial_z \bar{Q}_j,
$$

$$
S_{\text{bdy}}^{3d} = \int d^3 x d^2 \theta d^2 \bar{\theta} \left( \bar{B}^+ e^{gV} B^+ + \bar{B}^- e^{-gV} B^- \right)
+ \frac{ig}{\sqrt{2}} \left[ \int d^3 x d^2 \theta B^+ Q_2 B^- + c.c. \right]
$$
Non-renormalization

Write field theory action in $\mathcal{N} = 2, d = 3$ superspace

No contributions possible to divergence of supercurrent

$$\bar{D}^\alpha J_{\alpha\dot{\alpha}} = D_\alpha S , \quad S = 0 \text{ here!}$$

($V\Sigma$ is vector multiplet)

$SO(3,2)$ conformal symmetry preserved even in quantized theory
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Applications

Defect systems described are useful for

- uncovering universal behaviour of systems at quantum critical points

- due to conformal invariance and strong coupling

Recent developments

- Conductivities, specific heat, speed of sound [Myers, Wapler; Karch, Parnachev, .. ]
- BKT-Transitions [Karch, Son et al; Evans et al]
- Fractional Quantum Hall Effect [Keski-Vakkuri, Kraus et al ’09] [Fujita, Li, Ryu, Takayanagi ’09]
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Low–Energy descriptions of M2–Branes

\[ \text{(Aharony, Bergman, Jafferis, Maldacena, '08)} \]

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<th>( N_c )</th>
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\( N_c \) M2–Branes on \( C^4/Z_k \):

- two different low–energy descriptions for \( N_c \to \infty \) and \( N_c \gg k \):
  - **Gravity side**
    - For \( N_c \gg k^5 \): 11D Supergravity on asymptotically \( AdS_4 \times S^7/Z_k \)
    - For \( N_c \ll k^5 \): 10D IIA Supergravity on asymptotically \( AdS_4 \times CP^3 \)
  - **Gauge theory side**
    - \( U(N_c)_k \times U(N_c)_{-k} \) Chern-Simons Matter Theory (CSM)
    - \( \mathcal{N} = 6 \) supersymmetric for general \( k \)
    - conformal, invariant under parity, \( SU(4)_\mathcal{R} \simeq SO(6)_\mathcal{R} \)
Deriving AdS$_4$ / CFT$_3$ from type IIB setup

Four steps:

1. Write D3-brane theory, add 2 NS5-branes
2. Add $k$ D5-branes, form (1, $k$) 5-brane bound state ($\Rightarrow$ Chern-Simons theory)
3. Lift to M-theory
4. Low-energy limit
IIB Brane construction (Step 1)

Brane Setup

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IIB Picture

Low–Energy Field Theory

\( \mathcal{N} = 4, \ 3 + 1 \) dim. \( U(N_c) \times U(N_c) \) gauge theory + bifundamental fields, Vector Multiplet \((A^{0126}_{\mu}, 345789)\).

Dimensional reduction along 6 direction: Vector multiplet splits into

\( \mathcal{N} = 4, \ 2 + 1 \) dim. Vector \((A^{012}_{\mu}, 345)\) and

\( \mathcal{N} = 4, \ 2 + 1 \) dim. Hyper \((A_6, 789)\)
**IIB Brane construction (Step 1)**

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Dimensional reduction along 6 direction: Vector multiplet splits into $\mathcal{N} = 4$, $2 + 1$ dim. Vector ($A_{\mu}^{012, 345}$)

Hyper removed by NS5 – Branes
IIB Brane construction (Step 2)

Chern – Simons terms

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IIB Picture

Low – Energy Field Theory

- 3–5 strings $\Rightarrow$ "Flavour" Fields
- Supersymmetry broken down to $\mathcal{N} = 2$.
- Give mass to "flavour" fields and integrate them out.
- Via parity anomaly generate Chern–Simons terms.
IIB Brane construction (Step 2)

Maximally supersymmetric mass deformation: Bind $k$ D5–Branes to NS5 forming $(1, k)5$–Brane and rotate in $(37), (48), (59)$ plane.

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Low – Energy Field Theory

- Chern-Simons term generated
- $\mathcal{N} = 3 \ U(N_c)_k \times U(N_c)_{-k}$ Yang-Mills theory with a Chern-Simons term
- 4 massless bifundamental matter fields $(A_a, B_a)$
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</table>

**Low – Energy Field Theory**

- Chern-Simons term generated
- \( \mathcal{N} = 3 \ U(N_c)_k \times U(N_c)_{-k} \) Yang-Mills theory with a Chern-Simons term
- 4 massless bifundamental matter fields \((A_a, B_a)\)

**IIB Picture**
Uplift to M-theory (Step 3)

**T–Dualize in $x^6$–direction**

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<tr>
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<td>●</td>
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<td>●</td>
<td>[3, 7]$_\theta$</td>
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**Lift to M-theory**

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<td>$X_8$</td>
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<td>●</td>
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<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
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where $X_8$ is the intersection of two KK monopoles.

**Field Theory**

Still $\mathcal{N} = 3$ SYM + CS + matter
Near–Horizon Limit (Step 4)

Enhancement to $\mathcal{N} = 6$ supersymmetry:

Gravity side

- $X_8$ has singularity; near singularity spacetime locally $C^4/Z_k$.
- take "near-horizon" limit

Field Theory side

- low–energy limit
- write effective theory at scales below $\sim g_{YM}^2 k$
  $\Rightarrow$ discard YM terms, only CS terms survive
- $\mathcal{N} = 6$ supersymmetric.
Chern-Simons Matter Theory

Field content

- Two $\mathcal{N} = 2$ vector superfields $V_i$, one for each gauge group,
- Two $\mathcal{N} = 2$ chiral superfields $\Phi_i$ in the adjoint representation,
- Four $\mathcal{N} = 2$ chiral superfields, $A_1$, $A_2$, $B_1$ and $B_2$, where $A_k$ in $(N_c, N_c)$ and $B_k$ in $(\overline{N_c}, N_c)$ representation.

Action

$$S_{\text{ABJM}} = S_{\text{CS}} + S_{\text{bifund}} + S_{\text{pot}}$$

with

- $S_{\text{CS}} = kS_{\text{CS},1} - kS_{\text{CS},2}$,
- $S_{\text{bifund}} = \int d^3x d^4\theta \left[ \bar{A}_a e^{-V_1} A_a e^{V_2} + \bar{B}_a e^{-V_2} B_a e^{V_1} \right]$,
- $S_{\text{pot}} = \int d^3x d^2\theta W + c.c.$,

and superpotential $W = -\frac{k}{8\pi} \text{Tr} (\Phi_1^2 - \Phi_2^2) + \text{Tr} (B_a \Phi_1 A_a) + \text{Tr} (A_a \Phi_2 B_a)$
Field content

- Two $\mathcal{N} = 2$ vector superfields $V_i$, one for each gauge group,
- Two $\mathcal{N} = 2$ chiral superfields $\Phi_i$ in the adjoint representation,
- Four $\mathcal{N} = 2$ chiral superfields, $A_1$, $A_2$, $B_1$ and $B_2$, where $A_k$ in $(N_c, \overline{N_c})$ and $B_k$ in $(\overline{N_c}, N_c)$ representation.

Action

$$S_{ABJM} = S_{CS} + S_{bifund} + S_{pot}$$

with

- $S_{CS} = kS_{CS,1} - kS_{CS,2}$, $S_{CS,k} = -\frac{i}{4\pi} \int d^3 x d^4 \theta \int_0^1 dt \, \text{Tr} \, V_k \bar{D}^\alpha (e^{tV_k} D_\alpha e^{-tV_k})$
- $S_{bifund} = \int d^3 x d^4 \theta \left[ \bar{A}_a e^{-V_1} A_a e^{V_2} + \bar{B}_a e^{-V_2} B_a e^{V_1} \right]$,
- $S_{pot} = \int d^3 x d^2 \theta \, W + c.c.$,

and superpotential $W = -\frac{k}{8\pi} \text{Tr} \left( \Phi_1^2 - \Phi_2^2 \right) + \text{Tr} \left( B_a \Phi_1 A_a \right) + \text{Tr} \left( A_a \Phi_2 B_a \right)$.
Chern-Simons Matter Theory

**Field content**

- Two $\mathcal{N} = 2$ vector superfields $V_i$, one for each gauge group,
- Two $\mathcal{N} = 2$ chiral superfields $\Phi_i$ in the adjoint representation,
- Four $\mathcal{N} = 2$ chiral superfields, $A_1$, $A_2$, $B_1$ and $B_2$, where $A_k$ in $(N_c, \overline{N_c})$ and $B_k$ in $(\overline{N_c}, N_c)$ representation.

**Action**

$$S_{\text{ABJM}} = S_{\text{CS}} + S_{\text{bifund}} + S_{\text{pot}}$$

with

- $S_{\text{CS}} = kS_{\text{CS},1} - kS_{\text{CS},2}$, \quad $S_{\text{CS},k} = -\frac{i}{4\pi} \int d^3x \, d^4\theta \int_0^1 dt \, \text{Tr} \, V_k \bar{D}^\alpha (e^{tV_k} D_\alpha e^{-tV_k})$
- $S_{\text{bifund}} = \int d^3x d^4\theta \left[ \bar{A}_a e^{-V_1} A_a e^{V_2} + \bar{B}_a e^{-V_2} B_a e^{V_1} \right]$, 
- $S_{\text{pot}} = \int d^3x \, d^2\theta \, W + \text{c.c.}$,

and superpotential $W = \frac{2\pi}{k} \varepsilon^{ab} \varepsilon^{cd} \text{Tr} (A_a B_c A_b B_d)$ after integrating out $\Phi_1$ and $\Phi_2$. 
Add in type IIB flavour branes and follow the four steps:

### Supersymmetric flavour branes in type IIB

<table>
<thead>
<tr>
<th>Type IIB</th>
<th>Type IIA</th>
<th>M theory</th>
<th>codim</th>
<th>wrapping</th>
<th>SUSY</th>
<th>SUSY (anti)</th>
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<td>D2</td>
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<td>D6</td>
<td>KK</td>
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<td>012(347)</td>
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<tr>
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<td>D6</td>
<td>KK</td>
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</table>

[Ammon, J.E., Meyer, O'Bannon, Wrase 2009]
Codimension zero Flavour, Step 1

Consider D5–Brane in 012789 direction.

[Hohenegger, Kirsch], [Hikida, Li, Takayanagi], [Gaiotto, Jafferis]

\[ \begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline
  & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
N_c D3 & \bullet & \bullet & \bullet & - & - & - & \bullet & - & - & - \\
N_f D5 & \bullet & \bullet & \bullet & - & - & - & - & \bullet & \bullet & \bullet \\
\hline
\end{array} \]

- 2+1 dimensional \( \mathcal{N} = 4 \) supersymmetry
- Action of Flavour degrees in \( \mathcal{N} = 2 \) superspace

\[ S_{fl} = \int d^3x \, d^4\theta \left( \bar{Q}e^V Q + \bar{\tilde{Q}}e^{-V} \tilde{Q} \right) + \int d^3x \, d^2\theta \bar{Q}\Phi Q , \]

where \( (V, \Phi) \) is the \( \mathcal{N} = 4 \) Vector Multiplet
The ABJM model: AdS$^4$/CFT$^3$

Codimension zero Flavour, Step 1

Add NS5–Branes

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</tr>
<tr>
<td>1 NS5</td>
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</tr>
</tbody>
</table>

- Dimensionally reduce on $x_6$, set hypermultiplet to zero

$\Rightarrow S_{fl}$ unchanged

Compactify $x_6$

Add $N_f$ D5–Branes intersecting each stack of $N_c$ D3–Branes

$\Rightarrow$ massless flavour in each gauge group.

$$S_{fl} = \int d^3 x \, d^4 \theta \left( \bar{Q}_k e^{V_k} Q_k + \bar{Q}_k e^{-V_k} \bar{Q}_k \right) + \int d^3 x \, d^2 \theta \bar{Q}_k (-1)^k \Phi_k Q_k,$$
Codimension zero Flavour, Step 2+3

(1, k)5–Brane
- Supersymmetry broken to $\mathcal{N} = 3$.
- Flavour action unchanged

T-duality along $x_6$ and Lift to M-theory
- type IIA configuration:
  $N_f \, D5 \rightarrow N_f \, D6$–Branes.
- M-theory configuration:
  $N_f \, D6 \rightarrow KK$–Monopole associated with M–theory circle.
- action $S_{fl}$ unchanged.
Codimension zero Flavour, Step 4

Gravity side

- zoom in on $C^4/Z_k$.
- For $N_c \gg k^5$ : $KK$–Monopole wrapping a three cycle on $S^7/Z_k$.
- For $N_c \ll k^5$ : $D6$–Brane wrapping $AdS_4 \times RP^3$.
- preserves 12 supercharges, i.e. $\mathcal{N} = 3$ in 2+1 dimensions, as well as $U(1)_b$ and $SU(2)_R \times SU(2)_D \simeq SO(4) \subset SO(6)_R$.

Field theory side

- Determine effective theory valid on scales $\ll g^2_{YM} k$.
- In $S_{fl}$, write down all terms consistent with 2+1 dimensional $\mathcal{N} = 3$ supersymmetry and $SO(3)_R$.
  - $\Rightarrow$ No such terms! $\Rightarrow S_{fl}$ unchanged.
- Integrate out $\Phi_1$ and $\Phi_2$. [Gaiotto, Yin '07]
The ABJM model: AdS$_4$/CFT$_3$

Adding flavour

**Codimension zero Flavour, Action**

**Action**

\[ S = S_{fl} + S_{ABJM} = S_{fl} + S_{CS} + S_{bifund} + S_{pot}, \]

where

- $S_{CS}$ and $S_{bifund}$ unchanged,
- $S_{pot} = \int d^3 x \, d^2 \theta \, W + c.c.$, with
  \[ W = \frac{2\pi}{k} \text{Tr} \left[ (A_a B_a + Q_1 \tilde{Q}_1)^2 - (B_a A_a - Q_2 \tilde{Q}_2)^2 \right]. \]

- $S_{fl} = \int d^3 x \, d^4 \theta \left( \bar{Q}_k e^{V_k} Q_k + \tilde{Q}_k e^{-V_k} \tilde{Q}_k \right)$,

**Symmetries of the action**

- preserves 12 supersymmetry charges, i.e. $\mathcal{N} = 3$ in 2+1 D
- $U(1)_b$ Symmetry as well as $SU(2)_D \times SU(2)_R = SO(4)_R$
- Symmetries on gravity and field theory side match
Example: codimension one $\mathcal{N} = (0, 6)$ chiral flavour

D7 brane probe
Repeat the four steps given above
Consider D7–Brane in 01345789 direction. [Fujita, Li, Ryu, Takayanagi]

### $N_c$ D3–Branes and $N_f$ D7–Branes

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<tr>
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</table>

8 supercharges, Flavour fields confined to 1+1 dimensional defect.

### Flavour fields

- study spectrum of 3–7 strings $\rightarrow$ single 1+1 dim. Weyl fermion $\psi$.
- Fermions are *left–handed*, preserved supercharges *right–handed*, i.e. $\mathcal{N} = (0, 8)$.
- $S_{\text{def}} = \int dx_+ dx_- \psi^\dagger (i\partial_- - A_-) \psi$.

[Harvey, Royston ’08]
Codimension one Flavour, Step 1

Add NS5–Branes

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</table>

- Dimensionally reduce on $x_6$, set $\mathcal{N} = 4$ Hypermultiplet to zero
  Supersymmetry broken down to $\mathcal{N} = (0, 4)$.

$S_{\text{def}}$ unchanged

Compactify $x_6$

Add $N_f$ D7–Branes intersecting each stack of $N_c$ D3–Branes

$$S_{\text{def}} = \int dx_+ dx_- \psi^\dagger_k (i \partial_- - A_k - ) \psi_k.$$
(1, k)5–Brane

Bind $k$ D5 and NS5 into (1, k)5–Brane

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</table>

Supersymmetry broken to $\mathcal{N} = (0, 3)$.

Flavour action $S_{def}$ unchanged.

T-duality along $x_6$ and Lift to M-theory

- type IIA configuration: $N_f D7 \rightarrow N_f D8$–Branes.
- M-theory configuration: $N_f D8 \rightarrow "M9"$–Branes.
- action $S_{def}$ unchanged.
Codimension one Flavour, Step 4

Gravity side

- zoom in on $C^4/Z_k$.
- For $N_c \gg k^5$: "M9"–Branes wrapping $AdS_3 \times S^7/Z_k$.
- For $N_c \ll k^5$: D8–Branes wrapping $AdS_3 \times CP^3$.
- preserves 6 real supercharges, as well as $U(1)_b$ and $SU(4)_R$.

Field theory side

- Determine effective theory valid on scales $\ll g_{YM}^2 k$.
- For $S_{def}$, write down all terms consistent with 1+1 dimensional $\mathcal{N} = (0, 3)$ supersymmetry, $SO(3)_R$, 1+1 D Lorentz- and gauge invariance $\Rightarrow$ No such terms! $\Rightarrow S_{def}$ unchanged.
- Integrate out $\Phi_1$ and $\Phi_2$ (trivial) $\Rightarrow$ action $S = S_{ABJM} + S_{def}$.
Codimension one Flavour, Step 4

Gravity side

- zoom in on $C^4/Z_k$.
- For $N_c \gg k^5$: "M9"–Branes wrapping $AdS_3 \times S^7/Z_k$.
- For $N_c \ll k^5$: D8–Branes wrapping $AdS_3 \times CP^3$.
- preserves 6 real supercharges, as well as $U(1)_b$ and $SU(4)_{\mathcal{R}}$.

Field theory side

- Determine effective theory valid on scales $\ll g^2_{YM} k$.
- For $S_{\text{def}}$, write down all terms consistent with 1+1 dimensional $\mathcal{N} = (0,3)$ supersymmetry, $SO(3)_{\mathcal{R}}$, 1+1 D Lorentz- and gauge invariance $\Rightarrow$ No such terms! $\Rightarrow S_{\text{def}}$ unchanged.
- Integrate out $\Phi_1$ and $\Phi_2$ (trivial) $\Rightarrow$ action $S = S_{\text{ABJM}} + S_{\text{def}}$.
- $\mathcal{N} = (0,6)$ susy, $SU(4)_{\mathcal{R}} \times U(1)_b$
Codimension one Flavour, Step 4

**Gravity side**

- zoom in on $C^4/Z_k$.
- For $N_c \gg k^5$: "M9"–Branes wrapping $AdS_3 \times S^7/Z_k$.
- For $N_c \ll k^5$: D8–Branes wrapping $AdS_3 \times CP^3$.
- preserves 6 real supercharges, as well as $U(1)_b$ and $SU(4)_\mathcal{R}$.

**Field theory side**

- Determine effective theory valid on scales $\ll g^2_{YM} k$.
- For $S_{def}$, write down all terms consistent with 1+1 dimensional $\mathcal{N} = (0, 3)$ supersymmetry, $SO(3)_\mathcal{R}$, 1+1 D Lorentz- and gauge invariance $\Rightarrow$ No such terms! $\Rightarrow S_{def}$ unchanged.
- Integrate out $\Phi_1$ and $\Phi_2$ (trivial) $\Rightarrow$ action $S = S_{ABJM} + S_{def}$.
- $\mathcal{N} = (0, 6)$ susy, $SU(4)_\mathcal{R} \times U(1)_b \Rightarrow$ Symmetries match!
Generalizations

Further examples

- D3-brane in type IIB (D4-brane in type IIA, M5-brane in M-theory) → codimension one, non-chiral, $\mathcal{N} = (3, 3)$ flavour fields.
- D3-brane in type IIB (D2-brane in type IIA, M2-brane in M-theory) → codimension two, $\mathcal{N} = 4$ flavour fields.

Applications of chiral codimension one theory

Chiral fermions on 1+1-dimensional defect coupled to Chern-Simons theory

⇒ compare to Quantum Hall theory with edge states
⇒ Calculate Phase diagram at finite temperature and density
D6 brane embedding at finite temperature and density

Lines of constant density
Conclusion

Summary

General *recipe* for adding flavour to $AdS_4/CFT_3$, in particular

- codimension zero $\mathcal{N} = 3$ flavour

Defect theories:

- codimension one $\mathcal{N} = (0, 6)$ chiral flavour
- codimension one $\mathcal{N} = (3, 3)$ non-chiral flavour
- codimension two $\mathcal{N} = 4$ flavour
**Conclusion**

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**Future Directions**

- More examples, complete classification!
- Introduce mass, compute meson spectra.
- Study thermodynamics and hydrodynamics.
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