



One loop calculations with SAMURAI

Francesco Tramontano
CERN Theory Group

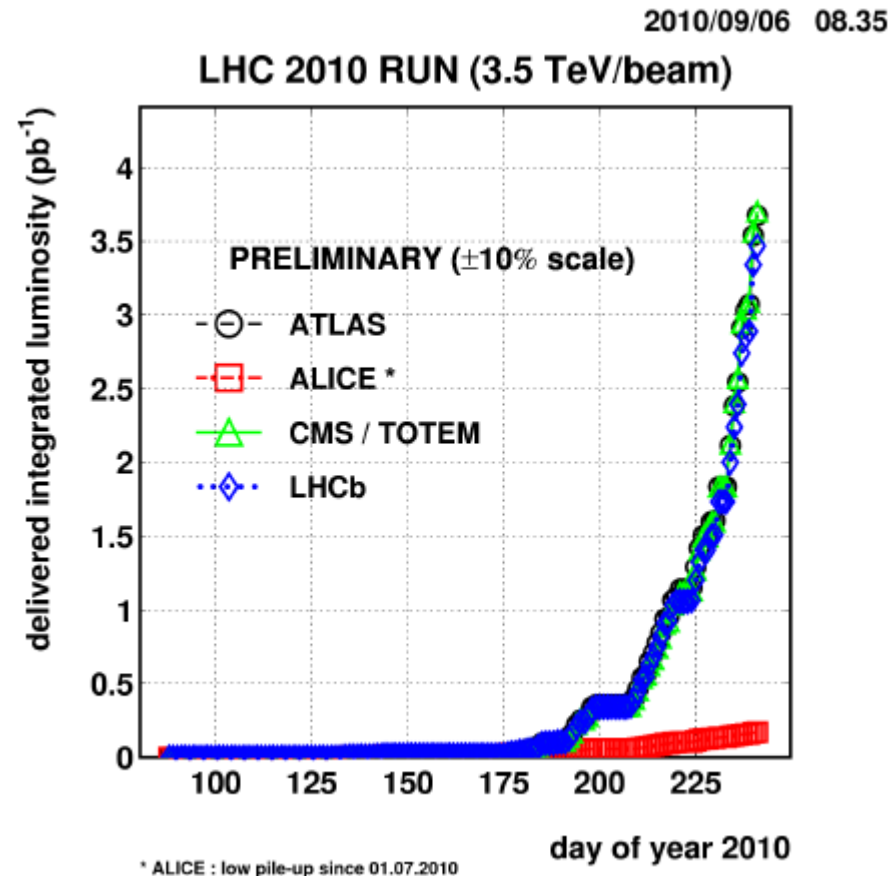
work done in collaboration with
P. Mastrolia, G. Ossola and T. Reiter

OVERVIEW

- Introduction
- Methods
- Running SAMURAI
- Examples
- Conclusions and outlook

Introduction

- ❑ LHC successfully started collisions at 7 TeV on March 30th 2010
- ❑ The need of Next to Leading Order multi-particle scattering predictions is more pressing
- ❑ New ideas in the field of loop corrections give the possibility to perform the automatic generation of NLO predictions for multi-leg processes



Status of the art

Numerical calculations:

- ❑ EW corr. $e^+e^- \rightarrow 4$ fermions
Denner and Dittmaier (2005)
- ❑ $pp \rightarrow W + 3\text{jets}$
Ellis et al, Berger et al (2009)

- ❑ $pp \rightarrow Z + 3\text{jets}$
Berger et al (2009)

- ❑ $pp \rightarrow t\bar{t}b\bar{b}$
Bredenstein et al, Bevilacqua
et al (2009)

- ❑ $pp \rightarrow t\bar{t} + 2\text{jets}$ Czakon et al (2010)

- ❑ $pp \rightarrow 4b$ Binoth et al (2010)

Analytic calculations:

- ❑ $W/Z/\gamma + 2\text{jets}$ Bern et al (1998)

- ❑ $H + 2\text{jets}$ (eff. coupling)
Badger, Berger, Campbell, Del Duca,
Dixon, Ellis, Glover, Mastrolia,
Risager, Sofianatos, Williams
(2006–2009)

We combined some of the recent techniques into a new computer program we
called **SAMURAI**

Basic features of SAMURAI:

Scattering **A**mplitudes from **U**nitariness based
Reduction **A**lgorithm at the **I**ntegrand-level

- ❑ Is a fortran90 library for the calculation of the one loop corrections downloadable at the URL: www.cern.ch/samurai
- ❑ Main purpose was to provide a flexible and easy to use tool for the evaluation of the virtual corrections
- ❑ It works with any number/kind of legs
- ❑ Can process integrands written either as numerator of Feynman diagrams or as product of tree level amplitudes
- ❑ Rational terms are produced/processed together with the cut-constructible one

And further:

- ❑ **SAMURAI** can be compiled in **2x** or **4x** precision, a version working in multiple precision is available on request
- ❑ It has a **modular** structure that allows for quick local updates
- ❑ It could also be useful to perform fast **numerical check** of analytic results
- ❑ Details and examples of applications can be found in arXiv:**1006.0710**

Methods

SAMURAI: a numerical implementation of the **OPP/D-dimensional** generalized unitarity cuts technique

$$\mathcal{A}_n = \int d^d \bar{q} A(\bar{q}, \epsilon) ,$$

$$A(\bar{q}, \epsilon) = \frac{\mathcal{N}(\bar{q}, \epsilon)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{n-1}} , \quad \mathcal{N}(\bar{q}, \epsilon) = N_0(\bar{q}) + \epsilon N_1(\bar{q}) + \epsilon^2 N_2(\bar{q}).$$

$$\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2 = (q + p_i)^2 - m_i^2 - \mu^2 ,$$

$$\bar{q}^\dagger = q^\dagger + \mu \quad \bar{q}^2 = q^2 - \mu^2$$

- ❑ **OPP polynomials** (n-ple cut, n=1,2,3,4) **extended** to the framework of D-dim unitarity [Ellis, Giele, Kunszt, Melnikov]
- ❑ **5-ple cut** residue depending only on **mu²** [Melnikov, Schultze]
- ❑ Integrand **sampling** with **DFT** for 3-ple and 2-ple cuts [Mastrolia, Ossola, Papadopoulos, Pittau]

OPP integrand decomposition: 4-dim

- Any amplitude can be expressed as a linear combination of scalar integrals: boxes, triangles, bubbles, tadpoles plus rational terms

$$\int A = \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} d(i_0 i_1 i_2 i_3) D_0(i_0 i_1 i_2 i_3) + \sum_{i_0 < i_1 < i_2}^{m-1} c(i_0 i_1 i_2) C_0(i_0 i_1 i_2) \\ + \sum_{i_0 < i_1}^{m-1} b(i_0 i_1) B_0(i_0 i_1) + \sum_{i_0}^{m-1} a(i_0) A_0(i_0) + \text{rational terms}$$

- At integrand level the structure is enriched by polynomial terms that integrate to zero

$$N(q) = \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} [d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0 < i_1 < i_2}^{m-1} [c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\ + \sum_{i_0 < i_1}^{m-1} [b(i_0 i_1) + \tilde{b}(q; i_0 i_1)] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} [a(i_0) + \tilde{a}(q; i_0)] \prod_{i \neq i_0}^{m-1} D_i$$

- The power of the OPP method is the fact that for each phase space point the only requirement for the reduction is the knowledge of the numerical value of the numerator function N for a finite set of values of the loop momentum variable, solutions of the multiple cut conditions

Extension to D-dim

- Once fixed a parametrization for the loop momentum in terms of a linear combination of known four-vectors (p_0, e_i) the vanishing term are polynomials of x_i and μ^2

$$\bar{q} = q + \mu \quad \bar{q}^2 = q^2 - \mu^2 \quad q = -p_0 + x_1 e_1 + x_2 e_2 + x_3 e_3 + x_4 e_4$$

- The problem is to fit the coefficients of the Δ -polynomials

$$\begin{aligned} N(\bar{q}) = & \sum_{i \ll m}^{n-1} \Delta_{ijklm}(\bar{q}) \prod_{h \neq i,j,k,\ell,m}^{n-1} \bar{D}_h + \sum_{i \ll \ell}^{n-1} \Delta_{ijkl}(\bar{q}) \prod_{h \neq i,j,k,\ell}^{n-1} \bar{D}_h + \\ & + \sum_{i \ll k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h \neq i,j,k}^{n-1} \bar{D}_h + \sum_{i < j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h \neq ij}^{n-1} \bar{D}_h + \sum_i^{n-1} \Delta_i(\bar{q}) \prod_{h \neq i}^{n-1} \bar{D}_h \end{aligned}$$

For example the 3-ple cut residue (function of the unfrozen components) reads:

$$\begin{aligned} \Delta_{ijk}(\bar{q}) = & c_{3,0}^{(ijk)} + c_{3,7}^{(ijk)} \mu^2 - \left((c_{3,1}^{(ijk)} + c_{3,8}^{(ijk)} \mu^2) x_4 + (c_{3,4}^{(ijk)} + c_{3,9}^{(ijk)} \mu^2) x_3 \right) (e_1 \cdot e_2) + \\ & + \left(c_{3,2}^{(ijk)} x_4^2 + c_{3,5}^{(ijk)} x_3^2 \right) (e_1 \cdot e_2)^2 - \left(c_{3,3}^{(ijk)} x_4^3 + c_{3,6}^{(ijk)} x_3^3 \right) (e_1 \cdot e_2)^3 . \end{aligned}$$

5-ple cut residue

$$\Delta_{ijklm}(\bar{q}) = c_{5,0}^{(ijklm)} + c_{5,1}^{(ijklm)} \mu^2 + c_{5,2}^{(ijklm)} \mu^4$$

$$\int \frac{d^4 p}{(2\pi)^4} \int \frac{d^{-2\epsilon} \mu}{(2\pi)^{-2\epsilon}} (\mu^2)^r f(p^\alpha, \mu^2) = -\epsilon(1-\epsilon)(2-\epsilon)\cdots(r-1-\epsilon) (4\pi)^r \int \frac{d^{4+2r-2\epsilon} P}{(2\pi)^{4+2r-2\epsilon}} f(p^\alpha, \mu^2)$$

$$I_n^{D=6-2\epsilon} = \frac{1}{(n-5+2\epsilon)c_0} \left[2I_n^{D=4-2\epsilon} - \sum_{i=1}^n c_i I_{n-1}^{(i), D=4-2\epsilon} \right]$$

$$I_n^{D=8-2\epsilon} = \frac{1}{(n-7+2\epsilon)c_0} \left[2I_n^{D=6-2\epsilon} - \sum_{i=1}^n c_i I_{n-1}^{(i), D=6-2\epsilon} \right]$$

Linear dependence

Best choice: $\Delta_{ijklm}(\bar{q}) = +c_{5,1}^{(ijklm)} \mu^2$

- ✓ avoid scalar pentagon decomposition
- ✓ avoid pentagon subtraction for tadpoles
- ✓ numerically more stable

Numerical Sampling

$$P(x) = \sum_{\ell=0}^n c_{\ell} x^{\ell}$$

1. generate the set of discrete values P_k ($k = 0, \dots, n$),

$$P_k = P(x_k) = \sum_{\ell=0}^n c_{\ell} \rho^{\ell} e^{-2\pi i \frac{k}{n+1} \ell}$$

by sampling $P(x)$ at the points

$$x_k = \rho e^{-2\pi i \frac{k}{n+1}}$$

2. using the orthogonality relation

$$\sum_{n=0}^{N-1} e^{2\pi i \frac{k}{N} n} e^{-2\pi i \frac{k'}{N} n} = N \delta_{kk'}$$

each coefficient c_{ℓ} finally reads,

$$c_{\ell} = \frac{\rho^{-\ell}}{n+1} \sum_{k=0}^n P_k e^{2\pi i \frac{k}{n+1} \ell}$$

- ❑ Cut-5: completely frozen
- ❑ Cut-4: mu2 sampling
- ❑ Cut-3,2: mu2 sampling + DFT:
- ❑ Cut-1: trivial

- straightforward extension to multi-variate DFT projection
- Sampling on different circles for stable solutions
- number of the integrand samplings = number of the unknowns
- dynamical mu2-sampling

Amplitudes & Master Integrals

$$\begin{aligned}
 \mathcal{A}_n = & \sum_{i < j < k < \ell}^{n-1} \left\{ c_{4,0}^{(ijkl)} I_{ijkl}^{(d)} + \frac{(d-2)(d-4)}{4} c_{4,4}^{(ijkl)} I_{ijkl}^{(d+4)} \right\} & \int d^d \bar{q} \frac{\bar{q} \cdot e_2}{\bar{D}_i \bar{D}_j} = J_{ij}^{(d)} \\
 & + \sum_{i < j < k}^{n-1} \left\{ c_{3,0}^{(ijk)} I_{ijk}^{(d)} - \frac{(d-4)}{2} c_{3,7}^{(ijk)} I_{ijk}^{(d+2)} \right\} & \int d^d \bar{q} \frac{(\bar{q} \cdot e_2)^2}{\bar{D}_i \bar{D}_j} = K_{ij}^{(d)} \\
 & + \sum_{i < j}^{n-1} \left\{ c_{2,0}^{(ij)} I_{ij}^{(d)} + c_{2,1}^{(ij)} J_{ij}^{(d)} + c_{2,2}^{(ij)} K_{ij}^{(d)} - \frac{(d-4)}{2} c_{2,9}^{(ij)} I_{ij}^{(d+2)} \right\} \\
 & + \sum_i^{n-1} c_{1,0}^{(i)} I_i^{(d)}
 \end{aligned}$$

The sources of rational terms are the integrals with μ^2 powers in the numerator

They are generated by the reduction algorithm, but could also be present ab initio in the numerator function as a consequence of the algebraic manipulations

$$\begin{aligned}
 \int d^d \bar{q} \frac{\mu^2}{\bar{D}_i \bar{D}_j} &= -\frac{(d-4)}{2} I_{ij}^{(d+2)} \\
 \int d^d \bar{q} \frac{\mu^4}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell} &= \frac{(d-2)(d-4)}{4} I_{ijkl}^{(d+4)} \\
 \int d^d \bar{q} \frac{\mu^2}{\bar{D}_i \bar{D}_j \bar{D}_k} &= -\frac{(d-4)}{2} I_{ijk}^{(d+2)}
 \end{aligned}$$

Running
SAMURAI

calls:

```
call initsamurai(imeth,isca,verbosity,itest)
call InitDenominators(nleg,Pi,msq,v0,m0,v1,m1,...,vlast,mlast)
call samurai(xnum,tot,totr,Pi,msq,nleg,rank,istop,scale2,ok)
call exitsamurai
```

A dedicated module (kinematic) is also available in the release that contains useful functions to evaluate:

- ✓ Polarization vectors for massless vectors
- ✓ Scalar and spinor products with both real and complex four vectors as arguments

call `initsamurai(imeth,isca,verbosity,itest)`

- ✓ `imeth = 'diag'` for an integrand given as numerator of a Feynman diagram
 - `'tree'` for an integrand given as the product of tree level amplitudes

- ✓ `isca = 1`, scalar integrals evaluated with the QCDLoop package (Ellis and Zanderighi)
 - 2, scalar integrals evaluated with the AVH-OL0 package (van Hameren)

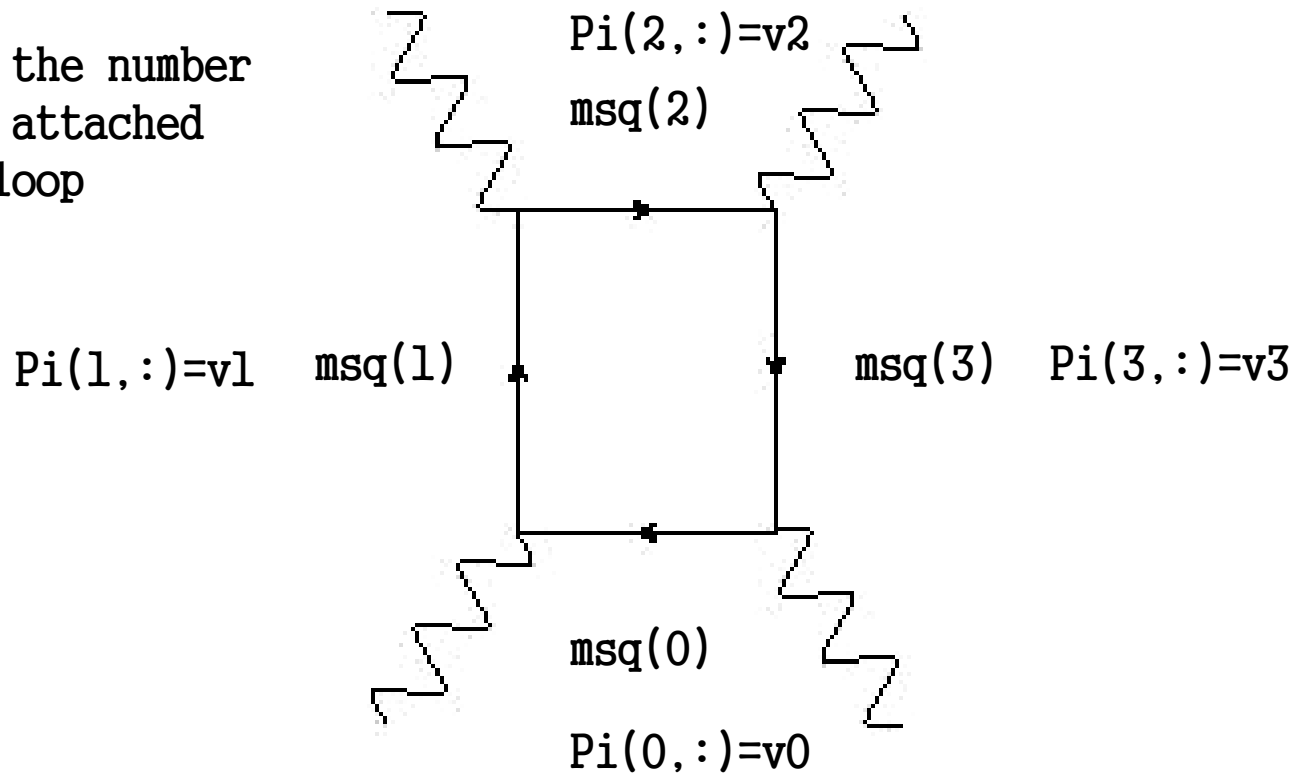
- ✓ `verbosity = 0`, nothing is printed by the reduction
 - 1, the coefficients are printed out
 - 2, also the value of the MI are printed out
 - 3, also the results of the tests are printed out

- ✓ `itest = 0`, none test
 - 1, global $n=n$ test is performed (not avail. for `imeth= 'tree'`)
 - 2, local $n=n$ test is performed
 - 3, power test is performed (not avail. for `imeth= 'tree'`)
 - new - based on the mismatch of the polynomial degree of the given integrand and the reconstructed one

Optionally, to fill the denominators

```
call InitDenominators(nleg,Pi,msq,v0,m0,v1,m1,...,vlast,mlast)
```

nleg is the number
of legs attached
to the loop



$$\text{Denominator}(j) = [q + Pi(j,:)]^2 - \mu^2 - msq(j)$$

```
call samurai(xnum,tot,totr,Pi,msq,nleg,rank,istop,scale2,ok)
```

- ✓ xnum [i]= the name of the function to reduce with arguments xnum(cut, q, mu2)
for imeth=tree the cut play a selective role to use the relative tree product
- ✓ tot [o] = contains the result of the reduction convoluted with the MI
- ✓ totr [o]= contains the rational part only
- ✓ rank [i] = the rank of the numerator, useful to speed up the reduction
- ✓ istop [i] = when stop the reduction, i.e. after pentuple cut (5) quadruple (4)...
- ✓ scale2 [i] = the value of the renormalization scale (square)
- ✓ ok [o] = a logical variable giving the result of the test if they are evaluated

About the precision

- ✓ Gram Determinant -> induce large cancellations between contributions from the MI that carry such a factor (the tests coded in SAMURAI detect the associated instabilities)
- ✓ Big cancellations between diagrams -> on-shell methods seems to be the best option
- ✓ If running with big internal masses -> big cancellations between cut-constructible and rational part

Quadruple precision solves these issues

For numerical studies and checks SAMURAI compiles also in quad

A simple option to treat instabilities:

[with G. Heinrich, G. Ossola, T. Reiter (2010)]

switch to a Tensorial Reconstruction paired with an efficient numerical evaluation of tensor integrals

New

$$\mathcal{N}(q) = \sum_{r=0}^R C_{\mu_1 \dots \mu_r} q_{\mu_1} \dots q_{\mu_r}$$

$$C_{\mu_1 \dots \mu_r} q_{\mu_1} \dots q_{\mu_r} = \sum_{(i_1, i_2, i_3, i_4) \vdash r} \hat{C}_{i_1 i_2 i_3 i_4}^{(r)} \cdot (q_1)^{i_1} (q_2)^{i_2} (q_3)^{i_3} (q_4)^{i_4}$$

Level-0

$$q = (0, 0, 0, 0)$$

$$\mathcal{N}(0, 0, 0, 0) \equiv \mathcal{N}^{(0)} = C_0$$

Level-1

Sampling monomial with one component of q

$$\mathcal{N}^{(1)}(q) \equiv \mathcal{N}(q) - \mathcal{N}^{(0)}$$

4 SYSTEMS $l_1 = 4R$

$$q = (x, 0, 0, 0) \Rightarrow \mathcal{N}^{(1)}(x, 0, 0, 0) \equiv x C_1 + x^2 C_{11} + \dots + x^R \underbrace{C_{11 \dots 1}}_{R \text{ times}}$$

$$q = (0, y, 0, 0) \Rightarrow \mathcal{N}^{(1)}(0, y, 0, 0) \equiv y C_2 + y^2 C_{22} + \dots + y^R \underbrace{C_{22 \dots 2}}_{R \text{ times}}$$

.....

Level-2

$$\mathcal{N}^{(2)}(q) \equiv \mathcal{N}^{(1)}(q) - \sum_{j=1}^4 C_j q_j - \sum_{j=1}^4 C_{jj} q_j^2 - \dots - \sum_{j=1}^4 \underbrace{C_{jj\dots j}}_{R \text{ times}} q_j^R$$

Sampling monomial with two components of q

6 SYSTEMS

$$l_2 = 3R(R - 1)$$

$$q = (x, y, 0, 0) \Rightarrow \mathcal{N}^{(2)}(x, y, 0, 0) \equiv xy C_{12} + x^2 y C_{112} + x y^2 C_{122} + \dots + \sum_{a,b:(a+b=R)} x^a y^b \hat{C}_{ab00}^{(R)}$$

$$q = (x, 0, z, 0) \Rightarrow \mathcal{N}^{(2)}(x, 0, z, 0) \equiv xz C_{13} + x^2 z C_{113} + x z^2 C_{133} + \dots + \sum_{a,b:(a+b=R)} x^a z^b \hat{C}_{a0b0}^{(R)}$$

.....

.....

Level-3

$$\mathcal{N}^{(3)}(q) \equiv \mathcal{N}^{(2)}(q) - \sum_{r=2}^R \sum_{a,b:(a+b)=r} \sum_{k < j=1}^4 \underbrace{C_{j\dots j}}_{a \text{ times}} \underbrace{k\dots k}_{b \text{ times}} q_j^a q_k^b$$

Sampling monomial with three components of q

4 SYSTEMS

$$l_3 = 2R(R - 1)(R - 2)/3$$

$$q = (x, y, z, 0) \Rightarrow \mathcal{N}^{(3)}(x, y, z, 0) \equiv xyz C_{123} + x^2 yz C_{1123} + \dots + \sum_{a,b,c:(a+b+c=R)} x^a y^b z^c \hat{C}_{abc0}^{(R)}$$

.....

.....

Level-4 $\mathcal{N}^{(4)}(q) \equiv \mathcal{N}^{(3)}(q) - \sum_{r=3}^R \sum_{a,b,c:(a+b+c)=r} \sum_{m < k < j=1}^4 C_{\underbrace{j \dots j}_a \text{ times} \underbrace{k \dots k}_b \text{ times} \underbrace{m \dots m}_c \text{ times}} q_j^a q_k^b q_m^c$
Sampling monomial with four components of q

1 SYSTEM $l_4 = R(R-1)(R-2)(R-3)/24$

$$q = (x, y, z, w) \quad \rightarrow \quad \mathcal{N}^{(4)}(x, y, z, w) \equiv xyzw C_{1234} + \dots + \sum_{a,b,c,d:(a+b+c+d=R)} x^a y^b z^c w^d \hat{C}_{abcd}^{(R)}$$

mu2-part $\mathcal{N}(\bar{q}) = \langle N(q) \rangle + G^{(1)} \mu^2 + G^{(2)} \mu^4 + G_{\alpha}^{(3)} q^{\alpha} \mu^2 + G_{\alpha\beta}^{(4)} q^{\alpha} q^{\beta} \mu^2$

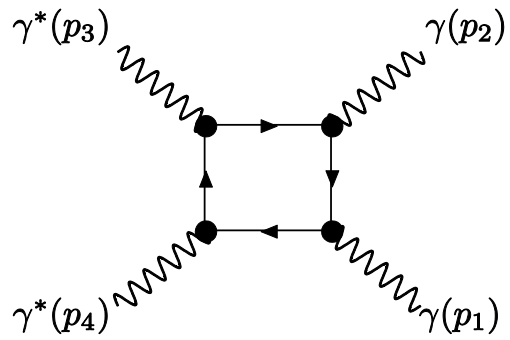
- ✓ Once reconstructed tensors that does not involve mu2, $\langle N(q) \rangle$, one can subtract it and sample the rest as above taking mu2.ne.0
- ✓ Not all components relevant
- ✓ For several diagrams the mu2 part can be inferred from $\langle N(q) \rangle$

BUBBLE: $\mathcal{N}_2(\bar{q}) = \langle N(q) \rangle + G^{(1)} \mu^2$,

TRIANGLE: $\mathcal{N}_3(\bar{q}) = \langle N(q) \rangle + G^{(1)} \mu^2 + G_{\alpha}^{(3)} q^{\alpha} \mu^2$,

BOX: $\mathcal{N}_4(\bar{q}) = \langle N(q) \rangle + G^{(1)} \mu^2 + G^{(2)} \mu^4 + G_{\alpha}^{(3)} q^{\alpha} \mu^2 + G_{\alpha\alpha}^{(4)} q^{\alpha} q^{\alpha} \mu^2$

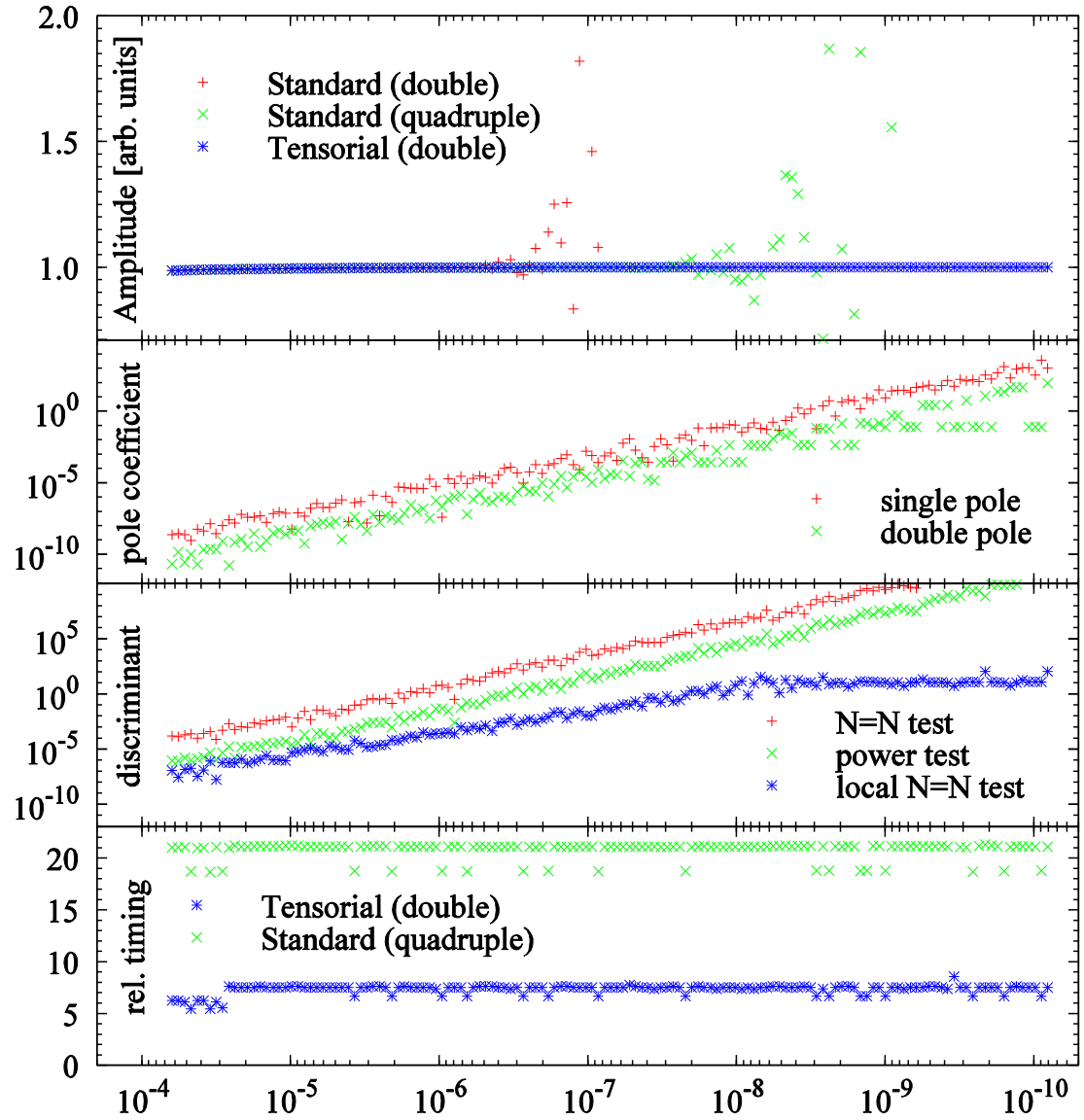
Example:



Standard(double):
2x SAMURAI

Standard(quadruple):
2x Kin + integrals
4x Algorithm

Tensorial(double):
Reconstruction paired
with numerical evaluation
of tensor integrals with
GOLEM95



$$\begin{aligned}
 p_{1,2} &= (E, 0, 0, \pm E) & p_{1,2}^2 &= 0 & \det G &= 32 E^4 Q^2 \sin^2 \theta \\
 p_{3,4} &= (E, 0, \pm Q \sin \theta, \pm Q \cos \theta) & p_{3,4}^2 &= m^2 \\
 E &= \sqrt{m^2 + Q^2} & \theta &= \frac{35}{180} \pi \text{ and } m^2 = 7
 \end{aligned}$$

Also useful for:

- **Speed up the computation** if the numerator has a long expression: added time for tensorial reconstruct compensated by a faster reduction

✓ **Example:**
dummy numerator of one line of code sitting on 6 denominators and repeated N times

# Lines	Time ratio "hybrid"/standard	
N	Rank = 4	Rank = 6
1	1.3	1.6
10	1.1	1.4
100	0.51	0.85
1000	0.30	0.59
10000	0.27	0.55

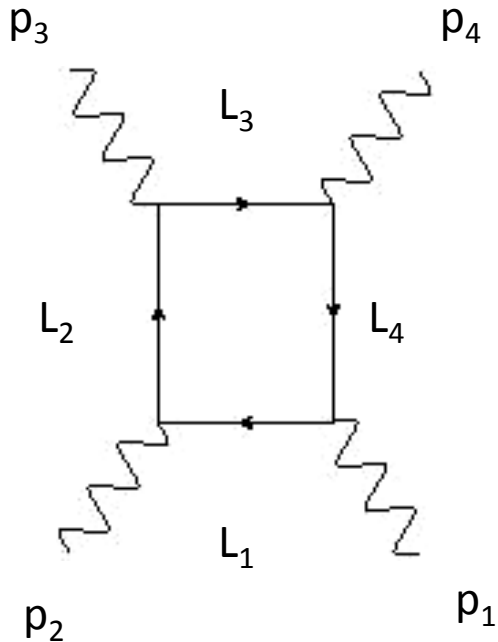
- **Real loop variable** match well with the automatic generation of integrand directly from tree level generators like MadGraph and HELAC

Examples

Note:

- ❑ Are chosen to **address** typical **technical issues** that one encounter performing one loop virtual calculation
- ❑ The aim is to show the **flexibility** of the framework
- ❑ Are part of the release and could also be used as **templates** for other calculations

4-photons



- imeth= 'diag'
- nleg = 4, rank = 4
- 6 permutations, only 3 relevant

$$\bar{L}_1 = \bar{q}, \quad \bar{L}_2 = \bar{q} + p_2, \quad \bar{L}_3 = \bar{q} + p_{23}, \quad \bar{L}_4 = \bar{q} + p_{234}$$

$$N(\bar{q}) = -\text{Tr}\left[(\bar{L}_1 + m)\not{\epsilon}_2(\bar{L}_2 + m)\not{\epsilon}_3(\bar{L}_3 + m)\not{\epsilon}_4(\bar{L}_4 + m)\not{\epsilon}_1\right]$$

$$\begin{aligned} N(q, \mu^2) = & -(m^4 - \mu^2 m^2 + \mu^4) \text{Tr}[\not{\epsilon}_2 \not{\epsilon}_3 \not{\epsilon}_4 \not{\epsilon}_1] \\ & - (m^2 - \mu^2) \left(\text{Tr}[\not{\epsilon}_2 \not{\epsilon}_3 \not{\epsilon}_4 \not{L}_4 \not{\epsilon}_1 \not{L}_1] + \text{Tr}[\not{\epsilon}_2 \not{\epsilon}_3 \not{L}_3 \not{\epsilon}_4 \not{\epsilon}_1 \not{L}_1] \right. \\ & + \text{Tr}[\not{\epsilon}_2 \not{\epsilon}_3 \not{L}_3 \not{\epsilon}_4 \not{L}_4 \not{\epsilon}_1] + \text{Tr}[\not{\epsilon}_2 \not{L}_2 \not{\epsilon}_3 \not{\epsilon}_4 \not{\epsilon}_1 \not{L}_1] \\ & + \text{Tr}[\not{\epsilon}_2 \not{L}_2 \not{\epsilon}_3 \not{\epsilon}_4 \not{L}_4 \not{\epsilon}_1] + \text{Tr}[\not{\epsilon}_2 \not{L}_2 \not{\epsilon}_3 \not{L}_3 \not{\epsilon}_4 \not{\epsilon}_1] \left. \right) \\ & - \text{Tr}[\not{L}_1 \not{\epsilon}_2 \not{L}_2 \not{\epsilon}_3 \not{L}_3 \not{\epsilon}_4 \not{L}_4 \not{\epsilon}_1], \end{aligned}$$

Denominators:

$$(\bar{L}_1^2 - m^2) (\bar{L}_2^2 - m^2) (\bar{L}_3^2 - m^2) (\bar{L}_4^2 - m^2)$$

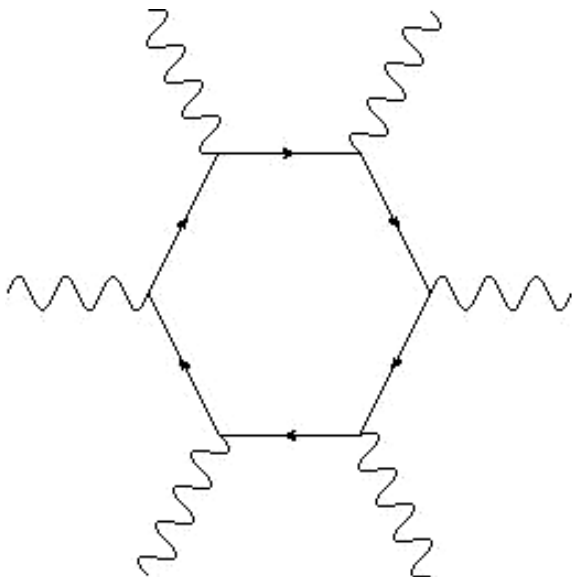
$$Pi = (0, p_2, p_2 + p_3, p_2 + p_3 + p_4), \quad msq = (m^2, m^2, m^2, m^2)$$

- mu2 terms give zero contribution
- mu2 q^aq^β cancel in the sum
- mu2² gives rise to the correct rational part

Results numerically checked vs. Gounaris et al (1999)

6-photons

- imeth = 'diag'
- nleg = 6, rank = 6
- 120 permutations, only 60 relevant



$$N(q, \mu^2) = N(q) = -\text{Tr} [L_1 \not{\epsilon}_2 L_2 \not{\epsilon}_3 L_3 \not{\epsilon}_4 L_4 \not{\epsilon}_5 L_5 \not{\epsilon}_6 L_6 \not{\epsilon}_1].$$

Bernicot et al (2007,2008)

$$\frac{s}{\alpha^3} A(-, -, +, +, +, +) = 11075.04009210435 ,$$

$$\frac{s}{\alpha^3} A(+, -, -, +, +, -) = 7814.762085902767 ,$$

SAMURAI with istop=2

$$\frac{s}{\alpha^3} A(-, -, +, +, +, +) = \underline{11075.040174990} ,$$

$$\frac{s}{\alpha^3} A(+, -, -, +, +, -) = \underline{7814.7623429908} .$$

SAMURAI with istop=3, subtracting tot

$$\frac{s}{\alpha^3} A(-, -, +, +, +, +) = \underline{11075.040092102} ,$$

$$\frac{s}{\alpha^3} A(+, -, -, +, +, -) = \underline{7814.7620859084} .$$

PS point as in Nagy and Soper (2006)

$$\vec{p}_3 = (33.5, 15.9, 25.0)$$

$$\vec{p}_4 = (-12.5, 15.3, 0.3)$$

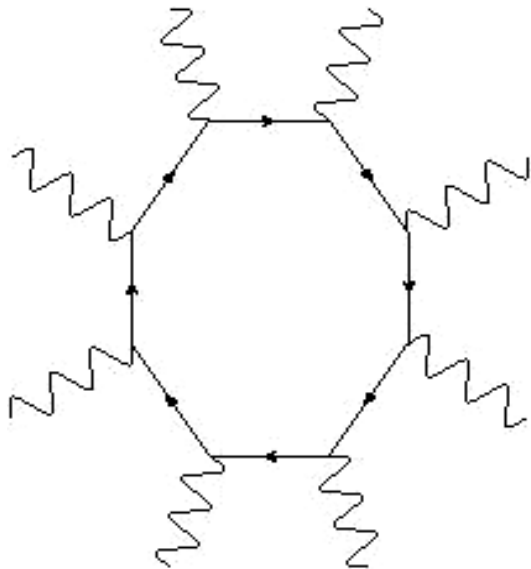
$$\vec{p}_5 = (-10.0, -18.0, -3.3)$$

$$\vec{p}_6 = (-11.0, -13.2, -22.0)$$

Results numerically checked vs. Bernicot et al (2007,2008)

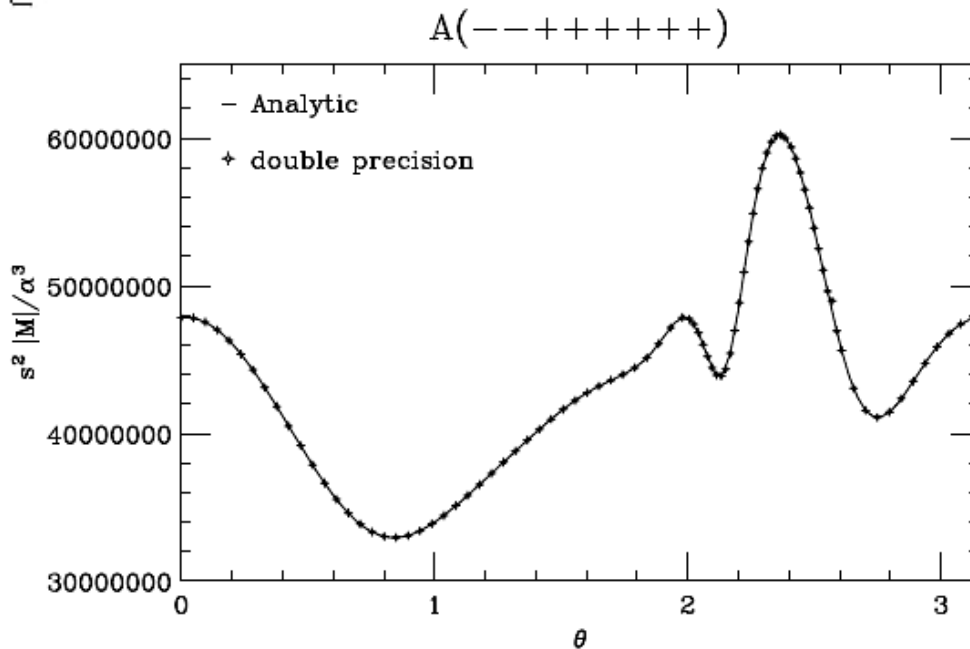
8-photons

- `imeth = 'diag'`
- `nleg = 8, rank = 8`
- 5040 permutations, only 2520 relevant
- sampling set as in Gong et al (2008)



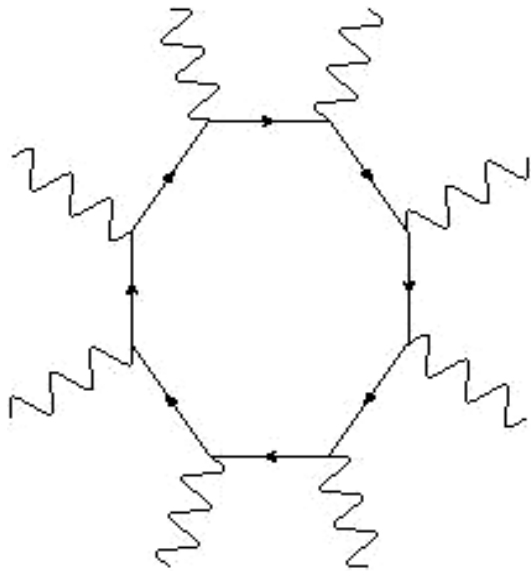
$$N(q, \mu^2) = -\text{Tr} \left[\cancel{L_0 \not{\epsilon}_1} \cancel{L_1 \not{\epsilon}_2} \cancel{L_2 \not{\epsilon}_3} \cancel{L_3 \not{\epsilon}_4} \cancel{L_4 \not{\epsilon}_5} \cancel{L_5 \not{\epsilon}_6} \cancel{L_6 \not{\epsilon}_7} \cancel{L_7 \not{\epsilon}_8} \right]$$

MHV result numerically checked vs. Mahlon (1993)



8-photons

- imeth = 'diag'
- nleg = 8, rank = 8
- 5040 permutations, only 2520 relevant
- sampling set as in Gong et al (2008)

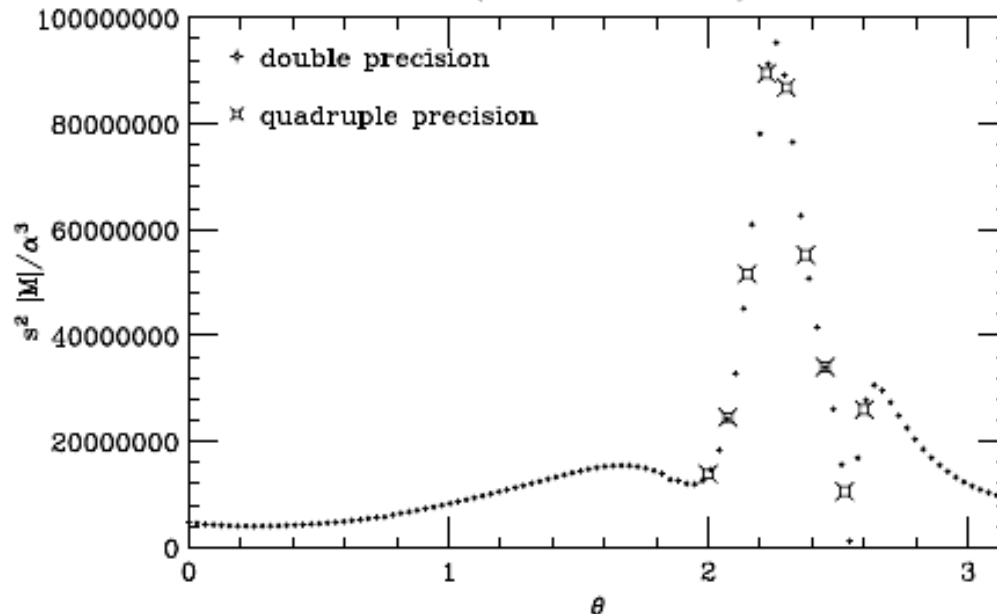


$$N(q, \mu^2) = -\text{Tr} \left[\cancel{L_0 \cancel{\epsilon_1}} \cancel{L_1 \cancel{\epsilon_2}} \cancel{L_2 \cancel{\epsilon_3}} \cancel{L_3 \cancel{\epsilon_4}} \cancel{L_4 \cancel{\epsilon_5}} \cancel{L_5 \cancel{\epsilon_6}} \cancel{L_6 \cancel{\epsilon_7}} \cancel{L_7 \cancel{\epsilon_8}} \right]$$

NMHV result (new) numerically confirm the structure in Badger et al (2009)

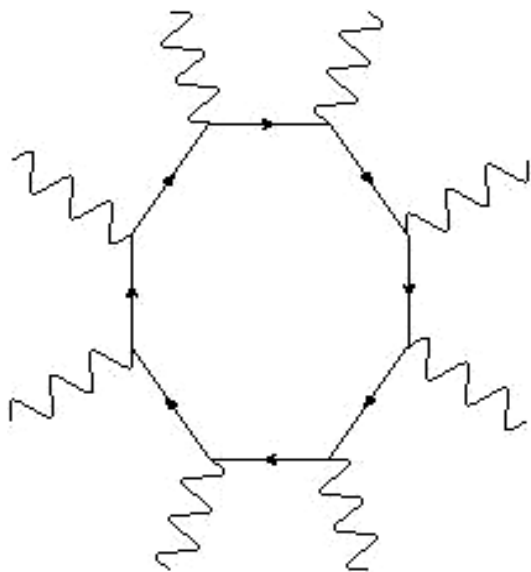
The points in quadruple precision (x) have been calculated with istop=2, i.e. retaining all the cut constructible and rational pieces

$$A(-++-++-+)$$



8-photons

- imeth = 'diag'
- nleg = 8, rank = 8
- 5040 permutations, only 2520 relevant
- sampling set as in Gong et al (2008)

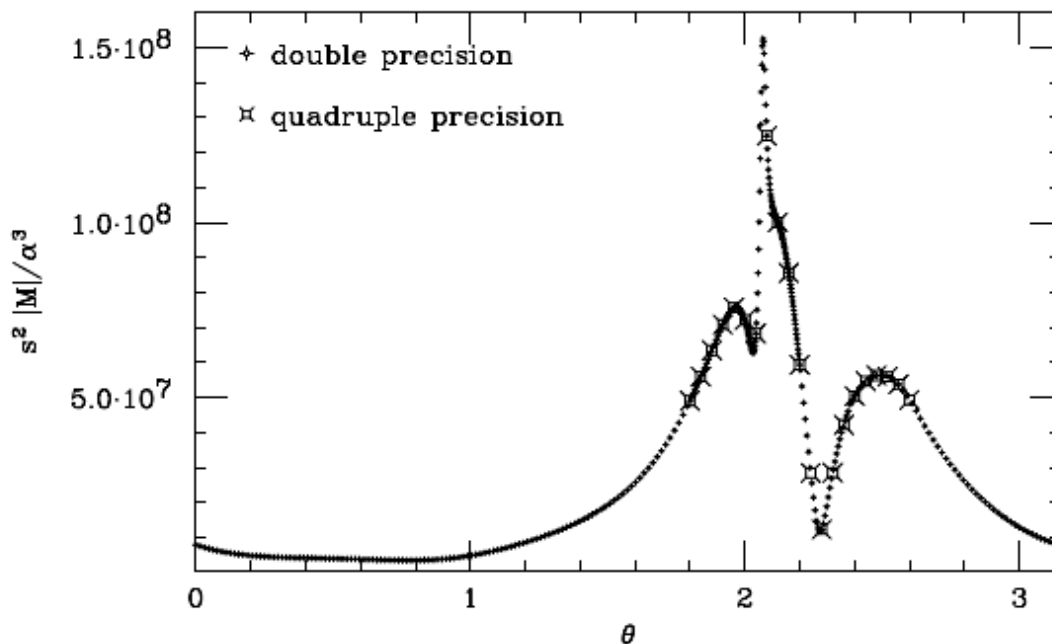


$$N(q, \mu^2) = -\text{Tr} \left[\cancel{L_0} \cancel{e_1} \cancel{L_1} \cancel{e_2} \cancel{L_2} \cancel{e_3} \cancel{L_3} \cancel{e_4} \cancel{L_4} \cancel{e_5} \cancel{L_5} \cancel{e_6} \cancel{L_6} \cancel{e_7} \cancel{L_7} \cancel{e_8} \right]$$

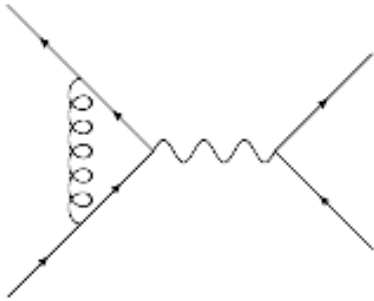
NNMHV result (new) numerically confirm the structure in Badger et al (2009)

The points in quadruple precision (x) have been calculated with istop=2, i.e. retaining all the cut constructible and rational pieces

A(-----++++)



Drell-Yan



If one wants to consider regularization schemes giving rise to $O(\epsilon)$ terms and reduce them, then one needs to process N_0 and N_1 below separately

$$\mathcal{N}(\bar{q}, \epsilon) = N_0(\bar{q}) + \epsilon N_1(\bar{q}) + \epsilon^2 N_2(\bar{q}).$$

- imeth = 'diag'
- nleg = 3, rank = 2

d=4 → Dim Red
d=4-2ε → CDR



$$N(q, \mu^2) = C_F g_s^2 e^2 \bar{u}(p_{e^-}) \gamma^\mu v(p_{e^+}) \bar{v}(p_{\bar{u}}) [2(2-d) \bar{q}^\mu \not{\bar{q}} + [(d-2) \bar{q}^2 + 4(p_u \cdot \bar{q} - p_{\bar{u}} \cdot \bar{q} - p_u \cdot p_{\bar{u}})] \gamma^\mu] u(p_u)$$

Denominators: $\bar{q}^2 \quad (\bar{q} + p_u)^2 \quad (\bar{q} + p_u + p_{e^-} + p_{e^+})^2$

- msq = { 0, 0, 0 }
- Pi = { $\underline{0}$, p_u , $p_u + p_{e^-} + p_{e^+}$ }
- N_1 generate a rational term = $-g_s^2 C_F L_0$

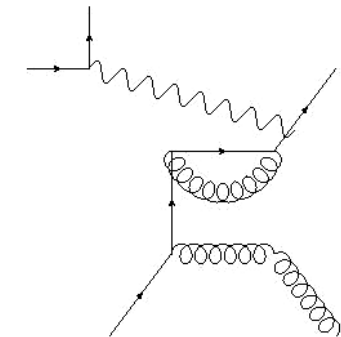
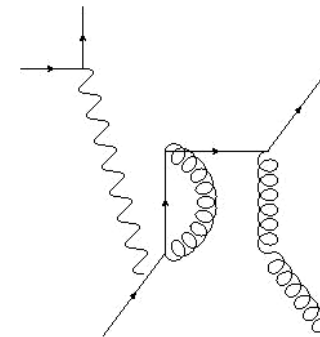
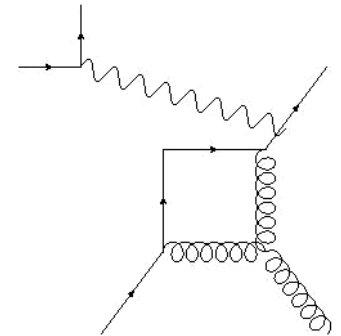
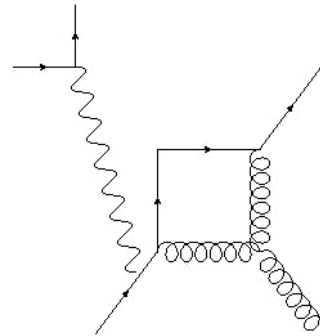
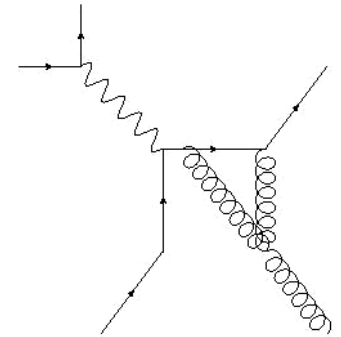
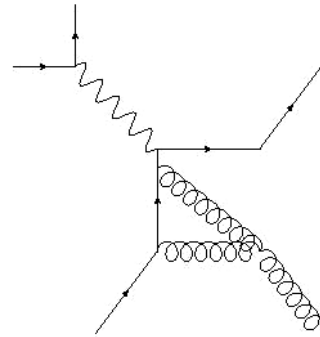
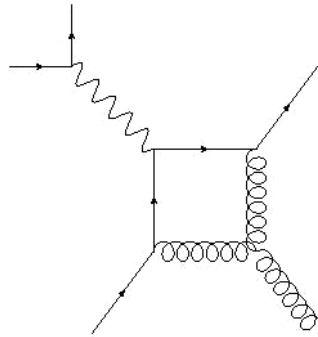
VB+lj: *leading color*

- `imeth = 'diag'`

- 1 Box nleg=4, rank=3
- 4 Tri nleg=3, rank=2
- 2 Bub nleg=2, rank=1

- Diagrams can be collected on a common box denominator

- Studying Left-handed current needs of a prescription for γ_5 :
adopting DR w/anticommuting γ_5
we added $-N_c/2$ times the Tree Level amplitude

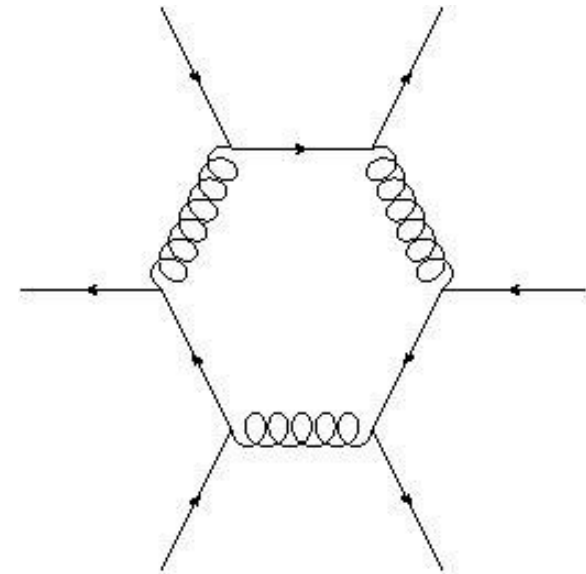


Results numerically checked vs. Bern et al (1997)

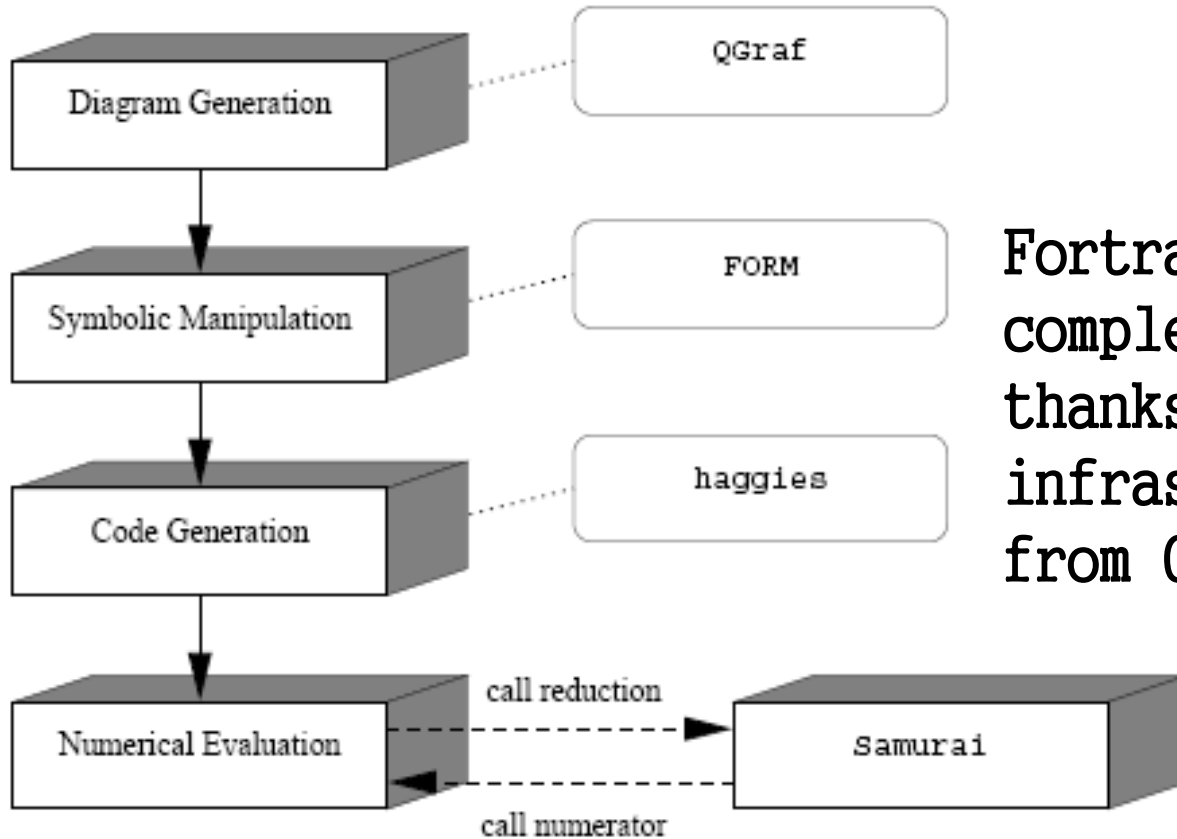
6q amplitudes

8 hexagons
24 pentagons
42 boxes
70 triangles
114 bubbles

258 Feynman Diagrams



$$q_1 \bar{q}_1 \rightarrow q_2 \bar{q}_2 q_3 \bar{q}_3$$



Fortran Code generation completely automated thanks to an infrastructure derived from Golem-2.0

6q amplitudes

- $A(-+-+--)$
- ren scale = 1GeV
- uv renormalization included

$$\vec{p}_3 = (33.5, 15.9, 25.0)$$

$$\vec{p}_4 = (-12.5, 15.3, 0.3)$$

$$\vec{p}_5 = (-10.0, -18.0, -3.3)$$

$$\vec{p}_6 = (-11.0, -13.2, -22.0)$$

GOLEM-2.0 + GOLEM95

$$a_{\text{LO}} = 0.9686295685264447 \times 10^{-6} ,$$

$$a_{-2} = -8.0000000000048633 ,$$

$$a_{-1} = 46.40675046335535 ,$$

$$a_0 = -233.8908276457752 ;$$

Infrared poles calculated
from the integrated dipoles

$$a_{\text{LO}} = \mathcal{A}_{\text{LO}}^\dagger \mathcal{A}_{\text{LO}}$$

$$\mathcal{A}_{\text{virt}}^\dagger \mathcal{A}_{\text{LO}} + h.c. = a_{\text{LO}} \cdot \frac{\alpha_s}{2\pi} \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left(\frac{a_{-2}}{\epsilon^2} + \frac{a_{-1}}{\epsilon^1} + a_0 \right)$$

Automatic generated fortran code
based on **SAMURAI** reduction more then
10 times shorter and faster
already without any optimization

GOLEM-2.0 + SAMURAI

$$a_{\text{LO}} = 0.9686295685264458 \times 10^{-6} ,$$

$$a_{-2} = -7.9999999999999935 ,$$

$$a_{-1} = 46.40675045992446 ,$$

$$a_0 = -233.8908276128404 ,$$

$$a_{-2} = 8.0000000000000000 ,$$

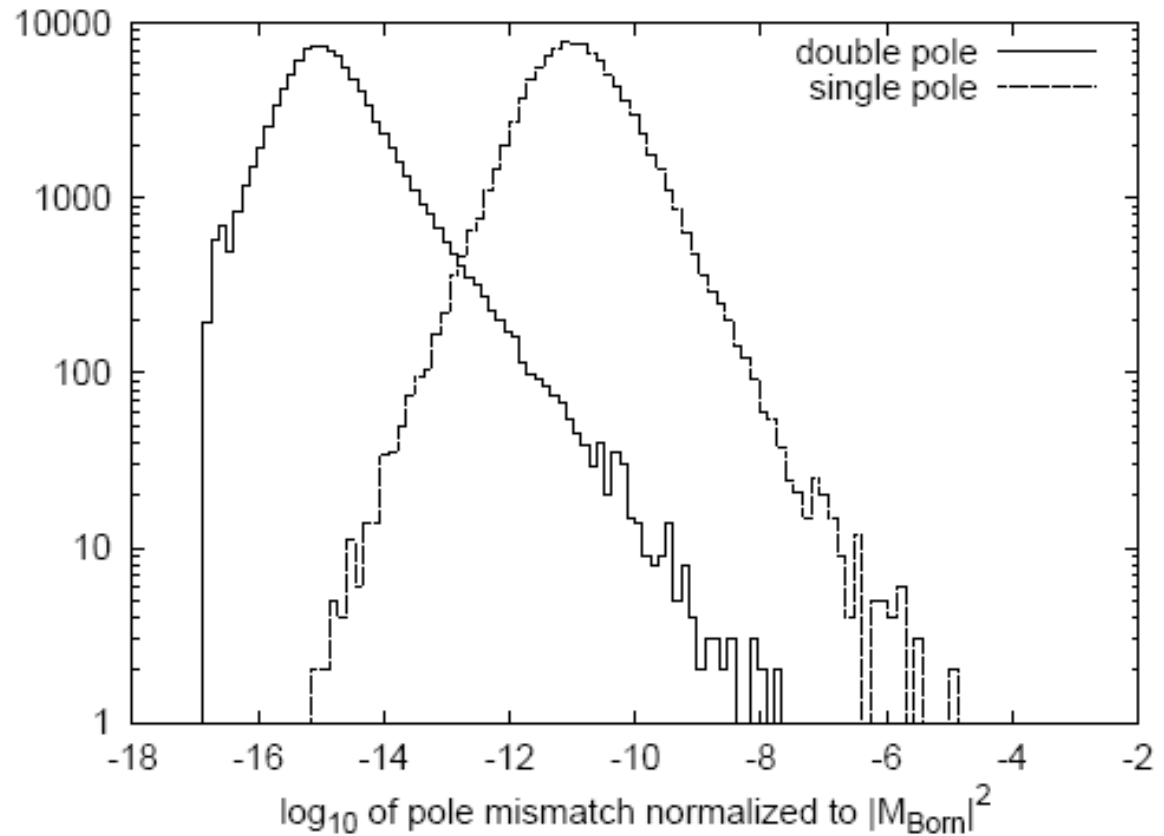
$$a_{-1} = -46.40675046319159 .$$

6q amplitudes

$$p_T > 30 \text{ GeV}$$

$$|\eta| < 2.5$$

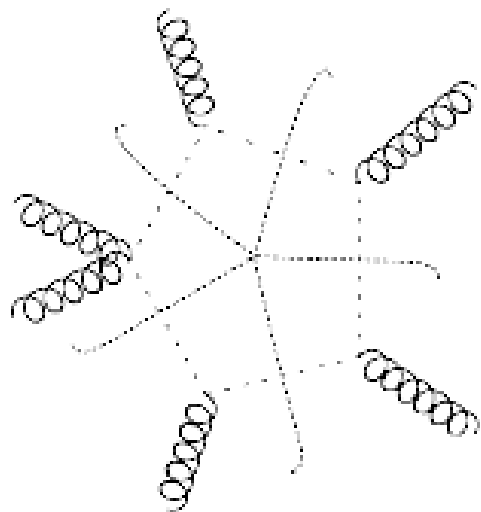
$$\Delta R > 0.8$$



Difference between the single (double) virtual poles and those of the integrated dipoles for 10^5 phase space points

5 and 6-gluons all plus: *massive scalar loop*

- imeth= 'tree'
- nleg = 6, rank = 6



$$A_3^{\text{tree}}(1_s; 2^+, 3_s) = \frac{[2|1|r_2]}{\langle 2r_2 \rangle},$$

$$A_4^{\text{tree}}(1_s; 2^+, 3^+, 4_s) = \frac{\mu^2 [23]}{\langle 23 \rangle (p_{12}^2 - \mu^2)},$$

$$A_5^{\text{tree}}(1_s; 2^+, 3^+, 4^+, 5_s) = \frac{\mu^2 [2|1(2+3)|4]}{\langle 23 \rangle \langle 34 \rangle (p_{12}^2 - \mu^2) (p_{45}^2 - \mu^2)},$$

$$N(q, \mu^2) = A_4(L_1; 1^+, 2^+; -L_2) \times A_3(L_2; 3^+; -L_3) \times A_3(L_3; 4^+; -L_4) \\ \times A_3(L_4; 5^+; -L_5) \times A_3(L_5; 6^+; -L_1)$$

**For this helicity choice
the result is purely rational**

numerically checked vs. S. Badger's table

Conclusions

- SAMURAI is a fortran library for the automatic evaluation of the NLO virtual correction to scattering processes, once the integrand is given in the form of Feynman diagrams or as products of tree level amplitudes
- We produced several examples to show its main features
- We tried to make things as effective and simple as possible to allow for interfaces with other tools
- We provided also a “rescue system” pairing a tensorial reconstruction to the numerical evaluation of tensor integrals with GOLEM95 (with G. Heinrich)
- SAMURAI is mature for physical implementations!