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Towards Jet Cross Sections at NNLO for hadron colliders

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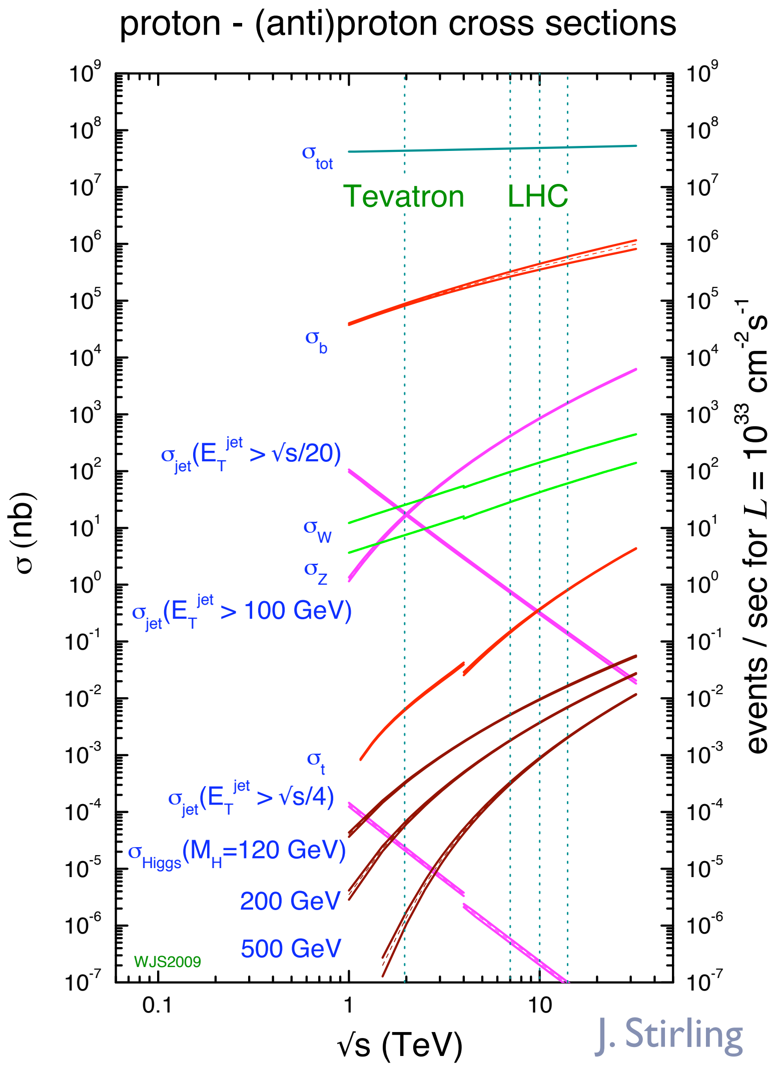
Expectations at LHC

- Large production rates for Standard Model processes
  - jets
  - top quark pairs
  - vector bosons
- Allow precision measurements
  - masses
  - couplings
  - parton distributions
- Require precise theory: NNLO



J. Stirling

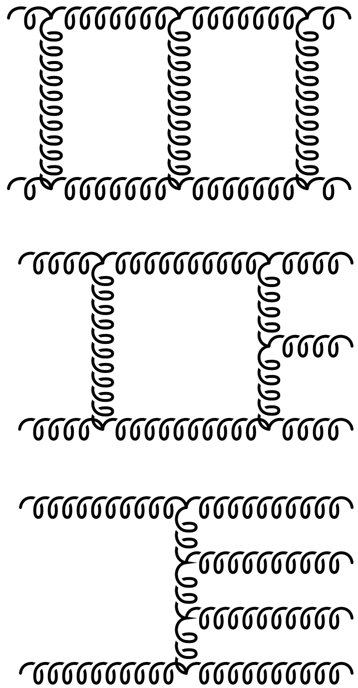
Where are NNLO corrections needed?

- Processes measured to few per cent accuracy
  - $e^+e^- \to 3$ jets, 2+1 jet production in DIS
  - hadron collider processes:
    - jet production
    - vector boson (+jet) production
    - top quark pair production

- Processes with potentially large perturbative corrections
  - Higgs or vector boson pair production
    - prediction stable only at NNLO

NNLO calculations

- Require three principal ingredients (here: pp → 2j)
  - two-loop matrix elements
    - explicit infrared poles from loop integral
      - known for all massless 2 → 2 processes
  - one-loop matrix elements
    - explicit infrared poles from loop integral
    - and implicit poles from soft/collinear emission
      - usually known from NLO calculations
  - tree-level matrix elements
    - implicit poles from two partons unresolved
      - known from LO calculations



- Challenge: combine contributions into parton-level generator
- need method to extract implicit infrared poles

NNLO calculations

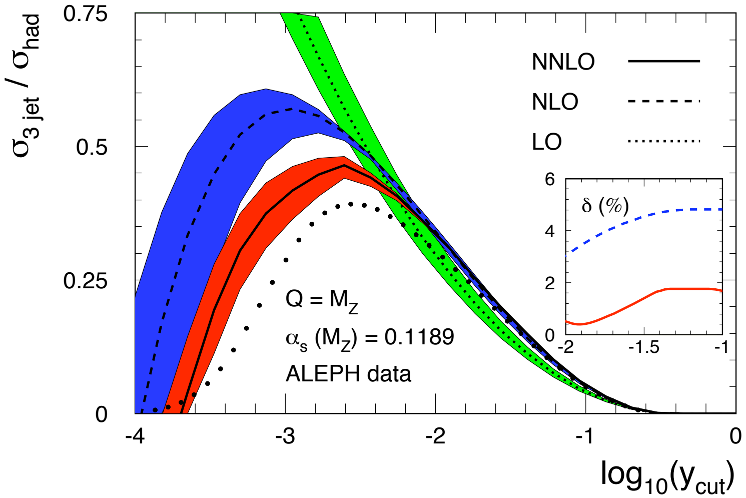
- Solutions
  - sector decomposition: expansion in distributions, numerical integration (T. Binoth, G. Heinrich; C. Anastasiou, K. Melnikov, F. Petriello; M. Czakon)
    - applied to Higgs and vector boson production (C. Anastasiou, K. Melnikov, F. Petriello)
  - subtraction: add and subtract counter-terms: process-independent approximations in all unresolved limits, analytical integration
    - several well-established methods at NLO
    - q_T subtraction applied to Higgs and vector boson production (S. Catani, M. Grazzini; with L. Cieri, G. Ferrera, D. de Florian)
    - antenna subtraction for jet observables in e^+e^- processes (T. Gehrmann, E.W.N. Glover, AG)

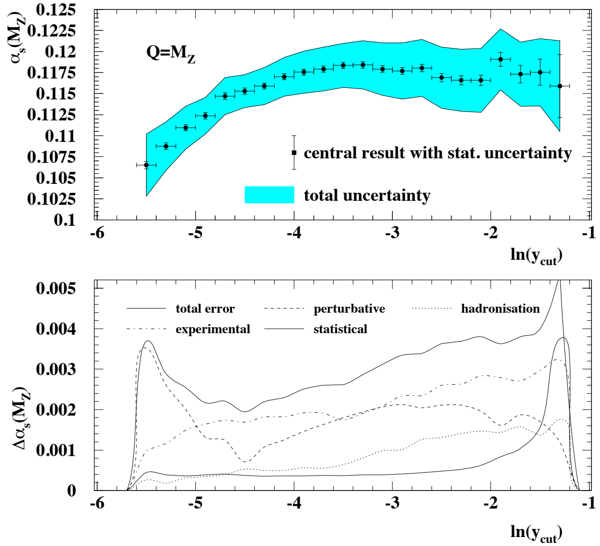
α_s from three-jet rate at NNLO

- NNLO corrections small
  (T. Gehrmann, E.W.N. Glover, G. Heinrich, AG; S.Weinzierl)
  - stable perturbative prediction
  - resummation not needed
  - theory error below 2%
- hadronization corrections
  - much smaller than for event shapes
- data with different jet resolution correlated
  - fit at y_{cut} = 0.02
  - consistent results with other resolution

α_s = 0.1175±0.0020(exp)±0.0015(th)

(G. Dissertori, T. Gehrmann, E.W.N. Glover, G. Heinrich, H. Stenzel, AG)





NNLO Subtraction

- Structure of NNLO m-jet cross section at hadron colliders

$$
\begin{aligned}
d\hat{\sigma}_{NNLO} &= \int_{d\Phi_{m+2}} \left(d\hat{\sigma}^{R}_{NNLO} - d\hat{\sigma}^{S}_{NNLO}\right) \\
&+ \int_{d\Phi_{m+1}} \left(d\hat{\sigma}^{V,1}_{NNLO} + d\hat{\sigma}^{MF,1}_{NNLO} - d\hat{\sigma}^{VS,1}_{NNLO}\right) \\
&+ \int_{d\Phi_{m}} \left(d\hat{\sigma}^{V,2}_{NNLO} + d\hat{\sigma}^{MF,2}_{NNLO}\right) + \int_{d\Phi_{m+2}} d\hat{\sigma}^{S}_{NNLO} + \int_{d\Phi_{m+1}} d\hat{\sigma}^{VS,1}_{NNLO}
\end{aligned}
$$

- with:
  - Partonic contributions: $d\hat{\sigma}^{R}_{NNLO}$ $d\hat{\sigma}^{V,1}_{NNLO}$ $d\hat{\sigma}^{V,2}_{NNLO}$
  - Subtraction terms: $d\hat{\sigma}^{S}_{NNLO}$ $d\hat{\sigma}^{VS,1}_{NNLO}$
  - Mass factorization terms: $d\hat{\sigma}^{MF,1}_{NNLO}$ $d\hat{\sigma}^{MF,2}_{NNLO}$

- Challenge: construction and integration of subtraction terms

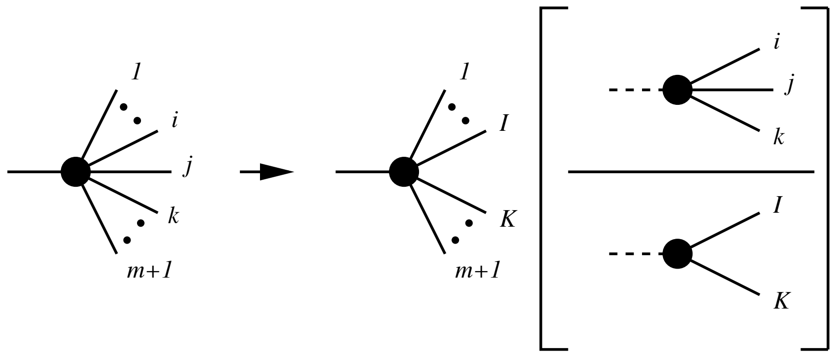
Antenna subtraction at NLO

- real radiation contribution to m-jet cross section

$$d\sigma^R = \mathcal{N} \int d\Phi_{m+1} |\mathcal{M}_{m+1}|^2 J_m^{(m+1)}(p_1, \ldots, p_{m+1})$$

- antenna subtraction term:

$$d\sigma^S_{NLO} = \mathcal{N} \int d\Phi_{m+1}(p_1, \ldots, p_{m+1}; q)$$
$$\sum_j X^0_{ijk} \, |\mathcal{M}_m|^2 \, J_m^{(m)}(p_1, \ldots, \tilde{p}_I, \tilde{p}_K, \ldots, p_{m+1})$$



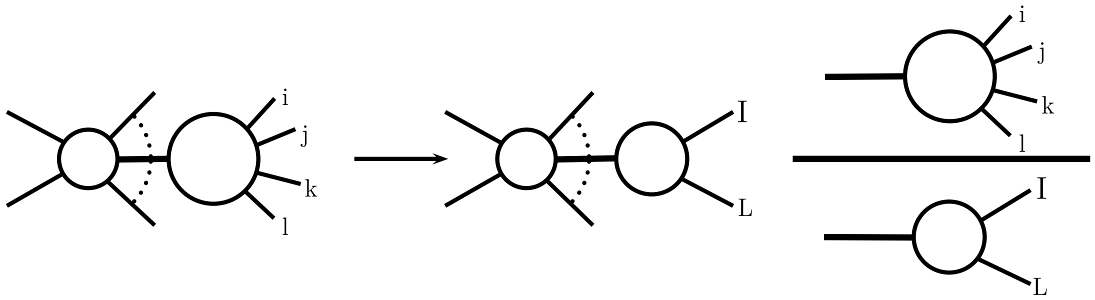
  - antenna X^0_{ijk} describes soft and collinear radiation off a hard parton pair

Colour-ordered antenna functions

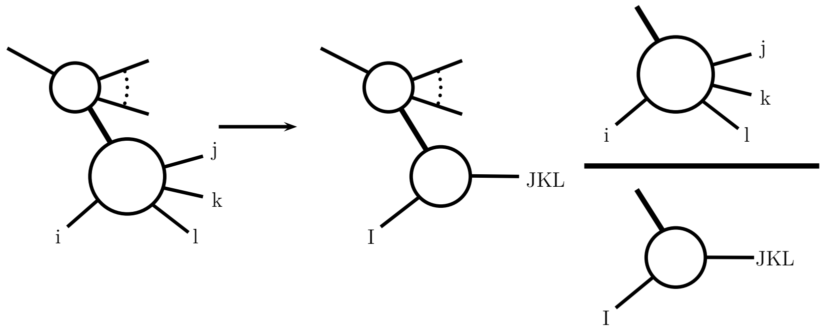
- colour-ordered pair of hard partons (radiators)
  - quark-antiquark pair
  - quark-gluon pair
  - gluon-gluon pair
- three-parton antenna $\rightarrow$ one unresolved parton
- four-parton antenna $\rightarrow$ two unresolved partons
- at tree-level or at one loop
- radiators in initial or final state:
  - three types of antennae: final-final, initial-final, initial-initial
- all antenna functions derived from physical $|\mathcal{M}|^2$

Subtraction for hadronic processes at NNLO

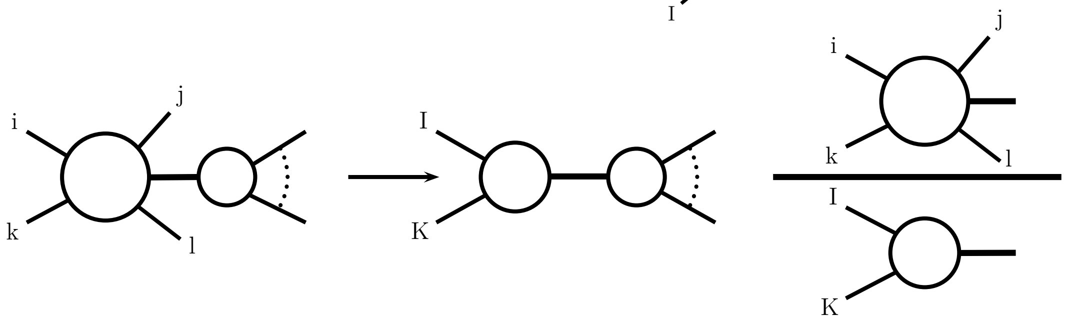
- Colour-connected double unresolved case
  - final-final



  - initial-final



  - initial-initial

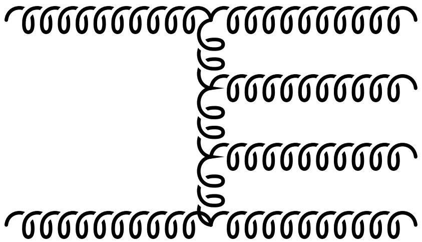


Hadron collider processes at NNLO

Double real radiation at NNLO for $pp \to 2j$

- Contributions from all tree-level $2 \to 4$ processes
- Test case: $gg \to gggg$ (E.W.N. Glover, J. Pires)

$$d\sigma^R_{NNLO} = N^2\, N_{born} \left(\frac{\alpha_s}{2\pi}\right)^2 d\Phi_4(p_3,\ldots,p_6;p_1,p_2)\Bigg(\frac{2}{4!}\sum_{P(i,j,k,l)\in(3,4,5,6)} A_6^0(\hat{1}_g,\hat{2}_g,i_g,j_g,k_g,l_g)J_2^{(4)}(p_i,\ldots,p_l)$$
$$+\frac{2}{4!}\sum_{P(i,j,k,l)\in(3,4,5,6)} A_6^0(\hat{1}_g,i_g,\hat{2}_g,j_g,k_g,l_g)J_2^{(4)}(p_i,\ldots,p_l)$$
$$+\frac{2}{4!}\sum_{P_C(i,j,k,l)\in(3,4,5,6)} A_6^0(\hat{1}_g,i_g,j_g,\hat{2}_g,k_g,l_g)J_2^{(4)}(p_i,\ldots,p_l)\Bigg)$$



  - three topologies according to initial state gluon positions
- antenna subtraction terms constructed, implemented and tested in all unresolved limits

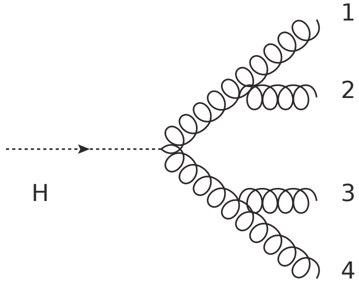
Antenna subtraction for gg → gggg

- Subtraction terms involve only gluon-gluon antennae

  - F_4^0 in final-final, initial-final, initial-initial, for colour-connected double unresolved limits

  - $F_3^0 \otimes F_3^0$ in all configurations, for oversubtracted single unresolved limits and colour unconnected double unresolved limits

  - F_3^0 for single unresolved limits



- Need to identify hard radiators for phase space mapping

Antenna subtraction for gg → gggg

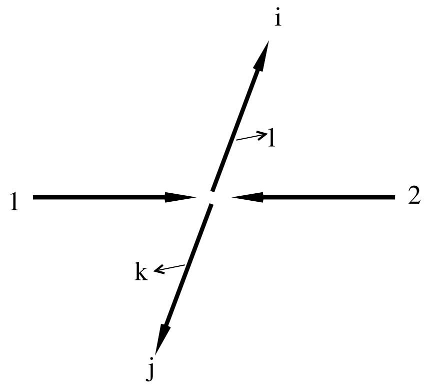
- Identification of hard radiators
- Problem: each parton is radiator or unresolved
  - due to colour cyclicity of quark-gluon and gluon-gluon antennae
  - only in final-final case (initial state fixes radiators)
  - decompose into sub-antennae, e.g.

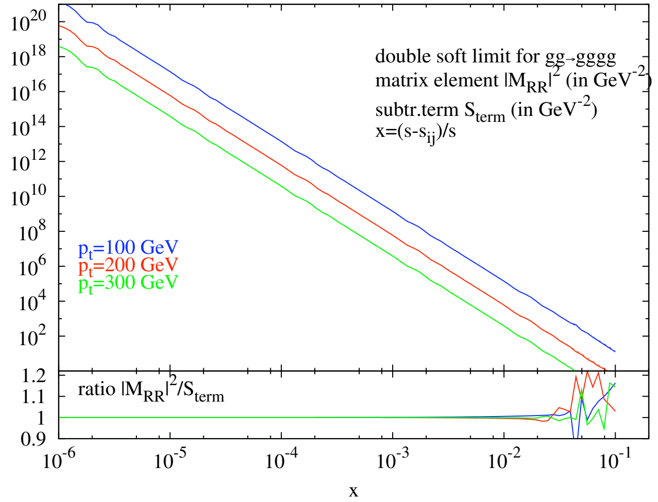
$$F_3^0(1,2,3) = f_3^0(1,3,2) + f_3^0(3,2,1) + f_3^0(2,1,3)$$

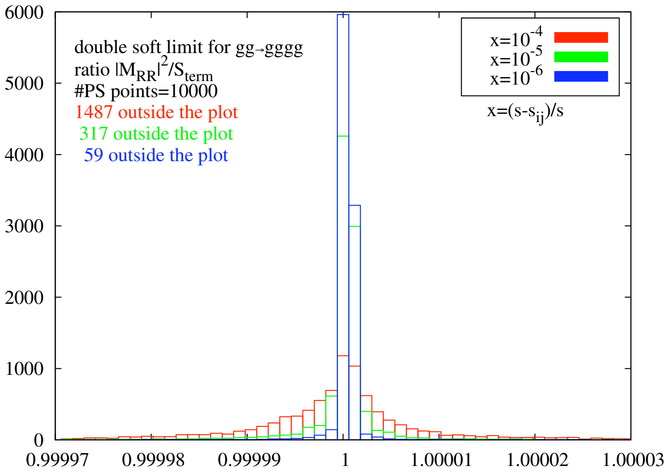
  - each sub-antenna has
    - different phase space mapping, fixed hard and unresolved partons
  - was done for four-parton quark-gluon antenna functions previously
    - based on N=1 SUSY relations among splitting functions
  - achieved now for gluon-gluon antenna F_4^0 (E.W.N. Glover, J. Pires)
  - eight sub-antennae contained in F_4^0

Antenna subtraction for $gg \to gggg$

- Check of the subtraction terms (E.W.N. Glover, J. Pires)
  - choose scaling parameter x for each limit
  - generate phase space trajectories into each limit
  - require reconstruction of two hard jets
  - compute ratio (matrix element)/(subtraction term): $|M_{RR}|^2/S_{term}$
  - Example: double soft limit : $s_{ij} \simeq s$



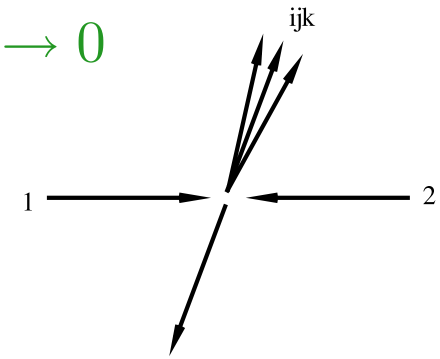


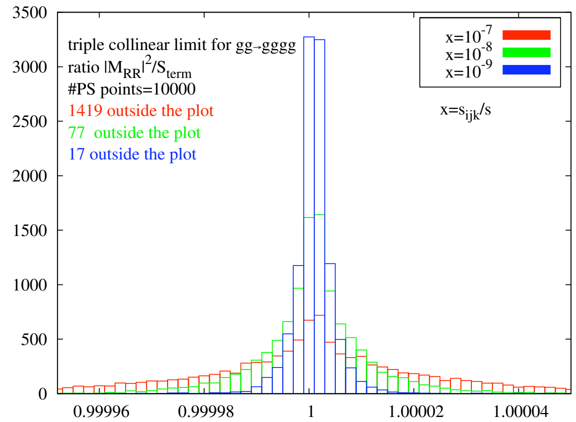


Antenna subtraction for gg → gggg

- Check of the subtraction terms (E.W.N. Glover, J. Pires)
  - Example: triple collinear final state limit

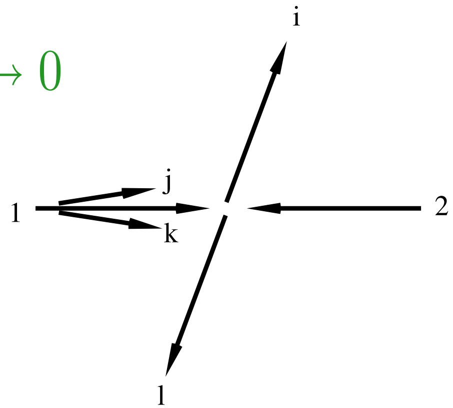
$s_{ijk} \to 0$

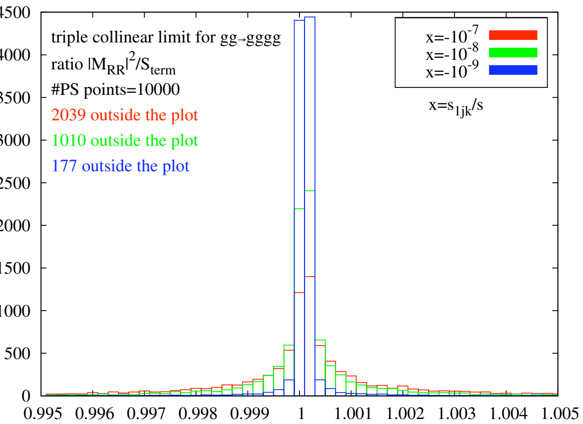




  - Example: triple collinear initial state limit

$s_{1jk} \to 0$





Jet production at hadron colliders

- Antenna subtraction for $gg \to gggg$
  - successful proof-of-principle of antenna subtraction
  - starting point for implementation of all $2 \to 4$ processes
- Next steps
  - implementation of virtual single unresolved $2 \to 3$ processes
  - integration of antenna functions
    - Final-final known (T. Gehrmann, E.W.N. Glover, AG)
    - Initial-final known (A. Daleo, T. Gehrmann, G. Luisoni, AG)
    - Initial-initial in progress (R. Boughezal, M. Ritzmann, AG)

Integrated NNLO antenna functions

- Analytical integration over unresolved part of phase space only
  - phase space integrals reduced to masters (C. Anastasiou, K. Melnikov)
  - Final-final: $q \to k_1 + k_2 + k_3 (+k_4)$, one scale: q^2
    - 1 → 4 tree level (4 master integrals)
    - 1 → 3 one loop (3 master integrals)
  - Initial-final: $q + p_1 \to k_1 + k_2 (+k_3)$, two scales: q^2, x
    - 2 → 3 tree level (9 master integrals) (→See talk G. Luisoni)
    - 2 → 2 one loop (6 master integrals)
  - Initial-initial: $p_1 + p_2 \to q + k_1 (+k_2)$, three scales: q^2, x_1, x_2
    - 2 → 3 tree level (32 master integrals)
    - 2 → 2 one loop (5 master integrals)

Initial-initial antenna functions

- are crossings of final-final antennae: four-parton case

quark-antiquark antennae	
A_4^0	$A_4^0(\widehat{q},\widehat{g},g,\overline{q})$, $A_4^0(\widehat{q},g,\widehat{g},\overline{q})$, $A_4^0(\widehat{q},g,g,\widehat{\overline{q}})$, $A_4^0(q,\widehat{g},\widehat{g},\overline{q})$
$\widetilde{A}_4^0$	$\widetilde{A}_4^0(\widehat{q},\widehat{g},g,\overline{q})$, $\widetilde{A}_4^0(\widehat{q},g,g,\widehat{\overline{q}})$, $\widetilde{A}_4^0(q,\widehat{g},\widehat{g},\overline{q})$
B_4^0	$B_4^0(\widehat{q},\widehat{q'},\overline{q}',\overline{q})$, $B_4^0(\widehat{q},q',\overline{q}',\widehat{\overline{q}})$, $B_4^0(q,\widehat{q'},\widehat{\overline{q}'},\overline{q})^*$
C_4^0	$C_4^0(\widehat{q},\widehat{\overline{q}},q,\overline{q})$, $C_4^0(\widehat{q},\overline{q},\widehat{q},\overline{q})$, $C_4^0(q,\widehat{\overline{q}},\widehat{q},\overline{q})^*$, $C_4^0(q,\overline{q},\widehat{q},\widehat{\overline{q}})^*$

quark-gluon antennae	
D_4^0	$D_4^0(\widehat{q},\widehat{g},g,g)$, $D_4^0(\widehat{q},g,\widehat{g},g)$, $D_4^0(q,\widehat{g},\widehat{g},g)$, $D_4^0(q,\widehat{g},g,\widehat{g})$
E_4^0	$E_4^0(\widehat{q},\widehat{q'},\overline{q}',g)$, $E_4^0(\widehat{q},q',\overline{q}',\widehat{g})$, $E_4^0(q,\widehat{q'},\widehat{\overline{q}'},g)$, $E_4^0(q,\widehat{q'},\overline{q}',\widehat{g})$,
$\widetilde{E}_4^0$	$\widetilde{E}_4^0(\widehat{q},\widehat{q'},\overline{q}',g)$, $\widetilde{E}_4^0(\widehat{q},q',\overline{q}',\widehat{g})$, $\widetilde{E}_4^0(q,\widehat{q'},\widehat{\overline{q}'},g)$, $\widetilde{E}_4^0(q,\widehat{q'},\overline{q}',\widehat{g})$

gluon-gluon antennae	
F_4^0	$F_4^0(\widehat{g},\widehat{g},g,g)$, $F_4^0(\widehat{g},g,\widehat{g},g)$
G_4^0	$G_4^0(\widehat{g},\widehat{q},\overline{q},g)$, $G_4^0(\widehat{g},q,\widehat{\overline{q}},g)$, $G_4^0(\widehat{g},q,\overline{q},\widehat{g})$, $G_4^0(g,\widehat{q},\widehat{\overline{q}},g)$
$\widetilde{G}_4^0$	$\widetilde{G}_4^0(\widehat{g},\widehat{q},\overline{q},g)$, $\widetilde{G}_4^0(\widehat{g},q,\overline{q},\widehat{g})$, $\widetilde{G}_4^0(g,\widehat{q},\widehat{\overline{q}},g)$
H_4^0	$H_4^0(\widehat{q},\widehat{\overline{q}},q',\overline{q}')$, $H_4^0(\widehat{q},\overline{q},\widehat{q'},\overline{q}')$

Integrated initial-initial antenna functions

- Double real radiation $p_1 + p_2 \longrightarrow q + k_j + k_k$
  - phase space factorization (A. Daleo, T. Gehrmann, D. Maitre)

$$\begin{aligned} d\Phi_{m+2}(k_1,\ldots,k_{m+2};p_1,p_2) &= d\Phi_m(\tilde{k}_1,\ldots,\tilde{k}_i,\tilde{k}_l,\ldots,\tilde{k}_{m+2};x_1p_1,x_2p_2) \\ &\quad \times\delta(x_1-\hat{x}_1)\,\delta(x_2-\hat{x}_2)\,[dk_j]\,[dk_k]dx_1\,dx_2 \end{aligned}$$

$$\begin{aligned} \hat{x}_1 &= \left(\frac{s_{12}-s_{j2}-s_{k2}}{s_{12}}\,\frac{s_{12}-s_{1j}-s_{1k}-s_{j2}-s_{k2}+s_{jk}}{s_{12}-s_{1j}-s_{1k}}\right)^{\frac{1}{2}} \\ \hat{x}_2 &= \left(\frac{s_{12}-s_{1j}-s_{1k}}{s_{12}}\,\frac{s_{12}-s_{1j}-s_{1k}-s_{j2}-s_{k2}+s_{jk}}{s_{12}-s_{j2}-s_{k2}}\right)^{\frac{1}{2}} \end{aligned}$$

  - require collinear rescaling: $p_1 \longrightarrow x_1p_1 \qquad p_2 \longrightarrow x_2p_2$
  - use Lorentz boost: $q \longrightarrow \tilde{q} = x_1p_1 + x_2p_2$
  - x_1, x_2 constrained with right behaviour in all unresolved limits

Integrated initial-initial antennae

(R. Boughezal, M. Ritzmann, AG)

$$\mathcal{X}^0_{il}(x_i,x_l) = \frac{1}{C^2(\epsilon)} \int [\mathrm{d}k_j]\,[\mathrm{d}k_k]\,\delta(x_i-\hat{x}_i)\delta(x_l-\hat{x}_l)X^0_{il,jk}(p_i,p_j,p_k,p_l)$$

- are linear combinations of 32 master integrals
  - coefficients contain poles in ε and rational factors in x_1,x_2
  - endpoint behaviour: $(1-x_1)^{-1-2\varepsilon}(1-x_2)^{-1-2\varepsilon}\,R(x_1,x_2)$
  - expansion in distributions around endpoints $x_1,x_2 = 1$
  - pole structure up to ε^{-4}
  - need to know the masters a priori up to transcendentality 4

Initial-initial integrated antenna

- Masters are calculated in different regions
  - Hard region ($x_1 \neq 1, x_2 \neq 1$)
    - up to transcendentality 2, yielding GHPL of weight 2 in x_1, x_2
  - Collinear regions ($x_1 = 1, x_2 \neq 1$ or $x_2 = 1, x_1 \neq 1$)
    - up to transcendentality 3, yielding HPL of weight 3 in x_1 or x_2
  - Soft region ($x_1 = 1$ and $x_2 = 1$)
    - up to transcendentality 4, yielding constants
- use differential equations in x_1, x_2 to compute masters in hard and collinear regions

Initial-initial integrated antenna

- First step: integrated antennae with two quark flavours
  - crossings of:
    - quark-antiquark antenna: $B_4^0(q, q', \bar{q}', \bar{q})$
    - quark-gluon antenna: $\tilde{E}_4^0(q, q', \bar{q}', g)$
    - gluon-gluon antenna: $H_4^0(q, \bar{q}, q', \bar{q}')$
  - contain 12 (out of 32) master integrals
- Full set in progress

Initial-initial antenna functions

- One-loop single unresolved real radiation: $p_1 + p_2 \to q + k_j$
  - phase-space overconstrained $\to$ no integrals, just expansions

$$\mathrm{d}\Phi_{m+1}(k_1,\ldots,k_{m+1};p_1,p_2) = \mathrm{d}\Phi_m(\tilde{k}_1,\ldots,\tilde{k}_{j-1},\tilde{k}_{j+1},\ldots,\tilde{k}_{m+1};x_1p_1,x_2p_2)$$
$$\delta(x_1-\hat{x}_1)\,\delta(x_2-\hat{x}_2)\,[\mathrm{d}k_j]\,\mathrm{d}x_1\,\mathrm{d}x_2$$

$$\hat{x}_1 = \left(\frac{s_{12}-s_{j2}}{s_{12}}\,\frac{s_{12}-s_{1j}-s_{j2}}{s_{12}-s_{1j}}\right)^{\frac{1}{2}} \qquad \hat{x}_2 = \left(\frac{s_{12}-s_{1j}}{s_{12}}\,\frac{s_{12}-s_{1j}-s_{j2}}{s_{12}-s_{j2}}\right)^{\frac{1}{2}}$$

  - analytically continue master integrals from final-final kinematics
    - one-loop boxes and bubbles
  - expand in distributions

- in progress

Antenna subtraction with massive particles

Towards subtraction for top quark pair production at NNLO

(G. Abelof, AG)

- First step: NLO antenna subtraction for $t\bar{t}$ and $t\bar{t}+j$
  - previous NLO results in dipole subtraction (A. Bredenstein, A. Denner, S. Dittmaier, S. Pozzorini; G. Bevilacqua et al., K. Melnikov, M. Schulze)
  - require massive phase space mappings
  - require massless antennae: final-final, initial-final, final-final
  - require massive antennae: final-final (M. Ritzmann, AG), initial-final (new)
    - need flavour-violating quark-antiquark antennae (new)
  - constructed colour-ordered antenna subtraction terms for

$$q\bar{q} \to t\bar{t}g \qquad gg \to t\bar{t}q \qquad gg \to t\bar{t}g$$
$$q\bar{q} \to t\bar{t}gg \qquad q\bar{q} \to t\bar{t}q\bar{q} \qquad gg \to t\bar{t}gg$$

Conclusions

- Towards jet cross sections at NNLO for hadron colliders: Progress on the antenna subtraction formalism
  - Implementation of antenna subtraction for double real radiation corrections to $gg \to gg$
    - Subtraction terms constructed and tested in all unresolved limits
  - Status of integrated NNLO antennae
    - Remaining NNLO initial-initial antennae under way
  - Massive antenna formalism under development
    - Subtraction terms for top pair production at NNLO under construction

Backup slides

$e^+e^- \to$ 3 jets and event shapes at NNLO

- NNLO results triggered
  - Progress on resummation
    - N^3LL for 1-T (T. Becher, M. Schwartz, R. Abbate et al.) and M_H (Y. Chien, M. Schwartz)
  - Progress on hadronization
    - Shape function approach for 1-T (R. Abbate et al.)
    - Dispersive model to NNLO (T. Gehrmann, G. Luisoni, M. Jaquier)
  - Reanalysis of data from LEP/PETRA/Tristan