Plain text:

Shape variables at hadron colliders

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Work done in collaboration with

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and
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Final states at the LHC are characterised by large hadron multiplicities.

Shape variables are IR and collinear (IRC) safe observables obtained from suitable combinations of hadron momenta (e.g. event shapes).

**:grinning_smiley: IRC safety** \(\Rightarrow\) **Hadronic final states can be described with PT QCD!**
1. Event shapes at hadron colliders

2. Jet shapes and non-global logarithms

3. Shape variables for New Physics
Outline

1. Event shapes at hadron colliders
2. Jet shapes and non-global logarithms
3. Shape variables for New Physics
Event shapes in hadron-hadron collisions

Event shapes explore the geometry of hadronic energy-momentum flow (i.e. if hadronic events are planar, spherical, etc.)

- Two examples: transverse thrust and thrust minor

\[ T_t \equiv \max_{\vec{n}_t} \frac{\sum_i |\vec{q}_{ti} \cdot \vec{n}_t|}{\sum_i q_{ti}} \]

\[ T_m \equiv \frac{\sum_i |\vec{q}_{ti} \times \vec{n}_t|}{\sum_i q_{ti}} \]

- Event shapes can involve also longitudinal momenta, e.g. total and heavy-jet mass \( \rho_T, \rho_H \), total and wide-jet broadening \( B_T, B_W \), three-jet resolution parameter \( y_{23} \)

- All event shapes we consider vanish in the two-jet limit
Resummation vs fixed order: the example of $T_m$

- **Fixed order** predictions (3 jets at NLO) diverge at small $T_m$
  
  [Nagy PRD 68 (2003) 094002]

- **Resummation** of large logarithms
  
  $\exp\{\alpha_s^n \ln^{n+1} T_m + \alpha_s^n \ln^n T_m\}$
  
  (NLL) restores correct physical behaviour for $T_m \to 0$
  
  [AB Salam Zanderighi JHEP 1006 (2010) 038]

![Graph showing the comparison of LO, NLO, NLL, and NLL+NLO predictions for $T_m$ distribution at Tevatron, $p_{t1} > 200$ GeV.]

**Peak of $T_m$ distribution** where $d/dT_m (d\sigma/dT_m) = 0 \Rightarrow \alpha_s \ln T_m \sim 1$

Peak position and height stabilised by NLL resummation
General NLL resummation for any suitable event shape is possible with the **Computer Automated Expert Semi-Analytical Resummer**

[AB Salam Zanderighi JHEP **0503** (2005) 073, qcd-caesar.org]

Given a computer subroutine that computes $V(k_1, \ldots, k_n)$, CAESAR:

1. checks whether $V$ is resummable within NLL accuracy
2. performs the NLL resummation using a general master formula

The core of the automation lies in:

- high-precision arithmetic to take soft and collinear limits
- methods of Experimental Mathematics to verify or falsify hypotheses

⚠️ **CAESAR is not one more parton shower**

- the produced results have the quality of analytical predictions
- an answer is provided only if NLL accuracy is guaranteed
An event shape $V(k_1, \ldots, k_n)$ is resummable at NLL accuracy if

1. $V(k)$ has a specific functional dependence on a single soft and emission $k$ collinear to a leg $\ell$

$$V(k) = \left( \frac{k_t}{Q} \right)^{a_\ell} e^{-b_\ell \eta} g_\ell(\phi)$$

2. it is (continuously) global, i.e. it is sensitive to soft/collinear emissions in the whole of the phase space

3. it is recursively IRC safe, i.e. it has good scaling properties with respect to multiple emissions

Globalness + rIRC safety + QCD coherence $\Rightarrow$ angular ordered parton branching accounts for all LL and NLL contributions
Classes of global event shapes

In spite of limited detector acceptance $|\eta| < \eta_0$ ($\sim 5$ at the LHC), it is possible to devise global event shapes even in hadron collisions

[AB Salam Zanderighi JHEP 0408 (2004) 062]

- **Directly global**: measure all hadrons up to $\eta_0$
  - NLL valid up to $v \sim e^{-c_V\eta_0}$, e.g. $T_m \sim e^{-\eta_0}$

- **Exponentially suppressed**: event shape in central region $C$ + exponentially suppressed forward term $\mathcal{E}_\bar{C}$
  - [Similar to recent proposal by Stewart Tackmann Waalewijn PRD 81 (2010) 094035]
  - % potentially affected by coherence violating logarithms?
  - [Forshaw Kyrieleis Seymour JHEP 0608 (2006) 059]

- **Recoil**: event shape in central region $C$ + recoil term $\mathcal{R}_{t,C}$
  - NLL predictions diverge at small $v$

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Diagram:

- $\mathcal{E}_{\bar{C}} \sim \sum_{i \notin C} q_{ti} e^{-|\eta_i - \eta_C|}$
- $\mathcal{R}_{t,C} \sim \left| \sum_{i \in C} \vec{q}_{ti} \right| = \sum_{i \notin C} \vec{q}_{ti}$
Estimate of theoretical uncertainties

Theoretical uncertainties are under control and within ±20%

- Asymmetric variation of $\mu_R$ and $\mu_F$ around $p_t = (p_{t1} + p_{t2})/2$
  
  \[
  p_t/2 \leq \mu_R \leq 2p_t
  \]
  
  \[
  \mu_R/2 \leq \mu_F \leq 2\mu_R
  \]

- Rescaling of the argument of the logs to be resummed
  
  \[
  \ln T_m \to \ln(X T_m), \quad 1/2 \leq X \leq 2
  \]

- Change the procedure to match NLL with NLO

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Shapes at hadron colliders
Sensitivity to hadronisation and underlying event

Three-jet fractions are hardly affected by hadronisation and UE

- PT predictions directly compared to data ⇒ PT consistency checks
- Suitable for tunings of parton shower parameters

Event-shape distributions get large corrections from UE

- Comparison to parton level MC for tests of parton shower
- Suitable for tests and tunings of UE models
Agreement between NLL and parton level MC is good for quark-dominated samples
Sizable differences in gluon dominated samples ⇒ new tests of initial state gluon branching?
Future developments for global observables

1. Straightforward extension to event shapes in processes with massive particles (Drell-Yan, Higgs, top, SUSY, etc.)
   - Characterisation of boson+jets with hadronic final states (out-of-plane radiation, jet mass, etc.)
   - Suitable event-shape distributions as central-jet vetoes
     [Stewart Tackmann Waalewijn PRL 105 (2010) 092002]

2. Resummation of transverse momentum distributions
   - Globalness and rIRC safety $\Rightarrow$ angular ordered branching at NLL
   - LL do not exponentiate in variable space $\Rightarrow$ CAESAR’s automated predictions diverge for small transverse momentum
   - Check resummability conditions and perform analytic resummation in impact parameter space (see e.g. $Z$-boson $a_T$ distribution)
     [AB Dasgupta Duran-Delgado JHEP 0912 (2009) 022]

3. Automated NNLL resummation $\Rightarrow$ new physical picture needed due to interplay between logarithms $\alpha_s^n L^m$ and constants $\alpha_s^m$
   - Precision determination of $\alpha_s(M_Z)$ using $e^+ e^-$ event shapes
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Event shapes inside a jet

Jet shapes are defined using hadrons in a single jet

- Less sensitive to initial-state radiation and underlying event
- Their distributions depend strongly (at the LL level) on the underlying jet flavour (quark or gluon jet)

Example: angularities of the observed jet, with jet minimum transverse energy $E_0$

[Ellis Hornig Lee Vermilion Walsh PLB 689 (2010) 82]

Example of angularity: distribution in jet invariant mass $M_{j_1}^2$

$$\Sigma(\rho, E_0) = \text{Prob} \left[ \frac{M_{j_1}^2}{Q^2} < \rho, \sum_{i \notin \text{jets}} k_{ti} < E_0 \right]$$
Jet-shape distributions like $\Sigma(\rho, E_0)$ are non-global, because no hadrons are measured inside the unobserved jets $j_2, j_3, \ldots, j_N$

Non-global observables receive extra NLL contributions from soft large-angle gluons

- **Non-global logarithms**: gluons inside a jet coherently emitting a softer gluon in the interjet region (or vice versa)

These non-abelian contributions are resummed only in the large-$N_c$ limit by solving a non-linear evolution equation

New sources of NLL contributions

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Non-global observables receive extra NLL contributions from soft large-angle gluons

- **Jet-clustering logarithms**: gluons independently emitted in two different angular regions get recombined in the same jet

Resummed numerically with a generalisation of CAESAR branching algorithm from soft large-angle gluons

[AB Dasgupta PLB 628 (2005) 49]

In the anti-$k_t$ algorithm (i.e. jets = circular cones of radius $R$), jet-clustering logarithms are absent
NLL resummation for jet shapes

General NLL resummation of jet shapes for well-separated jets with the scale hierarchy $p_{t,jets} \sim Q \gg E_0 \gg \rho Q / R^2$

$$\Sigma(\rho, E_0) = \Sigma^{sc} \left( \frac{R^2}{\rho}, \frac{Q}{E_0} \right) S^{ng} \left( \frac{R^2}{\rho}, \frac{Q}{E_0} \right) \Sigma^{\text{cluster}}(\rho)$$

- $\Sigma^{sc}$ is the jet-shape distribution obtained with only soft and collinear real emissions (the CAESAR’s way)
- $S^{ng}$ is the contribution from non-global logarithms
- $S^{ng} \left( \frac{R^2}{\rho}, \frac{Q}{E_0} \right) = S_{j_1} \left( \frac{E_0}{\rho Q / R^2} \right) \prod_{i=2}^{N} S_{j_i} \left( \frac{Q}{E_0} \right)$
- $S^{ng}$ is the product of individual contributions of each jet
- $\Sigma^{\text{cluster}}(\rho)$ is the contribution of jet-clustering logs

Both non-global and jet-clustering logarithms are finite for $R \rightarrow 0$
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[AB Dasgupta Khelifa-Kerfa Marzani JHEP 1008 (2010) 064]

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Both non-global and jet-clustering logarithms are finite for $R \to 0$
Impact of non-global logarithms

Toy example: two jets in $e^+ e^-$ annihilation with the anti-$k_t$ algorithm

$$\Sigma^{ng} \left( \frac{R^2}{\rho}, \frac{Q}{E_0} \right) = S_{\text{meas}} \left( \frac{E_0}{\rho Q / R^2} \right) S_{\text{unmeas}} \left( \frac{Q}{E_0} \right)$$

Non-global logarithms arise when emissions in two different angular regions have widely separated characteristic scales

- Non-global logs modify the peak height in distributions
- It is not possible to play with scales so as to get rid simultaneously of all non-global logarithms
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Which shapes for new physics?

New Physics events are generally broader than dijet events

SUSY multi-jet event  
Black hole production

Use event shapes to discriminate among different topologies?
Discrimination between two- and multi-jet events

Consider a maximally symmetric event in the transverse plane

N = 2  
N = 3  
N = 4  
.......  
N = ∞

Event shapes can discriminate between two- and multi-jet events

Current event shapes are not monotonic with number of jets ⇒ no distinction among different multi-jet samples
Consider two selected 3-jet events at $\eta = 0$ with Herwig parton shower

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- IRC safe shape variables give better resolution in discriminating among different topologies in a given $n$-jet sample.

- Variables like $B_{T,C}$ or $\rho_{T,C}$, equally sensitive to transverse and longitudinal degrees of freedom, better suited for identification of massive particle decays.

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Shapes at hadron colliders
Sensitivity to spherical topologies

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Phenomenology of **global event shapes** at hadron colliders is very rich and challenging

- First ever **NLL+NLO predictions** with full theoretical uncertainties
- Event shapes extremely useful for tuning of **MC shower and UE**

**Event-shape measurements** have been performed at the Tevatron and are being performed at the LHC

Resummation of **non-global observables** remains extremely tricky

- Non-global logarithms are well understood in the large-$N_c$ limit
- Jet-clustering logarithms can be computed at all orders but very little general features are known (e.g. behaviour with jet radius)
- Coherence violating logarithms might have large impact

**Important research directions**

- Better event shapes for **New Physics** searches
- Transverse momentum resummations (e.g. $t\bar{t}$, dijets)
- Automated NNLL for global observables