



Mueller-Navelet Jets at LHC: Complete NLL calculation

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In collaboration with

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Motivation and Outline

- Motivations
 - One of the important longstanding theoretical questions raised by QCD is its behaviour in the **high-energy** (Regge) **limit** $s \gg -t$
 - E.g., to establish the Regge behaviour of QCD, predict the Regge trajectories (growth exponents of particular amplitudes)
- Outline
 - Identify processes suited for study of high energy QCD dynamics (among which Mueller-Navelet jets);
 - Review the most important aspects of PT QCD at high energies (BFKL approach);
 - Describe details of the factorization formula for MN jets and the factors for a complete NLL study;
 - Numerical results: predictions for LHC phenomenology of MN jets;
 - Unexpected conclusions stemming from our results.

How to test QCD in the Regge limit?

Look for high-energy observables

- accessible at present and (near) future colliders (Tevatron, LHC, ...)
- calculable within perturbative QCD (large scales: hard γ^* , heavy mesons (J/ψ , Υ), energetic [forward] jets)
- insensitive to partonic content of hadrons, to hadronization, and to standard collinear evolution (DGLAP)

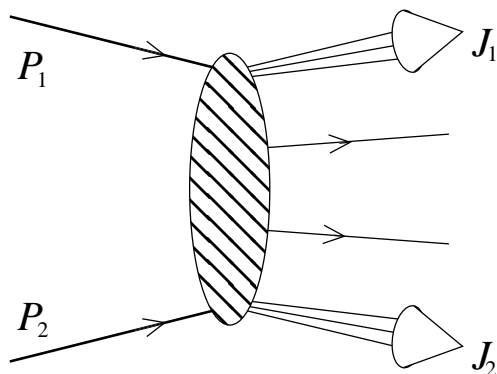
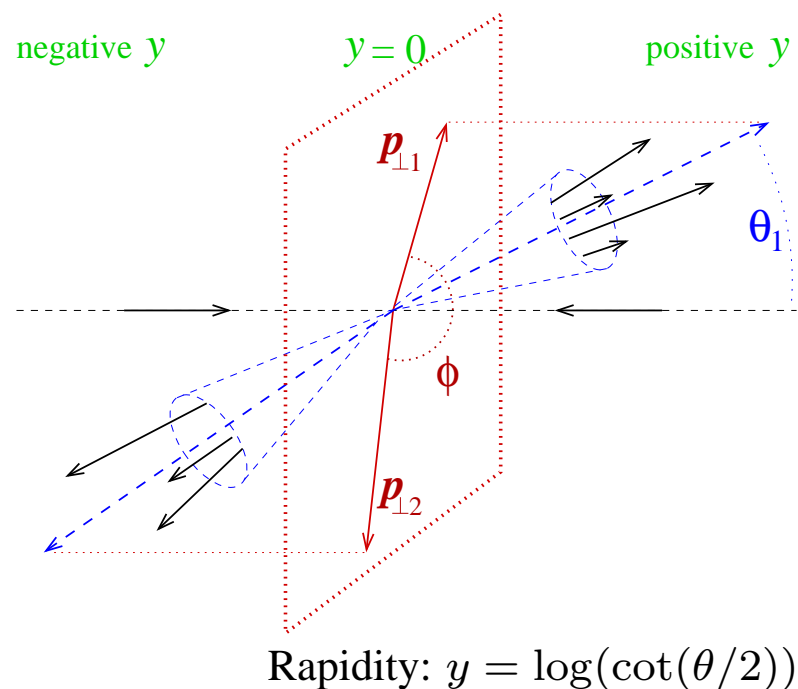
Criteria met by semi-hard processes with $s \gg k_i^2 \gg \Lambda_{\text{QCD}}^2$, where k_i^2 are typical transverse scales, all of the same order

Mueller-Navelet jets

One of most famous testing processes for studying PT high-energy QCD at hadron colliders [Mueller Navelet 1987]

Final states with two jets with similar E_T and large rapidity separation

- Comparable hard scales (jet energies)
limit the logarithms of collinear type $\log(E_1/E_2)$
- Big separation in rapidity $Y \equiv y_1 - y_2 \Rightarrow$ large $\log(s/E_J^2) \sim Y$

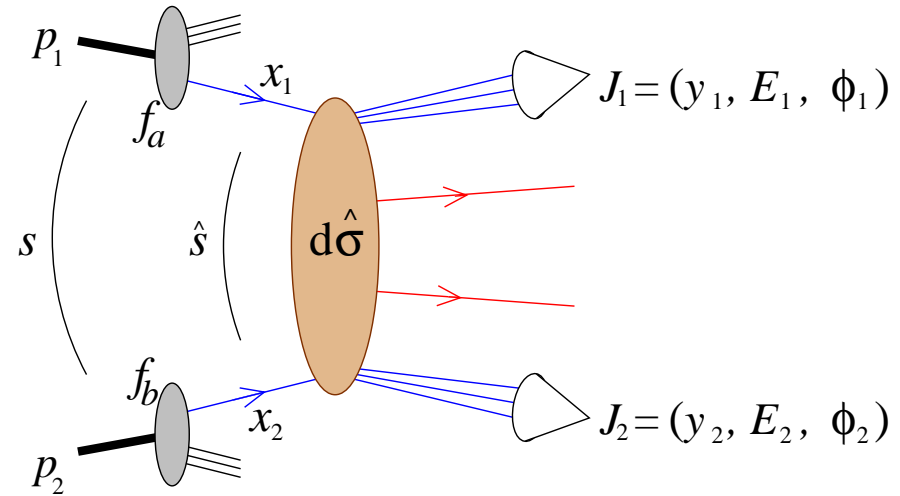


Anything can be emitted between the jets

Factorization of NP effects

MN propose use of factorization theorem:

$$\frac{d\sigma}{(dy_1 dE_1 d\phi_1)(dy_2 dE_2 d\phi_2)} = \sum_{a,b=g,u,d,s,\dots} \int_0^1 dx_1 dx_2 f_a(x_1, E_{J_1}^2) f_b(x_2, E_{J_2}^2) \frac{d\hat{\sigma}(x_1, x_2)}{dJ_1 dJ_2}$$



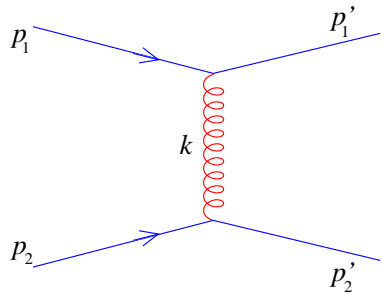
Factorization formula justified because:

- semi-inclusive observable (jets + anything)
- large transferred momenta ($E_J \gg \Lambda_{\text{QCD}}$)

QCD at large s

Factorization theorem \Rightarrow perturbatively compute partonic cross sections

At high energy, gluon exchanges dominate



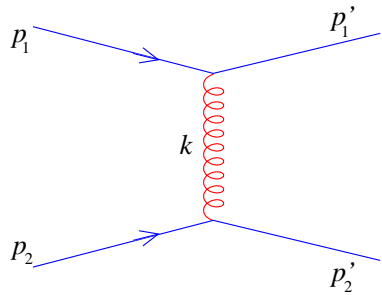
Lowest order:

constant (in s) cross section

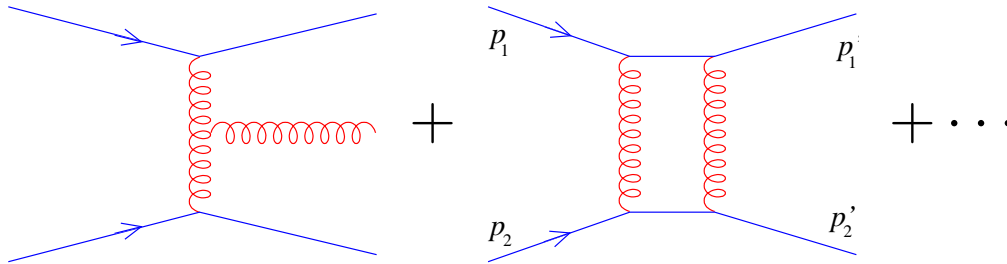
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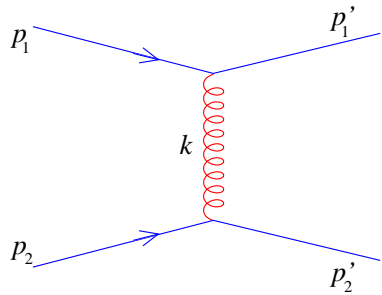


First order corrections show
increasing cross section $\sim \alpha_s \log s$

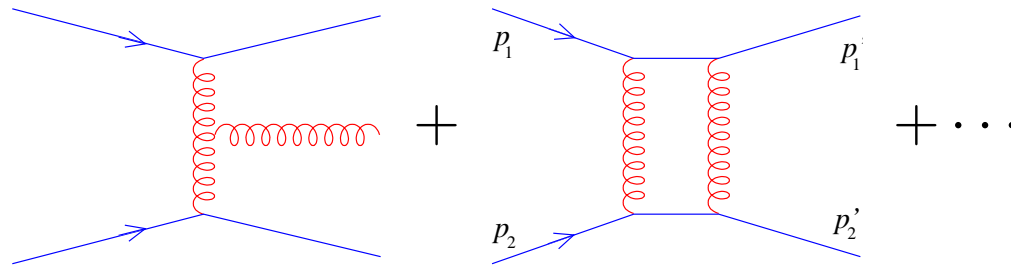
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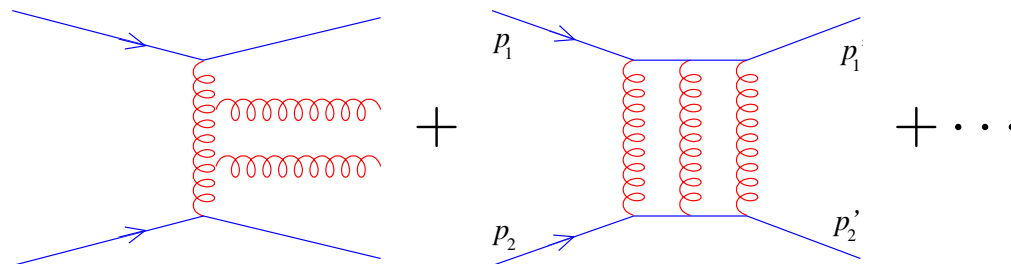
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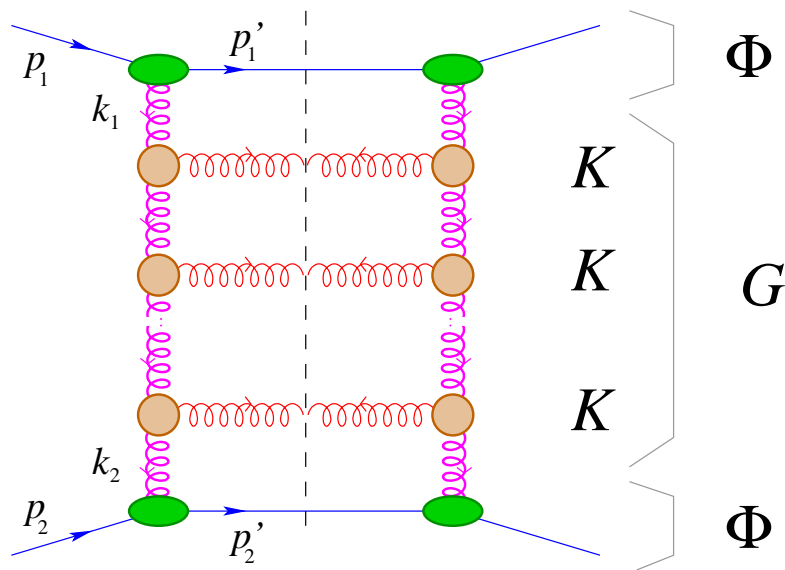
First order corrections show
increasing cross section $\sim \alpha_s \log s$



Second order corrections
yield $\sim (\alpha_s \log s)^2$ contributions
and so on

BFKL theory

[BFKL '78] QCD amplitudes have $\sim (\alpha_s \log s)^n$ enhancements to all perturbative orders (LL approx)



$$\sigma_{12}(s) = \int d\mathbf{k}_1 d\mathbf{k}_2 \Phi_1(\mathbf{k}_1) G(s, \mathbf{k}_1, \mathbf{k}_2) \Phi_2(\mathbf{k}_2)$$

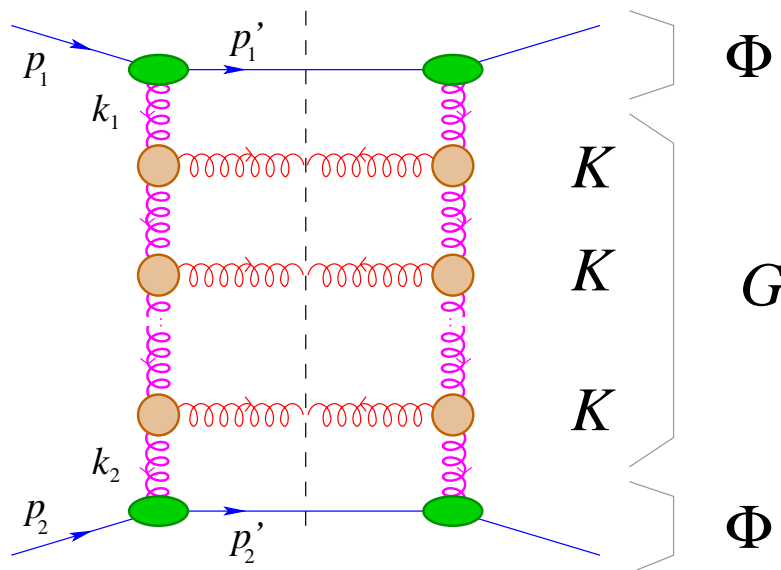
$$\frac{\partial}{\partial \log s} G(s, \mathbf{k}_1, \mathbf{k}_2) = \int d\mathbf{k} K(\mathbf{k}_1, \mathbf{k}) G(s, \mathbf{k}, \mathbf{k}_2)$$

$$K = \alpha_s K_0 + \alpha_s^2 K_1 + \dots$$

$$G(s, \mathbf{k}, \mathbf{k}) = G(s_0, \mathbf{k}, \mathbf{k}) \left(\frac{s}{s_0} \right)^{\omega_{\mathbb{P}}}$$

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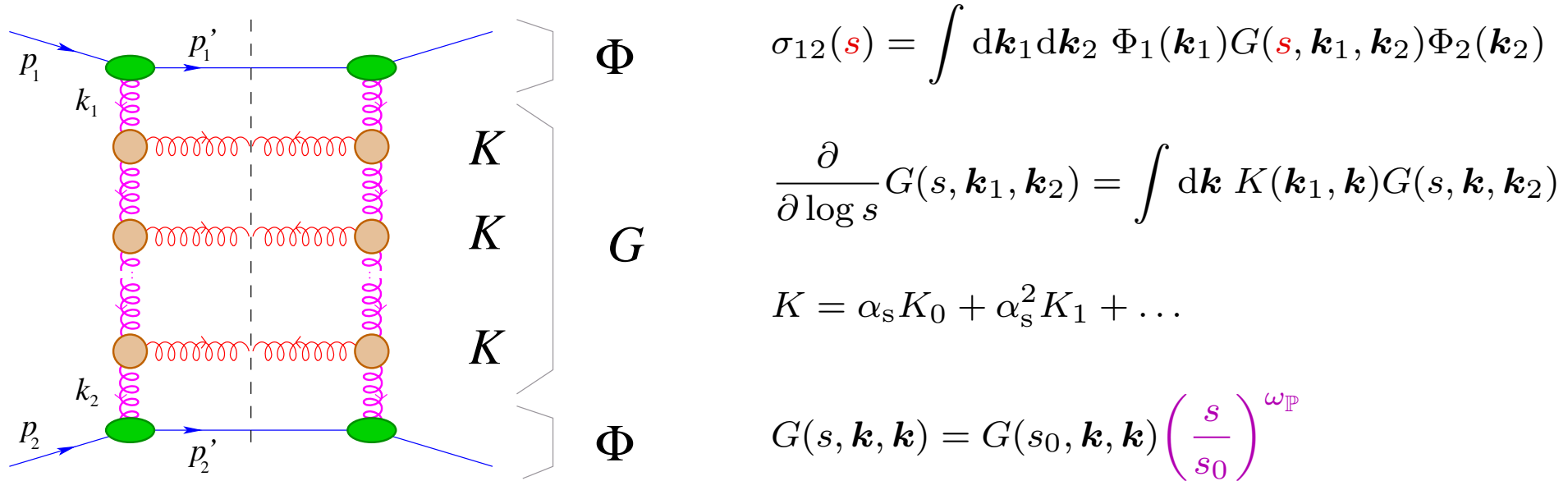
$$G(s, \mathbf{k}, \mathbf{k}) = G(s_0, \mathbf{k}, \mathbf{k}) \left(\frac{s}{s_0} \right)^{\omega_{\mathbb{P}}}$$

The **Pomeron** (?) shows up as a colour-singlet state of two (Reggeized) gluons.

But: $\omega_{\mathbb{P}} = 4 \log(2) N_c \alpha_s / \pi \simeq 0.5 \gg 0.08$ (for $\alpha_s \simeq 0.2$)

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NLL approx resums $\alpha_s^n \log^{n-1} s$ [Fadin-Lipatov, Camici-Ciafaloni '98]

Even worse: $\omega_{\mathbb{P}} = 4 \log(2) N_c \alpha_s / \pi (1 - 6.7 \alpha_s) \simeq -0.15$ (for $\alpha_s \simeq 0.2$)

Possible issues:

- $s^{\omega_{\mathbb{P}}}$ is asymptotic behaviour. At present energies preasymptotic behaviour, exact kinematics and RG improvements are important
- need full NL treatment of impact factors (also for estimate of errors)

Known NLL impact factors

Impact factors are known in some cases at NLL

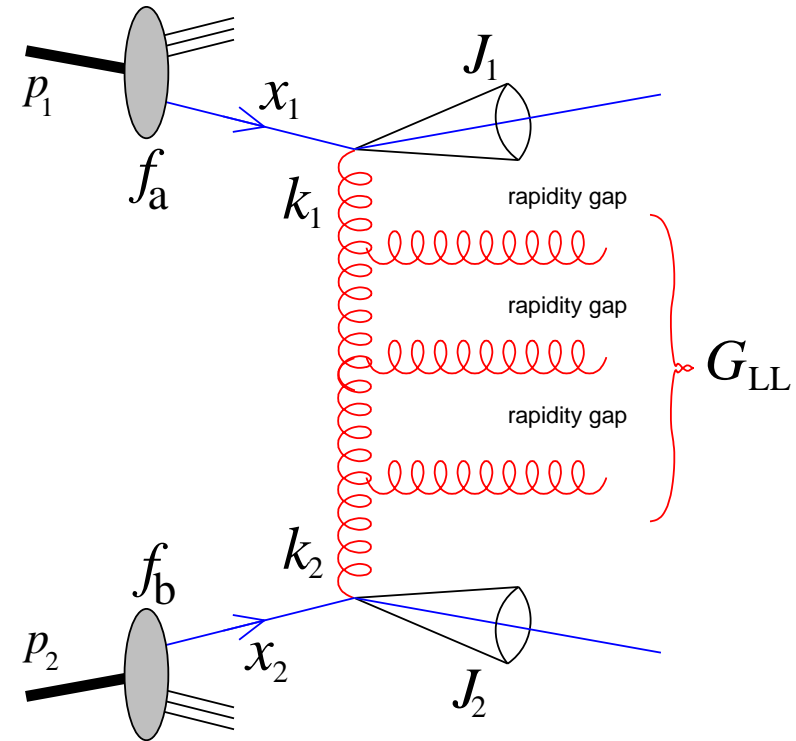
- $\gamma^* \rightarrow$ anything [*Bartels, Colferai, Gieseke, Kyrieleis, Qiao*]
- $\gamma^* \rightarrow \rho$ in forward limit [*Ivanov, Kotsky, Papa*]
- forward jet production [*Bartels, Colferai, Vacca*]

MN Jets in NL approximation

[Bartels, DC, Vacca '02] computed NLL calculations of impact factors for Mueller-Navelet jets

Starting from LL factorization formula

$$\begin{aligned} \frac{d\sigma(s)}{dJ_1 dJ_2} = & \sum_{a,b} \int_0^1 dx_1 dx_2 \int d\mathbf{k}_1 d\mathbf{k}_2 \\ & \times f_a(x_1) \\ & \times V_a^{(0)}(x_1, \mathbf{k}_1; J_1) \\ & \times G_{LL}(x_1 x_2 s, \mathbf{k}_1, \mathbf{k}_2) \\ & \times V_b^{(0)}(x_2, \mathbf{k}_2; J_2) \\ & \times f_b(x_2) \end{aligned}$$



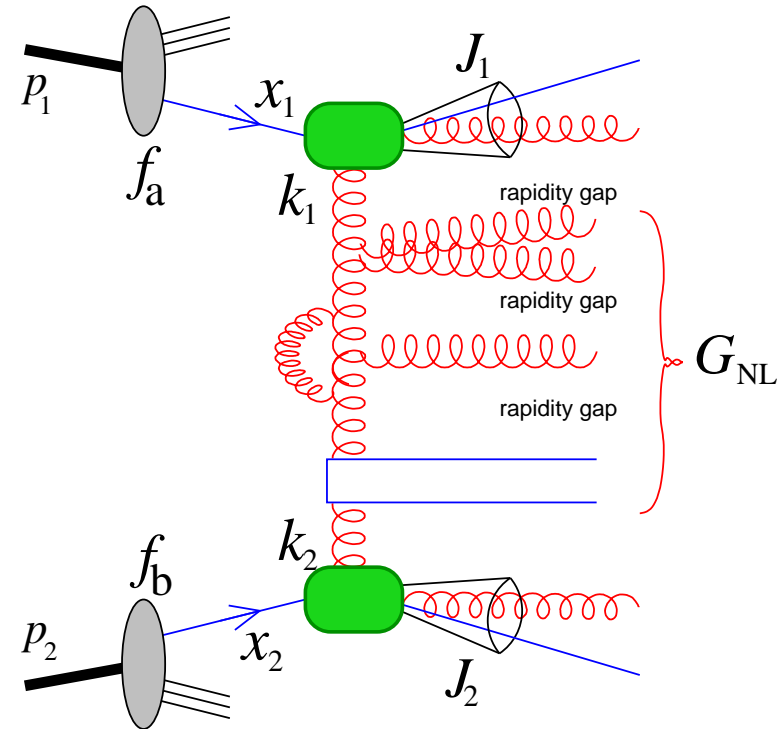
where $V_a^{(0)}(x, \mathbf{k}; J) = \alpha_s C_a \delta(\mathbf{k} - \mathbf{p}_J) \delta(x - x_J)$ and $x_J = |\mathbf{p}_J| e^{y_J} / \sqrt{s}$

MN Jets in NL approximation

[Bartels, DC, Vacca '02] computed NLL calculations of impact factors for Mueller-Navelet jets

Assume NLL factorization formula

$$\begin{aligned} \frac{d\sigma(s)}{dJ_1 dJ_2} &= \sum_{a,b} \int_0^1 dx_1 dx_2 \int d\mathbf{k}_1 d\mathbf{k}_2 \\ &\times f_a(x_1) \\ &\times V_a^{(1)}(x_1, \mathbf{k}_1; J_1) \\ &\times G_{\text{NL}}(x_1 x_2 s, \mathbf{k}_1, \mathbf{k}_2) \\ &\times V_b^{(1)}(x_2, \mathbf{k}_2; J_2) \\ &\times f_b(x_2) \end{aligned}$$



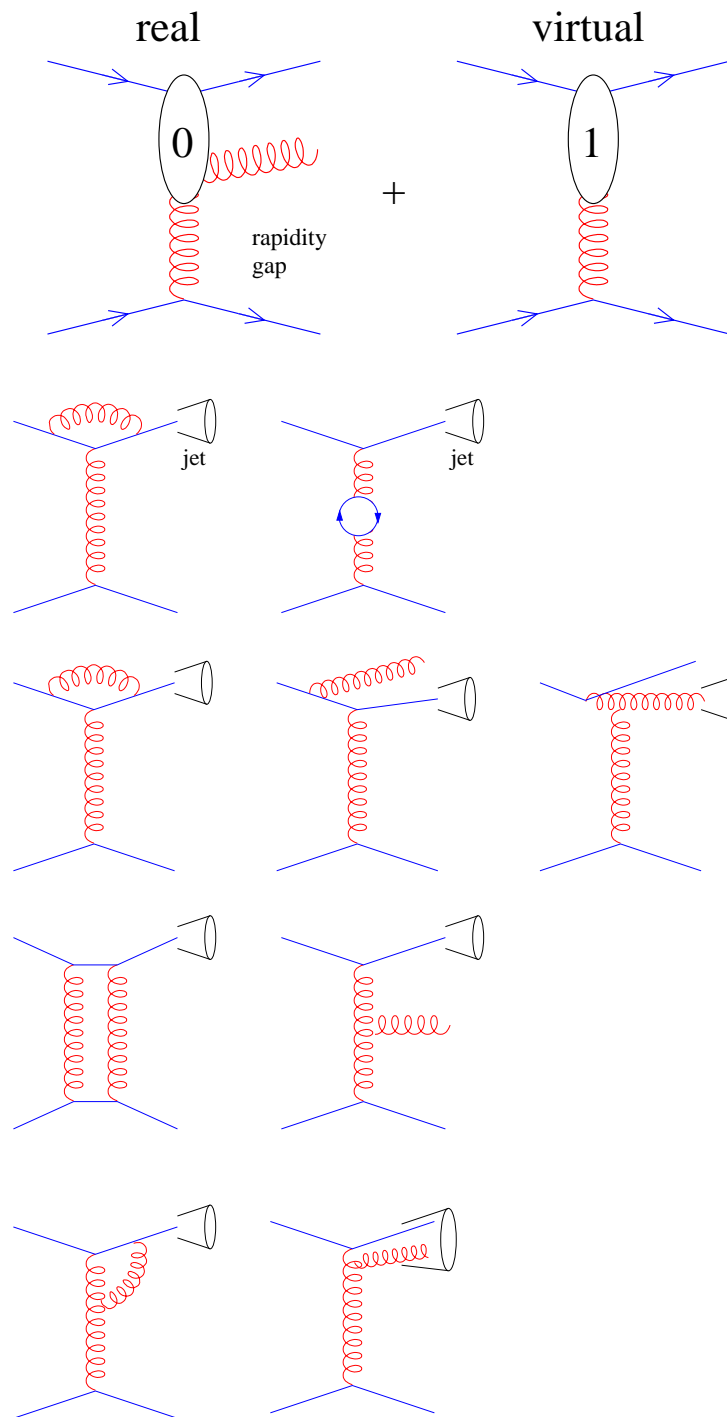
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- At NL level the jet vertex condition is non-trivial (e.g. depends on jet radius R)
- Factorization formula must be proven [Bartels, DC, Vacca '02]

NL jet vertex

Proof of NL factorization formula require disentangling various structures from 1-loop diagrams:

- UV divergencies (absorbed by running coupling)
- IR collinear divergencies (absorbed by PDFs)
- $\log(s)$ enhanced contributions (to build GGF)
- all remaining IR singularities must cancel, to yield finite (in ϵ) and constant (in s) terms to be identified with $V^{(1)}$

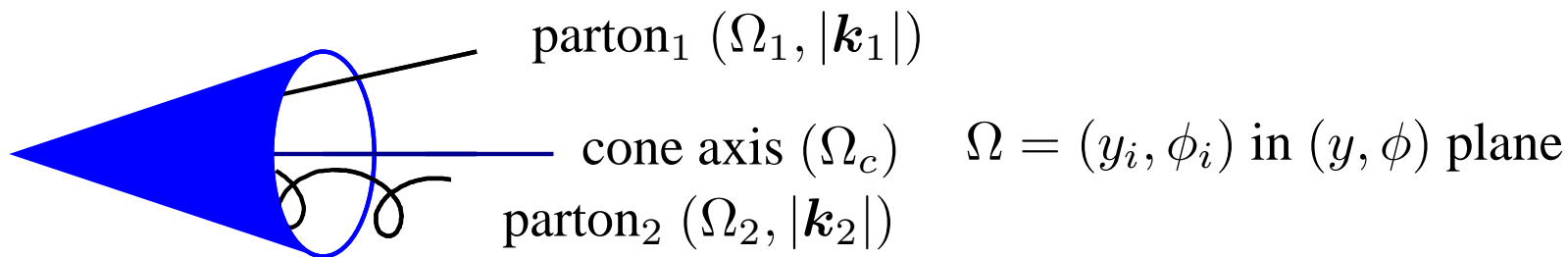


Cone jet algorithm

- Should partons $(|\mathbf{k}_1|, \phi_1, y_1)$ and $(|\mathbf{k}_2|, \phi_2, y_2)$ combined in a single jet?
 $|\mathbf{k}_i|$ =transverse energy deposit in the calorimeter cell i of parameter $\Omega = (y_i, \phi_i)$ in (y, ϕ) plane
- define transverse energy of the jet: $E_J = |\mathbf{k}_1| + |\mathbf{k}_2|$

- jet axis:

$$\Omega_c \begin{cases} y_J = \frac{|\mathbf{k}_1| y_1 + |\mathbf{k}_2| y_2}{E_J} \\ \phi_J = \frac{|\mathbf{k}_1| \phi_1 + |\mathbf{k}_2| \phi_2}{E_J} \end{cases}$$



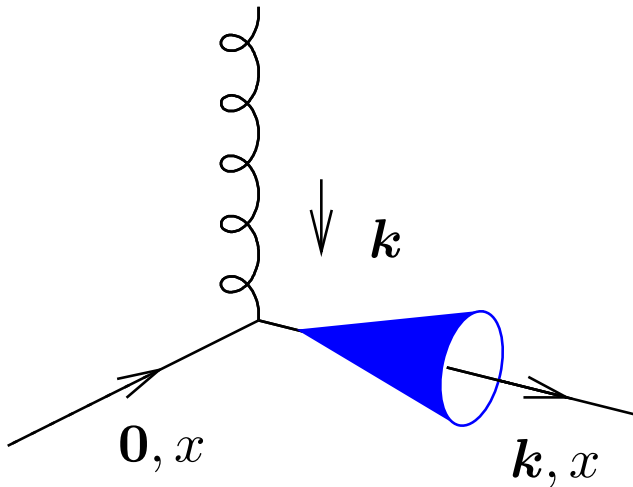
If distances $|\Omega_i - \Omega_c|^2 \equiv (y_i - y_c)^2 + (\phi_i - \phi_c)^2 < R^2$ ($i = 1$ and $i = 2$)

\implies partons 1 and 2 are in the same cone Ω_c [Ellis, Kunszt, Soper]

combined condition: $|\Omega_1 - \Omega_2| < \frac{|\mathbf{k}_1| + |\mathbf{k}_2|}{\max(|\mathbf{k}_1|, |\mathbf{k}_2|)} R$

Jet algorithm: LL

$\mathbf{k}, \mathbf{k}' =$ Euclidian two dimensional vectors

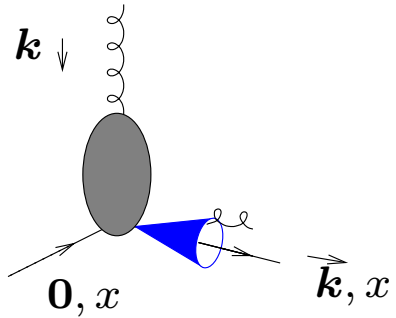


$$S_J^{(2)}(\mathbf{k}; x) = x_J \delta(x - x_J) |\mathbf{k}| \delta^{(2)}(\mathbf{k} - \mathbf{k}_J)$$

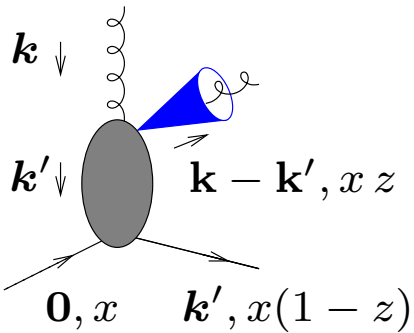
Jet algorithm: **NLL**

$\mathbf{k}, \mathbf{k}' =$ Euclidian two dimensional vectors

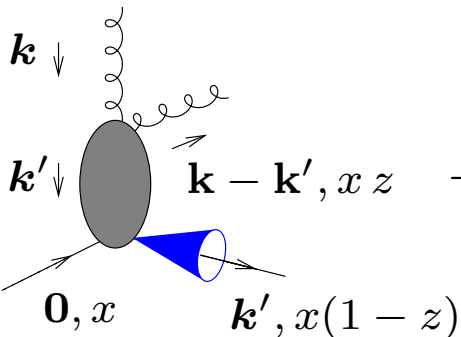
$$S_J^{(3, \text{cone})}(\mathbf{k}', \mathbf{k} - \mathbf{k}', xz; x) =$$



$$S_J^{(2)}(\mathbf{k}, x) \Theta \left(\left[\frac{|\mathbf{k} - \mathbf{k}'| + |\mathbf{k}'|}{\max(|\mathbf{k} - \mathbf{k}'|, |\mathbf{k}'|)} R_{\text{cone}} \right]^2 - [\Delta y^2 + \Delta \phi^2] \right)$$



$$+ S_J^{(2)}(\mathbf{k} - \mathbf{k}', xz) \Theta \left([\Delta y^2 + \Delta \phi^2] - \left[\frac{|\mathbf{k} - \mathbf{k}'| + |\mathbf{k}'|}{\max(|\mathbf{k} - \mathbf{k}'|, |\mathbf{k}'|)} R_{\text{cone}} \right]^2 \right)$$



$$+ S_J^{(2)}(\mathbf{k}', x(1-z)) \Theta \left([\Delta y^2 + \Delta \phi^2] - \left[\frac{|\mathbf{k} - \mathbf{k}'| + |\mathbf{k}'|}{\max(|\mathbf{k} - \mathbf{k}'|, |\mathbf{k}'|)} R_{\text{cone}} \right]^2 \right),$$

NL (quark-initiated) jet vertex

$$\begin{aligned}
 V_q^{(1)}(\mathbf{k}, x) = & \left[\left(\frac{3}{2} \log \frac{\mathbf{k}^2}{\mu_F^2} - 2 \right) \frac{C_F}{\pi} + \left(\frac{85}{36} + \frac{\pi^2}{4} \right) \frac{C_A}{\pi} - \frac{5}{18} \frac{N_f}{\pi} - b_0 \log \frac{\mathbf{k}^2}{\mu_R^2} \right] V_q^{(0)}(\mathbf{k}, x) \\
 & + \int dz V_q^{(0)}(\mathbf{k}, xz) \left\{ \frac{C_F}{\pi} \left[\frac{1-z}{2} + \left(\frac{\log(1-z)}{1-z} \right)_+ (1+z^2) \right] + \frac{C_A}{\pi} \frac{z}{2} \right\} \\
 & + \frac{C_A}{\pi} \int \frac{d\mathbf{k}'}{\pi} \int dz \left[\frac{1}{2} P_{qq}(z) \left((1-z) \frac{\mathbf{q} \cdot (\mathbf{q} - \mathbf{k})}{\mathbf{q}^2 (\mathbf{q} - \mathbf{k})^2} h_q^{(0)}(\mathbf{k}') \mathcal{S}_J^{(3)}(\mathbf{k}', \mathbf{q}, xz; x) + \right. \right. \\
 & \left. \left. - \frac{1}{\mathbf{k}'^2} \Theta(\mu_F^2 - \mathbf{k}'^2) V_q^{(0)}(\mathbf{k}, xz) \right) - \frac{1}{z\mathbf{q}^2} \Theta(|\mathbf{q}| - z(|\mathbf{q}| + |\mathbf{k}'|)) V_q^{(0)}(\mathbf{k}', x) \right] \\
 & + \frac{C_F}{2\pi} \int dz \frac{1}{(1-z)_+} (1+z^2) \int \frac{d\mathbf{l}}{\pi l^2} \left[\frac{\mathcal{N} C_F}{l^2 + (\mathbf{l} - \mathbf{k})^2} \right. \\
 & \times \left(\mathcal{S}_J^{(3)}(z\mathbf{k} + (1-z)\mathbf{l}, (1-z)(\mathbf{k} - \mathbf{l}), x(1-z); x) \right. \\
 & \left. \left. + \mathcal{S}_J^{(3)}(\mathbf{k} - (1-z)\mathbf{l}, (1-z)\mathbf{l}, x(1-z); x) \right) \right. \\
 & \left. - \Theta(\mu_F^2 - \mathbf{l}^2) \left(V_q^{(0)}(\mathbf{k}, xz) + V_q^{(0)}(\mathbf{k}, x) \right) \right]
 \end{aligned}$$

Implementation

[DC, Schwennsen, Szymanowski, Wallon '09] used:

- MSTW 2008 PDFs (available as Mathematica packages)
- $\mu_R = \mu_F$ (this is imposed by the MSTW 2008 PDFs)
- two-loop running coupling $\alpha_s(\mu_R^2)$
- all numerical calculations are done in Mathematica
- we use Cuba integration routines (in practice Vegas):
precision 10^{-2} for 500.000 max points per integration

Numerical Results

$$\sqrt{s} = 14 \text{ TeV}$$

$$E_J = 35, 50 \text{ GeV}$$

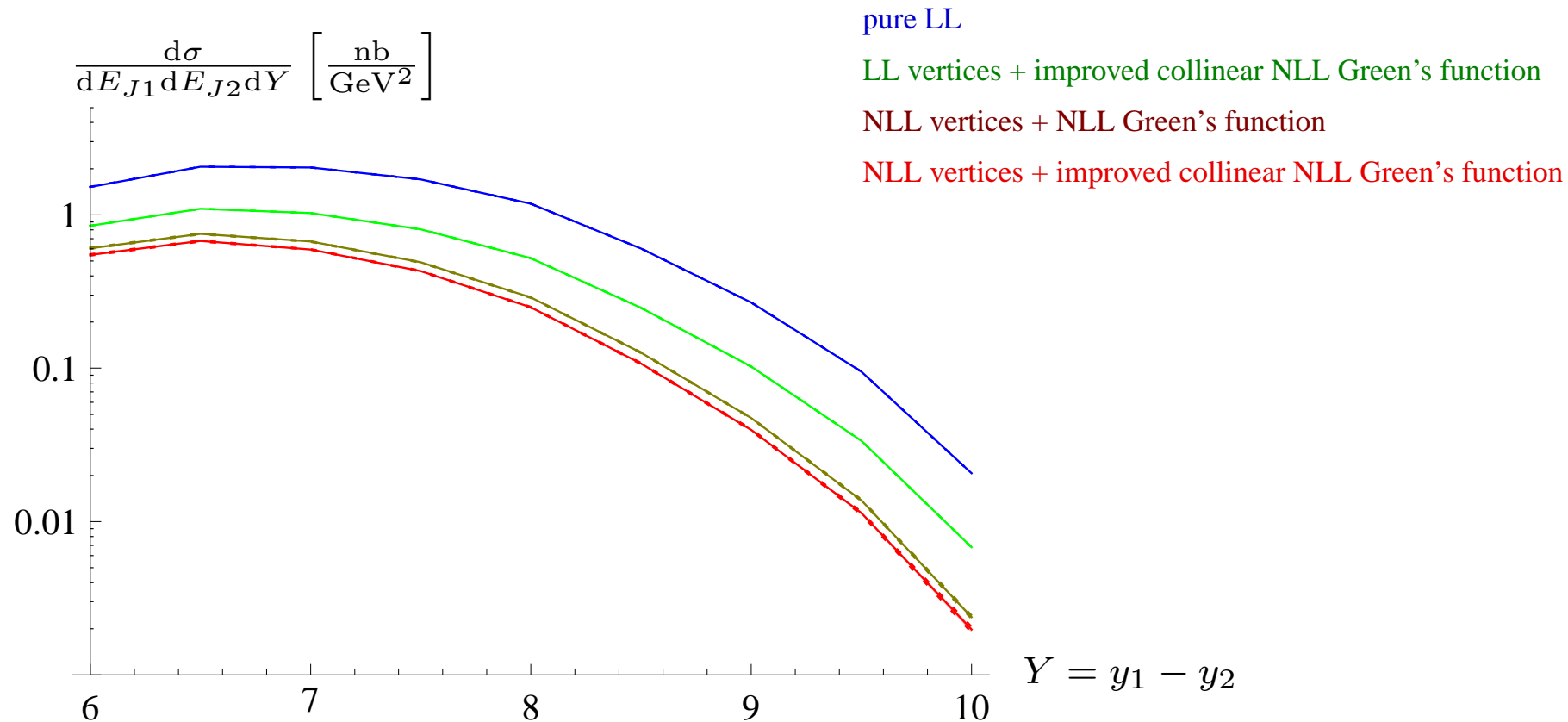
$$3 \leq y_J \leq 5$$

$$Y \equiv y_1 - y_2$$

$$6 \leq Y \leq 10$$

Symmetric configuration ($E_{J_1} = E_{J_2} = 35 \text{ GeV}$)

Cross-section



Differential cross section in dependence on Y for $E_{J_1} = E_{J_2} = 35 \text{ GeV}$.

error bands = errors due to the Monte Carlo integration (2% to 5%)

The effect of NLL vertex correction is very sizeable, comparable with NLL Green's function effects

Symmetric configuration ($E_{J_1} = E_{J_2} = 35 \text{ GeV}$)

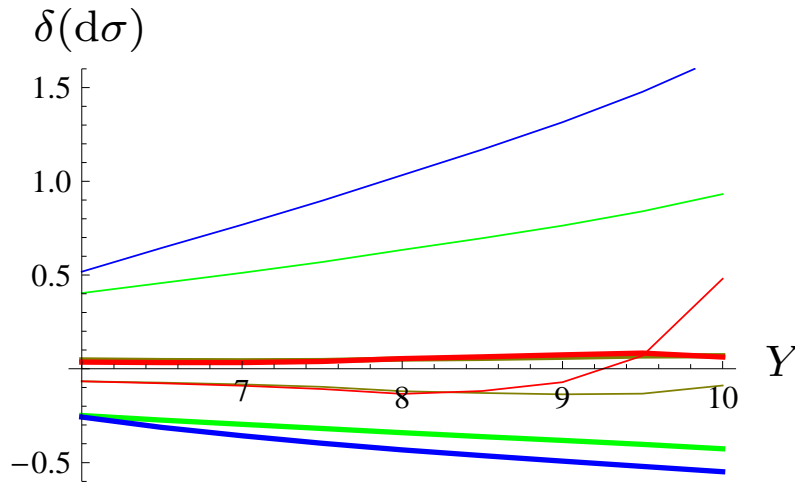
Cross-section: stability with respect to $\mu_R = \mu_F$ and s_0 changes

pure LL

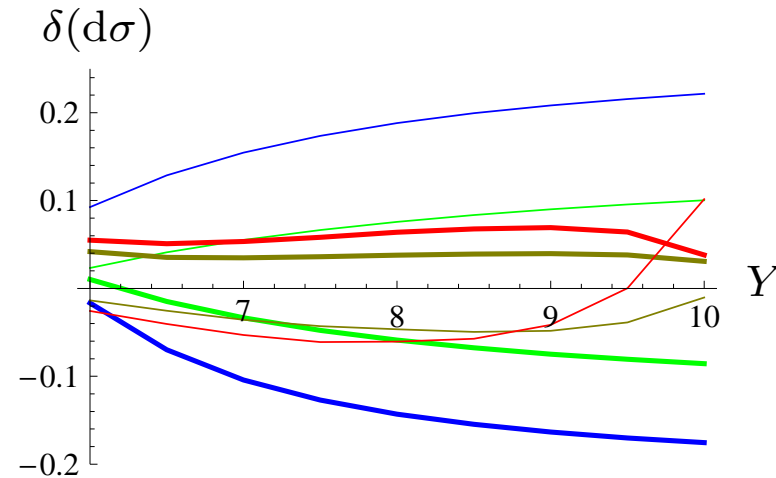
LL vertices + improved collinear NLL Green's function

NLL vertices + NLL Green's function

NLL vertices + improved collinear NLL Green's function



Relative effect of changing $\mu_R = \mu_F$ by factors 2 (thick) and 1/2 (thin)



Relative effect of changing $\sqrt{s_0}$ (in $\log(\frac{s}{s_0})$) by factors 2 (thick) and 1/2 (thin)

Symmetric configuration ($E_{J_1} = E_{J_2} = 35 \text{ GeV}$)

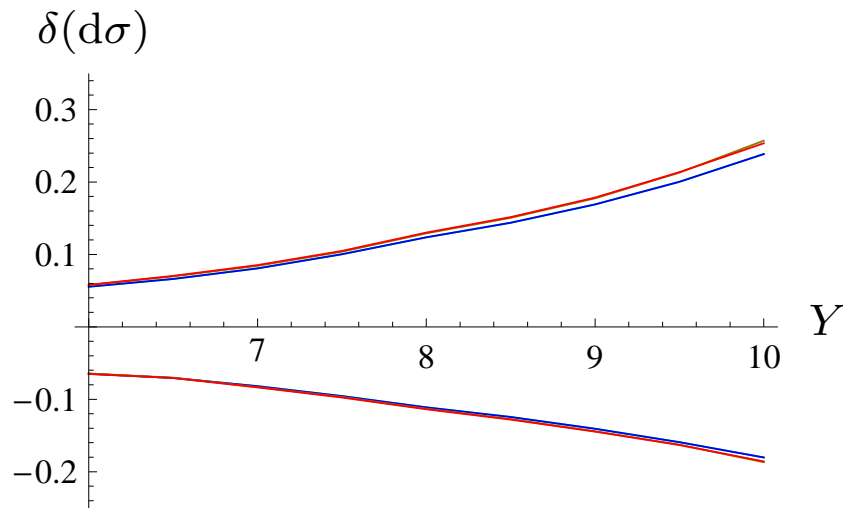
Cross-section: PDF and Monte Carlo errors

pure LL

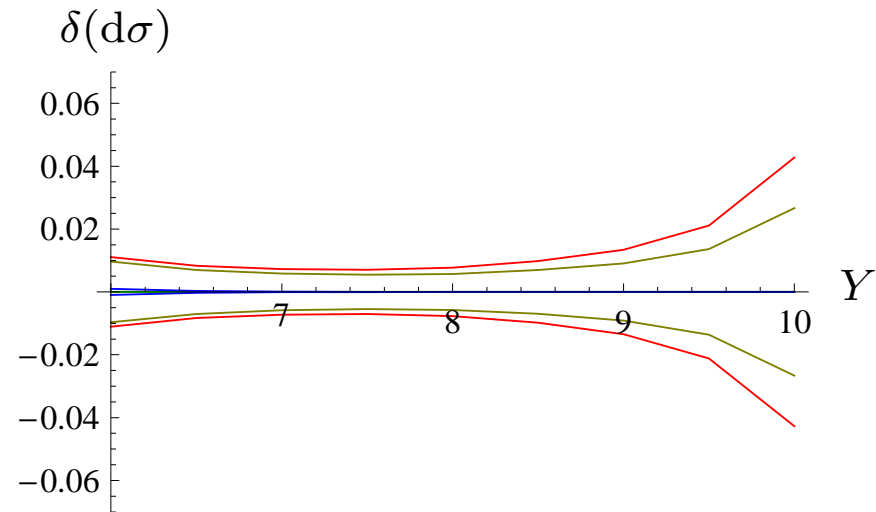
LL vertices + improved collinear NLL Green's function

NLL vertices + NLL Green's function

NLL vertices + improved collinear NLL Green's function



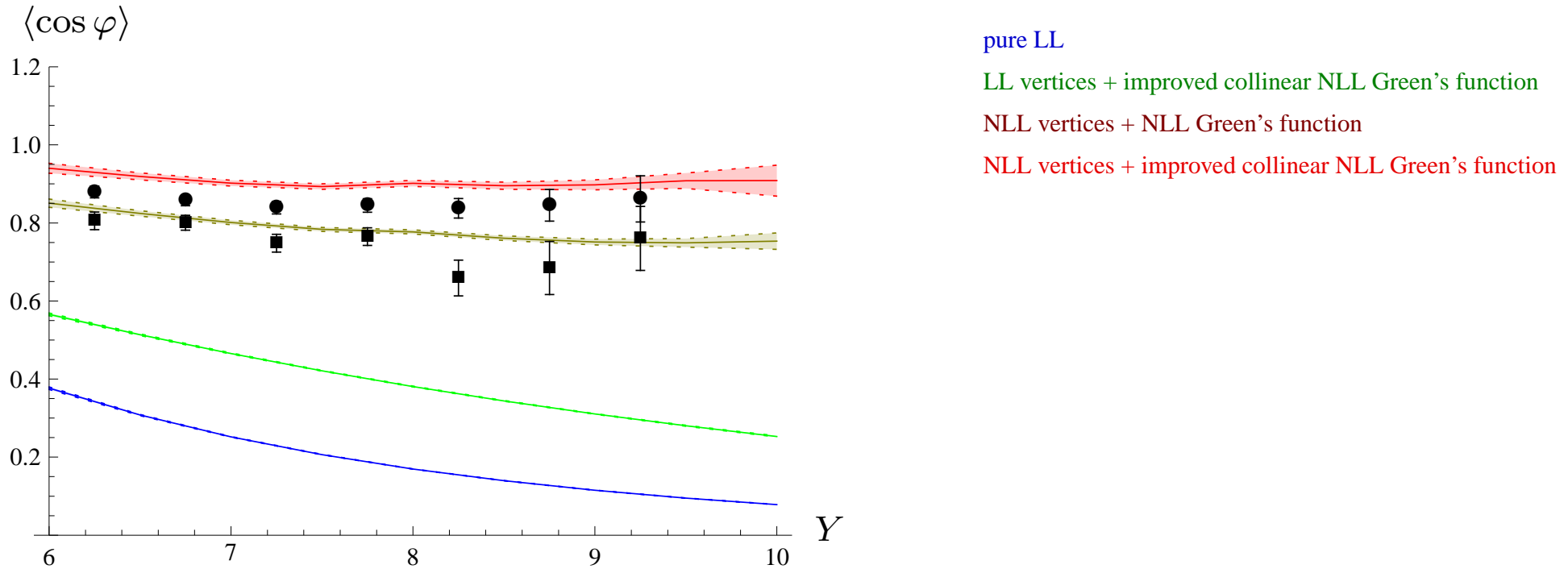
Relative effect of the PDF errors



Relative effect of the Monte Carlo errors

Symmetric configuration ($E_{J_1} = E_{J_2} = 35 \text{ GeV}$)

Azimuthal correlation: $\phi \equiv \phi_1 - \phi_2 - \pi$

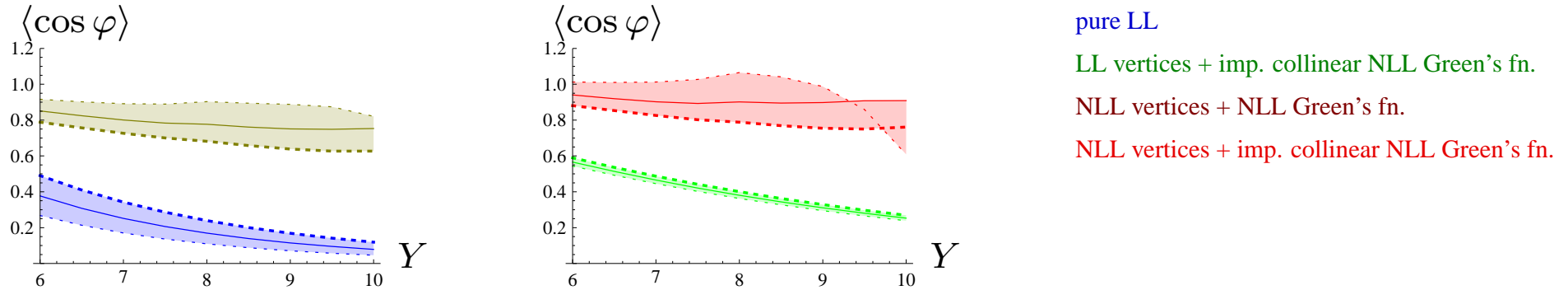


- error bands = errors due to the Monte Carlo integration
- dots = results obtained with PYTHIA (DGLAP LO MC)
- squares = results obtained with HERWIG (DGLAP LO MC)

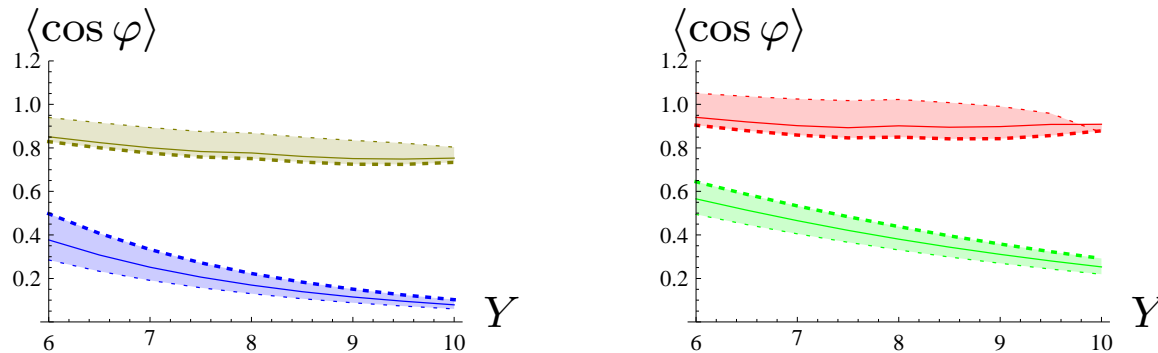
- NLL \rightarrow LL vertices change results dramatically
- At NLL, the decorrelation is very flat and close to LO DGLAP type of Monte Carlo

Symmetric configuration ($E_{J_1} = E_{J_2} = 35 \text{ GeV}$)

Azimuthal correlation: dependency with respect to $\mu_R = \mu_F$ and s_0 changes



Effect of changing $\mu_R = \mu_F$ by factors 2 (thick) and 1/2 (thin)

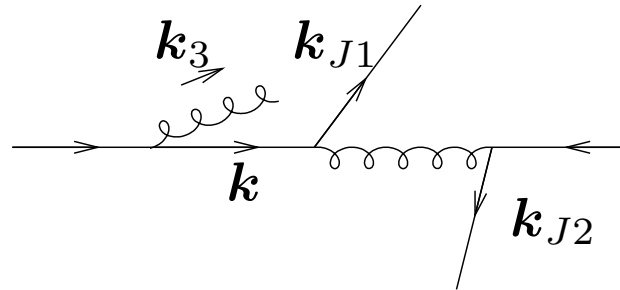


Effect of changing $\sqrt{s_0}$ by factors 2 (thick) and 1/2 (thin)

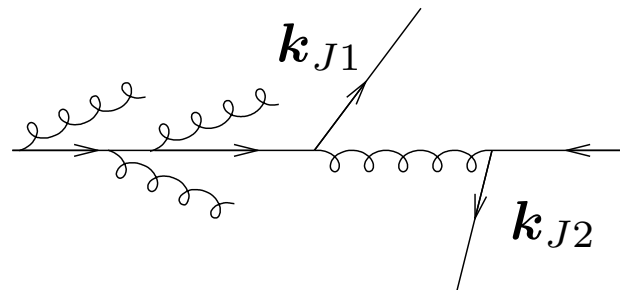
- $\langle \cos \varphi \rangle$ is still rather $\mu_R = \mu_F$ and s_0 dependent
- collinear resummation can lead to $\langle \cos \varphi \rangle > 1$ (!) for small $\mu_R = \mu_F$
- based on NLL double- ρ production [Ivanov, Papa] one can expect that small scales is disfavored

Motivation for asymmetric configurations

- Initial state radiation (unseen) produces divergencies if one touches the collinear singularity $k \rightarrow 0$

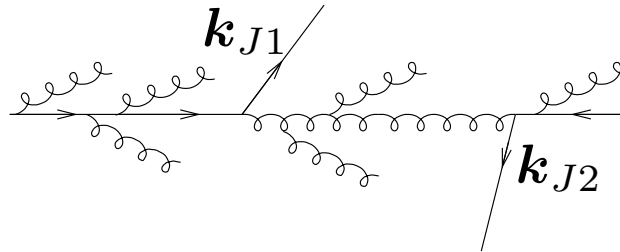


- they are compensated by virtual corrections
- this compensation is in practice difficult to implement when for some reason this additional emission is in a "corner" of phase space (dip in the differential cross-section)
- this is the case when $k_{J1} + k_{J2} \rightarrow 0$ (at NLO, $d\sigma$ is finite but has a log cusp)
- this calls for a **Sudakov resummation** of large logs



Motivation for asymmetric configurations

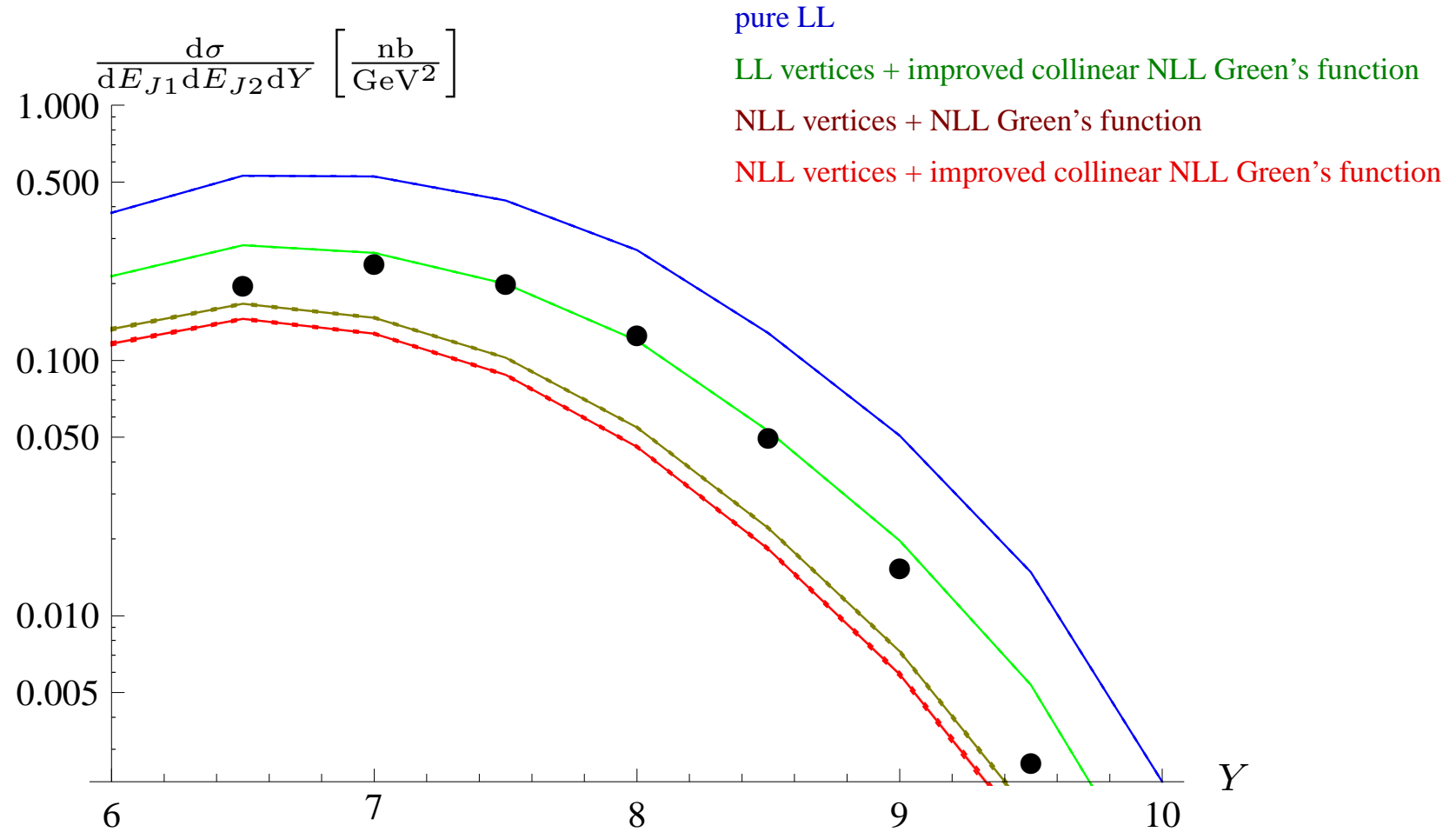
- since these resummation have never been investigated in this context, one should better avoid that region
- note that for **BFKL**, due to additional emission between the two jets, one may expect a less severe problem (at least a smearing in the dip region $|\mathbf{k}_{J1}| \sim |\mathbf{k}_{J2}|$)



- this may however not mean that the region $|\mathbf{k}_{J1}| \sim |\mathbf{k}_{J2}|$ is perfectly trustable even in a **BFKL** type of treatment
- we now investigate a region where NLL **DGLAP** is under control

Asymmetric configuration ($E_{J_1} = 35 \text{ GeV}, E_{J_2} = 50 \text{ GeV}$)

Cross-section

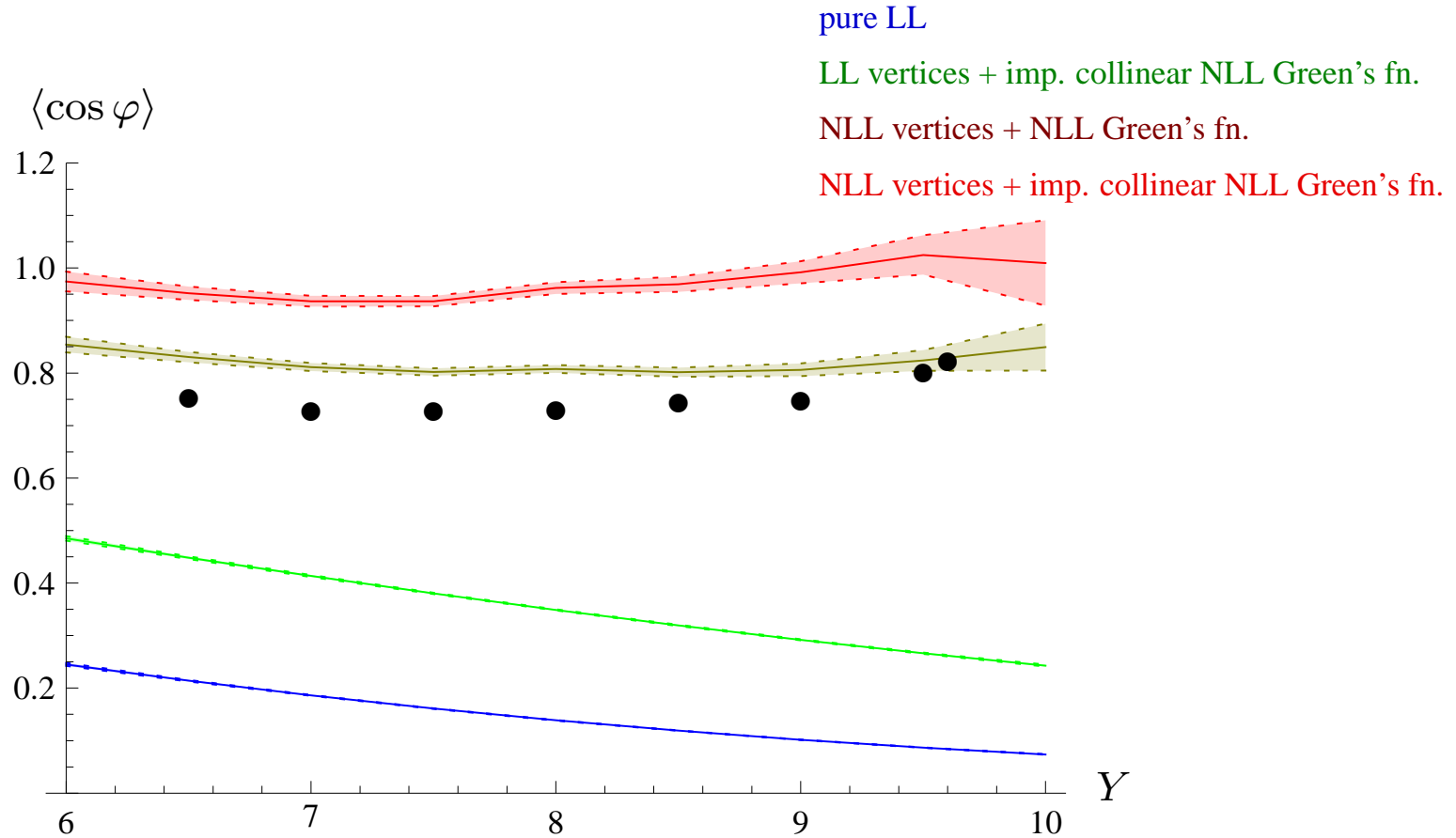


bands = errors due to the Monte Carlo integration

dots = based on the NLO **DGLAP** parton generator DIJET (thanks to *[Fontannaz]*)

Asymmetric configuration ($E_{J_1} = 35 \text{ GeV}, E_{J_2} = 50 \text{ GeV}$)

Azimuthal correlation: $\langle \cos \varphi \rangle$



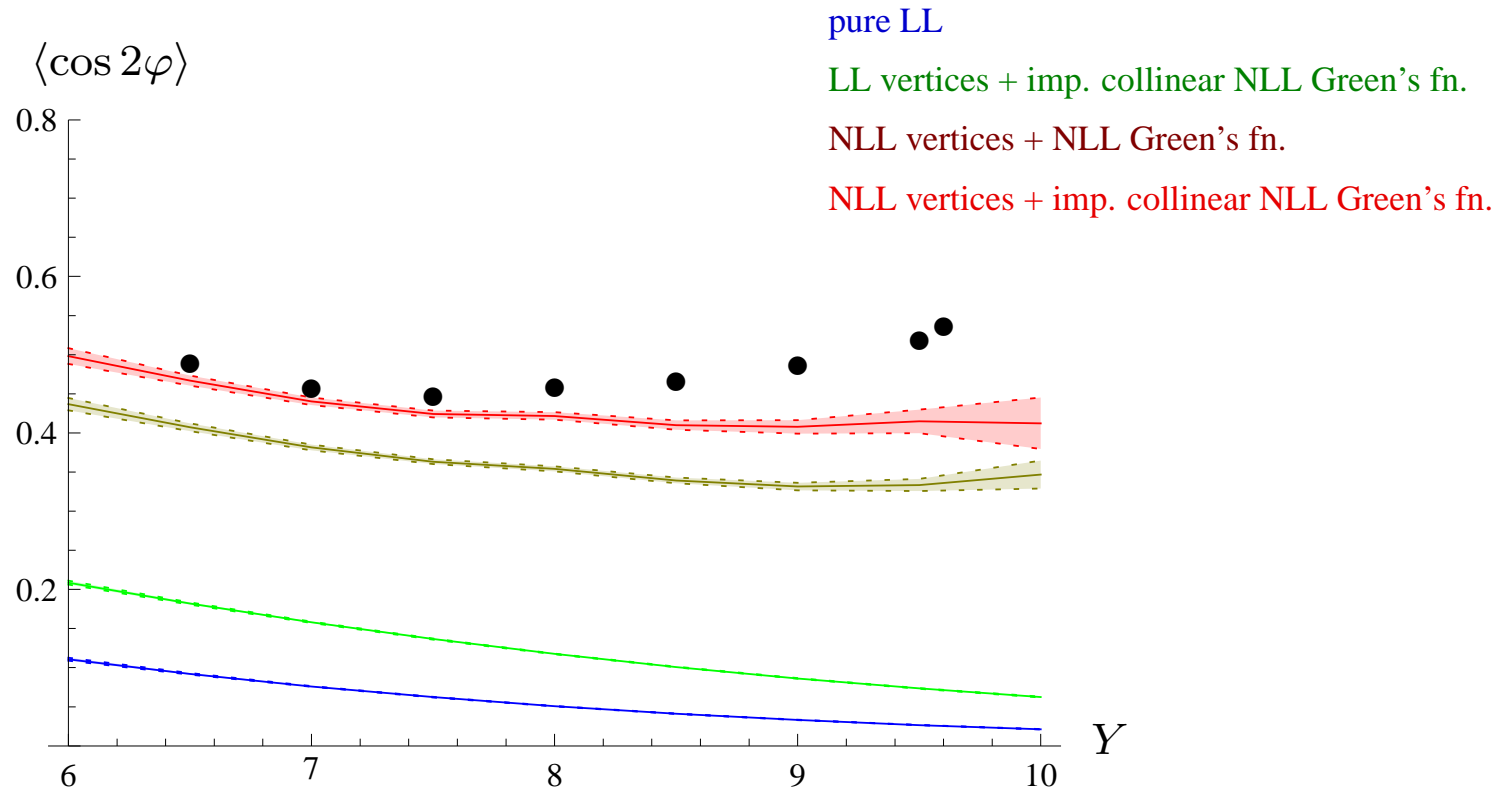
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- Both NLL and improved NLL results are almost flat in Y
- no significant difference between NLL BFKL and NLO DGLAP

Asymmetric configuration ($E_{J1} = 35 \text{ GeV}, E_{J2} = 50 \text{ GeV}$)

Azimuthal correlation: $\langle \cos 2\varphi \rangle$



bands = errors due to the Monte Carlo integration

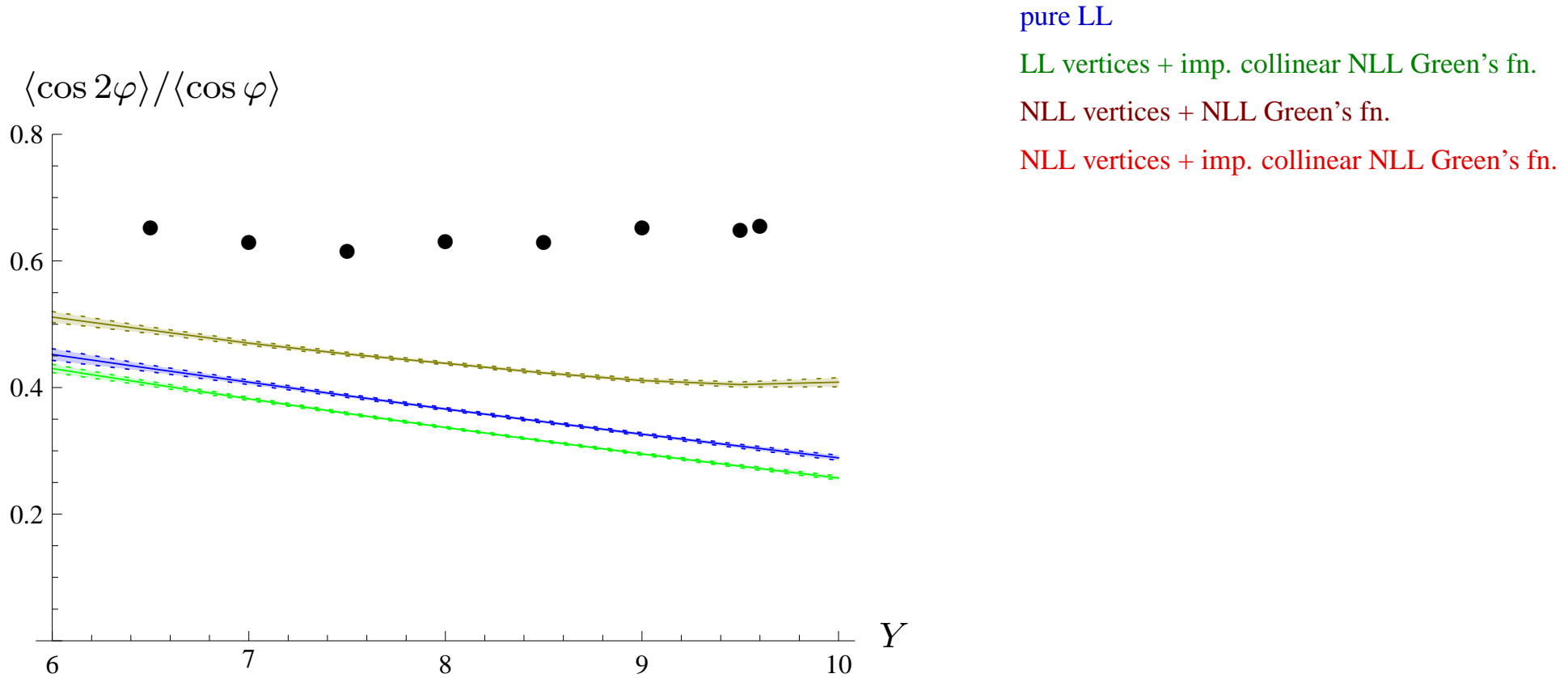
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Same conclusions:

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- no significant difference between NLL BFKL and NLO DGLAP

Asymmetric configuration ($E_{J1} = 35 \text{ GeV}, E_{J2} = 50 \text{ GeV}$)

Ratio of azimuthal correlations $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$



bands = errors due to the Monte Carlo integration

dots = based on the NLO **DGLAP** parton generator DIJET (thanks to *[Fontannaz]*)

NB: NLL collinear improved changed nothing wrt pure NLL

This is the only observable which might still differ between NLL **BFKL** and NLO **DGLAP** scenarii

Conclusions

- We have performed for the first time a **complete NLL analysis of Mueller-Navelet jets**
- **Corrections due to NLL jet vertices corrections** have a **large** effect, similar to the NLL Green function corrections
- for the **cross-section**:
 - it makes the **prediction much more stable** with respect to variation of parameters (factorization scale, scale s_0 entering the rapidity definition, Parton Distribution Functions)
 - it is close to **DGLAP** (although surprisingly a bit below!)
- the **decorrelation** effect is **very small**:
 - it is very close to **DGLAP**
 - it is very flat in rapidity Y
 - it is still rather dependent with respect to scale parameters
- pure NLL **BFKL** and collinear improved NLL **BFKL** lead to similar results
- collinear improved NLL **BFKL** faces some puzzling behaviour for the azimuthal correlation
- except for $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$, there is almost no difference between NLL **BFKL** and NLO **DGLAP** based observables
- **Mueller Navelet jets are thus probably not such a conclusive observable to see the perturbative Regge effect of QCD**

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- **Mueller Navelet jets are thus probably not such a conclusive observable to see the perturbative Regge effect of QCD**
- a serious study of Sudakov type of effects is still missing, both in DGLAP and BFKL approaches