Threshold resummation for Drell-Yan production: theory and phenomenology

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In collaboration with:

Stefano Forte, Giovanni Ridolfi

Outline

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  - the minimal prescription
  - the Borel prescription
  - subleading terms
- new phenomenological results
  - rapidity distributions at NNLO $+$ NNLL

For which τ is resummation important?

$z \sim 1$: logarithmic enhancement $\longrightarrow$ resummation of $\frac{\log^k(1-z)}{1-z}$

$$\sigma(\tau) = \int_\tau^1 \frac{dz}{z}\,\mathcal{L}\left(\frac{\tau}{z}\right)\,\hat{\sigma}(z)\,, \qquad \tau = \frac{Q^2}{s}\,, \qquad z = \frac{Q^2}{\hat{s}}$$

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N**–space analysis**
and saddle point argument

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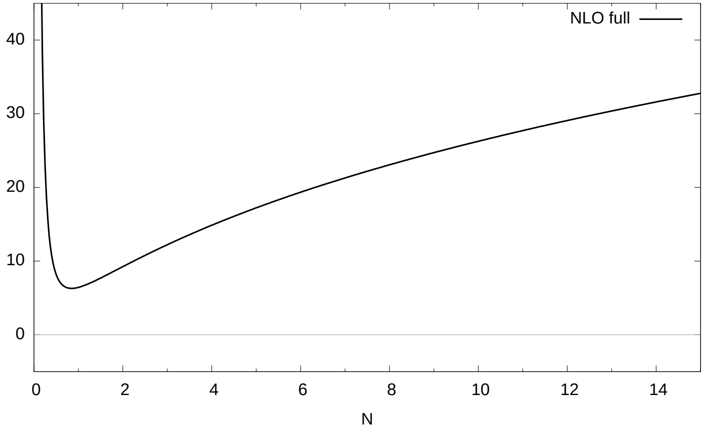
Drell-Yan $q\bar{q}$ at NLO in N–space

$$\frac{\alpha_s}{\pi}\,4C_F\left\{\left[\frac{\log(1-z)}{1-z}\right]_+-\frac{\log\sqrt{z}}{1-z}-\frac{1+z}{2}\log\frac{1-z}{\sqrt{z}}+\left(\frac{\pi^2}{12}-1\right)\delta(1-z)\right\}$$



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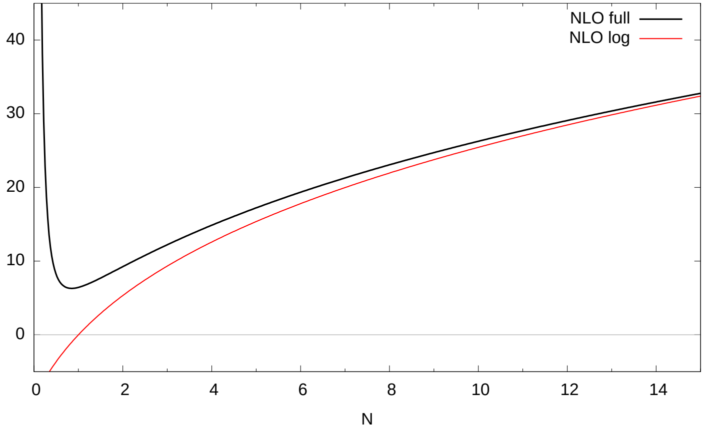

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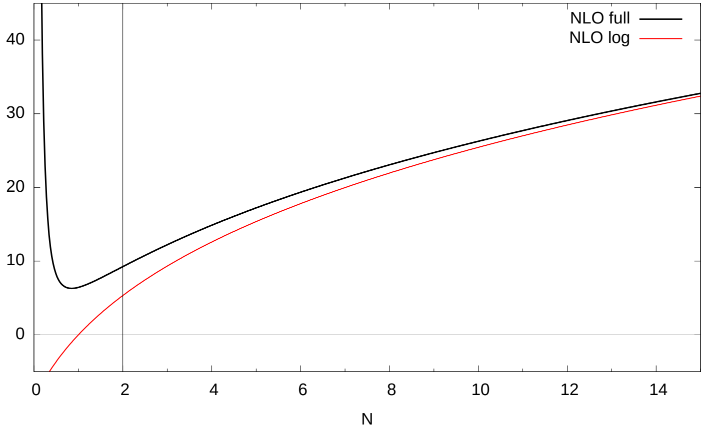

For $N \gtrsim 2$ more than 50% of the NLO is given by the log term

Saddle point argument

$$\sigma(\tau) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\ \tau^{-N}\,\mathcal{L}(N)\,\hat{\sigma}(N)$$

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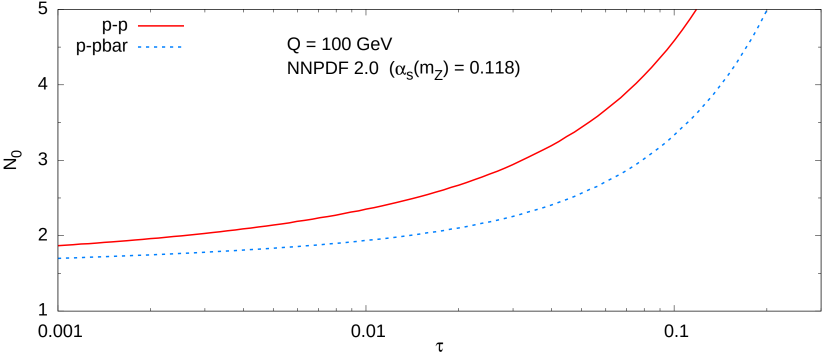
How small?

Saddle point N_0 vs τ



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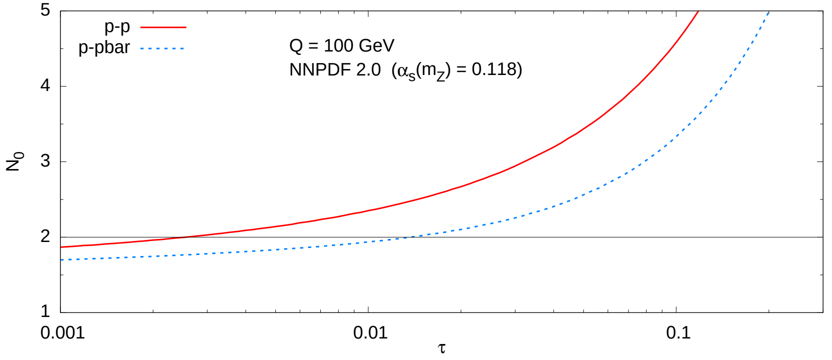

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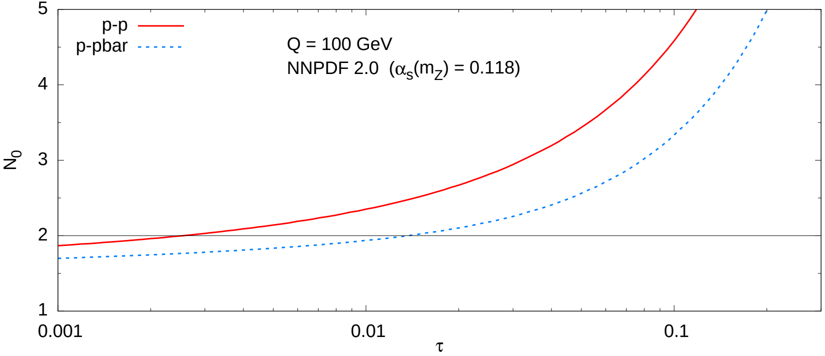

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$$\tau \gtrsim \begin{cases} 0.003 & \text{for } pp \text{ colliders (LHC)} \\ 0.02 & \text{for } p\bar{p} \text{ colliders (Tevatron)} \end{cases}$$

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Much smaller than expected!

Resummation

Resummation is performed in N–space $\quad (L = 2\beta_0 \alpha_s \log \frac{1}{N})$

$$\hat{\sigma}^{\rm res}(N) = g_0(\alpha_s)\, \exp\left[\frac{1}{\alpha_s} g_1(L) + g_2(L) + \alpha_s g_3(L) + \alpha_s^2 g_4(L) + \ldots\right]$$

known up to g_4 ($\mathrm{N^3LL}$): S.Moch, J.A.M.Vermaseren, A.Vogt (hep-ph/0506288)

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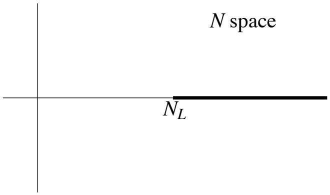
Branch cut due to the Landau singularity for $N > N_L = \exp\frac{1}{2\beta_0\alpha_s}$



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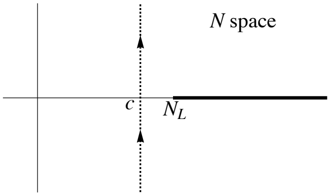

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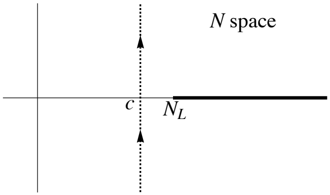

The Mellin inverse does not exist

Minimal prescription

S.Catani, M.L.Mangano, P.Nason, L.Trentadue (hep-ph/9604351)

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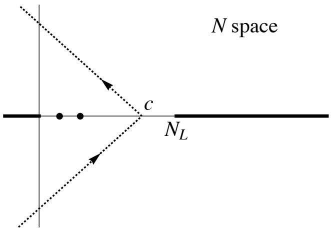

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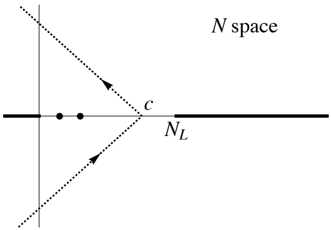

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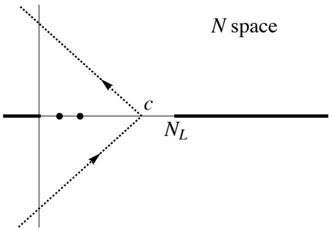

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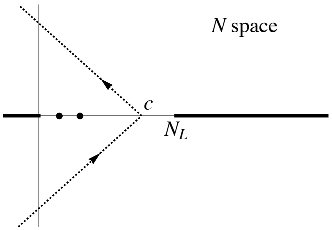

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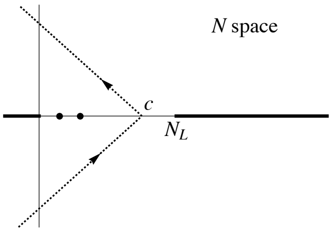

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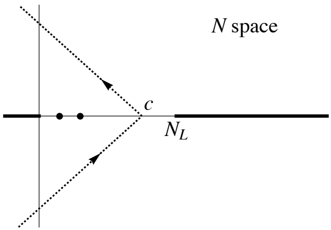

But...

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- difficult numerical implementation

Minimal prescription: non-physical contribution

$$\sigma_{\rm MP}(\tau) = \int_\tau^{+\infty} \frac{dz}{z}\, \mathcal{L}\left(\frac{\tau}{z}\right)\, \hat\sigma_{\rm MP}(z)$$

The integral extends to $+\infty$, not to 1!

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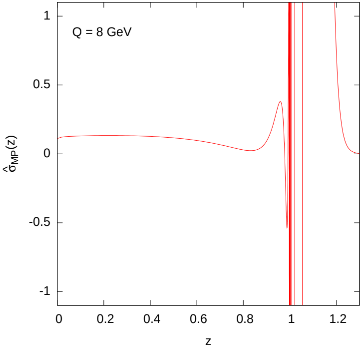

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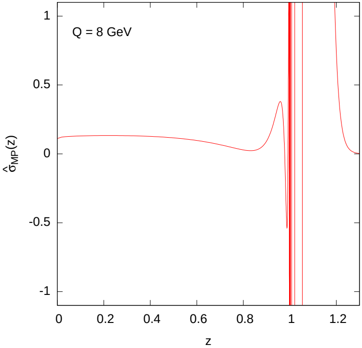

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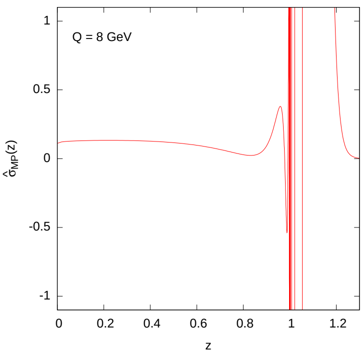
Need for $\mathcal{L}(N)$, for values of N where the Mellin transform of $\mathcal{L}(x)$ does not converge



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Comparison with fixed order: Drell-Yan $q\bar{q}$ at NLO

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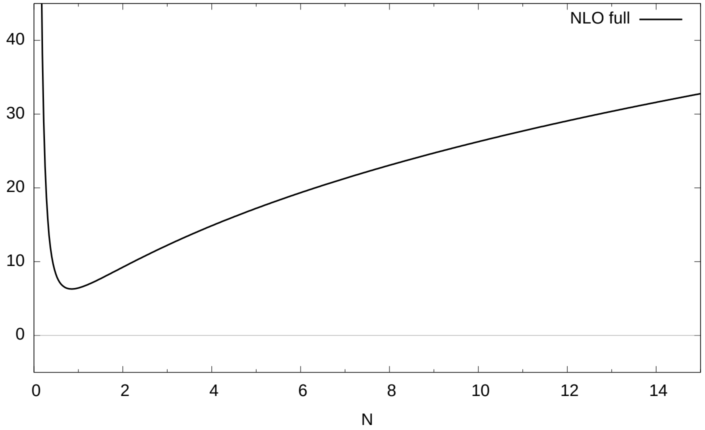

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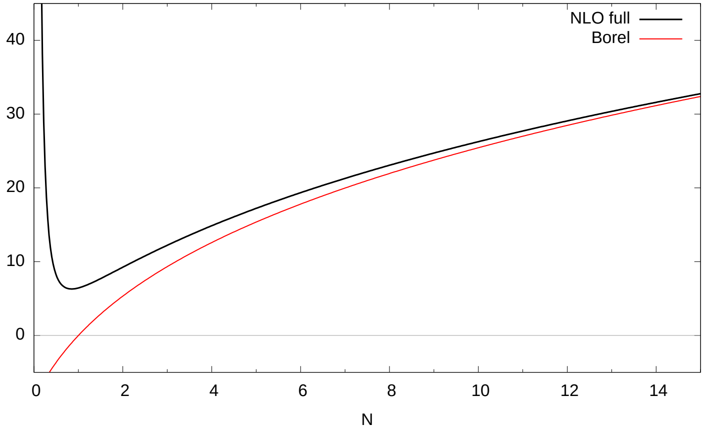

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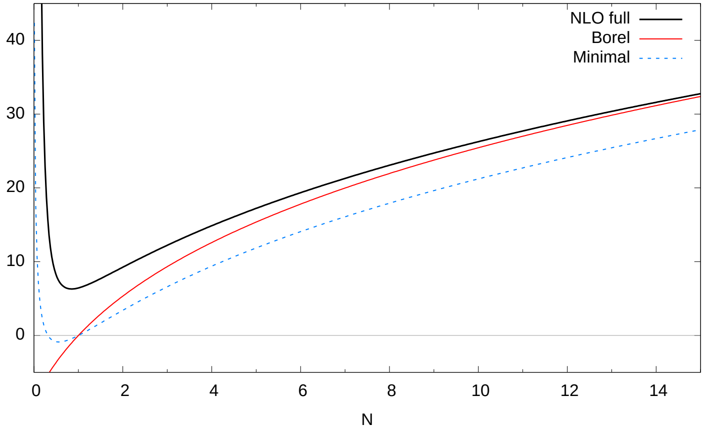

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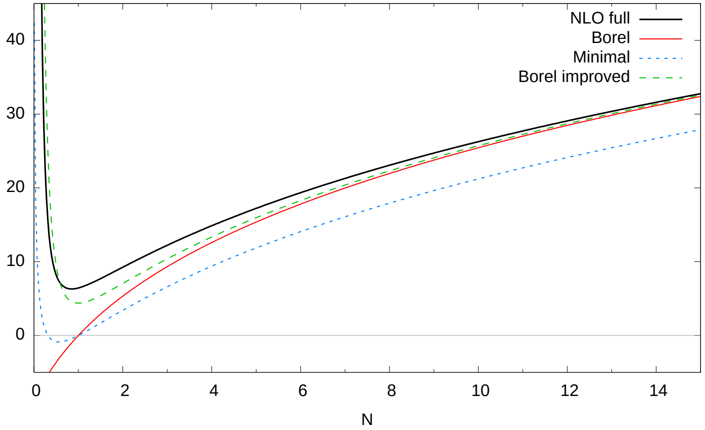

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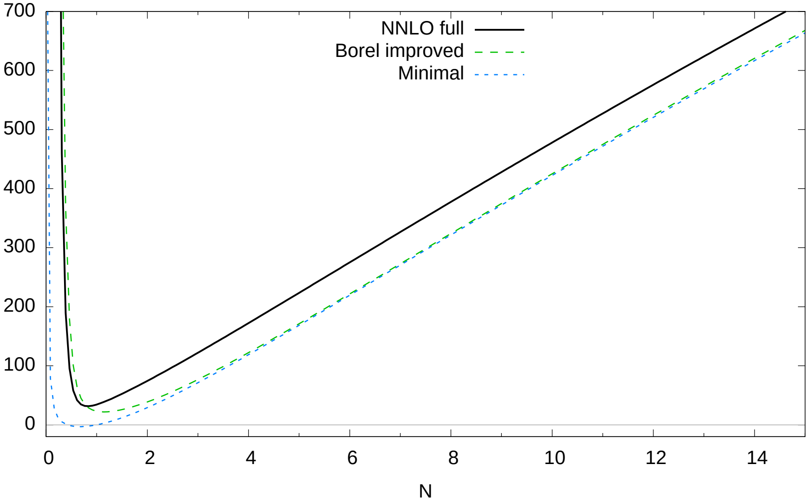

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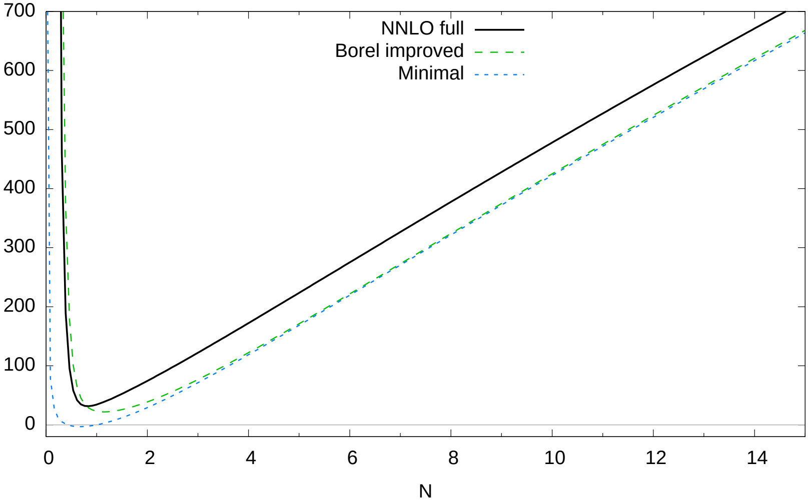

Discrepancy due to terms like $\log^k(1-z) \to \frac{\log^k N}{N}$ $\Rightarrow$ subleading

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- the difference in the included subleading terms is useful to estimate the importance of these terms

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or, back to y space,

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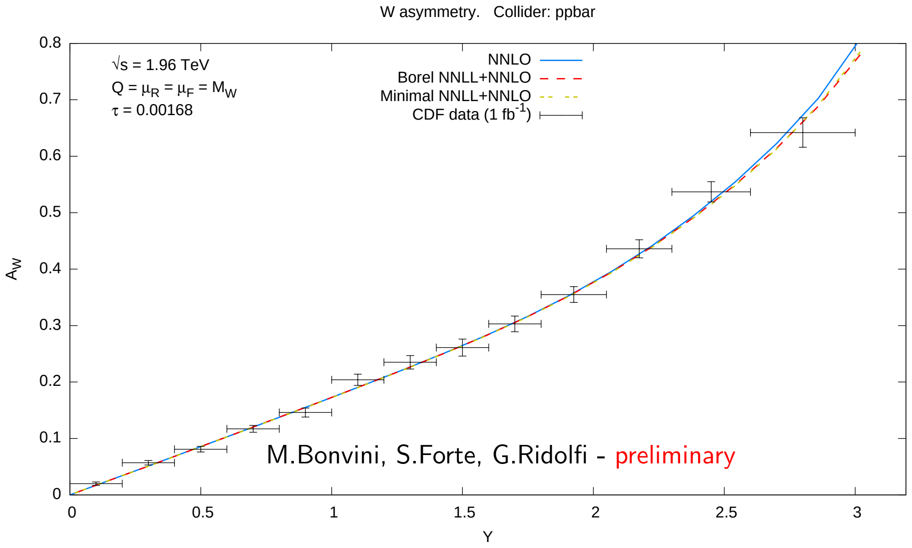
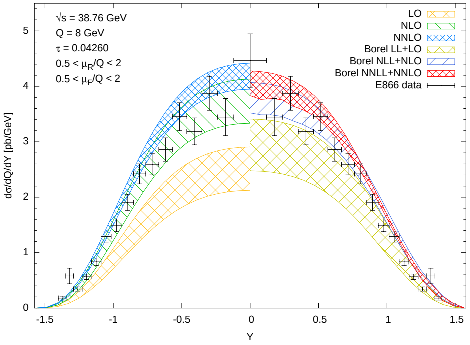
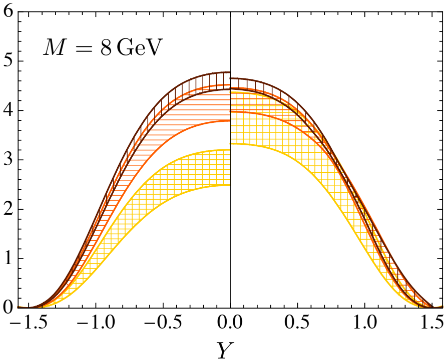
W asymmetry at Tevatron with NNPDF2.0



$$\tau \simeq 0.04$$





M.Bonvini, S.Forte, G.Ridolfi
preliminary

NNPDF2.0

T.Becher, M.Neubert, G.Xu
(hep-ph/0710.0680)

MRST04NNLO

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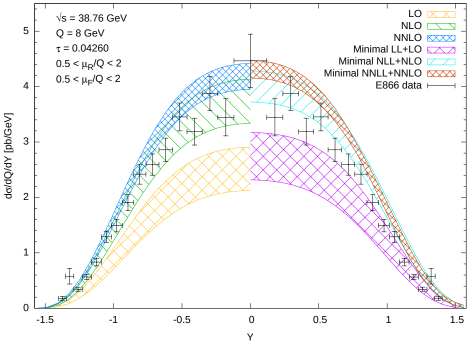

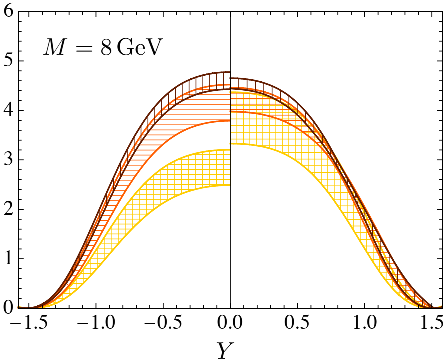

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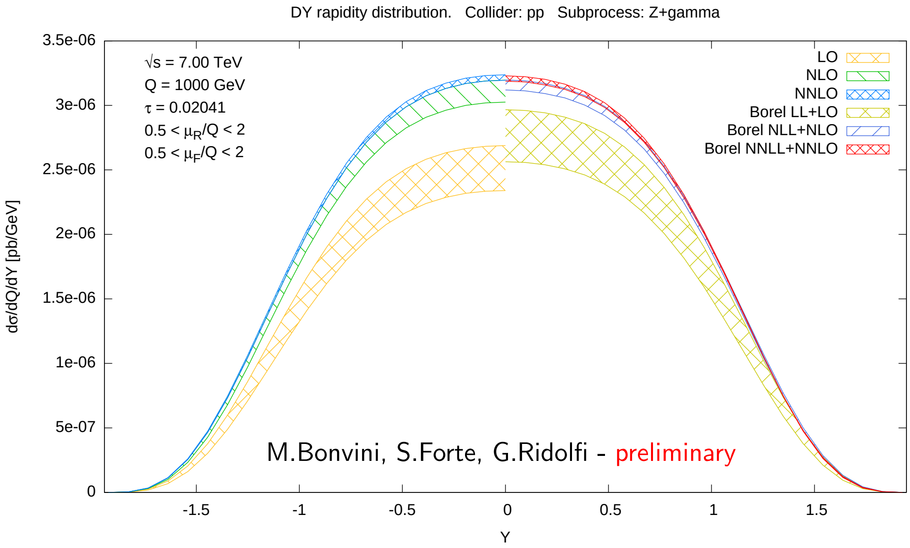
Rapidity distribution: DY (1 TeV) at LHC with NNPDF2.0



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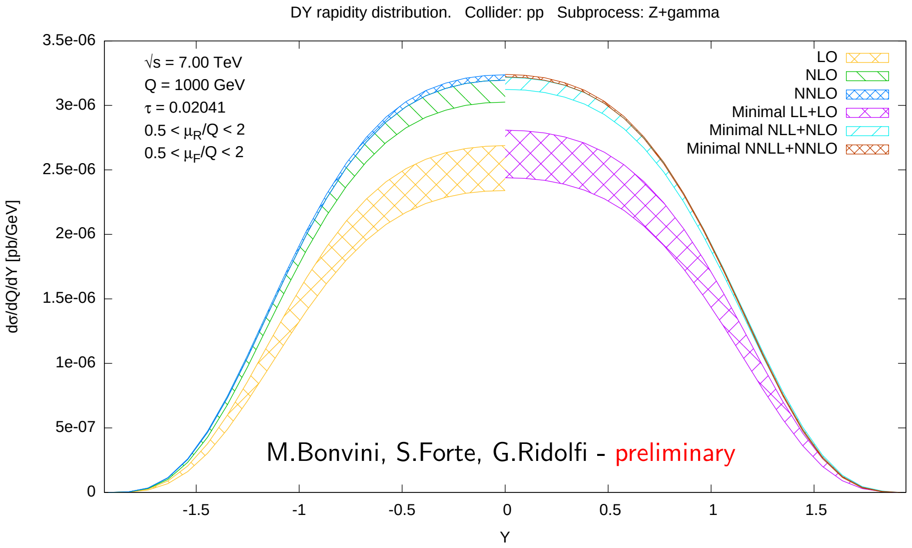
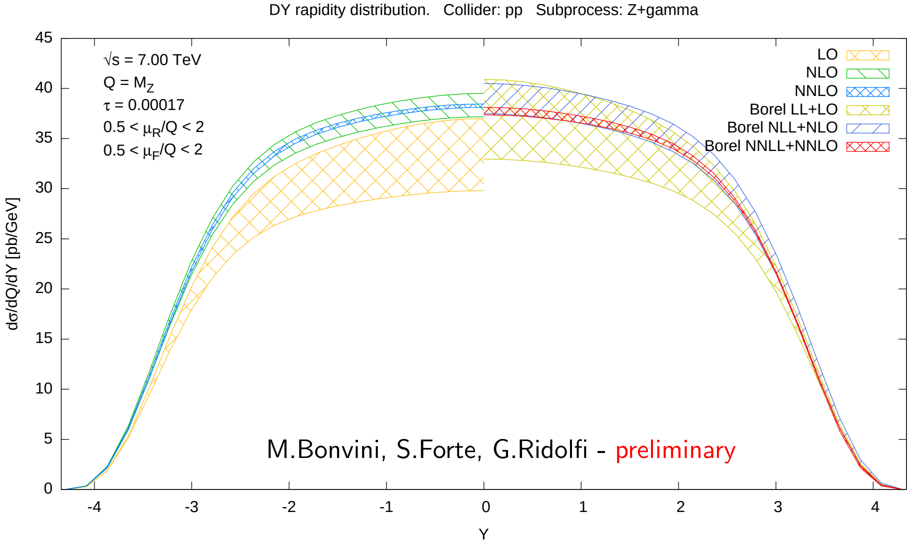


Rapidity distribution: Z at LHC with NNPDF2.0



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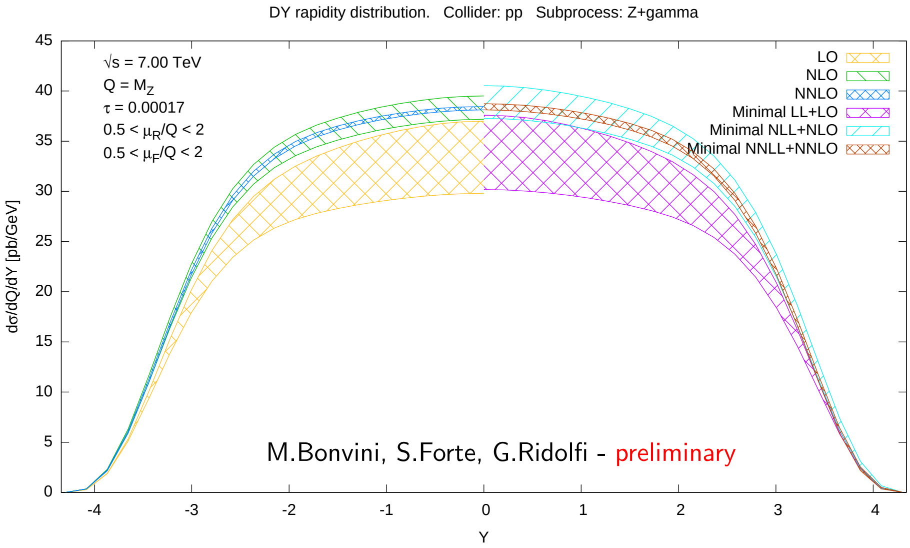
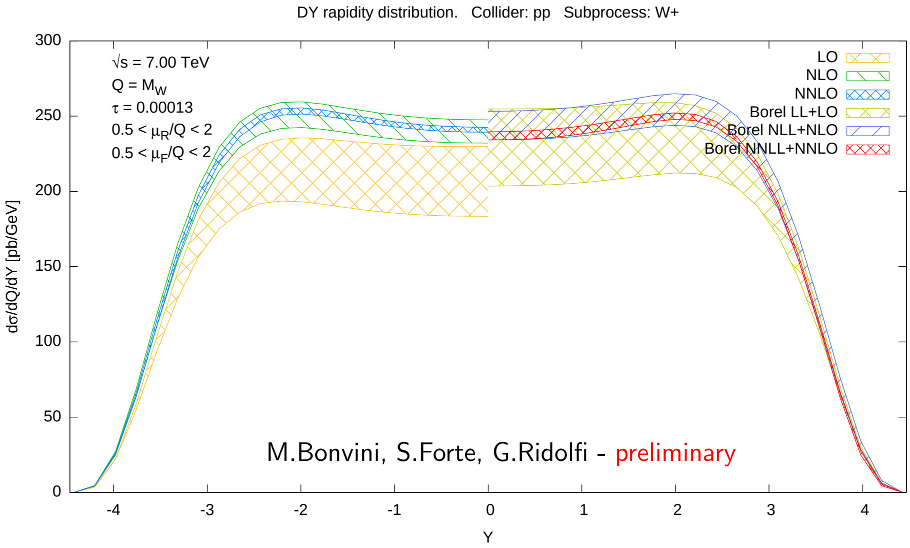
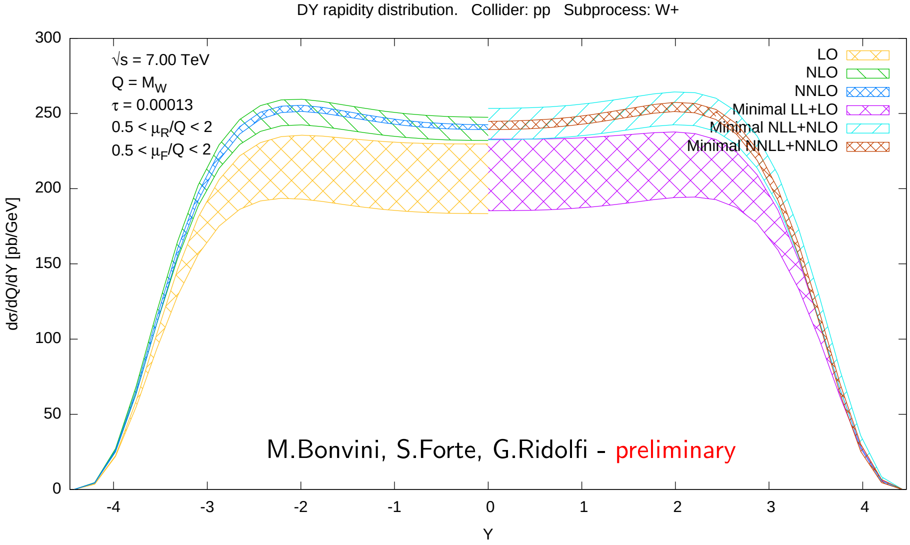


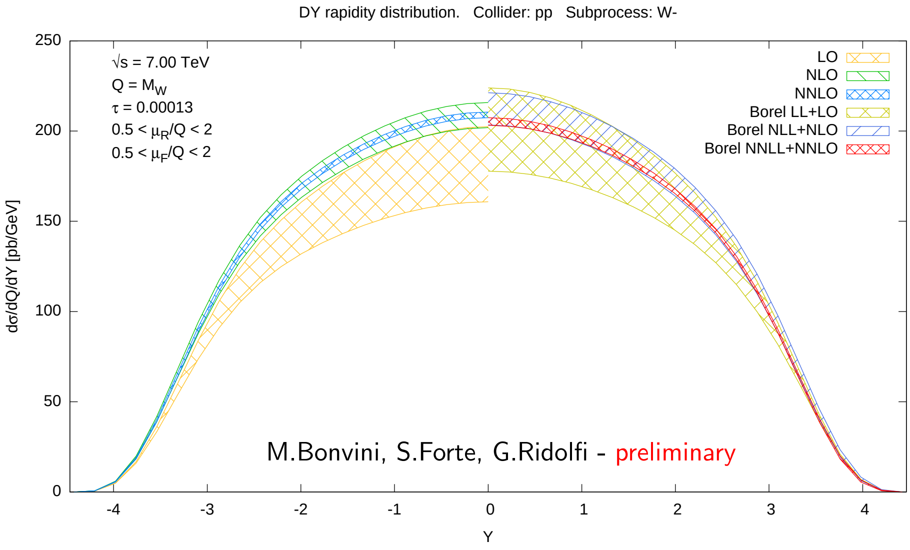
Rapidity distribution: W^+ at LHC with NNPDF2.0



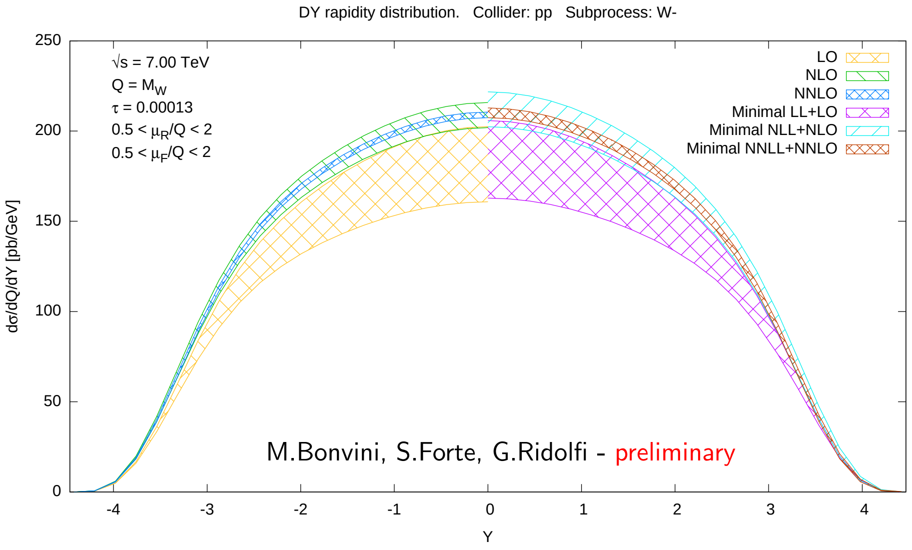
Rapidity distribution: W^+ at LHC with NNPDF2.0



Rapidity distribution: W^- at LHC with NNPDF2.0



Rapidity distribution: W^- at LHC with NNPDF2.0



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- Include subdominant $1/N$ contributions
  (S.Moch, A.Vogt: hep-ph/0909.2124 and today talk)

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- New phenomenological results: rapidity distributions

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- Include subdominant $1/N$ contributions
  (S.Moch, A.Vogt: hep-ph/0909.2124 and today talk)
- Apply to other processes such as Higgs production

Backup slides

Minimal prescription: practical implementation

- Expand the function

$$\frac{z^{\alpha}}{(1-z)^{\beta}}\,\mathcal{L}(z)$$

  on a polynomial basis (with suitable $\alpha, \beta > 0$)
- Compute the Mellin transform of $\mathcal{L}(z)$ analytically
- Compute the complex Mellin inversion integral numerically

Borel prescription: practical implementation

- Compute the convolution integral

$$\int_\tau^1 \frac{dz}{z}\mathcal{L}\left(\frac{\tau}{z}\right)\left[(1-z)^{\xi-1}\right]_+$$

  It is convenient to expand on a polynomial basis the function

$$\frac{1}{1-z}\left[\frac{1}{z}\mathcal{L}\left(\frac{\tau}{z}\right)-\mathcal{L}(\tau)\right]$$

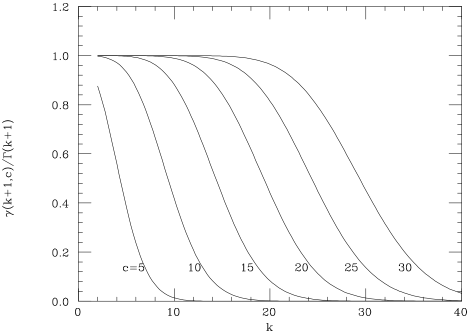
  and compute the integral analytically

- Compute the complex ξ integral numerically

How BP works

Apply the BP to a power of $\log \frac{1}{N}$

$$\mathcal{M}^{-1}\left(\log^k \frac{1}{N}\right)\Bigg|_{\mathrm{BP}} = \frac{\gamma(k+1, C/\bar{\alpha})}{\Gamma(k+1)}\,\mathcal{M}^{-1}\left(\log^k \frac{1}{N}\right)$$



The BP essentially truncates the divergent sum