UNIVERSITÄT BERN

DRELL-YAN PRODUCTION AT SMALL Q_T

transverse PDFs, the collinear anomaly, and resummation at NNLL

arXiv:1007.4005 with Matthias Neubert

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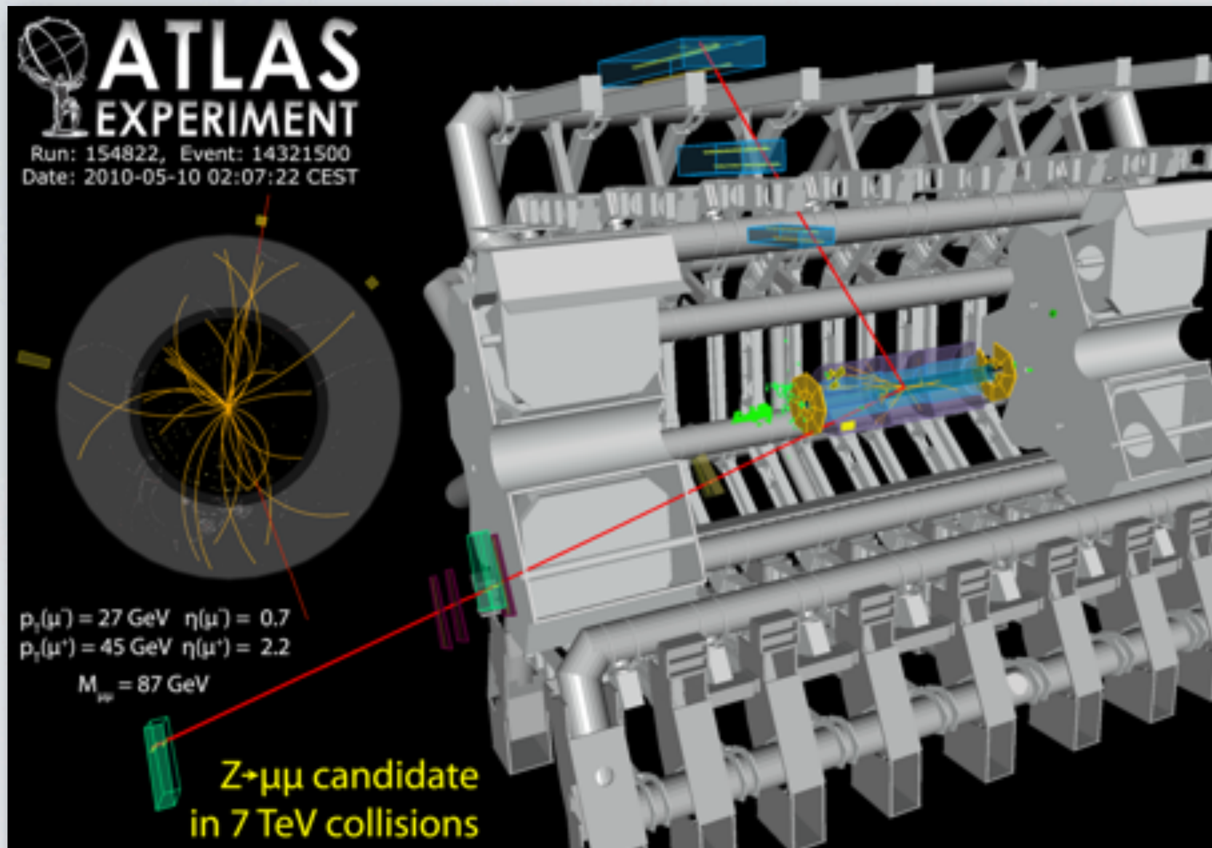
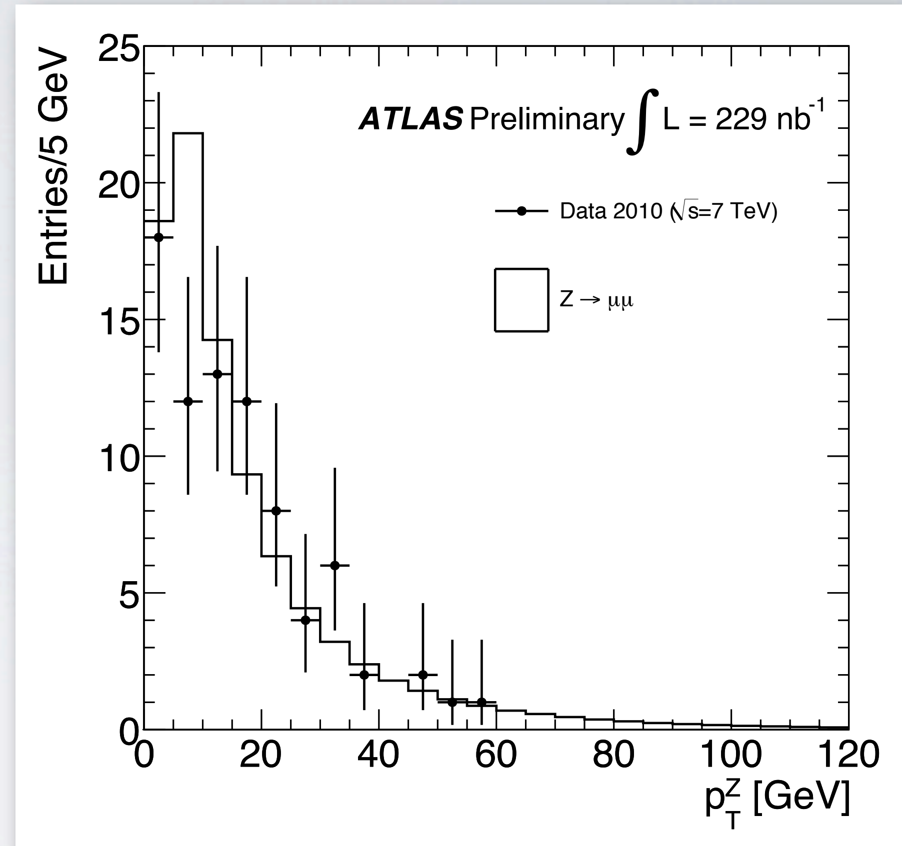
OUTLINE

- Factorization of the cross section at low transverse momentum q_T
- The collinear anomaly and the definition of transverse momentum dependent PDFs
- Resummation of large log's
- Relation to the Collins Soper Sterman (CSS) formalism
- Numerical results to NNLL





The production of a lepton pair with large invariant mass is the most basic hard-scattering process at a hadron collider. This is the place for HP^2 at hadron colliders!

ATLAS has 3.5 pb^{-1} of data: ~ 7 ×10^5 W's and 2x10^5 Z's !

PERTURBATIVE EXPANSION

The perturbative expansion of the q_T spectrum contains singular terms of the form (M is the invariant mass of the lepton pair)

$$\frac{d\sigma}{dq_T^2} = \frac{1}{q_T^2}\Big[A_1^{(1)}\alpha_s \ln\frac{M^2}{q_T^2} + \alpha_s A_0^{(1)} + A_3^{(2)}\alpha_s^2 \ln^3\frac{M^2}{q_T^2} + \dots$$

$$+A_{2n-1}^{(n)}\alpha_s^n \ln^{2n-1}\frac{M^2}{q_T^2} + \dots\Big] + \dots$$

which ruin the perturbative expansion at $q_T \ll M$ and must be resummed to all orders.

classic example of an observable which needs resummation!

RESUMMATION

A formula which allows for the resummation of the logarithmically enhanced terms at small q_T to arbitrary precision was first obtained by Collins, Soper and Sterman (CSS) in ‘84, based on earlier work of Collins and Soper.

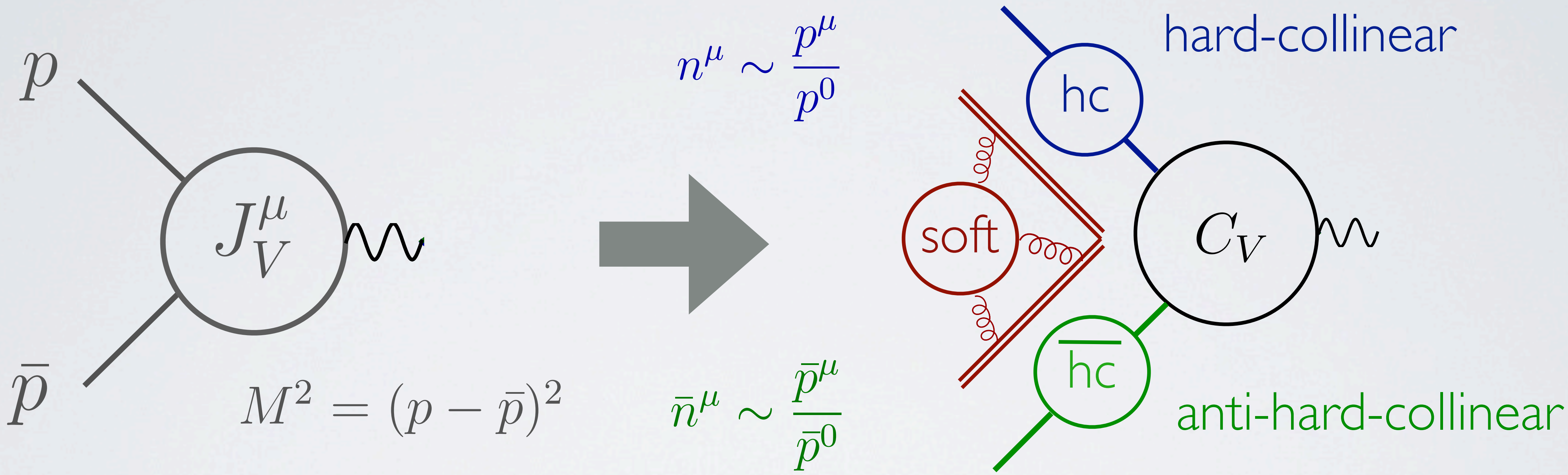
A corresponding expression for the simpler case of soft-gluon resummation was derived only later by Sterman in ’87, and by Catani and Trentadue in ‘89.

We will now analyze the factorization properties of the cross section in these kinematic configurations.

FACTORIZATION

SOFT-COLLINEAR FACTORIZATION

- Starting point is the factorization of the electroweak current in the Sudakov limit



- In Soft-Collinear Effective Theory (SCET) this can be written in operator form as

$$\bar{\psi}\gamma_\mu\psi \rightarrow C_V(M^2, \mu)\, \bar{\chi}_{\overline{hc}}\, S_{\bar{n}}^\dagger\, \gamma^\mu\, S_n\, \chi_{hc}$$

SCET hc quark field

The Drell-Yan cross section is obtained from the matrix element of two currents

$$(-g_{\mu\nu})\,\langle N_1(p)\,N_2(\bar{p})|\,J_V^{\mu\dagger}(x)\,J_V^{\nu}(0)\,|N_1(p)\,N_2(\bar{p})\rangle \to \frac{1}{2N_c}|C_V(M^2,\mu)|^2$$

$$\times\,\hat{W}_{\rm DY}(x)\,\langle N_1(p)|\,\bar{\chi}_{hc}(x)\,\frac{\not{\bar{n}}}{2}\,\chi_{hc}(0)\,|N_1(p)\rangle\,\langle N_2(\bar{p})|\,\bar{\chi}_{\overline{hc}}(0)\,\frac{\not{n}}{2}\,\chi_{\overline{hc}}(x)\,|N_2(\bar{p})\rangle$$

n and $\bar{n}$ are light-cone reference vectors along p and $\bar{p}$.

The soft function contains a product of four Wilson lines along the directions of large energy flow

$$\hat{W}_{\rm DY}(x) = \frac{1}{N_c}\,\langle 0|\,{\rm Tr}\big[S_n^\dagger(x)\,S_{\bar{n}}(x)\,S_{\bar{n}}^\dagger(0)\,S_n(0)\big]\,|0\rangle$$

$$S_n(x) = \mathbf{P}\exp\left[i\int_{-\infty}^{0} ds\,n\cdot A_s(x+sn)\right]$$

DERIVATIVE EXPANSION

Final step is to expand the matrix elements in small momentum components, i.e. to perform a derivative expansion.

The light-cone components $(n \cdot k, \bar{n} \cdot k, k_\perp)$ scale as

$$p_{hc} \sim M\,(\lambda^2, 1, \lambda)\,, \qquad p_{\overline{hc}} \sim M\,(1, \lambda^2, \lambda)\,.$$

$$p_s \sim M\,(\lambda^2, \lambda^2, \lambda^2)\,.$$

expansion parameter

$$\lambda = \frac{q_T}{M}$$

while the separation between the two currents scales as

$$x \sim M^{-1}(1, 1, \lambda^{-1})$$

$$\longrightarrow \quad A_s^\mu(x) = A_s^\mu(0) + x \cdot \partial A_s^\mu(0) + \ldots$$

power suppressed, can be dropped

NAIVE FACTORIZATION

Dropping power suppressed *x*-dependence leads to the result

$$\hat{W}_{\rm DY}(0)\,\langle N_1(p)|\,\bar{\chi}_{hc}(x_+ + x_\perp)\,\frac{\bar{n}\!\!\!/}{2}\,\chi_{hc}(0)\,|N_1(p)\rangle\,\langle N_2(\bar{p})|\,\bar{\chi}_{\overline{hc}}(0)\,\frac{n\!\!\!/}{2}\,\chi_{\overline{hc}}(x_- + x_\perp)\,|N_2(\bar{p})\rangle$$

$\underbrace{}_{1}$ × "transverse PDF" × "transverse PDF"

KLN cancellation!

this spells trouble: well known that transverse PDF not well defined w/o additional regulators

For comparison: for soft-gluon resummation, the result is

$$\hat{W}_{\rm DY}(x_0)\,\langle N_1(p)|\,\bar{\chi}_{hc}(x_+)\,\frac{\bar{n}\!\!\!/}{2}\,\chi_{hc}(0)\,|N_1(p)\rangle\,\langle N_2(\bar{p})|\,\bar{\chi}_{\overline{hc}}(0)\,\frac{n\!\!\!/}{2}\,\chi_{\overline{hc}}(x_-)\,|N_2(\bar{p})\rangle$$

"soft" × "standard PDF" × "standard PDF"

CROSS SECTION

In terms of the hadronic matrix elements

$$\mathcal{B}_{q/N}(z, x_T^2, \mu) = \frac{1}{2\pi} \int dt \, e^{-izt\bar{n}\cdot p} \, \langle N(p)| \, \bar{\chi}(t\bar{n} + x_\perp) \, \frac{\bar{\not{n}}}{2} \, \chi(0) \, |N(p)\rangle$$

on then obtains the DY cross section

$$\frac{d^3\sigma}{dM^2 \, dq_T^2 \, dy} = \frac{4\pi\alpha^2}{3N_c M^2 s} \left| C_V(-M^2, \mu) \right|^2 \frac{1}{4\pi} \int d^2x_\perp \, e^{-iq_\perp \cdot x_\perp}$$

$$\times \sum_q e_q^2 \left[\mathcal{B}_{q/N_1}(\xi_1, x_T^2, \mu) \, \mathcal{B}_{\bar{q}/N_2}(\xi_2, x_T^2, \mu) + (q \leftrightarrow \bar{q}) \right] + \mathcal{O}\left(\frac{q_T^2}{M^2} \right)$$

with

$$\xi_1 = \sqrt{\tau} \, e^{y} \, , \qquad \xi_2 = \sqrt{\tau} \, e^{-y} \, , \qquad \text{with} \quad \tau = \frac{m_\perp^2}{s} = \frac{M^2 + q_T^2}{s} \, .$$

CROSS SECTION

$$\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \frac{4\pi\alpha^2}{3N_cM^2s}\big|C_V(-M^2,\mu)\big|^2\frac{1}{4\pi}\int d^2x_\perp\,e^{-iq_\perp\cdot x_\perp}$$

$$\times\sum_q e_q^2\left[\mathcal{B}_{q/N_1}(\xi_1,x_T^2,\mu)\,\mathcal{B}_{\bar{q}/N_2}(\xi_2,x_T^2,\mu)+(q\leftrightarrow\bar{q})\right]+\mathcal{O}\left(\frac{q_T^2}{M^2}\right)$$

The resummation would then be obtained by solving the RG equation

$$\frac{d}{d\ln\mu}\,C_V(M^2,\mu) = \left[\Gamma^F_{\text{cusp}}(\alpha_s)\,\ln\frac{-M^2}{\mu^2}+2\gamma^q(\alpha_s)\right]C_V(M^2,\mu)$$

see SCET papers: Gao, Li, Liu 2005; Idilbi, Ji, Yuan 2005; Mantry, Petriello 2009

This cannot be correct! If $\mathcal{B}_{q/N_1}(\xi_1,x_T^2,\mu)\,\mathcal{B}_{\bar{q}/N_2}(\xi_2,x_T^2,\mu)$ is independent of M, the above cross section is μ dependent!

COLLINEAR ANOMALY

RG invariance of cross section implies that the product of transverse PDFs $\mathcal{B}_{q/N_1}(\xi_1, x_T^2, \mu)\, \mathcal{B}_{\bar{q}/N_2}(\xi_2, x_T^2, \mu)$ must contain hidden M^2 dependence.

Analyzing the relevant diagrams, one finds that an additional regulator is needed to make transverse PDFs well defined. In the product of the two PDFs, this regulator can be removed, but anomalous M^2 dependence remains. Can refactorize

$$\left[\mathcal{B}_{q/N_1}(z_1, x_T^2, \mu)\, \mathcal{B}_{\bar{q}/N_2}(z_2, x_T^2, \mu)\right]_{M^2} = \left(\frac{x_T^2 M^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2, \mu)} B_{q/N_1}(z_1, x_T^2, \mu)\, B_{\bar{q}/N_2}(z_2, x_T^2, \mu)\,,$$

with $\dfrac{dF_{q\bar{q}}(x_T^2, \mu)}{d\ln\mu} = 2\Gamma_{\text{cusp}}^F(\alpha_s)$

Note that M^2 dependence exponentiates!

TRANSVERSE PDFs

What God has joined together, let no man separate...

The "operator definition of TMD PDFs is quite problematic [...] and is nowadays under active investigation".

quote from Cherednikov and Stefanis '09. For reviews, see Collins '03, '08

Regularization of the individual transverse PDFs is delicate, but the product is well defined, and has specific dependence on the hard momentum transfer M^2.

Anomaly: Classically, $\langle N_1(p)|\,\bar{\chi}_{hc}(x_+ + x_\perp)\not{\bar{n}}\,\chi_{hc}(0)\,|N_1(p)\rangle$ is invariant under a rescaling of the momentum of the other nucleon N_2. Quantum theory needs regularization. Symmetry cannot be recovered after removing regulator.

Not an anomaly of QCD, but of the low energy theory.

RESUMMATION

SIMPLIFICATION FOR $q_T^2 \gg \Lambda_{\rm QCD}$

For perturbative values of q_T we can perform an operator product expansion

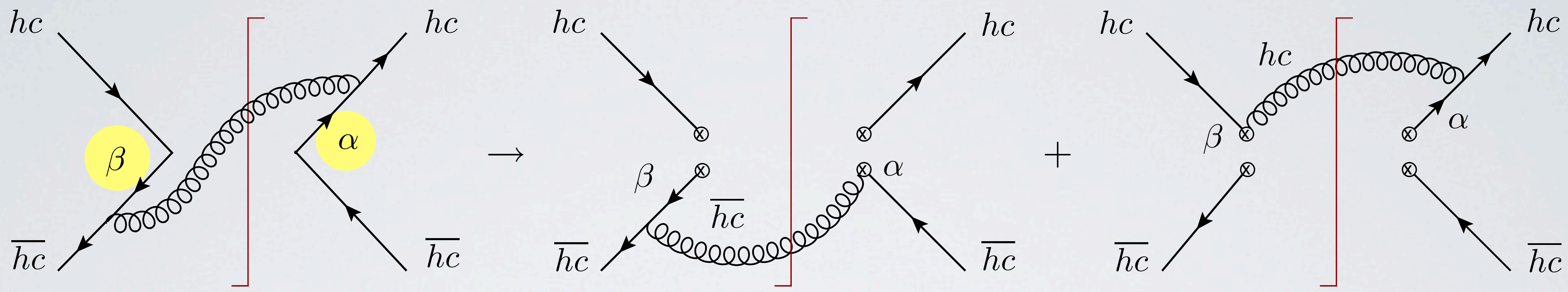
$$\mathcal{B}_{i/N}(\xi, x_T^2, \mu) = \sum_j \int_\xi^1 \frac{dz}{z}\, \mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu)\, \phi_{j/N}(\xi/z, \mu) + \mathcal{O}(\Lambda_{\rm QCD}^2\, x_T^2)$$

Again, only the product of two $\mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu)$ functions is well defined.

$$\left[\mathcal{I}_{q\leftarrow i}(z_1, x_T^2, \mu)\, \mathcal{I}_{\bar{q}\leftarrow j}(z_2, x_T^2, \mu)\right]_{q^2} = \left(\frac{x_T^2 q^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2, \mu)} I_{q\leftarrow i}(z_1, x_T^2, \mu)\, I_{\bar{q}\leftarrow j}(z_2, x_T^2, \mu)$$

Effective theory diagrams for $\mathcal{I}_{i\leftarrow j}(z, x_T^2, \mu)$ are not well-defined in dim. reg.. Following Smirnov '83, we use additional *analytical regularization*, which is very economical, since it does not introduce additional scales into the problem.

ANALYTICAL REGULARIZATION

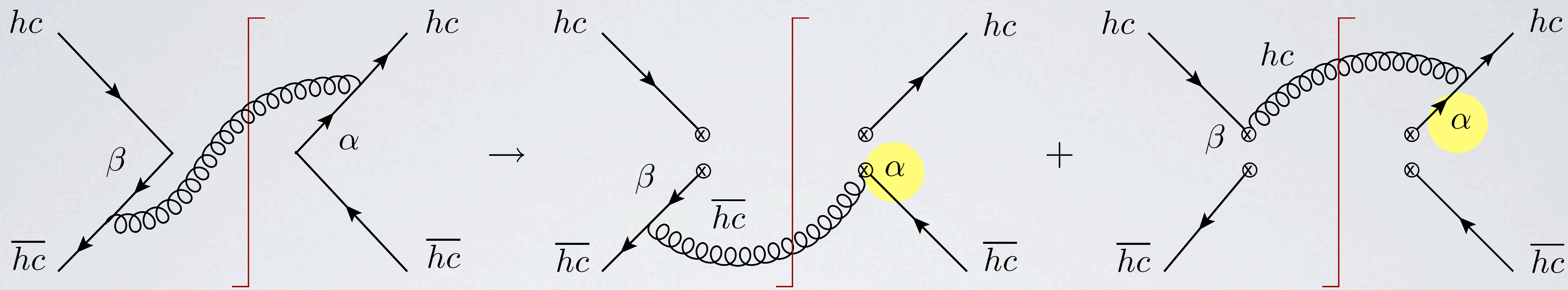


- Raise QCD propagators carrying large momentum p, $(\bar{p})$ to fractional powers α, (β):

$$\frac{1}{-(p-k)^2 - i\varepsilon} \to \frac{\nu_1^{2\alpha}}{[-(p-k)^2 - i\varepsilon]^{1+\alpha}}$$

- Limit $\alpha \to 0$, $\beta \to 0$ is trivial for QCD, but effective theory diagrams have poles, which only cancel in the sum of collinear and anti-collinear diagrams.

ANALYTICAL REGULARIZATION



- Regulators play double role. E.g. α regulates *hc* propagators and $\overline{hc}$ Wilson line

$$\frac{n^\mu}{n\cdot k - i\varepsilon} \to \frac{\nu_1^{2\alpha}\; n^\mu\, \bar{n}\cdot p}{(n\cdot k\, \bar{n}\cdot p - i\varepsilon)^{1+\alpha}}$$

- Regulator breaks invariance of anti-hard-collinear sector under a rescaling of the hard-collinear momentum $p \longrightarrow \lambda p$.

1-LOOP RESULT

Taking first $\beta \longrightarrow 0$, then $\alpha \longrightarrow 0$, one finds ($L_\perp = \ln \frac{x_T^2 \mu^2}{4e^{-2\gamma_E}}$)

$$\mathcal{I}_{q\leftarrow q}(z, x_T^2, \mu) = \delta(1-z) - \frac{C_F \alpha_s}{2\pi} \left\{ \left(\frac{1}{\epsilon} + L_\perp \right) \left[\left(\frac{2}{\alpha} - 2 \ln \frac{\mu^2}{\nu_1^2} \right) \delta(1-z) + \frac{1+z^2}{(1-z)_+} \right] + \delta(1-z) \left(-\frac{2}{\epsilon^2} + L_\perp^2 + \frac{\pi^2}{6} \right) - (1-z) \right\}$$

anomalous M^2 dep.

↓

$$\mathcal{I}_{\bar{q}\leftarrow \bar{q}}(z, x_T^2, \mu) = \delta(1-z) - \frac{C_F \alpha_s}{2\pi} \left\{ \left(\frac{1}{\epsilon} + L_\perp \right) \left[\left(-\frac{2}{\alpha} + 2 \ln \frac{M^2}{\nu_1^2} \right) \delta(1-z) + \frac{1+z^2}{(1-z)_+} \right] - (1-z) \right\}$$

In the product the $1/\alpha$ divergences vanish, but anomalous M^2 dependence remains.

- For the functions I and F, one then obtains

$$F_{q\bar{q}}(L_\perp, \alpha_s) = \frac{C_F \alpha_s}{\pi} L_\perp + \mathcal{O}(\alpha_s^2)$$

$$I_{q\leftarrow q}(z, L_\perp, \alpha_s) = I_{\bar{q}\leftarrow\bar{q}}(z, L_\perp, \alpha_s) = \delta(1-z)\left[1 + \frac{C_F\alpha_s}{4\pi}\left(L_\perp^2 + 3L_\perp - \frac{\pi^2}{6}\right)\right] - \frac{C_F\alpha_s}{2\pi}\Big[L_\perp P_{q\leftarrow q}(z) - (1-z)\Big] + \mathcal{O}(\alpha_s^2)$$

$$I_{q\leftarrow g}(z, L_\perp, \alpha_s) = I_{\bar{q}\leftarrow g}(z, L_\perp, \alpha_s) = -\frac{T_F\alpha_s}{2\pi}\Big[L_\perp P_{q\leftarrow g}(z) - 2z(1-z)\Big] + \mathcal{O}(\alpha_s^2)$$

- Solving its RG and using Davies, Stirling and Webber '84 and de Florian and Grazzini '01, we extract the two-loop *F*

$$F_{q\bar{q}}(L_\perp, \alpha_s) = \frac{\alpha_s}{4\pi}\Gamma_0^F L_\perp + \left(\frac{\alpha_s}{4\pi}\right)^2\left[\frac{\Gamma_0^F\beta_0}{2}L_\perp^2 + \Gamma_1^F L_\perp + d_2^q\right]$$

$$d_2^q = C_F\left[C_A\left(\frac{808}{27} - 28\zeta_3\right) - \frac{224}{27}T_F n_f\right]$$

Casimir scaling

$$\frac{F_{q\bar{q}}(L_\perp, \alpha_s)}{C_F} = \frac{F_{gg}(L_\perp, \alpha_s)}{C_A}$$

All the necessary input for NNLL resummation!

RESUMMED RESULT

$$\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \frac{4\pi\alpha^2}{3N_cM^2s}\sum_q e_q^2 \sum_{i=q,g}\sum_{j=\bar{q},g}\int_{\xi_1}^1 \frac{dz_1}{z_1}\int_{\xi_2}^1 \frac{dz_2}{z_2}$$
$$\times\left[C_{q\bar{q}\to ij}\left(\frac{\xi_1}{z_1},\frac{\xi_2}{z_2},q_T^2,M^2,\mu\right)\phi_{i/N_1}(z_1,\mu)\,\phi_{j/N_2}(z_2,\mu) + (q,i\leftrightarrow\bar{q},j)\right]$$

The hard-scattering kernel is

$$C_{q\bar{q}\to ij}(z_1,z_2,q_T^2,M^2,\mu) = \left|C_V(-M^2,\mu)\right|^2 \frac{1}{4\pi}\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\left(\frac{x_T^2M^2}{4e^{-2\gamma_E}}\right)^{-F_{q\bar{q}}(x_T^2,\mu)}$$
$$\times\, I_{q\leftarrow i}(z_1,x_T^2,\mu)\, I_{\bar{q}\leftarrow j}(z_2,x_T^2,\mu)$$

- Two sources of *M* dependence: hard function and anomaly
- Fourier transform can be evaluated numerically or analytically, if higher-log terms are expanded out.

RESULT IN MOMENTUM SPACE

Using the relation ($\eta = \frac{C_F \alpha_s}{\pi} \ln \frac{M^2}{\mu^2}$)

$$\frac{1}{4\pi}\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\, L_\perp^n \left(\frac{x_T^2\mu^2}{4e^{-2\gamma_E}}\right)^{-\eta} = (-\partial_\eta)^n\, \frac{1}{q_T^2}\left(\frac{q_T^2}{\mu^2}\right)^\eta \frac{\Gamma(1-\eta)}{e^{2\eta\gamma_E}\,\Gamma(\eta)}$$

one obtains an analytic expression for the rate in momentum space.

However, the *x*-space result contains terms $\exp\left(-\alpha_s c L_\perp^2\right)$. Since $L_\perp$ is a small log for the proper scale choice, this can formally be expanded, but higher order terms are factorially enhanced. Frixione, Nason, Ridolfi '98

→ Keep the double log terms in exponent, perform Fourier transform numerically.

BEHAVIOR AT VERY LOW q_T

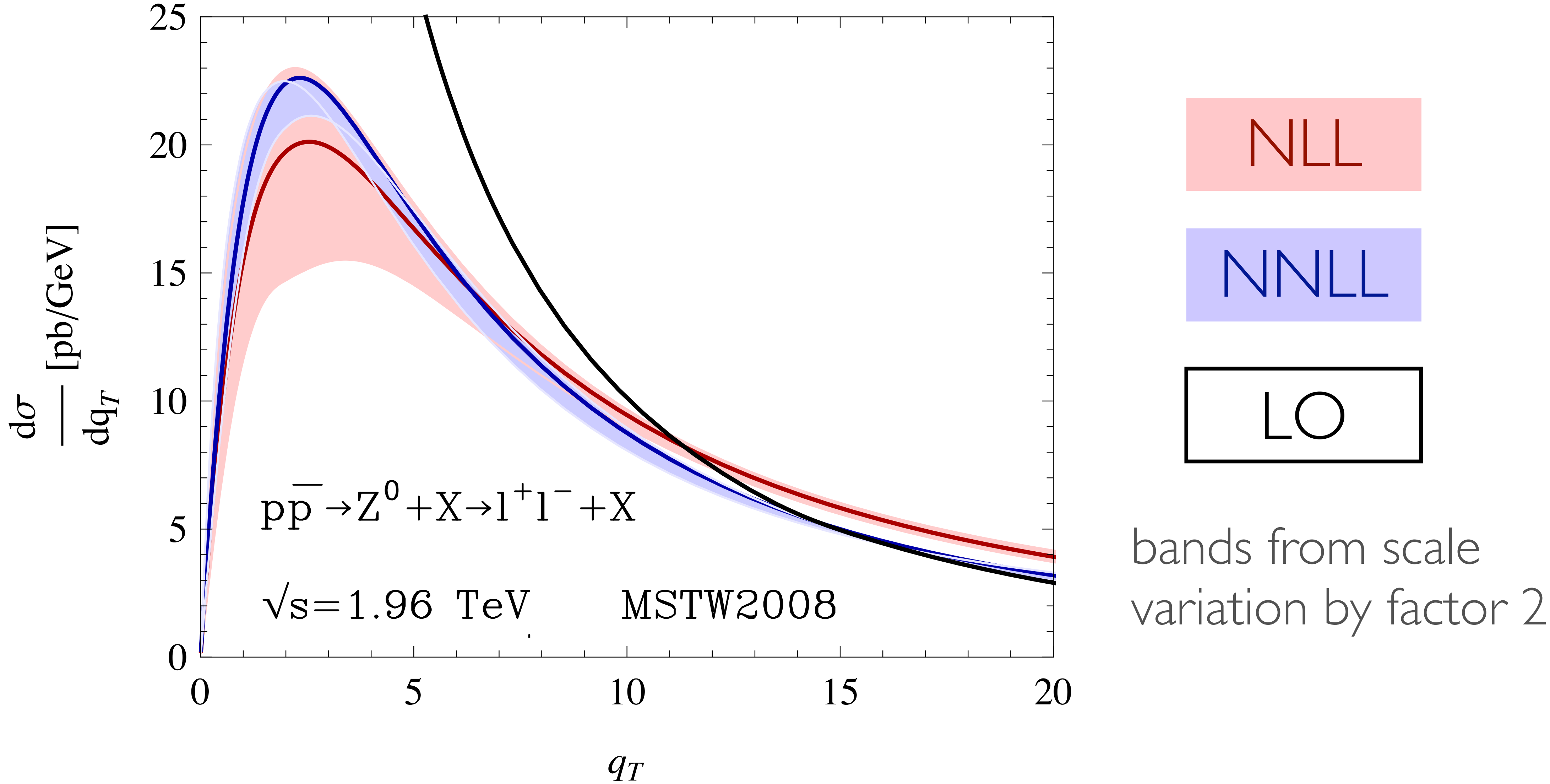
For moderate q_T, the natural scale choice is $\mu = q_T$. This is no longer true at very small q_T. Detailed analysis shows that near $q_T \approx 0$ the Fourier integral is dominated by

$$\langle x_T^{-1} \rangle = q_* = M \exp\left(-\frac{\pi}{2C_F \alpha_s(q_*)} \right) = 1.75\,\mathrm{GeV} \quad \text{for } M = M_Z$$

Small scale q_T drops out of integral! Spectrum is perturbative even at very low q_T! Parisi and Petronzio '80

We choose $\mu = \max(q_*, q_T)$ for general q_T.

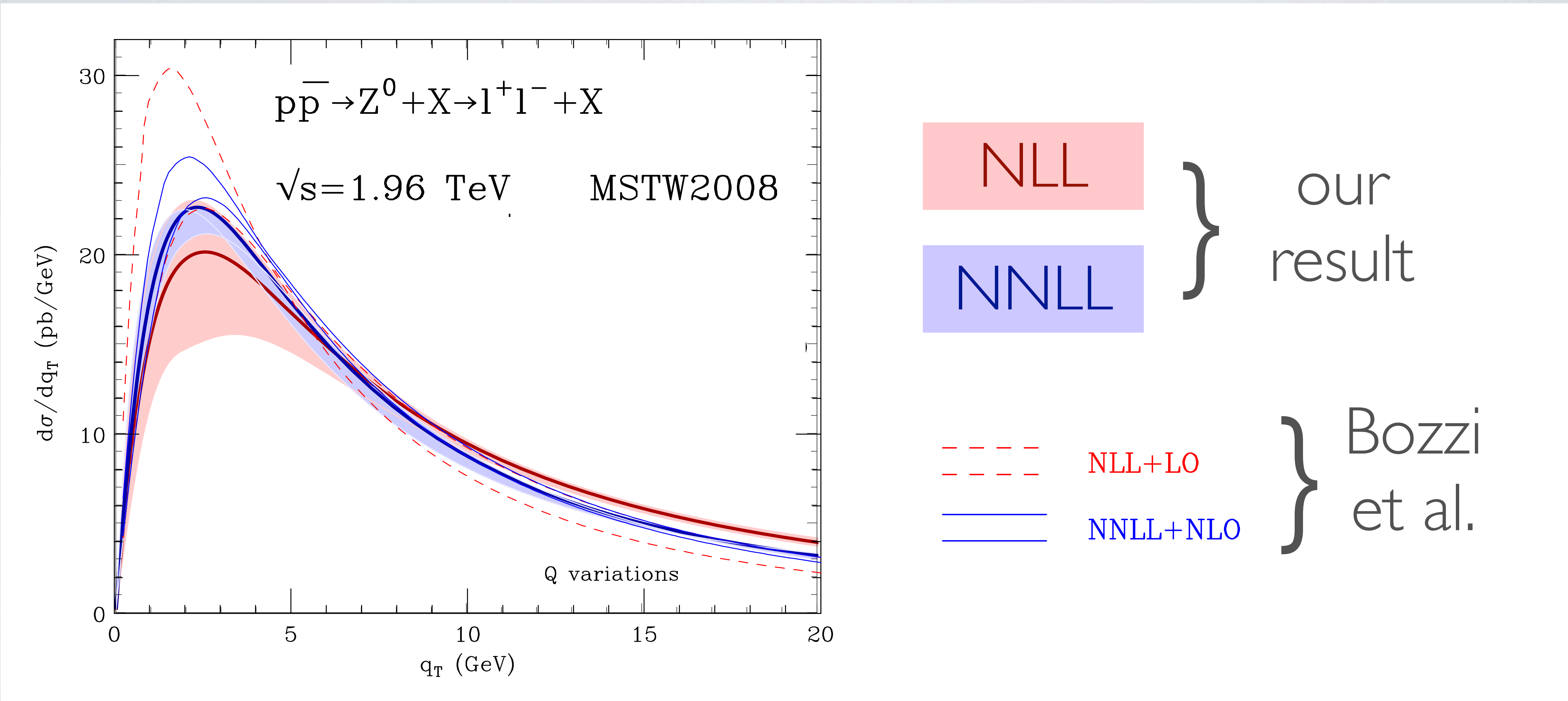
NUMERICAL RESULTS



Preliminary: no matching to fixed order, narrow width approximation, no photon contribution.

ROUGH COMPARISON WITH BOZZI ET AL.

Bozzi, Catani, Ferrera, de Florian and Grazzini, arXiv:1007.2351



Our result does not (yet!) include the matching to fixed order, which is about 2pb/GeV for Bozzi et al. at the peak.

Taking this into account, the results are in good agreement.

COLLINS SOPER STERMAN FORMULA

$$\frac{d^3\sigma}{dM^2\,dq_T^2\,dy} = \frac{4\pi\alpha^2}{3N_cM^2s}\,\frac{1}{4\pi}\int d^2x_\perp\, e^{-iq_\perp\cdot x_\perp}\sum_q e_q^2\sum_{i=q,g}\sum_{j=\bar{q},g}\int_{\xi_1}^1\frac{dz_1}{z_1}\int_{\xi_2}^1\frac{dz_2}{z_2}$$

$$\times\,\exp\left\{-\int_{\mu_b^2}^{M^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\frac{M^2}{\bar{\mu}^2}\,A\big(\alpha_s(\bar{\mu})\big)+B\big(\alpha_s(\bar{\mu})\big)\right]\right\}$$

$$\times\,\Big[C_{qi}\big(z_1,\alpha_s(\mu_b)\big)\,C_{\bar{q}j}\big(z_2,\alpha_s(\mu_b)\big)\,\phi_{i/N_1}(\xi_1/z_1,\mu_b)\,\phi_{j/N_2}(\xi_2/z_2,\mu_b)+(q,i\leftrightarrow\bar{q},j)\Big]$$

- The low scale is $\mu_b = b_0/x_T$, and we set $b_0 = 2e^{-\gamma_E}$.
- Landau-pole singularity in the Fourier transform. To use the formula, one needs additional prescription to deal with this.

RELATION TO CSS

If adopt the choice $\mu = \mu_b = 2e^{-\gamma_E}/x_\perp$ in our result reduces to CSS formula, provided we identity

$$A(\alpha_s) = \Gamma^F_{\rm cusp}(\alpha_s) - \frac{\beta(\alpha_s)}{2}\,\frac{dg_1(\alpha_s)}{d\alpha_s}\,,$$

$$B(\alpha_s) = 2\gamma^q(\alpha_s) + g_1(\alpha_s) - \frac{\beta(\alpha_s)}{2}\,\frac{dg_2(\alpha_s)}{d\alpha_s}\,,$$

$$C_{ij}\big(z,\alpha_s(\mu_b)\big) = \big|C_V\big(-\mu_b^2,\mu_b\big)\big|\, I_{i\leftarrow j}\big(z,0,\alpha_s(\mu_b)\big)\,,$$

anomaly contribution

$$g_1(\alpha_s) = F(0,\alpha_s)$$

$$g_2(\alpha_s) = \ln\big|C_V(-\mu^2,\mu)\big|^2$$

Use these relations to derive unknown three-loop coefficient, necessary for NNLL resummation

$$A^{(3)} = \Gamma_2^F + 2\beta_0 d_2^q = 239.2 - 652.9$$

Not equal to the cusp anom. dim. as was usually assumed!

CONCLUSION

- Have performed a factorization analysis of the Drell-Yan cross section at low transverse momentum and derived a resummed result for the spectrum, free of large logarithms
  - No Landau-pole ambiguity. Have analytic expression in momentum space, which allows for simple matching to fixed order.
- Reduces to the known CSS result for a special scale choice.
  - Have derived unkown three-loop coefficient $A^{(3)}$, the last missing piece needed for NNLL accuracy.
- The product of two transverse PDFs is well defined but has anomalous dependence on the large momentum transfer
  - we show that this dependence exponentiates
- Phenomenological analysis at NNLL+NLO is in progress.