Introduction to Electroweak theory and Higgs-boson physics at the LHC

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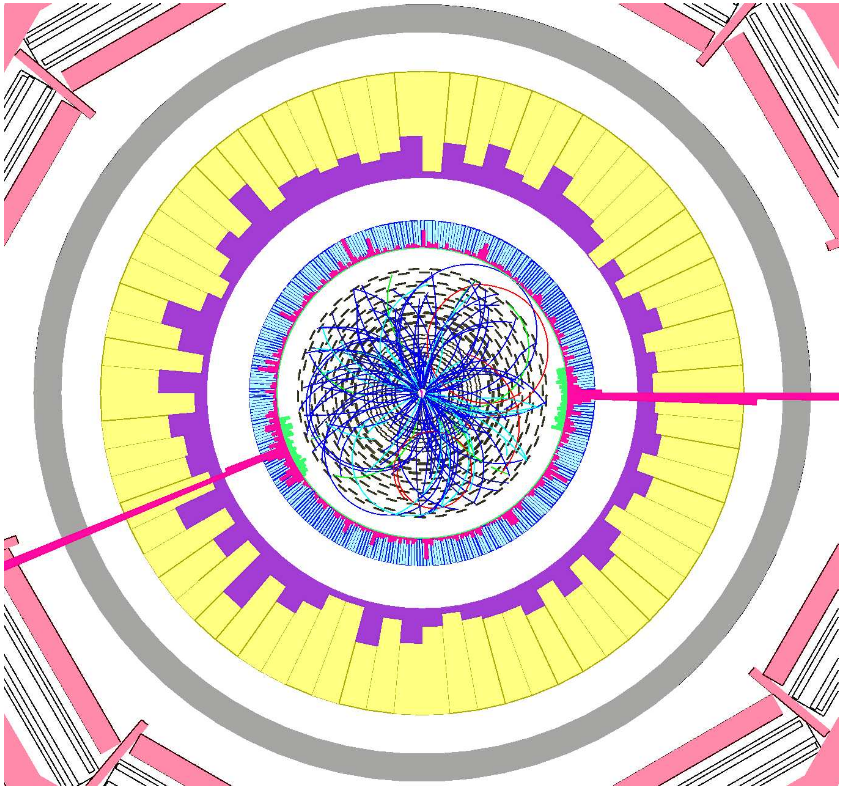
GGI, Firenze, September 2007

- Theoretical introduction
- Constraints on the Higgs boson
- Higgs boson signals at the LHC



The Standard Model (SM)

- A quick introduction to non-Abelian gauge theories: many formulae but they will look familiar!
  - QED
  - Yang-Mills theories
  - Electroweak interactions

- Spontaneous symmetry breaking and mass generation: the Higgs boson

- Theoretical bounds on the mass of the Higgs boson

- Experimental bounds on the mass of the Higgs boson

Exercise: Please, do the exercises! You will be given all the elements to solve them.

Abelian gauge theory: QED

We start with a Lagrangian (density)

$$\mathcal{L}_0 = \bar{\psi}(x)\,(i\partial\!\!\!/ - m)\,\psi(x)$$

invariant under a GLOBAL U(1) symmetry (θ is constant)

$$\begin{aligned} \psi(x) &\quad\rightarrow\quad e^{iq\theta}\psi(x) \\ \partial_\mu\psi(x) &\quad\rightarrow\quad e^{iq\theta}\partial_\mu\psi(x) \end{aligned}$$

From Noether's theorem, there is a conserved current:

$$J_\mu(x) = q\bar{\psi}(x)\gamma_\mu\psi(x) \qquad \Longrightarrow \qquad \partial^\mu J_\mu(x) = 0$$

To gauge this theory, we promote the GLOBAL U(1) symmetry to local symmetry:

$$\begin{aligned} \psi(x) &\quad\rightarrow\quad e^{iq\theta(x)}\psi(x) \\ \partial_\mu\psi(x) &\quad\rightarrow\quad e^{iq\theta(x)}\partial_\mu\psi(x) + iqe^{iq\theta(x)}\psi(x)\partial_\mu\theta(x) \end{aligned}$$

Covariant derivative

Invent a new derivative D_μ such that

$$\begin{aligned} \psi(x) &\rightarrow e^{iq\theta(x)}\psi(x) = U(x)\,\psi(x) \\ D_\mu\psi(x) &\rightarrow e^{iq\theta(x)}D_\mu\psi(x) = U(x)D_\mu\psi(x) \end{aligned}$$

i.e. both $\psi(x)$ and $D_\mu\psi(x)$ transform the same way under the U(1) local symmetry

$$D_\mu \equiv \partial_\mu + iqA_\mu$$

where A_μ transforms under the local gauge symmetry as

$$A_\mu \rightarrow A_\mu - \partial_\mu\theta(x)$$

The commutator of the covariant derivatives gives the electric and the magnetic fields, i.e. the field strength tensor

$$F_{\mu\nu}(x) = \frac{1}{iq}\left[D_\mu, D_\nu\right] = \frac{1}{iq}\left[\partial_\mu + iqA_\mu, \partial_\nu + iqA_\nu\right] = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

$F_{\mu\nu}$ is invariant under a gauge transformation.

From global to local symmetry

From

$$\mathcal{L}_0 = \bar{\psi}(x)\,(i\partial\!\!\!/ - m)\,\psi(x)$$

invariant under GLOBAL U(1), to

$$\begin{aligned}\mathcal{L}_1 &= \bar{\psi}(x)\,(iD\!\!\!\!/ - m)\,\psi(x)\\ &= \bar{\psi}(x)\,(i\partial\!\!\!/ - m)\,\psi(x) - q\bar{\psi}(x)\gamma_\mu\psi(x)A^\mu(x)\end{aligned}$$

invariant under LOCAL U(1) and interpret $A^\mu(x)$ as the photon field and $J_\mu = q\bar{\psi}\gamma_\mu\psi$ as the electromagnetic current. The only missing ingredient is the kinetic term for the photon field

$$\mathcal{L}_2 = \mathcal{L}_1 - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x)$$

$\mathcal{L}_2$ cannot contain a term proportional to $A_\mu A^\mu$ (a mass term for the photon field) since this term is not gauge invariant under the local U(1)

$$A_\mu \longrightarrow A_\mu - \partial_\mu\theta(x)$$

Non-Abelian (Yang-Mills) gauge theories

The starting point is a Lagrangian of free or self-interacting fields, that is symmetric under a GLOBAL symmetry

$$\mathcal{L}_\psi(\psi, \partial_\mu \psi)$$

where

$$\psi = \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix} = \text{multiplet of a compact Lie group } G$$

The Lagrangian is symmetric under the transformation

$$\psi \to \psi' = U(\theta)\psi \qquad U(\theta) = \exp(igT^a\theta_a) \qquad \text{unitary matrix} \qquad UU^\dagger = U^\dagger U = 1$$

If U is unitary, the T^a are hermitian, and are called group generators (they "generate" infinitesimal transformation around the unity

$$U(\theta) = 1 + igT^a\theta_a + \mathcal{O}\left(\theta^2\right)$$

If $U \in \mathrm{SU}(N)$ matrix (unitary and $\det U = 1$), then there are $N^2 - 1$ traceless, hermitian generators $T^a = \lambda^a/2$.

Exercise: Show this.

Gauging the symmetry

The generators satisfy the relation

$$\left[T^a, T^b\right] = i f^{abc} T^c$$

and the f^{abc} are called the structure functions of the group G. The starting hypothesis is that $\mathcal{L}$ is invariant under G

$$\mathcal{L}_\psi(\psi, \partial_\mu \psi) = \mathcal{L}_\psi(\psi', \partial_\mu \psi') \qquad\qquad \psi' = U(\theta)\psi$$

Gauging the symmetry means to allow the parameters θ^a to be function of the space-time coordinates $\theta^a \to \theta^a(x)$ so that $\Longrightarrow U \to U(x)$

$$U(x) = 1 + igT^a\theta_a(x) + \mathcal{O}\left(\theta^2\right)$$

From $\partial_\mu \longrightarrow D_\mu$

We obtain a LOCAL invariant Lagrangian if we make the substitution

$$\mathcal{L}_\psi(\psi, \partial_\mu \psi) \rightarrow \mathcal{L}_\psi(\psi, D_\mu \psi) \qquad D_\mu = \partial_\mu - ig A^a_\mu(x) T_a \equiv \partial_\mu - ig A_\mu(x)$$

with the transformation properties

$$\begin{aligned} \psi(x) \quad &\rightarrow \quad U(x)\,\psi(x) = \left(1 + ig\,\theta^a\,T_a + \mathcal{O}(\theta^2)\right)\psi(x) \\ D_\mu \quad &\rightarrow \quad U(x) D_\mu \psi(x) = U(x) D_\mu U^{-1}(x)\,U(x)\,\psi(x) \end{aligned}$$

i.e. the covariant derivative must transform as

$$D_\mu \rightarrow U(x) D_\mu U^{-1}(x) \qquad \text{implying} \qquad A^a_\mu \rightarrow A^a_\mu + \partial_\mu \theta^a(x) + g f^{abc} A^b_\mu \theta^c + \mathcal{O}(\theta^2)$$

We can build a kinetic term for the A^a_μ fields from

$$F_{\mu\nu} = F^a_{\mu\nu}\,T^a = \frac{i}{g}\,[D_\mu, D_\nu] \qquad \text{with} \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

which transforms homogeneously under a local gauge transformation

$$F_{\mu\nu} \rightarrow U F_{\mu\nu} U^{-1} \quad \Longrightarrow \quad F^a_{\mu\nu} F^{\mu\nu}_a \equiv \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} \rightarrow \mathrm{Tr}\, U F_{\mu\nu} U^{-1}\, U F^{\mu\nu} U^{-1} = \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu}$$

where $F^a_{\mu\nu} F^{\mu\nu}_a$ is gauge invariant ($F^a_{\mu\nu}$ in not singularly gauge-invariant).

The Lagrangian for gauge and matter field

Gauge invariant Yang-Mills (YM) Lagrangian for gauge and matter fields

$$\mathcal{L}_{YM} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \mathcal{L}_\psi(\psi, D_\mu \psi)$$

where

$$\begin{aligned} D_\mu &= \partial_\mu - i g A^a_\mu T_a \\ F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \\ \left[T^a, T^b\right] &= i f^{abc} T^c \end{aligned}$$

Remarks on Yang-Mills theories

- Mass terms $A_\mu^a A_a^\mu$ for the gauge bosons are NOT gauge invariant!
  No mass term is allowed in the Lagrangian.

  Gauge bosons of (unbroken) YM theories are massless.

- From the $F_{\mu\nu}^a F_a^{\mu\nu} = \left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c\right)\left(\partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc} A_b^\mu A_c^\nu\right)$ part of the Lagrangian, we have cubic and quartic gauge boson self interactions

- gauge invariance, Lorentz structure and renormalizability (absence of higher powers of fields and covariant derivatives in $\mathcal{L}$) determines gauge-boson/matter couplings and gauge-boson self interaction

- if G =SU$_c(N=3)$ and the fermion are in triplets,

$$\psi = \begin{pmatrix} \psi_{\text{red}} \\ \psi_{\text{blue}} \\ \psi_{\text{green}} \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

  we have the QCD Lagrangian and $N^2 - 1 = 8$ gauge bosons = gluons.

Exercise: Derive the form of the three- and four-gluon vertex starting from gauge invariance, Lorentz structure and renormalizability of the Lagrangian.

Electroweak sector

From experimental facts (charged currents couple only with left-handed fermions, the existence of a massless photon and a neutral $Z\ldots$), the electroweak group is chosen to be $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$.

$$\psi_L \equiv \frac{1}{2}(1-\gamma_5)\psi \qquad \psi_R \equiv \frac{1}{2}(1+\gamma_5)\psi \qquad \psi = \psi_L + \psi_R$$

$$L_L \equiv \frac{1}{2}(1-\gamma_5)\begin{pmatrix} \nu_e \\ e \end{pmatrix} = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \qquad \nu_{eR} \equiv \frac{1}{2}(1+\gamma_5)\nu_e \qquad e_R \equiv \frac{1}{2}(1+\gamma_5)e$$

- $\mathrm{SU}(2)_L$: weak isospin group. Three generators $\Longrightarrow$ three gauge bosons: W^1, W^2 and W^3.
  The generators for doublets are $T^a = \sigma^a/2$, where σ^a are the 3 Pauli matrices (when acting on the gauge singlet e_R and ν_R, $T^a \equiv 0$), and they satisfy $\left[\sigma^a, \sigma^b\right] = i\epsilon^{abc}\sigma^c$.
  The gauge coupling will be indicated with g.
- $\mathrm{U}(1)_Y$: weak hypercharge Y. One gauge boson B with gauge coupling g'.
  One generator (charge) $Y(\psi)$, whose value depends on the corresponding field.

Gauging the symmetry: fermionic Lagrangian

Following the gauging recipe (for one generation of leptons. Quarks work the same way)

$$\mathcal{L}_\psi = i\,\bar{L}_L\,\not{D}\,L_L + i\,\bar{\nu}_{eR}\,\not{D}\,\nu_{eR} + i\,\bar{e}_R\,\not{D}\,e_R$$

where

$$D^\mu = \partial^\mu - igW_i^\mu T^i - ig'\frac{Y(\psi)}{2}B^\mu \qquad T^i = \frac{\sigma^i}{2} \quad \text{or} \quad T^i = 0 \qquad i = 1,2,3$$

$$\mathcal{L}_\psi \equiv \mathcal{L}_{kin} + \mathcal{L}_{CC} + \mathcal{L}_{NC}$$

$$\begin{aligned}
\mathcal{L}_{kin} &= i\,\bar{L}_L\,\not{\partial}\,L_L + i\,\bar{\nu}_{eR}\,\not{\partial}\,\nu_{eR} + i\,\bar{e}_R\,\not{\partial}\,e_R \\
\mathcal{L}_{CC} &= g\,W_\mu^1\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_1}{2}\,L_L + g\,W_\mu^2\,\bar{L}_L\,\gamma^\mu\,\frac{\sigma_2}{2}\,L_L = \frac{g}{\sqrt{2}}\,W_\mu^+\,\bar{L}_L\,\gamma^\mu\,\sigma^+\,L_L + \frac{g}{\sqrt{2}}\,W_\mu^-\,\bar{L}_L\,\gamma^\mu\,\sigma^-\,L_L \\
&= \frac{g}{\sqrt{2}}\,W_\mu^+\,\bar{\nu}_L\,\gamma^\mu\,e_L + \frac{g}{\sqrt{2}}\,W_\mu^-\,\bar{e}_L\,\gamma^\mu\,\nu_L \\
\mathcal{L}_{NC} &= \frac{g}{2}\,W_\mu^3\,[\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} - \bar{e}_L\,\gamma^\mu\,e_L] + \frac{g'}{2}\,B_\mu\Big[Y(L)\,(\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} + \bar{e}_L\,\gamma^\mu\,e_L) \\
&\quad + Y(\nu_{eR})\,\bar{\nu}_{eR}\,\gamma^\mu\,\nu_{eR} + Y(e_R)\,\bar{e}_R\,\gamma^\mu\,e_R\Big]
\end{aligned}$$

with

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right) \qquad \sigma^\pm = \frac{1}{2}\left(\sigma^1 \pm i\sigma^2\right)$$

Electroweak unification

$$\mathcal{L}_{NC} = \frac{g}{2} W^3_\mu \left[\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} - \bar{e}_L\,\gamma^\mu\,e_L\right] + \frac{g'}{2} B_\mu \Big[Y(L)\left(\bar{\nu}_{eL}\,\gamma^\mu\,\nu_{eL} + \bar{e}_L\,\gamma^\mu\,e_L\right)$$
$$+Y(\nu_{eR})\,\bar{\nu}_{eR}\,\gamma^\mu\,\nu_{eR} + Y(e_R)\,\bar{e}_R\,\gamma^\mu\,e_R\Big]$$

Neither W^3_μ nor B_μ can be interpreted as the photon field A_μ, since they couple to neutral fields.

$$\Psi \equiv \begin{pmatrix} \nu_{eL} \\ e_L \\ \nu_{eR} \\ e_R \end{pmatrix} \qquad \mathcal{T}_3 \equiv \begin{pmatrix} 1/2 & 0 & & \\ 0 & -1/2 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \qquad \mathcal{Y} \equiv \begin{pmatrix} Y(L) & & & \\ & Y(L) & & \\ & & Y(\nu_{eR}) & \\ & & & Y(e_R) \end{pmatrix}$$

$$\mathcal{L}_{NC} = g\,\bar{\Psi}\,\gamma^\mu\,\mathcal{T}_3\,\Psi\,W^3_\mu + g'\,\bar{\Psi}\,\gamma^\mu\,\frac{\mathcal{Y}}{2}\,\Psi\,B_\mu$$

Weak mixing angle

We perform a rotation of an angle θ_W, the Weinberg angle, in the space of the two neutral gauge fields (W^3_μ and B_μ). We use an orthogonal transformation in order to keep the kinetic terms diagonal in the vector fields

$$\begin{aligned} B_\mu &= A_\mu \cos\theta_W - Z_\mu \sin\theta_W \\ W^3_\mu &= A_\mu \sin\theta_W + Z_\mu \cos\theta_W \end{aligned}$$

so that

$$\mathcal{L}_{NC} = \bar{\Psi}\gamma^\mu \left[g\, \sin\theta_W\, \mathcal{T}_3 + g'\, \cos\theta_W\, \frac{\mathcal{Y}}{2} \right] \Psi\, A_\mu + \bar{\Psi}\gamma^\mu \left[g\, \cos\theta_W\, \mathcal{T}_3 - g'\, \sin\theta_W\, \frac{\mathcal{Y}}{2} \right] \Psi\, Z_\mu$$

We can identify A_μ with the photon field provided

$$e\mathcal{Q} = g\, \sin\theta_W\, \mathcal{T}_3 + g'\, \cos\theta_W\, \frac{\mathcal{Y}}{2} \qquad\qquad \mathcal{Q} = \text{electromagnetic charge}$$

The weak hypercharges $\mathcal{Y}$ appear only through the combination $g'\,\mathcal{Y}$. We use this freedom to fix

$$Y(L) = -1$$

Weak mixing angle

With this choice, the doublet of left-handed leptons gives $\left(e\mathcal{Q} = g\,\sin\theta_W\,\mathcal{T}_3 + g'\,\cos\theta_W\,\frac{\mathcal{Y}}{2}\right)$

$$\begin{aligned} 0 &= \frac{g}{2}\sin\theta_W - \frac{g'}{2}\cos\theta_W \\ -e &= -\frac{g}{2}\sin\theta_W - \frac{g'}{2}\cos\theta_W \end{aligned}$$

so that

$$g\sin\theta_W = g'\cos\theta_W = e$$

and

$$\mathcal{Q} = \mathcal{T}_3 + \frac{\mathcal{Y}}{2} \qquad \text{Gell-Mann–Nishijima formula.}$$

From this formula we have $Y(\nu_{eR}) = 0$ and $Y(e_R) = -2$.

Notice that the right-handed neutrino has zero charge, zero hypercharge and it is in a SU(2) singlet: it does not take part in electroweak interactions.

Exercise: Verify that, with the previous hypercharge assignments, one can generate the correct electromagnetic current.

The neutral current

$$
\begin{aligned}
\mathcal{L}_{NC} &= \bar{\Psi}\gamma^{\mu}\left[g\,\sin\theta_W\,\mathcal{T}_3 + g'\,\cos\theta_W\,\frac{\mathcal{Y}}{2}\right]\Psi\,A_{\mu} + \bar{\Psi}\gamma^{\mu}\left[g\,\cos\theta_W\,\mathcal{T}_3 - g'\,\sin\theta_W\,\frac{\mathcal{Y}}{2}\right]\Psi\,Z_{\mu} \\
&= e\,\bar{\Psi}\gamma^{\mu}\mathcal{Q}\Psi\,A_{\mu} + \bar{\Psi}\gamma^{\mu}\mathcal{Q}_Z\Psi\,Z_{\mu}
\end{aligned}
$$

where $\mathcal{Q}_Z$ is a diagonal matrix given by

$$
\mathcal{Q}_Z = \frac{e}{\cos\theta_W\sin\theta_W}\left(\mathcal{T}_3 - \mathcal{Q}\sin^2\theta_W\right)
$$

Exercise: Show this.

We can proceed, in a similar way, with quarks (see more later)

$$
Q_L^i = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \qquad \begin{aligned} u_R^i &= u_R, c_R, t_R \\ d_R^i &= d_R, s_R, b_R \end{aligned}
$$

Fermion fields of the SM and gauge quantum numbers

				$SU(3)$	$SU(2)$	$U(1)_Y$	$Q = T_3 + \frac{Y}{2}$
$Q_L^i =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{3}$	$\frac{2}{3}$ $-\frac{1}{3}$
$u_R^i =$	u_R	c_R	t_R	3	1	$\frac{4}{3}$	$\frac{2}{3}$
$d_R^i =$	d_R	s_R	b_R	3	1	$-\frac{2}{3}$	$-\frac{1}{3}$
$L_L^i =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	-1	0 -1
$e_R^i =$	e_R	μ_R	τ_R	1	1	-2	-1
$\nu_R^i =$	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	1	1	0	0

Electroweak gauge-boson sector

Gauge invariance and renormalizability completely determine the kinetic terms for the gauge bosons

$$\mathcal{L}_{YM} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu}$$

$$\begin{aligned} B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu \\ W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\, \epsilon^{abc}\, W_{b,\mu}\, W_{c,\nu} \end{aligned}$$

The gauge symmetry does NOT allow any mass terms for $W^\pm$ and Z.

Mass terms for gauge bosons

$$\mathcal{L}_{mass} = \frac{1}{2}\, m_A^2\, A_\mu\, A^\mu$$

are not invariant under a gauge transformation

$$A^\mu \;\rightarrow\; U(x) \left(A^\mu + \frac{i}{g}\, \partial^\mu \right) U^{-1}(x)$$

However, the gauge bosons of weak interactions are massive (short range of weak interactions).

Symmetries and conservation laws

To any continuous symmetry of the Lagrangian we can associate a conservation law and a conserved current.

Noether's theorem: if, without using the equation of motion, one can show that the Lagrangian density changes by a total divergence under and infinitesimal transformation

$$\phi \to \phi + \delta\phi \sim \phi + i\,\delta\theta\,\phi \qquad \left(\phi_j \to \phi_j + i\,\delta\theta^a\, t^a_{jk}\,\phi_k\right) \qquad \delta\theta \ll 1$$

$$\delta\mathcal{L}\,(\phi, \partial\phi) = \delta\theta\,\partial^\mu K_\mu \qquad \delta S = 0$$

then

$$J^\mu = \frac{\delta\mathcal{L}}{\delta\,\partial_\mu\phi}\delta\phi - K^\mu \quad \text{is conserved} \quad \partial_\mu J^\mu = 0$$

Important consequences

✓
$$Q = \int d^3x\, J^0(\vec{x}, t)$$
is conserved $(dQ/dt = 0)$ and is a Lorentz scalar

✓ After canonical quantization, $i\,\delta\theta\,[Q, \phi] = \delta\phi$, hence Q generates the symmetry acting on the fields

Symmetries in quantum field theories

Two ways of realizing symmetries in a QFT. Suppose we have a charge Q (obtained from Noether's theorem) that commutes with the Hamiltonian $[Q, H] = 0$. Then

- Wigner–Weyl

$$[Q, H] = 0 \qquad\qquad Q|0\rangle = 0$$

  The spectrum falls in explicit multiplets of the symmetry group (the vacuum $|0\rangle$ is the state of lowest energy)

- Nambu–Goldstone

$$[Q, H] = 0 \qquad\qquad Q|0\rangle \neq 0$$

  The symmetry is not manifest in the spectrum.

There is a third way too: the anomalous symmetries. In this case, the classical theory respects the symmetry, that is violated by quantum fluctuations

$$\partial^\mu J_\mu = 0 + \mathcal{O}(\hbar)$$

As we have stressed up to now, another important distinction is between global and local symmetries.

Spontaneous symmetry breaking

A symmetry is said to be spontaneously broken when the vacuum state is not invariant

$$\exp\left(i\,\delta\theta^{a}\,t^{a}\right)|0\rangle \neq |0\rangle \qquad \Longrightarrow \qquad Q^{a}|0\rangle \neq 0$$

This condition is equivalent to the existence of some set of fields operators ϕ_k with non-trivial transformation property under that symmetry transformation, and non-vanishing vacuum expectation values

$$\langle 0|\phi_k|0\rangle = v_k \neq 0$$

Proof

If the set of fields ϕ_j transforms non-trivially

$$\phi_j \to \left(e^{i\,\delta\theta^{a}\,t^{a}}\right)_{jk}\phi_k \sim \phi_j + \underbrace{i\,\delta\theta^{a}\,t^{a}_{jk}\,\phi_k}_{\delta\phi_j} = \phi_j + i\,\delta\theta^{a}\left[Q^{a},\phi_j\right]$$

Taking the expectation value on the vacuum

$$t^{a}_{jk}\,\langle 0|\phi_k|0\rangle = \langle 0|\left[Q^{a},\phi_j\right]|0\rangle \neq 0 \qquad \Longleftrightarrow \qquad Q^{a}|0\rangle \neq 0$$

Spontaneous symmetry breaking

Observations

- Experimentally, the space is isotropic, so ϕ_k must be a scalar, otherwise its vacuum expectation value would be frame-dependent.

- Experimentally, the space is homogeneous, so that $\langle 0|\phi_k|0\rangle$ is a constant. In fact, if the vacuum state is invariant under translations

$$\langle 0|\phi_k(x)|0\rangle = \langle 0|e^{iPx}\,\phi_k(0)e^{-iPx}|0\rangle = \langle 0|\phi_k(0)|0\rangle$$

- ϕ_k is not necessarily an elementary field

Spontaneous symmetry breaking in the SM

- ✓ Experimentally, the weak bosons have masses.
- ✓ The only way to introduce masses for the W and Z vector bosons, without spoiling unitarity and renormalizability, is spontaneous breaking of the gauge symmetry.
- ✓ The simplest way is through the (minimal) Higgs mechanism.

Spontaneous symmetry breaking in the SM

We give mass to the gauge bosons through the Higgs mechanism: generate mass terms from the kinetic energy term of a scalar doublet field Φ that undergoes a broken-symmetry process.

Introduce a complex scalar doublet: four scalar real fields (why will become clear later)

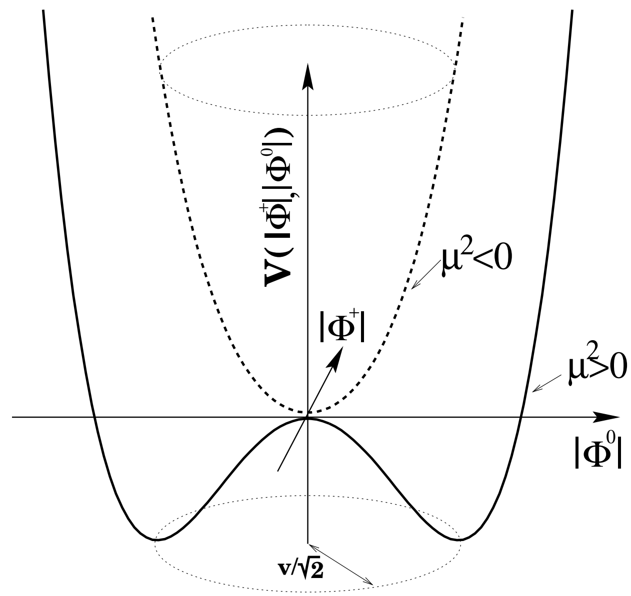
$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad Y(\Phi) = 1$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu\Phi)^\dagger(D^\mu\Phi) - V\left(\Phi^\dagger\Phi\right)$$

$$D^\mu = \partial^\mu - igW_i^\mu\frac{\sigma^i}{2} - ig'\frac{Y(\Phi)}{2}B^\mu$$

$$V\left(\Phi^\dagger\Phi\right) = -\mu^2\Phi^\dagger\Phi + \lambda\left(\Phi^\dagger\Phi\right)^2, \qquad \mu^2, \lambda > 0$$

Notice the "wrong" mass sign.



$V\left(\Phi^\dagger\Phi\right)$ is $SU(2)_L \times U(1)_Y$ symmetric.

- The reason why $Y(\Phi) = 1$ is not to break electric-charge conservation.
- Charge assignment for the Higgs doublets is done according to $Q = T_3 + Y/2$.

Spontaneous symmetry breaking

The potential has a minimum in correspondence of

$$|\Phi|^2 = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}$$

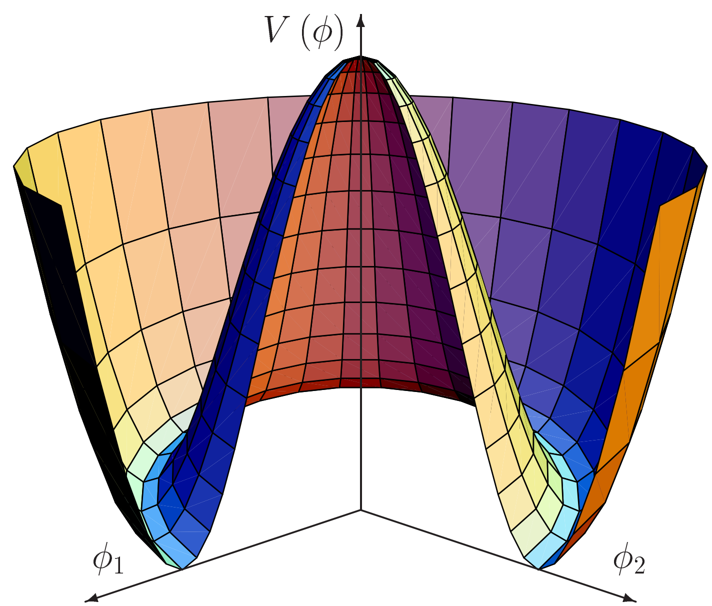
All these minimum configurations (ground states) are connected by gauge transformations, that change the phase of the complex field Φ, without affecting its modulus.

v is called the vacuum expectation value (VEV) of the neutral component of the Higgs doublet.

When the system chooses one of the minimum configurations, this configuration is no longer symmetric under the the gauge symmetry.

This is called spontaneous symmetry breaking.

The Lagrangian is still gauge invariant and all the properties connected with that (such that current conservation) are still there!



Expanding Φ around the minimum

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}\left[v + H(x) + i\chi(x)\right] \end{pmatrix} = \frac{1}{\sqrt{2}} \exp\left[\frac{i\sigma_i \theta^i(x)}{v}\right] \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

We can rotate away the fields $\theta^i(x)$ by an $\mathrm{SU}(2)_L$ gauge transformation

$$\Phi(x) \to \Phi'(x) = U(x)\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

where $U(x) = \exp\left[-\frac{i\sigma_i \theta^i(x)}{v}\right]$.

This gauge choice is called unitary gauge, and is equivalent to absorbing the Goldstone modes $\theta^i(x)$. Three would-be Goldstone bosons "eaten up" by three vector bosons $(W^\pm, Z)$ that acquire mass. This is why we introduced a complex scalar doublet (four elementary fields).

The vacuum state can be chosen to correspond to the vacuum expectation value

$$\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$\mathbf{SU(2)}_L \times \mathbf{U(1)}_Y \longrightarrow \mathbf{U(1)}_{\rm em}$

We can easily verify that the vacuum state breaks the gauge symmetry.

A state $\tilde{\Phi}$ is invariant under a symmetry operation $\exp(igT^a\theta_a)$ if

$$\exp(igT^a\theta_a)\tilde{\Phi} = \tilde{\Phi}$$

This means that a state is invariant if (just expand the exponent)

$$T^a\tilde{\Phi} = 0$$

For the $SU(2)_L \times U(1)_Y$ case we have

$$\sigma_1\Phi_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \qquad \text{broken}$$

$$\sigma_2\Phi_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \qquad \text{broken}$$

$\mathbf{SU(2)}_L \times \mathbf{U(1)}_Y \longrightarrow \mathbf{U(1)}_{\text{em}}$

$$\sigma_3 \Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \qquad \text{broken}$$

$$Y\Phi_0 = Y(\Phi) \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = +1 \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \qquad \text{broken}$$

But, if we examine the effect of the electric charge operator $\hat{Q} = Y/2 + T_3$ on the (electrically neutral) vacuum state, we have ($Y(\Phi) = 1$)

$$\hat{Q}\Phi_0 = \frac{1}{2}\left(\sigma_3 + Y\right)\Phi_0 = \frac{1}{2}\begin{pmatrix} Y(\Phi)+1 & 0 \\ 0 & Y(\Phi)-1 \end{pmatrix}\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

the electric charge symmetry is unbroken!

Consequences for the scalar field H

The scalar potential

$$V\left(\Phi^\dagger\Phi\right) = -\mu^2\Phi^\dagger\Phi \,+\, \lambda\left(\Phi^\dagger\Phi\right)^2$$

expanded around the vacuum state

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

becomes

$$V \;=\; \frac{1}{2}\left(2\lambda v^2\right)H^2 \,+\, \lambda v H^3 \,+\, \frac{\lambda}{4}H^4 - \frac{\lambda}{4}v^4$$

- the scalar field H gets a mass

$$m_H^2 = 2\lambda v^2$$

- there is a term of cubic and quartic self-coupling.
- a constant term: the cosmological constant (irrelevant in the Standard Model)

$$\rho_H \equiv \frac{\lambda}{4}v^4 = \frac{v^2 m_H^2}{8}$$

Cosmological constant

Up to now, we don't have a theory of gravitation. Gravitational interactions are commonly introduced by replacing ∂_μ by an appropriate derivative D_μ, containing the gravitation field

$$g_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu} \qquad \eta_{\mu\nu} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Furthermore, the Lagrangian must be given the overall factor $\sqrt{-\det(g_{\mu\nu})}$. At this point, the addition of a constant to the Lagrangian is of physical consequence.

The coefficient of the term that contains no other field dependence other than $\sqrt{-\det(g_{\mu\nu})}$ is the cosmological constant.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \mathcal{C} g_{\mu\nu} = -8\pi G_N T_{\mu\nu}$$

where $R_{\mu\nu}$ is the curvature tensor, and $T_{\mu\nu}$ is the energy-matter tensor. A non-zero value implies that a curved Universe in the absence of energy-matter. The cosmological constant defines the curvature of the vacuum.

Cosmological constant

Experimentally the Universe is known to be very flat, with a very tiny vacuum energy density

$$\rho_{\text{vac}} \leq 10^{-46}\ \text{GeV}^4$$

Inserting the current experimental lower bound for the Higgs boson mass, $m_H \geq 114$ GeV, and the value of $v = 246.22$ GeV (see more later), we find

$$\rho_H \geq 10^8\ \text{GeV}^4$$

some 54 order of magnitude larger than the upper bound inferred from the cosmological constant!

The smallness of the cosmological constant needs to be explained.

Either we must find a separate principle to zero the vacuum energy density of the Higgs field, or we may suppose that a proper quantum theory of gravity, in combination with the other interactions, will resolve the puzzle of the cosmological constant.

The vacuum energy problem must be an important clue. But to what?

Kinetic terms

$$
\begin{aligned}
D^\mu \Phi &= \left(\partial^\mu - igW_i^\mu \frac{\sigma^i}{2} - ig'\frac{1}{2}B^\mu\right)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \\
&= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2\sqrt{2}}\left[g\begin{pmatrix} W_3^\mu & W_1^\mu - iW_2^\mu \\ W_1^\mu + iW_2^\mu & -W_3^\mu \end{pmatrix} + g'B^\mu\right]\begin{pmatrix} 0 \\ v + H \end{pmatrix} \\
&= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2}(v+H)\begin{pmatrix} g\left(W_1^\mu - iW_2^\mu\right) \\ -gW_3^\mu + g'B^\mu \end{pmatrix}\right] \\
&= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2}\left(1 + \frac{H}{v}\right)\begin{pmatrix} gvW^{\mu+} \\ -v\sqrt{(g^2+g'^2)/2}\,Z^\mu \end{pmatrix}\right]
\end{aligned}
$$

$$
\left(D^\mu\Phi\right)^\dagger D_\mu\Phi = \frac{1}{2}\partial^\mu H\partial_\mu H + \left[\left(\frac{gv}{2}\right)^2 W^{\mu+}W_\mu^- + \frac{1}{2}\frac{\left(g^2+g'^2\right)v^2}{4}Z^\mu Z_\mu\right]\left(1+\frac{H}{v}\right)^2
$$

Exercise: Show this.

Consequences

- The W and Z gauge bosons have acquired masses

$$m_W^2 = \frac{g^2 v^2}{4} \qquad m_Z^2 = \frac{\left(g^2 + g'^2\right) v^2}{4} = \frac{m_W^2}{\cos^2\theta_W}$$

From the measured value of the Fermi constant G_F

$$\frac{G_F}{\sqrt{2}} = \left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{m_W^2} \qquad \Longrightarrow \qquad v = \sqrt{\frac{1}{\sqrt{2}G_F}} \approx 246.22 \text{ GeV}$$

- the photon stays massless
- HWW and HZZ couplings from $2H/v$ term (and $HHWW$ and $HHZZ$ couplings from H^2/v^2 term)

$$\mathcal{L}_{HVV} = \frac{2m_W^2}{v} W_\mu^+ W^{-\mu} H + \frac{m_Z^2}{v} Z^\mu Z_\mu H \equiv g m_w W_\mu^+ W^{-\mu} H + \frac{1}{2} \frac{g m_Z}{\cos\theta_W} Z^\mu Z_\mu H$$

Higgs coupling proportional to mass

- tree-level HVV (V = vector boson) coupling requires VEV!
Normal scalar couplings give $\Phi^\dagger \Phi V$ or $\Phi^\dagger \Phi V V$ couplings only.

Fermion mass generation

A direct mass term is not invariant under $SU(2)_L$ or $U(1)_Y$ gauge transformation

$$m_f \bar{\psi}\psi = m_f \left(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R \right)$$

Generate fermion masses through Yukawa-type interactions terms

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\Gamma_d^{ij} \bar{Q}_L'^{\,i} \Phi d_R'^{\,j} - \Gamma_d^{ij*} \bar{d}_R'^{\,i} \Phi^\dagger Q_L'^{\,j} \\
&\quad -\Gamma_u^{ij} \bar{Q}_L'^{\,i} \Phi_c u_R'^{\,j} + \text{h.c.} \\
&\quad -\Gamma_e^{ij} \bar{L}_L^i \Phi e_R^j + \text{h.c.} \\
&\quad -\Gamma_\nu^{ij} \bar{L}_L^i \Phi_c \nu_R^j + \text{h.c.}
\end{aligned}$$

$$\Phi_c = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

where Q', u' and d' are quark fields that are generic linear combination of the mass eigenstates u and d and Γ_u, Γ_d and Γ_e are 3×3 complex matrices in generation space, spanned by the indices i and j.

$\mathcal{L}_{\text{Yukawa}}$ is Lorentz invariant, gauge invariant and renormalizable, and therefore it can (actually it must) be included in the Lagrangian.

Notice: neutrino masses can be implemented via the Γ_ν term. Since $m_\nu \approx 0$, we neglect it.

Expanding around the vacuum state

In the unitary gauge we have

$$\bar{Q}'^{\,i}_L \, \Phi \, d'^{\,j}_R = \begin{pmatrix} \bar{u}'^{\,i}_L & \bar{d}'^{\,i}_L \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} d'^{\,j}_R = \frac{v+H}{\sqrt{2}} \, \bar{d}'^{\,i}_L \, d'^{\,j}_R$$

$$\bar{Q}'^{\,i}_L \, \Phi_c \, u'^{\,j}_R = \begin{pmatrix} \bar{u}'^{\,i}_L & \bar{d}'^{\,i}_L \end{pmatrix} \begin{pmatrix} \frac{v+H}{\sqrt{2}} \\ 0 \end{pmatrix} u'^{\,j}_R = \frac{v+H}{\sqrt{2}} \, \bar{u}'^{\,i}_L \, u'^{\,j}_R$$

and we obtain

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} &= -\Gamma^{ij}_d \frac{v+H}{\sqrt{2}} \, \bar{d}'^{\,i}_L \, d'^{\,j}_R - \Gamma^{ij}_u \frac{v+H}{\sqrt{2}} \, \bar{u}'^{\,i}_L \, u'^{\,j}_R - \Gamma^{ij}_e \frac{v+H}{\sqrt{2}} \, \bar{e}^i_L \, e^j_R + \text{h.c.} \\ &= -\left[M^{ij}_u \, \bar{u}'^{\,i}_L \, u'^{\,j}_R + M^{ij}_d \, \bar{d}'^{\,i}_L \, d'^{\,j}_R + M^{ij}_e \, \bar{e}^i_L \, e^j_R + \text{h.c.} \right] \left(1 + \frac{H}{v} \right) \end{aligned}$$

$$M^{ij} = \Gamma^{ij} \frac{v}{\sqrt{2}}$$

A little help from linear algebra

Theorem: For any generic complex squared matrix C, there exist two unitary matrices U, V such that

$$D = U^\dagger C V$$

is diagonal with real positive entries

Diagonalizing M_f

Using the previous theorem, we know that we can diagonalize the matrix M_f ($f = u, d, e$) with the help of two unitary matrices, U_L^f and U_R^f

$$\left(U_L^f\right)^\dagger M_f\, U_R^f = \text{diagonal with real positive entries}$$

For example:

$$(U_L^u)^\dagger M_u\, U_R^u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \qquad \left(U_L^d\right)^\dagger M_d\, U_R^d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

Mass terms

We can make the following change of fermionic fields

$$f'_{Li} = \left(U_L^f\right)_{ij} f_{Lj} \qquad\qquad f'_{Ri} = \left(U_R^f\right)_{ij} f_{Rj}$$

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\sum_{f',i,j} \bar{f}_L'^{\,i} M_f^{ij} f_R'^{\,j} \left(1+\frac{H}{v}\right) + \text{h.c.} \\
&= -\sum_{f,i,j} \bar{f}_L^i \left[\left(U_L^f\right)^\dagger M_f\, U_R^f\right]_{ij} f_R^j \left(1+\frac{H}{v}\right) + \text{h.c.} \\
&= -\sum_f m_f \left(\bar{f}_L f_R + \bar{f}_R f_L\right)\left(1+\frac{H}{v}\right)
\end{aligned}$$

- We succeed in producing fermion masses and we got a fermion-antifermion-Higgs coupling proportional to the fermion mass.
- Notice that the fermionic field redefinition preserves the form of the kinetic terms in the Lagrangian ($\bar{\psi}\,\partial\!\!\!/\,\psi = \bar{\psi}_R\,\partial\!\!\!/\,\psi_R + \bar{\psi}_L\,\partial\!\!\!/\,\psi_L$ invariant for left and right field unitary transformation).
- The Higgs Yukawa couplings are flavor diagonal: no flavor changing Higgs interactions.

Mass diagonalization and charged current interaction

The charged current interaction is given by

$$\frac{e}{\sqrt{2}\sin\theta_W}\,\bar{u}'^{\,i}_L\, \not{W}^+\, d'^{\,i}_L + \text{h.c.}$$

After the mass diagonalization described previously, this term becomes

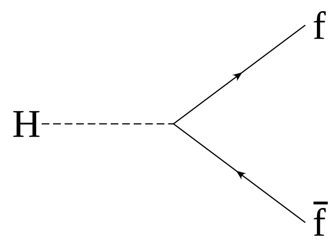
$$\frac{e}{\sqrt{2}\sin\theta_W}\,\bar{u}^{i}_L \left[(U^u_L)^\dagger\, U^d_L\right]_{ij} \not{W}^+ d^j_L + \text{h.c.}$$

and we define the Cabibbo-Kobayashi-Maskawa matrix V_{CKM}

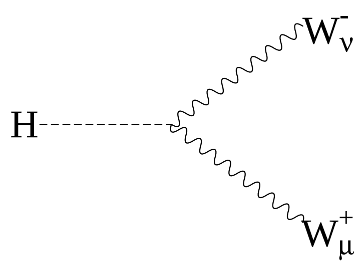
$$V_{CKM} = (U^u_L)^\dagger\, U^d_L$$

- V_{CKM} is a complex not diagonal matrix and then it mixes the flavors of the different quarks.
- For N flavour families, V_{CKM} depends on $(N-1)^2$ parameters. $(N-1)(N-2)/2$ of them are complex phases. For $N=3$ there is one complex phase and this implies violation of the **CP** symmetry (first observed in the K^0-$\bar{K}^0$ system in 1964).
- It is a unitary matrix and the values of its entries must be determined from experiments.

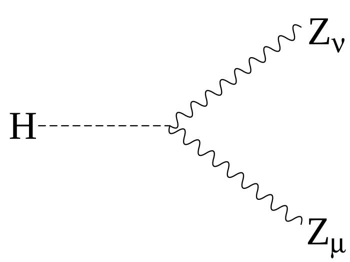
Feynman rules for Higgs couplings



$$-i\frac{m_f}{v}$$



$$ig\, m_W\, g_{\mu\nu}$$



$$i\, g\, \frac{1}{\cos\theta_W}\, m_Z\, g_{\mu\nu}$$

Within the Standard Model, the Higgs couplings are almost completely constrained. The only free parameter (not yet measured) is the Higgs mass

$$m_H^2 = 2\lambda v^2$$

Constraints on the Higgs boson mass

We have found that the Higgs boson mass is related to the value of the quartic Higgs coupling λ

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V\left(\Phi^\dagger \Phi\right) \qquad\qquad V\left(\Phi^\dagger \Phi\right) = -\mu^2 \Phi^\dagger \Phi + \lambda \left(\Phi^\dagger \Phi\right)^2$$

leads to

$$m_H^2 = 2\lambda v^2$$

So far we have measured neither m_H nor λ $\implies$ no direct experimental information.

This raises several questions

- Can we get rid of the Higgs boson by setting $m_H = \infty$ and $\lambda = \infty$? Can we eliminate the Higgs boson from the SM?
- Does consistency of the SM as a renormalizable field theory provide constraints?
- Is there indirect information on m_H, e.g. from precision observables and loop effects?

The perturbative unitary bound

A very severe constraint on the Higgs boson mass comes from unitarity of the scattering amplitude.

$$\text{unitarity} \Longleftrightarrow \text{probability}$$

and probability is the link between the theoretical calculations and reality!

Considering the elastic scattering of longitudinally polarized Z bosons

$$Z_L Z_L \to Z_L Z_L$$

$$\mathcal{M} = -\frac{m_H^2}{v^2}\left[\frac{s}{s-m_H^2} + \frac{t}{t-m_H^2} + \frac{u}{u-m_H^2}\right] \qquad \text{in the } s \gg m_Z^2 \text{ limit}$$

where s, t and u are the usual Mandelstam variables.

The perturbative unitary bound on the $J = 0$ partial amplitude takes the form

$$|\mathcal{M}_0|^2 = \left[\frac{3}{16\pi}\frac{m_H^2}{v^2}\right]^2 < 1 \qquad \Longrightarrow \qquad m_H < \sqrt{\frac{16\pi}{3}}v \approx 1\ \text{TeV}$$

More restrictive bounds ($\sim$ 800 GeV) are obtained with other scattering processes, such as $Z_L W_L \to Z_L W_L$

The perturbative unitary bound

If the bound is respected, weak interactions remain weak at all energies, and perturbation theory is everywhere reliable.

If the bound is violated, perturbation theory breaks down, and weak interactions among $W^{\pm}$, Z and H become strong on the 1 TeV scale.

Running of λ

The one-loop renormalization group equation (RGE) for $\lambda(\mu)$ is

$$\frac{d\lambda(\mu)}{d\log\mu^2} = \frac{1}{16\pi^2}\left[12\lambda^2 + \frac{3}{8}g^4 + \frac{3}{16}\left(g^2+g'^2\right)^2 - 3h_t^4 - 3\lambda g^2 - \frac{3}{2}\lambda\left(g^2+g'^2\right) + 6\lambda h_t^2\right]$$

where

$$m_t = \frac{h_t v}{\sqrt{2}} \qquad\qquad m_H^2 = 2\lambda v^2$$

This equation must be solved together with the one-loop RGEs for the gauge and Yukawa couplings, which, in the Standard Model, are given by

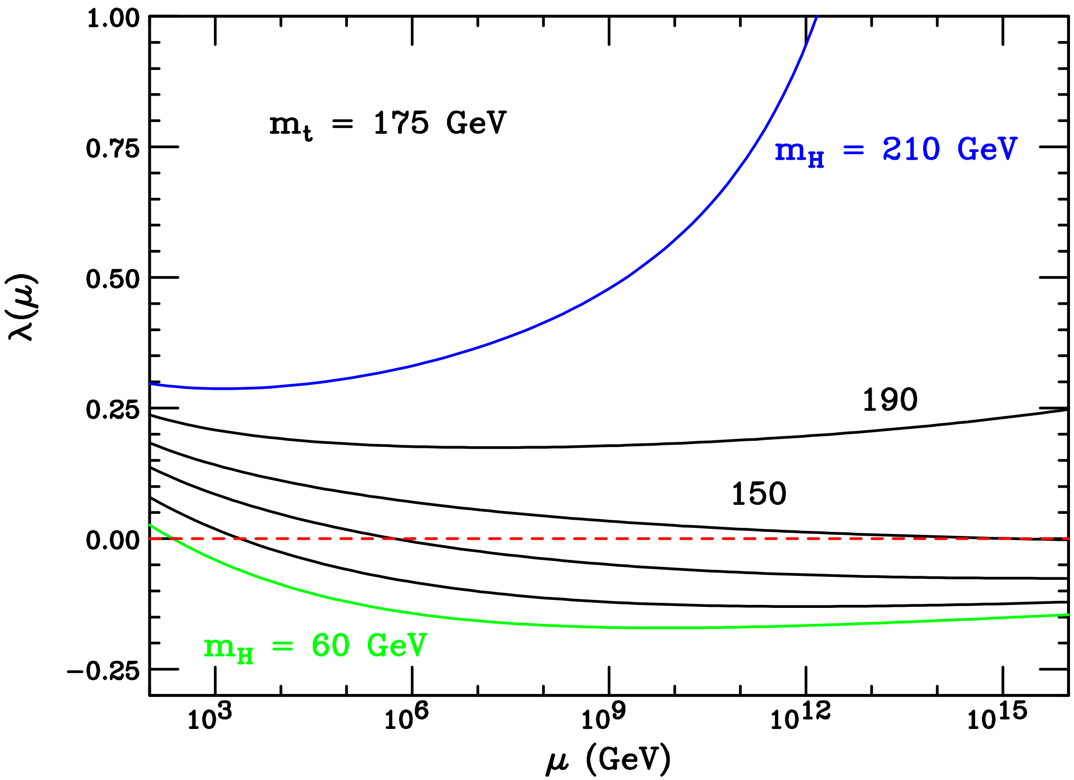
$$\begin{aligned}
\frac{dg(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\left(-\frac{19}{6}g^3\right)\\
\frac{dg'(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\frac{41}{6}g'^3\\
\frac{dg_s(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\left(-7g_s^3\right)\\
\frac{dh_t(\mu)}{d\log\mu^2} &= \frac{1}{32\pi^2}\left[\frac{9}{2}h_t^3 - \left(8g_s^2 + \frac{9}{4}g^2 + \frac{17}{12}g'^2\right)h_t\right]
\end{aligned}$$

here g_s is the strong interaction coupling constant, and the $\overline{\mathrm{MS}}$ scheme is adopted.

Solutions for $\lambda(\mu)$

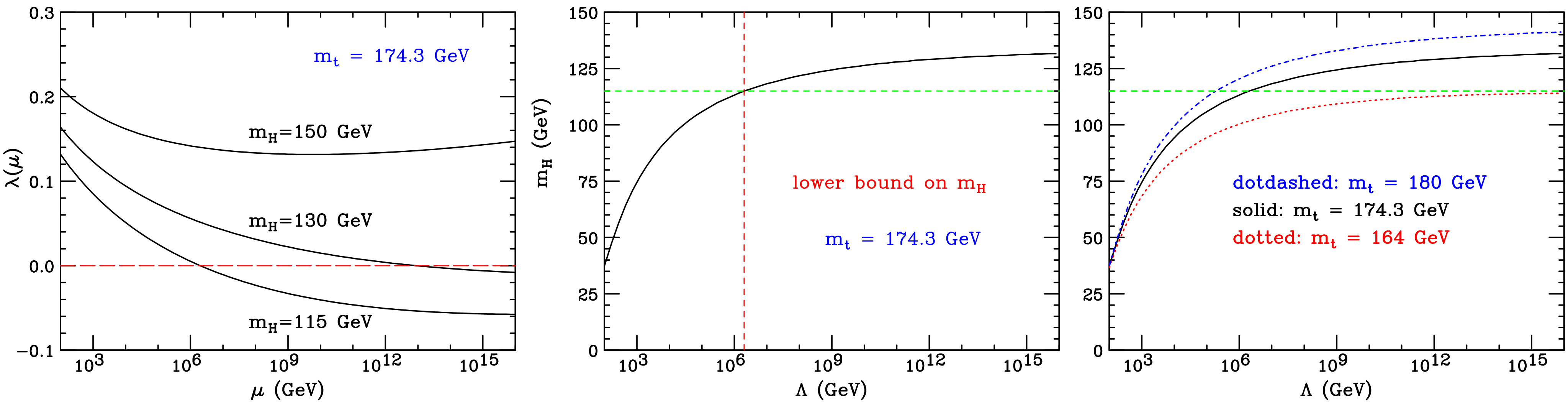
Solving this system of coupled equations with the initial condition

$$\lambda\left(m_H\right) = \frac{m_H^2}{2v^2}$$



Lower bound for m_H: vacuum stability

It can be shown that the requirement that the Higgs potential be bounded from below, even after the inclusion of radiative corrections, is fulfilled if $\lambda(\mu)$ stays positive, at least up to a certain scale $\mu \approx \Lambda$, the maximum energy scale at which the theory can be considered reliable (use effective action).

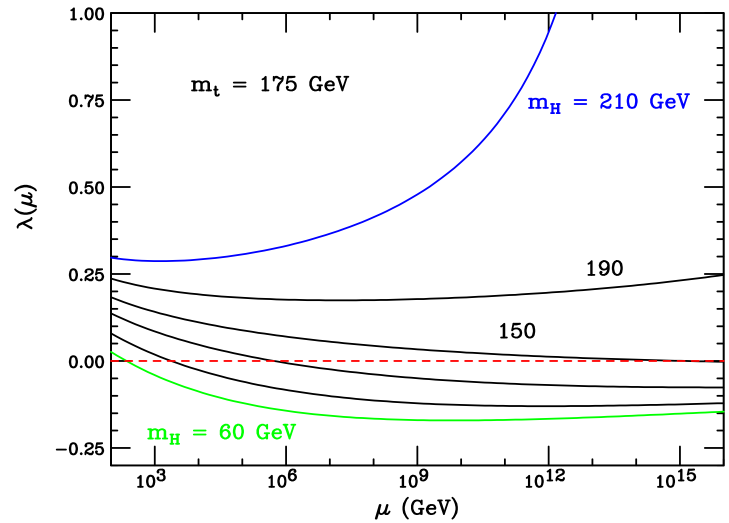


✗ This limit is extremely sensitive to the top-quark mass.

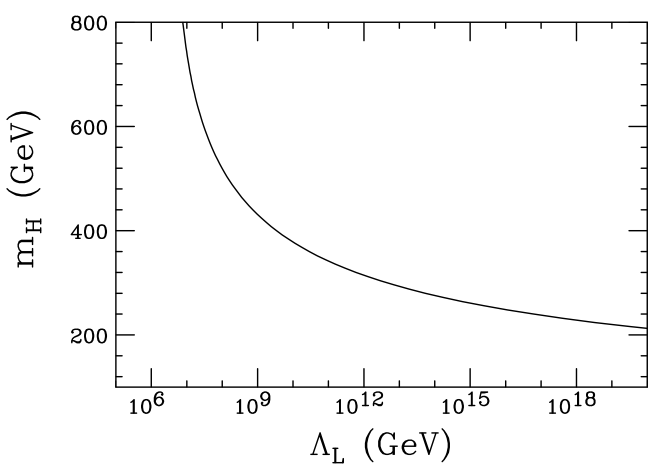
✓ The stability lower bound can be relaxed by allowing metastability

Upper bound for m_H: triviality bound

For large values of the Higgs boson mass, the coupling $\lambda(\mu)$ grows with increasing μ, and eventually leaves the perturbative domain ($\lambda \lesssim 1$): the solution has a singularity in μ , known as the Landau singularity.
For the theory to make sense up to a scale Λ, we must ask $\lambda(\mu) \lesssim 1$ (or something similar), for $\mu \leq \Lambda$.
Neglecting gauge and Yukawa coupling, we have



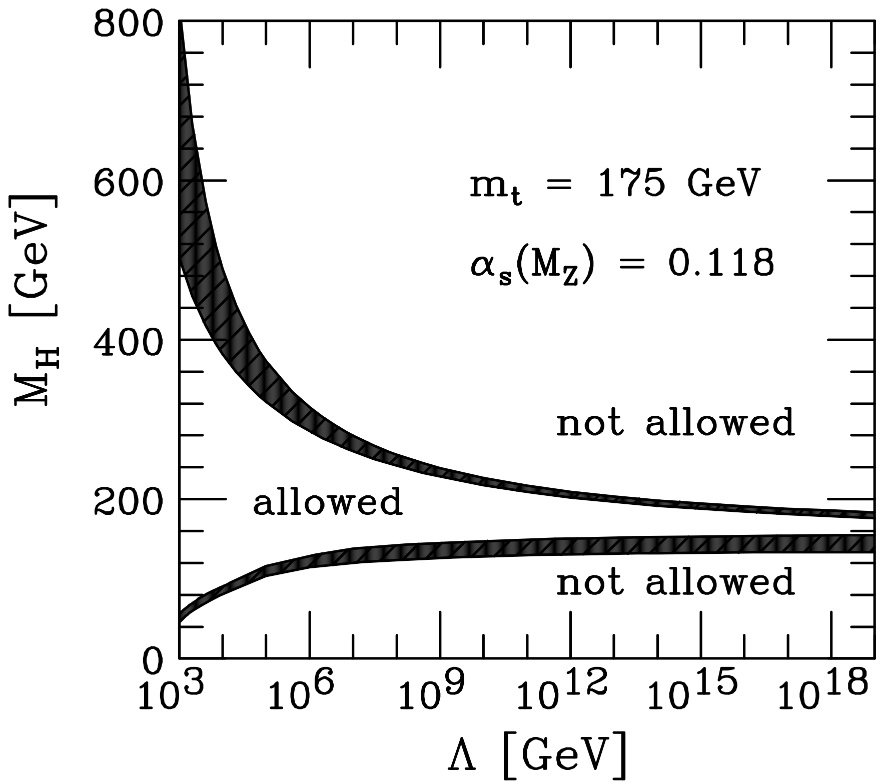
$$\lambda(\mu^2) = \frac{\lambda(m_H^2)}{1 - \frac{3}{4\pi^2}\lambda(m_H^2)\log\frac{\mu^2}{m_H^2}} \qquad \text{singular when} \qquad \mu^2 \approx \Lambda_L^2 \equiv m_H^2 \exp\left[\frac{4\pi^2}{3\lambda(m_H^2)}\right]$$



- For any value of $\lambda\left(m_H^2\right)$ the theory has an upper scale Λ of validity.
- $\Lambda \to \infty$ for pure scalar theory possible only if $\lambda(m_H^2) \equiv 0$, i.e. no scalar self-coupling $\Longrightarrow$ free or "trivial" theory

Higgs boson mass bounds

Riesselmann, hep-ph/9711456



Notice the small window 150 GeV $< m_H <$ 180 GeV, where the theory is valid up to the Planck scale $M_{\text{Planck}} = (\hbar c / G_{\text{Newton}})^{1/2} \approx 1.22 \times 10^{19}$ GeV.

Hierarchy, naturalness and fine tuning

Apart from the considerations made up to now, the SM must be considered as an effective low-energy theory: at very high energy new phenomena take place that are not described by the SM (gravitation is an obvious example) $\Longrightarrow$ other scales have to be considered.

Why the weak scale ($\sim 10^2$ GeV) is much smaller than other relevant scales, such as the Planck mass ($\approx 10^{19}$ GeV) or the unification scale ($\approx 10^{16}$ GeV) (or why the Planck scale is so high with respect to the weak scale $\Longrightarrow$ extra dimensions)?

This is the **hierarchy problem**.

And this problem is especially difficult to solve in the SM because of the un-naturalness of the Higgs boson mass.

As we have seen and as the experimental data suggest, the Higgs boson mass is of the same order of the weak scale. However, it's not naturally small, in the sense that there is no approximate symmetry that prevent it from receiving large radiative corrections.

As a consequence, it naturally tends to become as heavy as the heaviest degree of freedom in the underlying theory (Planck mass, unification scale).

Toy model

Two scalars interacting through the potential

$$V(\varphi, \Phi) = \frac{m^2}{2}\varphi^2 + \frac{M^2}{2}\Phi^2 + \frac{\lambda}{4!}\varphi^4 + \frac{\sigma}{4!}\Phi^4 + \frac{\delta}{4}\varphi^2\Phi^2$$

which is the most general renormalizable potential, if we require the symmetry under $\varphi \to -\varphi$ and $\Phi \to -\Phi$. We assume that $M^2 \gg m^2$. Let's check if this hierarchy is conserved at the quantum level. Compute the one-loop radiative corrections to the pole mass m^2

$$m^2_{\text{pole}} = m^2(\mu^2) + \frac{\lambda m^2}{32\pi^2}\left(\log\frac{m^2}{\mu^2} - 1\right) + \frac{\delta M^2}{32\pi^2}\left(\log\frac{M^2}{\mu^2} - 1\right)$$

where the running mass $m^2(\mu^2)$ obeys the RGE

$$\frac{dm^2(\mu^2)}{d\log\mu^2} = \frac{1}{32\pi^2}\left(\lambda m^2 + \delta M^2\right)$$

Corrections to m^2 proportional to M^2 appear at one loop. One can choose $\mu^2 \approx M^2$ to get rid of them, but they reappear through the running of $m^2(\mu^2)$.

Toy model, cont'd

The only way to preserve the hierarchy $m^2 \ll M^2$ is carefully choosing the coupling constants

$$\lambda m^2 \approx \delta M^2$$

and this requires fixing the renormalized coupling constants with and unnaturally high accuracy

$$\frac{\lambda}{\delta} \approx \frac{m^2}{M^2}$$

This is what is usually called the fine tuning of the parameters.

The same happens if the theory is spontaneously broken ($m^2 < 0$, $M^2 \gg |m^2| > 0$).

Therefore, without a suitable fine tuning of the parameters, the mass of the scalar Higgs boson naturally becomes as large as the largest energy scale in the theory. And this is related to the fact that no extra symmetry is recovered when the scalar masses vanish, in contrast to what happens to fermions, where the chiral symmetry prevents the dependence from powers of higher scales, and gives a typical logarithmic dependence.

Solutions to the naturalness problem?

Leaving the toy model and back to the Standard Model, the corrections to m_H^2 due to a top-quark loop is given by

$$\delta m_H^2 = \frac{3G_F m_t^2}{\sqrt{2}\pi^2}\Lambda^2 \approx (0.27\Lambda)^2$$

where we are assuming that the scale Λ that characterizes non-standard physics acts as a cut-off for the loop momentum.

So, how can we prevent these large corrections to the Higgs boson mass?

- SUperSYmmetry offers a solution to the naturalness problem: exploiting the fact that fermion loops contribute with an overall minus sign (because of Fermi statistics), SUSY balances the contributions of fermion and boson loops.

  In the limit of unbroken SUSY, in which the masses of bosons are degenerate with those of their fermion counterparts, the cancellation is exact.

  If the supersymmetry is broken (as it must be in our world), the contribution of the integrals may still be acceptably small if the fermion-boson mass splittings ΔM are not too large. The condition that $g^2 \Delta M^2$ be "small enough" leads to the requirement that superpartner masses be less than about 1 TeV.

Solutions to the naturalness problem?

- A second solution is offered by theories of dynamical symmetry breaking such as technicolor. In technicolor models, the Higgs boson is composite, and new physics arises on the scale of its binding, $\Lambda_{\rm TC} \simeq \mathcal{O}\left(1 \text{ TeV}\right)$. Thus the effective range of integration is cut off, and mass shifts are under control.

- A third possibility is that the gauge sector becomes strongly interacting. This would give rise to WW resonances, multiple production of gauge bosons, and other new phenomena at energies of 1 TeV or so.

Constraints from precision data

$$\begin{aligned}
\alpha &= \frac{1}{4\pi}\frac{g^2 g'^2}{g^2+g'^2} = \frac{1}{137.03599976(50)} \\
G_F &= \frac{1}{\sqrt{2}v^2} = 1.16637(1)\times 10^{-5}\ \mathrm{GeV}^{-2} \\
m_Z &= \frac{1}{2}\sqrt{g^2+g'^2}\,v = 91.1875(21)\ \mathrm{GeV}\,,
\end{aligned}$$

where the uncertainty is given in parentheses. The value of α is extracted from low-energy experiments, G_F is extracted from the muon lifetime, and m_Z is measured from e^+e^- annihilation near the Z mass.

We can express m_W as

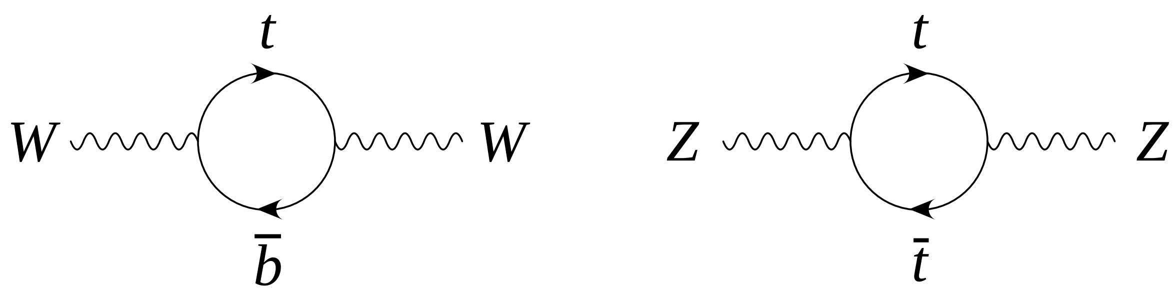
$$m_W^2 = \frac{1}{\sin^2\theta_W}\frac{\pi\alpha}{\sqrt{2}G_F}$$

where

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2}$$

Clues to the Higgs boson mass

From the sensitivity of electroweak observables to the mass of the top quark, we are able to measure its mass, even without directly producing it

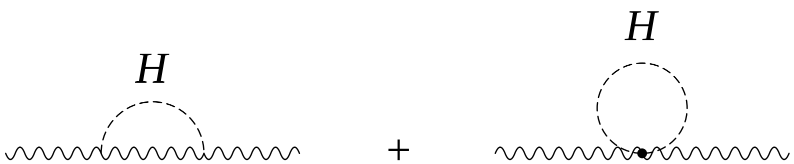


These quantum corrections alter the link between W and Z boson masses

$$m_W^2 = \frac{1}{\sin^2\theta_W\,(1-\Delta\rho)}\frac{\pi\alpha}{\sqrt{2}G_F} \qquad \Delta\rho_{(\mathrm{top})} \approx -\frac{3G_F}{8\pi^2\sqrt{2}}\frac{1}{\tan^2\theta_W}m_t^2$$

The strong dependence on m_t^2 accounts for the precision of the top-quark mass estimates derived from electroweak observables.

The Higgs boson quantum corrections are typically smaller than the top-quark corrections, and exhibit a more subtle dependence on m_H than the m_t^2 dependence of the top-quark corrections.



$$\Delta\rho_{(\text{Higgs})} = \frac{11 G_F m_Z^2 \cos^2\theta_W}{24\sqrt{2}\pi^2} \log\left(\frac{m_H^2}{m_W^2}\right)$$

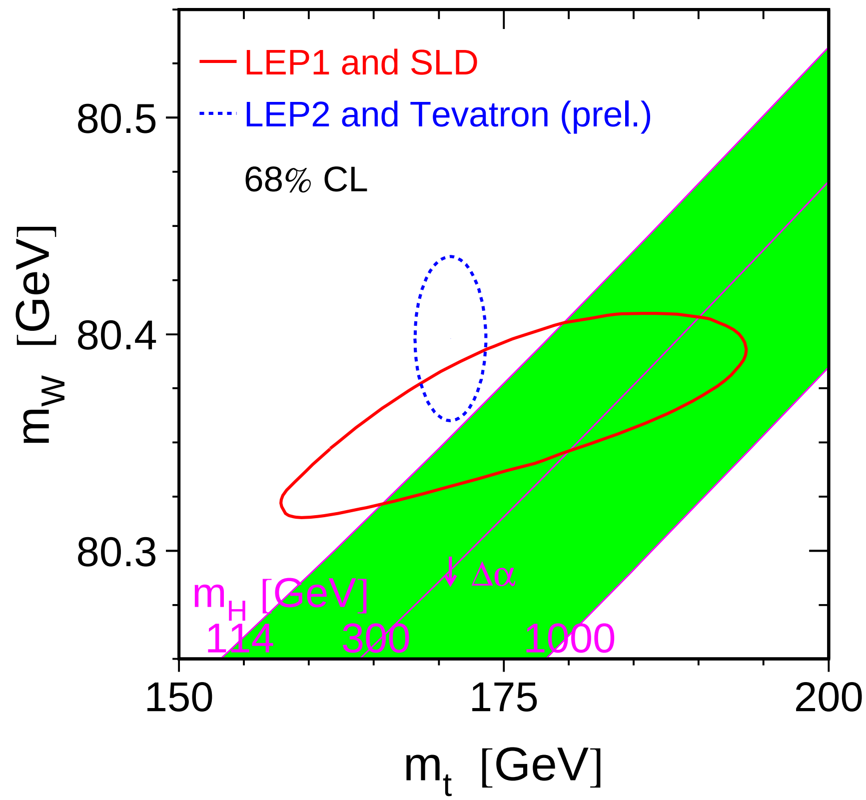
Since m_Z has been determined at LEP to 23 ppm, it is interesting to examine the dependence of m_W upon m_t and m_H.

Indirect measurements of m_W and m_t (solid line)

Direct measurements of m_W and m_t (dotted line)

$m_t = 170.9 \pm 1.8$ GeV
$m_W = 80.398 \pm 0.025$ GeV

both shown as one-standard-deviation regions.



The indirect and direct determinations are in reasonable agreement and both favor a light Higgs boson, within the framework of the SM.

Summary of EW precision data

Better estimates of the SM Higgs boson mass are obtained by combining all available data.

Summary of electroweak precision measurements (status winter 2007) are given on LEP-EWWG page
`http://lepewwg.web.cern.ch/LEPEWWG`

Exercise: Derive the slope of the lines of constant Higgs mass of the previous slide and compare numerically with the plot.

Blue band plot

The indication for a light Higgs boson becomes somewhat stronger when all the electroweak observables are examined.

$$m_H = 76^{+33}_{-24}\ \text{GeV}$$

Including theory uncertainty

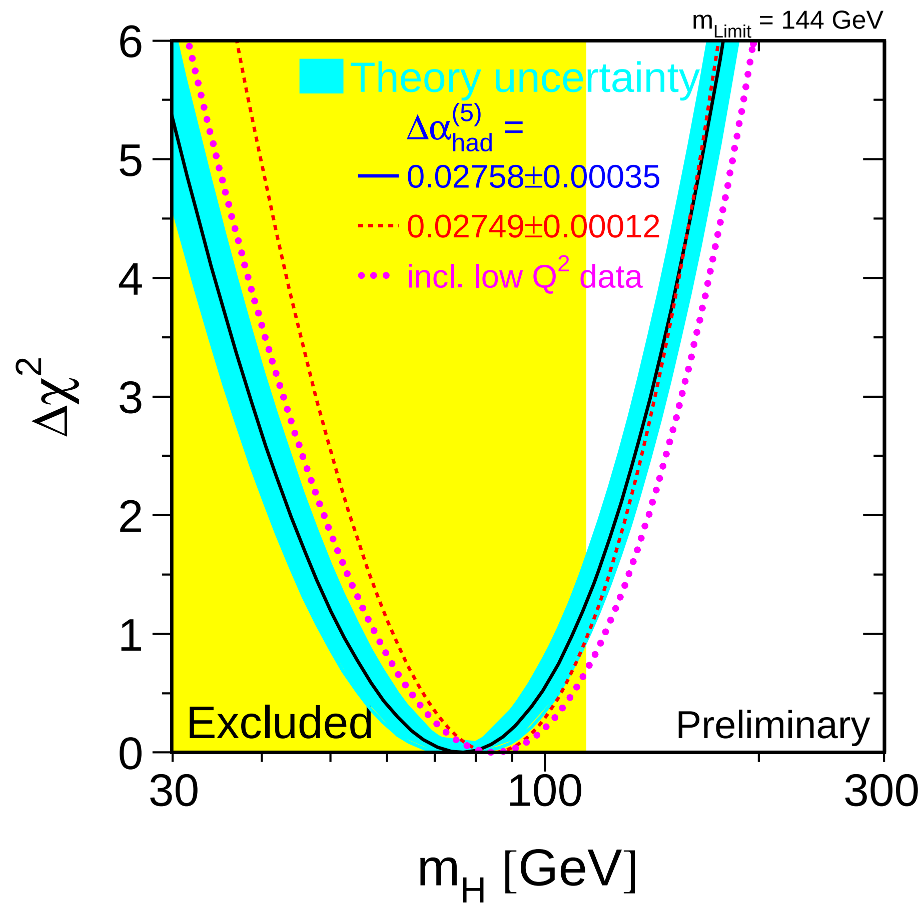
$$m_H < 144\ \text{GeV} \qquad (95\%\text{CL})$$

Direct search limit from LEP

$$m_H > 114.4\ \text{GeV} \qquad (95\%\text{CL})$$

But the χ^2 of the fit is very bad!

$$\chi^2/\text{dof} = 25.4/15$$
$$\chi^2/\text{dof} = 16.8/14 \quad \text{without NuTeV}$$



Up to now…

Peter W. Higgs, University of Edinburgh

⇐ **Only unambiguous example of observed Higgs**

(D. Froidevaux, HCP School, 2007)

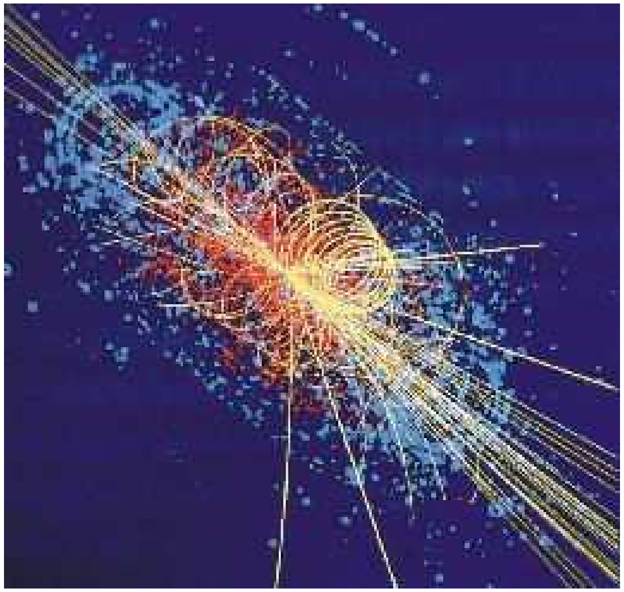
Final remarks

The Standard Model is not the whole story

Open questions

- ✗ gravity
- ✗ neutrino masses and oscillations (heavy sterile neutrinos + see-saw mechanism)
- ✗ dark matter/dark energy
- ✗ baryogenesis

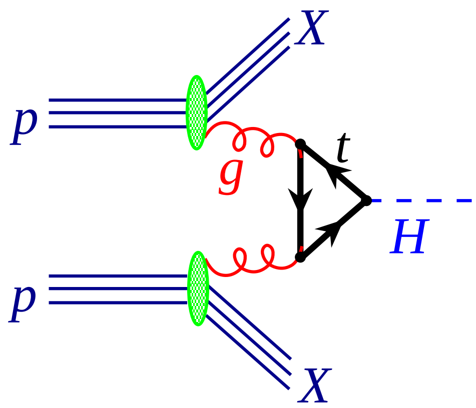
Higgs boson at the LHC



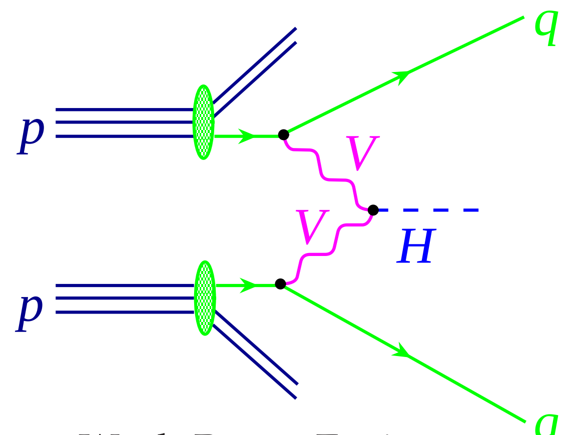
Two steps

- Production of the Higgs boson
- Detection of the decay products of the Higgs boson and identification of the events

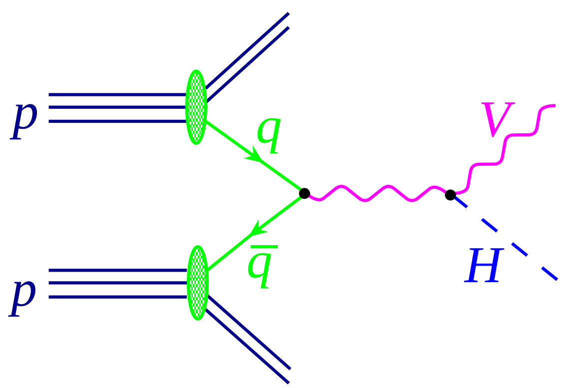
Production Modes



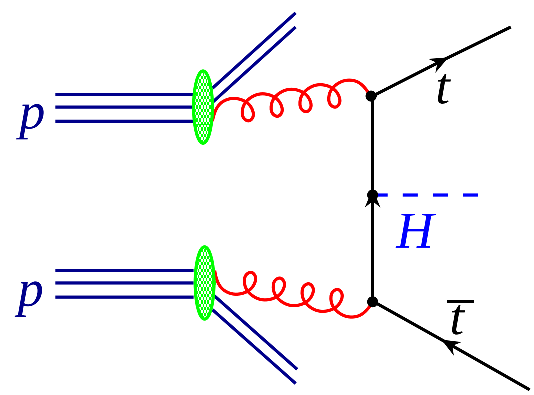
Gluon fusion



Weak-Boson Fusion

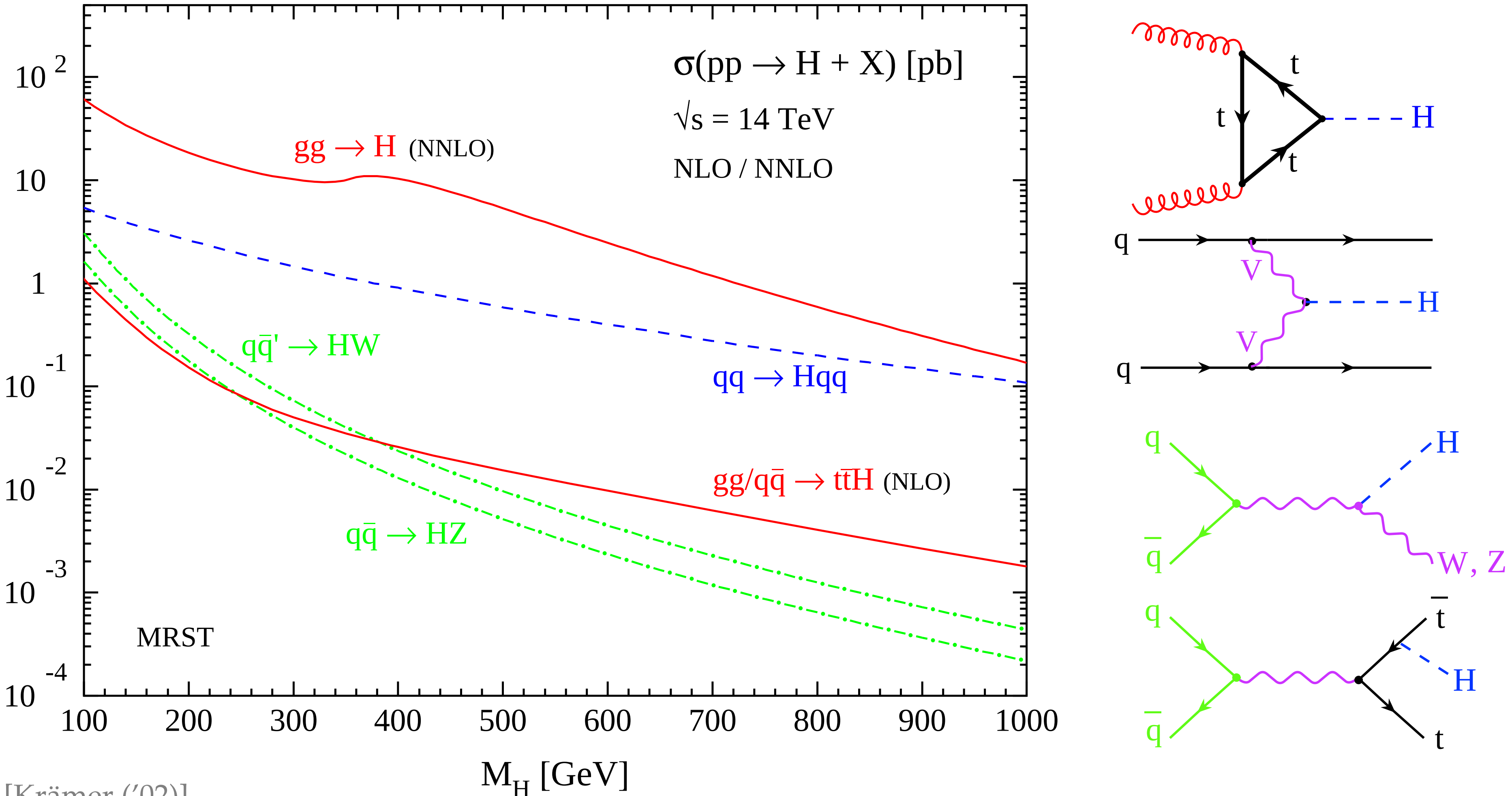


Higgs Strahlung



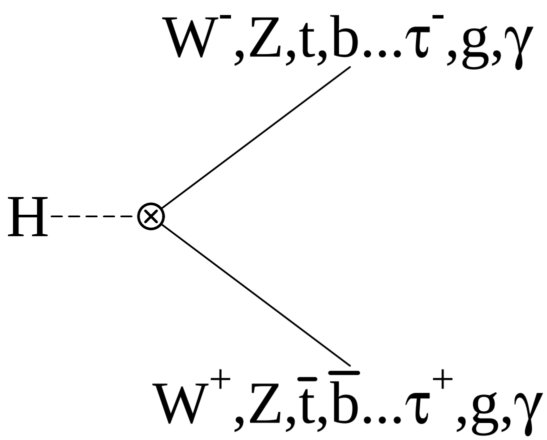
$t\bar{t}H$

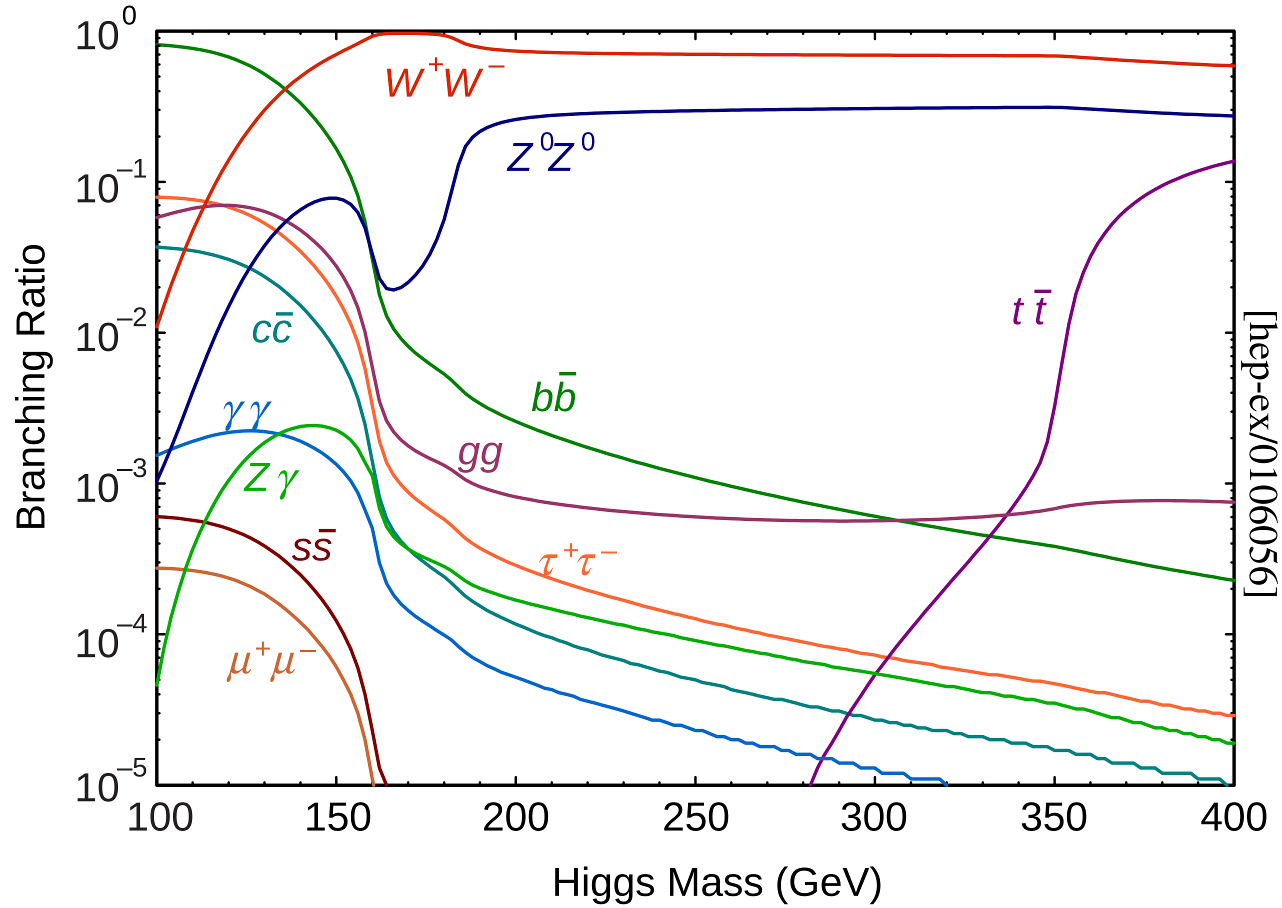
Total cross sections at the LHC



[Krämer ('02)]

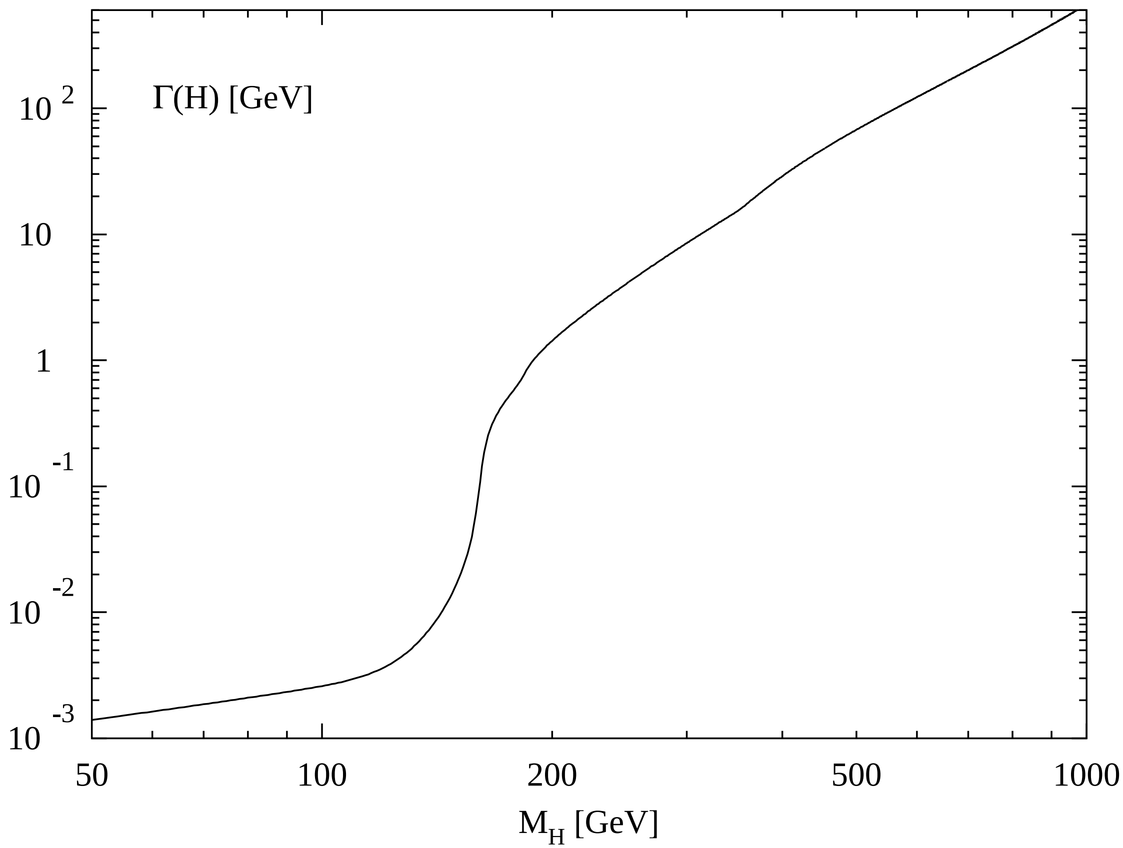
Branching fractions of the SM Higgs





Exercise: compute, at leading order, $\Gamma(H \to f\bar{f})$ and $\Gamma(H \to VV)$. More challenging (one-loop integral) $\Gamma(H \to gg)$ and $\Gamma(H \to \gamma\gamma)$. [Spira (hep-ph/9705337)]

Total decay width



[Spira and Zerwas]

Inclusive search channels

- inclusive search for

$$H \to \gamma\gamma$$

invariant-mass peak, for $m_H < 150$ GeV

- inclusive search for

$$H \to ZZ^* \to \ell^+\ell^-\ell^+\ell^-$$

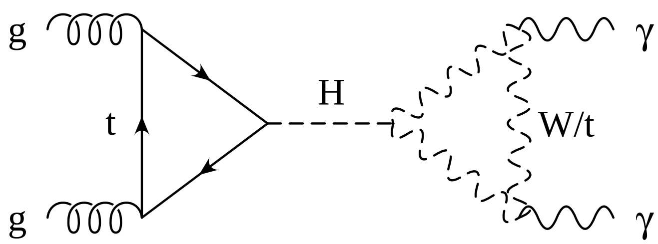
for $m_H \geq 130$ GeV and $m_H \neq 2m_W$.

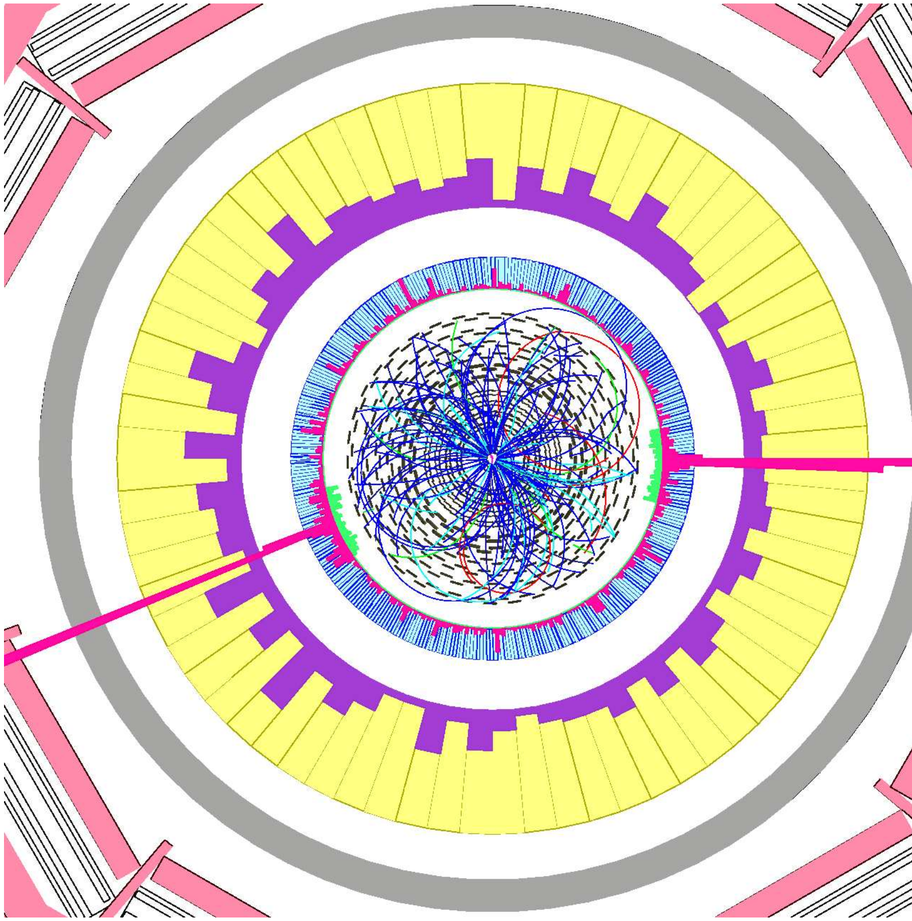
- inclusive search for

$$H \to W^+W^- \to \ell^+\bar{\nu}\ell^-\nu$$

for $140 \text{ GeV} \leq m_H \leq 200$ GeV

$H \to \gamma\gamma$

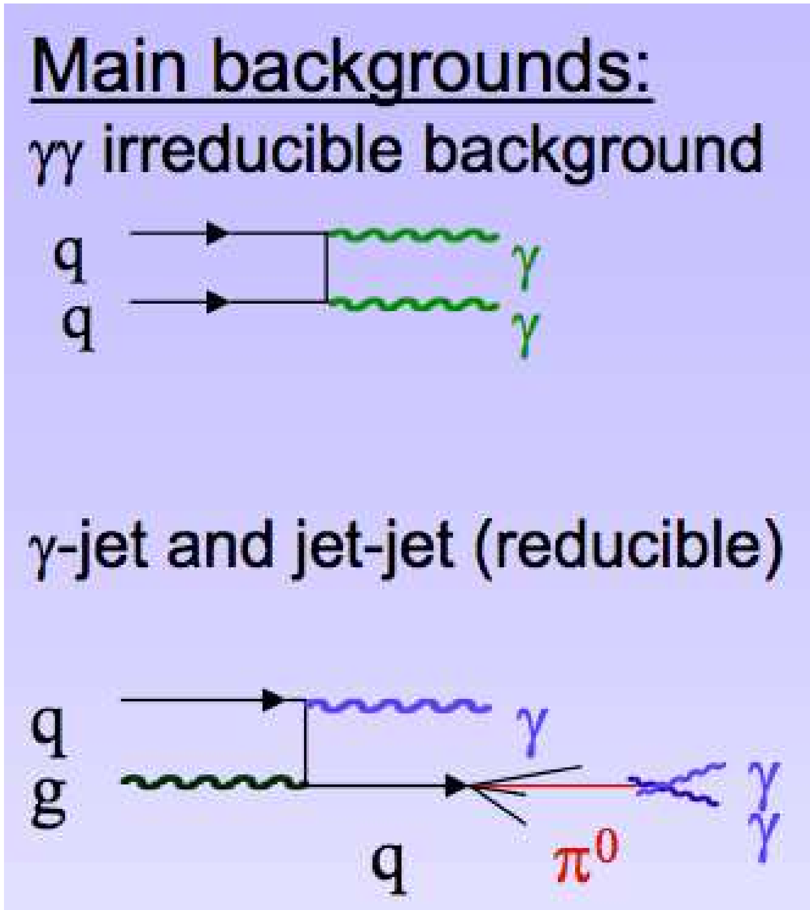




- ✗ BR$(H \to \gamma\gamma) \approx 10^{-3}$
- ✗ large backgrounds from $q\bar{q} \to \gamma\gamma$ and $gg \to \gamma\gamma$
- ✓ but CMS and ATLAS will have excellent photon-energy resolution (order of 1%)

Look for two isolated photons.

$H \longrightarrow \gamma\gamma$

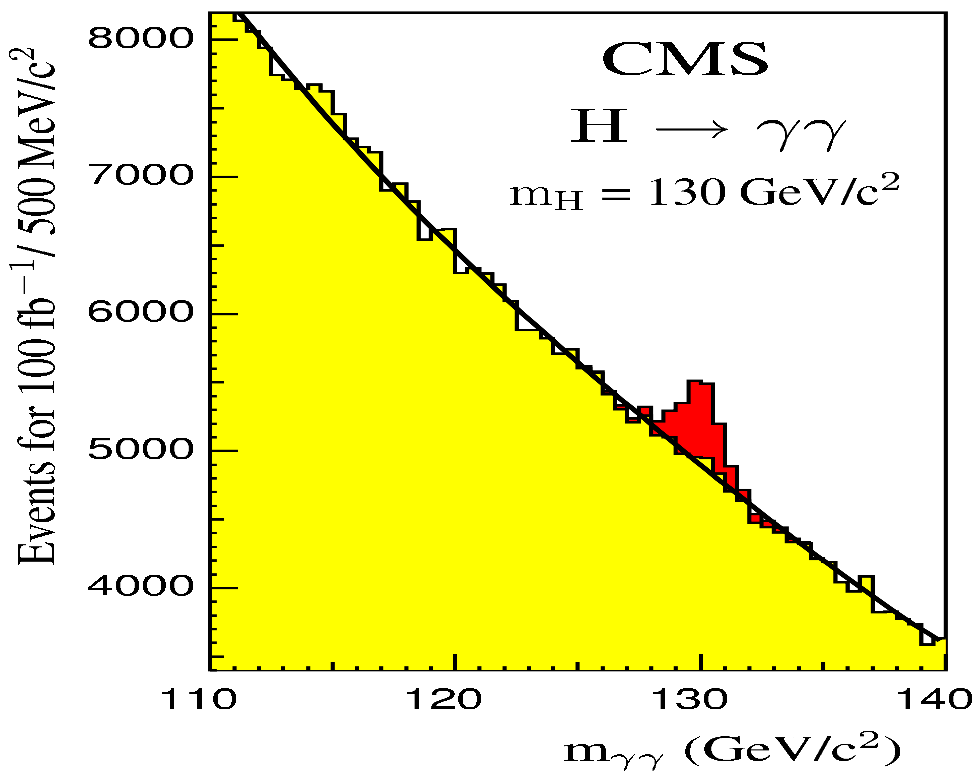


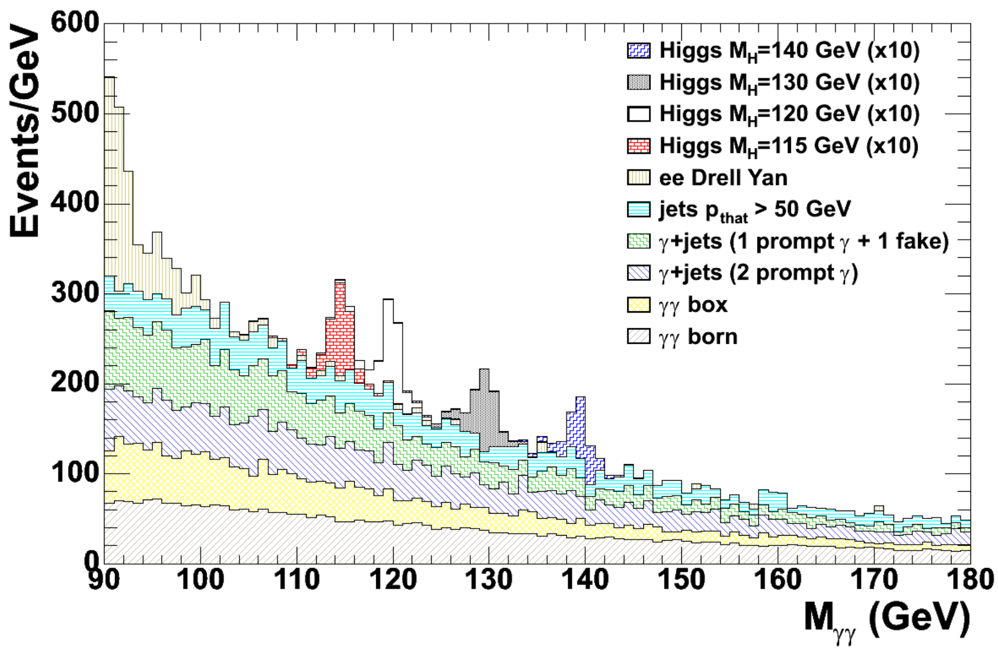
K. Jakobs, CSS07

- ✓ $\sigma_{\gamma \mathrm{j}} \sim 10^6\, \sigma_{\gamma\gamma}$ with large uncertainties
- ✓ we can at most misidentify 1 jet in 10^3
- ✓ we need an efficiency $\epsilon_\gamma \sim 80\%$ to get $\sigma_{\gamma \mathrm{j+jj}} \ll \sigma_{\gamma\gamma}$

$H \longrightarrow \gamma\gamma$

- ✓ Look for a narrow $\gamma\gamma$ invariant mass peak
- ✓ extrapolate background into the signal region from sidebands.

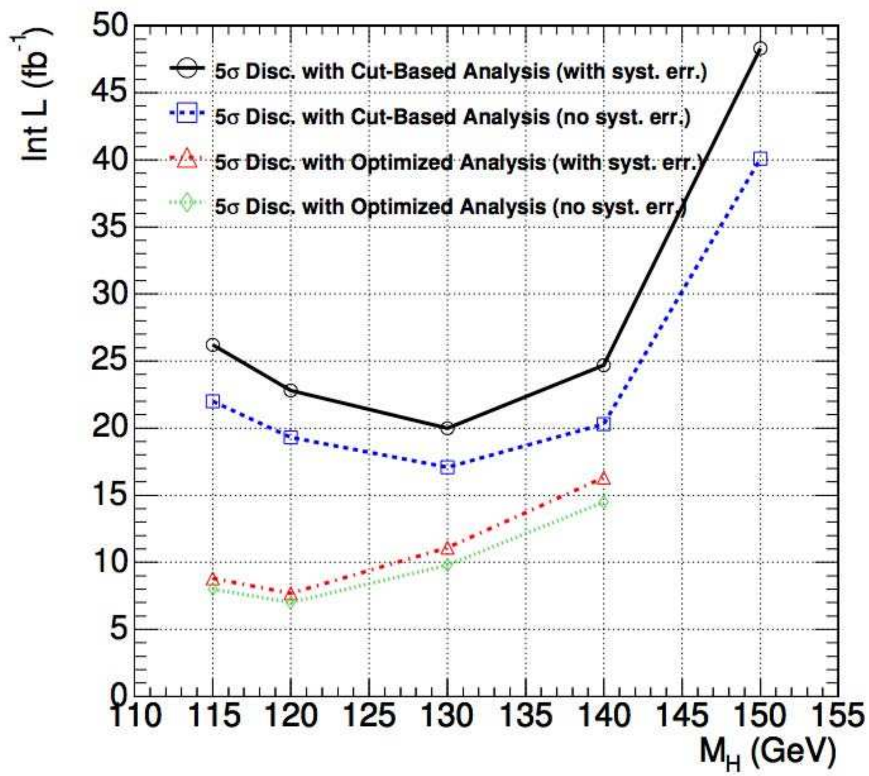




- ✓ discovery with less than 30 fb^{-1}
- ✓ assumes ECAL intercalibration, for which 10 fb^{-1} are needed
- ✓ optimized analysis: assumes perfect understanding of detector. Uses Neural Net

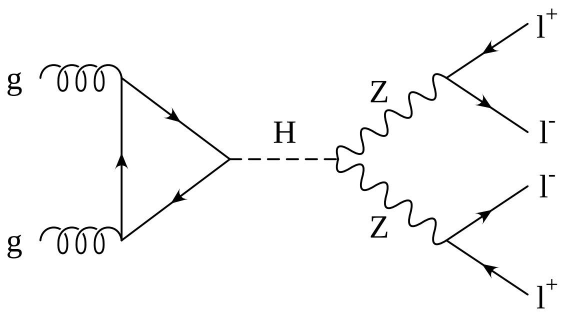
CMS PTDR

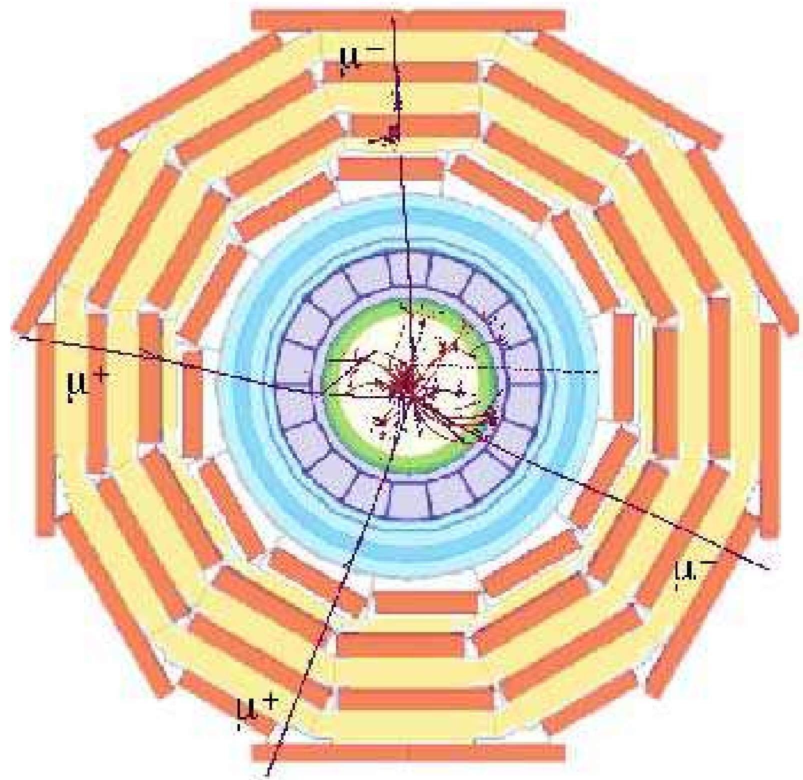
- ✓ 1 fb^{-1}
- ✓ cut-based analysis



$H \to ZZ \to \ell^+\ell^-\ell^+\ell^-$

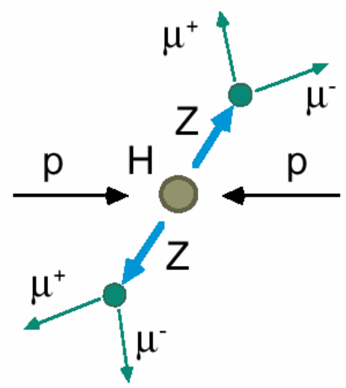
The gold-plated mode

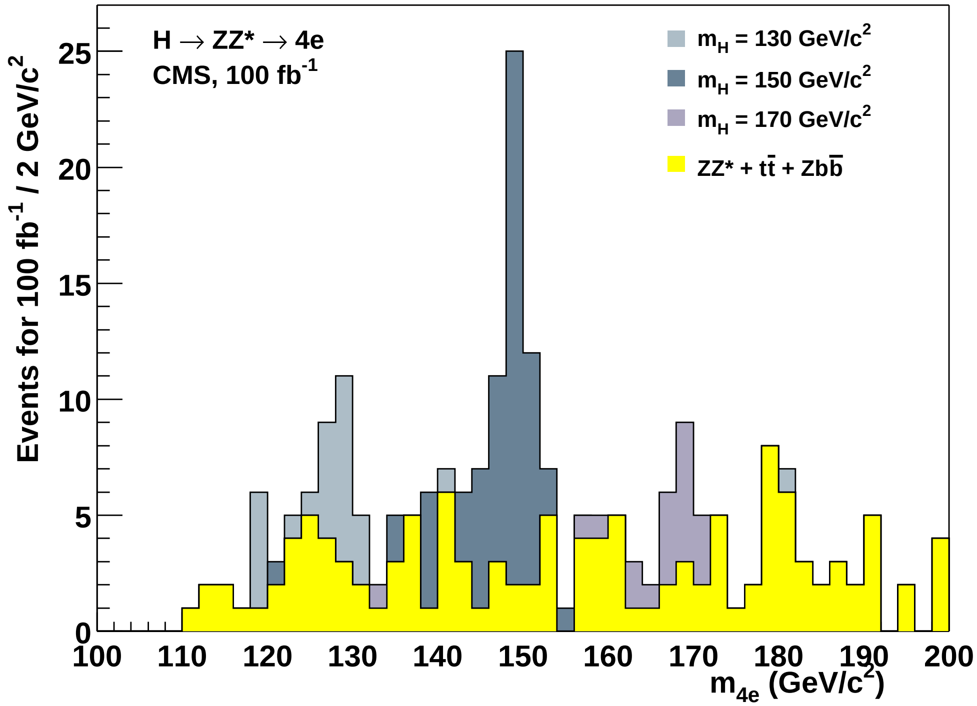




- ✓ This is the most important and clean search mode for $2m_Z < m_H < 600$ GeV.
- ✓ continuum, limited, irreducible background from $q\bar{q} \to ZZ$
- ✗ small BR$(H \to \ell^+\ell^-\ell^+\ell^-) \approx 0.15\%$ (even smaller when $m_H < 2m_Z$)

$H \to ZZ \to \ell^+\ell^-\ell^+\ell^-$





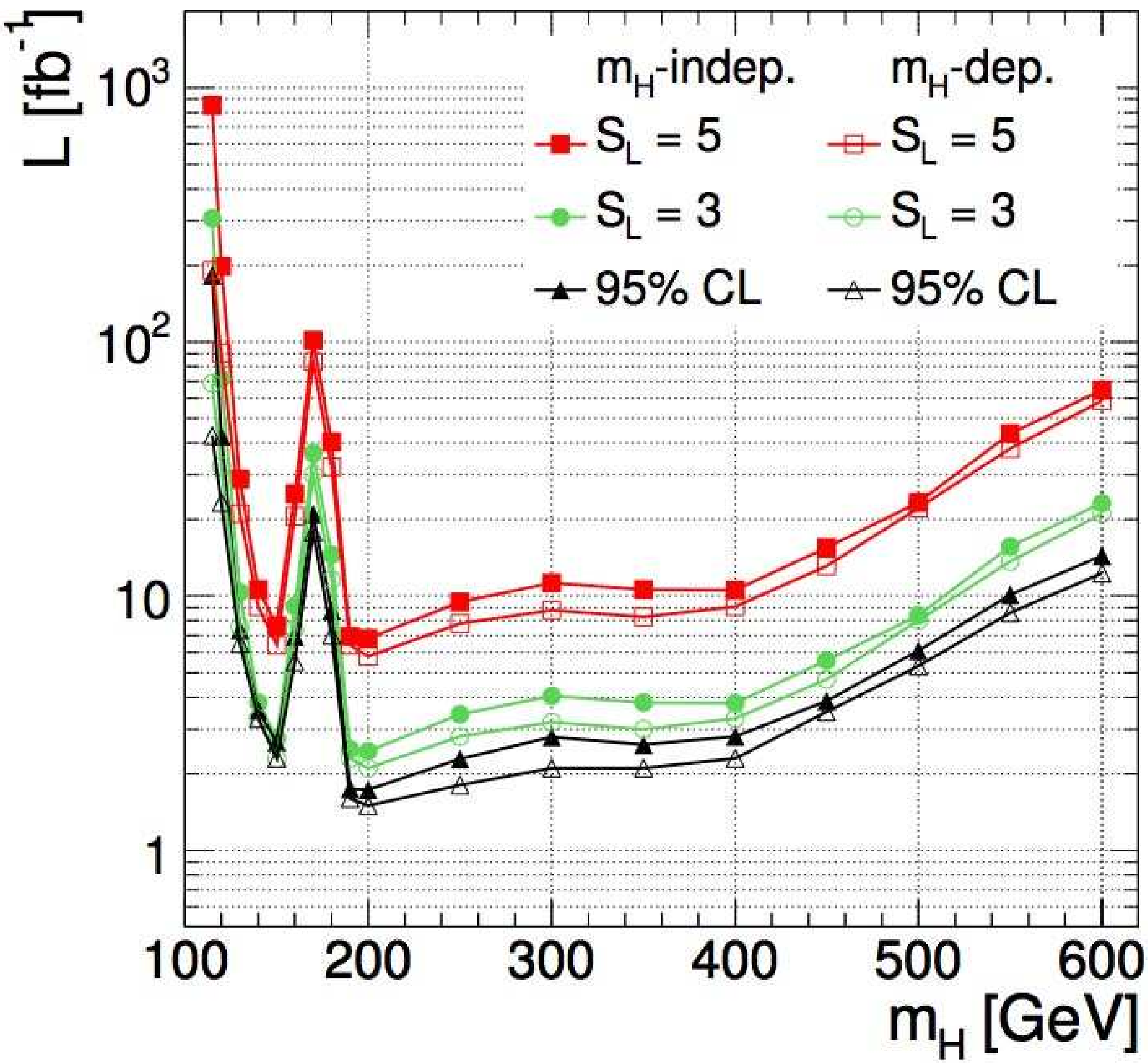
✓ invariant mass of the charged leptons fully reconstructed

For $m_H \approx 0.6\text{-}1$ TeV, use the "silver-plated" mode $H \to ZZ \to \nu\bar{\nu}\ell^+\ell^-$

✓ $\mathrm{BR}(H \to \nu\bar{\nu}\ell^+\ell^-) = 6\ \mathrm{BR}(H \to \ell^+\ell^-\ell^+\ell^-)$

✓ the large E_T missing allows a measurement of the transverse mass

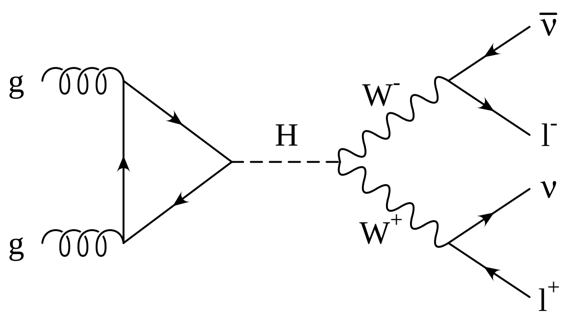
$H \to ZZ \to \mu^+\mu^-\mu^+\mu^-$

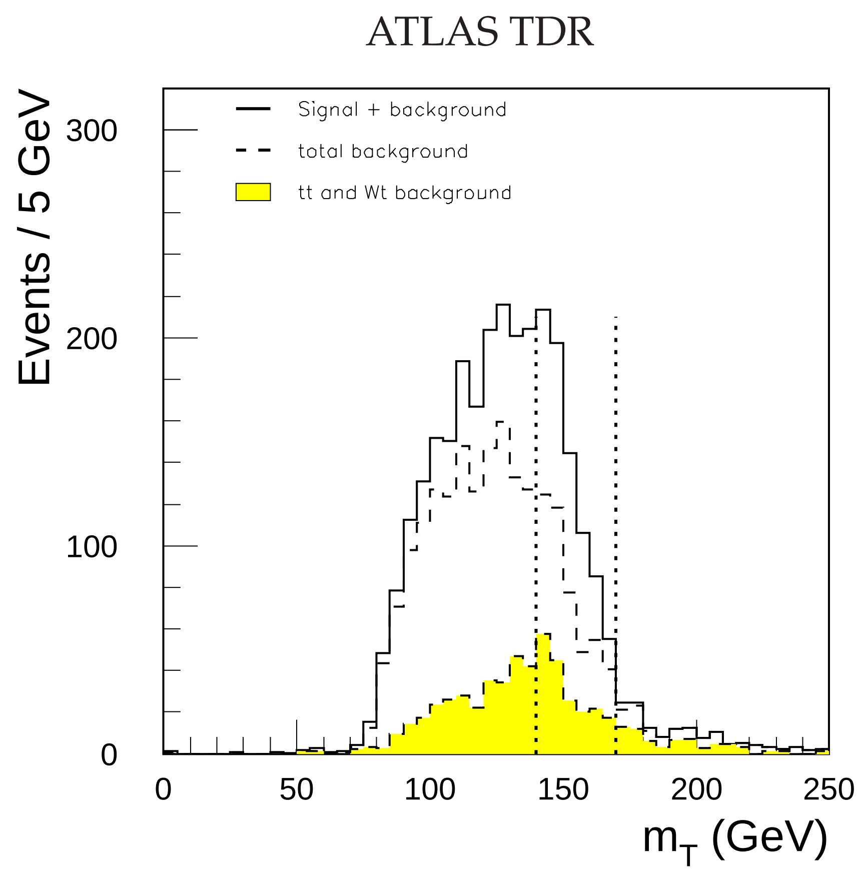


with $30\ \mathrm{fb}^{-1}$

- ✓ m_H measured with $0.1 \div 5\%$ precision
- ✓ production cross section known at 30% precision

$H \to WW \to \ell^+ \bar{\nu} \ell^- \nu$





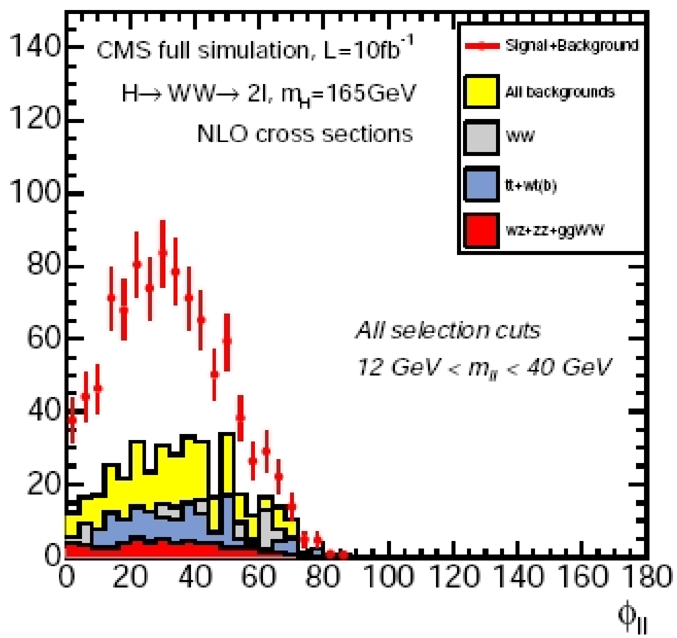
- ✓ No reconstruction of clear mass peak. Measure the transverse mass with a Jacobian peak at m_H

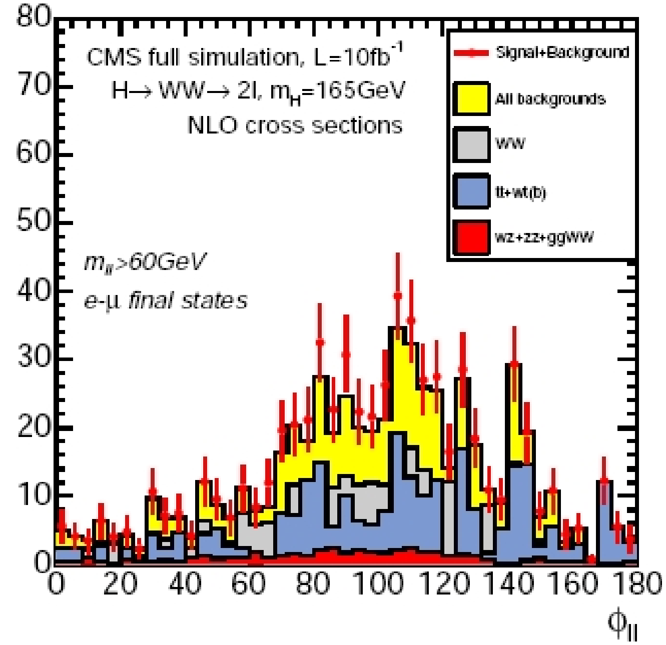
$$m_T = \sqrt{2\, p_T^{\ell\ell}\, \not{E}_T\, (1 - \cos(\Delta\Phi))}$$

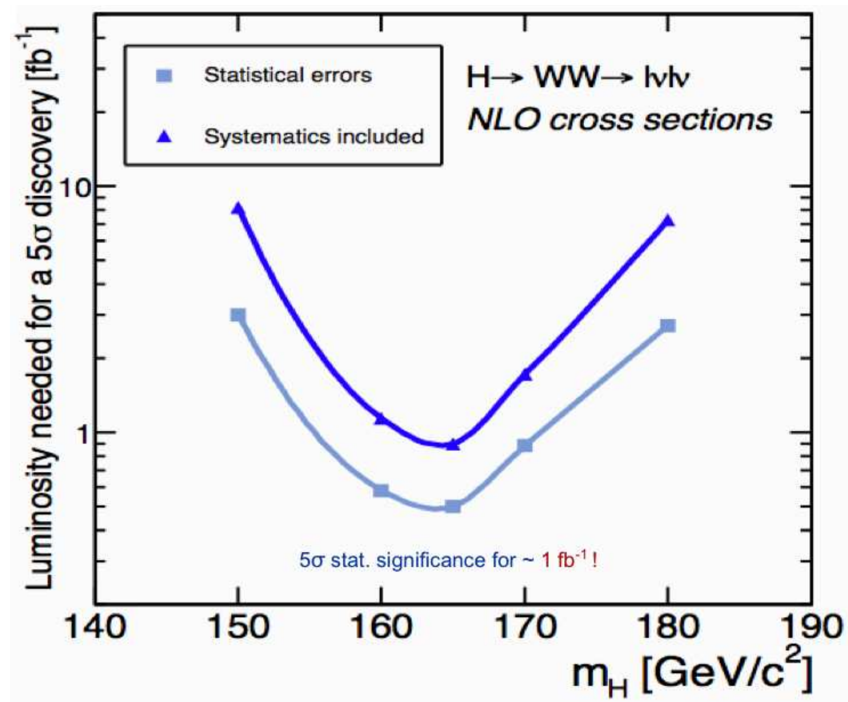
- ✓ Exploit $\ell^+\ell^-$ angular correlations
- ✗ Background and signal have similar shape $\Longrightarrow$ must know the background normalization precisely

$m_H = 170$ GeV

integrated luminosity = 20 fb^{-1}







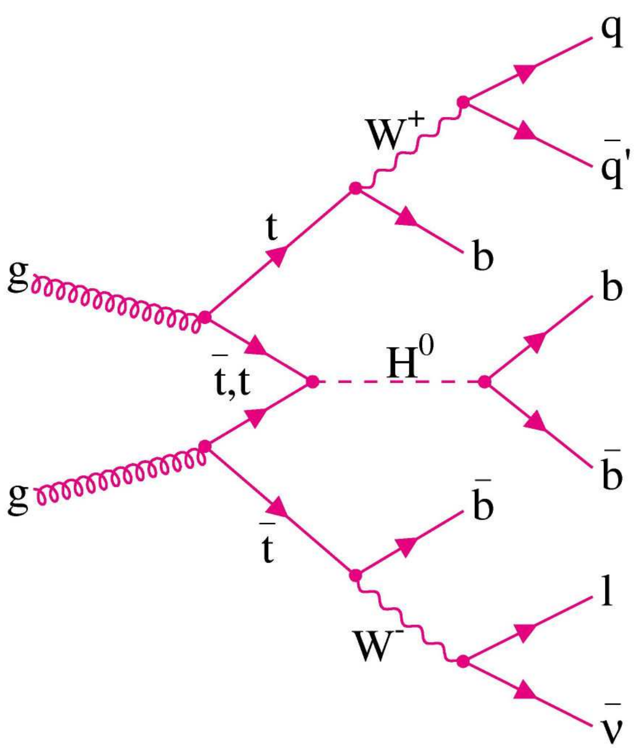
- ✓ best channel for $m_H \sim 160$ - 170 GeV
- ✓ systematic uncertainty 10 - 20%
- ✓ m_H can be determined to 2 - 2.5 GeV
- ✓ production cross section known at $\sim$ 10%

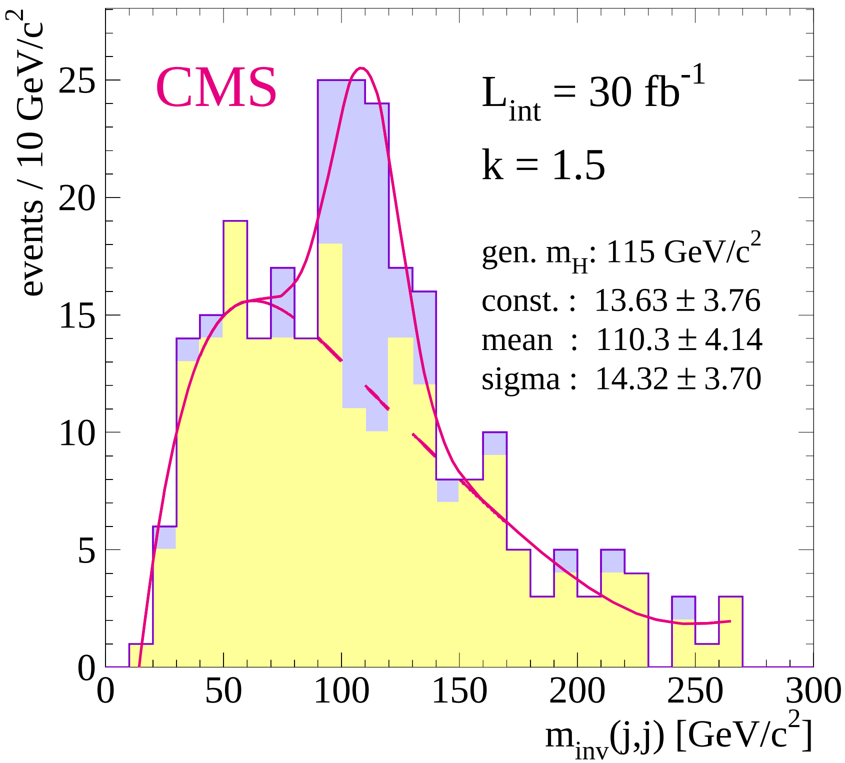
Associated production search channels

- $pp \to t\bar{t}H \to t\bar{t}b\bar{b}$
  for $m_H < 120 - 130$ GeV
- $qq \to Hqq$
  in vector-boson fusion (VBF)

The particles produced in association with the Higgs boson are the trigger of the event.

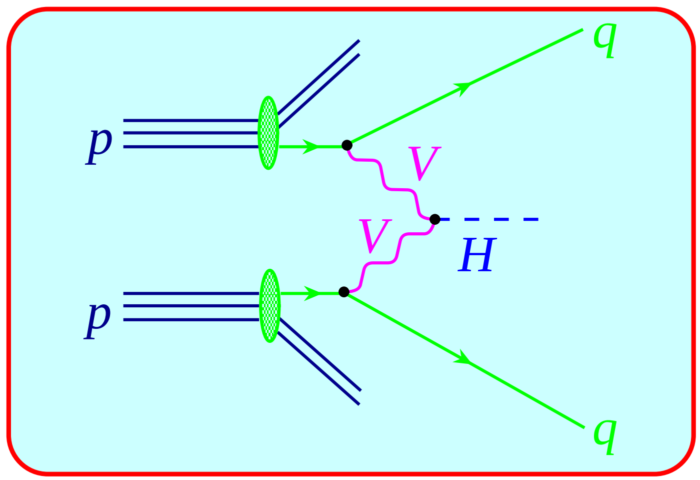
$t\bar{t}H \longrightarrow t\bar{t}b\bar{b}$

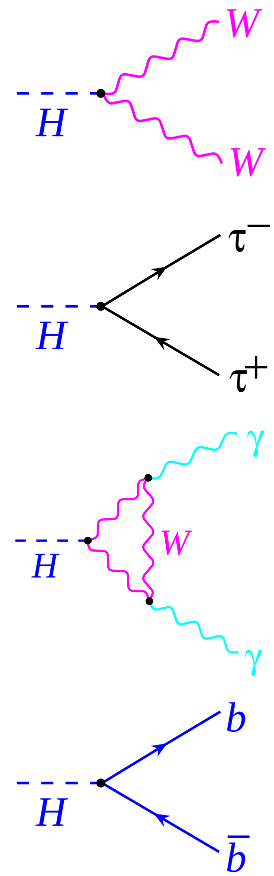




- ✓ $h_t = t\bar{t}H$ Yukawa coupling $\Longrightarrow$ measure h_t^2 BR$(H \to b\bar{b})$
- ✗ must know the background normalization precisely
- ✗ it has been shown recently that this channel is **no longer feasible**

Weak Boson Fusion





$m_H > 120$ GeV

$m_H < 140$ GeV

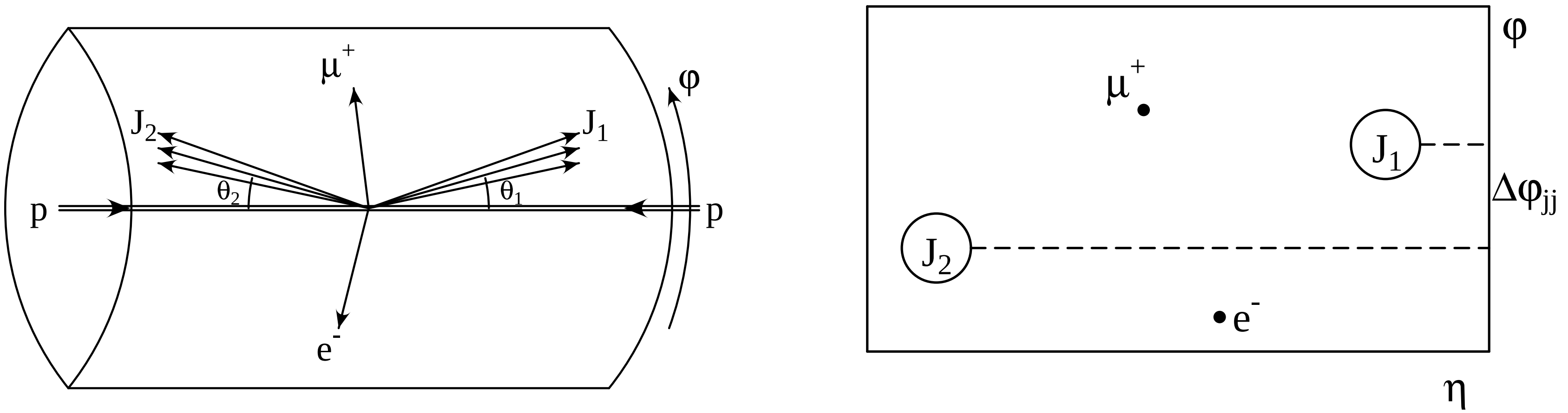
$m_H < 150$ GeV

$m_H < 140$ GeV

[Eboli, Hagiwara, Kauer, Plehn, Rainwater, Zeppenfeld . . .] [Mangano, Moretti, Piccinini, Pittau, Polosa ('03)]

These measurements can be performed at the LHC with statistical accuracies on the measured cross sections times decay branching ratios, $\sigma \times$ BR, of order 10% (sometimes even better).

VBF signature

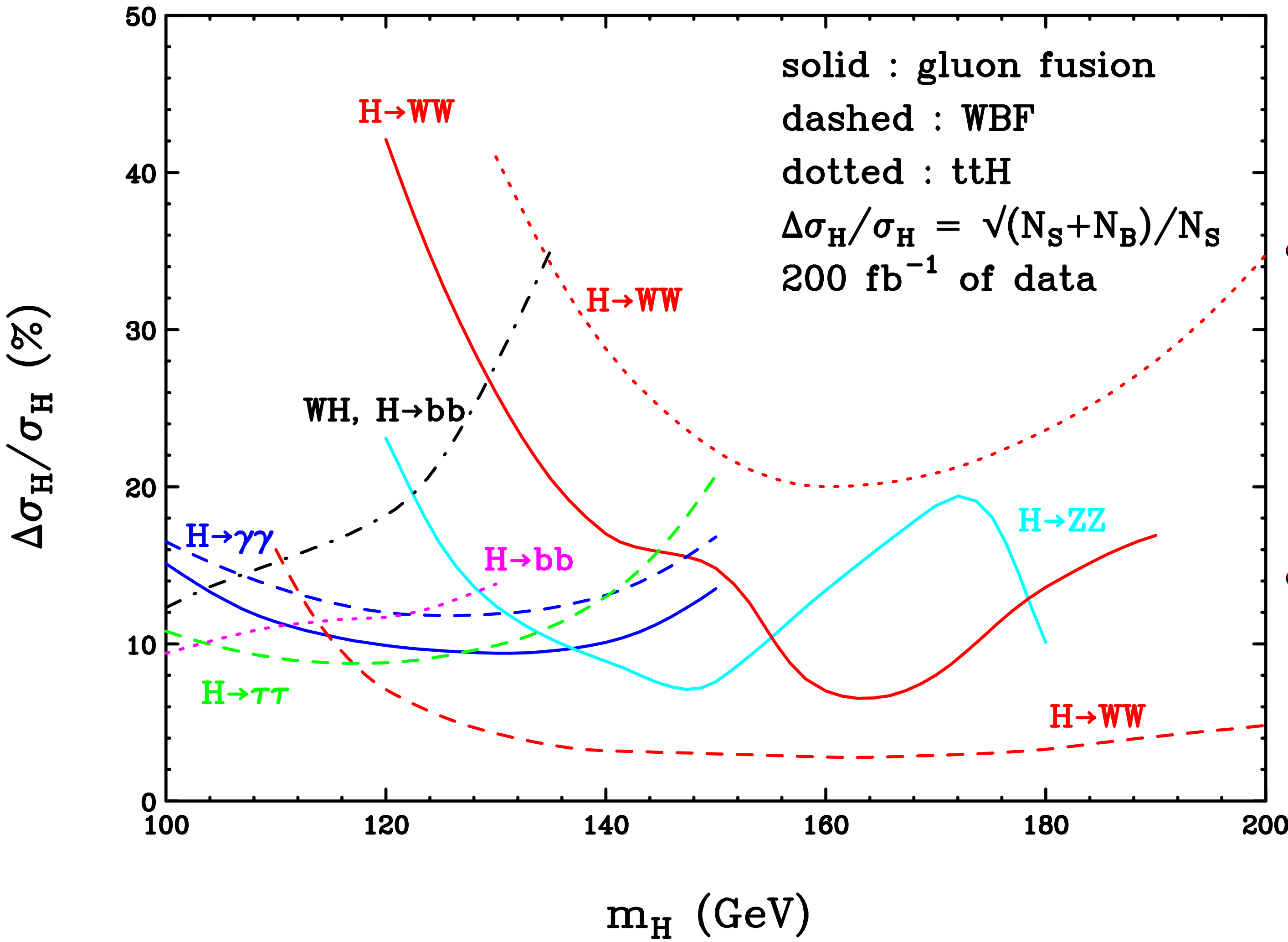


$$\eta = \frac{1}{2} \log \frac{1 + \cos\theta}{1 - \cos\theta}$$

Characteristics:

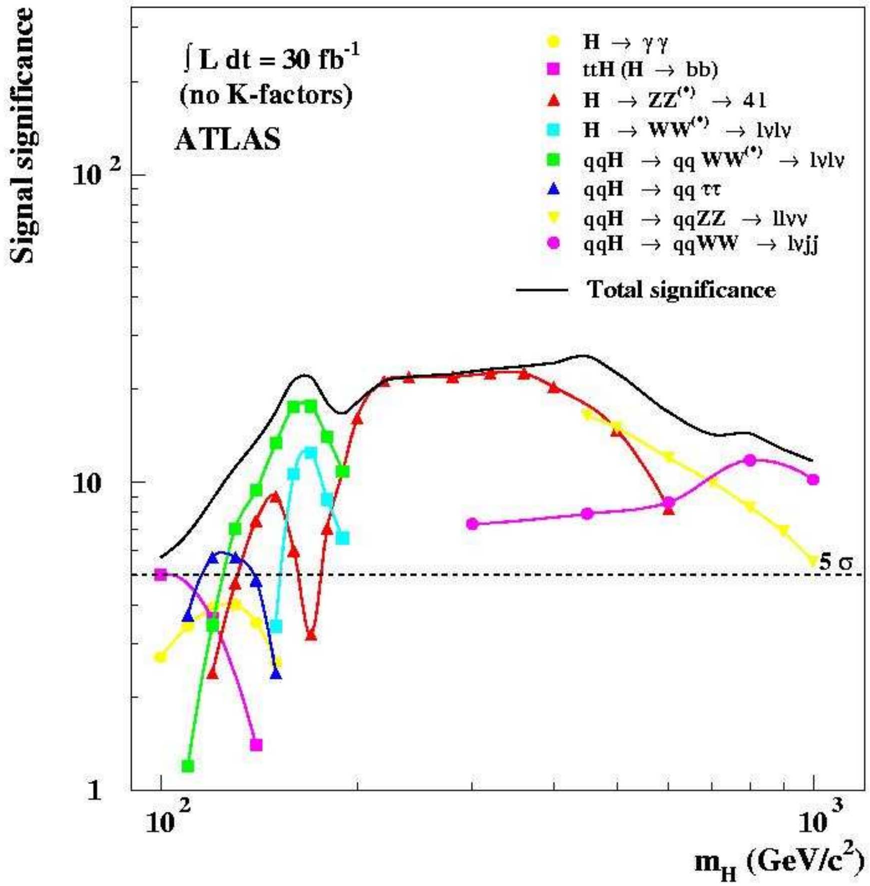
- energetic jets in the forward and backward directions ($p_T > 20$ GeV)
- large rapidity separation and large invariant mass of the two tagging jets
- Higgs decay products between tagging jets
- Little gluon radiation in the central-rapidity region, due to colorless W/Z exchange (central jet veto: no extra jets with $p_T > 20$ GeV and $|\eta| < 2.5$)

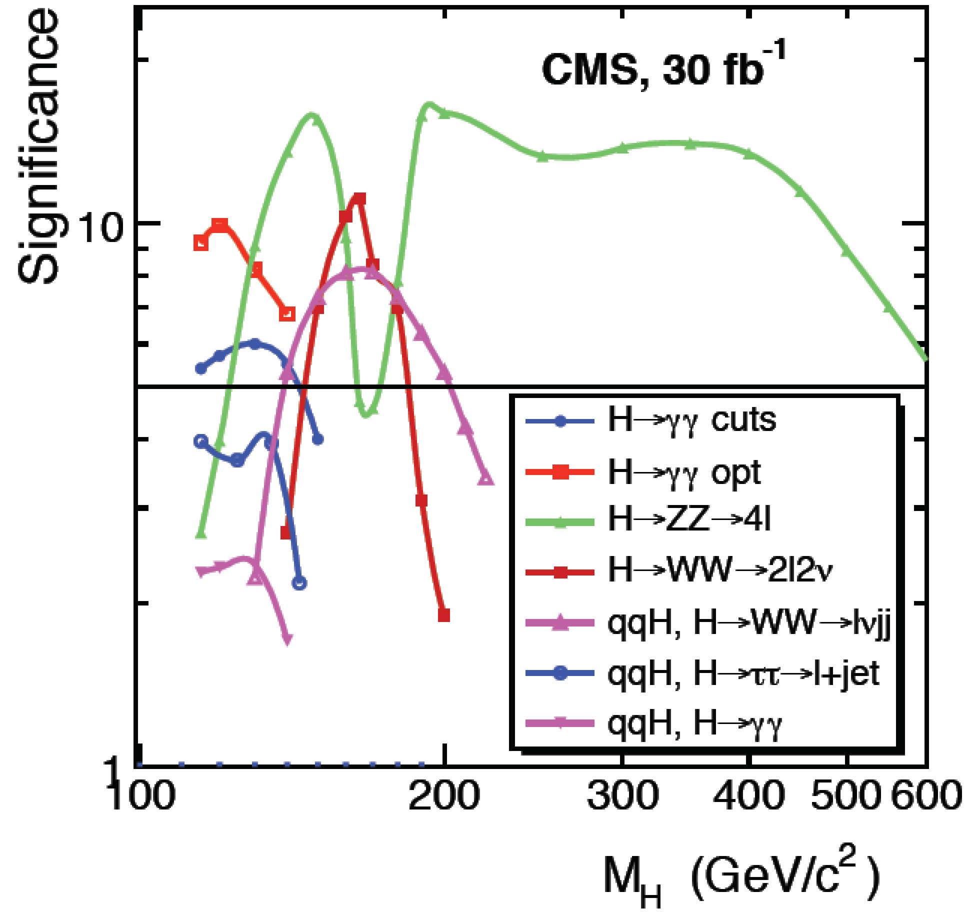
Statistical and systematic errors at LHC



- QCD/PDF uncertainties
  - ±5% for Weak Boson Fusion
  - ±20% for gluon fusion
- luminosity/acceptance uncertainties
  - ±5%

Higgs discovery potential with 30 fb^{-1}



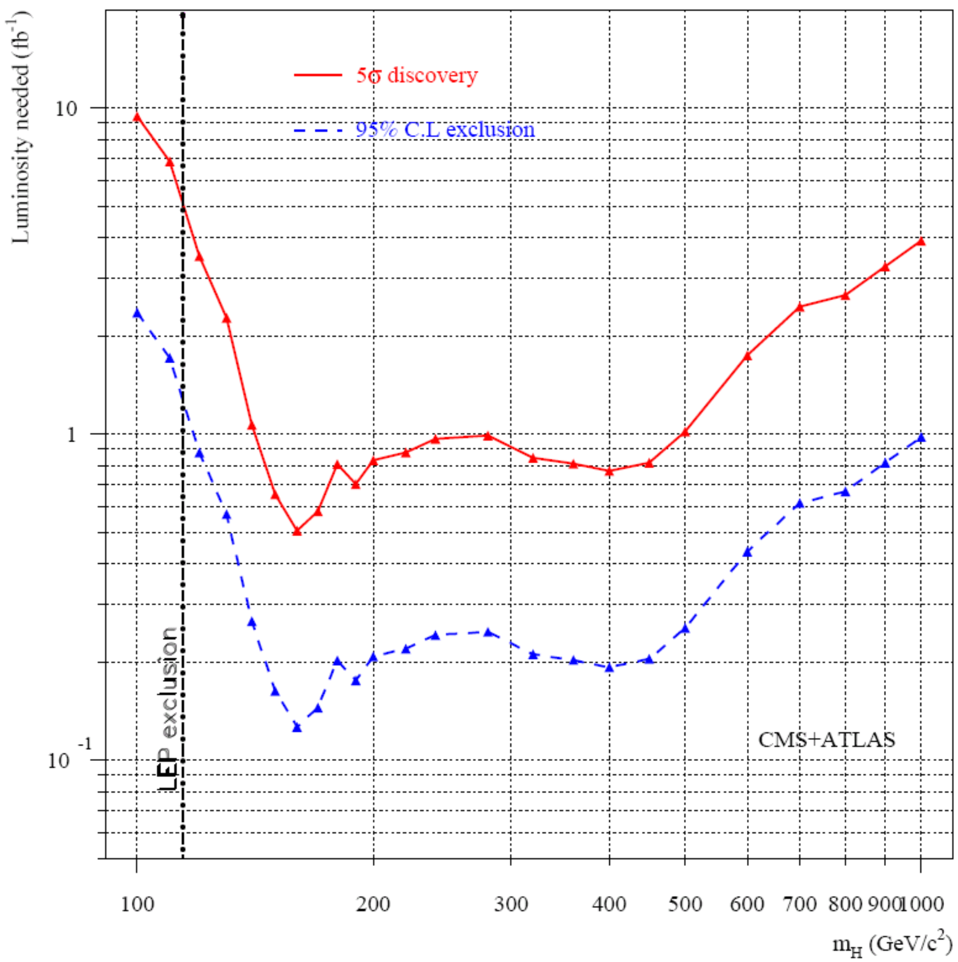


2003 **no K-factors** **2006** **K-factors included**

Full mass range can already be covered after a few years at low luminosity.

Vector-boson fusion channels play an important role at low mass!

ATLAS and CMS combined



K. Jakobs

Luminosity required for a 5 σ discovery or a 95% CL exclusion

- $\sim$ 5 fb^{-1} needed to achieve a 5 σ discovery (well-understood and calibrated detector)
- $<$ 1 fb^{-1} needed to set a 95% CL limit

Conclusions

More can be said about:

- Higgs boson couplings to bosons and fermions
- Higgs boson spin measurement from decay products and jet-angular correlations in VBF and gluon fusion
- CP properties
- Higgs boson self couplings
- SUSY Higgs bosons
- ...