# M2-branes at hypersurface singularities and their deformations


Dario Martelli

King's College London


Based on 0909.2036 [hep-th] with J. Sparks

Galileo Galilei Institute for Theoretical Physics

Mini-conference, 29 September 2010

# Plan of the talk

- Motivations
- A family of $\mathbf{d = 3}$ Chern-Simons quiver theories
- M-theory, Type IIA, and Type IIB duals
- Deformed supergravity solutions
- Deformed field theories

# Motivations

General:

- M-theory and M2-branes
- Dynamics of $\mathbf{d = (2+1)}$-dimensional SQFTs
- $AdS_4/CFT_3$ correspondence (possibly, AdS/CMT)

# Motivations

General:

- M-theory and M2-branes
- Dynamics of $\mathbf{d = (2+1)}$-dimensional SQFTs
- $AdS_4/CFT_3$ correspondence (possibly, AdS/CMT)

Particular:

- A candidate three dimensional cousin of the Klebanov-Strassler story

# Mini-review of Klebanov-Strassler

- Klebanov-Witten: $\mathbf{N}$ D3 branes at the conifold singularity

$$\mathrm{Con} = \{z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0\}$$

- $\mathcal{N} = 1$, $\mathbf{SU(N)} \times \mathbf{SU(N)}$ quiver gauge theory (strongly coupled)

- AdS/CFT dual to type IIB on $\mathrm{AdS}_5 \times \mathbf{T}^{1,1}$ ($\mathrm{Con} = \mathbf{C}(\mathbf{T}^{1,1})$) $\Rightarrow$ SCFT

- Can consider same field theory, but with $\mathbf{SU(N_1)} \times \mathbf{SU(N_2)}$.

Klebanov-Tseytlin: $\ell = |\mathbf{N_1} - \mathbf{N_2}|$ corresponds to adding $\ell$ fractional D5 branes to the $\mathbf{N}$ D3 branes

# Mini-review of Klebanov-Strassler

Field theory: conformal invariance is broken, beta function $\beta \propto \ell$

Gravity: three-form flux $\propto \ell$ at infinity $\Rightarrow$ background is not asymptotic to $AdS_5 \times \mathbf{T}^{1,1}$. There are logarithmic corrections

• Flux $\mathbf{r}$-dependent $\longrightarrow$ number of colours run $\longrightarrow$ cascade of Seiberg dualities! Very non trivial insight of Klebanov-Strassler

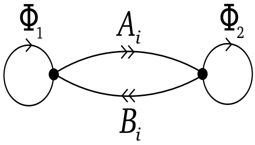
• Further insight: at the end of the cascade, $\mathbf{SU(2N)} \times \mathbf{SU(N)}$ theory develops a non perturbative superpotential $\longrightarrow$ geometry modified

$$z_1^2 + z_2^2 + z_3^2 + z_4^2 = \epsilon^2$$

deformed conifold

# A family of Chern-Simons theories: field content

Consider family of $\mathbf{d = 2 + 1}$, $\mathcal{N} = \mathbf{2}$ Chern-Simons-matter theories:




- Gauge group $\mathbf{U(N_1) \times U(N_2)}$, gauge fields $\mathscr{A}_{\mathbf{I}}$, adjoint scalars $\boldsymbol{\sigma}_{\mathbf{I}}$, Chern-Simons levels $\mathbf{k_I} \in \mathbb{Z}$, $\mathbf{I = 1, 2}$

- Chiral matter fields $\mathbf{A_i}$ in $\mathbf{N_1 \otimes \bar{N}_2}$, $\mathbf{B_i}$ in $\mathbf{\bar{N}_1 \otimes N_2}$, $\mathbf{i = 1, 2}$

- $\boldsymbol{\Phi}_{\mathbf{I}}$ in the adjoint of $\mathbf{U(N_I)}$, $\mathbf{I = 1, 2}$

# A family of Chern-Simons theories: interactions

- Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{CS}} + \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{potential}} + (\mathcal{L}_{\text{YM}})$$

where ($\mathbf{D_l}$ are auxiliary fields)

$$\mathcal{L}_{\text{CS}} = \sum_{l=1}^{2} \frac{k_l}{4\pi} \mathrm{Tr}\left( \mathscr{A}_l \wedge d\mathscr{A}_l + \frac{2}{3}\mathscr{A}_l^3 + 2D_l\sigma_l \right)$$

- Superpotential

$$\mathcal{W} = \mathrm{Tr}\left[\left((-1)^n \Phi_1^{n+1} + \Phi_2^{n+1}\right) + \Phi_2(A_1B_1 + A_2B_2) + \Phi_1(B_1A_1 + B_2A_2)\right]$$

- **n** is a positive integer. As we will see, $\mathbf{n = 1}$ and $\mathbf{n = 2}$ are special

# Remarks

- Specialize to Chern-Simons levels $(k_1, k_2) = (k, -k)$ (otherwise the duals are in massive IIA and will have no M-theory lift)

- Lagrangian has $SU(2)$ symmetry under which $A_i$, $B_i$ transform as doublets

- Also a $U(1)_b$ symmetry acting on the fields $(A_1, A_2, B_1, B_2, \Phi_1, \Phi_2)$ with charges $(1, 1, -1, -1, 0, 0)$

  $\Rightarrow$ Global symmetry: $SU(2) \times U(1)_b \times U(1)_R$, enhanced for $n = 1$ (ABJM) and $n = 2$

- For $n$ even there is a $\mathbb{Z}_2^{flip}$ symmetry: $\Phi_1 \leftrightarrow \Phi_2$, $A_i \leftrightarrow B_i$

Our family of CS theories is labelled by $N_1, N_2, n \in \mathbb{N}$ and $k \in \mathbb{Z}$

# The $\mathbf{n = 1}$ case is ABJ(M)

• When $\mathbf{n = 1}$ the adjoints $\boldsymbol{\Phi_1}$, $\boldsymbol{\Phi_2}$ are massive. Integrating them out in the IR leads to the the ABJM quartic superpotential

$$\mathcal{W}_{\mathbf{ABJM}} = \mathrm{Tr}\,(\mathbf{A_1 B_2 A_2 B_1} - \mathbf{A_1 B_1 A_2 B_2})$$

• This is the ABJM theory, and has $\mathcal{N} = \mathbf{6}$ superconformal symmetry

• For $\mathbf{N_1} = \mathbf{N_2}$, conjectured by Aharony-Bergman-Jafferis-Maldacena to be the low-energy theory on M2-branes transverse to $\mathbb{C}^{\mathbf{4}}/\mathbb{Z}_{\mathbf{k}}$

• ABJ: $\ell = |\mathbf{N_1} - \mathbf{N_2}| \neq \mathbf{0}$ corresponds to adding $\ell$ units of torsion (flat) $\mathbf{C}$ field

# Moduli space of supersymmetric vacua

- Begin with abelian moduli space: $\mathbf{N_1 = N_2 = 1}$, $\mathbf{k = 1}$:

$$\mathbf{X_n \equiv \{(n+1)\Phi_2^n + A_1B_1 + A_2B_2 = 0\} \subset \mathbb{C}^5}$$

- For $\mathbf{n = 1}$, $\mathbf{X_1 = \mathbb{C}^4}$, while for $\mathbf{n > 1}$ this is a four-fold isolated singularity

- For $\mathbf{k > 1}$ the moduli space is $\mathbf{X_n/\mathbb{Z}_k}$. Like for ABJM, $\mathbb{Z}_\mathbf{k}$ has weights $\mathbf{(1, 1, -1, -1)}$ on $\mathbf{(A_1, A_2, B_1, B_2)}$

- In general the classical moduli space is

$$\mathrm{Sym}^{\min(\mathbf{N_1, N_2})}(\mathbf{X_n/\mathbb{Z}_k})$$

# Different ranks

• Defining $\mathbf{N_1 = N + \ell}$, $\mathbf{N_2 = N}$, at generic point in the classical vacuum: $\mathbf{N}$ copies of the abelian theory, together with supersymmetric $\mathbf{U(\ell)_k}$ Chern-Simons with adjoint superpotential $\mathcal{W} = \Psi^{\mathbf{n+1}}$

• The quantum theory has no supersymmetric vacuum (Hanany-Witten, or Witten index) unless

$$\mathbf{0 \leq \ell \leq nk}$$

• We consider the $\mathbf{U(N + \ell)_k \times U(N)_{-k}}$ theories with $\mathbf{0 \leq \ell \leq nk}$, which have moduli space $\mathrm{Sym}^{\mathbf{N}}(\mathbf{X_n}/\mathbb{Z}_{\mathbf{k}})$

# M-theory interpretation

• The form of the moduli space plus ABJM results ($\mathbf{n = 1}$), suggest interpreting the $\mathbf{U(N)_k \times U(N)_{-k}}$ theories as arising from $\mathbf{N}$ M2-branes at the four-fold singularities $\mathbf{X_n}/\mathbb{Z}_\mathbf{k}$, where

$$\mathbf{X_n = \{z_0^n + \sum_{a=1}^{4} z_a^2 = 0\}} \subset \mathbb{C}^\mathbf{5}$$

• $\mathbf{X_n}/\mathbb{Z}_\mathbf{k}$ are Calabi-Yau singularities

• Topologically, $\mathbf{X_n}$ is a cone over a compact 7-manifold $\mathbf{Y_n}$

• Note $\mathbf{n = 2}$ is an eight dimensional version of the conifold

# Adding torsion C field

• In M-theory there is a four-form $\mathbf{G}$, locally $\mathbf{G} = \mathrm{d}\mathbf{C}$. Dirac quantization implies this is classified by $\mathbf{H^4(M, \mathbb{Z})}$

• One can compute $\mathbf{H^4(Y_n/\mathbb{Z}_k, \mathbb{Z}) \cong \mathbb{Z}_{nk} \cong H_3(Y_n/\mathbb{Z}_k, \mathbb{Z})}$. (Recall ABJM for $\mathbf{n = 1}$: $\mathbf{H_3(S^7/\mathbb{Z}_k, \mathbb{Z}) \cong \mathbb{Z}_k}$)

• Can turn on a flat $\mathbf{G}$ given by $\ell \in \mathbb{Z}_{\mathbf{nk}}$. Equivalently, a closed 3-form potential $\mathbf{C}$ satisfying

$$\int_{\Sigma_3} \frac{\mathbf{C}}{(2\pi l_p)^3} = \frac{\ell}{\mathbf{nk}} \quad \mathbf{mod\ 1}$$

where $\Sigma_3$ is the generator of $\mathbf{H_3(Y_n/\mathbb{Z}_k, \mathbb{Z})}$

• We identify the worldvolume theory on $\mathbf{N}$ M2-branes on $\mathbf{X_n/\mathbb{Z}_k}$ with $\ell$ units of $\mathbf{G}$-flux with the $\mathbf{U(N + \ell)_k \times U(N)_{-k}}$ theory

# Type IIA picture

- The IIA reduction leads to $\mathbf{N}$ D2 branes at the seven-dimensional singularity $\mathbf{Q_n = X_n/U(1)_b}$

- To get to this, we can start considering Type IIA on the 3-fold singularities ($\mathbf{n = 1}$ is precisely the conifold)

$$\mathbf{W_n} = \left\{ \mathbf{w_0^{2n}} + \sum_{\mathbf{i=1}}^{\mathbf{3}} \mathbf{w_i^2} = \mathbf{0} \right\} \subset \mathbb{C}^4$$

- Then consider placing $\mathbf{N}$ D2-branes at the origin of $\mathbb{R}_3 \times \mathbf{W_n}$

- Field theory on the D2-branes derived by Cachazo, Fiol, Intriligator, Katz, and Vafa. It is precisely the $\mathbf{U(N) \times U(N)}$ gauge theory we started with, but without any Chern-Simons interaction

- This is just the straight dimensional reduction of the parent $\mathbf{d = 4}$, $\mathcal{N} = \mathbf{1}$ field theory

# Turning on the Chern-Simons levels

• The $\mathbf{W_n}$ singularities admit a small Calabi-Yau resolution, in which one replaces the singular point by a $\mathbb{CP}^1$

• Wrapping $\ell$ D4-brane over this $\mathbb{CP}^1$, the gauge group becomes $\mathbf{U(N+\ell)\times U(N)}$ [familiar from Klebanov-Strassler]

• Turning on $\mathbf{k}$ units of RR 2-form flux $\mathbf{F_2}$ through the $\mathbb{CP}^1$, the Wess-Zumino coupling on D4-branes contains the term

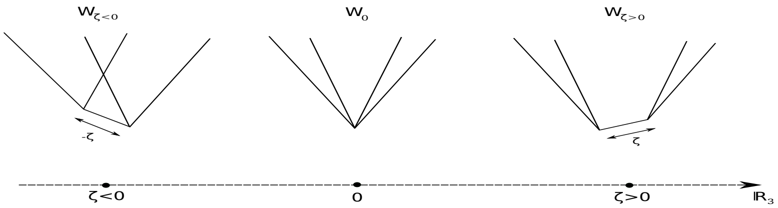
$$\begin{aligned}\int \mathbf{C_1} \wedge \mathrm{Tr}\mathscr{F} \wedge \mathscr{F} &= \int_{\mathbb{CP}^1} \mathbf{F_2} \int_{\mathbb{R}^{1,2}} \mathrm{Tr}(\mathscr{A} \wedge d\mathscr{A} + \tfrac{2}{3}\mathscr{A}^3) \\ &= \mathbf{k} \int_{\mathbb{R}^{1,2}} \mathrm{Tr}(\mathscr{A} \wedge d\mathscr{A} + \tfrac{2}{3}\mathscr{A}^3)\end{aligned}$$

# Turning on the Chern-Simons levels

• The first gauge group is that on D4-branes a wrapped on $\mathbb{CP}^{\mathbf{1}}$, while the second is an anti-D4-brane wrapped on $\mathbb{CP}^{\mathbf{1}}$ bound to a D2-brane at a point on $\mathbb{CP}^{\mathbf{1}}$

So turning on **k** units of $\mathbf{F_2}$ flux induces the Chern-Simons levels $(\mathbf{k}, -\mathbf{k})$ for the two gauge groups (Aganagic)

To preserve SUSY, one must also fibre the size of the $\mathbb{CP}^{\mathbf{1}}$ over the real line $\mathbb{R}_{\mathbf{3}}$:



# Connection to M-theory picture

• So we have that $\mathbf{Q_n = [W_n \longrightarrow \mathbb{R}]}$. Adding back the the M-theory circle $\mathbf{U(1)_b}$, this is precisely the four-fold $\mathbf{X_n}$

• Notice that in IIA $\ell$ is the number of D4-branes wrapped on $\mathbb{CP}^{\mathbf{1}}$. These lift to $\ell$ fractional M5-branes wrapped on an $\mathbf{S^3}/\mathbb{Z}_\mathbf{k} \subset \mathbf{X_n}/\mathbb{Z}_\mathbf{k}$. The (fractional) M5-brane is the magnetic source for (flat) **G** flux

• This is a Type IIA derivation of the field theories

# Type IIB picture

• Starting from the IIA description, perform a T-duality on $\mathbf{U(1)_6}$ where $\mathbf{W_n}$ is defined by the hypersurface $\mathbf{w_0^{2n} + w_1^2 + uv = 0}$ and $\mathbf{U(1)_6}$ has weights $\mathbf{(0, 0, 1, -1)}$ on $\mathbf{(w_0, w_1, u, v)}$

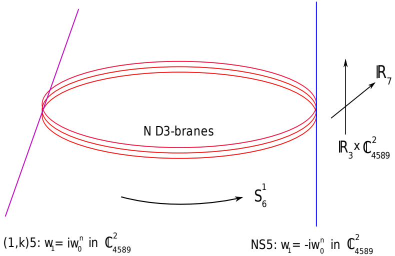
• The $\mathbf{N}$ D2-branes in $\mathbb{R}^{1,2}$ become $\mathbf{N}$ D3-branes wrapping $\mathbb{R}^{1,2}$ together with the T-dual circle $\mathbf{S^1_6}$

• There is a codimension 4 fixed point set $\mathbf{w_0^{2n} = -w_1^2}$, which become two 5-branes wrapping $\mathbf{w_0^n = \pm i w_1}$ in a copy of $\mathbb{C}^2$ spanned by $\mathbf{w_0, w_1}$. These are separated on the $\mathbf{S^1_6}$ circle by a distance depending on the period of $\mathbf{B}$ through the $\mathbb{CP}^1$

• Naively, these are both NS5-branes, but due to the $\mathbf{k}$ units of RR 2-form flux, one of them is $\mathbf{(1, k)}$ bound state with $\mathbf{k}$ D5-branes (Sen)

# Hanany-Witten-like brane picture

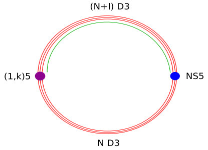
• This is a Hanany-Witten brane set-up, with D3-branes suspended between 5-branes

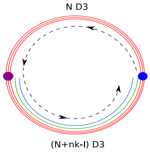



The two gauge groups and adjoints $\boldsymbol{\Phi}_{\mathbf{I}}$ are identified with the two segments of D3-brane. Where the D3s break on the 5-branes you get a bifundemental hypermultiplet, which accounts for the $\mathbf{A_i}$, $\mathbf{B_i}$ fields. The superpotential describes the non-trivial embedding of the 5-branes in the transverse $\mathbb{C}^{\mathbf{2}}_{\mathbf{4589}}$

# Field theory duality from brane creation effect

• This picture allows one to argue that the $\mathbf{U(N+\ell)_k \times U(N)_{-k}}$ theory is dual to the $\mathbf{U(N)_k \times U(N+nk-\ell)_{-k}}$ theory




• As the NS5 is moved past the $\mathbf{(1,k)}$5, $\mathbf{nk}$ D3 branes are created via the Hanany-Witten effect

# $AdS_4$ supergravity duals?

• ABJM conjectured their theory was AdS/CFT dual, in the large $N$ limit, to $AdS_4 \times S^7/\mathbb{Z}_k$ with the round Einstein metric on $S^7$ and

$$\frac{1}{(2\pi l_p)^6} \int_{S^7/\mathbb{Z}_k} *G = N$$

The $AdS_4$ radius is given by

$$\frac{R_{AdS}}{2\pi l_p} = \left( \frac{N}{6\mathrm{vol}(S^7/\mathbb{Z}_k)} \right)^{1/6}$$

• One might similarly conjecture that in the IR the theories we have written $\forall$ $n$ are conformal and are AdS/CFT dual to $AdS_4 \times Y_n/\mathbb{Z}_k$, with a Sasaki-Einstein metric on $Y_n$, where $X_n = C(Y_n)$...

# Problem with existence

Problem: for all $\mathbf{n > 2}$, $\mathbf{Y_n}$ does not admit a Sasaki-Einstein metric!

Proved by Gauntlett-DM-Sparks-Yau. Idea: any holomorphic function of definite scaling weight under the cone symmetry gives rise to an eigenfunction of the scalar Laplacian on $\mathbf{Y_n}$. For an Einstein metric, the smallest non-zero eigenvalue is bounded below by $\mathbf{7}$. For all $\mathbf{n > 3}$ the holomorphic function $\mathbf{z_0}$ violates this bound, so there cannot be an Einstein metric

This argument is "dual" to the unitarity bound in the field theory. Recall $\mathcal{W}$ contains the terms $\Phi_{\mathsf{I}}^{\mathbf{n+1}}$. If the theory is conformal, $\mathcal{W}$ must have scaling dimension 2, implying $\Phi_{\mathsf{I}}$ has scaling dimension $\Delta = \mathbf{2/(n+1)}$. But in any unitarity field theory in $\mathbf{d = 2+1}$, all gauge invariant scalar operators satisfy $\Delta \geq \mathbf{1/2}$, which is violated for $\mathbf{n > 3}$

In both cases, $\mathbf{n = 3}$ is marginal and can also be ruled out

# Problem with existence

• For $\mathbf{n > 2}$ it is natural to conjecture that the terms $\boldsymbol{\Phi}_{\mathbf{I}}^{\mathbf{n+1}}$ in $\boldsymbol{\mathcal{W}}$ are irrelevant in the IR, which then modifies the vacuum moduli space to

$$\mathbf{Sym^N(\mathbb{C} \times Con/\mathbb{Z}_k)}$$

where $\mathbf{Con = \{z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0\}}$ is the conifold 3-fold singularity

• This certainly has a Ricci-flat Kähler cone metric (albeit non-isolated singularity), and so there is an $\mathrm{AdS}_\mathbf{4} \times \mathbf{Y}/\mathbb{Z}_\mathbf{k}$ supergravity solution

# $\mathbf{n = 2}$: the eight dimensional conifold

• To have an $\mathbf{d = 11}$ supergravity Freund-Rubin dual, we need $\mathbf{n \leq 2}$. From now on, we focus on $\mathbf{n = 2}$

$$\mathbf{X_2 = \left\{ \sum_{a=0}^{4} z_a^2 = 0 \right\}}$$

is the natural 4-fold analogue of the conifold singularity

• The base of the cone $\mathbf{X_2 = C(Y_2)}$ is the homogeneous space $\mathbf{Y_2 = SO(5)/SO(3) \equiv V_{5,2}}$, which admits a (explicitly known) homogeneous Sasaki-Einstein metric

• It is possible to map Kaluza-Klein harmonics to gauge invariant operators in this theory, as well as certain wrapped M5-brane states

# The deformed supergravity solution

• The quadric singularity $\mathbf{X_2}$ may be deformed via

$$\mathcal{X} = \left\{ \sum_{a=0}^{4} z_a^2 = \gamma^2 \right\}$$

$\mathcal{X} \cong \mathbf{T}^*\mathbf{S}^4$, with $\mathbf{S}^4$ zero section

• This admits an explicit asymptotically conical Ricci-flat Kähler metric, called the Stenzel metric – analogue of the deformed conifold

• The $\mathrm{AdS}_4 \times \mathbf{V}_{5,2}$ supergravity solution may then be deformed to a smooth non-conformal background, first studied by Cvetic, Gibbons, Lu, Pope

# The deformed supergravity solution

$$\begin{aligned} ds^2_{11} &= H^{-2/3} ds^2_{\mathbb{R}^{1,2}} + H^{1/3}\gamma^2 ds^2_{\mathcal{X}} \\ G &= d^3x \wedge dH^{-1} + m\alpha \end{aligned}$$

where an orthonormal frame for $ds^2_{\mathcal{X}}$ is given by

$$e^0 = c(r)dr\ , \quad e^{\tilde{0}} = c(r)\nu\ , \quad e^i = a(r)\sigma_i\ , \quad e^{\tilde{i}} = b(r)\tilde{\sigma}_i\ ,$$

with $\nu$, $\sigma_i$, $\tilde{\sigma}_i$ $(i = 1, 2, 3)$ left-invariant one-forms on $SO(5)/SO(3)$ and

$$\begin{aligned} a^2 &= \frac{1}{3}(2+\cosh 2r)^{1/4}\cosh r\ , \quad b^2 = \frac{1}{3}(2+\cosh 2r)^{1/4}\sinh r \tanh r\ , \\ c^2 &= (2+\cosh 2r)^{-3/4}\cosh^3 r \end{aligned}$$

# The deformed supergravity solution

The four-form flux on $\mathcal{X}$ is

$$\alpha = \frac{3}{\cosh^4 r}\left(e^{\tilde{0}123} + e^{0\tilde{1}\tilde{2}\tilde{3}}\right) + \frac{1}{2}\frac{1}{\cosh^4 r}\epsilon_{ijk}\left(e^{0ij\tilde{k}} + e^{\tilde{0}\tilde{i}\tilde{j}\tilde{k}}\right)$$

which is a closed $L^2$-normalizable primitive $(2,2)$ form, which is hence harmonic

The warp factor is

$$H(y) = \frac{-24m^2}{\sqrt{2}}\int \frac{dy}{(y^4-1)^{5/2}}$$

where $y^4 = 2 + \cosh 2r$

# Flux quantization

Defining

$$\mathbf{N(r) \equiv \frac{1}{(2\pi l_p)^6} \int_{Y_r} *G}$$

we find

$$\mathbf{N(r) = \frac{\tilde{M}^2}{4} \tanh^4 r}$$

where

$$\mathbb{Z} \ni \mathbf{\tilde{M} \equiv \frac{1}{(2\pi l_p)^3} \int_{S^4} G = \frac{1}{(2\pi l_p)^3} \frac{m}{\sqrt{3}} \frac{8\pi^2}{3}}$$

- The solution is asymptotically $AdS_4 \times \mathbf{V_{5,2}}$ at large $\mathbf{r}$, with $\mathbf{N = N(\infty) = (\tilde{M}/2)^2}$. This implies we must set $\mathbf{\tilde{M} = 2M}$ even, which implies $\ell = \mathbf{0}$ mod $\mathbf{2}$ and there is no torsion $\mathbf{G}$-flux at infinity

# Flux quantization

For $\mathbf{k > 1}$, $\mathcal{X}/\mathbb{Z}_\mathbf{k}$ has two isolated $\mathbb{Z}_\mathbf{k}$ orbifold singularities

If we remove the singular points and quantize $\mathbf{G}$ in the usual way, we obtain $\mathbf{N = N(\infty) = kM^2}$ and zero torsion class for $\mathbf{G}$ at infinity

• This implies the UV SCFT theory at large $\mathbf{r}$ is the $\mathbf{U(kM^2)_k} \times \mathbf{U(kM^2)_{-k}}$ gauge theory with $\mathbf{n = 2}$

• On general grounds, the deformed solution corresponds either to deforming this UV SCFT by a relevant operator or to giving a VEV (SSB). Herzog-Klebanov argued the former, but we can be more precise

# Identifying the deformation

- On (asymptotic) $AdS_4$ in Fefferman-Graham coordinates

$$\mathbf{ds^2(AdS_4)_{FG} = \frac{1}{z^2}\left(dz^2 + dx_\mu dx^\mu\right)}$$

a scalar field $\varphi$ has modes

$$\varphi \sim \hat{\varphi} \mathbf{z}^{\boldsymbol{\Delta}} + \varphi_0 \mathbf{z}^{\mathbf{3}-\boldsymbol{\Delta}}$$

$\varphi_0$ is a perturbation by an operator of dimension $\boldsymbol{\Delta}$, while $\hat{\varphi}$ is a VEV

- The conformal dimension is related to the mass as

$$\boldsymbol{\Delta}(\boldsymbol{\Delta} - \mathbf{3}) = \mathbf{m}_\varphi^2$$

- If we have a mode $\varphi \sim \mathbf{z}^{\boldsymbol{\lambda}}$, is it the VEV of an operator of dimension $\boldsymbol{\Delta} = \boldsymbol{\lambda}$ or a deformation by an operator of dimension $\boldsymbol{\Delta} = \mathbf{3} - \boldsymbol{\lambda}$?

# Identifying the deformation

• Consider modes coming from the **G**-field. At large **r** the explicit **G**-flux has leading behaviour

$$\mathbf{G} = \mathrm{d}(\mathbf{r}^{-\nu}\beta)$$

where $\beta$ is a co-closed 3-form on $\mathbf{V_{5,2}}$ with $\Delta\beta = \nu^2\beta$ and $\nu = 4/3$. This leads to a KK pseudo-scalar mode with

$$\mathbf{m}^2 = \frac{\nu(\nu-6)}{4}\,, \qquad \Delta_\pm = \tfrac{1}{2}(3 \pm |3-\nu|)$$

Then $\nu = 4/3 \Rightarrow \Delta_+ = 7/3$, $\Delta_- = 2/3$

• Full KK multiplet spectrum computed by Ceresole, Dall'Agata, D'Auria, Ferrara: there is a mode with $\Delta = 7/3$, while $\Delta = 2/3$ is not realized

# Identifying the deformation

- At large $\mathbf{r}$ (small $\mathbf{z}$) we have $\mathbf{G} \sim \mathbf{z}^{2/3}$, implying that an operator of dimension $\Delta = 7/3$ is added to the Lagrangian

- To see which operator, we note that our four-form pseudo-scalar mode sits in a chiral multiplet whose top component has dimension $4/3 = 7/3 - 1$

- The background preserves $\mathbf{SU(2)}$ invariance: there are three chiral operators of dimension $4/3$ which are $\mathbf{SU(2)}$ invariant: $(\mathrm{Tr}\,\Phi_1^2 + \mathrm{Tr}\,\Phi_2^2)$, $(\mathrm{Tr}\,\Phi_1^2 - \mathrm{Tr}\,\Phi_2^2)$, $\mathrm{Tr}(A_1B_1 + A_2B_2)$.

- The $\mathbf{G}$-flux is odd under the $\mathbb{Z}_2^{\mathrm{flip}}$ symmetry that exchanges $\Phi_1 \leftrightarrow \Phi_2$, $A_i \leftrightarrow B_i$, leading us to identify uniquely the deformed background with the superpotential mass deformation

$$\mathcal{W} \rightarrow \mathcal{W} + \mu(\mathrm{Tr}\,\Phi_1^2 - \mathrm{Tr}\,\Phi_2^2)$$

# Matching the deformations

- This superpotential deformation changes the F-terms of the theory in such a way to precisely reproduce the deformation $\mathcal{X}$ as the abelian vacuum moduli space!

- Mass identified with the size of the $\mathbf{S}^4$ in the deformed Stenzel metric as

$$\gamma^2 = \frac{\mu^2}{12}$$

# Conclusions

• UV theory and its deformation well understood. All the remaining questions concern the resulting RG flow and the deep IR

- There is a "running" number of M2-branes $\mathbf{N(r)}$, suggesting interpreting the RG flow as a "cascade". In the IIA picture, the dilaton and $\mathbf{B}$-field through $\mathbb{CP}^1$ are the gauge couplings – in the deformed solution, these both run as a function of $\mathbf{r}$. In the IIB picture, the running $\mathbf{B}$-field suggests the 5-branes move around the $\mathbf{S}^1_6$ circle, leading to a possible cascade of dualities, à la Klebanov-Strassler
- Why is it necessary to start with $\mathbf{N = kM^2}$ M2-branes? Why must the ranks be equal? As opposed to Klebanov-Strassler, there are no fractional M5-branes here
- What is the field theory in the deep IR, near to $\mathbf{r = 0}$?