Strange metals from holography

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Hong Liu, John McGreevy, DV arXiv:0903.2477
Thomas Faulkner, Hong Liu, John McGreevy, DV arXiv:0907.2694
Thomas Faulkner, Gary Horowitz, John McGreevy, Matthew Roberts, DV arXiv:0911.3402
Thomas Faulkner, Nabil Iqbal, Hong Liu, John McGreevy, DV arXiv:1003.1728 and in progress
(see also: Sung-Sik Lee, arXiv:0809.3402 ; Cubrovic, Zaanen, Schalm, arXiv:0904.1933)

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Introduction

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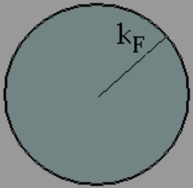
Holographic non-Fermi liquids
- $AdS_4 - BH$ geometry
- Fermi surfaces
- Near-horizon AdS_2 and emergent IR CFT

Fermions at finite density



- Landau Fermi liquid theory

  [Landau, 1957] [Abrikosov-Khalatnikov, 1963]

  stable RG fixed point [Polchinski, Shankar]

  (modulo BCS instability) [Benfatto-Gallivotti]

- (i) $\exists$ Fermi surface

  (ii) (weakly) interacting *quasiparticles*

  $\Rightarrow$ thermodynamics, transport properties

- appear as *poles* in the single-particle Green's function:

$$G_R(t, \vec{x}) = i\theta(t) \cdot \langle \{\psi^\dagger(t, \vec{x}), \psi(0, \vec{0})\} \rangle_{\mu, T}$$

$$G_R(\omega, \vec{k}) = \frac{Z}{\omega - v_F k_\perp - \Sigma} + \dots, \qquad k_\perp \equiv |\vec{k}| - k_F \qquad \Sigma \sim i\omega_\star^2$$

$$\rho(\omega, \vec{k}) \equiv \mathrm{Im}\, G_R(\omega, \vec{k}) \stackrel{k_\perp \to 0}{\longrightarrow} Z\delta(\omega - v_F k_\perp) \quad \text{with Z finite}$$

Non-Fermi liquids do exist

- sharp Fermi surface still present ✓
- no long-lived quasiparticles ×
  pole residue can vanish
- anomalous thermodynamic and transport properties

Examples

- 1+1d Luttinger liquid: interacting fermions $\rightarrow$ free bosons
- Mott metal-insulator transition
- heavy fermion compounds
- high-temperature superconductors [Müller, Bednorz, 1986]
  pseudogap, Fermi arcs and pockets, density waves
  optimally doped cuprates: $\rho \sim T$

Organizing principle for non-Fermi liquids?

Strategy

- set up finite density state
- look for a Fermi surface
- study low-energy excitations near FS
- transport properties

$AdS_4 - BH$ geometry

Relativistic CFT_3 with gravity dual and conserved $U(1)$ global symmetry

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left(\mathcal{R} + \frac{6}{R^2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \dots \right)$$

Charged black hole solution

$$ds^2 = \frac{r^2}{R^2} \left(-f(r) dt^2 + d\vec{x}^2 \right) + R^2 \frac{dr^2}{f(r) r^2}$$

$$f(r) = 1 + \frac{Q^2}{r^4} - \frac{M}{r^3} \qquad A = \mu \left(1 - \frac{r_0}{r} \right) dt$$

where $\mu =$ chemical potential, horizon at $r = r_0$.

Retarded spinor Green's function

Introduce ψ spinor field in the $AdS - BH$ background

$$S_{probe} = \int d^4x\sqrt{-g}\,(\bar{\psi}(\not{D} - m)\psi + interactions)$$

with $D_\mu = \partial_\mu + \frac{1}{4}\omega_{ab\mu}\Gamma^{ab} - iqA_\mu$.

Universality: for two-point functions, the interaction terms do not matter.
Results only depend on (q, Δ) $\qquad \Delta = \frac{3}{2} \pm mR$

Prescription [Henningson-Sfetsos] [Mück-Viswanathan] [Son-Starinets] [Iqbal-Liu]

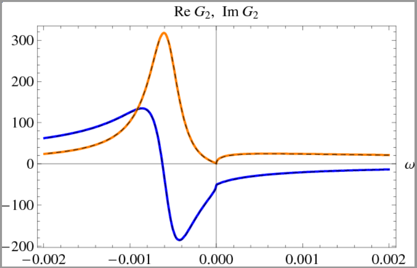
- Solve the Dirac equation for the bulk spinor in $AdS - BH$
- Impose infalling boundary conditions at the horizon
- Expand the solution at the boundary

$$\psi = (-gg^{rr})^{-1/4}e^{-i\omega t + ikx}\Psi \qquad \Phi_\alpha = \frac{1}{2}(1 - (-1)^\alpha\Gamma^r\Gamma^t\Gamma^x)\Psi$$

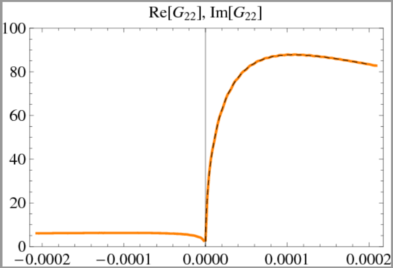
$$\Phi_\alpha \overset{r\to\infty}{\approx} a_\alpha r^m \begin{pmatrix} 0 \\ 1 \end{pmatrix} + b_\alpha r^{-m} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad G_\alpha(\omega, k) = \frac{b_\alpha}{a_\alpha} \qquad \alpha = 1, 2$$

Fermi surfaces

At $q = 1, \Delta = 3/2$ the numerical computation gives [Liu-McGreevy-DV]



$k = 0.9$



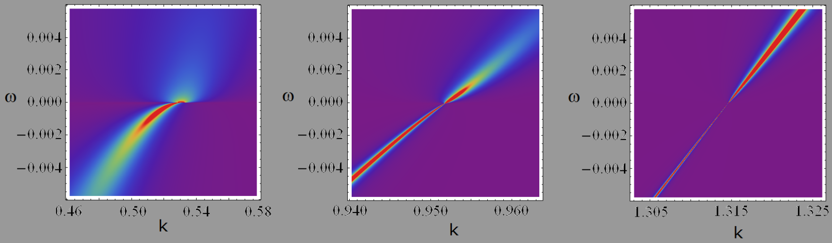
$k = 0.925$

$$k_F \approx 0.9185284990530$$

- Quasiparticle-like peaks for $k < k_F$ $\quad \omega \sim k_\perp^z$ with $z \approx 2.09$
- Bumps for $k > k_F$
- Scaling behavior: $G_R(\lambda k_\perp, \lambda^z \omega) = \lambda^{-\alpha} G_R(k_\perp, \omega)$ with $\alpha = 1$
- non-Fermi liquid!

Fermi surfaces

dispersion from ‘photoemission’ results

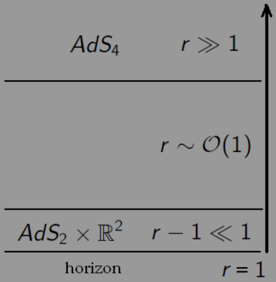


$q = 1.0 \quad k_F = 0.53$ $\qquad$ $q = 1.56 \quad k_F = 0.95$ $\qquad$ $q = 2.0 \quad k_F = 1.32$

Black hole geometry

$$ds^2 = \frac{r^2}{R^2}\left[-f(r)dt^2 + d\vec{x}^2\right] + R^2\frac{dr^2}{f(r)r^2} \qquad f(r) = 1 + \frac{3}{r^4} - \frac{4}{r^3}$$

- *Emergent "IR CFT"*
  $AdS_2 \Leftrightarrow$ "(0+1)-d CFT"
- At low frequencies, the parent theory is controlled by IR CFT.
- Each UV operator $\mathcal{O}$ is associated to a family of operators $\mathcal{O}_{\vec{k}}$ in IR.
- Conformal dimensions in IR
  $\delta_{\vec{k}} = \frac{1}{2} + \nu_{\vec{k}} \qquad \nu_{\vec{k}} = \frac{1}{\sqrt{6}}\sqrt{\left(\Delta - \frac{3}{2}\right)^2 + k^2 - \frac{q^2}{2}} \qquad \mathcal{G}_k(\omega) = c(k)\omega^{2\nu_k}$



Small ω expansion

- To understand scaling around FS, study low-ω behavior of correlators.
- Not straightforward:
  ω-dependent terms in DE are singular near the horizon.

Strategy [Faulkner-Liu-McGreevy-DV]

- Separate spacetime into UV region and the $AdS_2 \times \mathbb{R}^2$ IR region.
- Perform small ω expansions separately.
- Match them at the overlapping region.

Result

$$G_R(\omega, k) = \frac{b_+^{(0)} + \omega b_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left[b_-^{(0)} + \omega b_-^{(1)} + O(\omega^2)\right]}{a_+^{(0)} + \omega a_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left[a_-^{(0)} + \omega a_-^{(1)} + O(\omega^2)\right]}$$

$\mathcal{G}_k(\omega) \in \mathbb{C}$ retarded correlator for $\mathcal{O}_{\vec{k}}$ in IR CFT
$a_\pm^{(0)}, a_\pm^{(1)}, b_\pm^{(0)}, b_\pm^{(1)} \in \mathbb{R}$ are k-dependent functions (from UV region)

Fermi surfaces

$$G_R(\omega, k) = \frac{b_+^{(0)} + \omega b_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left[b_-^{(0)} + \omega b_-^{(1)} + O(\omega^2)\right]}{a_+^{(0)} + \omega a_+^{(1)} + O(\omega^2) + \mathcal{G}_k(\omega)\left[a_-^{(0)} + \omega a_-^{(1)} + O(\omega^2)\right]}$$

Suppose that for some k_F: $\quad a_+^{(0)}(k_F) = 0$
Then, at small $\omega, k_\perp$ we have

$$G_R(k, \omega) = \frac{h_1}{k_\perp - \frac{1}{v_F}\omega - h_2 \mathcal{G}_{k_F}(\omega)} + \dots$$

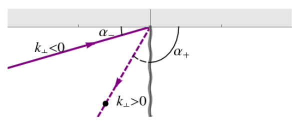
$$\mathcal{G}_k(\omega) = c(k)\omega^{2\nu_k} \qquad \nu_{\vec{k}} = \frac{1}{\sqrt{6}}\sqrt{m^2 + k^2 - \frac{q^2}{2}} \qquad h_1, h_2, v_F \in \mathbb{R}$$

This is the quasiparticle peak we saw earlier.

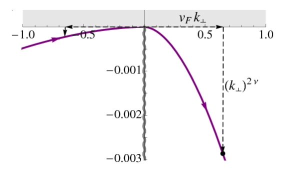
Fermi surfaces: singular and non-singular

Suppose $\nu_k < \frac{1}{2}$

$$G_R(k,\omega) = \frac{h_1}{k_\perp - h_2 \mathcal{G}_{k_F}(\omega)} + \dots$$



$$\omega_\star(k) \sim k_\perp^z \qquad z = \frac{1}{2\nu_{k_F}} > 1 \qquad \frac{\Gamma(k)}{\omega_\star(k)} = \mathrm{const} \qquad Z \sim k_\perp^{\frac{1-2\nu_{k_F}}{2\nu_{k_F}}} \to 0,\ k_\perp \to 0$$



Suppose $\nu_k > \frac{1}{2}$

$$G_R(k,\omega) = \frac{h_1}{k_\perp - \frac{1}{v_F}\omega - h_2 c(k_F)\omega^{2\nu_{k_F}}} + \dots$$

$$\omega_\star(k) \sim v_F k_\perp \qquad \frac{\Gamma(k)}{\omega_\star(k)} = k_\perp^{2\nu_{k_F}-1} \to 0 \qquad Z = h_1 v_F$$

Fermi surfaces: Marginal Fermi liquid

$$G_R(k,\omega) = \frac{h_1}{k_\perp - \frac{1}{v_F}\omega - h_2 c(k_F)\omega^{2\nu_{k_F}}} + \dots$$

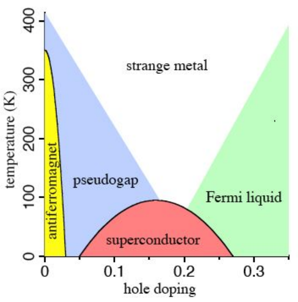
Suppose $\nu_k = \frac{1}{2}$

v_F goes to zero, $c(k_F)$ has a pole

$$G_R(k,\omega) = \frac{h_1}{k_\perp + c_1\omega + \tilde{c}_2\omega \log\omega + i c_2 \omega}$$

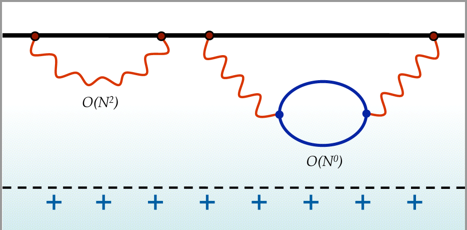
where $c \in \mathbb{R}$.

This is the Marginal Fermi liquid Green's function [Varma, 1989]

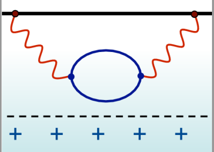


Charge transport

- Normal phase of optimally doped high-T_c: $\rho = (\sigma_{DC})^{-1} \sim T$
- impurities: $\rho \sim \text{const.}$
  e-e scattering: $\rho \sim T^2$
  e-phonon scattering: $\rho \sim T^5$
- Compute conductivity contribution of the holographic Fermi surfaces
  [Faulkner-Iqbal-Liu-McGreevy-DV]



Charge transport

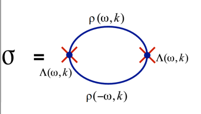


- tree-level conductivity dominates
  $\sigma_{DC} = \lim_{\omega\to 0} \frac{1}{\omega} Im\langle j_x j_x \rangle = N^2 \sigma^{tree} + N^0 \sigma^{FS} + \dots$
- gauge field a_x mixes with graviton
- trick: bulk spectral density factorizes

$$\mathrm{Im}\, D_{\alpha\beta}(\Omega, \vec{k}; r_1, r_2) = \frac{\psi_\alpha^{norm.}(\Omega, \vec{k}, r_1)\overline{\psi_\beta^{norm.}}(\Omega, \vec{k}, r_2)}{W} \rho(\Omega, \vec{k})$$

- we obtain

$$\sigma(\omega) = \frac{C}{i\omega} \int d\vec{k} \int \frac{d\omega_1}{2\pi} \frac{d\omega_2}{2\pi} \frac{f(\omega_1) - f(\omega_2)}{\omega_1 - \omega - \omega_2 - i\varepsilon} \times$$
$$\times \rho(\omega_1, \vec{k})\, \Lambda(\omega_1, \omega_2, \omega, \vec{k})\, \Lambda(\omega_2, \omega_1, \omega, \vec{k})\, \rho(\omega_2, \vec{k})$$



where $\rho(\omega, \vec{k})$ is the single-particle spectral function, $f(\omega) = \frac{1}{e^{\frac{\omega}{T}} + 1}$

$\Lambda(\omega_1, \omega_2, \Omega, \vec{k}) = \int dr \sqrt{g_{rr}}\, \overline{\Psi}(r; \omega_1, k) Q(r; \Omega, k) \Psi(r; \omega_2, k)$

Charge transport

- similar to Fermi liquid calculation
- $\sigma_{DC} \sim T^{-2\nu}$ where $\nu = \frac{1}{\sqrt{6}}\sqrt{m^2 + k_F^2 - \frac{q^2}{2}}$
- Marginal Fermi liquid: $\nu = \frac{1}{2}$ $\Rightarrow$ $\sigma_{DC} \sim \frac{1}{T}$
- optical conductivity

$\nu < \frac{1}{2}$

$$\sigma(\omega) \sim T^{-2\nu} F_1(\omega/T)$$

$$\text{for } \omega \gg T\text{: } \sigma(\omega) \sim (i\omega)^{-2\nu}$$

$\nu > \frac{1}{2}$

$$\text{for } \omega \sim T^{2\nu}\text{: } \sigma(\omega) \sim \frac{\sigma_0}{1+i\omega\tau} \text{ (Drude)}$$

$$\text{for } \omega \gg T\text{: } \sigma(\omega) \sim \frac{ia}{\omega} + b(i\omega)^{2\nu-2}$$

Summary

- charged black hole in AdS
- probe fermion spectral function shows Fermi surfaces
- (non-)Fermi liquids, marginal Fermi liquids
- conductivity: $\rho \sim T$ for MFL

- away from large N?
- backreaction of bulk fermions? Lifshitz spacetime
- quantum phase transitions