Peculiarities of String Theory on $AdS_4 \times CP^3$

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ArXiv:0811.1566 J. Gomis, D.S., L. Wulff
ArXiv:0903.5407 P.A.Grassi, D.S., L.Wulff
ArXiv:0911.5228 A.Cagnazzo, D.S., L.Wulff
ArXiv:1009.3498 D.S. and L. Wulff

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$AdS_4 \times CP^3$ versus $AdS_5 \times S^5$

(peculiarities and issues)

- Superstring theory on AdS_4xCP^3 is not maximally supersymmetric, it preserves 24 of 32 susy. The symmetry group is OSp(6|4)
  - The *complete* theory is not described by a supercoset sigma-model
  - The proof of it's classical integrability turned out to be much tricky
- some issues have not been completely settled on the boundary and bulk side of the AdS_4/CFT_3 holography:
  - Bethe ansatz CFT anomalous dimensions versus spinning string energy
  - subtleties in matching worldsheet degrees of freedom with those of S-matrix scattering theory (light and heavy worldsheet modes)
  - issue of the dual superconformal symmetry and fermionic T-duality
- The AdS_4xCP^3 theory admits string instantons wrapping 2-cycles of CP^3.

Green-Schwarz superstring

in a generic supergravity background

$$S = -\int d^2\xi \sqrt{-\det g_{ij}} - \int B_2$$

$Z^{\mathcal{M}} = (X^M, \Theta^{\underline{\alpha}}), \qquad M = 0,1,\ldots,9; \qquad \underline{\alpha} = 1,\ldots,32$

$B_2(X^M, \Theta^{\underline{\alpha}})$ - worldsheet pullback of the NS-NS 2-form guage field

$g_{ij} = E_i^A E_j^B \eta_{AB}$ - induced worldsheet metric

$E_i^A = \partial_i Z^{\mathcal{M}}(\xi)\, E_{\mathcal{M}}{}^A(X,\Theta)$ - pullback of the vector supevielbein of $D=10$ sugra

$A = 0,1,\ldots,9;$

$E^{\underline{\alpha}} = dZ^{\mathcal{M}} E_{\mathcal{M}}{}^{\underline{\alpha}}(X,\Theta)$ - spinor supervielbein of $D=10$ sugra

$$E_M^A(X,\Theta) = e_M^A(X) + \Psi_M(X)\Gamma^A\Theta + \omega_M^{BC}\,\Theta\Gamma^A\Gamma_{BC}\Theta + H_{MBC}\,\Theta\Gamma^A\Gamma^{BC}\Theta$$
$$+ e^{\Phi}F_{BC}\,\Theta\Gamma^A\Gamma^{BC}\Gamma_M\Theta + F_{BCDK}\,\Theta\Gamma^A\Gamma^{BCDK}\Gamma_M\Theta + \cdots$$

are obtained from superfield supergravity constraints = supergravity equations of motion :

$$T^A(X,\Theta) \equiv dE^A + \Omega^A{}_B E^B = 2i\, E^{\alpha}\Gamma^A_{\alpha\beta}E^{\beta}$$

Fermionic kappa-symmetry

Provided that the superbackground satisfies superfield supergravity constraints (or, equivalently, sugra field equations), the GS superstring action is invariant under the following local worldsheet transformations of the string coordinates $Z^{\mathcal{M}}(\xi)=(X^M, \Theta^\alpha)$, $\delta_\kappa Z^{\mathcal{M}}$:

$$\delta_\kappa Z^{\mathcal{M}} E_{\mathcal{M}}{}^{A}(X,\Theta) = 0, \qquad \delta_\kappa Z^{\mathcal{M}} E_{\mathcal{M}}{}^{\underline{\alpha}}(X,\Theta) = \frac{1}{2}(\mathrm{I}+\Gamma)^{\underline{\alpha}}{}_{\underline{\beta}}\,\kappa^{\underline{\beta}}(\xi),$$

$$\Gamma = \frac{1}{2\sqrt{-\det g}}\varepsilon^{ij} E_i{}^{A} E_j{}^{B} \Gamma_{AB}\Gamma^{11}, \qquad \Gamma^2 = \mathrm{I}, \qquad \mathrm{tr}\,\Gamma = 0$$

Due to the projector, the fermionic parameter $\kappa^{\underline{\alpha}}(\xi)$ has only 16 independent components. They can be used to gauge away 1/2 of 32 fermionic worldsheet fields $\Theta^{\underline{\alpha}}(\xi)$

$AdS_4 \times CP^3$ superbackground

- Preserves 24 of 32 susy in type IIA D=10 superspace

• The superstring action is not a supercoset sigma-model of OSp(6|4)

• the explicit proof of the classical integrability of the complete AdS4 x CP3 superstring has been lacking until recently (D.S.& L.Wulff, 09/2010)

- fermionic modes of the $AdS_4 \times CP^3$ superstring are of different nature: $\Theta_{32}(\xi) = (\vartheta_{24}, \upsilon_8)$

unbroken broken susy

$$\vartheta_{24} = \mathcal{P}_{24}\,\Theta, \qquad \upsilon_8 = \mathcal{P}_8\,\Theta, \qquad \mathcal{P}_{24} + \mathcal{P}_8 = I$$

$*F_4 = F_6$

$$\mathcal{P}_{24} = 1/8\,(6 - \Gamma^{a'b'} J_{a'b'} \Gamma_7), \qquad \Gamma_7 = \Gamma_1 \cdots \Gamma_6\, \varepsilon^{1\cdots 6}$$

a',b'=1,...,6 - CP^3 indices, $J_{a'b'}$ - Kaehler form on CP^3

F_2

OSp(6|4) supercoset sigma model

It is natural to try to get rid of the eight “broken susy” fermionic modes υ_8 using kappa-symmetry

$\upsilon_8 = 0$ - partial kappa-symmetry gauge fixing

Remaining string modes are:

10 ($AdS_4 \times CP^3$) bosons $x^a(\xi)$ (a=0,1,2,3), $y^{a'}(\xi)$ (a'=1,2,3,4,5,6)

24 fermions $\vartheta(\xi)$ corresponding to unbroken susy

they parametrize coset superspace $OSp(6|4)/U(3)xSO(1,3) \supset AdS4xCP3$

- similar to the $AdS_5 \times S^5$ string action on $SU(2,2|4)/SO(1,4)xSO(5)$

Cartan forms:

$\alpha=1,\ldots,4 \quad \alpha'=1,\ldots,6$

$$K^{-1}dK = E^a(x,y,\vartheta)\, P_a + E^{a'}(x,y,\vartheta)\, P_{a'} + E^{\alpha\alpha'}(x,y,\vartheta)\, Q_{\alpha\alpha'} + \Omega\,(x,y,\vartheta)\, M$$

Sigma-model action on $OSp(6|4)/U(3)xSO(1,3)$

(Arutyunov & Frolov; Stefanskij; D'Auria, Frè, Grassi & Trigiante, 2008)

$$S = \int d^2\xi\, (-\det E_i^A E_j^B \eta_{AB})^{1/2} + \int E^{\alpha\alpha'} \wedge E^{\beta\beta'} J_{\alpha'\beta'} \gamma^5_{\alpha\beta}$$

$OSp(6|4)$ supercoset sigma model

A problem with this model is that it does not describe all possible string configurations. E.g., it does not describe a string moving in AdS_4 only, or the string instanton in CP^3

Reason – kappa-gauge fixing $\upsilon_8 = \mathcal{P}_8 \Theta = 0$ is inconsistent in the AdS_4 region and for the string instanton in CP^3

$[(1+\Gamma_\kappa), \mathcal{P}_8] = 0$ ⟶ only ½ of υ_8 can be eliminated

To describe these string configurations the GS superstring action in $AdS_4 \times CP^3$ superspace with 32 fermionic coordinates is required (it is not a coset superspace)

- $AdS_4 \times CP^3$ sugra solution is related to $AdS_4 \times S^7$ (with 32 susy) in D=11 by dimenisonal reduction (*Nilsson and Pope; D.S., Tkach & Volkov 1984*)

- This superspace was constructed by performing the dimensional reduction of D=11 *coset* superspace $OSp(8|4)/SO(7)\times SO(1,3)$ which has 32 Θ and bose subspace $AdS_4 \times S^7$, $SO(2,3)\times SO(8) \subset OSp(8|4)$ (*Gomis, D.S., Wulff, 2008*)

Hopf fibration of $OSp(8|4)/SO(7)\times SO(1,3)$

- $K_{11,32}$ - D=11 superspace with the bosonic subspace $AdS_4\times S^7$ and 32 fermionic directions

$$K_{11,32} = \mathcal{M}_{10,32} \times S^1 \quad \text{(locally)}$$

base fiber

- $\mathcal{M}_{10,32}$ - D=10 superspace with the bosonic subspace $AdS_4\times CP^3$, 32 fermionic directions and $OSp(6|4)$ isometry (but it is not a coset space)

$\mathcal{M}_{10,32}$ *is the superspace we are looking for*

Classical Integrability of 2d dynamical systems

- The existence of ∞ # of conserved currents and charges

- The charges are generated by the Lax connection $\mathcal{L}$

$\mathcal{L}(\xi, z)$ – 2d one-form which depends on a spectral parameter z,

takes values in a symmetry algebra and

has zero curvature: $d\mathcal{L} + \mathcal{L} \wedge \mathcal{L} = 0$ (on the mass-shell)

The integrability is proven if one manages to construct $\mathcal{L}(\xi, z)$

No generic prescription exists how to do this

Classical Integrability of 2d dynamical systems

- G/H supercoset sigma-models with Z_4-grading, e.g.

  - $AdS_5 \times S^5$ superstring, $SU(2,2|4)/SO(1,4) \times SO(5)$, *Bena, Roiban, Polchinski '03*
  - $OSp(6|4)/SO(1,3) \times U(3)$ sigma-model

Cartan forms:

$$K^{-1}dK = \Omega(x,\vartheta)\, M_0 + E^2(x,\vartheta)\, P_2 + E^1(x,\vartheta)\, Q_1 + E^3(x,\vartheta)\, Q_3$$

$$[M_0, M_0] = M_0, \quad [P_2, P_2] = M_0, \quad \{Q_1, Q_1\} = P_2 = \{Q_3, Q_3\}, \quad \{Q_1, Q_3\} = M_0$$

Lax connection:

$$\mathcal{L} = \Omega(x,\vartheta) + \ell_1 E^2(x,\vartheta) + \ell_2 {*E^2}(x,\vartheta) + \ell_3 E^1(x,\vartheta) + \ell_4 E^3(x,\vartheta)$$

$$d\mathcal{L} + \mathcal{L} \wedge \mathcal{L} = 0 \quad \longrightarrow$$

on shell

Coefficients $\ell_i = f_i(z)$ are functions of the spectral parameter

Lax connection of the $AdS_4 \times CP^3$ superstring

(D.S. & L. Wulff, ArXiv:1009.3498)

- Use the $OSp(6|4)$ conserved Noether current

$$J = J_B + J_S$$

$SO(2,3) \times SU(4)$

$$J_B(X,\vartheta,\upsilon) = dX^M K_M(X) + J_1{}^A(X,\vartheta,\upsilon) K_A + J_2{}^{[AB]}(X,\vartheta,\upsilon) K_A K_B$$

$J_S(X,\vartheta,\upsilon)$ – susy current

<u>Lax connection:</u>

$$\mathcal{L} = \alpha_1 K(X) + \alpha_2 {}^*J_B + (\alpha_2)^2 J_2 + \alpha_1 \alpha_2 {}^*J_2 - \alpha_2(\beta_1 J_S - \beta_2 {}^*J_S)$$

$$+ \mathcal{O}(X,\vartheta,\upsilon^4) + \cdots$$

Superstring action in $AdS_4 \times CP^3$ superspace

(*up to the second order in fermions and Wick-rotated*)

[Cvetic' et al. 1999]

AdS$_4$ CP^3

$$S_E = \frac{1}{4\pi\alpha'} \int d^2\xi \sqrt{h}\, h^{ij} \left(e_i{}^a e_j{}^b \delta_{ab} + e_i{}^{a'} e_j{}^{b'} \delta_{a'b'} \right)$$

F_4 and F_2 fluxes

$$+ \frac{1}{2\pi\alpha'} \int d^2\xi \sqrt{h}\, h^{ij}\, \Theta(1-\Gamma) \left[i\, e_i{}^A \Gamma_A \nabla_j \Theta - \frac{1}{R} e_i{}^A e_j{}^B \Gamma_A \mathcal{P}_{24} \gamma^5 \Gamma_B \Theta \right]$$

Bosonic instanton solution

- In CP^3 there is a topologically nontrivial 2-cycle ~ S^2 associated with the Kahler 2-form J_2
- In the Wick-rotated theory, string worldsheet can wrap this $S^2 \subset CP^3$, thus forming a stringy instanton. It is ½ BPS

$\Theta = 0$ *and* $x^a = const$ (AdS-coordinates);

on CP^3 complex $y^I = y^I(z)$ are holomorphic functions of the worldsheet coordinates

the Virasoro constraints are identically satisfied: $g_{IJ}(y,\bar{y})\partial y^I \partial \bar{y}^J = 0$

String instanton on CP^3

A.Cagnazzo, L.Wulff and D.S. ArXiv:0911.5228

ABJ

Instanton action

$\int B_2, \quad B_2 \sim J_2, \quad d\, J_2 = d^* J_2 = 0$ – Kaehler form on CP^2

$$S_I = n\,(\frac{R^2_{CP^3}}{2\alpha'} - ia) + \int d^2\xi \sqrt{\det h}\,\left[i\vartheta\gamma^i\nabla_i\vartheta - \frac{2}{R}\vartheta\vartheta - 2(i\upsilon\gamma^i\nabla_i\vartheta - \frac{1}{R}\upsilon^2)\right]$$

$\Theta = 1/2(1-\Gamma)\Theta = (\vartheta, \upsilon)$ – gauged fixed kappa-symmetry, 16 physical fermions

Twelve fermionic zero modes, i.e. solutions of the 2d Dirac equation:

$\upsilon = 0$

8 zero modes are 4 pairs of Killing spinors on S^2

$$(\nabla_i^{S^2} + \frac{i}{R}\gamma_i)\,\vartheta_+ = 0$$

4 zero modes are massless charged fermions interacting with a monopole field on S^2

$$\gamma^i\,(\nabla_i^{S^2} + i\,A_i\gamma^3)\,\vartheta_- = 0$$

Fermionic zero modes

The 12 fermionic zero modes are goldstinos which manifest breaking of the ½ susy of the AdS_4 x CP^3 background by the string instanton

<u>24 target-space susy:</u> $\delta\vartheta = \varepsilon_{Killing}$, $\delta X^M e_M{}^A(X) = i\vartheta\Gamma^A \varepsilon_{Killing}$

AdS_4 x *CP^3* Killing spinor equation projected on the instanton

$$(1+\Gamma_\kappa)\,\vartheta = 0$$

$$(\nabla_M + \frac{i}{R}\Gamma_M\gamma_5)\vartheta = 0 \quad \Longrightarrow \quad \begin{cases} (\nabla_i^{S^2} + \frac{i}{R}\gamma_i)\vartheta_+ = 0 & \text{8 fermions} \\ (\nabla_i^{S^2} + iA_i\gamma_3)\vartheta_- = 0 & \text{4 fermions} \end{cases}$$

This fermionic zero modes solve also the complete non-linear string e.o.m

Discussion

- There also exists an NS5-brane instanton wrapping the whole CP^3

- What are the 3d CFT counterparts of the string/brane instantons?

- What are possible effects of the stringy instantons on the structure of the supergravity and superstring theory on $AdS_4 \times CP^3$? May their existence result in peculiarities of the AdS_4 / CFT_3 correspondence?

Drukker, Mariño & Putrov (arXiv:1007.3837) found contributions coming from world-sheet instanons to the partition function and Wilson loop observables computed in a matrix model description of ABJ(M) theory