AdS/CFT and Lovelock Gravity

Manuela Kulaxizi

University of Uppsala

Introduction

- The AdS/CFT correspondence provides a tool for studying large N_c gauge theories at strong coupling. Has been applied to several problems of interest from nuclear physics to condensed matter (chiral symmetry breaking, viscosity to entropy ratio, marginal fermi liquid description, superconductors etc.)

- Interesting to study higher derivative gravity theories in the context of the AdS/CFT correspondence. They provide a holographic example where $\mathrm{c} \neq \mathrm{a}$.

Introduction

Gravitational theories with higher derivative terms in general

- Have ghosts when expanded around flat space.
- Their equations of motion contain more than two derivatives of the metric. Hard to solve exactly.

  Additional degrees of freedom.

  In holography, this implies the existence of extra operators in the boundary CFT.

  [Skenderis, Taylor and van Rees].

Introduction

There exists a special class of gravitational theories with higher derivative terms, Lovelock gravity.

$$S = \int d^{d+1}x\sqrt{-g}\sum_{p=0}^{[\frac{d}{2}]}(-)^p\frac{(p-2d)!}{(p-2)!}\lambda_p\mathcal{L}_p$$

with $[\frac{d}{2}]$ the integral part of $\frac{d}{2}$, λ_p are the Lovelock parameters and the p-th order Lovelock term $\mathcal{L}_p$ is

$$\mathcal{L}_p = \frac{1}{2^p}\delta^{\mu_1\nu_1\cdots\mu_p\nu_p}_{\rho_1\sigma_1\cdots\rho_p\sigma_p}R^{\rho_1\sigma_1}{}_{\mu_1\nu_1}\cdots R^{\rho_p\sigma_p}{}_{\mu_p\nu_p}$$

$\mathcal{L}_p$ is the Euler density term in $2p$–dimensions.

Introduction

We choose $\lambda_0 = 1$ and $\lambda_1 = -1$ such that

$$\mathcal{L}_0 = \frac{d(d-1)}{L^2} \qquad\qquad \mathcal{L}_1 = R\,.$$

Examples:

- **2nd order Lovelock term $\Leftrightarrow$ Gauss-Bonnet**

$$\mathcal{L}_2 = R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2$$

- **3rd order Lovelock term**

$$\begin{aligned}\mathcal{L}_3 \;=\; & 2R^{\rho\sigma\kappa\lambda}R_{\kappa\lambda\mu\nu}R^{\mu\nu}{}_{\rho\sigma} + 8R^{\rho\sigma}{}_{\kappa\mu}R^{\kappa\lambda}{}_{\sigma\nu}R^{\mu\nu}_{\rho\lambda} + \\ & + 24R^{\rho\sigma\kappa\lambda}R_{\kappa\lambda\sigma\mu}R^{\mu}_{\rho} + 3RR^2_{\rho\sigma\kappa\lambda} + 24R^{\rho\kappa\sigma\lambda}R_{\sigma\rho}R_{\lambda\kappa} + \\ & + 16R^{\rho\sigma}R_{\sigma\kappa}R^{\kappa}_{\rho} - 12RR^2_{\rho\sigma} + R^3\end{aligned}$$

Introduction

Special Properties of the Lovelock action:

- **Equations of motion contain only up to second order derivatives of the metric $\Rightarrow$ No additional boundary data.**

  Black hole solutions can be found *exactly*.
- **No ghosts when expanded around Minkowski flat background.**
- **Palatini and Metric formulations equivalent [Exirifard, Sheikh–Jabbari].**

Introduction

Lovelock gravity admits AdS solutions with radius

$$L_{AdS}^2 = \alpha L^2 \qquad \textbf{where} \qquad \alpha = \alpha(\lambda_p)$$

Example: Gauss-Bonnet term $\lambda_2 \neq 0$

$$\alpha = \frac{1}{2}\left(1 + \sqrt{1 - 4\lambda_2}\right)$$

Asymptotically AdS black hole solutions exist

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 \sum_{i=1}^{d-1} dx_i^2$$

where $f(r)$ **satisfies the equation of motion**

$$\left[\sum_p (d-1)\lambda_p r^{d-2p} f^p\right]' = 0 \Rightarrow \sum_p \lambda_p \left(\frac{f}{r^2}\right)^p = \left(\frac{r_+}{r}\right)^d$$

Introduction

Study Lovelock theories of gravity in the context of the AdS/CFT correspondence. What new features does the boundary CFT acquire given the additional parameters of the theory λ_p? Can we learn something new?

In this talk:

Part 1: Energy Flux Positivity $\Rightarrow$ Absence of Ghosts

Part 2: Focus on holographic entanglement entropy. New features and tests [work in progress].

Outline

- *Part 1.*
  - **Review of causality and energy flux positivity correspondence**
  - **Absence of ghosts and energy flux positivity in field theory.**
- *Part 2.*
  - **Entanglement Entropy: A review**
  - **EE in four dimensional CFTs : Solodukhin's Result**
  - **Holographic Description of Entanglement Entropy**
  - **Fursaev's proposal and Generalizations**
  - **Summary, Conclusions and Open Questions**

Part 1.

Absence of ghosts and Positivity of the Energy Flux

Fluctuation Analysis

Study quasinormal modes of the AdS black hole solution $\Rightarrow$ Pole Structure of the retarded stress-energy tensor two point function.

- **Consider metric fluctuations $\delta g_{12} = \phi(r, t, x_{d-1})$ Corresponds to $\langle T_{12}(x) T_{12}(0) \rangle$ (scalar channel).**
- **Perform a Fourier Transform $\phi(t, r, x_{d-1}) = \int \frac{d\omega dq}{(2\pi)^2} \varphi(r) e^{-i\omega t + iqx_{d-1}}$, $k = (\omega, 0, 0, \cdots, 0, q)$. Express the equation of motion for φ in Schrodinger form**

$$-\frac{1}{q^2}\partial_y^2\Psi + \left[c_g^2(y) + \frac{V_1(y)}{q^2}\right]\Psi = \frac{\omega^2}{q^2}\Psi$$

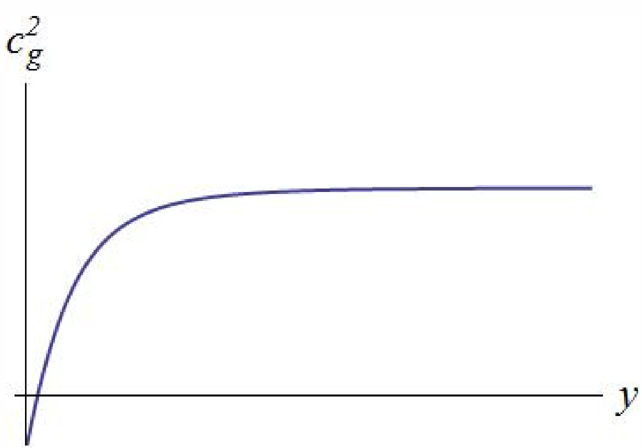
The horizon is now at $y = -\infty$ and the boundary at $y = 0$ whereas $\Psi \sim \varphi$.

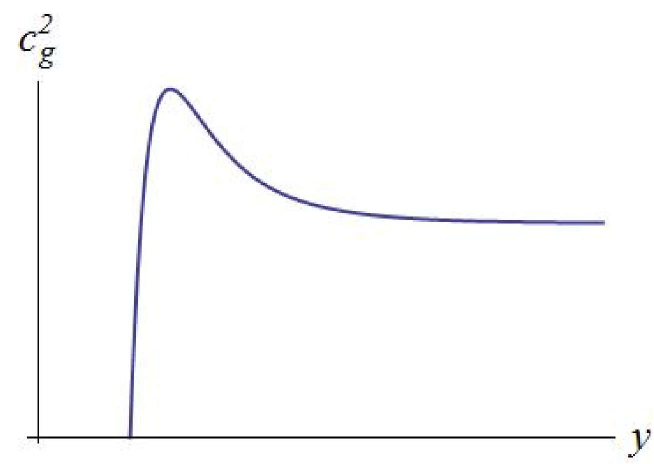
Fluctuation Analysis

What is the behavior of the potential?

$V_1(y)$ **is monotonically increasing function.**

Monotonicity properties of $c_g^2(y)$ **depend on** λ_p**. It is either monotonically increasing, reaching maximum at the boundary** $c_g^2 = 1$**, or develops a maximum in the bulk** $c_{g,max}^2 > 1$ **and metastable states may appear in the spectrum.**





Fluctuation Analysis

Consider the large q limit. Replace $V_1(y)$ by an infinite wall at $y = 0$. Use the WKB approximation to determine the group velocity of the states in the dual CFT.

$$U = \frac{d\omega}{dq} \longrightarrow c^2_{g,max}$$

Conclusion:

For values of the Lovelock parameters λ_p such that $c_g^2(y)$ attains a maximum greater than unity in the bulk, the boundary theory contains superluminal states, i.e., violates causality.

Method by [Brigante, Liu, Myers, Shenker, Yaida].

Causality Bounds

The specific form of the constraints on the Lovelock parameters λ_p are determined by the near boundary behavior of c_g^2

$$c_g^2 = 1 - C(\lambda_p)\frac{r_+^d}{r^d} + \cdots$$

where

$$C(\lambda_p) = -\frac{\sum_p p((d-2)(d-3) + 2d(p-1))\lambda_p \alpha^{p-1}}{\alpha\,(d-2)\,(d-3)\left(\sum_p p\lambda_p\alpha^{p-1}\right)^2}$$

Preserving causality in the dual theory

$$C(\lambda_p) \geq 0 \quad \Rightarrow \quad \sum_p p((d-2)(d-3) + 2d(p-1))\lambda_p\alpha^{p-1} < 0$$

[de Boer, Parnachev, M.K.] [Buchel, Escobedo, Myers, Paulos, Sinha, Smolkin] [Camanho, Edelstein]

Causality Bounds

Similar results can be obtained from studying graviton perturbations of different helicity. Each polarization gives a different constraint:

$C_1(\lambda_p) > 0, C_2(\lambda_p) > 0, C_3(\lambda_p) > 0$

[Myers, Buchel; Hofman; Camanho, Edelstein].

Examples:

Gauss–Bonnet gravity $d = 4$:

$$-\frac{7}{35} < \lambda_2 < \frac{9}{100}$$

3rd order Lovelock gravity $d = 6$:

$$C(\lambda_p) = \alpha \frac{5\alpha^2\lambda_2 + (9 - 8\alpha)}{[\alpha^2\lambda_2 + (3 - 2\alpha)]^2} \geq 0$$

Positivity of the Energy Flux

- **What do the Lovelock parameters λ_p correspond to in the boundary CFT? What are the corresponding constraints?**

The two- and three-point functions of the stress energy tensor are completely determined up to three independent coefficients $(\mathcal{A}, \mathcal{B}, \mathcal{C})$ [Osborn, Petkou].

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0)\rangle = \frac{(d-1)(d+2)\mathcal{A} - 2\mathcal{B} - 4(d+1)\mathcal{C}}{d(d+2)} \frac{\mathcal{I}_{\mu\nu,\rho\sigma}(x)}{x^{2d}}$$

$$\langle T_{\mu\nu}(x_3) T_{\rho\sigma}(x_2) T_{\tau\kappa}(x_1)\rangle = \mathcal{A}\frac{\mathcal{J}_{\mu\nu\rho\sigma\tau\kappa}(x)}{x_{12}^d x_{13}^d x_{23}^d} + \mathcal{B}\frac{\mathcal{K}_{\mu\nu\rho\sigma\tau\kappa}(x)}{x_{12}^d x_{13}^d x_{23}^d} + \\ + \mathcal{C}\frac{\mathcal{M}_{\mu\nu\rho\sigma\tau\kappa}(x)}{x_{12}^d x_{13}^d x_{23}^d}$$

Positivity of the Energy Flux

The Lovelock parameters λ_p can be expressed in terms of the CFT parameters $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$. Then holography predicts that $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ obey three independent constraints:

$$C_1(\mathcal{A},\, \mathcal{B},\, \mathcal{C}) > 0, \quad C_2(\mathcal{A},\, \mathcal{B},\, \mathcal{C}) > 0, \quad C_3(\mathcal{A},\, \mathcal{B},\, \mathcal{C}) > 0$$

These constraints precisely match the constraints derived from the positivity of the energy flux one-point function! [Hofman, Maldacena]

Note: Supersymmetry implies a linear relation between $\mathcal{A}, \mathcal{B}, \mathcal{C}$. Effectively, two independent parameters.

Example: the central charges a, c in $d = 4$.

Curiously, the Lovelock parameters satisfy this relation.

Positivity of the Energy Flux

Definition: The energy flux operator $\mathcal{E}(\widehat{n})$ per unit angle measured through a very large sphere of radius r is

$$\mathcal{E}(\widehat{n}) = \lim_{r\to\infty} r^{d-2} \int dt\, \widehat{n}^i\, T^0_i(t, r\widehat{n}^i)$$

n^i is a unit vector specifying the position on S^{d-2} where energy measurements may take place. Integrating over all angles yields the total energy flux at large distances.

Focus on the energy flux one-point function on states created by the stress–energy tensor operator

$$\mathcal{O}_q = \epsilon_{ij} T_{ij}(q)$$

with ϵ_{ij} a symmetric, traceless polarization tensor

Positivity of the Energy Flux

- Rotational symmetry fixes the form of the energy flux one–point function up to two independent parameters.

$$\langle \mathcal{E}(\widehat{n}) \rangle_{T_{ij}} = \frac{\langle \epsilon^*_{ik} T_{ik} \mathcal{E}(\widehat{n}) \epsilon_{lj} T_{lj} \rangle}{\langle \epsilon^*_{ik} T_{ik} \epsilon_{lj} T_{lj} \rangle} =$$

$$= \frac{E}{\Omega_{d-2}} \left[1 + t_2 \left(\frac{\epsilon^*_{il} \epsilon_{lj} n_i n_j}{\epsilon^*_{ij} \epsilon_{ij}} - \frac{1}{d-1} \right) + t_4 \left(\frac{|\epsilon_{ij} n_i n_j|^2}{\epsilon^*_{ij} \epsilon_{ij}} - \frac{2}{d^2-1} \right) \right]$$

By construction t_2, t_4 can be expressed in terms of the CFT parameters $\mathcal{A}, \mathcal{B}, \mathcal{C}$. The supersymmetric case: the linear relation between $\mathcal{A}, \mathcal{B}, \mathcal{C}$ is equivalent $t_4 = 0$.

Positivity of the Energy Flux

Demand positivity of the energy flux one point function, *i.e.*, $\langle \mathcal{E}(\widehat{n}) \rangle \geq 0$.

The positivity of the energy flux imposes constraints on t_2, t_4**:**

$$C_1(\mathcal{A},\mathcal{B},\mathcal{C}) \equiv 1 - \frac{1}{d-1}t_2 - \frac{2}{d^2-1}t_4 \geq 0$$

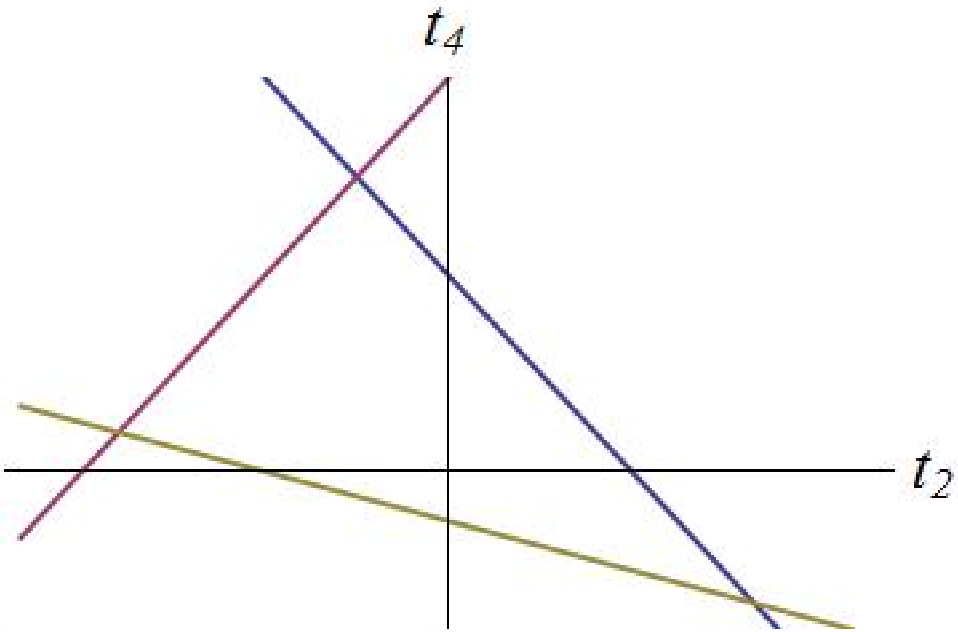
$$C_2(\mathcal{A},\mathcal{B},\mathcal{C}) \equiv 1 - \frac{1}{d-1}t_2 - \frac{2}{d^2-1}t_4 + \frac{t_2}{2} \geq 0$$

$$C_3(\mathcal{A},\mathcal{B},\mathcal{C}) \equiv 1 - \frac{1}{d-1}t_2 - \frac{2}{d^2-1}t_4 + \frac{d-2}{d-1}(t_2+t_4) \geq 0$$

When expressed in terms of $\mathcal{A},\mathcal{B},\mathcal{C}$ **these constraints precisely match the ones obtained from holography!**

Example: Bounds for a $d = 6$ dimensional SCFT

Parameter space t_2, t_4 of a consistent CFT. Values outside the triangle are forbidden.



Absence of ghosts and CFT constraints

The energy flux positivity constraints are related to causality in the gravity language. Can we see something similar in field theory?

Guide from the AdS/CFT analysis:

- Consider the Fourier transform of the two–point function of the stress energy tensor at finite temperature.
- Three independent polarizations; each polarization yields a different set of constraints.
- Focus on large momenta, small temperatures $\frac{k}{T} \gg 1$.

Absence of ghosts and CFT constraints

How do we compute the two–point function of the stress-energy tensor in an arbitrary CFT at finite temperature?

In the regime of small temperatures use the OPE:

$$T_{\mu\nu}(x)T_{\rho\sigma}(0) \sim \frac{\mathcal{I}_{\mu\nu,\rho\sigma}}{x^{2d}} + \mathcal{D}_{\mu\nu\rho\sigma\kappa\tau}(x)T^{\kappa\tau}(0) + \cdots$$

$\mathcal{D}_{\mu\nu\rho\sigma\kappa\tau}(x)$ **is related to the three point function of the stress energy tensor [Osborn, Petkou].**

Consider the three independent polarizations separately. Take the expectation value and Fourier transform.

Note: $\langle T_{00}\rangle = 3\langle T_{ii}\rangle \propto 3T^4$.

Absence of ghosts and CFT constraints

Example:

The two-point function in the "scalar channel" in $d = 4$.

$$G_{12,12}(w,q)_T \sim C_1(\mathcal{A},\mathcal{B},\mathcal{C})\frac{w^2+q^2}{w^2-q^2}T^4 + \cdots$$
$$C_1(\mathcal{A},\mathcal{B},\mathcal{C}) = \left(1 - \frac{t_2}{3} - \frac{t_4}{15}\right)$$

Note: C_1 **determines the sign of the residue of the pole. Absence of ghosts requires $C_1 \geq 0$. This is precisely the energy flux positivity constraint!**

The other two constraints are recovered by studying different polarizations.

Absence of ghosts and CFT constraints

What about other operators in the OPE? Relevant operators would dominate the low temperature limit!

- *Scalar Operators* $\langle O \rangle \sim T^{\Delta}$

  Their contribution to the OPE proportional to $T^{\Delta}(k^2)^{2-\frac{\Delta}{2}} \Rightarrow$ **not singular for** $\Delta \leq 4$.

- *Vector Operators* $\langle J_\mu \rangle \neq 0$

  Rotational invariance implies that only $\langle J_0 \rangle \neq 0$. **Rotation by** $\theta = \pi$ **in the** $x^0 - x^1$ **plane** $\Rightarrow \langle J_0 \rangle = 0$.

The argument breaks down when the theory contains more than one stress-energy tensors which do not decouple.

Summary and Open Questions

- AdS/CFT for Lovelock gravity helped to show that energy flux positivity is equivalent to the absence of ghosts.
- Unitarity constraints are derived from three point functions.
- Can we use this (and other lessons from Lovelock gravity) to understand conformal filed theories better? e.g.
  - Does scale invariance implies conformal invariance (proven only in $d = 2$) ?
  - Analog of Zamolodchikov's theorem in higher dimensions?

Part 2.

Entanglement Entropy and Lovelock Gravity

Entanglement Entropy: Review

- **Consider a quantum mechanical system at zero temperature in a pure state $|\Psi\rangle$. The density matrix is $\rho_0 = |\Psi\rangle\langle\Psi|$ and the von Neumman entropy vanishes**

$$S = -\mathrm{tr}\rho_0 \ln \rho_0 = 0\,.$$

- **"Divide" the system into two subsystems A, B with Hilbert spaces $\mathcal{H}_A, \mathcal{H}_B$. The reduced density matrix $\rho_A = \mathrm{tr}_B \rho_0$ is accessible only to A. The entanglement entropy for the subsystem A is the von Neumman entropy of the reduced density matrix ρ_A**

$$S_A = -\mathrm{tr}_A \rho_A \ln \rho_A$$

Entanglement Entropy: Review

The entanglement entropy, EE, measures how "quantum" a system is.

Example:

Consider two systems A, B with Hilbert spaces consisting of two states $\{|1\rangle, |2\rangle\}$. The total Hilbert space is the product of the Hilbert spaces $\mathcal{H}_A, \mathcal{H}_B$.

Product State:

$$|1_A 1_B\rangle \Rightarrow S_A = 0$$

Pure (non product) State:

$$\frac{1}{\sqrt{2}}(|1_A 2_B\rangle - |2_A 1_B\rangle) \Rightarrow S_A = \ln 2$$

Entanglement Entropy: Review

EE satisfies a number of different properties (pure state):

- For the subsystem V and its complement V^c entanglement entropy is equal.

$$S(V) = S(V^c)$$

- For any two subsystems A, B entanglement entropy satisfies the strong subadditivity property

$$S(A) + S(B) \geq S(A \cup B) + S(A \cap B)$$

Entanglement Entropy: Review

EE in a continuous system is UV divergent. The "Area Law" of EE refers to the form of the leading divergence

$$S(V) \sim \frac{Area(\partial V)}{\epsilon^{d-2}} + \cdots$$

Note: **The "Area Law" is violated for systems with a Fermi surface [Wolf, Gioev, Klich, ...].**

For a conformal field theory, CFT, in d-dimensions

$$S(V) = \frac{g_{d-2}[\partial V]}{\epsilon^{d-2}} + \cdots + \frac{g_1[\partial V]}{\epsilon} + g_0[\partial V] \ln \epsilon + s(V) \,.$$

If V has a single characteristic length scale, R, $g_i[\partial V]$ is a homogeneous function of degree i of R.

Entanglement Entropy: Review

Functions $g_i[\partial V]$ with $i \neq 0$ are non-physical, cutoff dependent.

- The coefficient of the logarithmically divergent term in the EE, $g_0[\partial V]$, is physical and universal.

In 2-dimensional CFTs the leading divergent term is logarithmic. Its coefficient is proportional to the central charge c of the CFT.

e.g: The EE of a line segment of length l

$$S(l) = \frac{c}{3} \ln \frac{l}{\epsilon}$$

[Casini, Huerta]: An alternative proof of the c-theorem in combining this result with the strong subadditivity property of EE.

Entanglement Entropy: Review

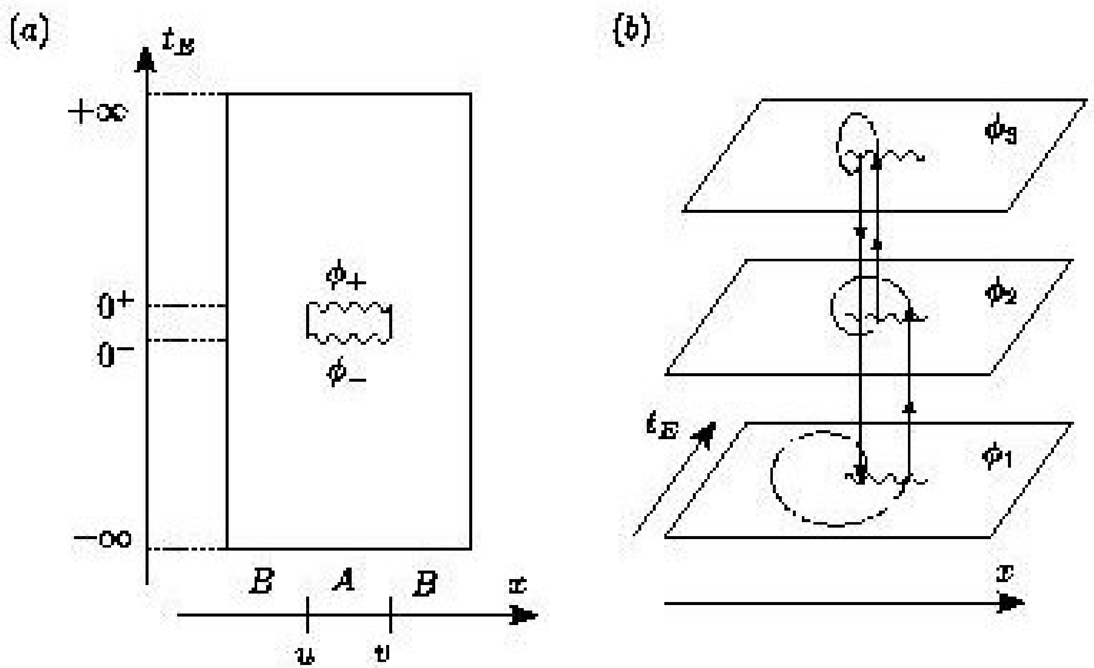
- **How to compute EE in quantum field theory?**

The replica trick:

$$S(V) = \lim_{n\to 1} \frac{\mathrm{tr}_V \rho_V^n - 1}{1-n} = -\frac{\partial}{\partial n} \ln \mathrm{tr}_V \rho_V^n |_{n=1}$$

In the path integral formalism $\mathrm{tr}_V \rho_V^n = \frac{Z_n}{Z_1^n}$ **and one computes the partition function** Z_n **by gluing together** n **copies of** R_d **along the boundary** (∂V)**.**

Entanglement Entropy: Review



(a) Path integral representation of the reduced density matrix,

(b) The n-sheeted surface, with $n = 3$ for simplicity.

Solodukhin's result for EE in $4d$-CFTs.

The coefficient of the logarithmic term in the EE of a subspace V with boundary ∂V of extrinsic curvature $k^i_{\mu\nu}$

$$g_0[\partial V] = \frac{\mathbf{c}}{720\pi} g_{0c}[\partial V] - \frac{\mathbf{a}}{720\pi} g_{0a}[\partial V]$$

$\mathbf{c}$, $\mathbf{a}$ are the CFT central charges defined through the Weyl anomaly on a curved manifold

$$\langle T^\mu_\mu \rangle = \frac{1}{90} \times \frac{1}{64\pi^2} \left(\mathbf{c}\mathrm{I}_2 - \mathbf{a}\mathcal{L}_{(2)} \right)$$

I_2 is the square of the Weyl tensor and $\mathcal{L}_{(2)}$ is the Euler density in four dimensions, *i.e.*, the Gauss–Bonnet term.

Solodukhin's result for EE in $4d$-CFTs.

g_{0c}, g_{0a} **depend on the details of the boundary** ∂V

$$g_{0c}[\partial V] = \int_{\partial V} R_{\mu\nu\sigma\tau}(n_i^{\mu} n_i^{\sigma})(n_j^{\nu} n_j^{\tau}) - R_{\mu\nu} n_i^{\mu} n_i^{\nu} + \frac{1}{3} R + \\ + \int_{\partial V} \left[\frac{1}{2} k^i k^i - (k^i_{\mu\nu})^2 \right]$$

$$g_{0a}[\partial V] = \int_{\partial V} R_{(\partial V)}$$

- n_i **with** $i = 1, 2$ **are vectors normal to the surface** (∂V)
- $k^i_{\mu\nu}$ **is the extrinsic curvature associated to** n^i **with** k^i **its trace.**

  $k^i_{\mu\nu} = -\gamma^{\rho}_{\mu} \gamma^{\sigma}_{\nu} D_{\rho} n^i_{\sigma}$ **where** $\gamma_{\mu\nu} = g_{\mu\nu} - n^i_{\mu} n^i_{\nu}$

Solodukhin's result for EE in $4d$-CFTs.

Corollary for the EE of any four dimensional CFT:

- **For V a ball $\mathcal{B}$ of radius of R**

$$g_0(\mathcal{B}) = \frac{\mathbf{a}}{90}$$

- **For V a cylinder $\mathcal{C}$ of radius R and "infinite" length l**

$$g_0(\mathcal{C}) = \frac{\mathbf{c}}{720}\frac{l}{R}$$

Solodukhin's result for EE in $4d$-CFTs.

Solodukhin's result for the coefficient of the logarithmically divergent term in the entanglement entropy of a ball was confirmed for the case of a free massless scalar field both numerically and analytically.

[Lohmayer, Neuberger, Schwimmer, Theisen / Casini, Huerta]

Note: **This result provides a** *new, distinct* **characterization of the central charges $(\mathbf{c}, \mathbf{a})$ of the CFT.**

Connection to Zamolodchikov's theorem? Generalization to arbitrary dimensions?

Holographic Description of EE

[Ryu-Takayanagi]

The EE in a CFT on R_d of a subspace V with arbitrary $(d-2)$-dimensional boundary $(\partial V) \in \mathrm{R}_{d-1}$ is given by

$$S(V) = \frac{1}{4G_N^{(d+1)}} \int_\Sigma \sqrt{\sigma}$$

Here Σ is the static d-dimensional minimal surface within AdS$_{d+2}$ which asymptotes to (∂V).

The proposal has been generalized to non-conformal cases and the near horizon limit of Dp–branes. A co-variant formulation has been proposed as well.

[Ryu, Takayanagi, Klebanov, Kutasov, Murugan, Hubeny, Rangamani]

Holographic Description of EE

Ryu-Takayanagi formula passed several tests:

- **It is trivially equal for V and its complement V^c (when evaluated at zero temperature).**
- **At zero temperature, in the limit of very large V the holographic EE vanishes. At finite temperature it asymptotes to the thermal entropy.**
- **Satisfies the strong subadditivity property.**
  [Headrick, Takayanagi]
- **Agreement with field theoretic results in 2-dimensional CFTs [Calabrese, Cardy].**

Fursaev's proposal and Generalizations

In all CFTs dual to Einstein-Hilbert gravity (with a cosmological constant): $\mathrm{a} = \mathrm{c}$.

- **Is there a way to distinguish between the two central charges in holography?**

Gauss-Bonnet gravity, **is a higher derivative gravity with this property.**

$$S_{GB} = \frac{1}{16\pi G_N^{(5)}} \int d^5x \sqrt{-g} \left(R + \frac{12}{L^2} + \frac{\lambda_{GB} L^2}{2} \mathcal{L}_{(2)} \right)$$

Fursaev's proposal and Generalizations

Gauss-Bonnet gravity admits two AdS solutions. One solution is unstable against small perturbations.

Consider the stable solution with radius:

$$L_{AdS}^2 = \frac{1+\sqrt{1-4\lambda_{GB}}}{2} L^2$$

Computation of the Weyl anomaly for Gauss-Bonnet gravity determines the CFT central charges in terms of the Gauss-Bonnet parameter λ_{GB} [Nojiri, Odintsov].

$$\mathbf{c} = 45\pi \frac{L_{AdS}^3}{G_N^{(5)}} \sqrt{1-4\lambda_{GB}}$$

$$\mathbf{a} = 45\pi \frac{L_{AdS}^3}{G_N^{(5)}} \left[-2+3\sqrt{1-4\lambda}\right]$$

Fursaev's proposal and Generalizations

A proposal for holographic EE in Gauss-Bonnet gravity [Fursaev].

$$S(V) = \frac{1}{4G_N^{(5)}} \int_\Sigma \sqrt{\sigma} \left(1 + \lambda_{GB} L^2 R_\Sigma\right)$$

Σ is the minimal surface ending on (∂V) which satisfies the e.o.m. derived from this action. R_Σ is the induced scalar curvature on Σ.

- **Coincides with Wald's entropy formula on AdS black holes.**
- **Satisfies all of the properties of EE, including strong subadditivity [Headrick, Takayanagi].**

Holographic EE for a ball of radius R

Finding the exact minimal surface is a difficult problem. Solving for the leading divergent terms in the EE is easy.

Consider the case of a ball. Write the AdS metric as

$$ds^2_{AdS} = L^2_{AdS}\left[\frac{d\rho^2}{4\rho^2} + \frac{1}{\rho}\left(-dt^2 + dr^2 + r^2 d\Omega_2^2\right)\right]$$

Symmetries indicate that Σ is determined by a single function $r(\rho)$. The e.o.m. in the vicinity of the boundary $\rho = 0$ are solved by

$$r(\rho) = R - \frac{\rho}{2R} + \cdots$$

Substitute into the "action" to arrive at

$$S(B) = \frac{\mathbf{a}}{90}\frac{R^2}{\epsilon^2} + \frac{\mathbf{a}}{90}\ln\epsilon + \cdots$$

Holographic EE for a cylinder and a belt

In similar manner, consider the EE of an infinite cylinder. Write the AdS metric as

$$ds^2_{AdS} = L^2_{AdS}\left[\frac{d\rho^2}{4\rho^2} + \frac{1}{\rho}\left(-dt^2 + dz^2 + dr^2 + r^2 d\phi^2\right)\right]$$

Solve the e.o.m. in the vicinity of the boundary $\rho = 0$ to find

$$r(\rho) = R - \frac{\rho}{4R} + \cdots$$

and substitute in the "action"

$$S(C) = \frac{\mathbf{a}}{90}\frac{2\pi R l}{4\pi\epsilon^2} + \frac{\mathbf{c}}{720}\frac{l}{R}\ln\epsilon + \cdots$$

The result once more agrees with Solodukhin's prediction for the lograthmically divergent term in the EE.

Holographic EE and Lovelock gravity

- Holographic results from Fursaev's proposal in perfect agreement with Solodukhin's.

A natural generalization of Fursaev's proposal to any Lovelock theory of gravity

$$S(V) = \frac{1}{4G_N^{(d+1)}} \sum_{p=0}^{[\frac{d}{2}]} (-)^{p+1} (p+1) \frac{(d-2p-2)!}{(d-2)!} \lambda_{p+1} \int_{\Sigma} \sqrt{\sigma} \mathcal{L}_{(p)}$$

Summary, Conclusions and Open Questions

- Fursaev's formula for the holographic calculation of EE in Gauss-Bonnet gravity agrees with Solodukhin's result.
- There is a natural generalization of this proposal for any Lovelock theory of gravity.

Open Questions:

- Generalization of Solodukhin's result to higher dimensional CFTs.
- EE in an arbitrary theory of higher derivative gravity?
- Helpful perhaps towards finding the analog of Zamolodchikov's theorem in higher dimensions?