CFT\ADS/CFT

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AdS4/CFT3 and the holographic states of matter

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OUTLINE

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- Free fields on AdS_4: boundary conditions
- $\mathcal{N} = 4$ SYM on AdS_4: Perturbative analysis
- $\mathcal{N} = 4$ SYM on AdS_4: Strong coupling results
- Discussion

Motivation

Field theory motivation:

- Would like to understand dynamics of strongly coupled fields in curved spacetime.
- Quantitative computations of expectation values of local operators, say $\langle T_{\mu\nu} \rangle$?

Why AdS?:

- Maximally symmetric spacetime.
- Not globally hyperbolic → role of boundary conditions and influence on dynamics?
- AdS spacetime provides a geometric IR cut-off.

Motivation

AdS/CFT motivation:

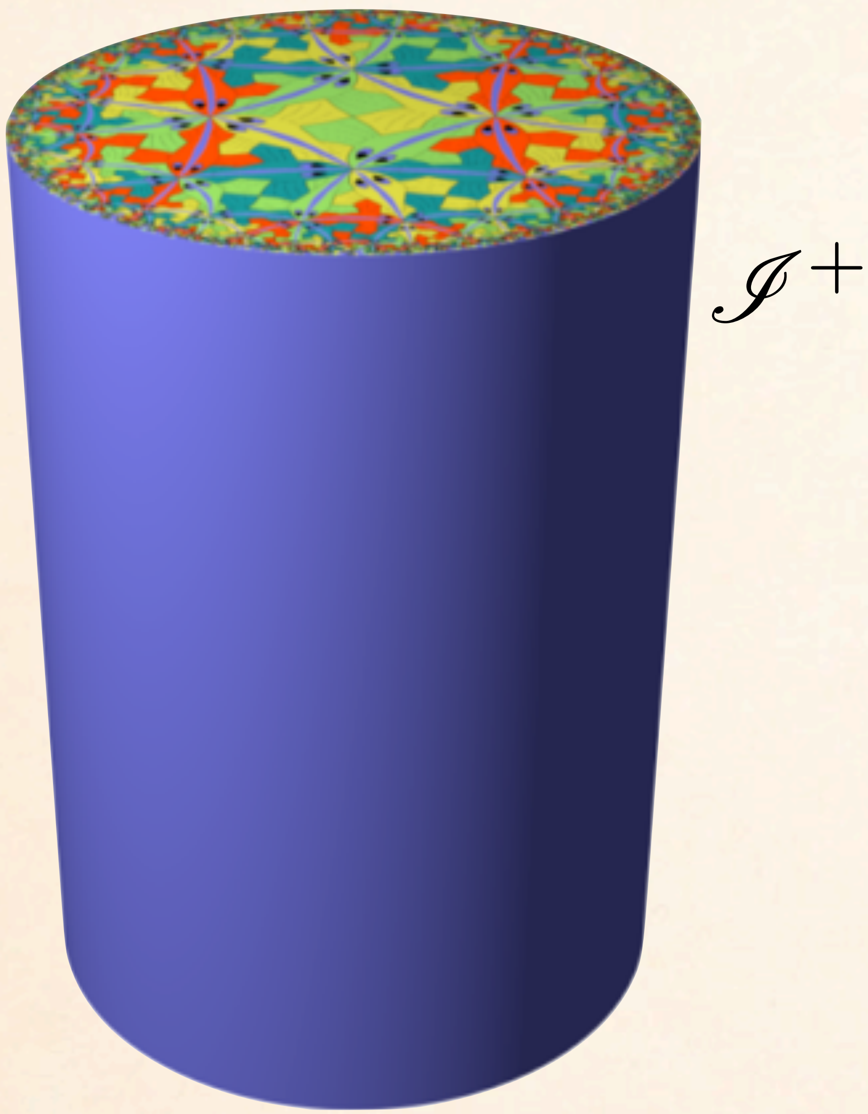
- `Russian doll' holography:

◆ What is the boundary dynamics of strongly interacting QFT + weak AdS gravity?

◆ Given a field theory, say $\mathcal{N} = 4$, what is the bulk dual when we consider this theory AdS_4?

- Better understanding of $\mathcal{N} = 4$ dynamics: rich story....

Global AdS spacetime



$$ds^2 = -\left(1 + \frac{r^2}{\ell^2}\right) dt^2 + \frac{dr^2}{\left(1 + \frac{r^2}{\ell^2}\right)} + r^2\, d\Omega^2$$

$$\partial(\mathrm{A}dS_d) = \mathbf{R} \times \mathbf{S}^{d-2}$$

Global AdS is a cylinder with a time-like boundary which is a copy of the Einstein Static Universe (Lorentzian cylinder).

Fields on AdS: Boundary conditions

- AdS is not globally hyperbolic, and has a timelike $\mathscr{I}^+$ $\Rightarrow$ boundary conditions are necessary to prescribe dynamics.

- For classical scalars in AdS_d satisfying Klein-Gordon equation with $m^2 l^2 = \Delta(\Delta - (d-1))$

$$\phi(r) \to A(x)\, r^{\Delta-(d-1)} + B(x)\, r^{-\Delta}$$

- Usual boundary conditions for $\Delta > d/2$:
  - fix $A(x)$ to be source for boundary operator
  - $B(x)$ is the vev of dual operator (response).

Fields on AdS: Boundary conditions

- It is however well known that in a special range of masses other boundary conditions are allowed.

$$m_{BF}^2 = -\frac{(d-1)^2}{4\,\ell^2}$$

- For $m^2{}_{BF} \leq m^2 \leq m^2{}_{BF} + 1$ can alternately fix B to be the source.
- Both fall-offs are normalizable in this range of masses.

- Nb: conformally coupled scalars always lie in this window.

$$m_c^2 = -\frac{d\,(d-2)}{4\,\ell^2}$$

Breitenlohner, Freedman

Gauge fields on AdS: Boundary conditions

- Vector fields can also admit different boundary conditions
- Vectors in AdS_4:

$$A_r = 0 \ , \qquad\qquad A_\mu(r,x) \to a_\mu(x) + \frac{b_\mu(x)}{r}$$

- Dirichlet/standard/electric boundary conditions: Witten
  - fix $a_\mu \Rightarrow$ source for boundary current $\mathcal{J}_\mu$ Ishibashi, Wald
- Neumann/modified/magnetic boundary conditions: Marolf, Ross
  - fix b_μ or integrate over all sources $A_\mu \Rightarrow \langle \mathcal{J}_\mu \rangle = 0$

- For abelian theories these boundary conditions are necessitated by electric-magnetic duality, which exchanges Dirichlet and Neumann.

Free gauge theories on AdS_4

- $SU(N)$ gauge theory with Dirichlet boundary conditions:
  - No Gauss law constraint
  - $O(N^2)$ excitations about the vacuum.

- $SU(N)$ gauge theory with Neumann boundary conditions:
  - No charged states allowed
  - $\exists$ Gauss law constraint
  - $O(1)$ excitations about the vacuum.
  - Theory undergoes a Hagedorn transition at $T_c l \sim 1$.

Spectrum of N = 4 SYM on AdS_4

- Field content (all adjoint valued):
  - 6 conformally coupled scalars
  - 4 Weyl fermions
  - Gauge fields

$$\omega\,\ell = \Delta + k + 2\,n\ , \qquad k, n \in \mathbb{Z}_+$$

- Lots of choices of boundary conditions:
  - scalars can have $\Delta = 1, 2$.
  - gauge fields can have Neumann/Dirichlet bc.
  - fermions have $\Delta = 3/2$.

Free theory partition functions

- Define the single particle partition sum

$$z(x) = \sum_{\text{single-particle states}} \sum_{\omega} e^{-\beta\,\omega} \,, \qquad x \equiv e^{-\beta\,\ell_4^{-1}}$$

- For $\mathcal{N} = 4$ SYM one has using the spectral data:

$$z^{\pm}_{\text{scalar}}(x) = x^{\Delta_\pm} \sum_{n,k=0}^{\infty} (2\,k+1)\, x^{2\,n+k} = \frac{x^{\Delta_\pm}}{(1-x)^3} \,, \qquad \Delta_\pm = 1,2$$

$$z_{\text{fermions}} = 2\,x^{\frac{3}{2}} \sum_{n,k=0}^{\infty} (2\,k+1)\, x^{2\,n+k} = 2\,\frac{x^{\frac{3}{2}}}{(1-x)^3}$$

$$z_{\text{gauge}}(x) = \sum_{k=1}^{\infty} \sum_{n=0}^{\infty} (2\,k+1)\,\left(x^2+x\right)\, x^{2\,n+k} = \frac{3\,x^2 - x^3}{(1-x)^3}$$

$$z_B(x) = \alpha\, z^{+}_{\text{scalar}}(x) + (6-\alpha)\, z^{-}_{\text{scalar}}(x) + z_{\text{gauge}}(x) \,, \qquad z_F(x) = 4\, z_{\text{fermions}}(x)$$

Partition functions for the simple bcs

- *Dirichlet bcs*: account for multi-particle states & statistics.

$$Z_{\text{boson}}(x) = \exp\left(\sum_{p=1}^{\infty} \frac{1}{p}\, z_{\text{boson}}(x^p)\right), \qquad Z_{\text{fermion}}(x) = \exp\left(\sum_{p=1}^{\infty} \frac{(-1)^{p+1}}{p}\, z_{\text{fermions}}(x^p)\right)$$

$$Z_{\mathcal{N}=4}(x) = \left[\left(Z^+_{\text{scalar}}(x)\right)^{\alpha}\, \left(Z^-_{\text{scalar}}(x)\right)^{6-\alpha}\, Z_{\text{gauge}}(x)\, Z^4_{\text{fermion}}(x)\right]^{\dim(\mathfrak{g})}$$

$$F = -15\,\zeta(4)\,T^4\,\ell_4^3\,\dim(\mathfrak{g})$$

- *Neumann bcs*: account for Gauss Law constraint & statistics.

$$Z(x) = \int [DU]\, \exp\left(\sum_{m=1}^{\infty} \frac{1}{m}\, \left[z_B(x^m) + (-1)^{m+1}\, z_F(x^m)\right]\, \chi^G_{\text{adj}}(U^m)\right)$$

Theory undergoes Hagedorn transition at:

$$T_\star = -\frac{1}{\ell_4\, \log(x_\star)}\,, \quad \text{with} \qquad z(x_\star) = z_B(x_\star) + z_F(x_\star) = 1$$

$\mathcal{N} = 4$ SYM on AdS_4 : Two puzzles

- $\mathcal{N} = 4$ has *SL(2,Z)* S-duality.
- Expect exchange Dirichlet and Neumann bcs (true for abelian theory), *but*
  - Dirichlet: $O(N^2)$ excitations about the vacuum
  - Neumann: $O(1)$ excitations about the vacuum
- Phase transition as a function of coupling?

- Expect to have holographic dual in AdS_5
  - how does bulk dual have N^2 dofs?
  - New asymptotically AdS spacetimes with degenerate horizons?

Susy bc for N = 4 SYM on AdS_4

- Useful to look at susy preserving bcs for $\mathcal{N} = 4$

Have to treat scalars asymmetrically: $SO(6) \rightarrow SO(3) \times SO(3)$

Breitenlohner, Freedman

In fact the theory has a plethora of $1/2$-BPS boundary conditions.

- Complete classification exists for theory on half-space $R^{2,1} \times R_+$.

Gaiotto, Witten

- Adapt these boundary conditions on AdS_4 as it is conformal to the half-space.
- Rich dynamics enabled by choice of bcs.

Abstract discussion of susy bc

- Gaiotto-Witten bcs are characterized by a triple (ϱ, H, B) for gauge group G.
  - ϱ (Nahm data): $SU(2) \rightarrow G$
  - H: commutant of Nahm data (preserved gauge group)
  - B: Boundary CFT living on $\boldsymbol{R} \; X \; \boldsymbol{S^2}$.

- Simpler understanding of boundary conditions in terms of D-branes for unitary G.

Bcs via brane constructions

- *Neumann bc:* N D3-branes ending on a single NS5-brane.
  - Gauge fields and 3 scalars: Neumann bc
  - Radial component + remaining 3 scalars: Dirichlet bc

- *Dirichlet bc:* N D3-branes ending on a N D5-branes.
  - Gauge fields and 3 scalars: Dirichlet bc
  - Radial component + remaining 3 scalars: generalized Neumann

$$\frac{dX^i}{dz} + \epsilon^{ijk}\left[X_j, X_k\right] = 0$$

  - each D3-brane ends on a D5-brane. SU(N) global symmetry comes from the D5 brane gauge fields.

S-duality for N=4 with boundary

Dirichlet bcs have explicit D5-brane sources and the N D5-branes actually realize the *SU(N)* global symmetry.

Can have D3 branes ending on a single D5-brane, but choice of solution to Nahm's equation breaks global symmetry completely. This is obtained by the *N*-dimensional irrep of *SU(2)*.

- S-dual of Dirichlet bc: Neumann bc coupled to a boundary CFT
- S-dual of Neumann bc: Nahm poles breaking G to trivial subgroup.

$$X^i \sim \frac{t^i}{z} \ , \qquad t^i \in \varrho \ , \quad \varrho : SU(2) \to G$$

Holographic duals for Simple bcs?

Boundary condition	**Realization**	**Dual boundary condition**
Neumann for $SU(N)$	N D3-branes ending on a NS5-brane	Dirichlet with Nahm pole $\rho = \mathbf{N}$
Dirichlet for $SU(N)$	D3-branes ending on N D5-branes	$(3+1)$-dim $\mathcal{N}=4$ SYM coupled to $(2+1)$-dim $T(SU(N))$ quiver

- What about holographic duals for these boundary conditions?
- Dirichlet: perhaps impossible to find weakly coupled gravity dual since we require to have large number of D5 sources.
- Neumann: potentially no obstruction, but no solution with desired properties is known.

Holographic Duals of the Simple bcs

- Dirichlet case:

- The D5s are expected to play an important role.
- Necessary for global symmetry $\Rightarrow$ can't replace them by their effective geometry (which would gauge the symmetry).
- They provide a place for the D3s to end $\Rightarrow$ effect will be strong near AdS boundary.
- S-dual involves complicated boundary CFT.

- Neumann case:

- NS5 has $O(1)$ backreaction in the sugra limit.
- S-dual is a single D5 $\Rightarrow$ Nahm data involving N-dim irrep of $SU(2)$.
- The W-bosons are all massive with $m \sim \sqrt{N}$. Gravity dual?

Summary: Strong coupling limits of simple bc

- The simplest set of susy bcs are disappointing.
- Hard to see how to construct appropriate duals in the Dirichlet case and the Neumann bcs seem just a bit out of reach.

- Main message: holographic duals possible if we have a boundary gauge symmetry of sufficient rank (scales with N).

- Strategy: Engineer new classes of boundary conditions that allow exploring the planar strong coupling limit.
- Preserve a large amount of boundary gauge symmetry.

- Lead us to study "quotient constructions".

- Alternately one can work with "transparent bcs". Hubeny, Marolf, MR

N = 4 SYM on AdS_4 : Quotients

- A-priori we can pick bcs that break G to subgroup H.
- Essentially these are Neumann for some subgroup.

- Obtain via a quotient:
- Mod out the theory on $\boldsymbol{R}^{3,1}$ by $\boldsymbol{Z}_2$ which acts as spacetime reflection together with an involution on G.

- Can get large class of such examples whose S-duals are also known.
- Focus on unitary G for simplicity.

Involution	**Quotient**	**Gauge group** $H \subset G$
Class I	$O5^+$	$SO(N)$
Class II	$O5^-$	$USp(N)$
Class III	$\mathcal{I}_4(-1)^{F_L}$	$SU(p) \times SU(N-p)$

$\mathcal{N} = 4$ SYM on AdS_4: Quotients

Gauge group H	**Quotient**	**Dual gauge group** $\widetilde{H}$	**Nahm data for dual**	**Boundary data (dual)**
$SU(\frac{N}{2}) \times SU(\frac{N}{2})$	$\mathcal{I}_4(-1)^{F_L}$	$USp(N)$	$\rho = N \times \mathbf{1}$	D5-brane localized on O-plane
$SU(p) \times SU(q)$	$\mathcal{I}_4(-1)^{F_L}$	$USp(2q)$	$\rho = (\mathbf{p}-\mathbf{q}) \oplus 2q \times \mathbf{1}$	Symmetry breaking Neumann bc
$SO(N)$	$O5^+$	$SU(N)$	$\rho = N \times \mathbf{1}$	Non-trivial $2+1$ SCFT on boundary
$USp(N)$	$O5^-$	$SU\left(\frac{N}{2}\right)_d \subset SU(N)$	$\rho = \frac{N}{2} \times \mathbf{2}$	None

Quotient theories at weak coupling

- All of the above examples have the essential features of Neumann bc:
- *O(1)* states at low temperature due to singlet constraint
- Hagedorn transition at some $T_c\, l \sim 1$.
- Curiously in all cases the Hagedorn transition occurs at a lower value of the temperature.

$$T_\star = -\frac{1}{\ell_4 \log(x_\star)} \text{, with} \qquad z(x_\star) = z_B(x_\star) + z_F(x_\star) = \frac{1}{2}$$

- Rationale: non-trivial matter (under $H \subset G$) implies that there are multiple ways to form singlets. More choices $\Rightarrow$ faster growth of operators.

Holographic duals of N = 4 SYM on AdS_4

- Expect holographic duals in terms of strings in AdS_5 X $\boldsymbol{S}^5$
- AdS_4 foliations AdS_5?
  - boundary is a double cover of AdS_4
  - identify two copies of AdS_4 via an orbifold/orientifold action.

$$ds^2 = dR^2 + \frac{L_5^2}{\ell_4^2}\cosh^2\left(\frac{R}{L_5}\right)\gamma_{\mu\nu}\,dx^\mu\,dx^\nu$$

- The boundaries are located at large positive and negative values of R respectively.
- Two copies of AdS_4 are joined across the equator of the $\boldsymbol{S}^3$.

Holographic duals of $N = 4$ SYM on AdS_4

- In fact, the quotient actions we saw above are precisely of the form we need to identify the two AdS boundaries.
- We identify the two AdS_4 factors and also act with a reflection on the S^5 to ensure that we have good string background.

- GW bcs with orbifolds/orientifolds preserving large enough gauge symmetry have duals in terms of strings on $AdS_5 \times S^5$ with orbifold/orientifold 5-plane wrapping $AdS_4 \times S^2$.

Strong coupling phase structure

- All static, spherically symmetric AdS_5 geometries can be rewritten in a form which makes the boundary AdS_4 manifest.

$$ds^2 = \frac{\bar\rho^2}{L_{d+1}^2\,(1+\frac{r^2}{\ell_d^2})}\left(-\frac{f(\bar\rho)}{\bar\rho^2}\,L_{d+1}^2\,\left(1+\frac{r^2}{\ell_d^2}\right)\,dt^2 + \frac{dr^2}{(1+\frac{r^2}{\ell_d^2})} + r^2\,d\Omega_{d-2}^2\right) + \frac{\ell_d^2}{L_{d+1}^2}\,\frac{d\bar\rho^2}{f(\bar\rho)}$$

- Phase structure tracks through to strong coupling regime.
  - low temperature phase is the thermal gas.
  - high temperature phases is the Schwarzschild-AdS_5 black hole.

$$T_c = \frac{\sqrt{2}}{\pi\,\ell_4}$$

- Transition temperature appears to be independent of involution.

Summary & Open issues

- Rich dynamics for field theories on AdS_4.
- Lots of the structure of the dynamics is due to the non-trivial nature of the boundary conditions.
- Possibility of various different bcs for scalars, vectors, fermions.

- The story for $\mathcal{N} = 4$ is very intricate.
- Discussed only susy (1/2-BPS) bcs, for which already there many interesting issues.
- Could look for duals for other bcs with less/no susy.
- Other CFTs; different dimensions?