Probing Holographic Superfluids with Solitons

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GGI Workshop on AdS4/CFT3 and the Holographic States of Matter

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0912.4280

AdS / CM

- There has been much recent effort to construct strongly coupled phases of matter using the AdS/CFT correspondence
  - superfluidity
  - superconductivity
- Really want to push on these systems and say how like/unlike known systems
  - probe calculations might be telling you about the probe, not the ground state
- Want an observable which is more intrinsic to the superfluid
  - use solitons

Outline

- Holographic Superfluids
- Holographic Solitons
    - construct “dark solitons” and vortices
- Lessons From Real World Solitons
    - suggests how organize our exploration of holographic solutions
- Outlook

Holographic Superfluids

- Many recent attempts to use gauge/gravity duality to study 2+1 dim. condensed matter system
- Focus on spontaneous breaking of an Abelian symmetry

Hartnoll, et. al. [0803.3295 and 0810.1563]

- AdS_4 Einstein-Maxwell-Higgs

$$\sqrt{-g}\mathcal{L} \sim \sqrt{-g}\left((R-12\Lambda)/2 - \frac{\kappa_4^2}{q^2}\left(F^2/4 + |D\Psi|^2 + m^2|\Psi|^2\right)\right)$$

  - probe limit: $\frac{\kappa_4}{q} \to 0$ (Schw-AdS b.h.)

$$ds^2 = L^2\left(-\frac{f dt^2}{z^2} + \frac{dz^2}{f z^2} + \frac{d\vec{x}^2}{z^2}\right), \;\; f(z) = 1 - z^3$$

- Begin with $m^2 = -2/L^2$, gauge fixed e.o.m.

$$fR'' + f'R' - zR + \partial_x^2 R + \frac{A_t^2}{f} R = 0$$
$$fA_t'' + \partial_x^2 A_t - R^2 A_t = 0$$

- use rescaled fields $R \equiv \sqrt{2}\frac{\Psi}{z}$

- asymptotic solution:

$$R \sim R^{(1)}(x) + zR^{(2)}(x) + ...$$
$$A_t \sim A_t^{(0)}(x) + zA_t^{(1)}(x) + ...$$

- for this bulk mass we may use Neumann or Dirichlet boundary conditions:

$$R^{(1)} \sim \langle\mathcal{O}_1\rangle, \quad R^{(2)} = 0 \qquad \text{or} \qquad R^{(1)} = 0, \quad R^{(2)} \sim \langle\mathcal{O}_2\rangle$$
$$A_t^{(0)} \sim \mu, \quad A_t^{(1)} \sim \rho_{\text{tot}} \qquad\qquad A_t^{(0)} \sim \mu, \quad A_t^{(1)} \sim \rho_{\text{tot}}$$

- both boundary conditions display spontaneous symmetry breaking (at fixed temperature) for $\mu \geq \mu_c$
  - charged operator has an expectation value as all charged external sources are removed
- can study how the order parameter behaves near μ_c

$$\langle \mathcal{O}_i \rangle \Big|_{\mu \to \mu_c} \sim \sqrt{\frac{\mu}{\mu_c} - 1}$$

- can use linear response to study sound modes, probe fermions,...

- Can we find an observable outside linear response theory which know about the some of the short distance physics?
  - a soliton's cores carry information about the short distance physics even at the mean field level

Holographic Dark Solitons

- Try to find "dark soliton" solutions to gravity equations

$$
\begin{aligned}
fR'' + f'R' - zR + \partial_x^2 R + \frac{A_t^2}{f} R = 0 \\
fA_t'' + \partial_x^2 A_t - R^2 A_t = 0
\end{aligned}
$$

  - want kink scalar field profiles external to black hole
  - cannot be stable
    - we will disallow any dependence on the coordinate along the soliton
      - can add this dependence later and study the soliton's decay
      - justified because real world kink solitons can be long lived

Aside on Numerical Methods

- Outside the horizon, the equations of motion are elliptic
  - like Poisson equation

$$-\nabla^2\phi_0 = \rho$$

- "Relaxation": extend to a diffusion equation with new variable t $\in$ [0,∞)

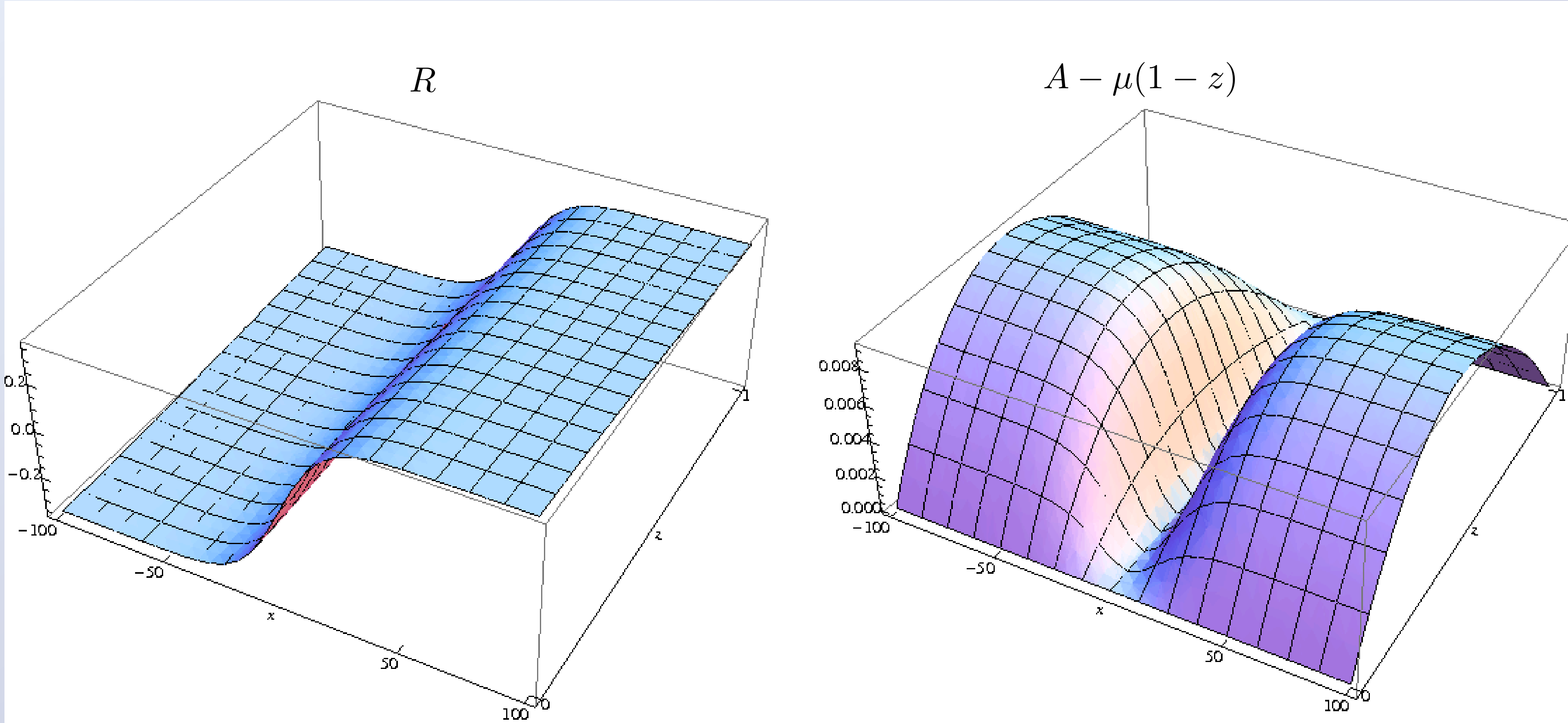
$$(\partial_t - \nabla^2)\phi(t) = \rho$$

  - now have a simple initial value problem: there is a flow to solution of Poisson equation
    - the error decreases monotonically in time

$$\mathcal{E}(t) \sim \phi_0 - \phi(t) \sim e^{-tk^2}$$

  - the "relaxation" scheme may be implemented on a lattice
  - topological information is contained in flow's initial conditions

Holographic Dark Solitons



$< O_1 >$ condensate

- Soliton for lower dimension scalar has a greater core depletion of the charge density
    - The fact that the depletion is not going to zero means that one cannot simply use a simple Landau-Ginzberg description

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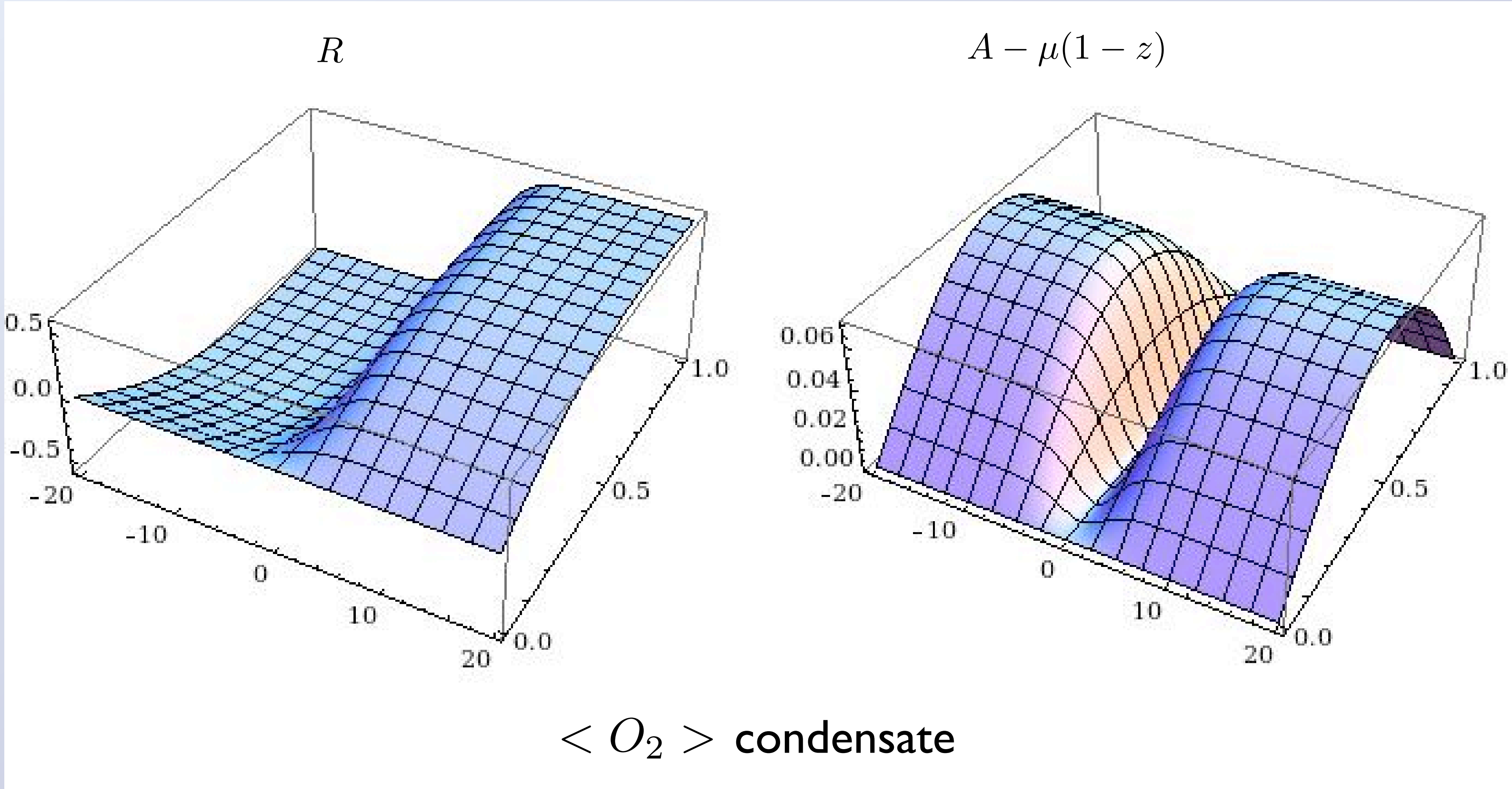

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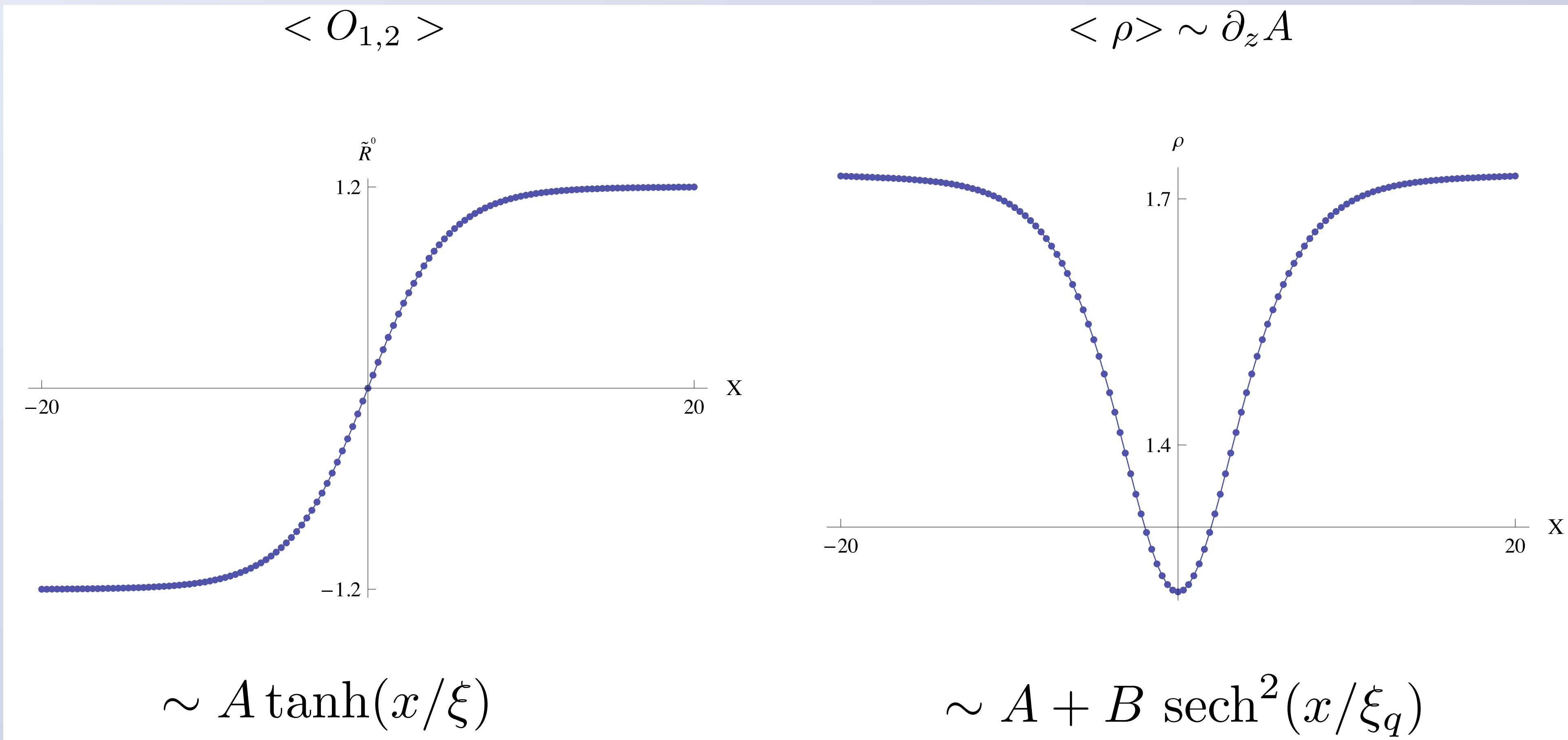

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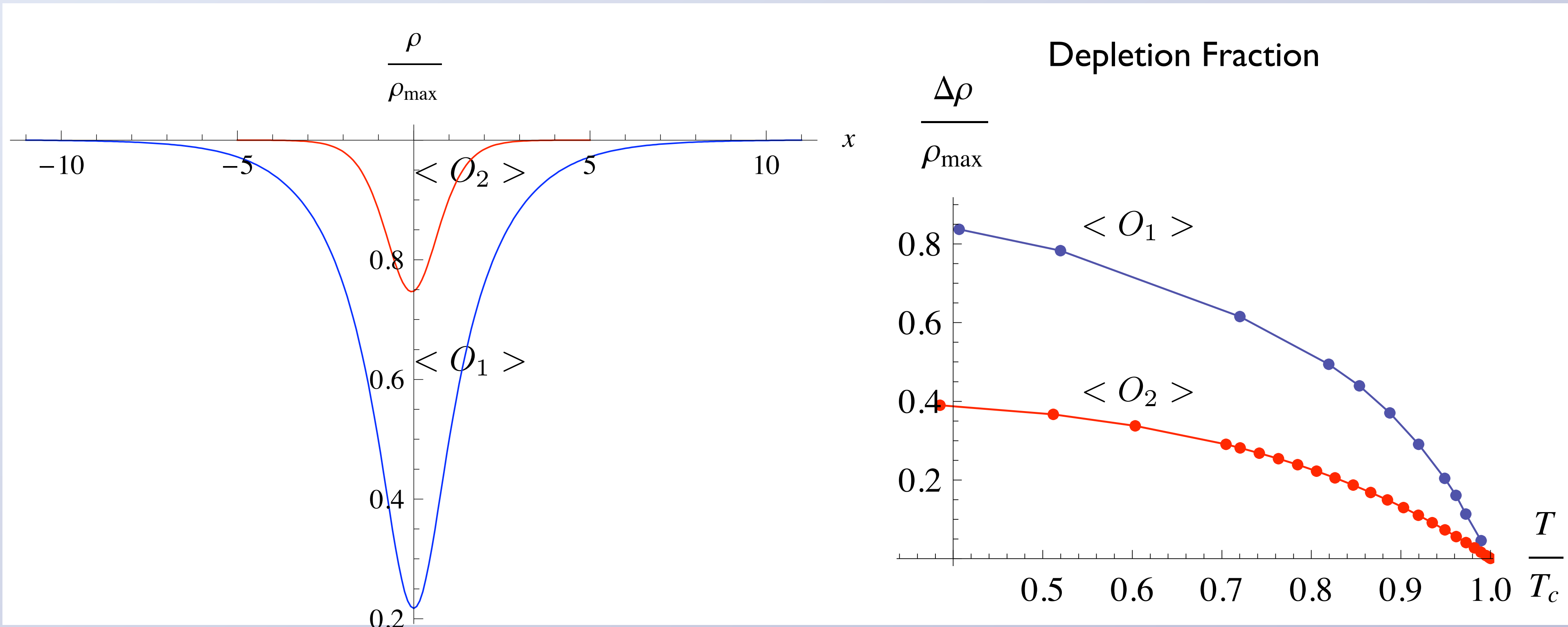

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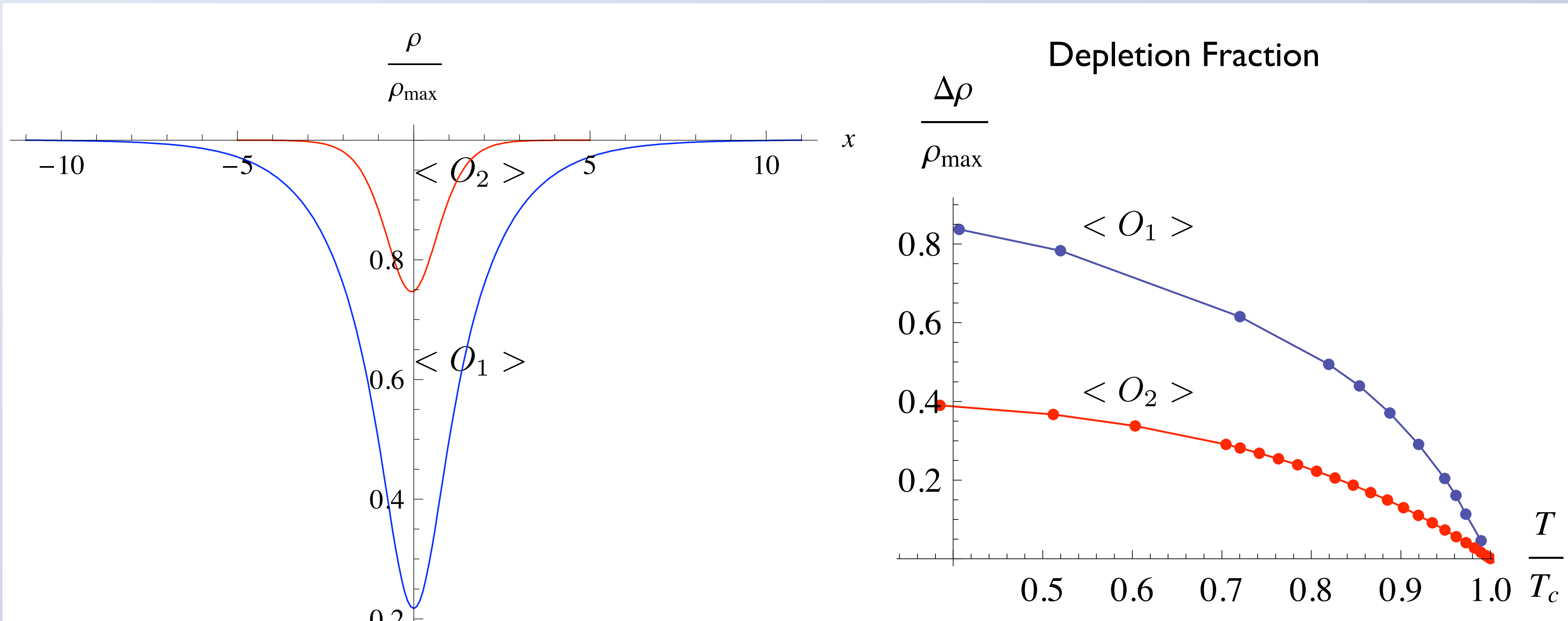

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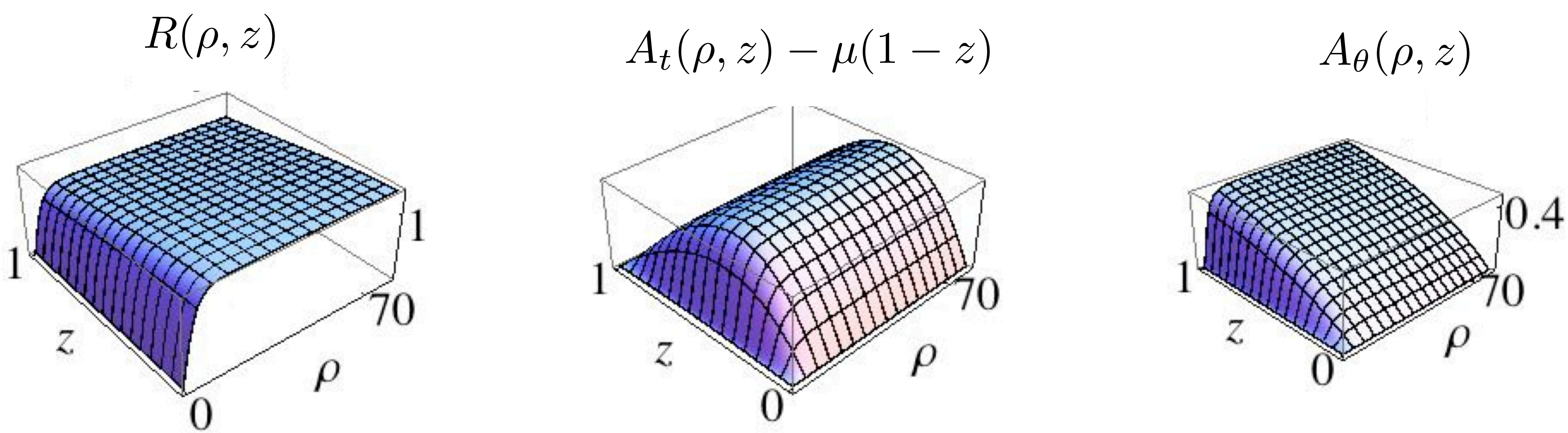
Holographic Vortices

- Seek vortex solutions which carry nontrivial winding
  - must include angular component of gauge field $A_\theta, \quad (\overbrace{\longrightarrow}^{z\to 0} 0)$
    - dual to the superfluid current density $\langle j_s \rangle$
    - now have 3 coupled nonlinear PDE's

$$
\begin{aligned}
0 &= f\partial_z^2 R + \partial_z f \partial_z R - zR + \frac{1}{\rho}\partial_\rho(\rho\partial_\rho R) - R(-\frac{1}{f}A_t^2 + \frac{(A_\theta - n)^2}{\rho^2}) \\
0 &= f\partial_z^2 A_t + \frac{1}{\rho}\partial_\rho(\rho\partial_\rho A_t) - R^2 A_t \\
0 &= \partial_z(f\partial_z A_\theta) + \rho\partial_\rho(\frac{1}{\rho}\partial_\rho A_\theta) - R^2(A_\theta - n)
\end{aligned}
$$

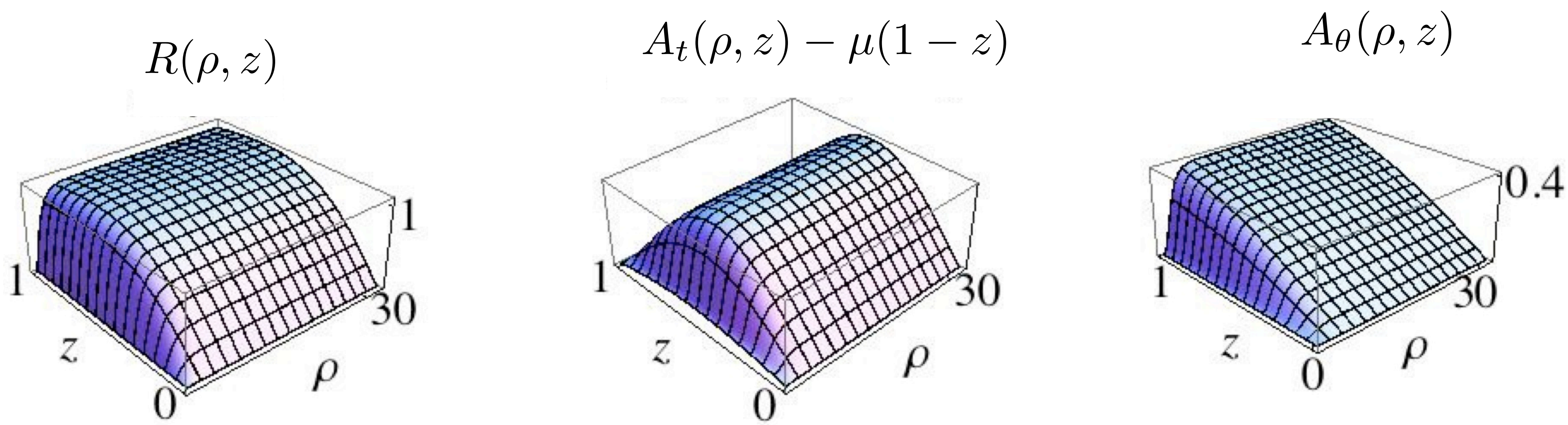
Holographic Vortices



$< O_1 >$ condensate

- Same pattern of charge depletion fractions

Holographic Vortices



$< O_2 >$ condensate

- Same pattern of charge depletion fractions

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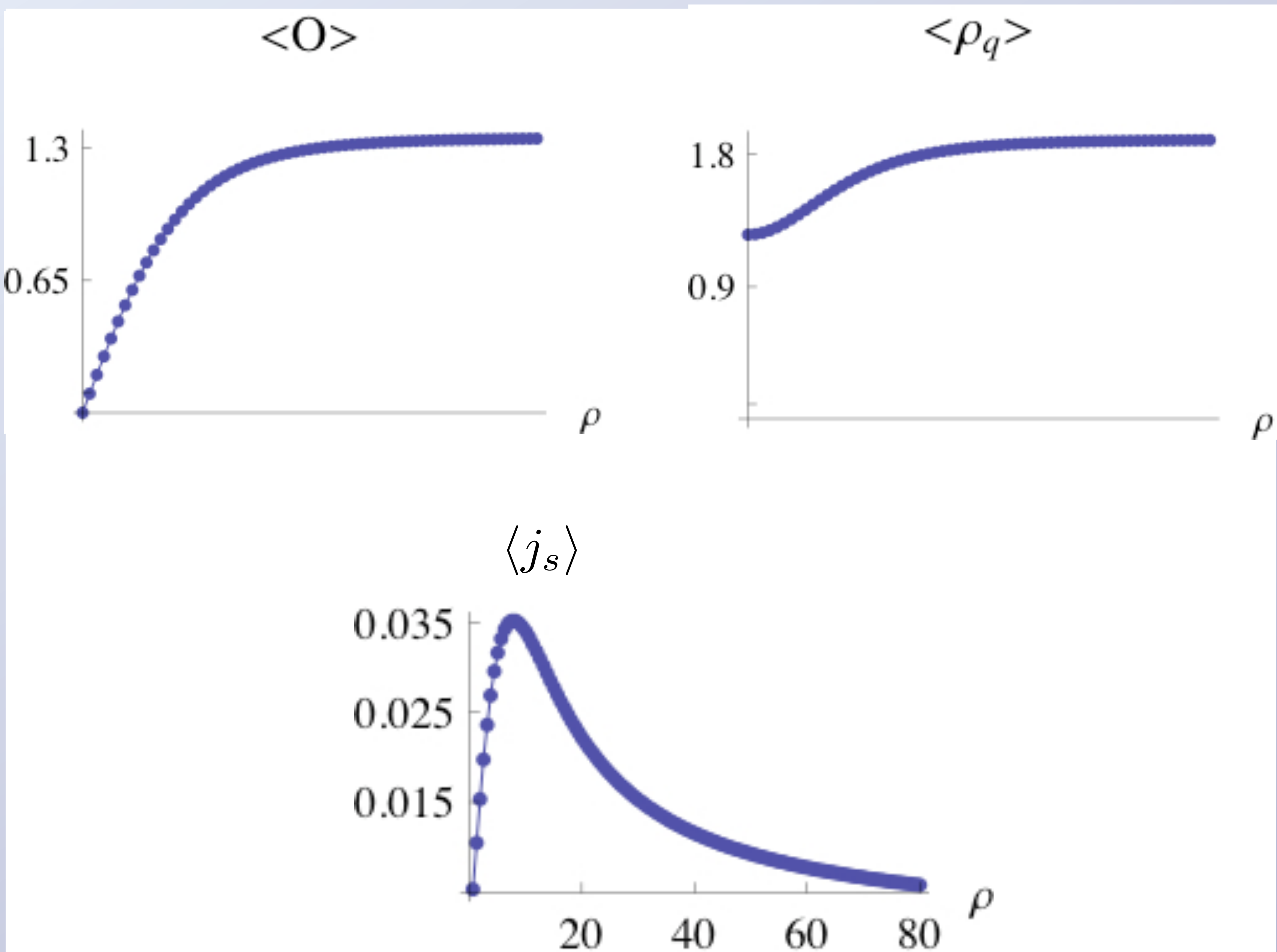

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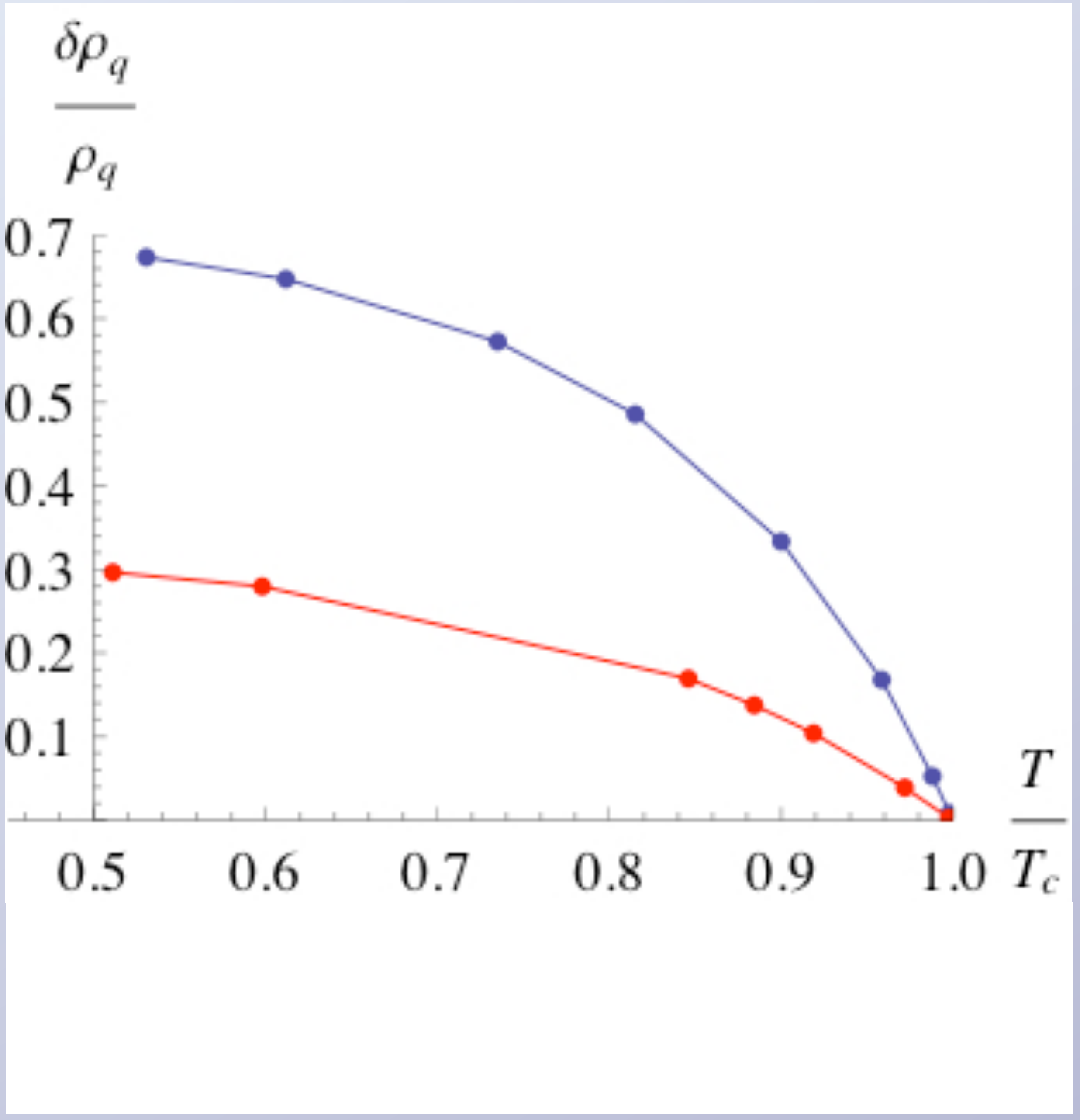

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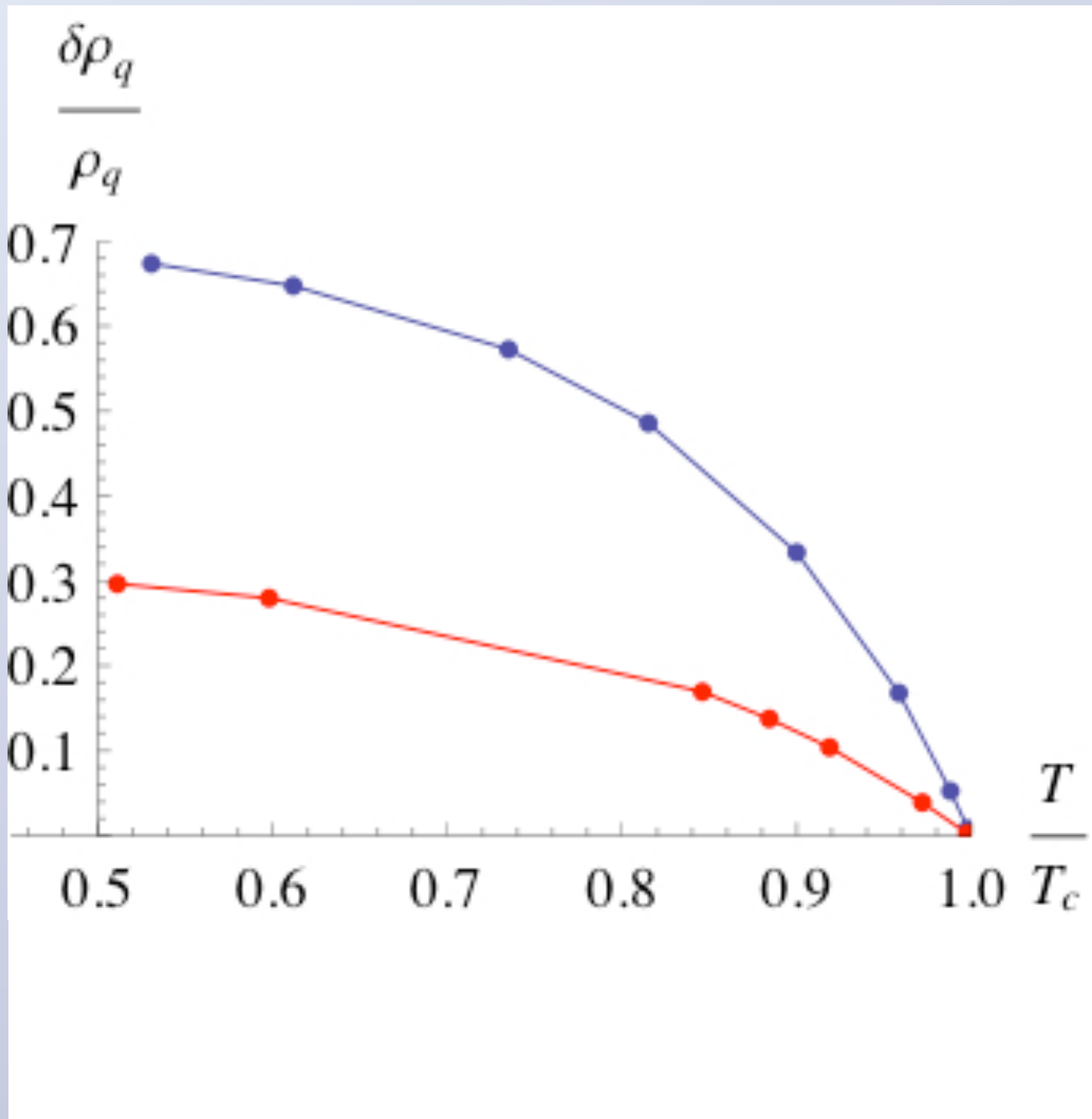

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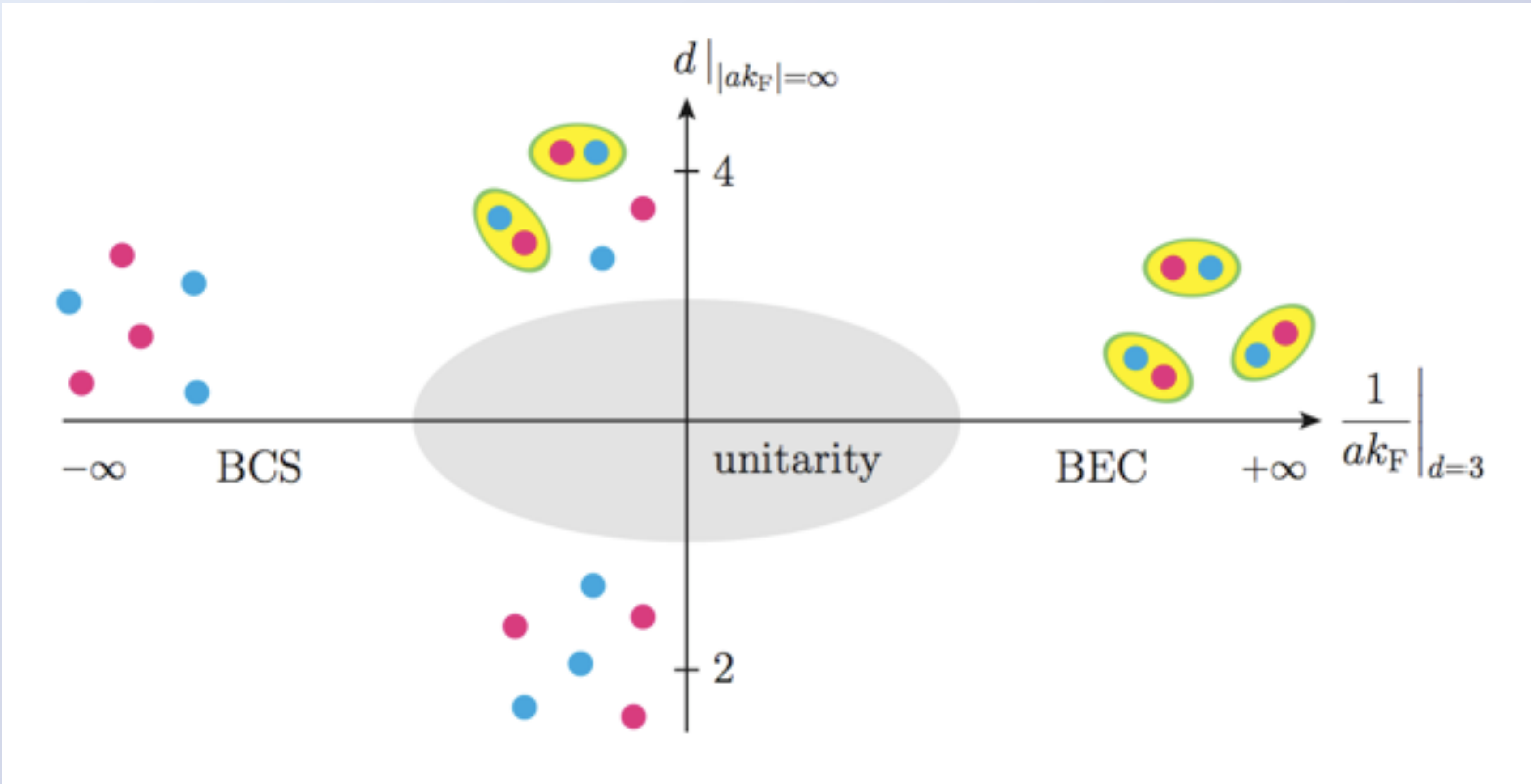
Lessons From (non-Rel.) Fermionic Superfluids



(from Nishida and Son [1004.3597])

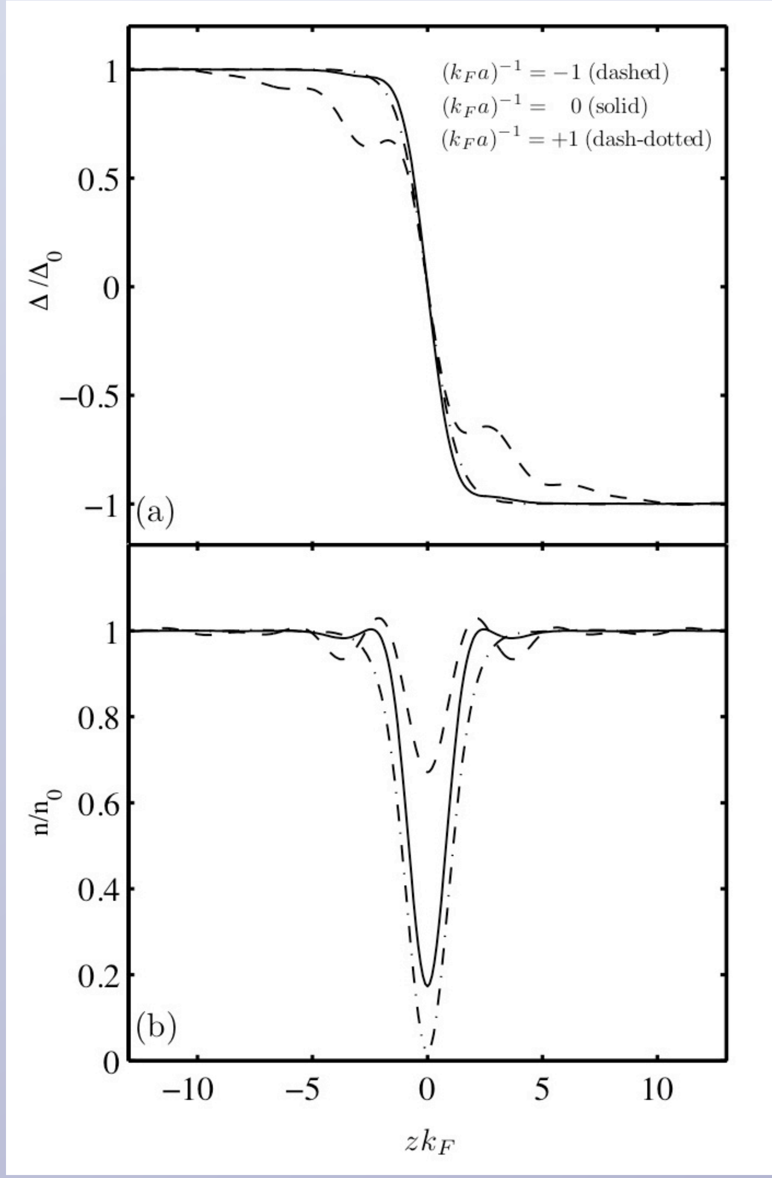
- modeled with a non-relativistic 4-Fermi interaction
  - may study in ε expansion without keeping the dimensionful scale ($ak_f \Rightarrow \infty$)
    - in this expansion we are tuning the scaling dimensions of fields
  - solitons in these systems remember the microscopic structure even at the mean field level

Solitons in the Crossover

- Non-Relativistic BCS - BEC crossover
  - display "dark soliton" solutions
    - unstable -- but observable
    - variable depletion fraction, bos. superfluid is near 100%, ferm. superfluid is much smaller
    - has oscillations which know about k_f on the BCS side

Dark Solitons

Soliton solution to gap equation

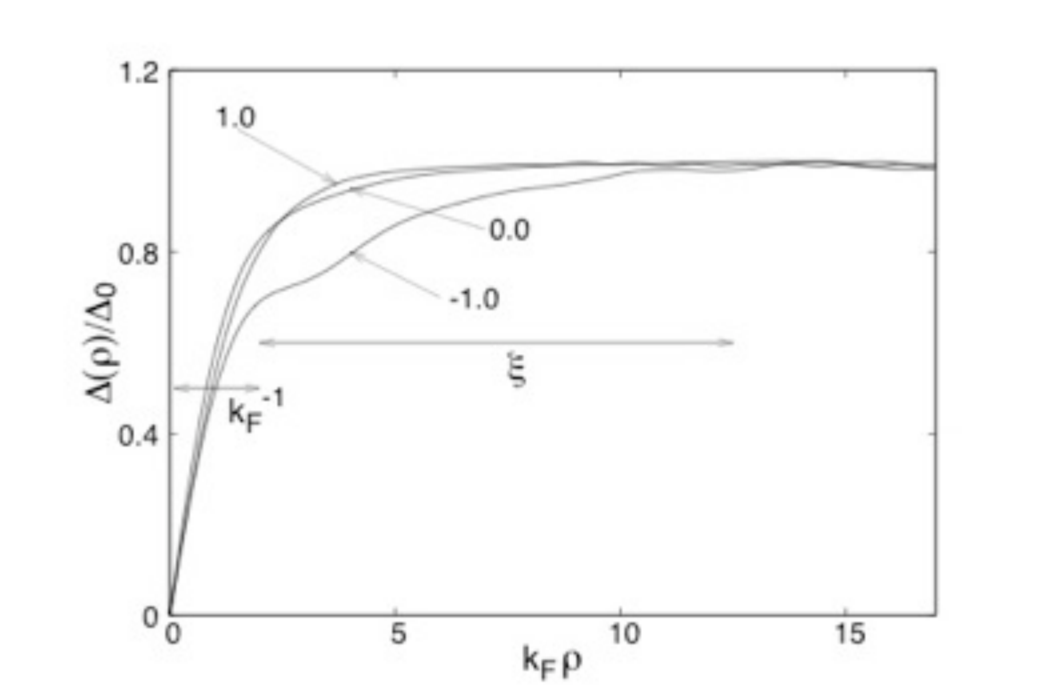


Dark soliton: BEC (dot-dashed), unitarity (cont.), and BCS (dashed) (from Antezza, et. al. [0706.0601])

Solitons in the Crossover

- Non-Relativistic BCS - BEC crossover
  - display "dark soliton" solutions
  - display vortex solutions
    - variable depletion fraction
    - same core and tail scales in BEC, different for BCS
    - also knows about k_f on BCS side

Vortices



Vortex Profiles: BEC ("1") and BCS ("-1")
(from Sensarma, et. al. [cond-mat/0510761])

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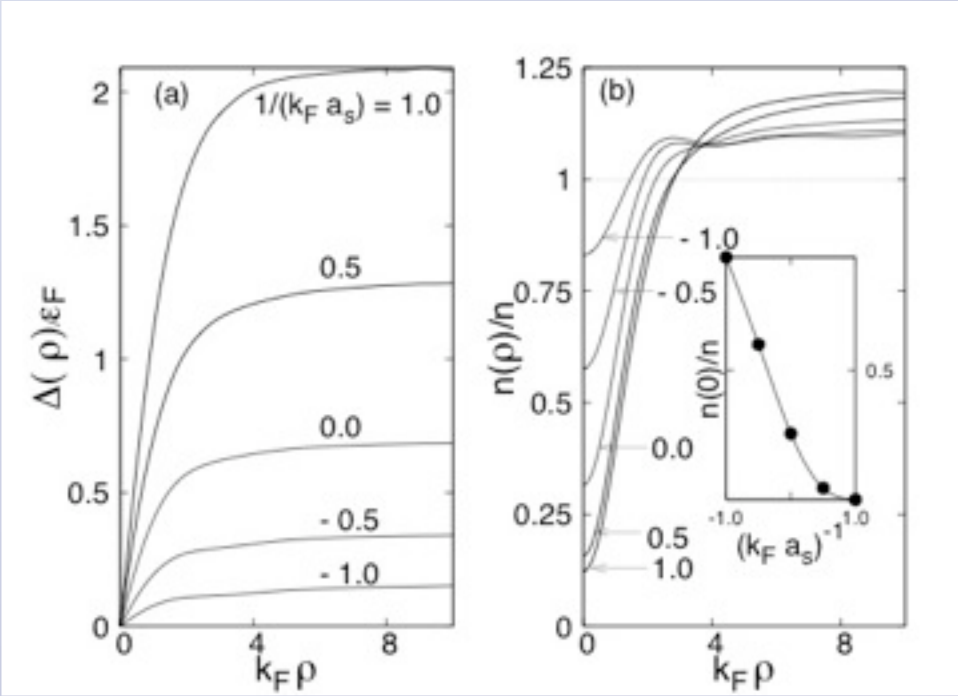

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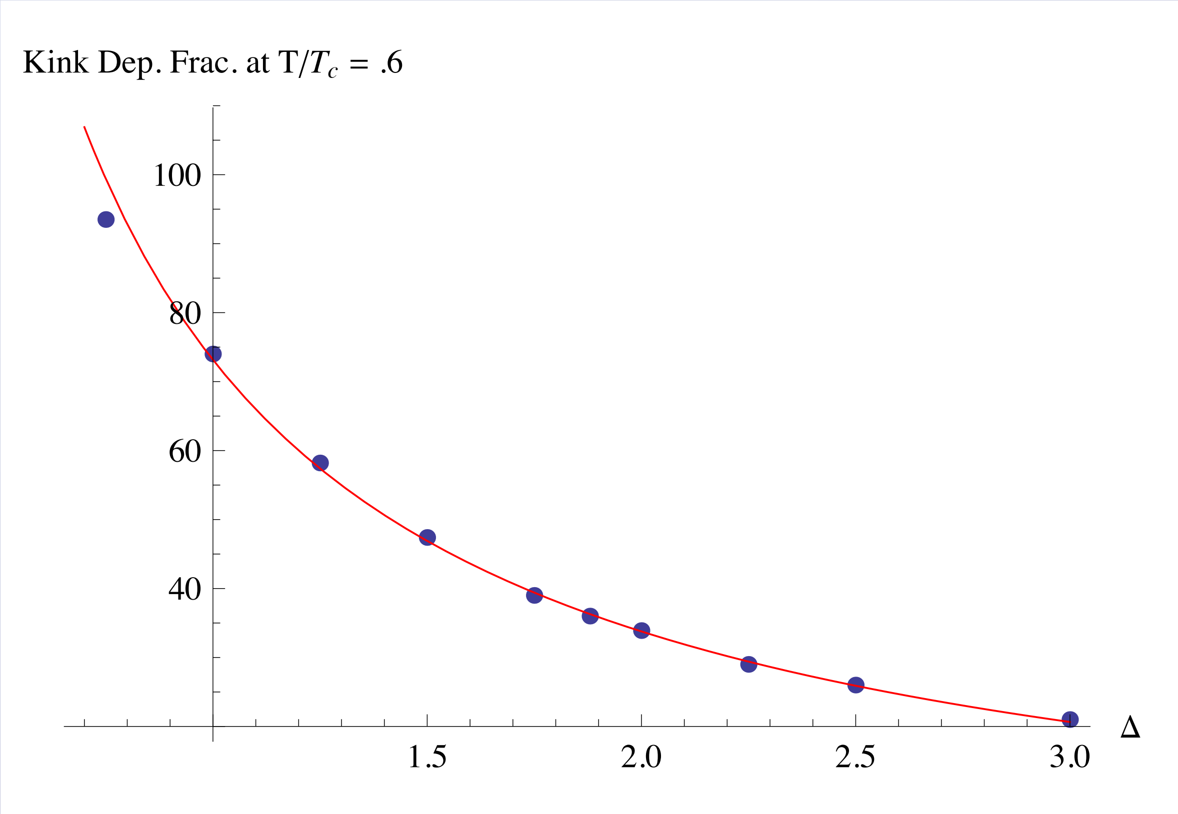
Things To Look For

- Suggests that we should study solitons as a function of the scaling dimension
- Number of independent lengthscales
- Friedel oscillations

Comparing Holographic Solitons

- Vary scaling dimension
  - monotonic dependence in charge density depletion on Δ
  - Multiple lengthscales in condensate (vortex)
    - temp. dependence ratio of lengthscales changes with Δ
  - Friedel oscillations
    - would give a measure of k_f without using a fermion probe
    - but.......



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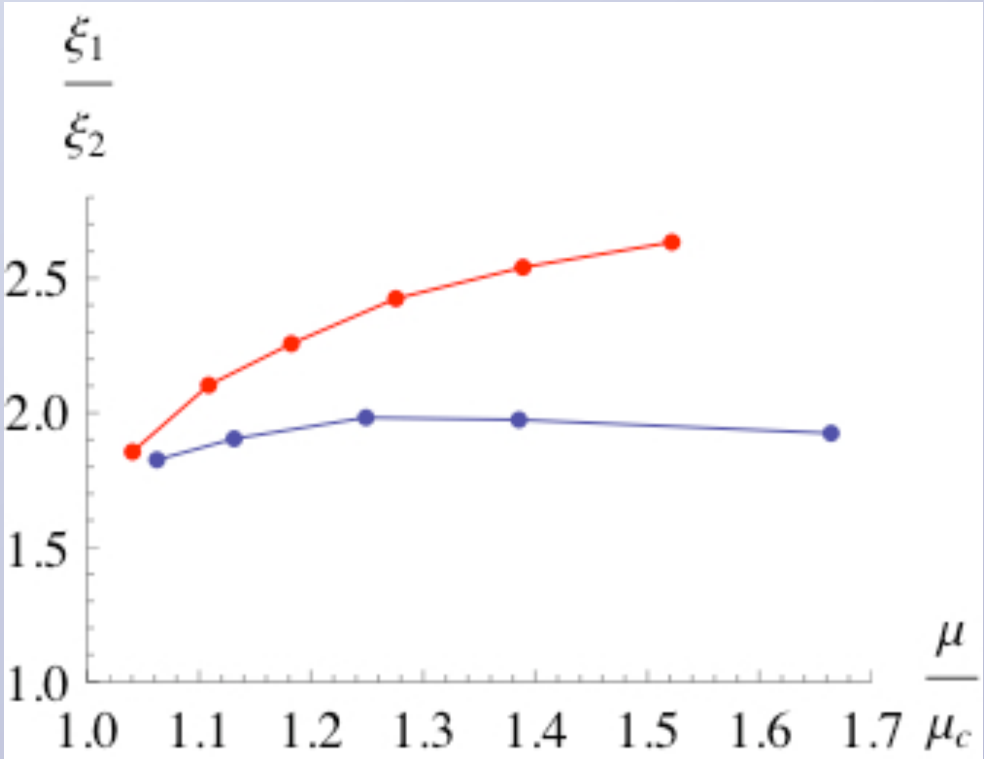

Ratio of core to tail length scales: $< O_1 >$ in blue and $< O_2 >$ in Red.

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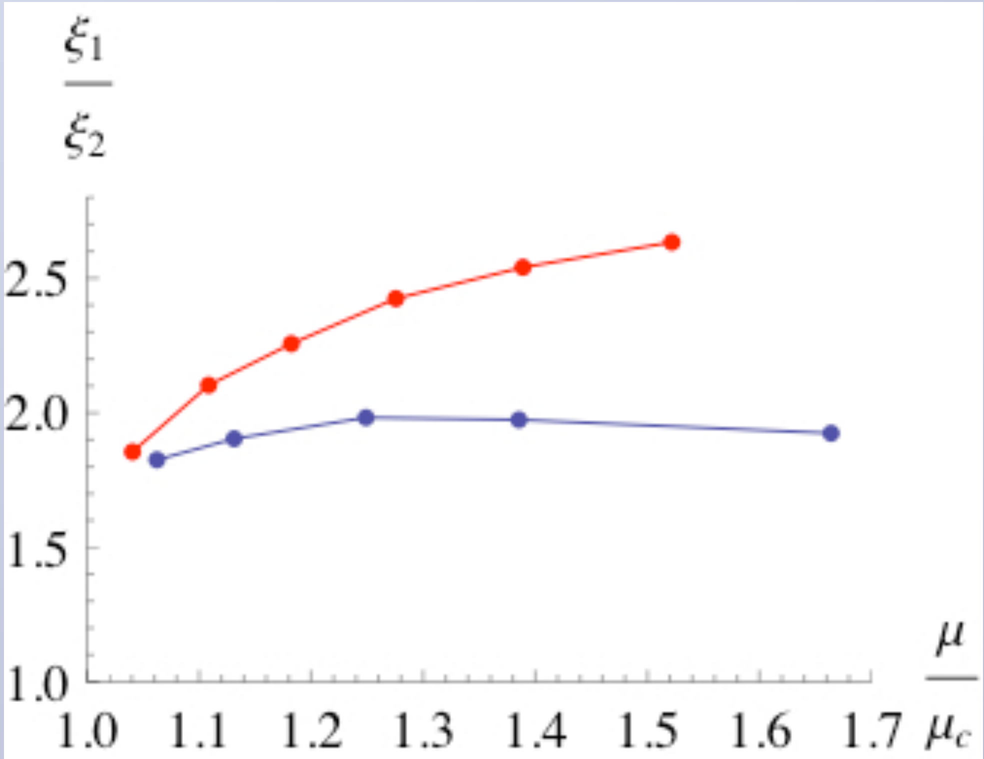

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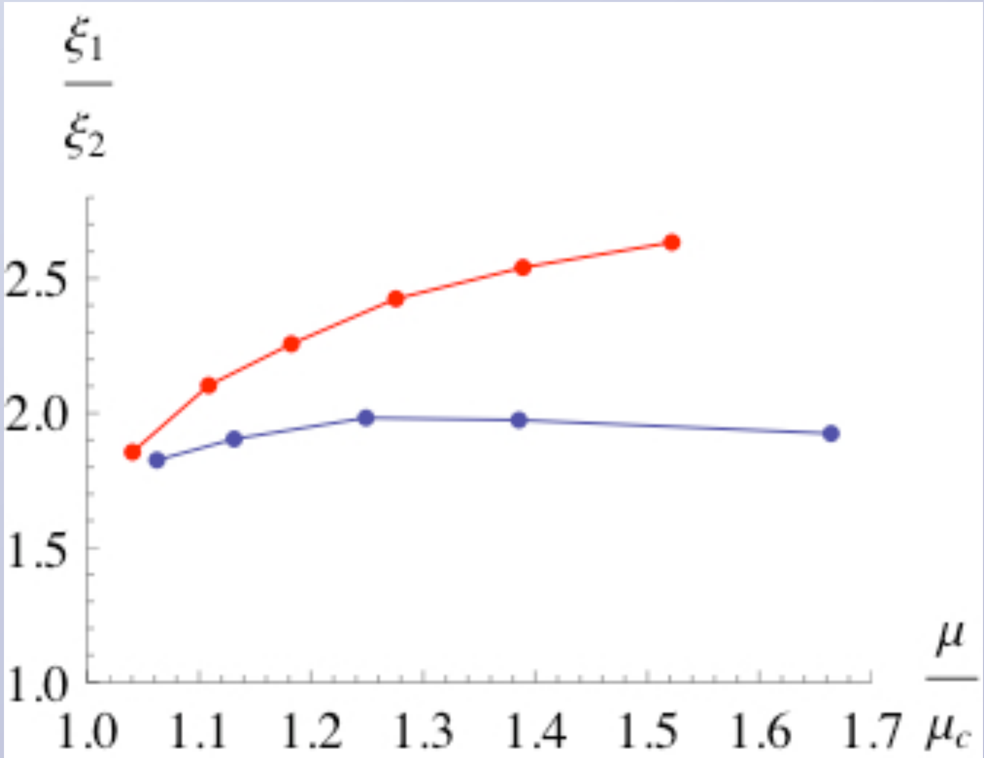

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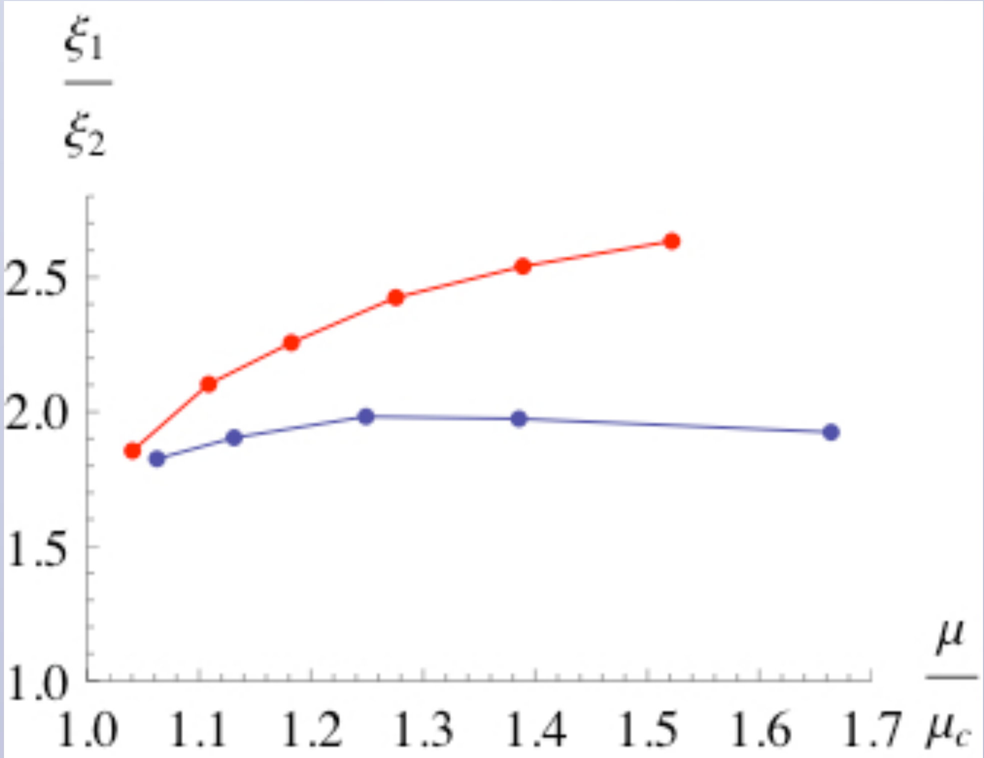

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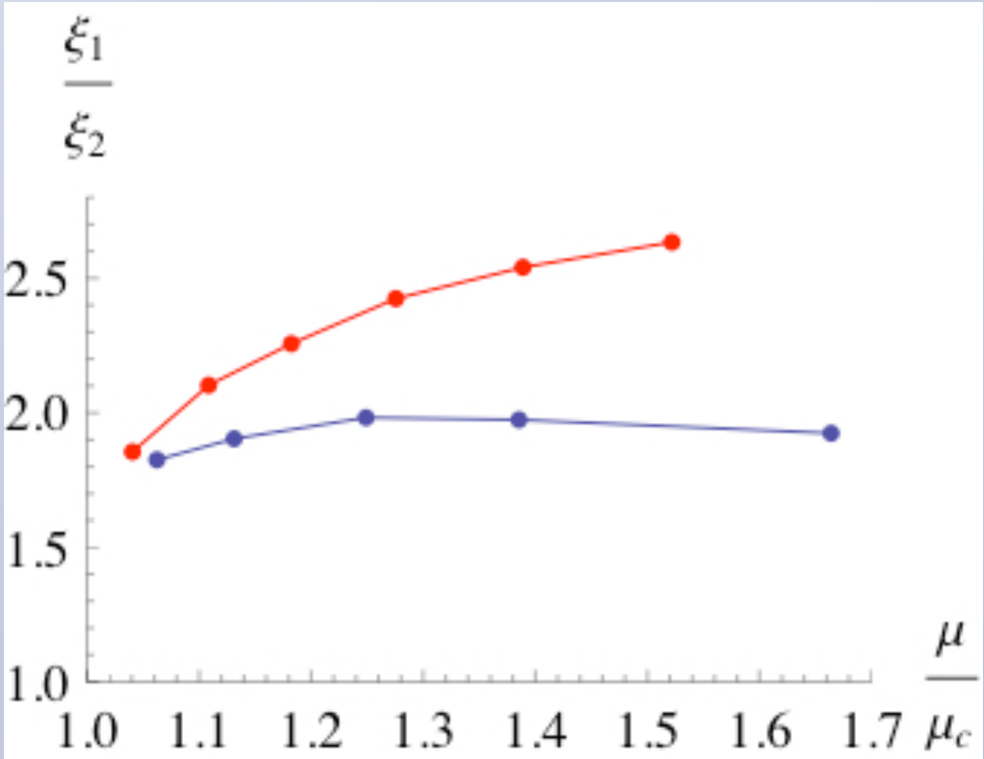

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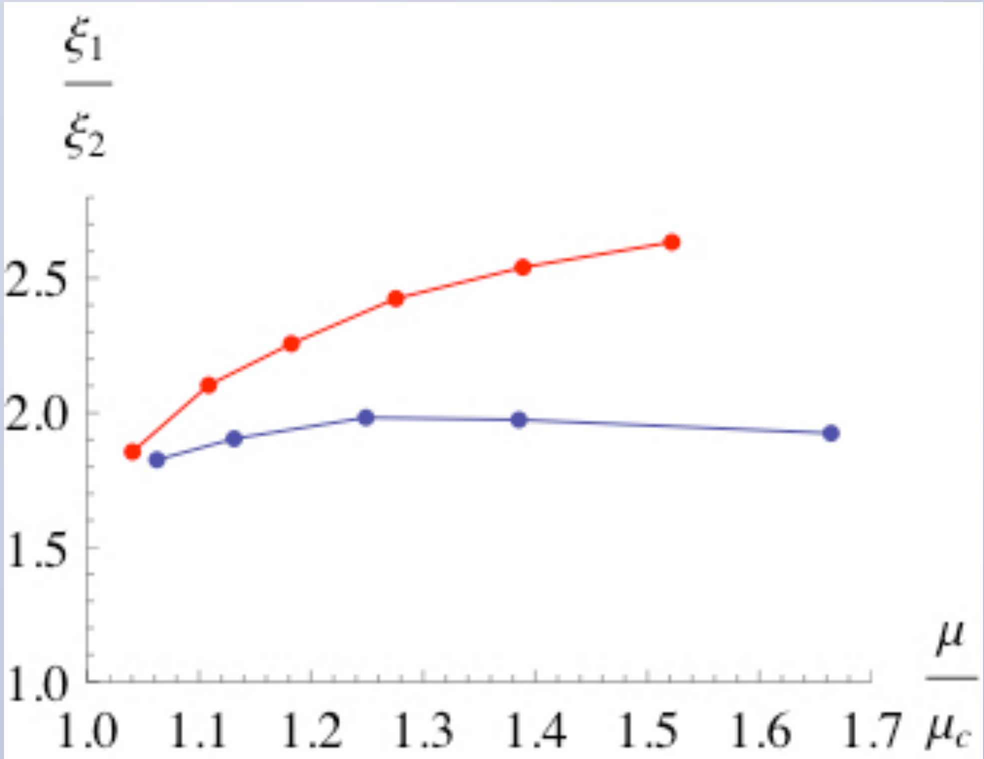
    - would give a measure of k_f without using a fermion probe
    - but....... not visible (at least for $T/T_c \sim .5$)



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Summary and Outlook

- Presented dark soliton and vortex solutions in holographic superfluid
  - the core charge density depletion is a strong function of m^2
- Holographic solitons are have features which are reminiscent solitons in the BCS-BEC crossover
  - suggests that we might vary the condensate type by changing the value of m^2
  - what is the analog of the crossover's "unitarity" regime?
  - but no evidence of Friedel oscillations yet
- Follow-up:
  - try to see any fermionic characteristics
    - cooling to lower temperatures or may need to use external probes
  - gravitational backreaction, transport properties, instabilities, non-relativistic symmetries...