Electron stars and metallic criticality

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Plan of talk

Breakdown of Landau's Fermi liquid theory

Finite density in holography

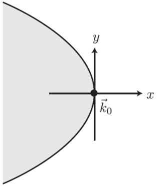
Electron star holography

Breakdown of Landau's Fermi liquid theory

Generic metals are weakly interacting

- Robustness of the 'billiard ball' picture of electrons in a metal explained by renormalisation group (Polchinski/Shankar $\sim$ 1993).

- Zoom in to a point on the Fermi surface



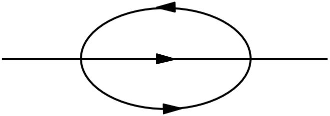
- Free action for excitations at that point

$$S_\psi \sim \int d^3x \psi^\dagger \left(\frac{\partial}{\partial \tau} - i v_F \frac{\partial}{\partial x} - \frac{\kappa}{2} \frac{\partial^2}{\partial y^2} \right) \psi \,.$$

- Lowest order nontrivial interaction, ψ^4, is irrelevant.

Non-Fermi liquids typically strongly interacting

- IR free Fermi liquid robustly predicts for instance DC resistivity



$$\rho(T) \sim \operatorname{Im}\Sigma(T) \sim T^2 \,,$$

- In e.g. heavy fermion compounds, high temperature superconductors or organic superconductors one observes

$$\rho(T) \sim T \,.$$

- Suggests (naively) $\operatorname{Im}\Sigma(T) \sim T$. Width comparable to energy.

- Quasiparticle is not stable anymore — effective theory unlikely to be weakly interacting.

Bosons and fermions

- Effective theories of non-Fermi liquids require additional fields. E.g.

$$S_\phi \sim \int d^3x \left[\phi\left(\frac{\partial^2}{\partial\tau^2} + c^2\frac{\partial^2}{\partial x^2} + c^2\frac{\partial^2}{\partial y^2} + r\right)\phi + \frac{u}{24}\phi^4\right] .$$

- Coupling to fermions is relevant

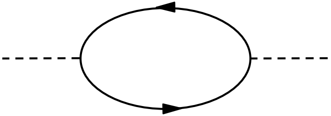
$$S_{\phi\psi^2} \sim \lambda \int d^3x \phi\psi^\dagger\psi .$$

- Typically run to strong coupling IR fixed point. How to compute?

Key physical ingredients

- Fermi surface: Fermionic gapless degrees of freedom with particular kinematics.

- Fermions Landau damp the boson $\longrightarrow$ critical exponent z



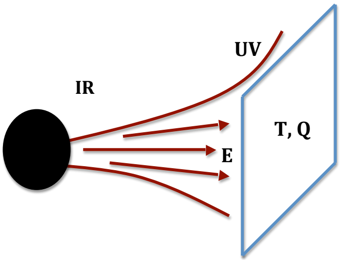
e.g. $z = 3$ in (uncontrolled and incorrect) Hertz-Millis

$$G^{-1} \sim k^2 + \gamma \frac{|\omega|}{k} \,.$$

Finite density holography

Finite density holography

- Does a nontrivial IR scaling emerge from finite density holography?
- Finite density $\Rightarrow$ electric flux at infinity.
- Traditional approach ($\sim$ last 10 years): extremal black holes.
  $\Rightarrow$ Does lead to an IR scaling, but a pathological one with $z = \infty$.



Comments on $z = \infty$

- From dimensional analysis at a critical point

$$s \sim T^{2/z} .$$

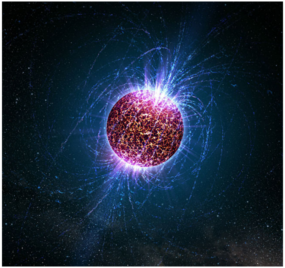
- Phenomenologically appealing: criticality at $\omega \sim k \sim 0$ is efficiently communicated to fermions at $k \sim k_F$ if $z = \infty$.
  [MIT, Polchinski-Faulkner, Sachdev et al.]

- Finite size horizon at $T = 0$:
  - Supported by massless flux: $F = \text{vol}_{AdS_2}$.
  - All the charge is hidden behind the horizon – we know nothing about what it is made of: fermions? bosons? neither?

3 ways to screen away AdS_2

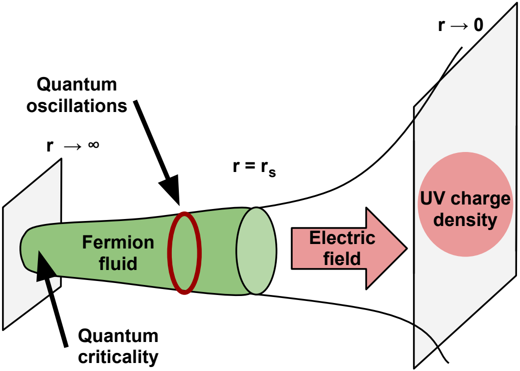
1. Dilatonic couplings [Kachru, Trivedi et al.] $\sim e^{\phi} F^2$. Violate naive Gauss's law.
    - Flux still emanates from behind horizon.
    - $\phi \sim \log r$: have not reached fixed point. In far IR higher derivative terms important. Fixed point likely AdS_2 [Sen]. Postpones rather than solves problem.
2. Sufficiently low dimension charged bosonic operator $\mathcal{O}$
    - Higgs the Maxwell field [Gubser, H^3, Roberts, Nellore,...].
    - Symmetry broken superfluid phase.
3. Sufficiently low dimension charged fermionic operator Ψ
    - Screen the Maxwell field [HPST].
    - Charge fully carried by fermion....

Electron stars



Basic properties of electron stars

- Solve Einstein-Maxwell-Ideal fermion fluid equations [HT]:

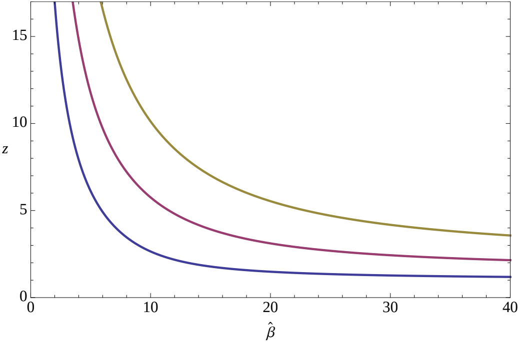


- All the charge is carried by the fermions.

Basic properties of electron stars

- Two free parameters
  - Fermion mass m.
  - Ratio of Maxwell and Newton couplings: $\hat{\beta} = \frac{e^4 L^2}{\kappa^2}$.

- Emergent IR criticality with nice Landau damping



Quantum oscillations

- Magnetic susceptibility in a magnetic field oscillates with period

$$\Delta \left(\frac{1}{B} \right) = \frac{1}{A_F} \,.$$

- Local magnetic field in the bulk

$$B_{\text{loc.}} = B r^2 \,.$$

- Local Fermi surface area

$$A_{F\,\text{loc.}} \propto \mu_{\text{loc}}^2 - m^2 \,.$$

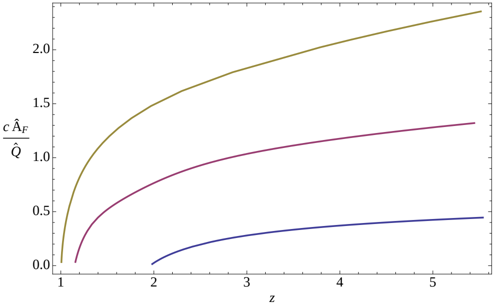
- Only fermions at the radius that maximises

$$\frac{\mu_{\text{loc}}^2 - m^2}{r^2} \,,$$

contribute to quantum oscillations.

The Luttinger count

- In a Fermi liquid, A_F over charge density is constant (Luttinger).



- Luttinger count restored by continuum of 'fractionalised' fermions, most of which don't contribute to oscillations

$$H_{\text{eff.}} = \int dM A(M) \int d^2k \,(\omega_k(M) - \mu)\, c_k^\dagger(M) c_k(M) \,,$$

Final comments

- Electron stars are the fermionic analogues of holographic superconductors.
- Naturally lead to finite z Landau damping at strong coupling.
- Charge is fully carried by fermions.
- Suggestive picture in terms of a continuum of ‘fractionalised’ fermions.
- Sharp ‘Kosevich-Lifshitz’ quantum oscillations without a Fermi liquid.