

Mottness and Holography: Spectral weight transfer  
and T-linear Resistivity

Thanks to: T.-P. Choy, R. G. Leigh, S. Chakraborty, M.  
Edalati, S. Hong

PRL, 99, 46404 (2007);

PRB, 77, 14512 (2008); *ibid*, 77, 104524 (2008) );

*ibid*, 79, 245120 (2009); RMP, 82, 1719 (2010)..., DMR/NSF

GGI Talk: Nov. 4, 2010

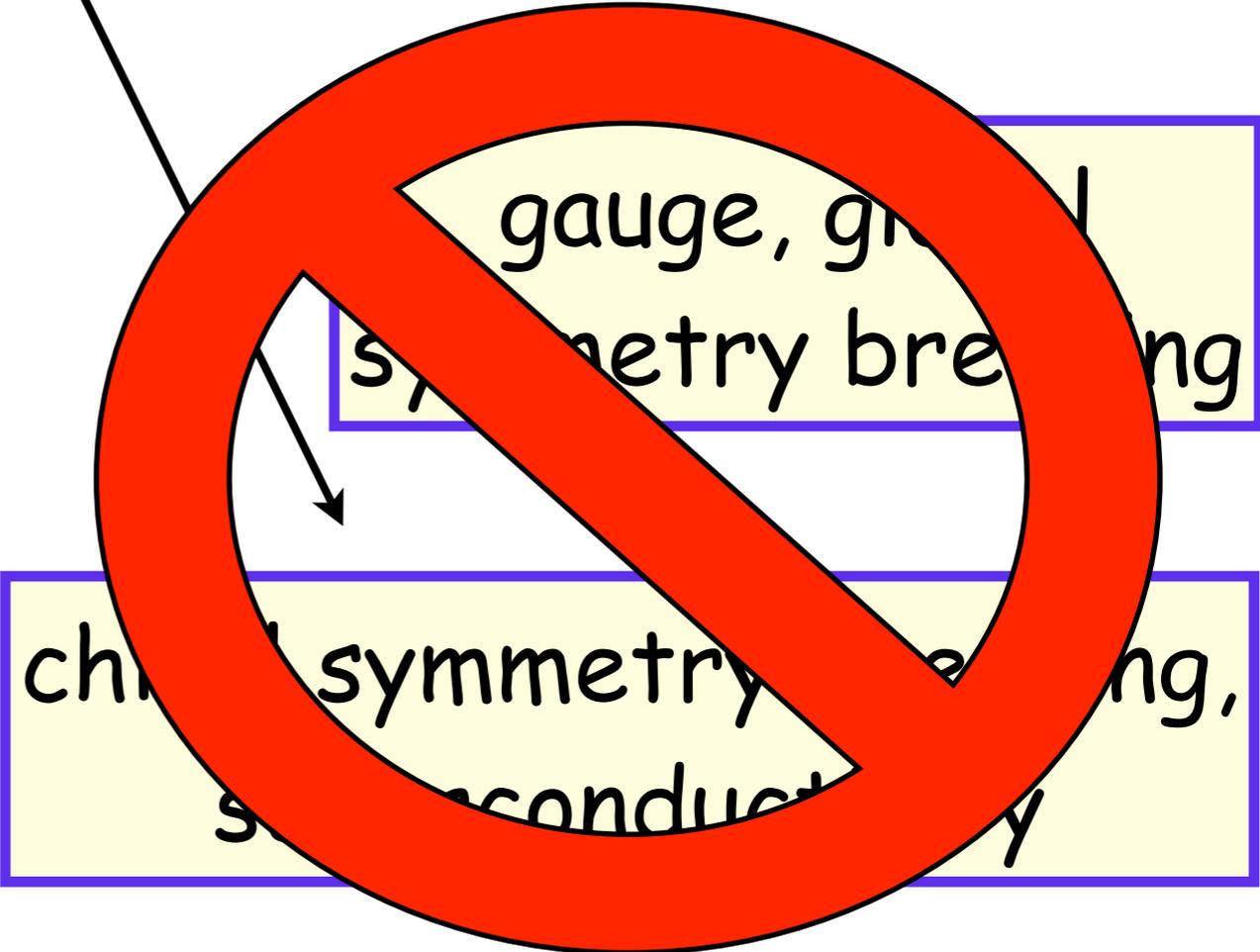
Dynamically Generated Gap

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graph TD; A[Dynamically Generated Gap] --> B[chiral symmetry breaking, superconductivity]; C[gauge, global symmetry breaking] --- A; C --- B;
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gauge, global  
symmetry breaking

chiral symmetry breaking,  
superconductivity

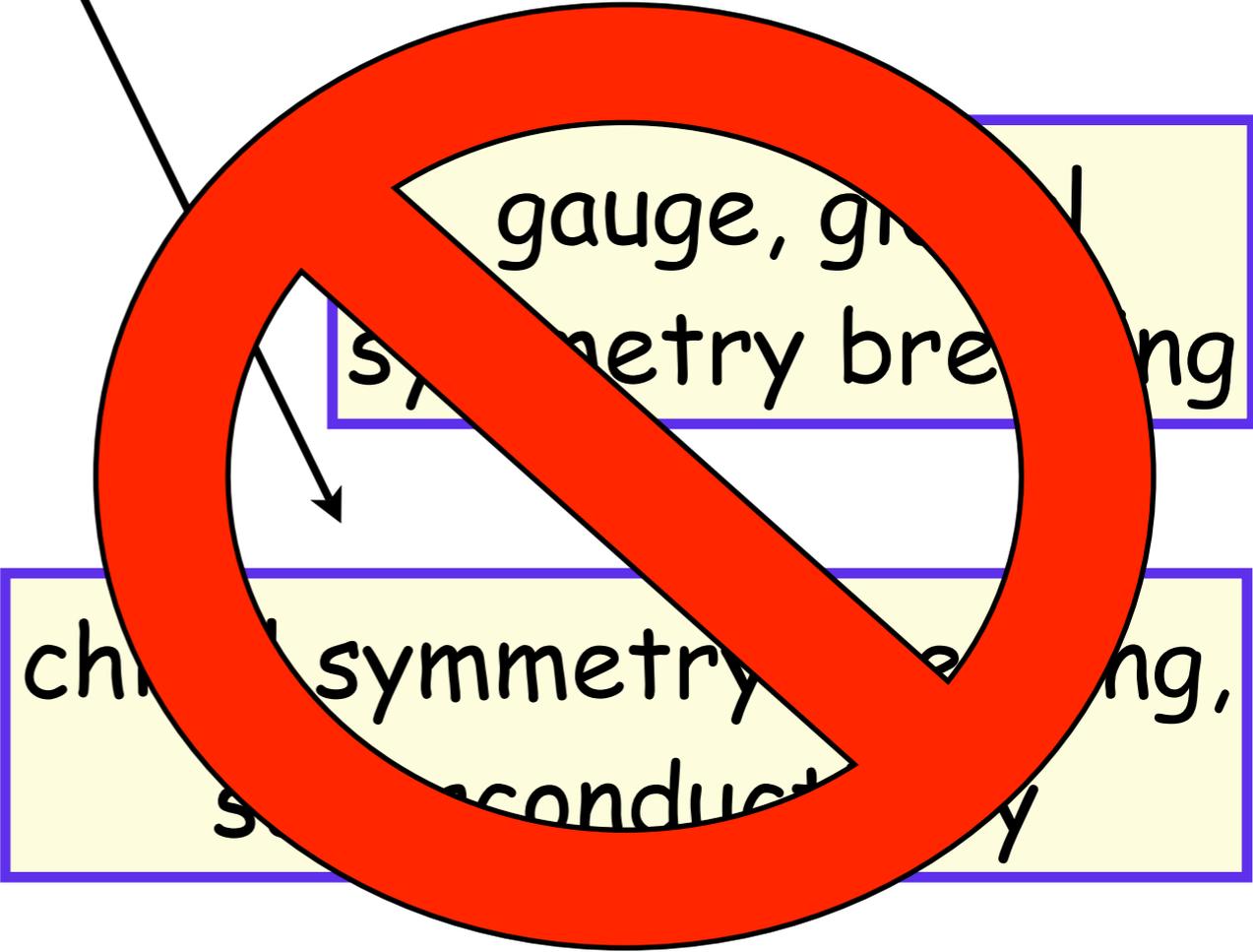
# Dynamically Generated Gap

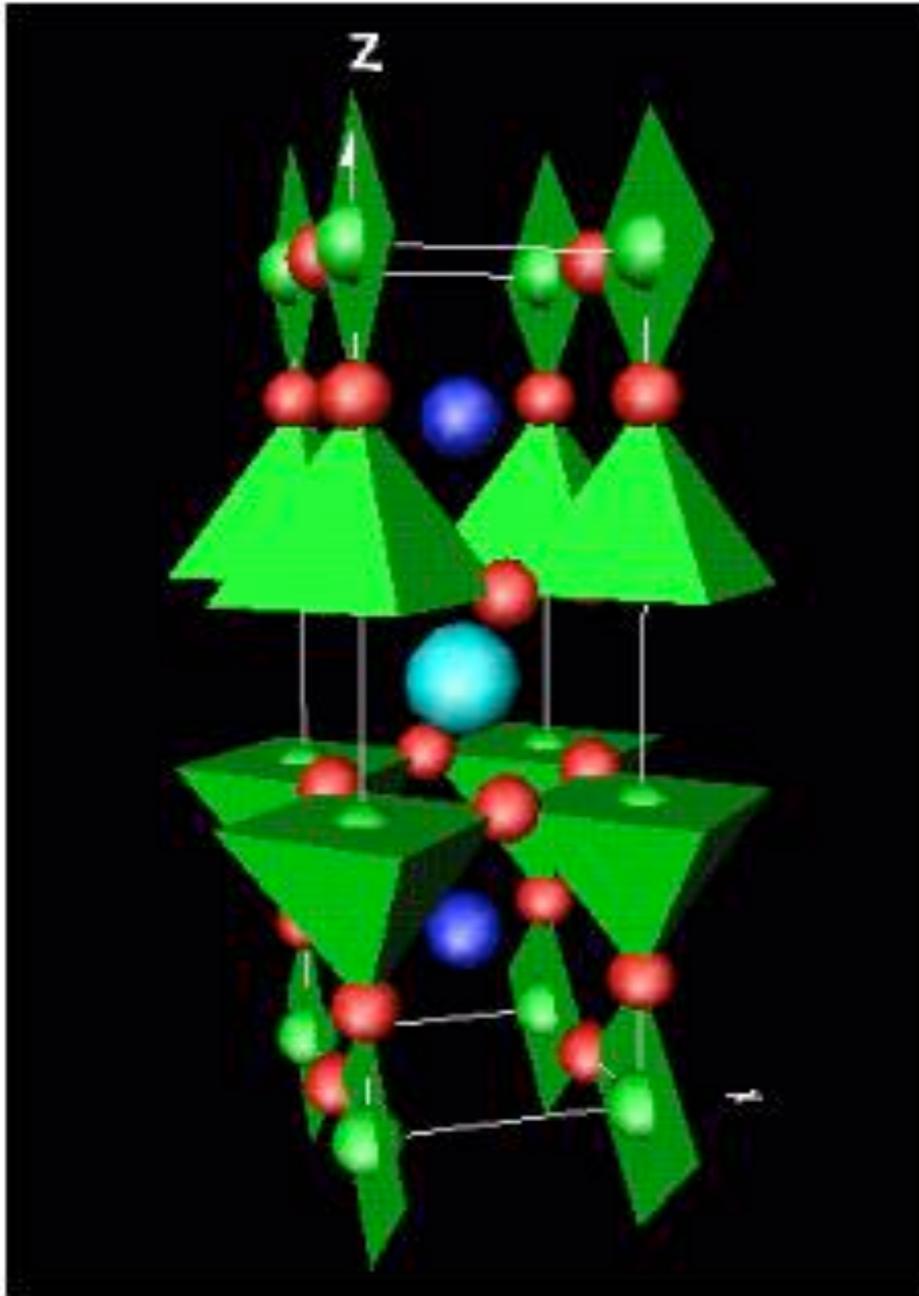


# Dynamically Generated Gap

No symmetry breaking

emergent bound states not in UV: QCD(pions), vulcanized rubber, Mott insulators

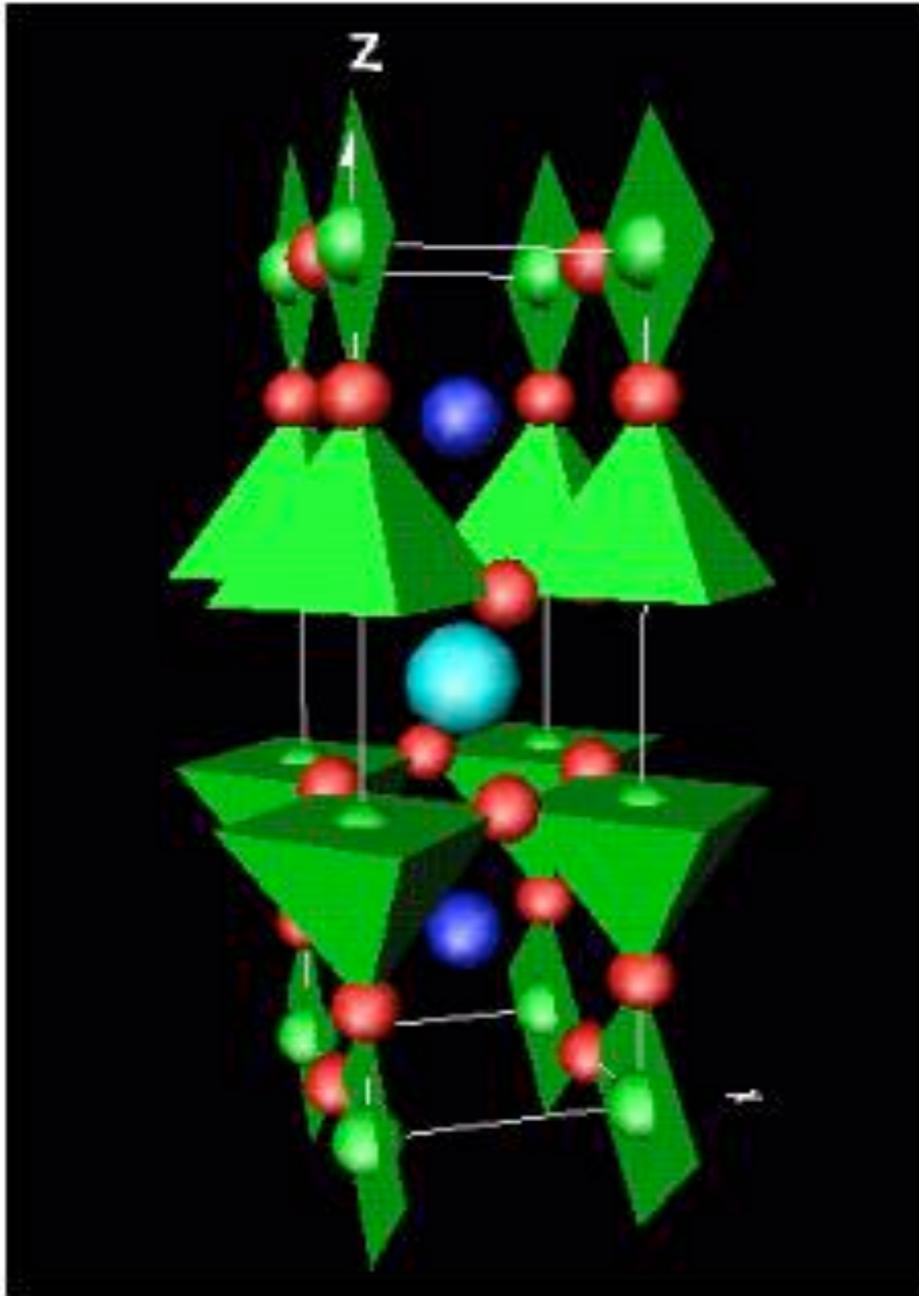




$$U/t = 10 \gg 1$$

interactions dominate:  
Strong Coupling Physics

$\text{Y Ba}_2 \text{Cu}_3 \text{O}_7$   
Cuprate Superconductors



## 2D Hubbard Model

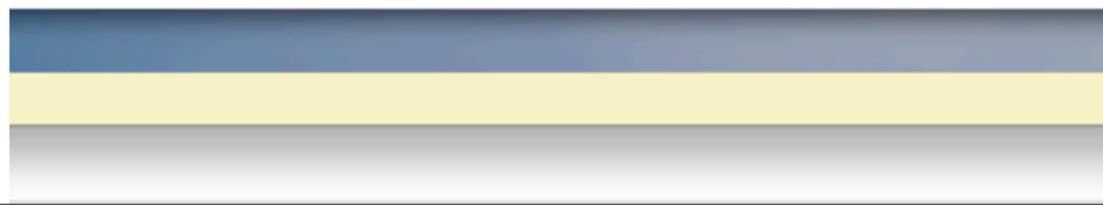
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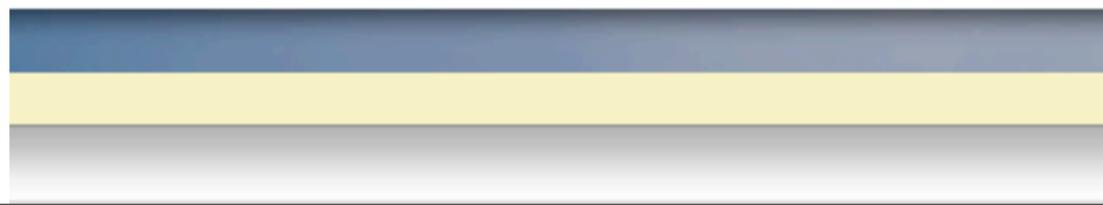
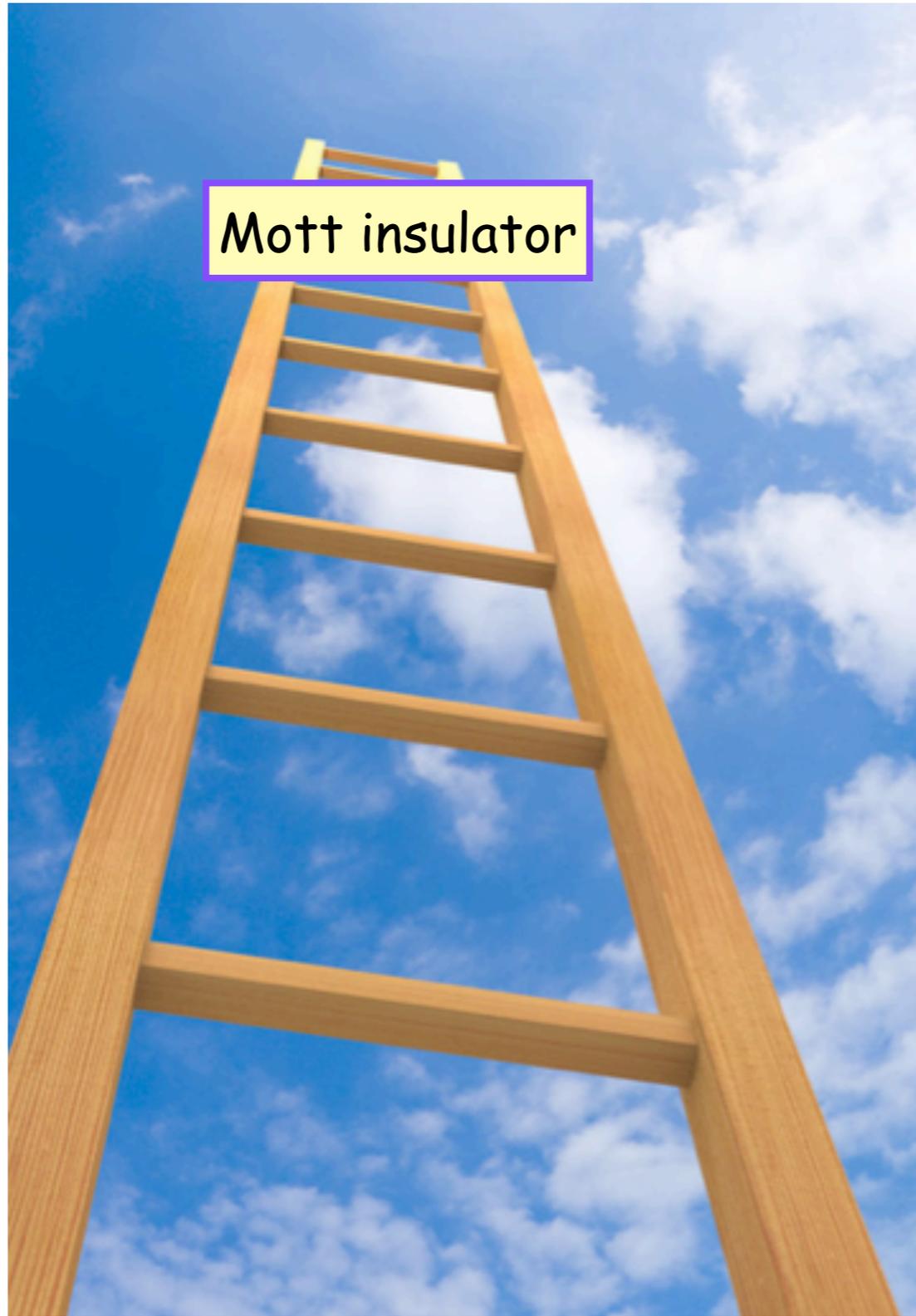
$$H = -t \sum_{i,j} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

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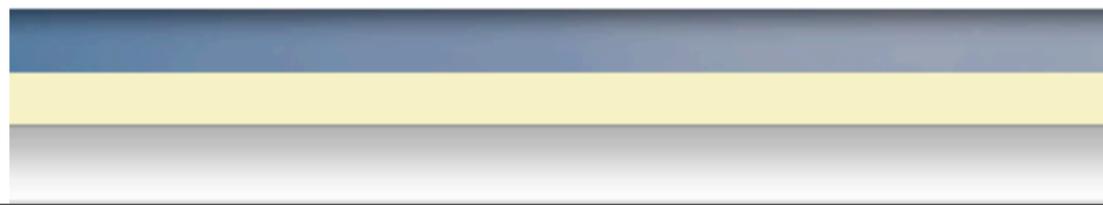
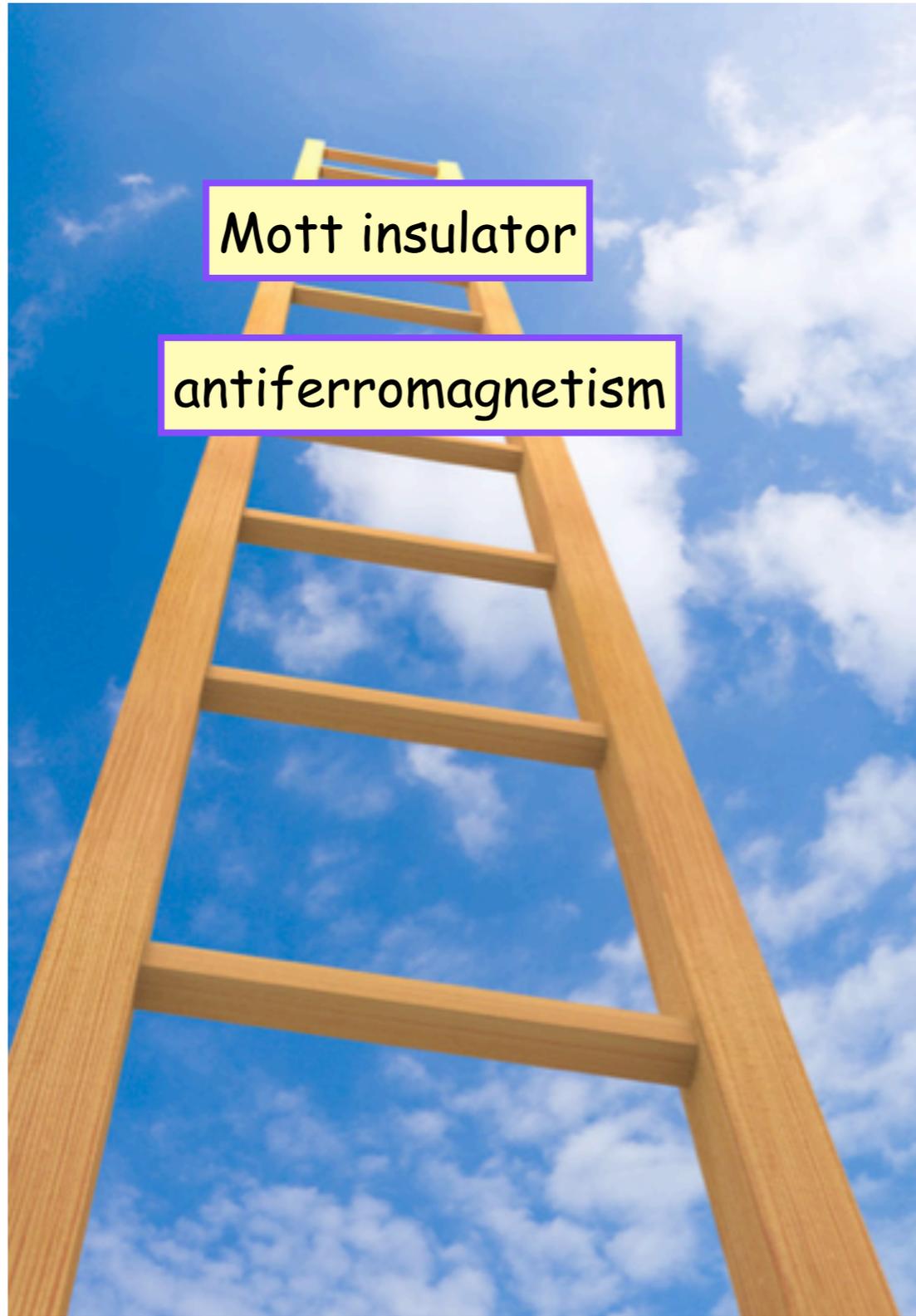
Mott insulator



$$H = -t \sum_{i,j} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Mott insulator

antiferromagnetism

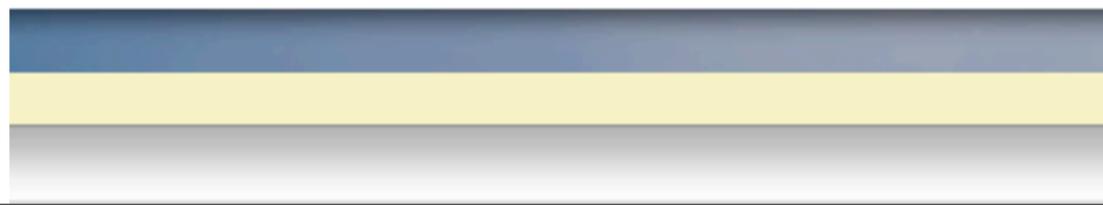
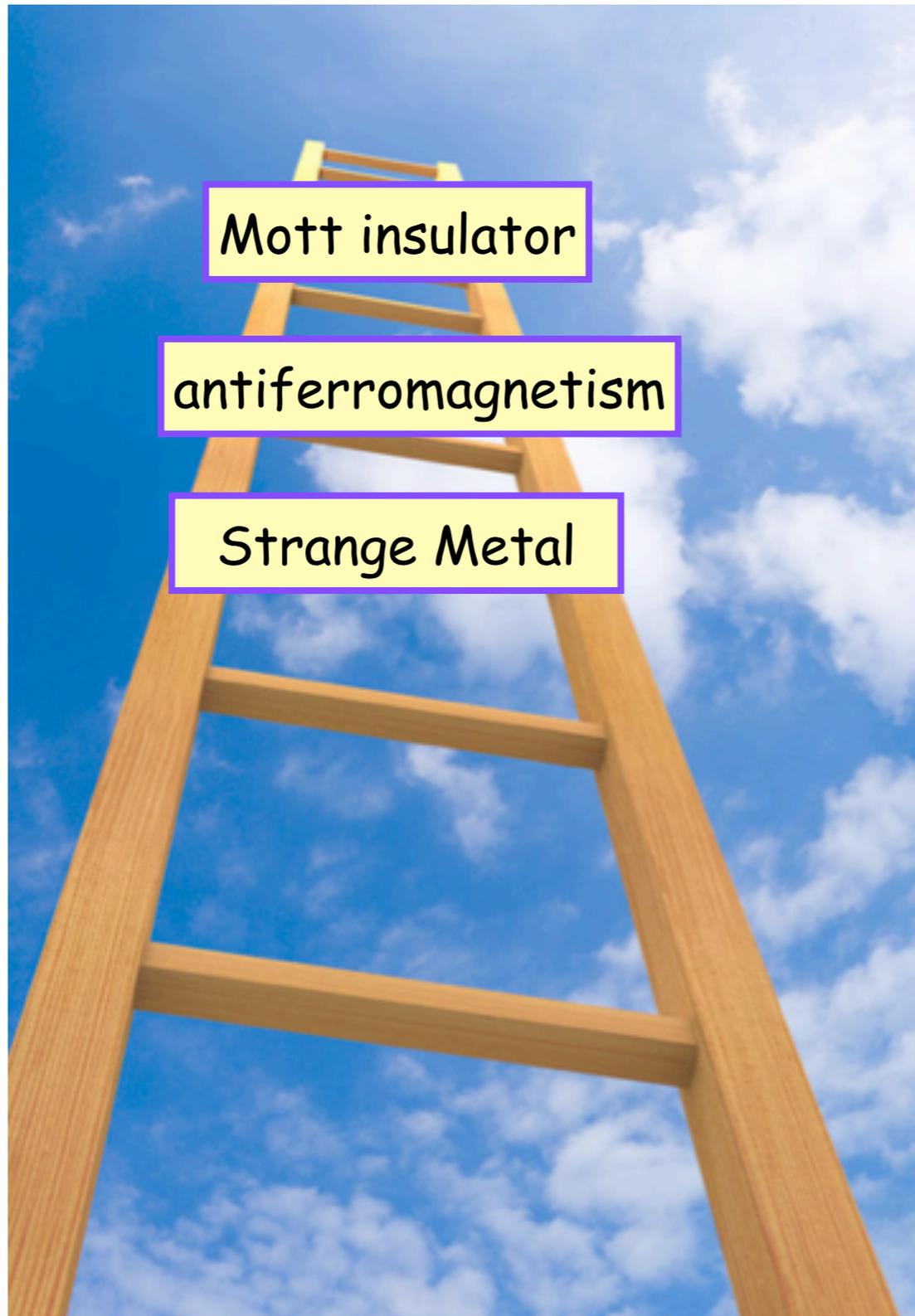


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Mott insulator

antiferromagnetism

Strange Metal



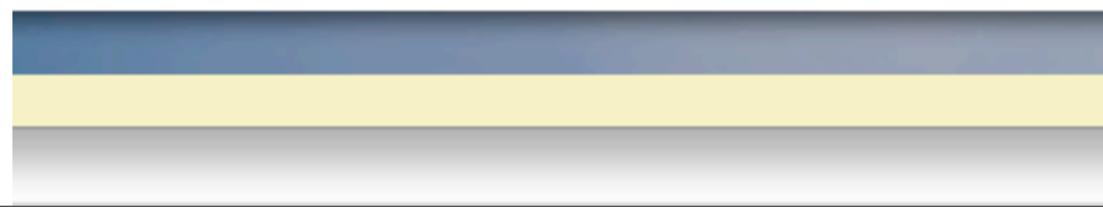
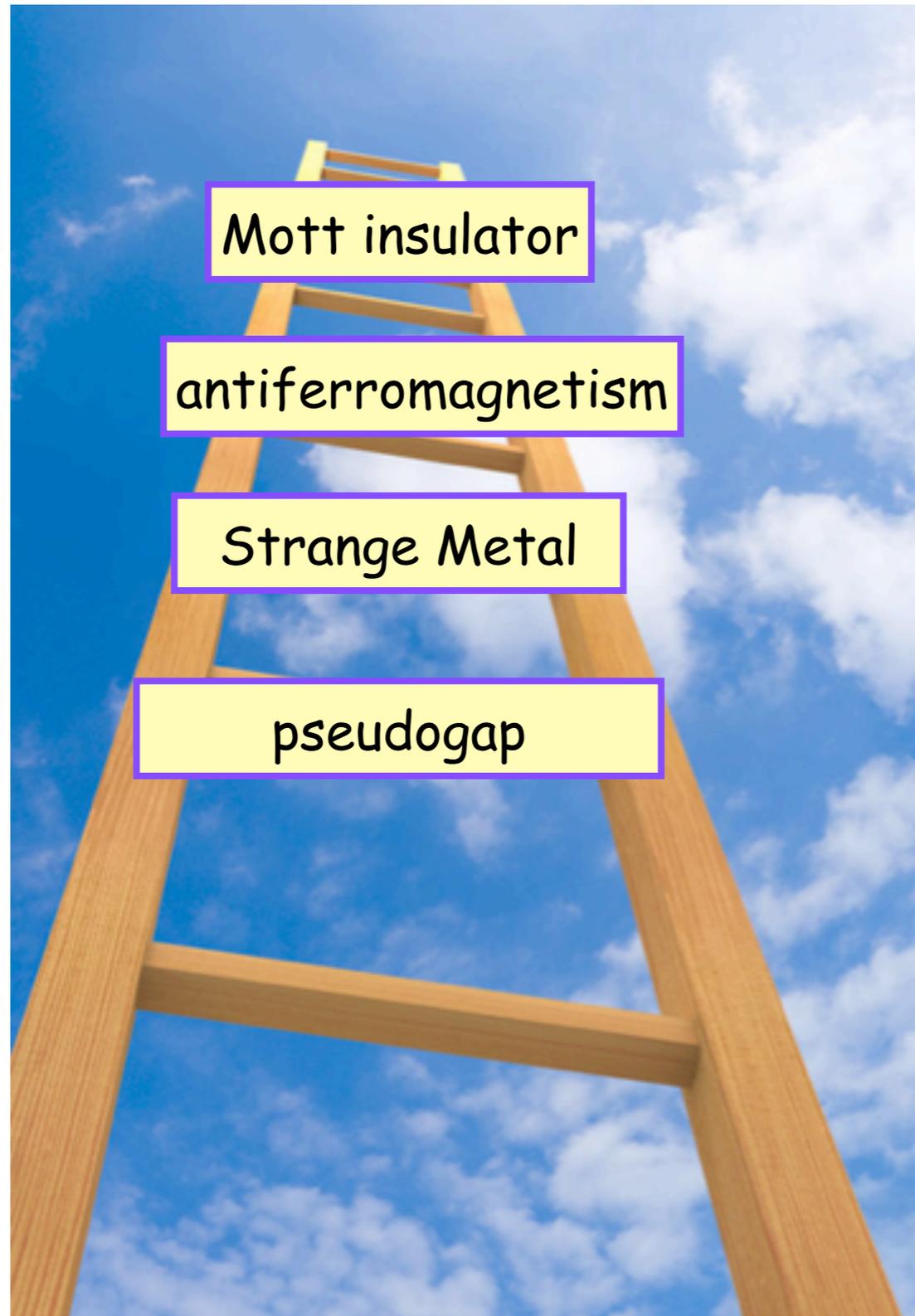
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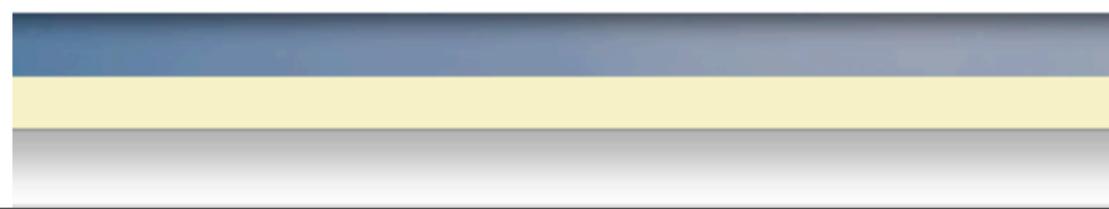
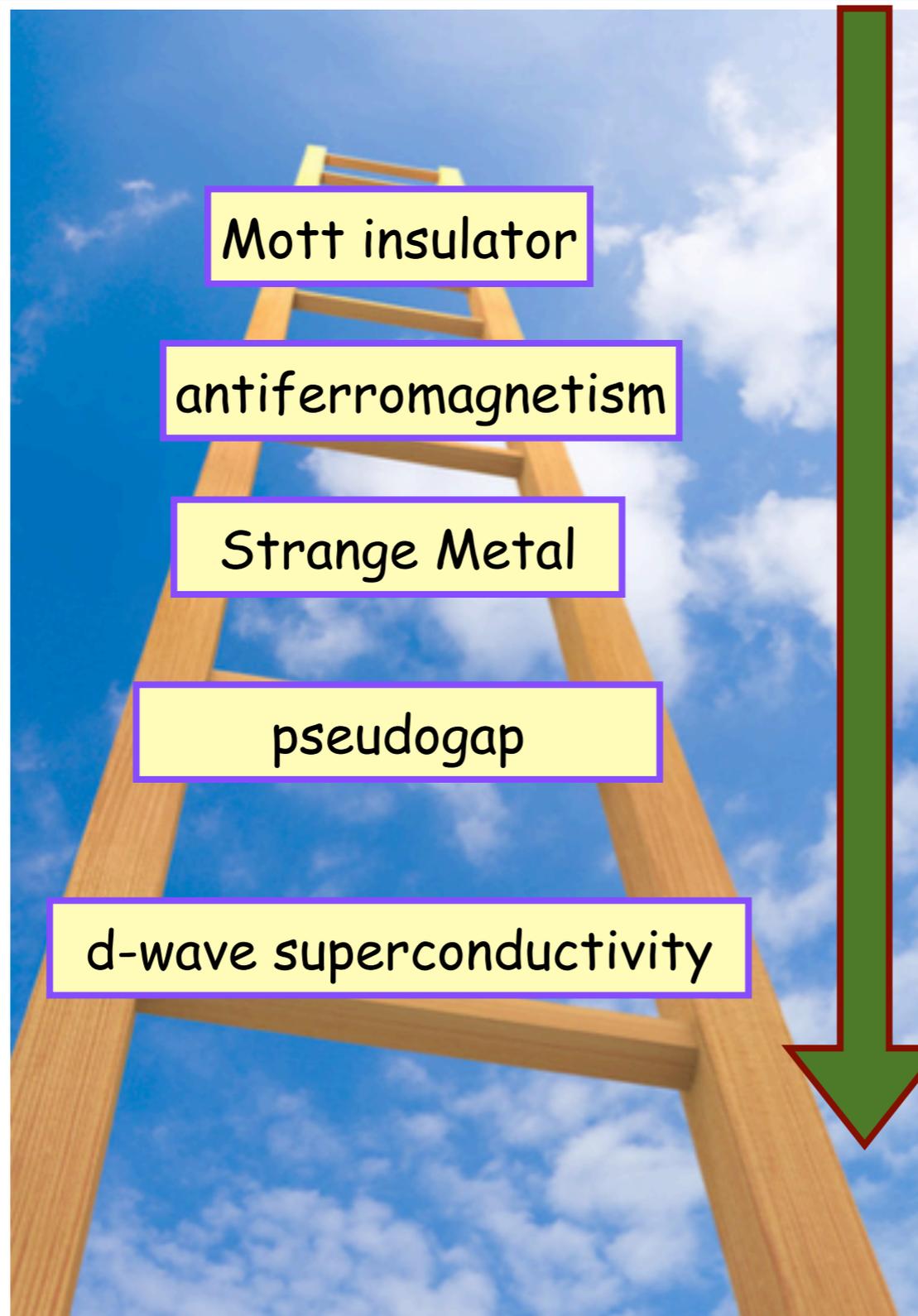
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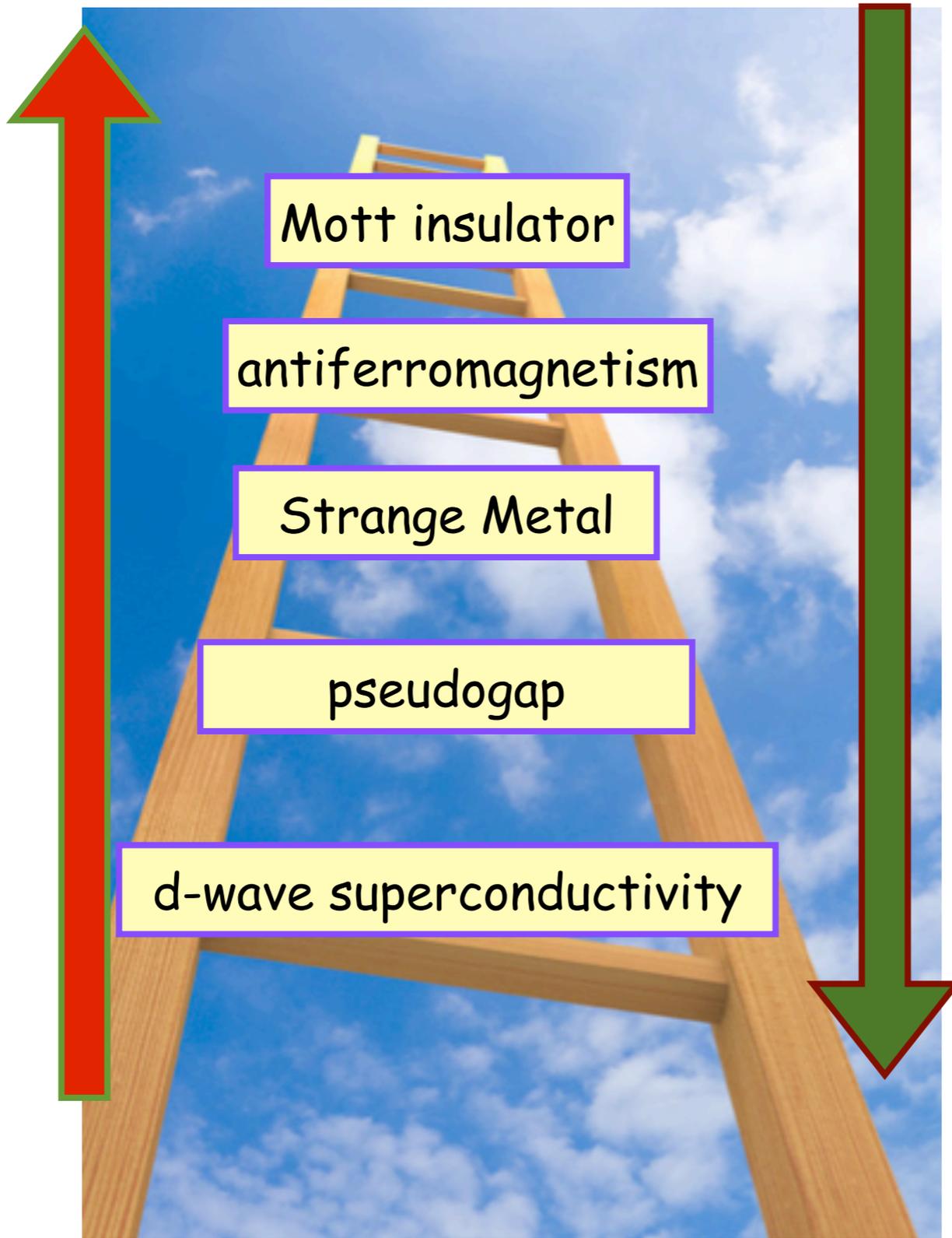
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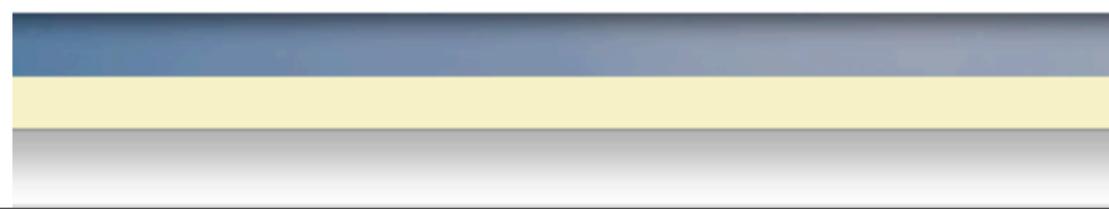
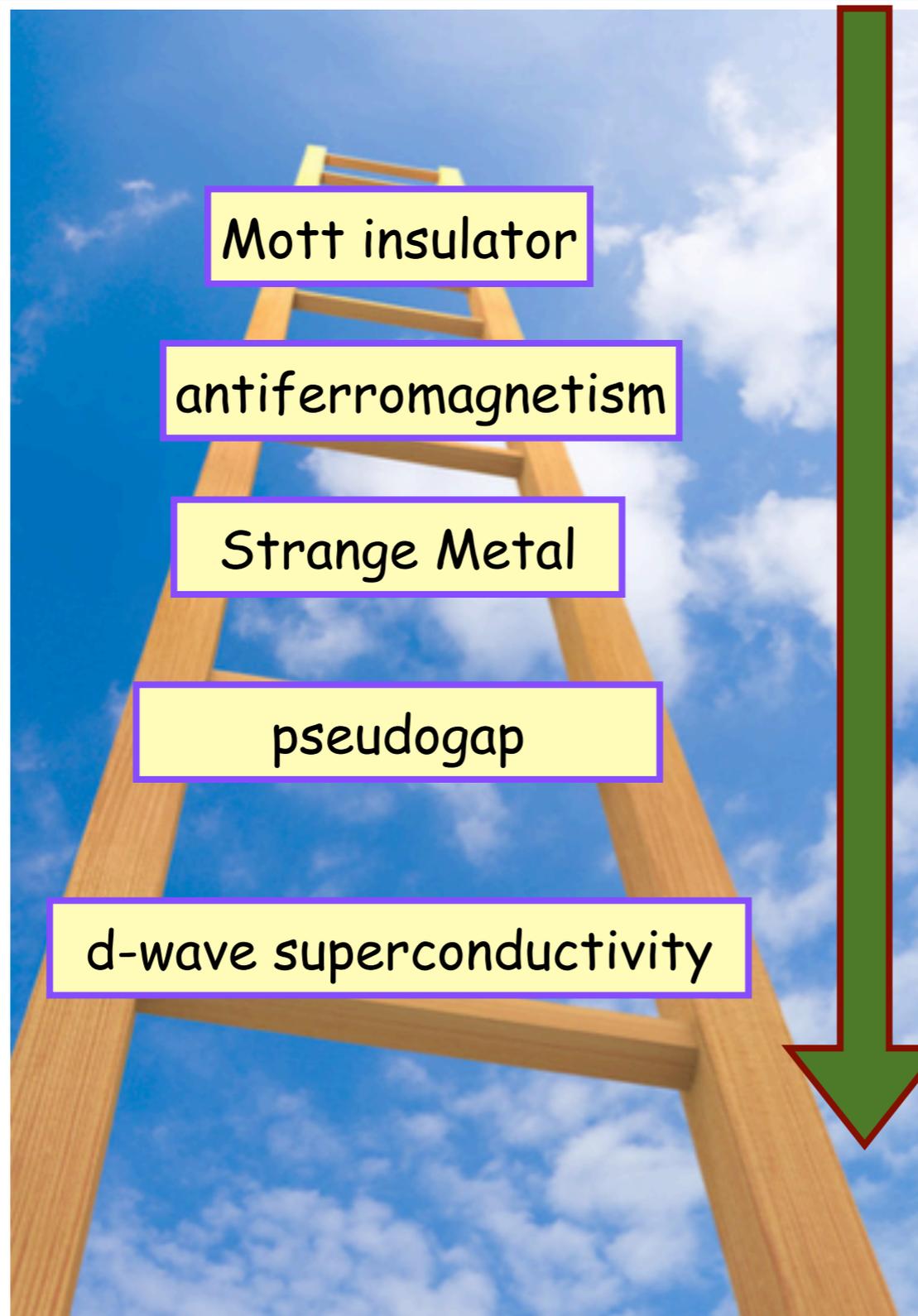
Mott insulator

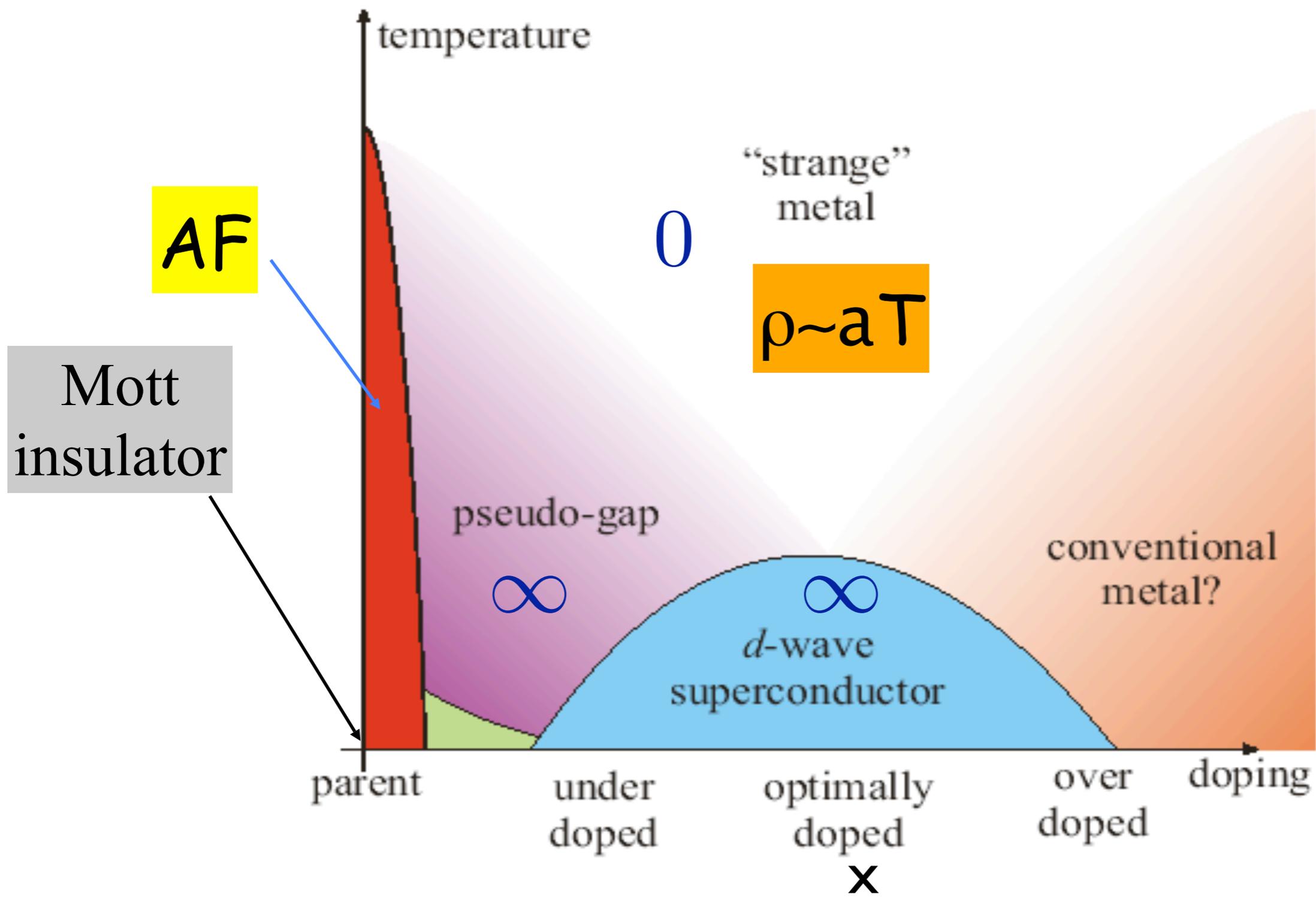
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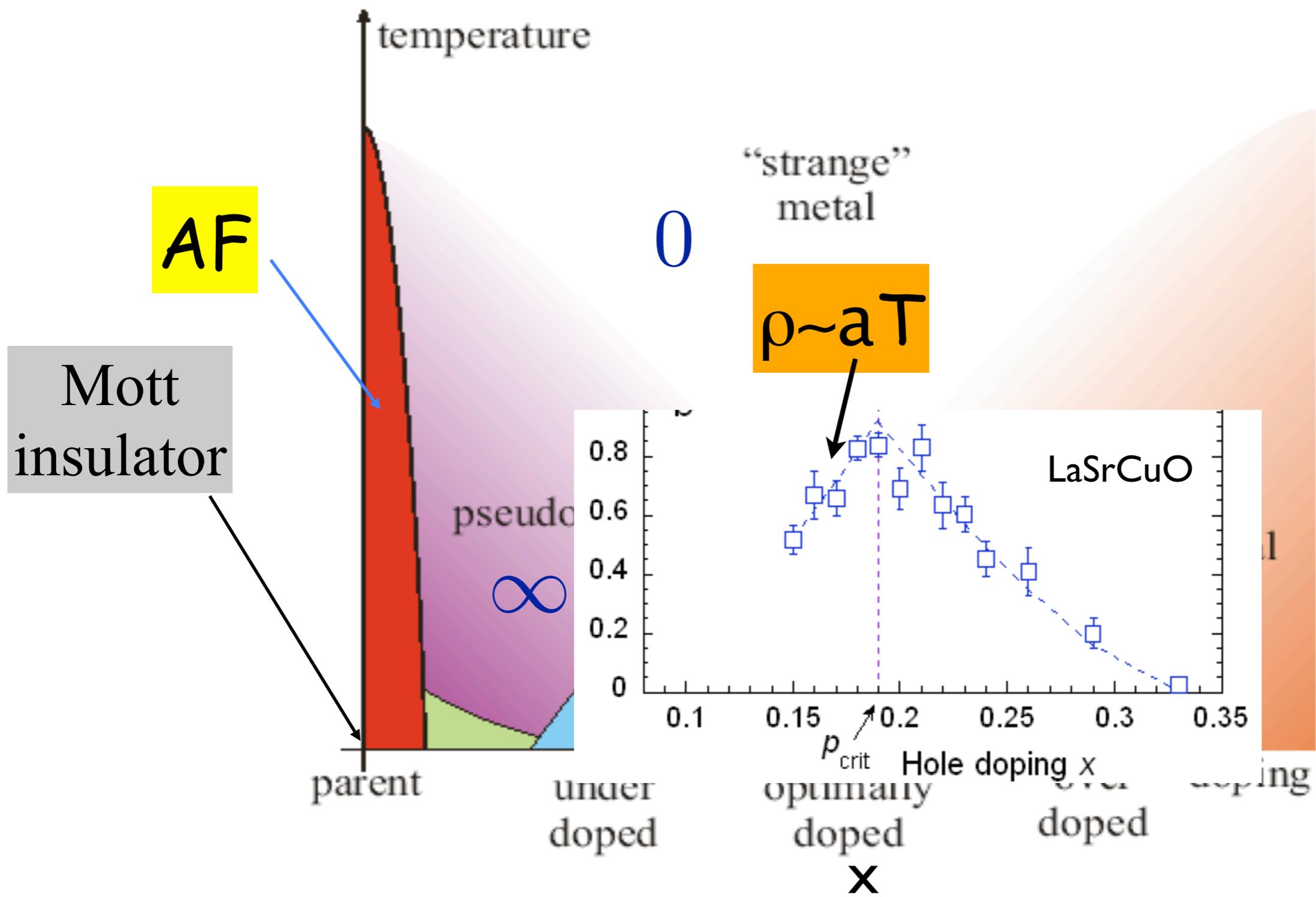
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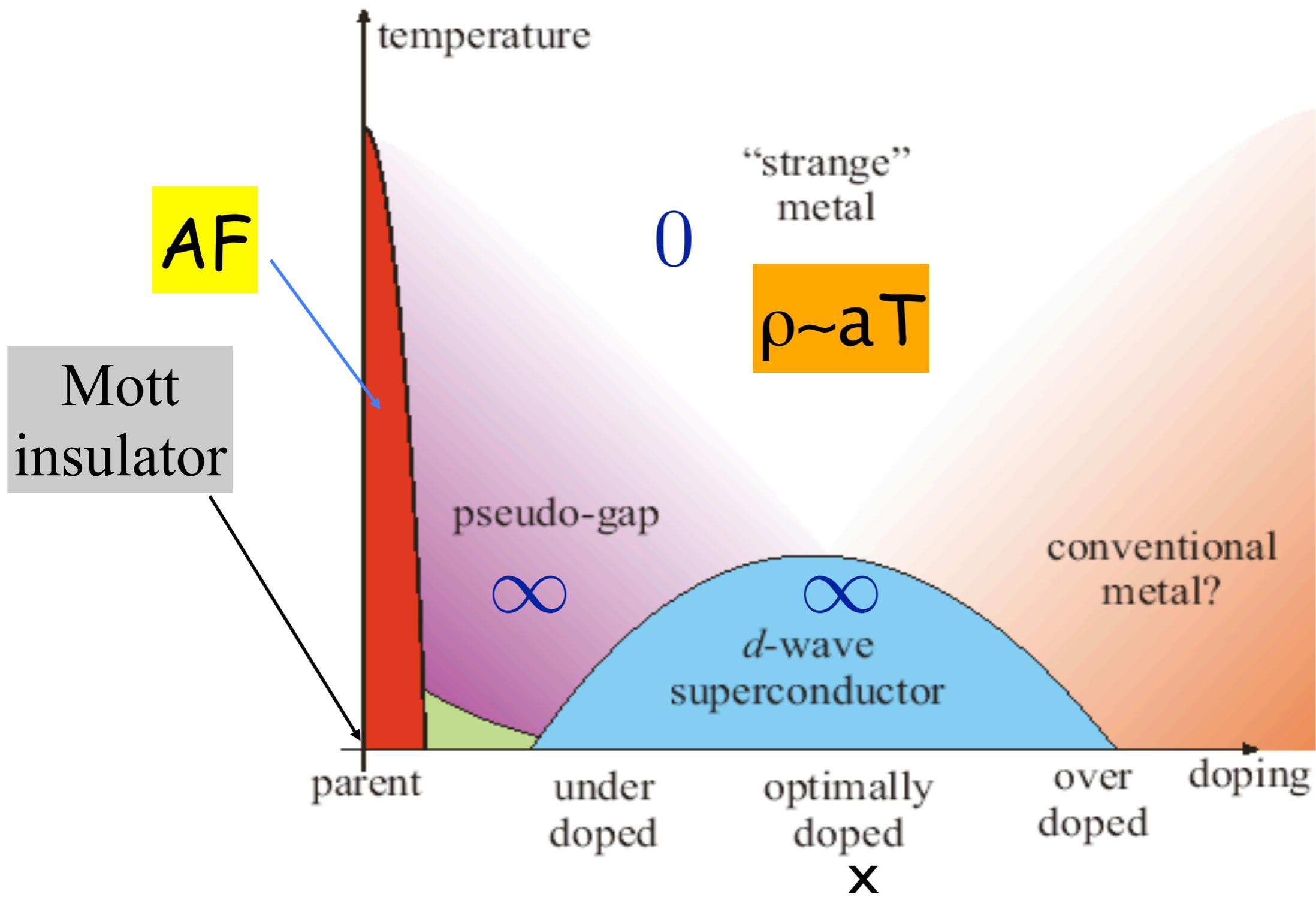
pseudogap

d-wave superconductivity



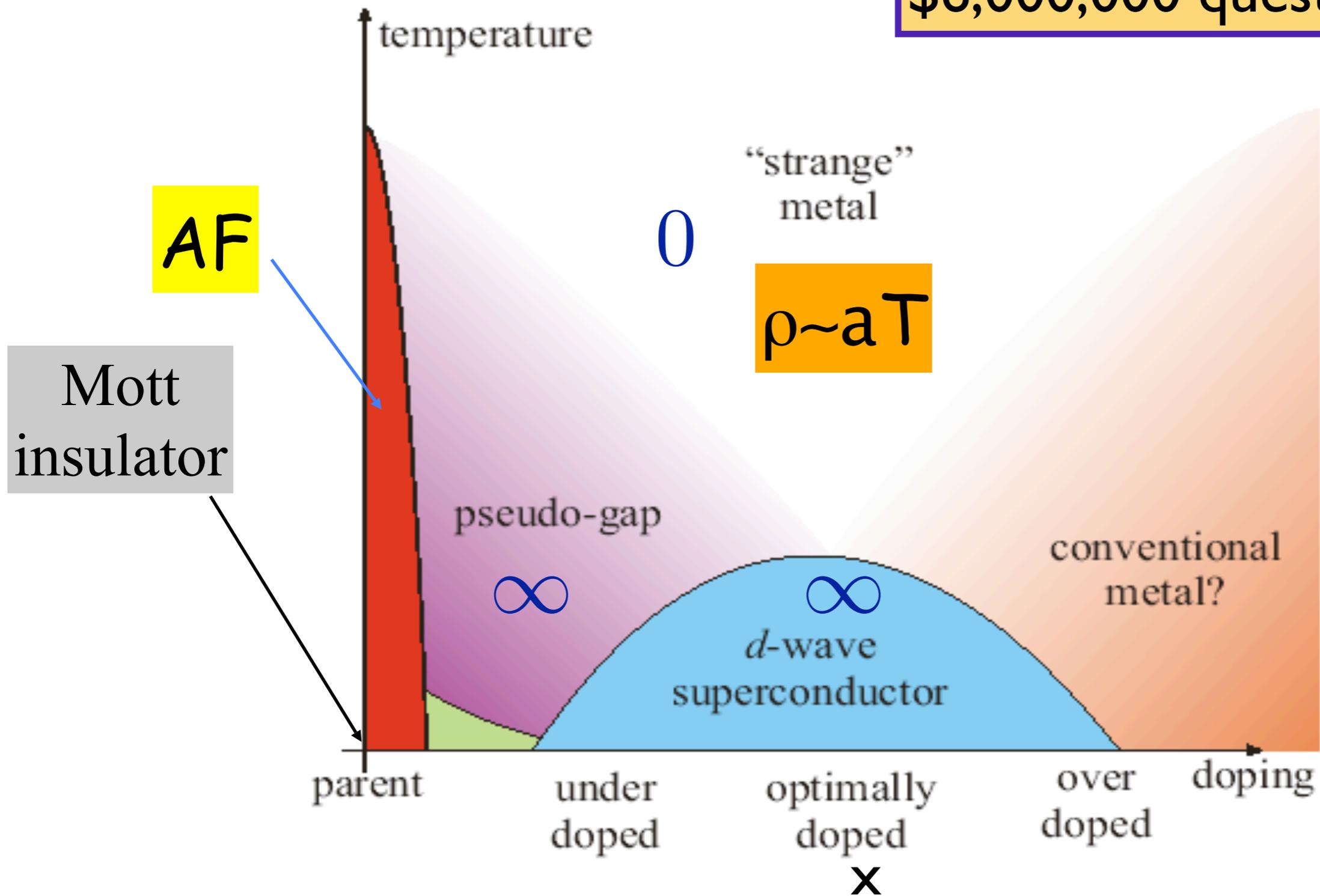






# How does Fermi Liquid Theory Breakdown?

\$6,000,000 question?



T-linear resistivity

T-linear resistivity

quantum criticality

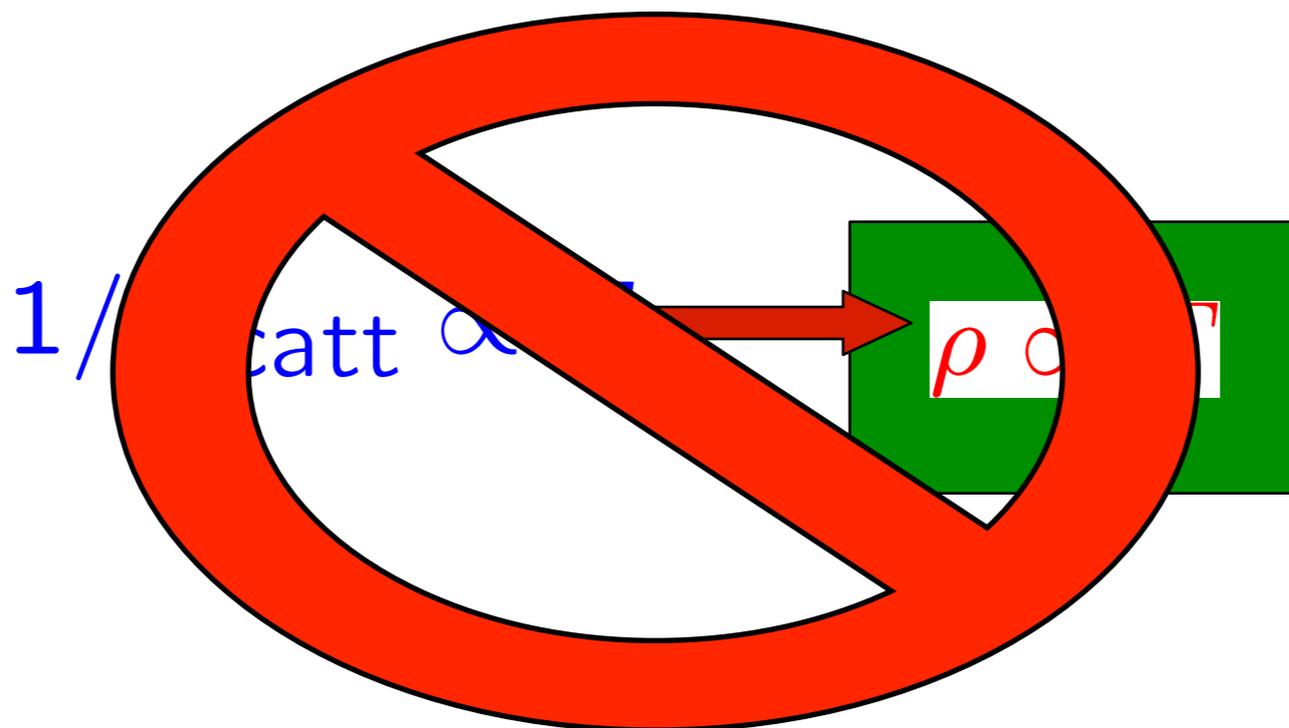
T-linear resistivity

quantum criticality

$$1/\tau_{\text{scatt}} \propto T \longrightarrow \rho \propto T$$

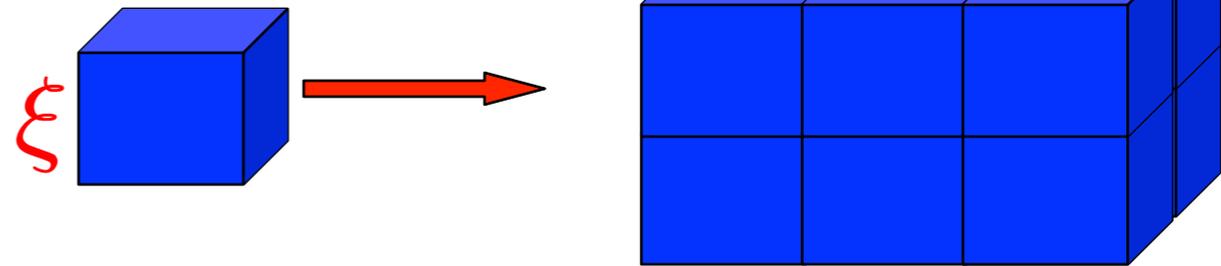
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# General Result

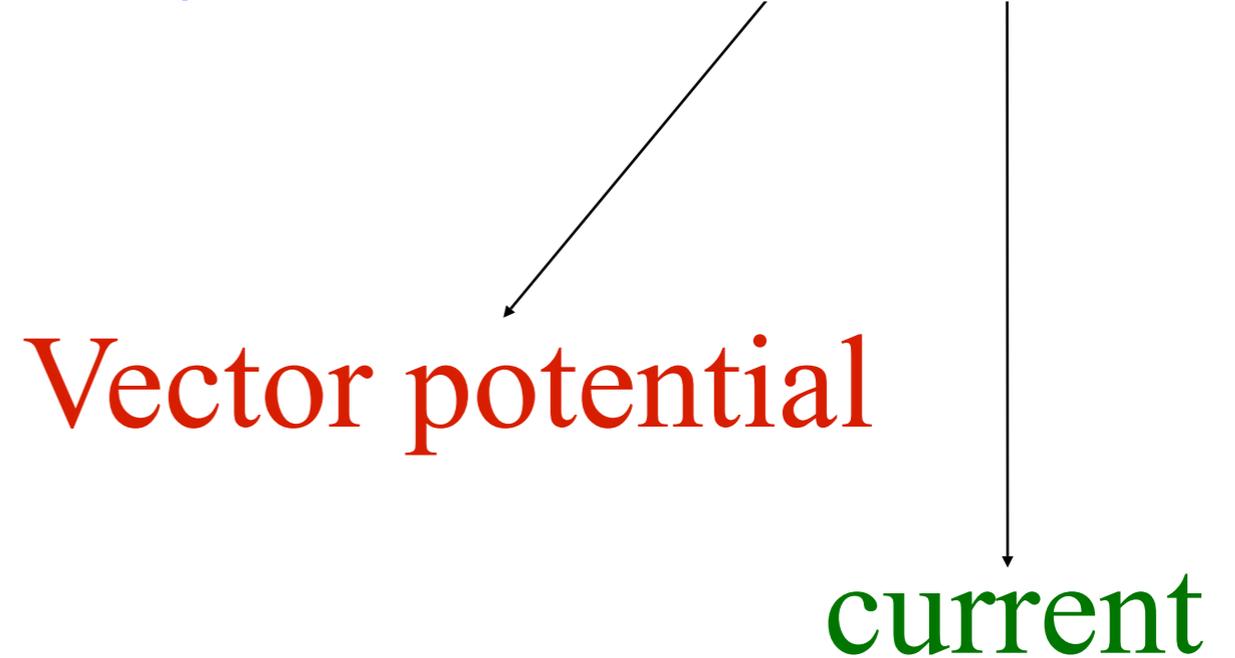
1.) one critical length scale



2.) charge carriers are critical

3.) charge conservation

$$S \rightarrow S_{\text{whatever}} + \int d\tau d^d x A^\mu j_\mu$$



Vector potential

current

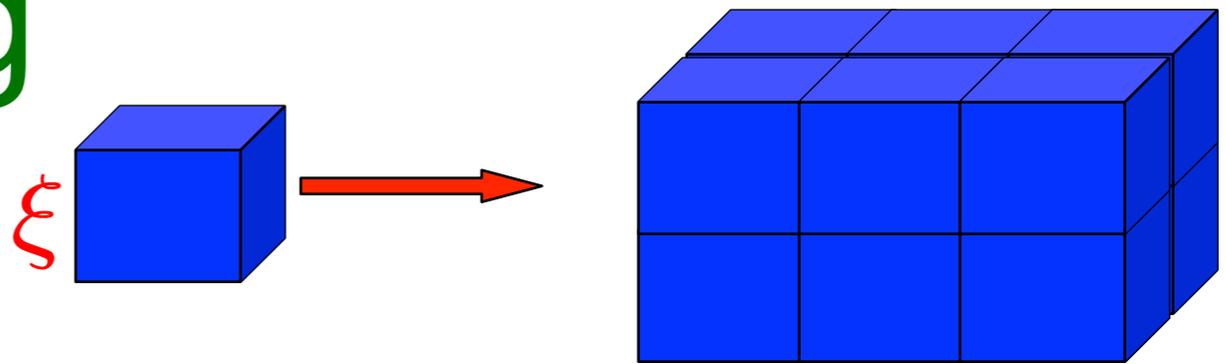
$$S \rightarrow S_{\text{whatever}} + \int d\tau d^d x A^\mu j_\mu$$

Vector potential

current

Charge conservation:  $d_A = 1$

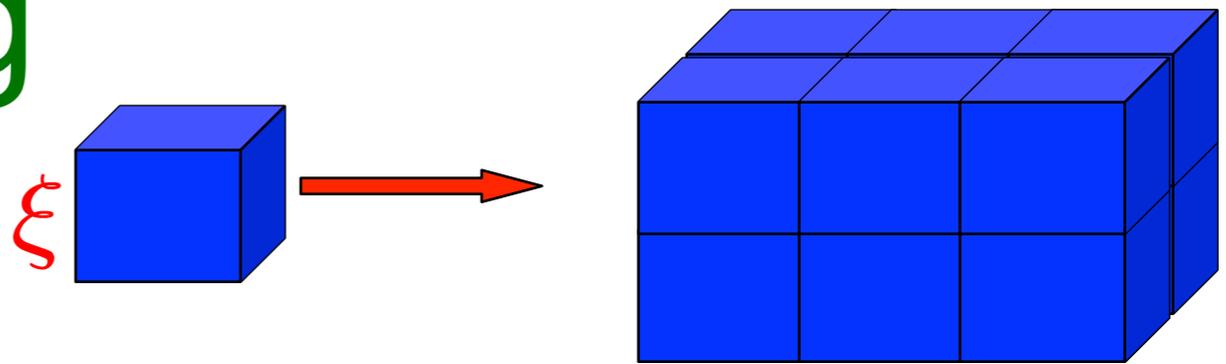
# scaling



$$\ln Z = \frac{L^d \beta}{\xi^d \xi_t} F(\delta \xi^{d_\delta}, \{A_\lambda^i \xi^{d_A}\})$$

$$A_\lambda^i = A^i(\omega = \lambda \xi_t^{-1})$$

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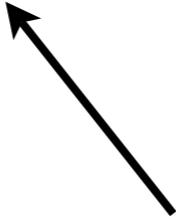
$$\sigma_{ij} = \frac{1}{L^d \beta} \frac{1}{\omega} \frac{\delta^2 \ln Z}{\delta A^i(-\omega) \delta A^j(\omega)}$$

PP, CC, PRL, vol. 95,  
107002 (2005)

$$\sigma(T) \propto T^{(d-2)/z}$$

$$E \propto p^z$$

dynamical exponent



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$d=3$



$$\rho \propto T$$

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only if  $z < 0$

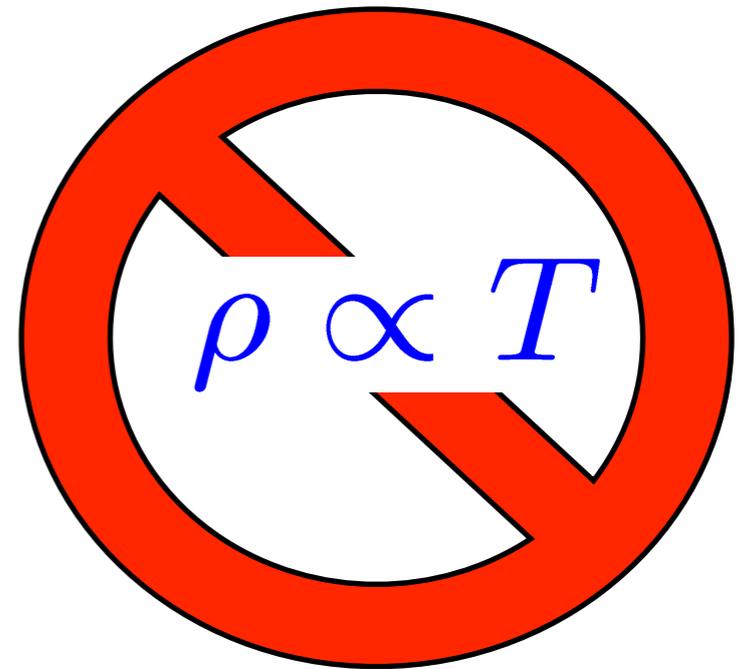
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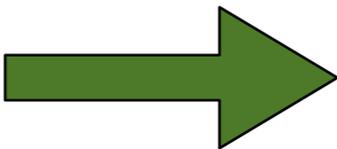
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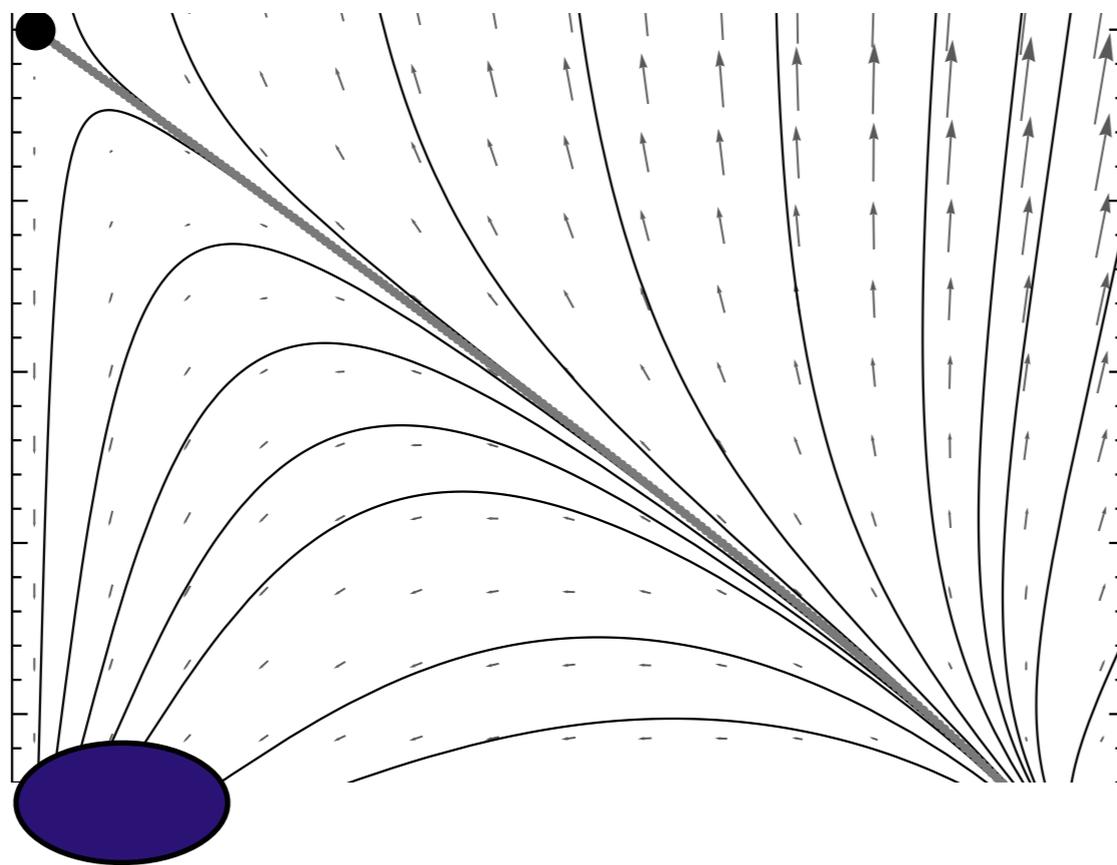
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# One-parameter Scaling breaks down

$\rho \propto T$   new length scale

new degree of freedom

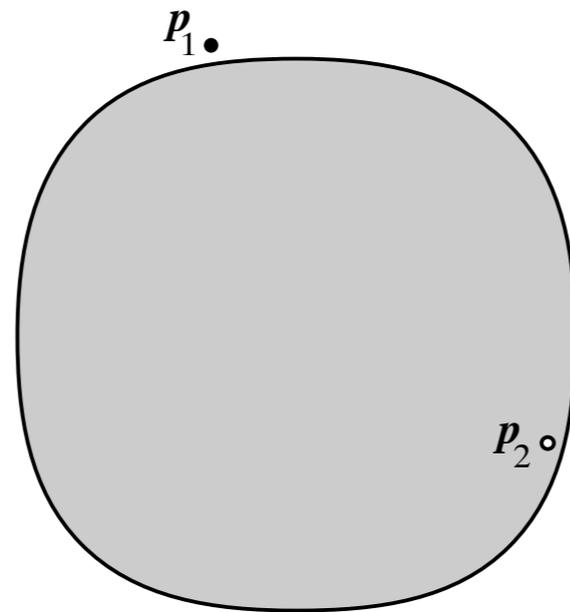
NFL?



FL

How to break Fermi liquid theory in  $d=2+1$ ?

# Polchinski, Shankar, others



$$\mathbf{p} = \mathbf{k} + \mathbf{l},$$

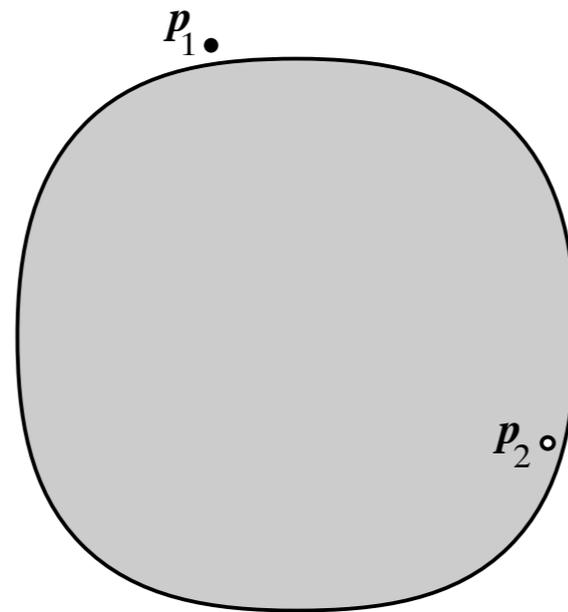
1.) e- charge carriers

2.) Fermi surface

$$\int dt d^2\mathbf{k}_1 d\mathbf{l}_1 d^2\mathbf{k}_2 d\mathbf{l}_2 d^2\mathbf{k}_3 d\mathbf{l}_3 d^2\mathbf{k}_4 d\mathbf{l}_4 V(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \psi_{\sigma}^{\dagger}(\mathbf{p}_1) \psi_{\sigma}(\mathbf{p}_3) \psi_{\sigma'}^{\dagger}(\mathbf{p}_2) \psi_{\sigma'}(\mathbf{p}_4) \delta^3(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4).$$

No relevant short-range 4-Fermi terms in  $d \geq 2$

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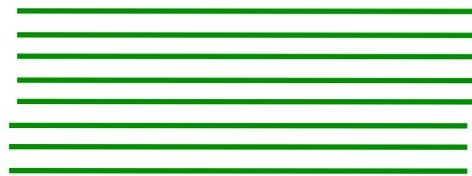
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No relevant short-range 4-Fermi terms in  $d \geq 2$   
Exception: Pairing

# Landau Correspondence

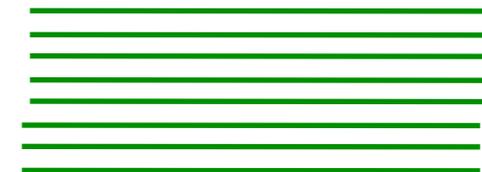
Free system



low-energy:  
one-to-one  
correspondence



Interacting  
system



How does this break down?

# Atomic Limit of Hubbard Model

$$H_{\text{Hubb}} \rightarrow U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$x = \#$  of holes

## Atomic Limit of Hubbard Model

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spectral weight:  $x$ -dependent

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density of states

$1+x$

$1-x$

PES

IPES

# Atomic Limit of Hubbard Model

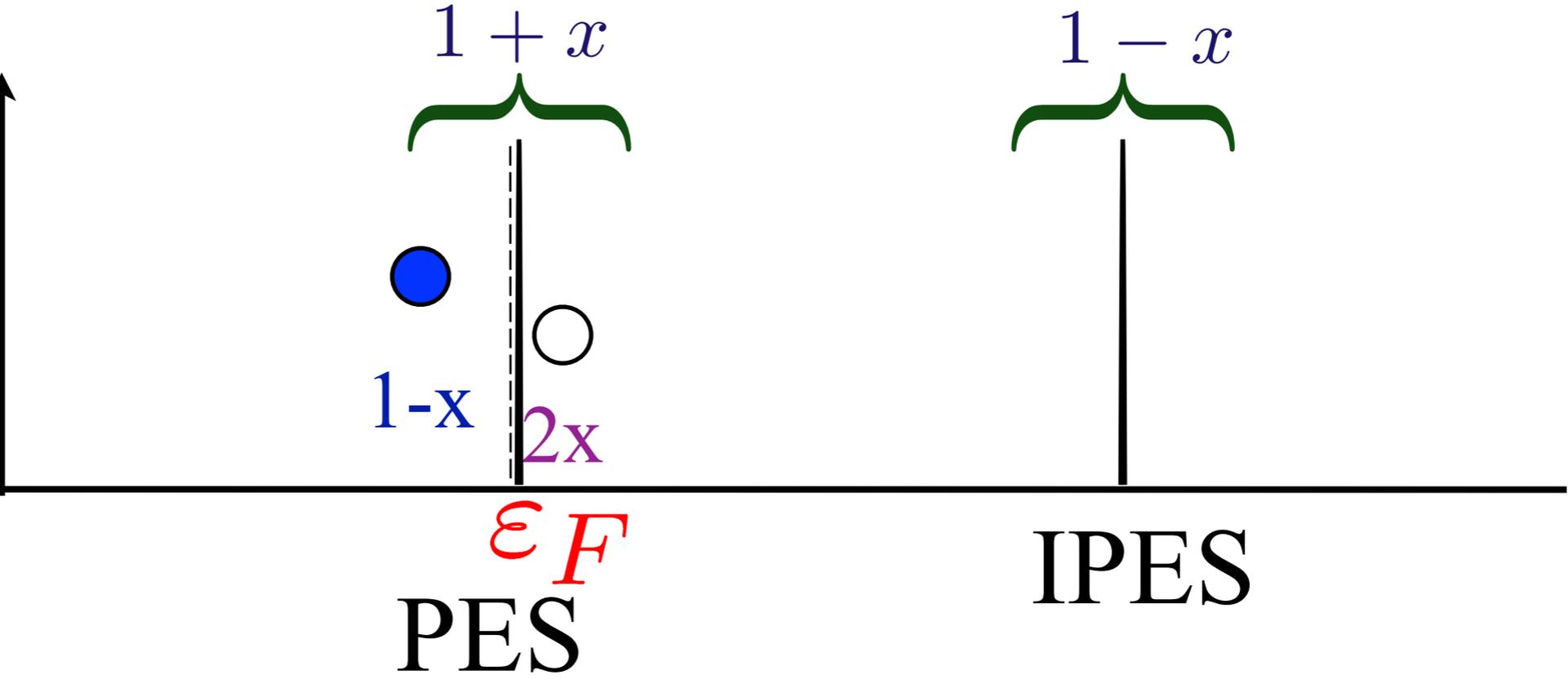
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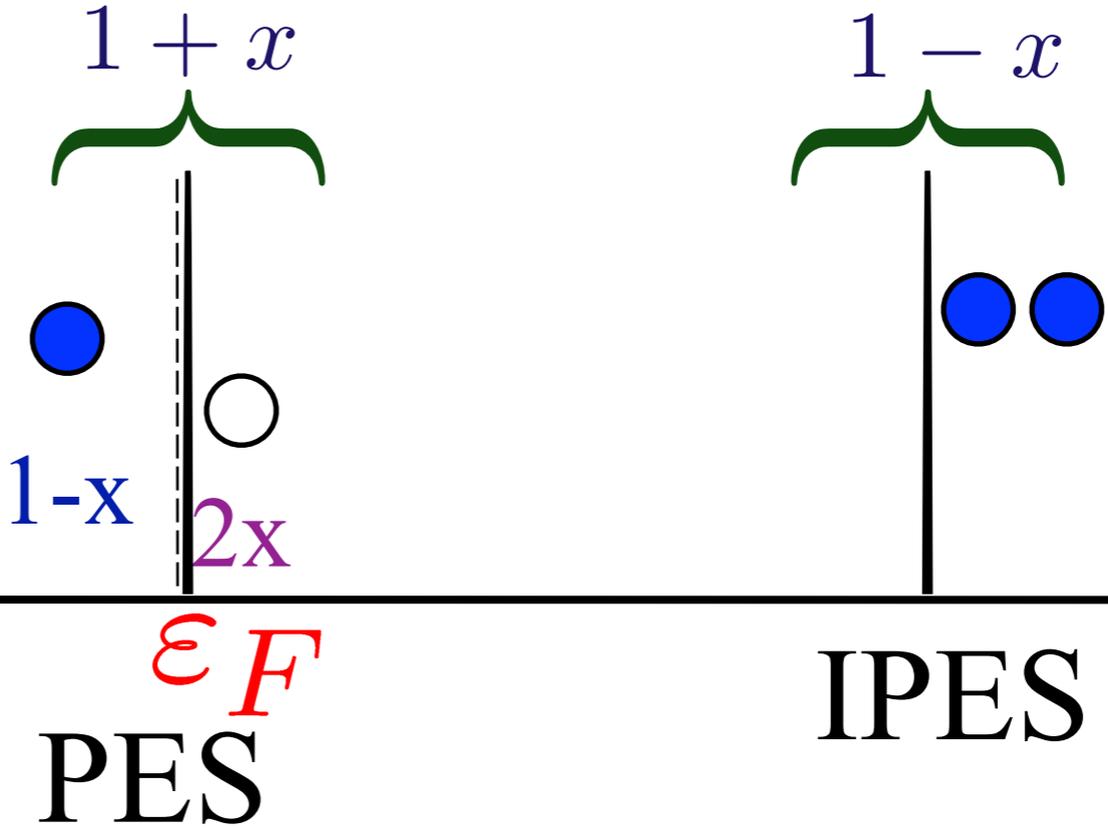
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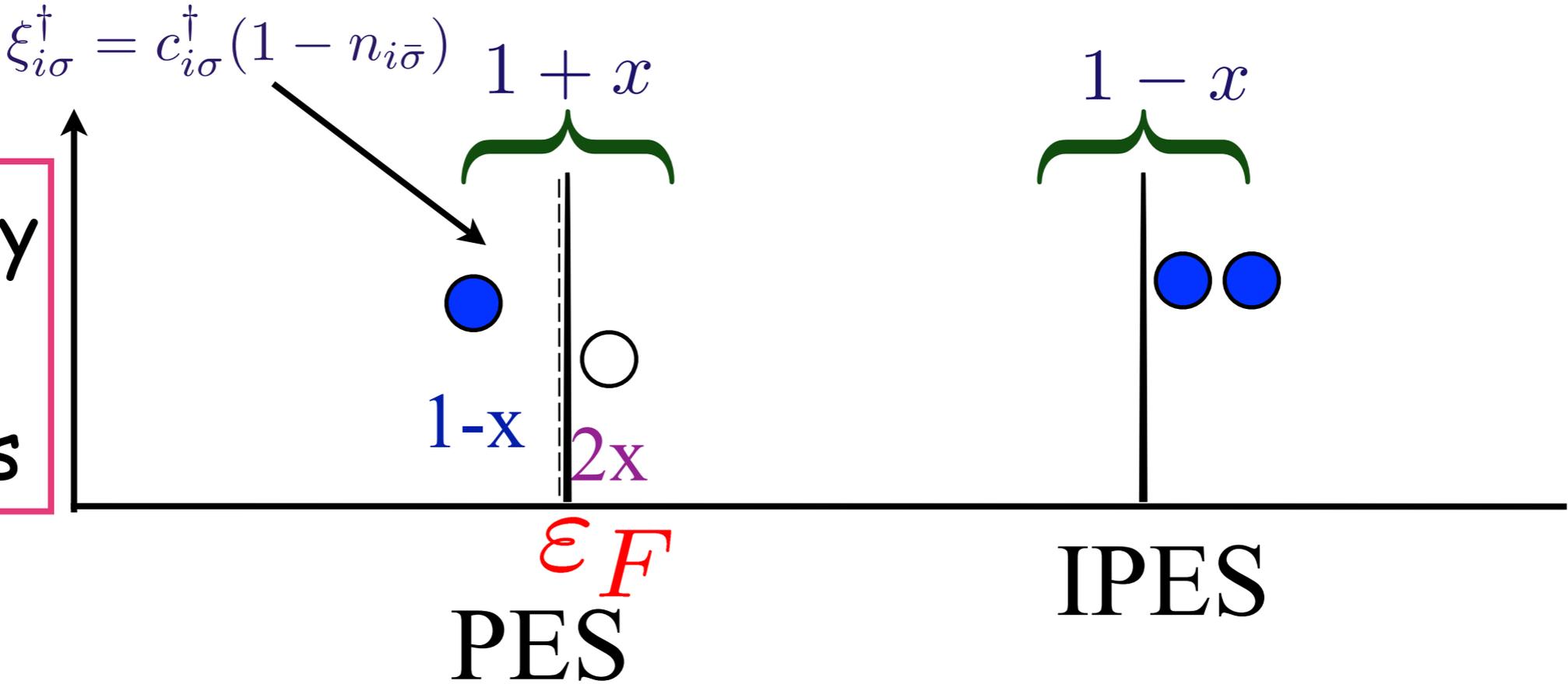
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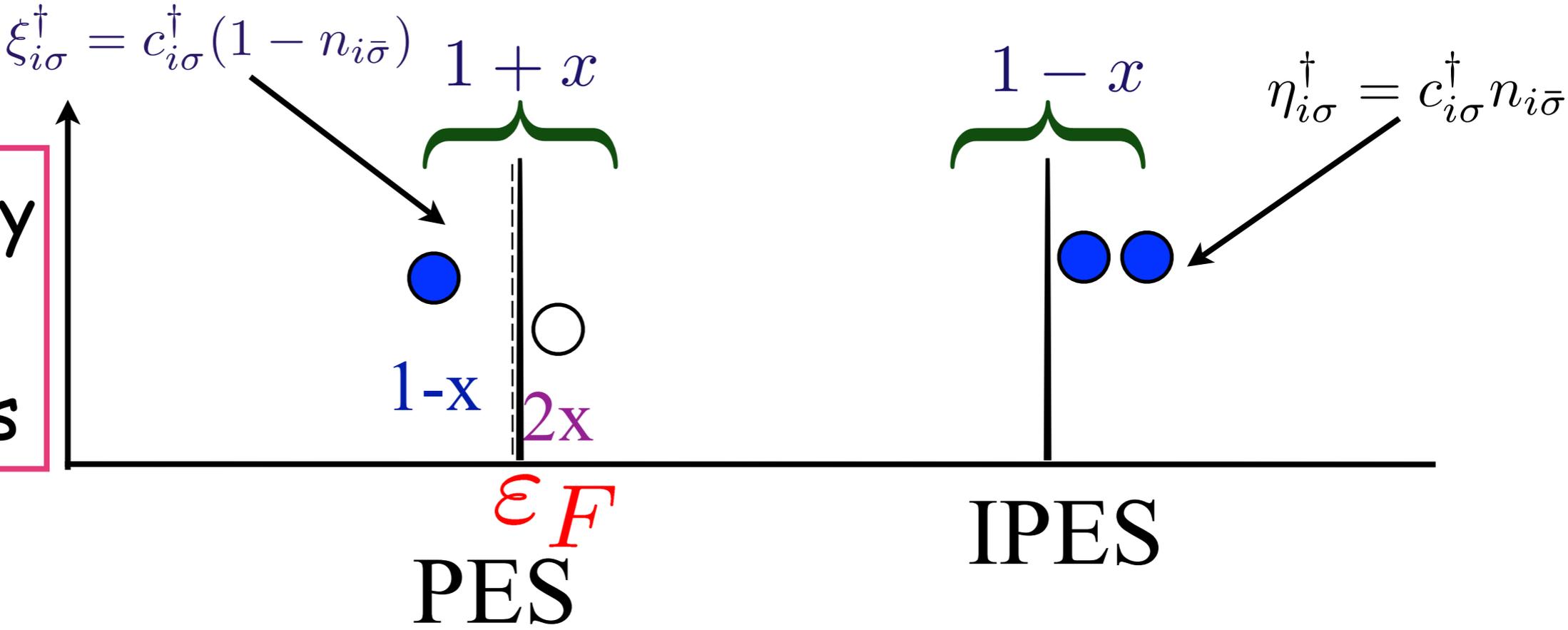
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atomic limit

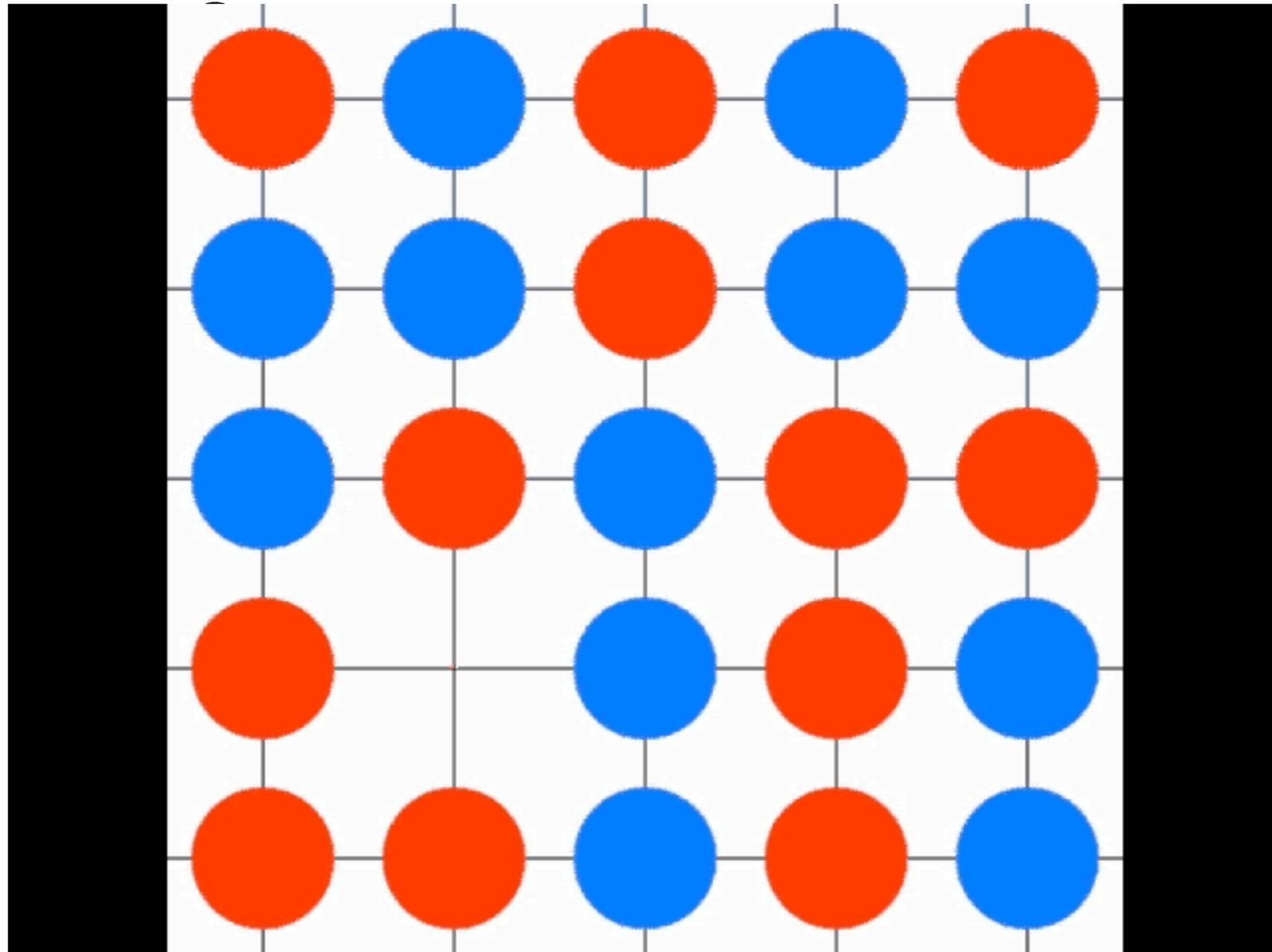
total weight =  $1+x$  = # of ways electrons  
can be added in lower band

intensity of lower band = # of electrons the  
band can hold

no problems yet!

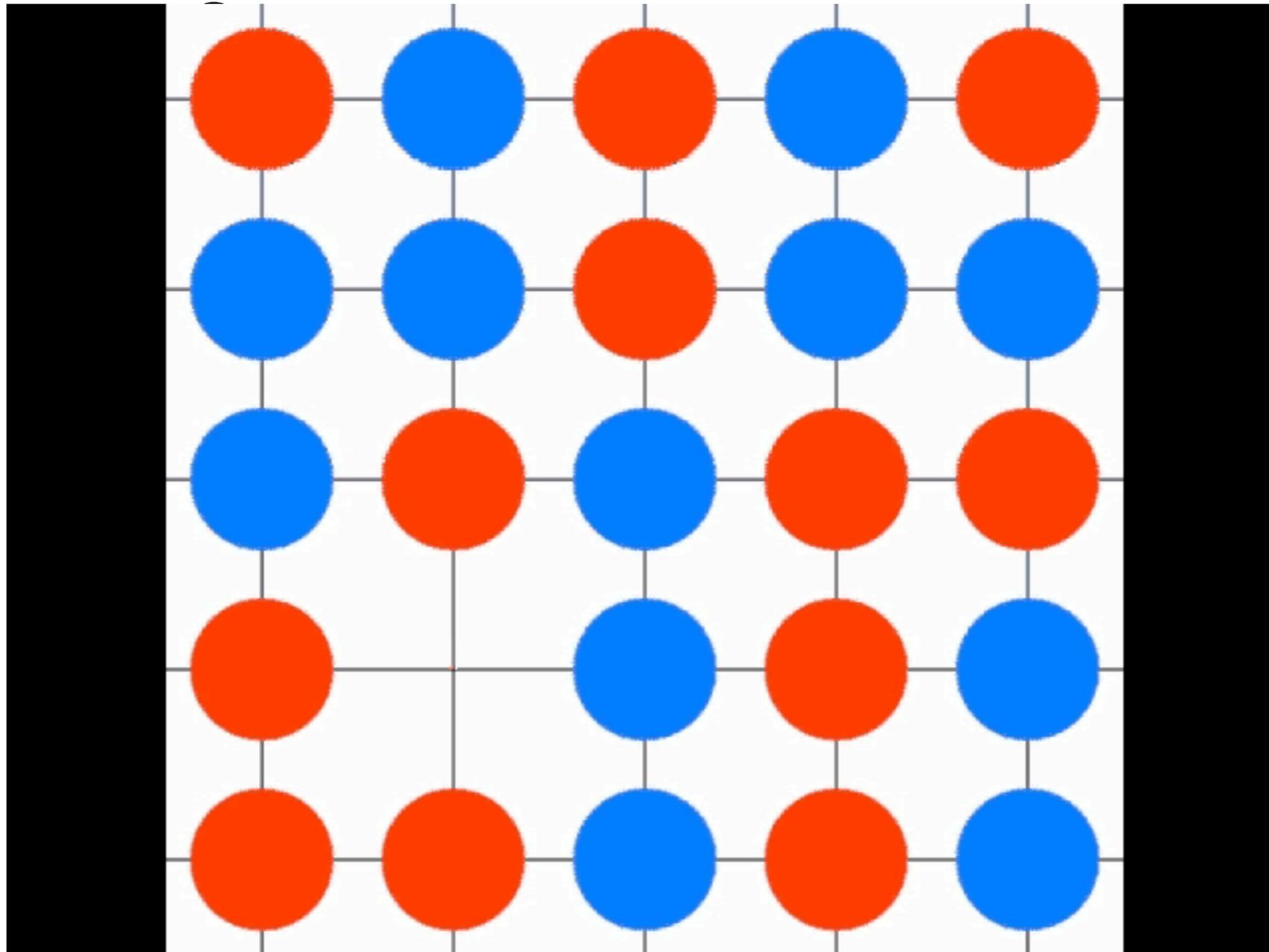
quantum Mottness:  $U$  finite

$$U \gg t$$



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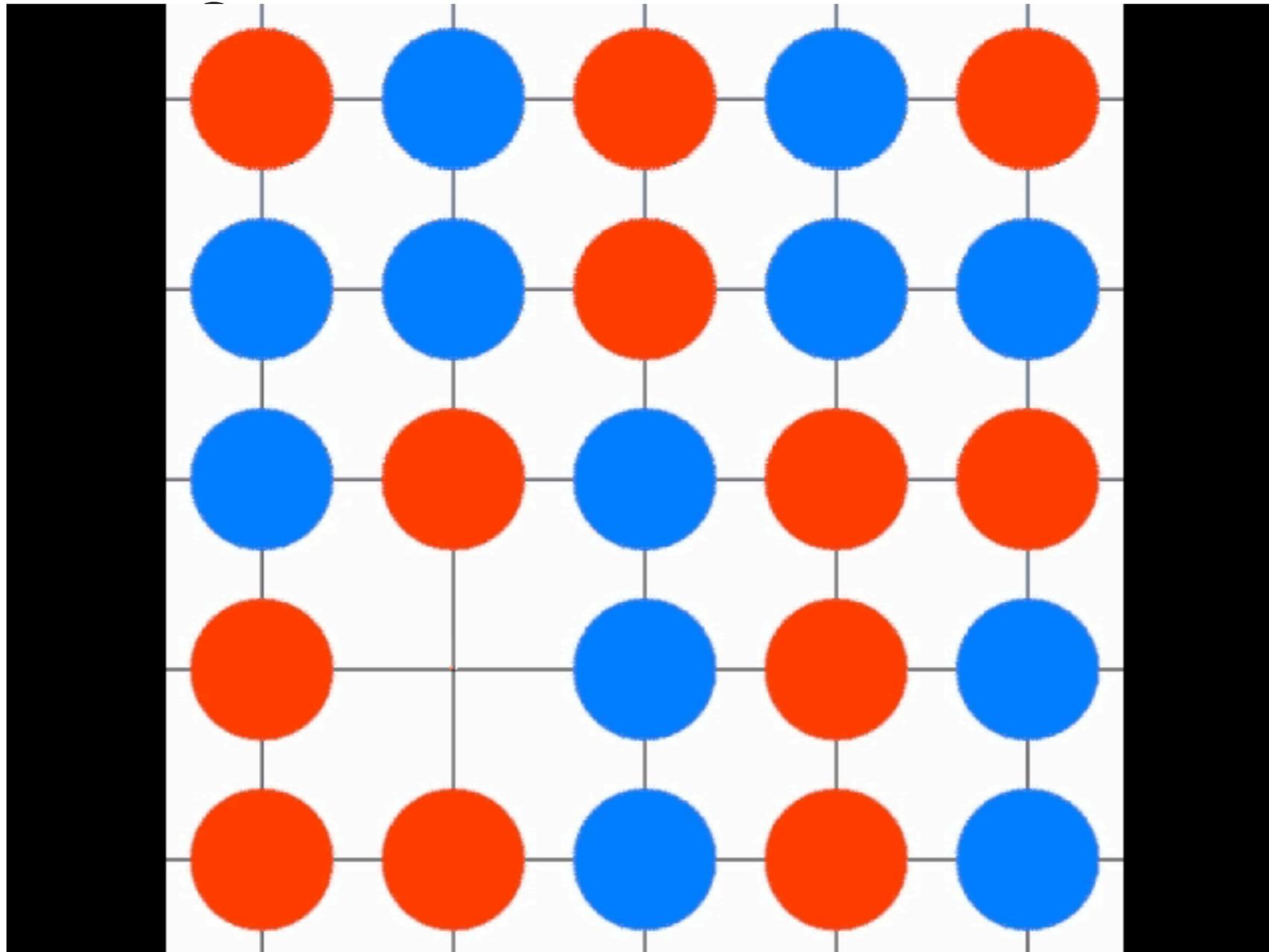
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double occupancy in ground (all)  
state!!

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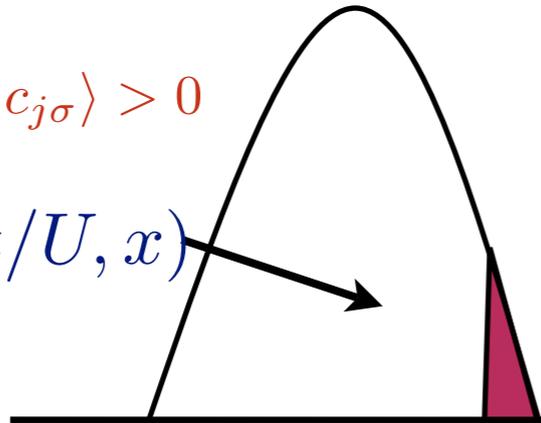
double occupancy in ground (all)  
state!!

$$W_{\text{PES}} > 1 + x$$

Harris & Lange, 1967

$$\alpha = \frac{t}{U} \sum_{ij} \langle c_{i\sigma}^\dagger c_{j\sigma} \rangle > 0$$

$$1 + x + \alpha(t/U, x)$$

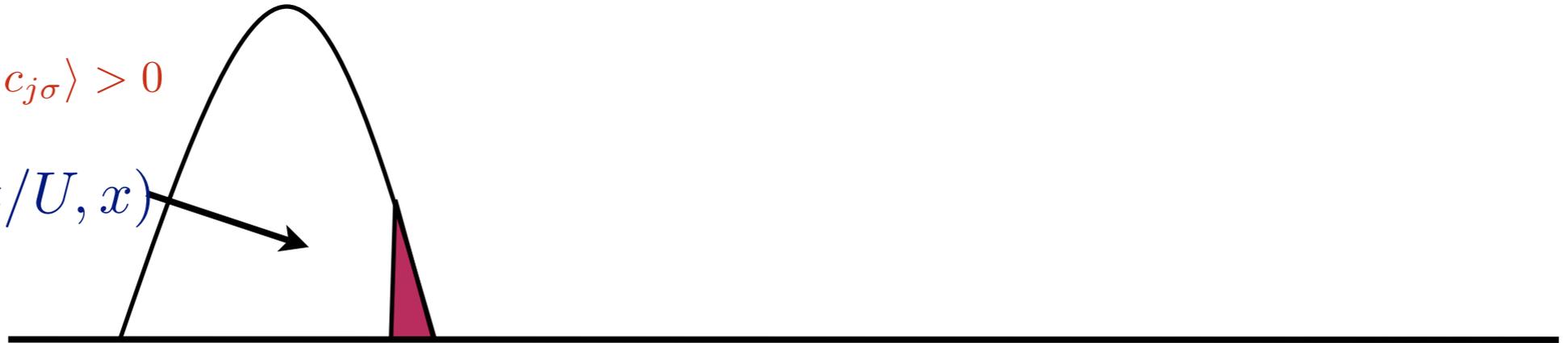


the rest of this state lives at high energy

Harris & Lange, 1967

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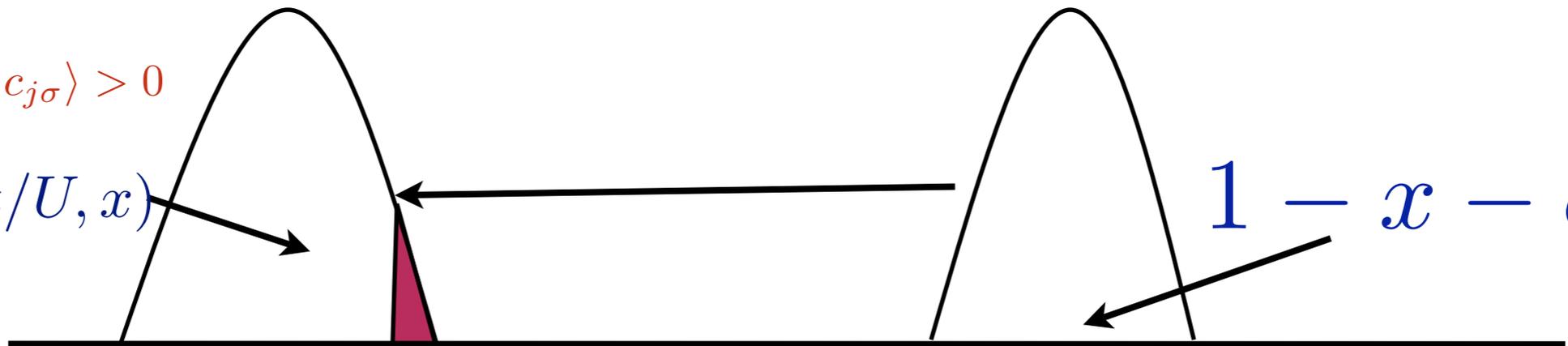


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$$1 - x - \alpha$$

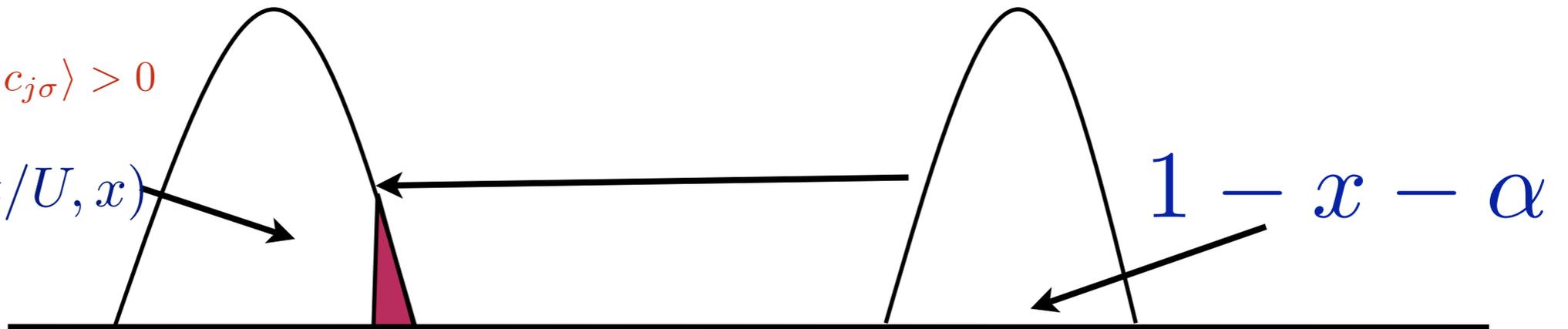


Harris & Lange, 1967

# dynamical spectral weight transfer

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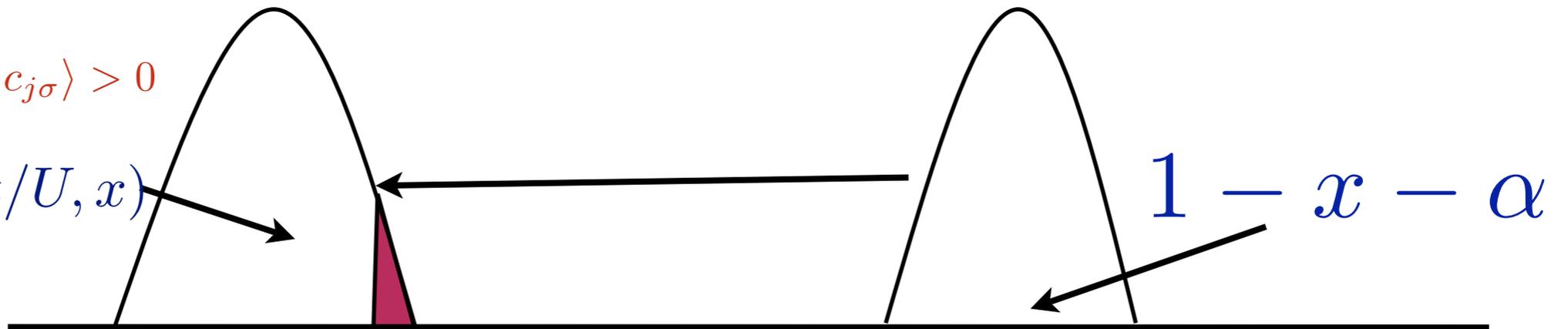


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Intensity  $> 1+x$



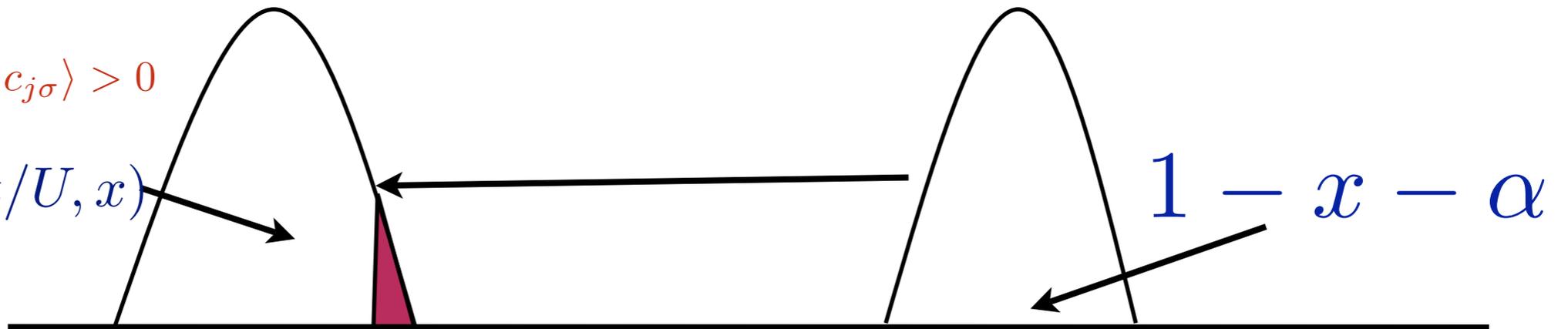
New non-electron degrees of freedom

Harris & Lange, 1967

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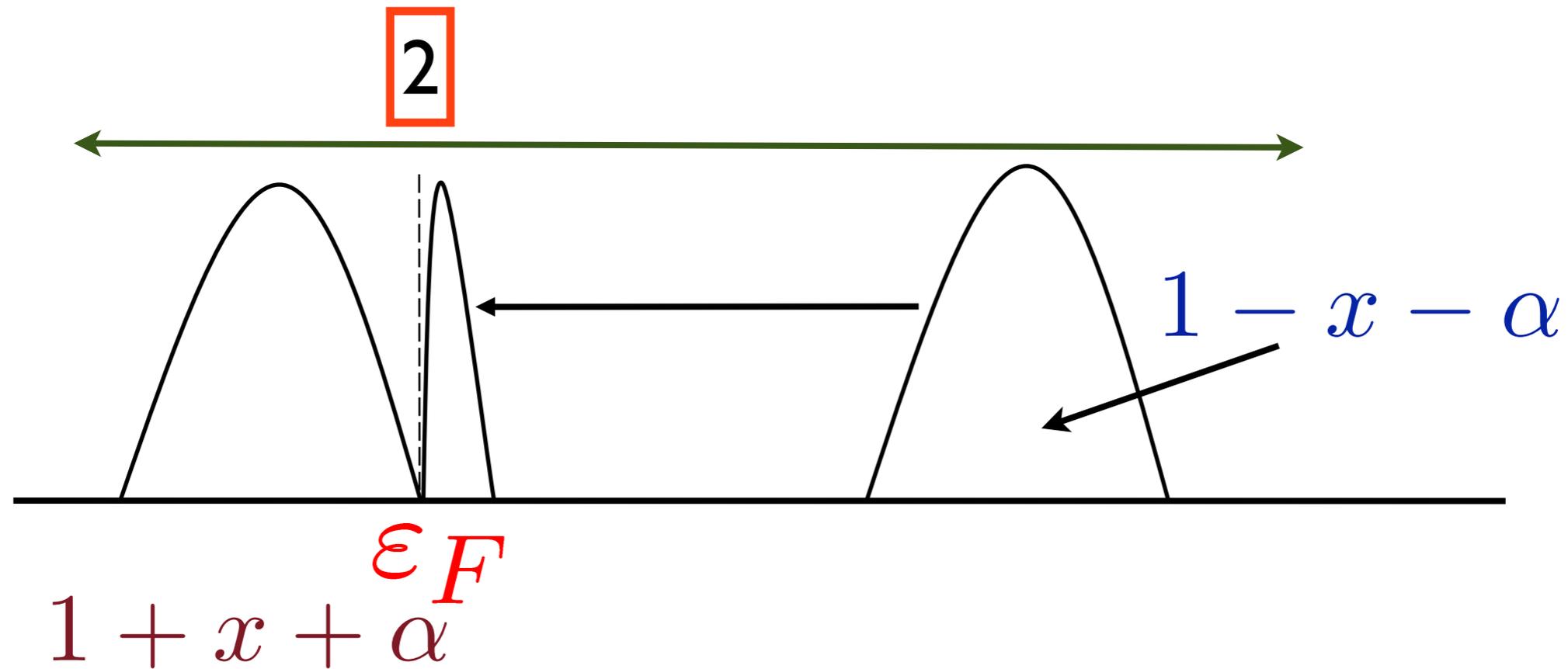


New non-electron degrees of freedom

Can this system be a Fermi liquid?

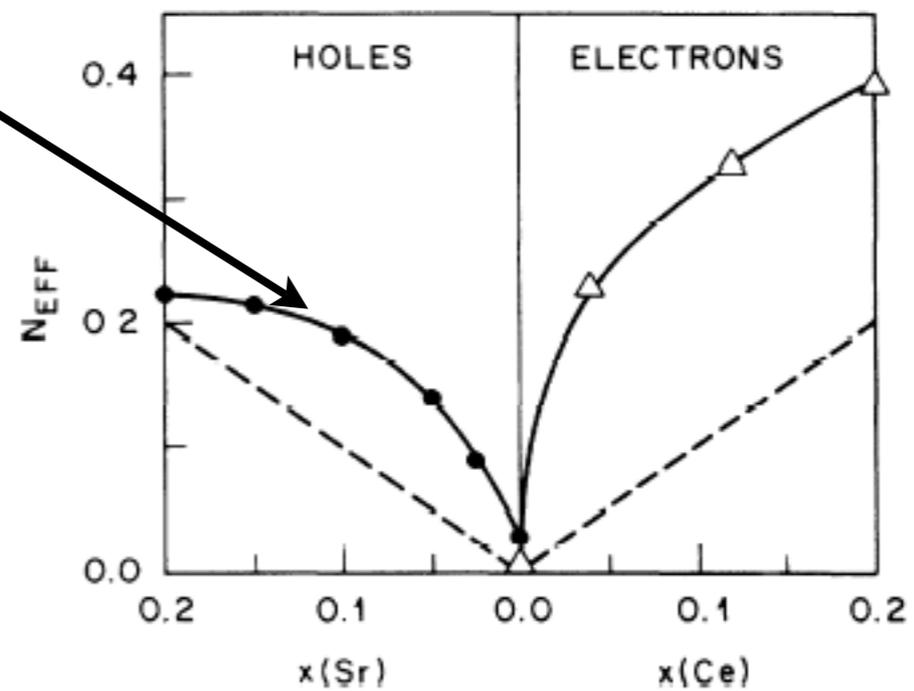
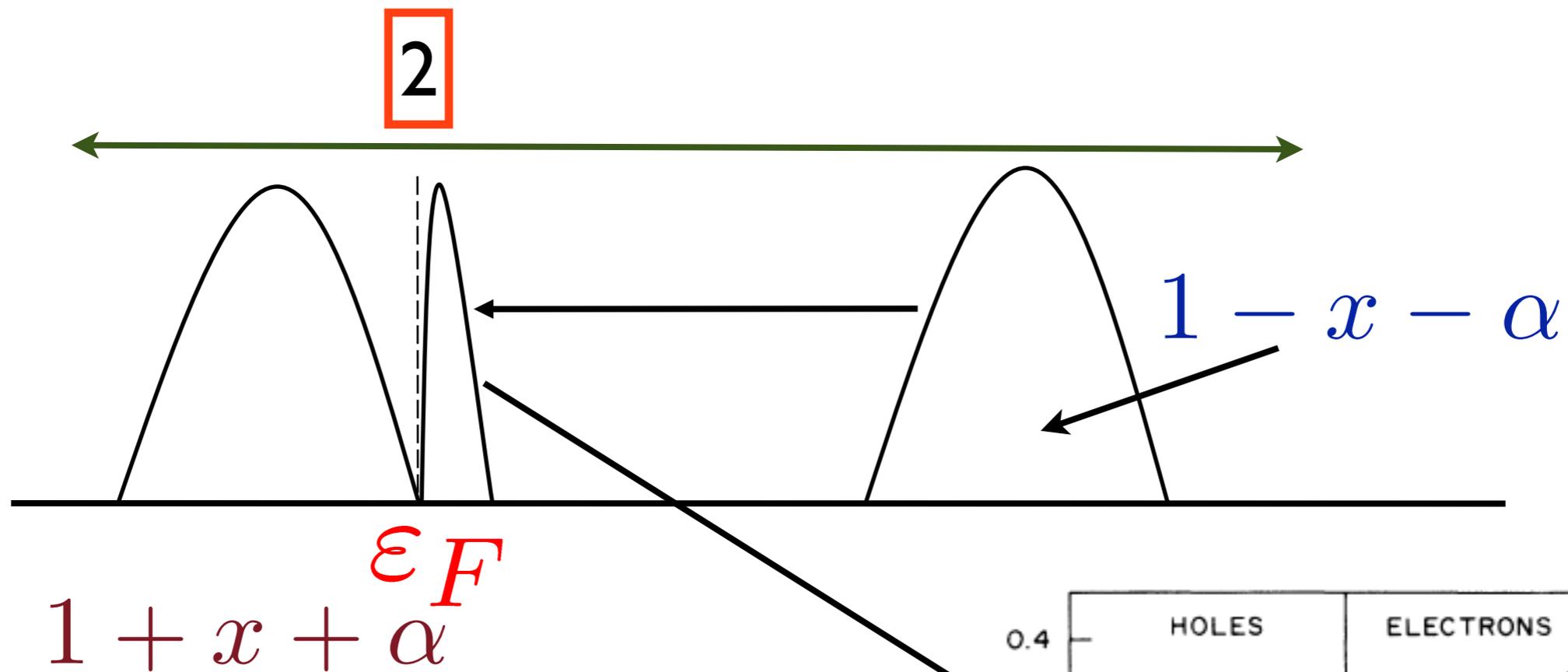
No

what are the low-energy fermionic excitations?



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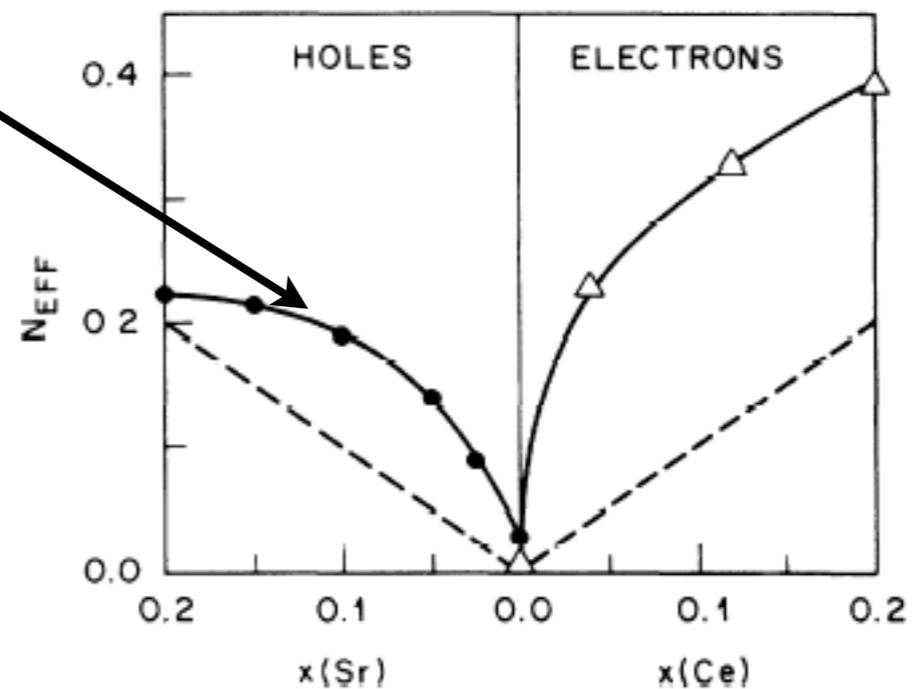
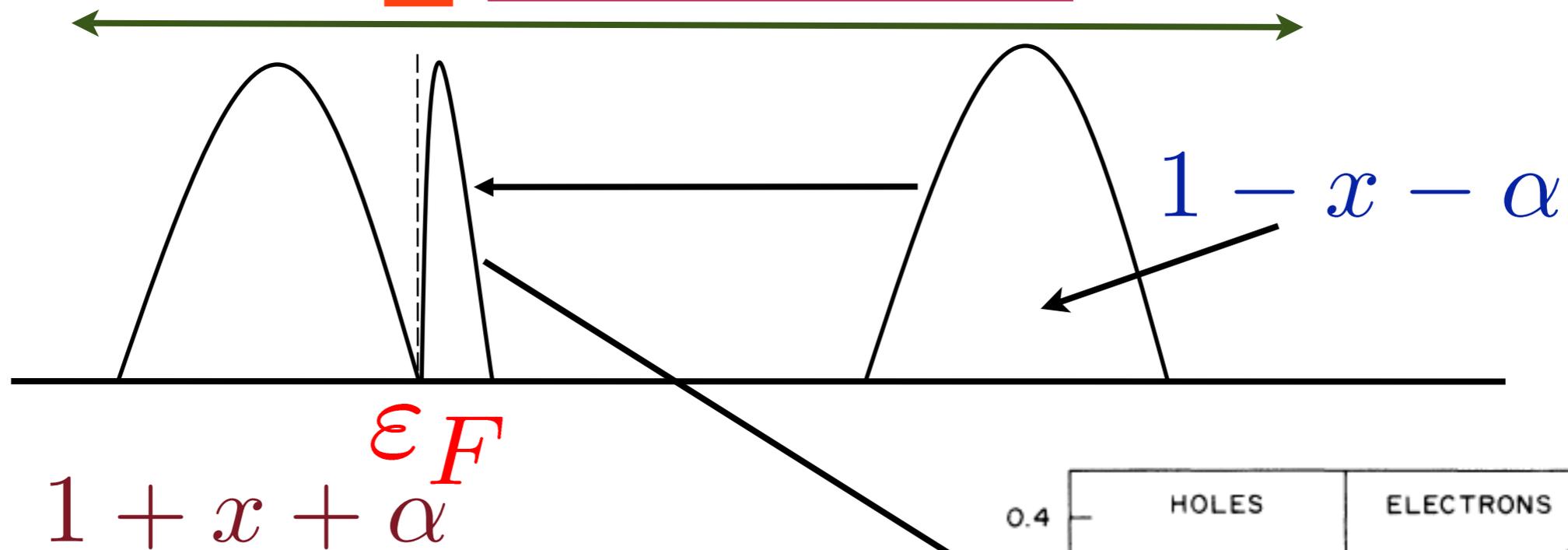
2



# what are the low-energy fermionic excitations?

2

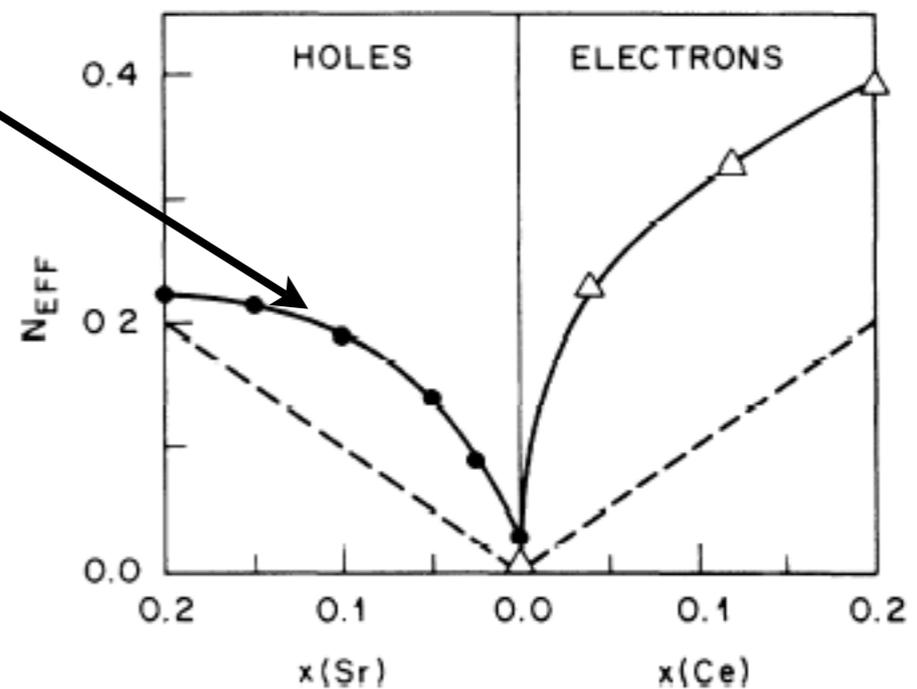
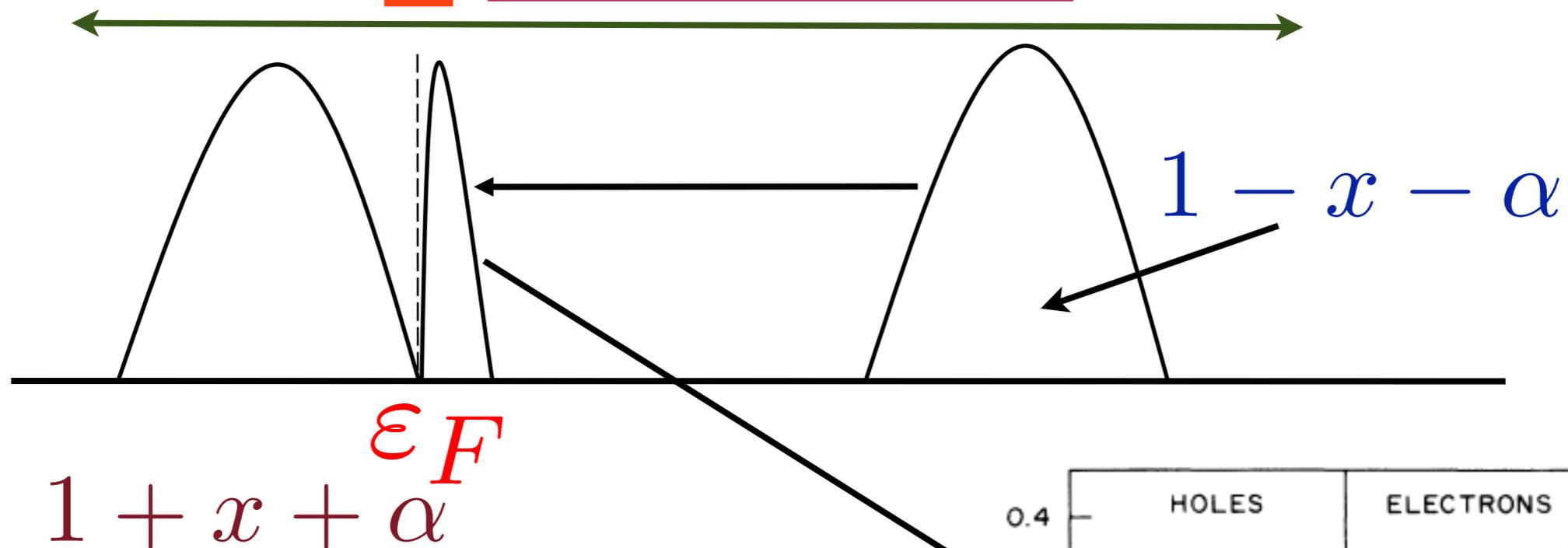
re-definition:  $x' = x + \alpha$



what are the low-energy fermionic excitations?

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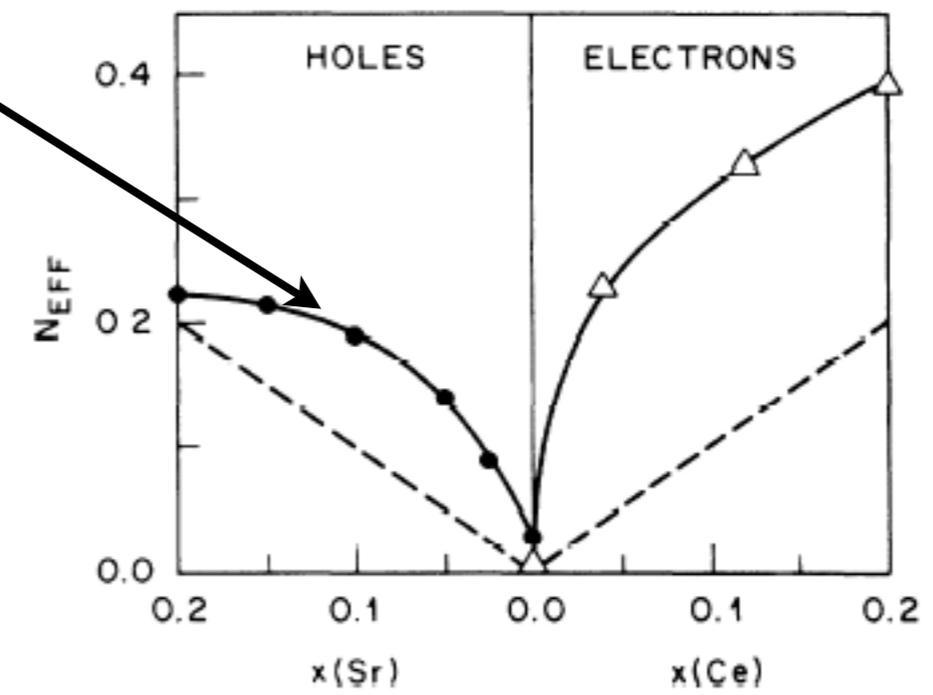
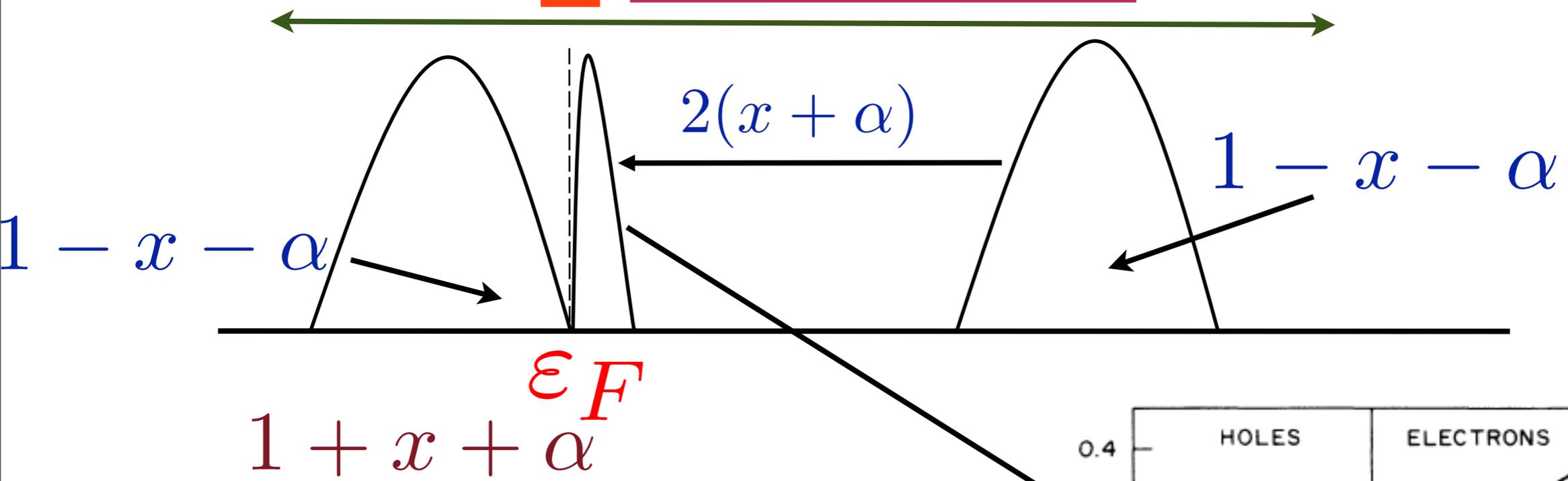
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get rid of dynamical mixing

what are the low-energy fermionic excitations?

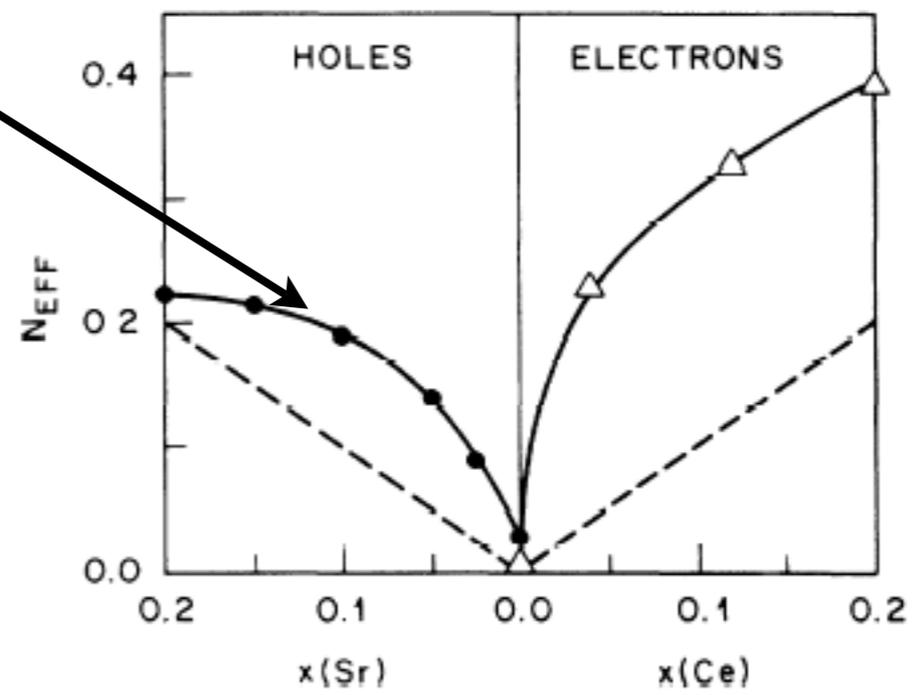
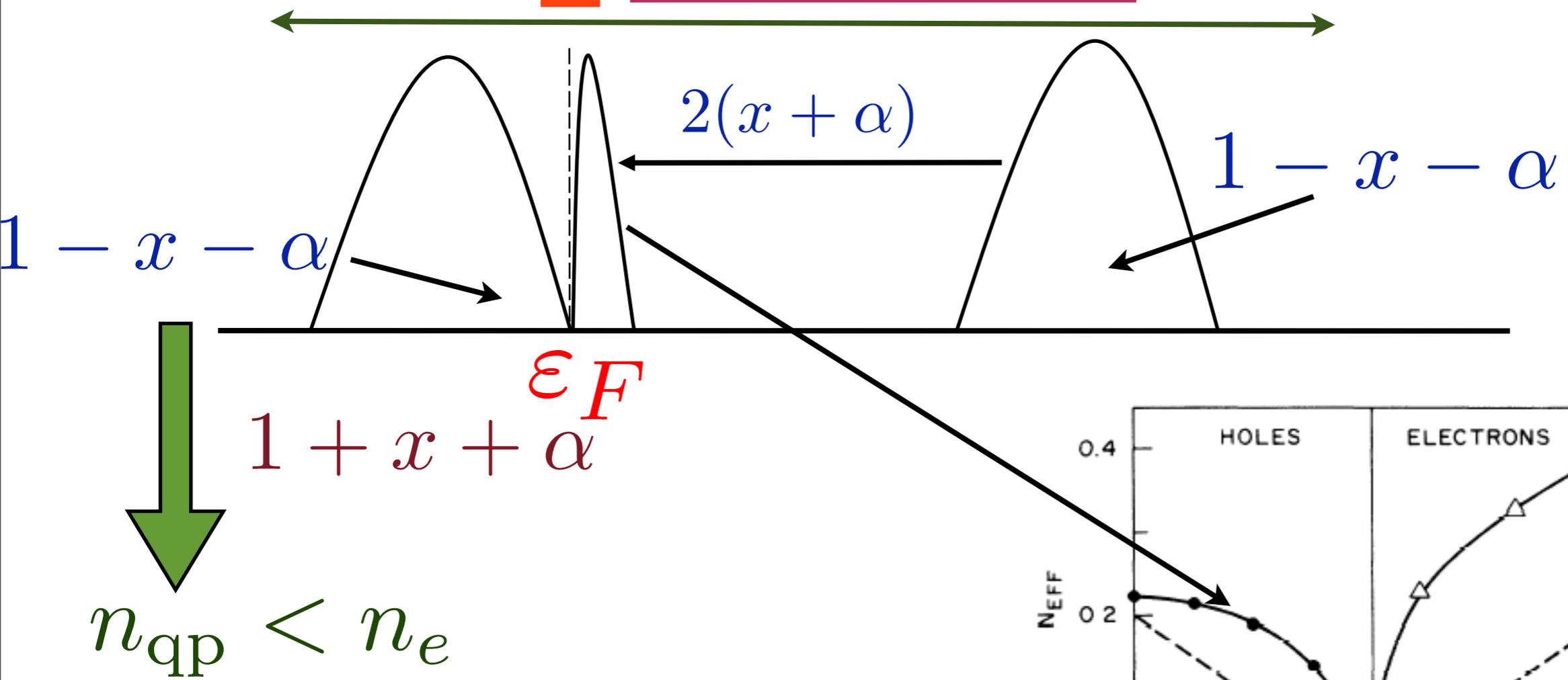
2 re-definition:  $x' = x + \alpha$



get rid of dynamical mixing

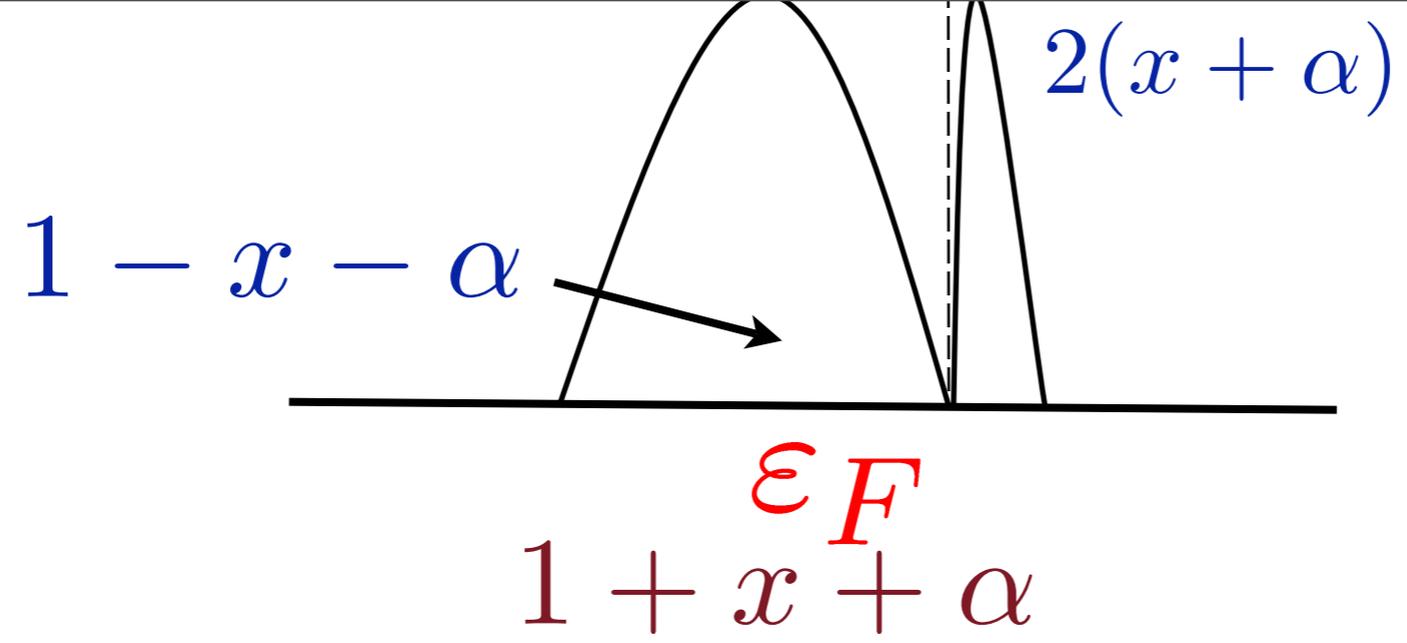
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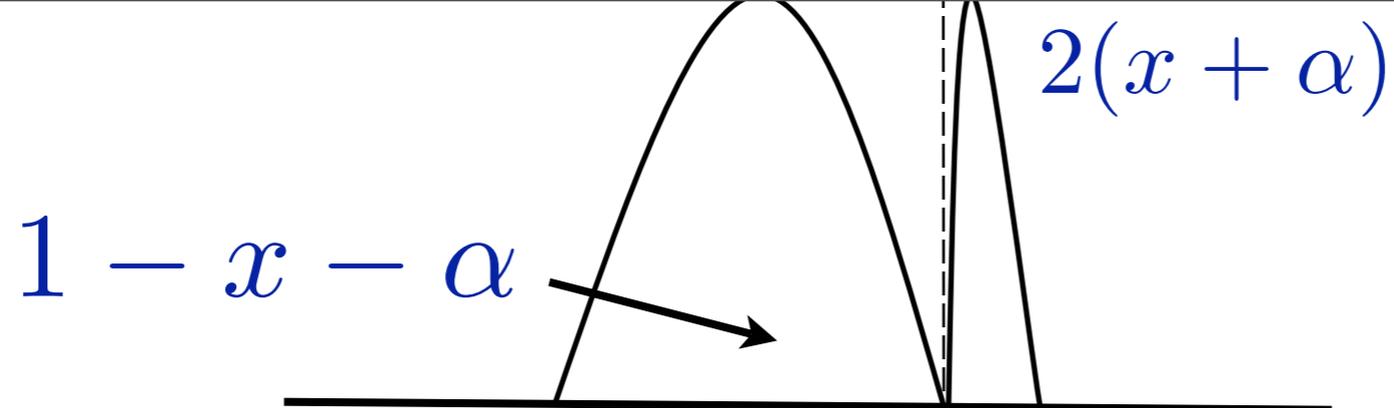
2 re-definition:  $x' = x + \alpha$



FLT breaks down

get rid of dynamical mixing

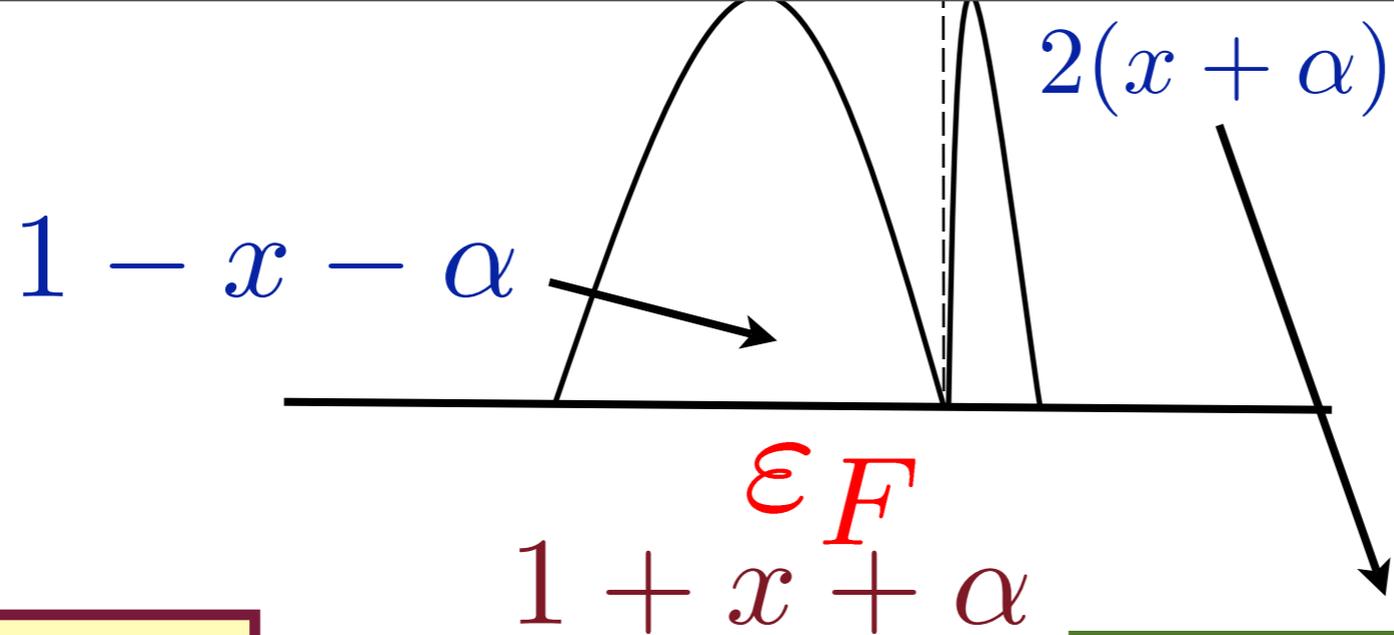




$$1 + x + \alpha$$

new degrees  
of freedom

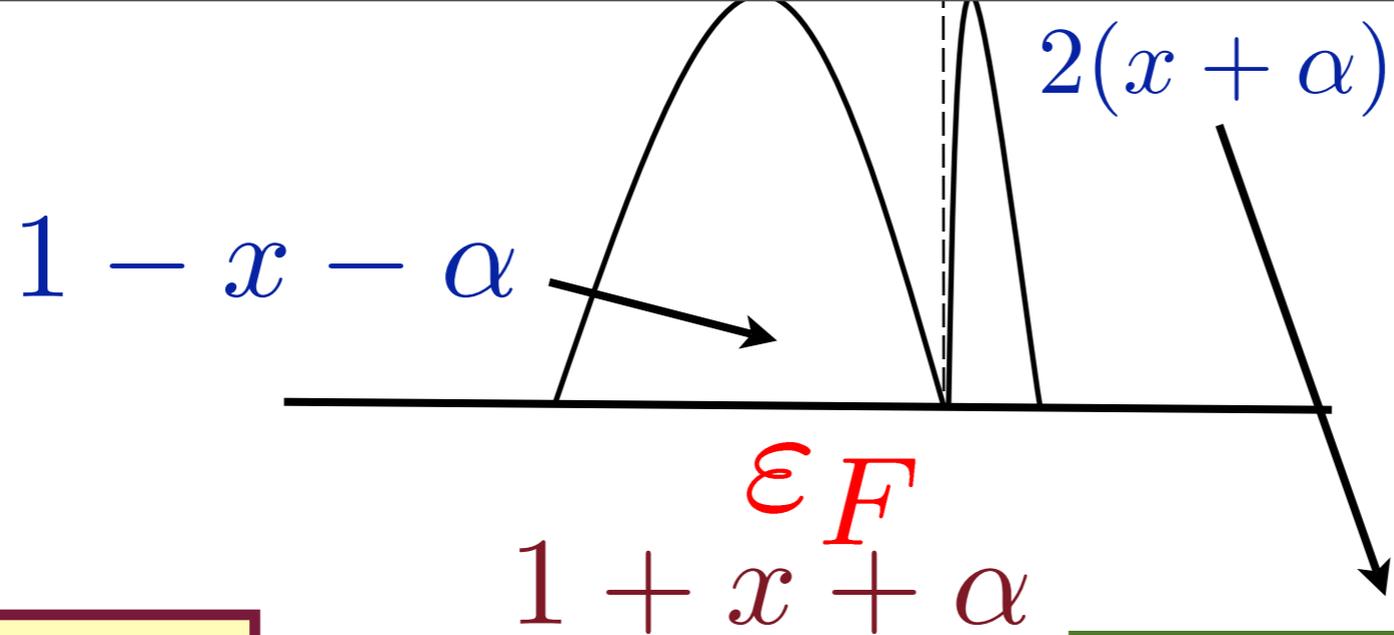
number of fermionic qp  
< number of bare  
electrons  $\Rightarrow$  FL theory  
breaks down



new degrees  
of freedom

# of addition states  
per e per spin > 1

number of fermionic qp  
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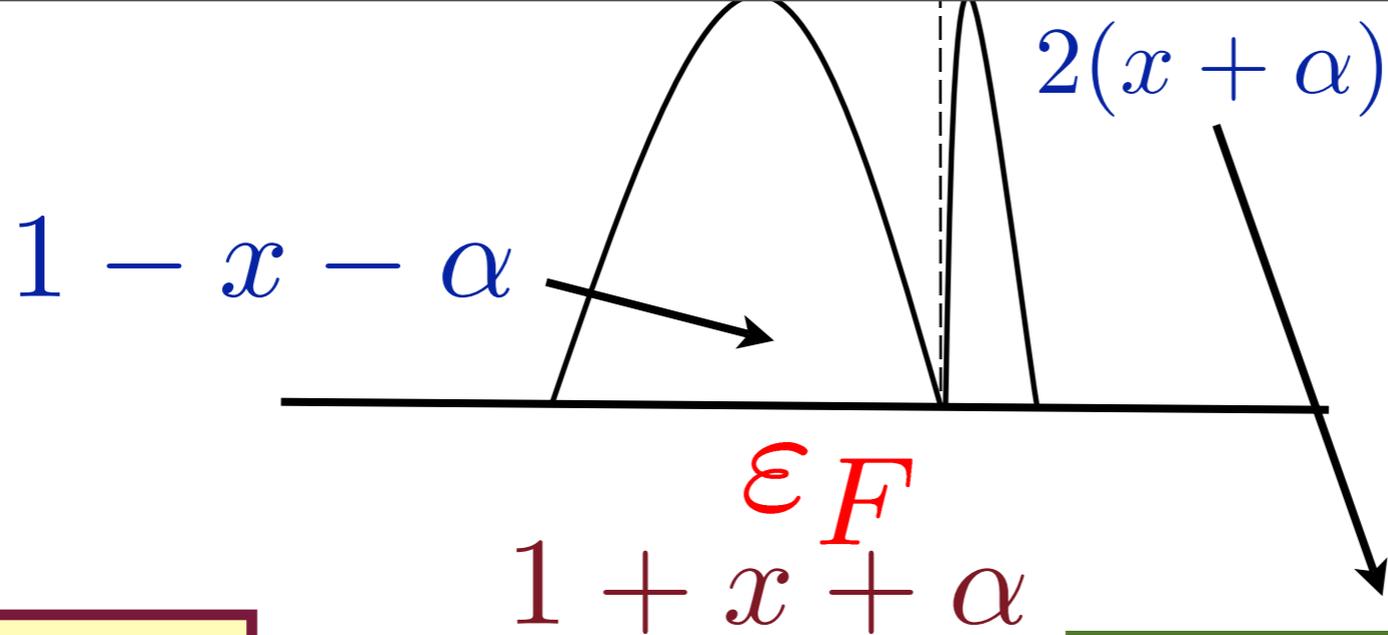


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number of fermionic qp < number of bare electrons => FL theory breaks down

ways to add a particle but not an electron (gapped spectrum)



new degrees of freedom

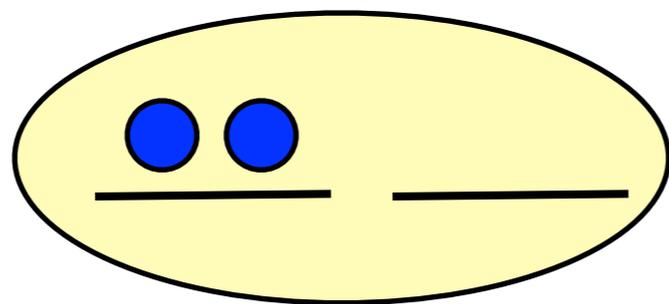
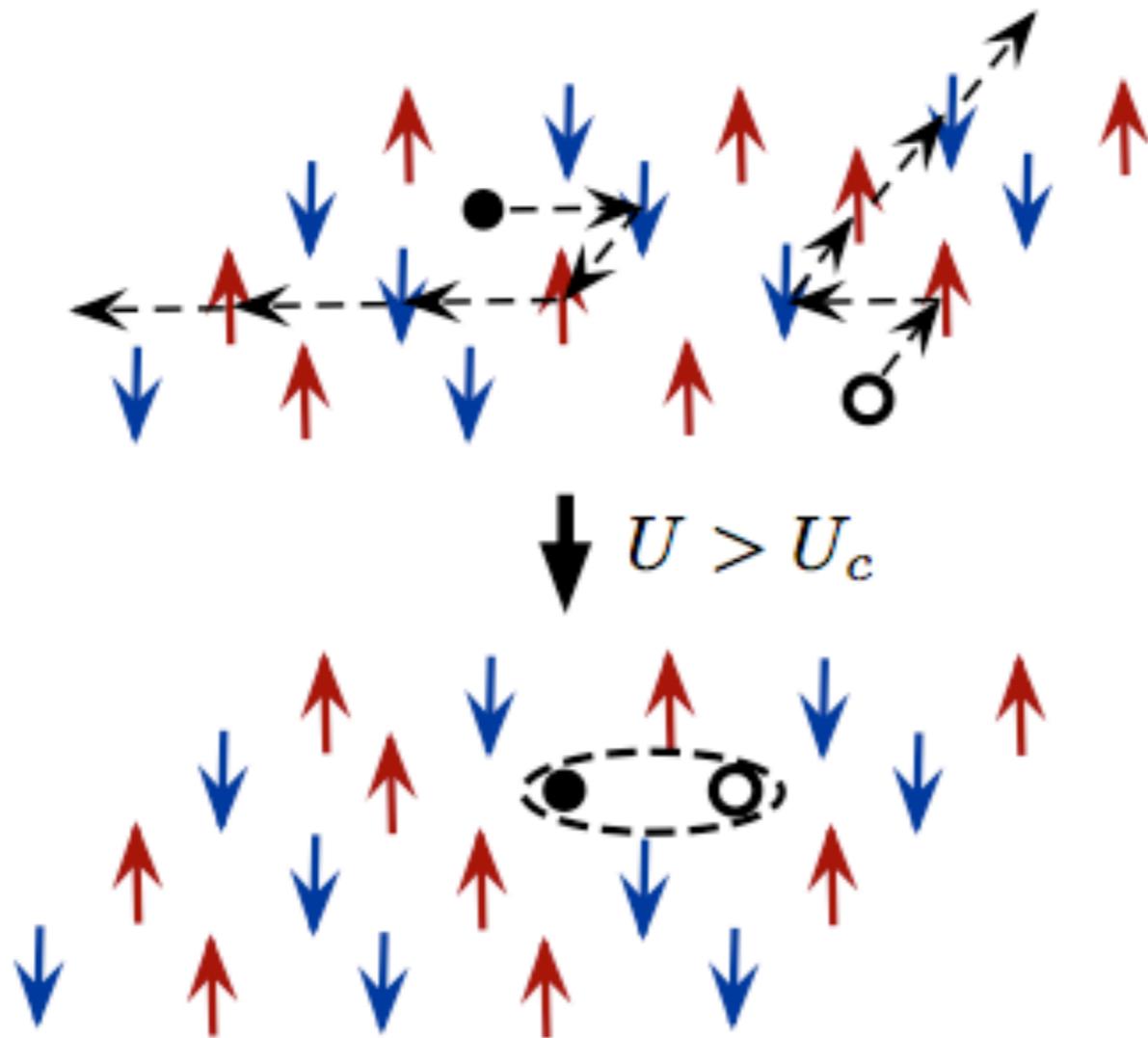
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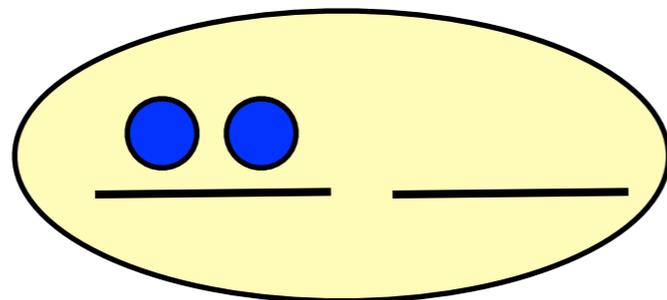
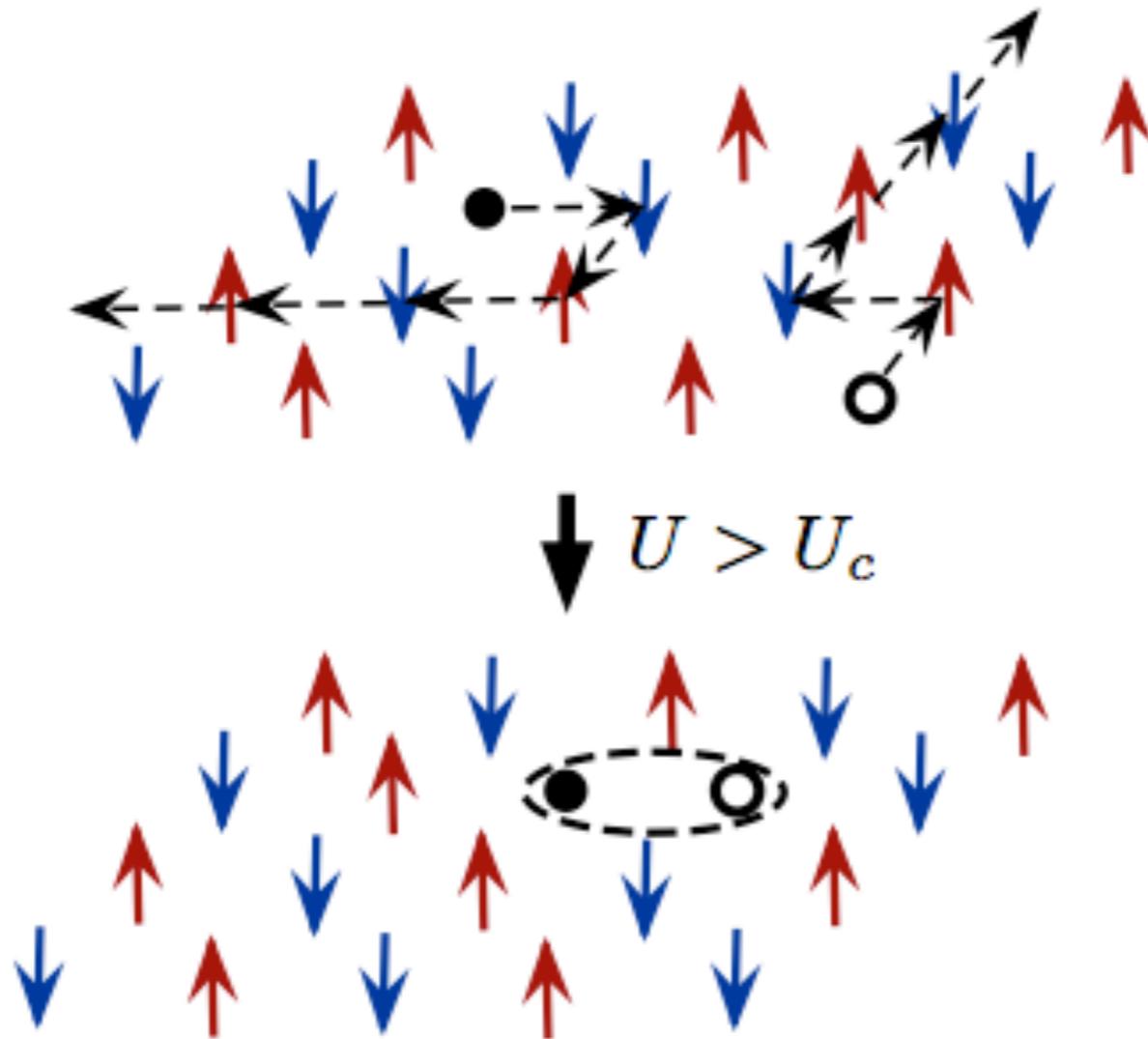
breakdown of electron quasi-particle picture: Mottness

# Same Physics at half-filling



No proof exists?  
Mottness is ill-defined

# Same Physics at half-filling



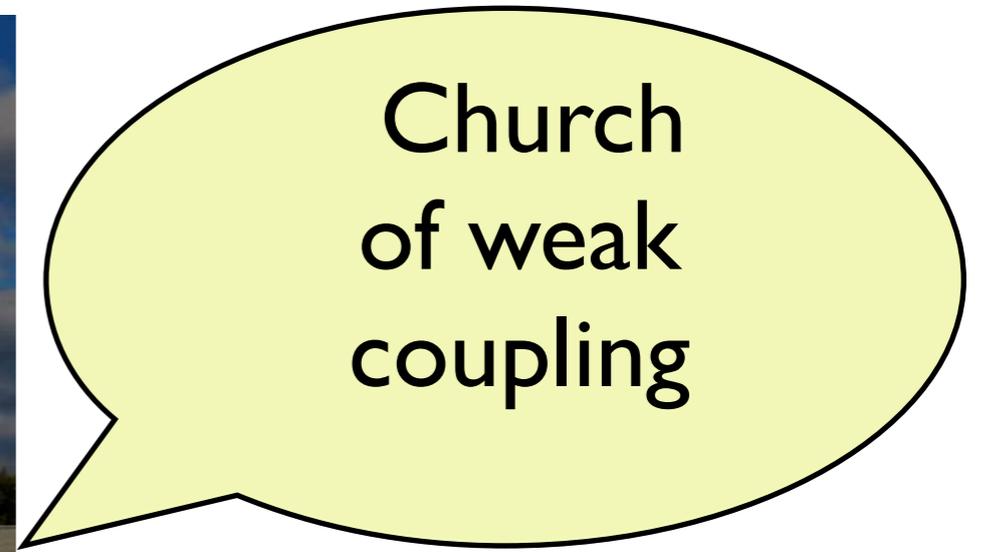
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## A Critique of Two Metals

R. B. Laughlin

idea is either missing or improperly understood. Another indicator that something is deeply wrong is the inability of anyone to describe the elementary excitation spectrum of the Mott insulator precisely even as pure phenomenology. Nowhere can one find a quantitative band structure of the elementary particle whose spectrum becomes gapped. Nowhere can one find precise information about the particle whose gapless spectrum causes the paramagnetism. Nowhere can one find information about the interactions among these particles or of their potential bound state spectroscopies. Nowhere can one find precise definitions of Mott insulator terminology. The upper and lower Hubbard bands, for example, are vague analogues of the valence and conduction bands of a semiconductor, except that they coexist and mix with soft magnetic excitations no one knows how to describe very well.



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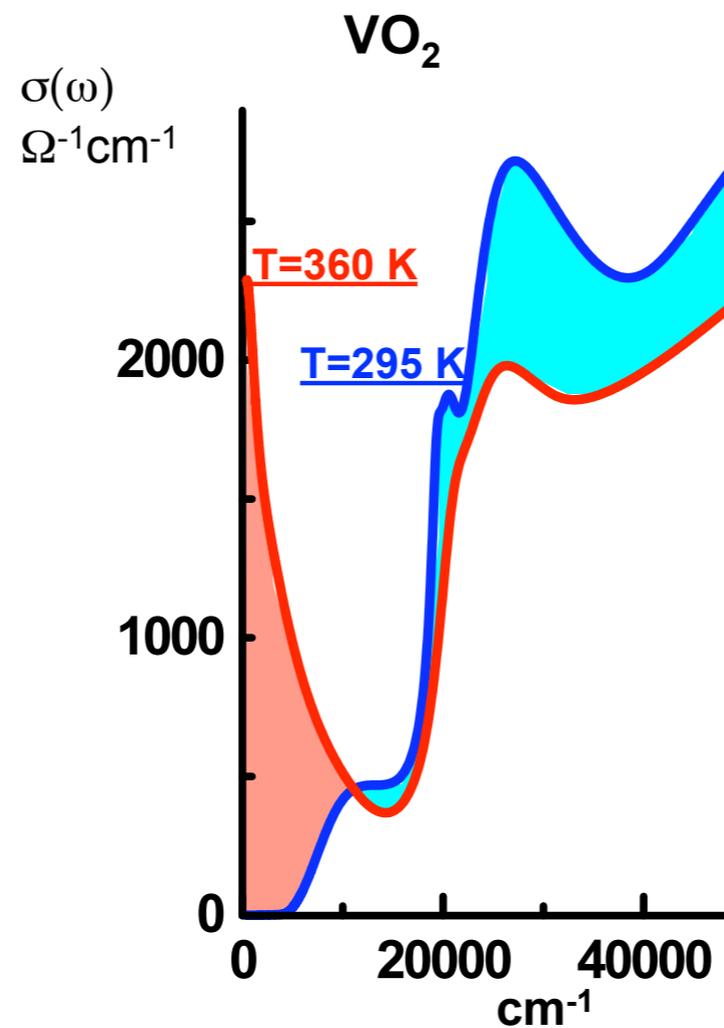
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Church  
of weak  
coupling

Beliefs:  
Mott gap is heresy?  
HF is the way!  
No UHB and LHB!

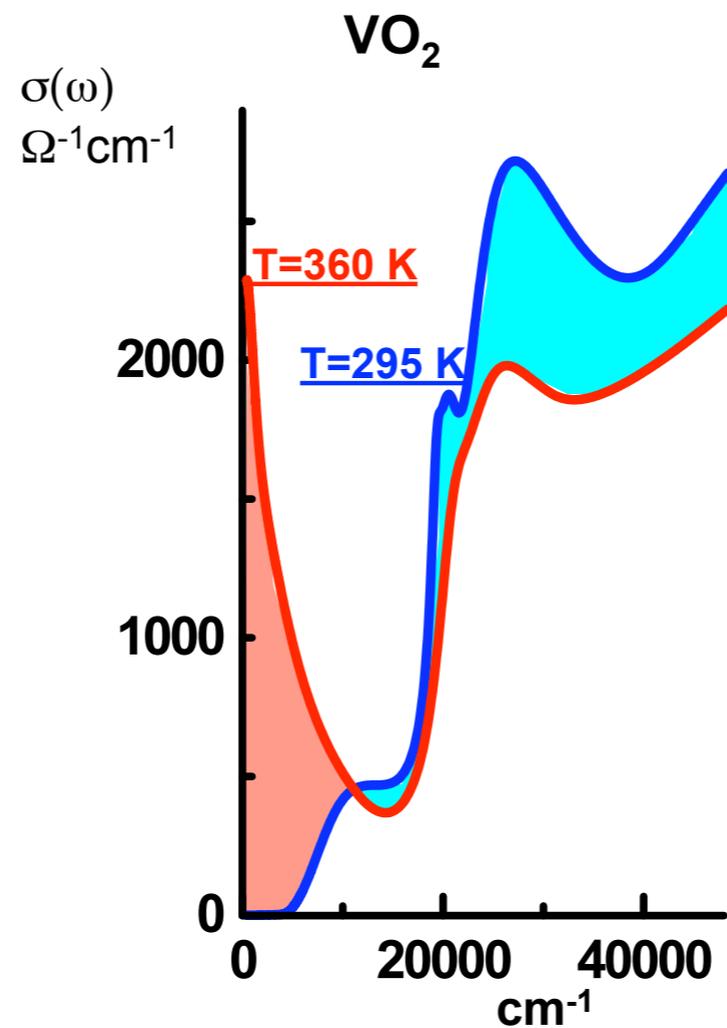
$$\Delta = 0.6eV > \Delta_{\text{dimerization}} \quad (\text{Mott, 1976}) \quad \frac{\Delta}{T_{\text{crit}}} \approx 20$$



transfer  
of spectral  
weight to  
high energies  
beyond any ordering  
scale

Recall,  $eV = 10^4 K$

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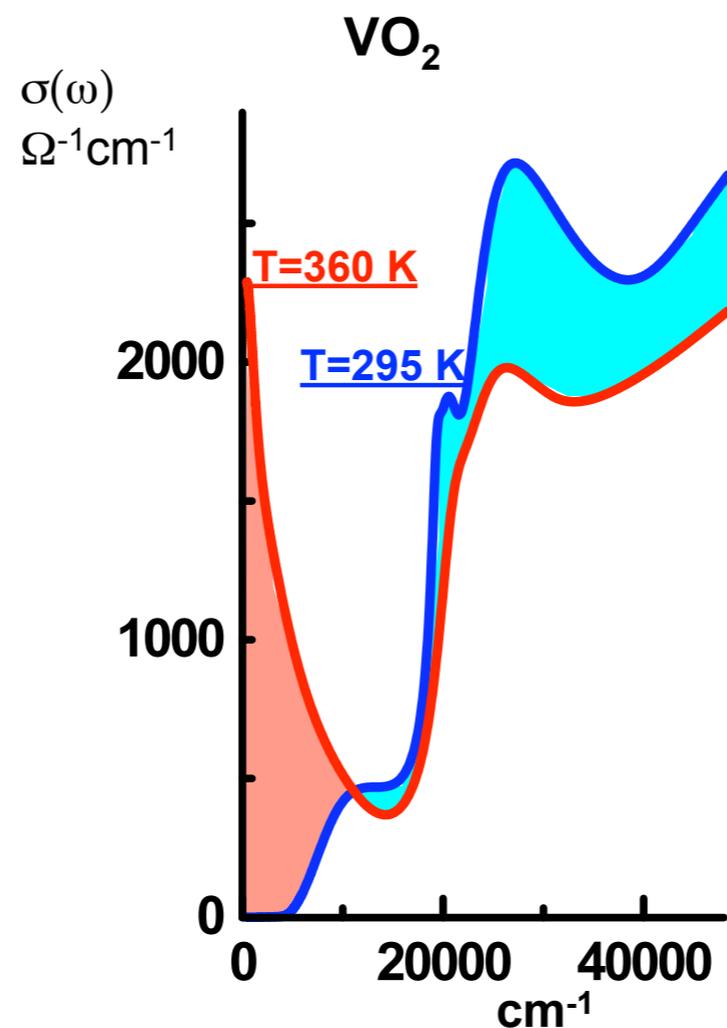
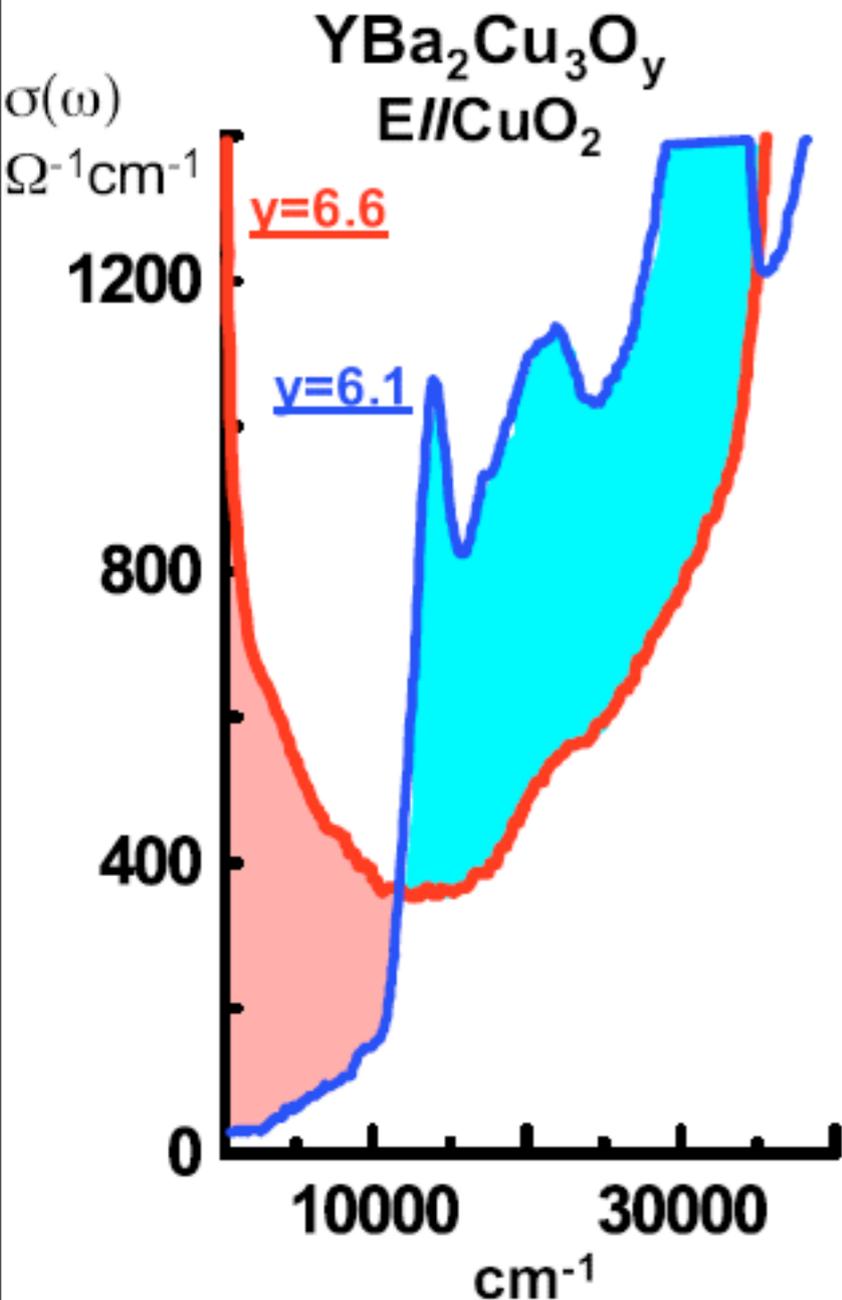


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*M. M. Qazilbash, K. S. Burch, D. Whisler,  
D. Shrekenhamer, B. G. Chae, H. T. Kim,  
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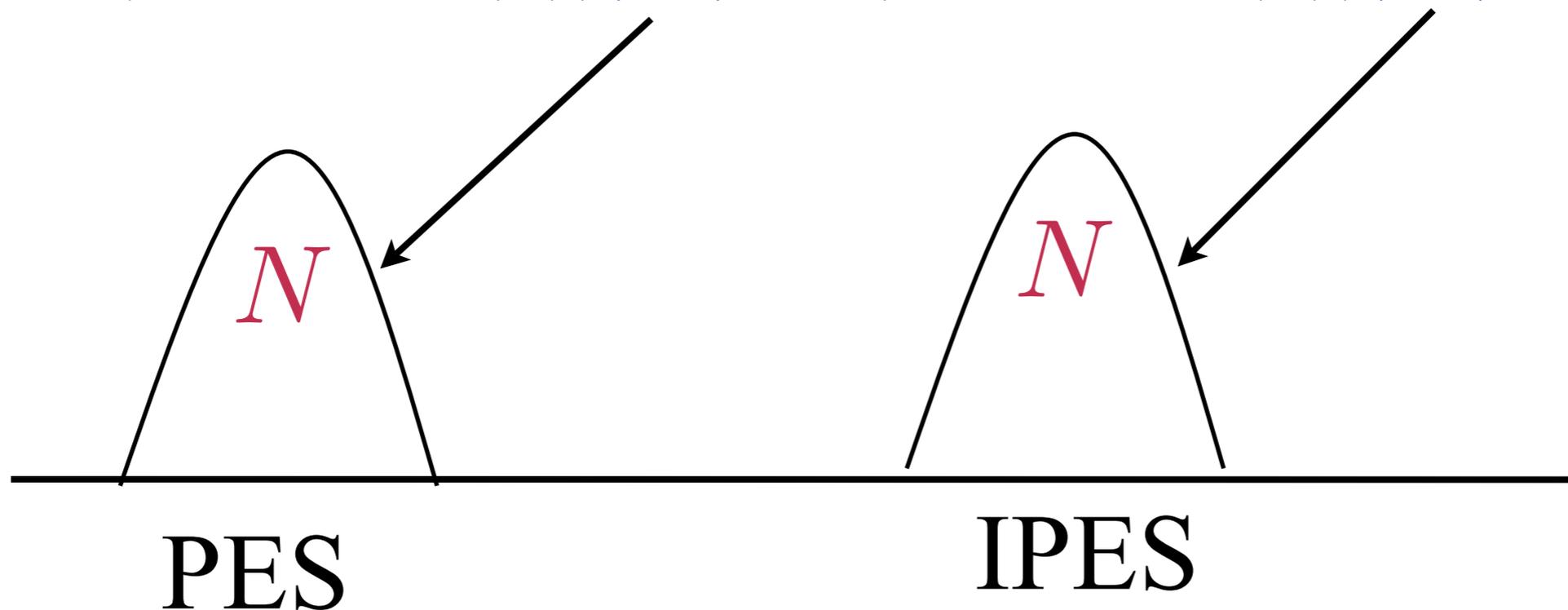
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# Fermi-liquid analogy

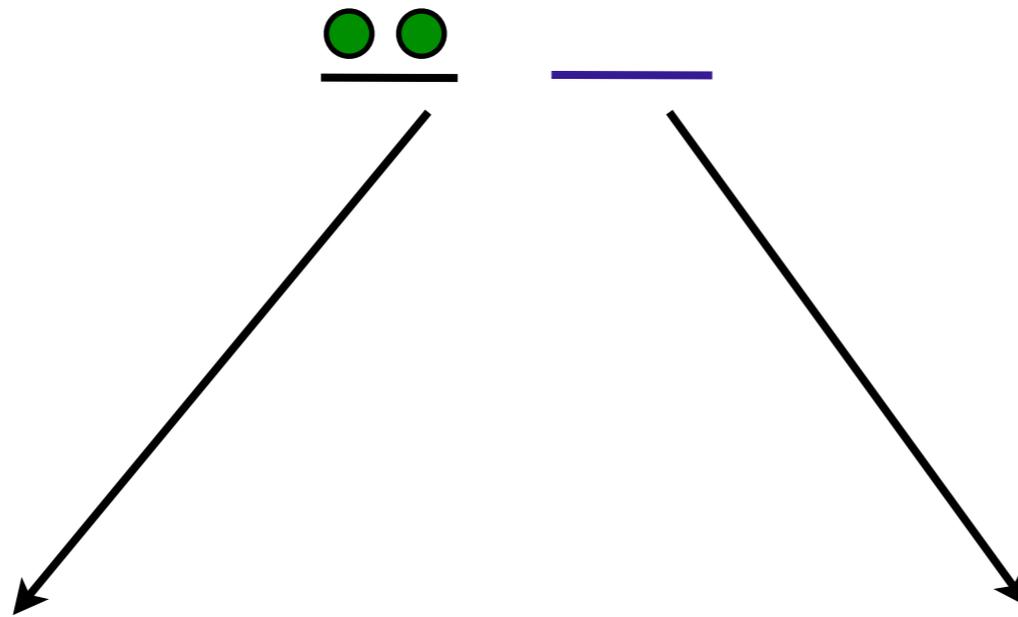
$$L_{\text{FL}} \propto (\omega - \epsilon_k) |\psi_k|^2$$

## Mott Problem?

$$L_{\text{MI}} = (\omega - E_{\text{LHB}}(k)) |\eta_k|^2 + (\omega - E_{\text{UHB}}(k)) |\tilde{\eta}_k|^2$$



composite excitation: bound state



half-filling:  
Mott gap

doping:  
SWT, pseudogap?

charge  $2e$  boson

$$W_{PES} > 1 + x$$

How?

Effective Theories:

$S(\phi)$  at half-filling

Integrate  
Out high  
Energy fields

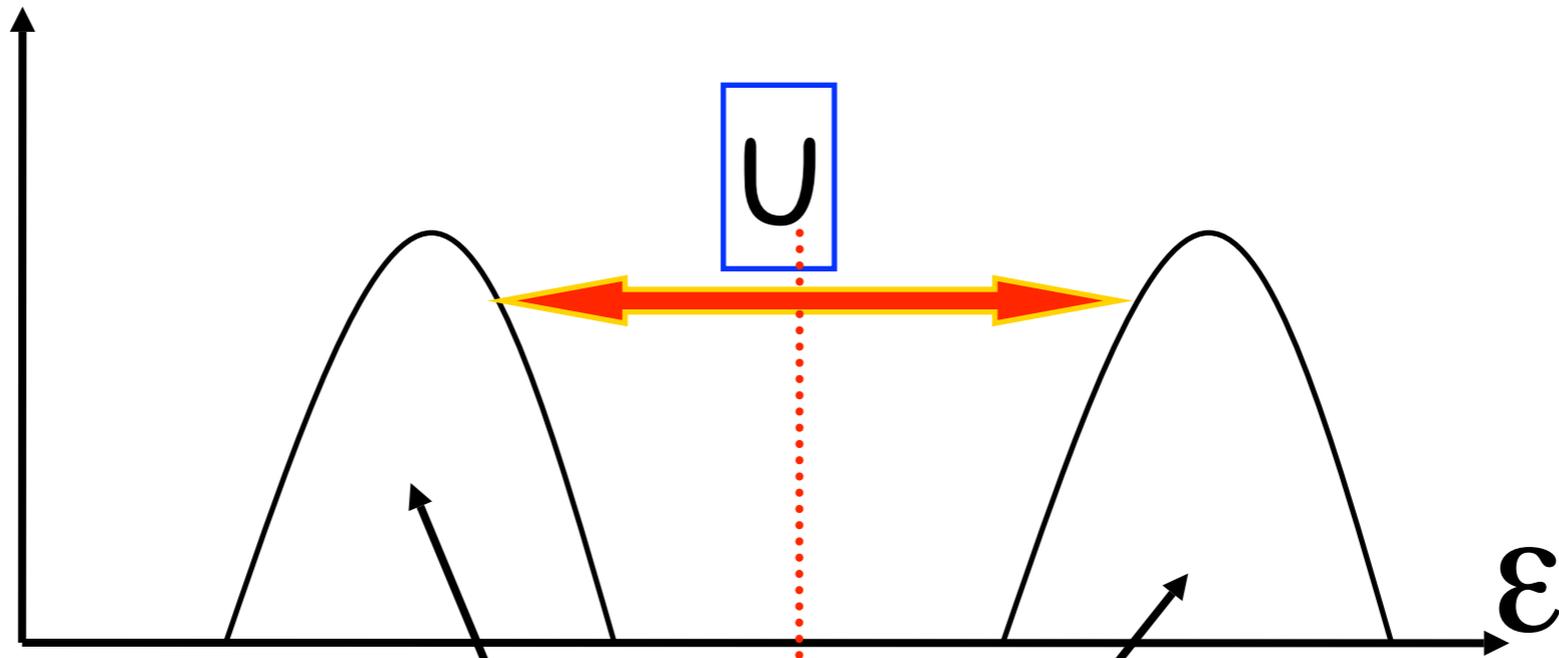
$$\phi = \phi_L + \phi_H$$

$$e^{-S_{\text{eff}}[\phi_L]} = \int d\phi_H \exp -S(\phi_L, \phi_H)$$

Low-energy theory of M I

# Half-filling

$N(\omega)$

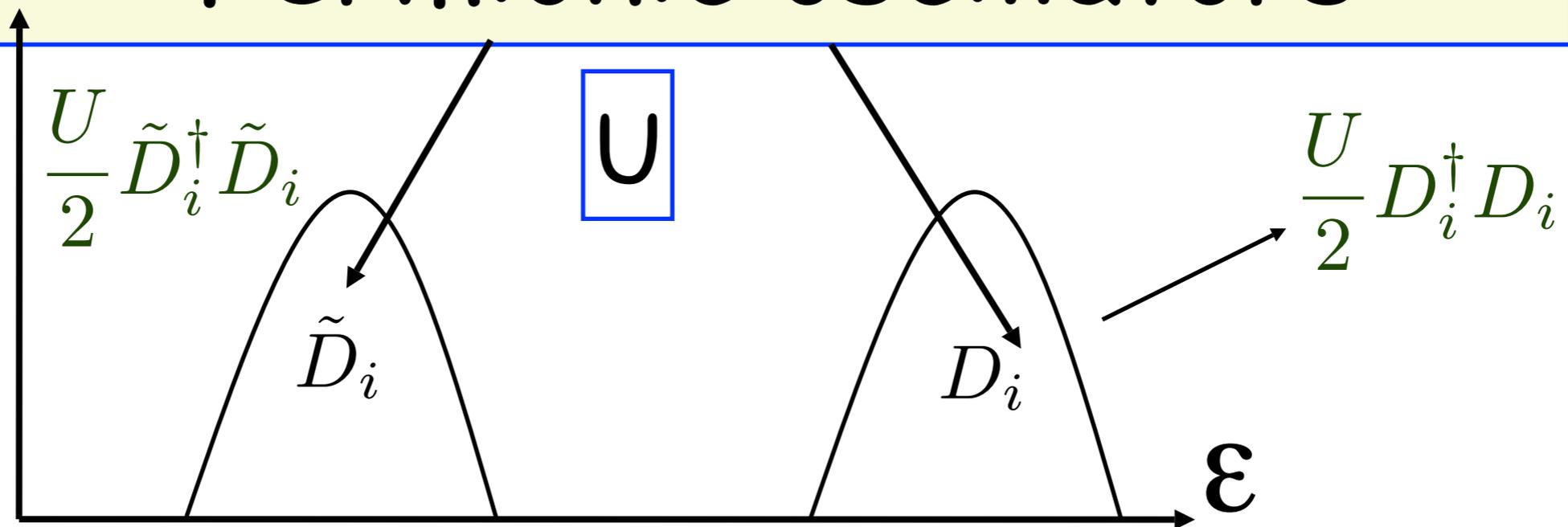


Integrate out both

Key idea: similar to Bohm/Pines

Extend the Hilbert space:  
Associate with U-scale new  
Fermionic oscillators

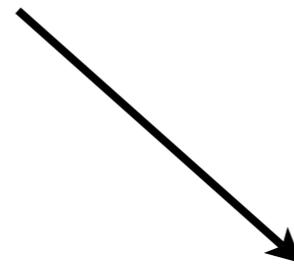
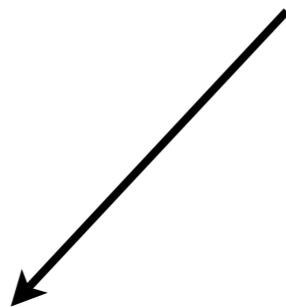
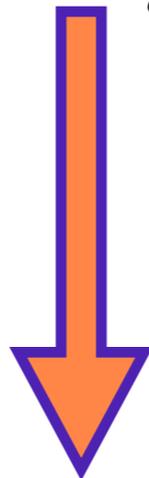
$N(\omega)$



$$D_i^\dagger$$

Fermionic

constraint



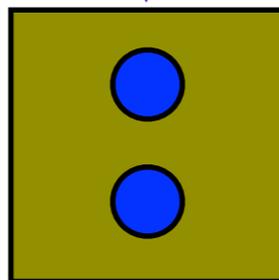
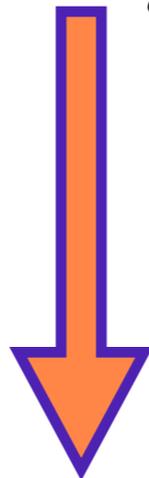
one per site  
(fermionic)

transforms as a boson

$$D_i^\dagger$$

Fermionic

constraint



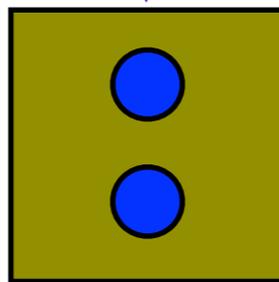
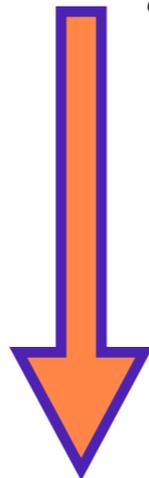
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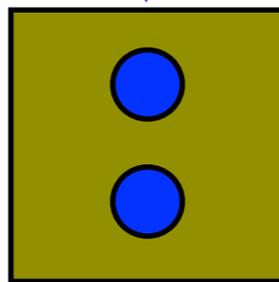
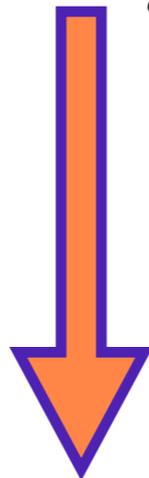
transforms as a boson

'supersymmetry'

$$D_i^\dagger$$

Fermionic

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one per site  
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Grassmann

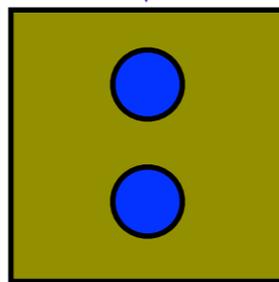
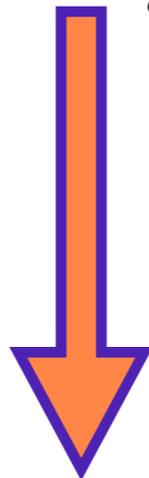
$$\delta(D_i - \theta c_{i\uparrow} c_{i\downarrow})$$



$$D_i^\dagger$$

Fermionic

constraint



one per site  
(fermionic)

transforms as a boson

'supersymmetry'

Grassmann

$$\delta(D_i - \theta c_{i\uparrow} c_{i\downarrow})$$

$$\bar{\theta}_\psi$$

constraint field



# Dual Theory

solve constraint

$$\varphi (Q_\varphi = 2e)$$

UV limit

$$\int d^2\theta \bar{\theta}\theta L_{\text{Hubb}} = \sum_{i,\sigma} c_{i,\sigma}^\dagger \dot{c}_{i,\sigma} + H_{\text{Hubb}},$$

integrate over  
heavy fields

Exact low-energy  
theory (IR limit)

# Dual Theory

solve constraint  
 $\varphi (Q_\varphi = 2e)$

integrate over  
heavy fields

UV limit

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Exact low-energy  
theory (IR limit)

$$\begin{aligned} L_{\text{UV}}^{\text{hf}} &= \int d^2\theta \left[ iD^\dagger \dot{D} - i\dot{\tilde{D}}^\dagger \tilde{D} - \frac{U}{2}(D^\dagger D - \tilde{D}\tilde{D}^\dagger) \right. \\ &+ \frac{t}{2}D^\dagger \theta b + \frac{t}{2}\bar{\theta}b\tilde{D} + h.c. + s\bar{\theta}\varphi^\dagger (D - \theta c_\uparrow c_\downarrow) \\ &\left. + \tilde{s}\bar{\theta}\tilde{\varphi}^\dagger (\tilde{D} - \theta c_\uparrow^\dagger c_\downarrow^\dagger) + h.c. \right] \end{aligned}$$

dynamics  
of  $\varphi$

# Exact IR Lagrangian

bare fields have no dynamics

$$L_{\text{IR}}^{\text{hf}} \rightarrow 2 \frac{|s|^2}{U} |\varphi_\omega|^2 + 2 \frac{|\tilde{s}|^2}{U} |\tilde{\varphi}_{-\omega}|^2 + \frac{t^2}{U} |b_\omega|^2$$

$$\left\{ \begin{aligned} &+ s \gamma_{\vec{p}}^{(\vec{k})}(\omega) \varphi_{\omega, \vec{k}}^\dagger c_{\vec{k}/2 + \vec{p}, \omega/2 + \omega', \uparrow} c_{\vec{k}/2 - \vec{p}, \omega/2 - \omega', \downarrow} \\ &+ \tilde{s}^* \tilde{\gamma}_{\vec{p}}^{(\vec{k})}(\omega) \tilde{\varphi}_{-\omega, \vec{k}} c_{\vec{k}/2 + \vec{p}, \omega/2 + \omega', \uparrow} c_{\vec{k}/2 - \vec{p}, \omega/2 - \omega', \downarrow} + h.c. \end{aligned} \right.$$

bosons  
and fermions  
are strongly coupled

$$\gamma_{\vec{p}}^{(\vec{k})}(\omega) = \frac{U - t \varepsilon_{\vec{p}}^{(\vec{k})} - 2\omega}{U} \sqrt{1 + 2\omega/U}$$

$$\tilde{\gamma}_{\vec{p}}^{(\vec{k})}(\omega) = \frac{U + t \varepsilon_{\vec{p}}^{(\vec{k})} + 2\omega}{U} \sqrt{1 - 2\omega/U}.$$

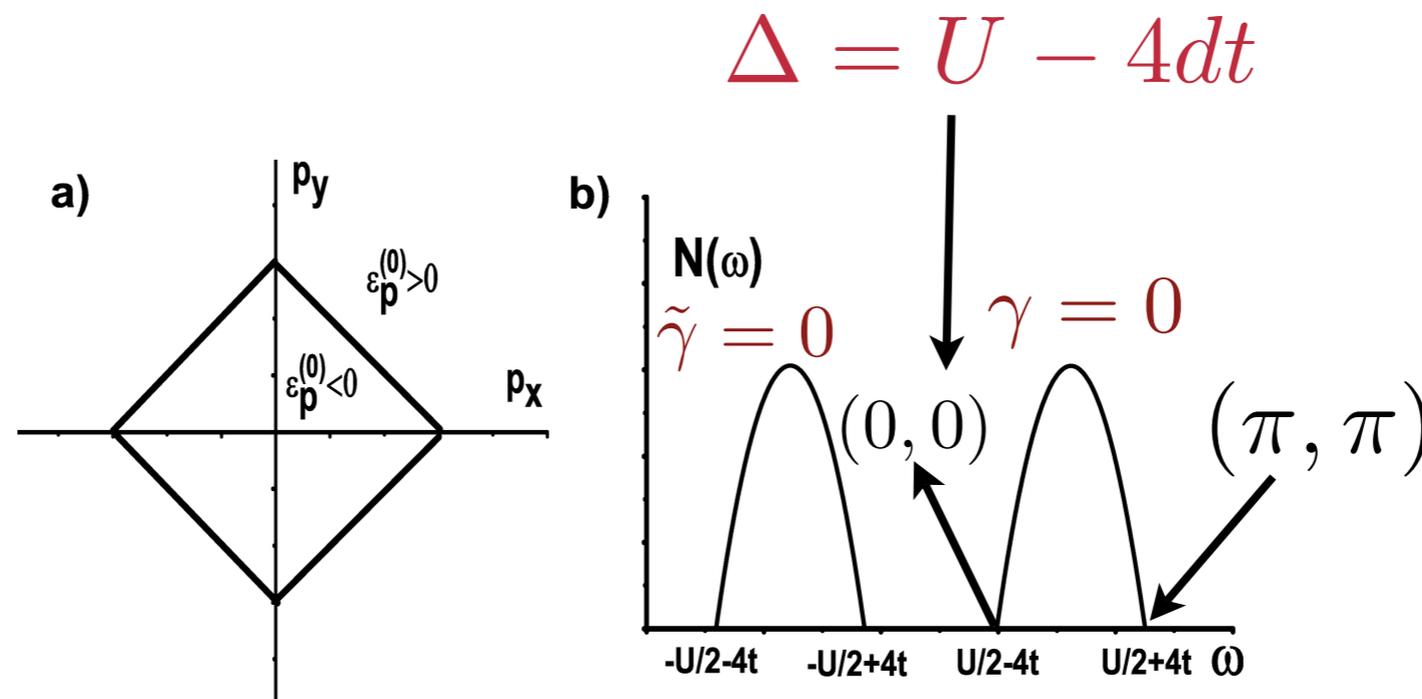
$$\varepsilon_{\vec{p}}^{(\vec{k})} = 4 \sum_{\mu} \cos(k_{\mu} a/2) \cos(p_{\mu} a)$$

turn-on of spectral  
weight governed  
by composite  
excitations (CEXONS)

# composite excitations determine spectral density

$$\gamma_{\vec{p}}^{(\vec{k})}(\omega) = \frac{U - t\varepsilon_{\vec{p}}^{(\vec{k})} - 2\omega}{U} \sqrt{1 + 2\omega/U}$$

$$\tilde{\gamma}_{\vec{p}}^{(\vec{k})}(\omega) = \frac{U + t\varepsilon_{\vec{p}}^{(\vec{k})} + 2\omega}{U} \sqrt{1 - 2\omega/U}.$$

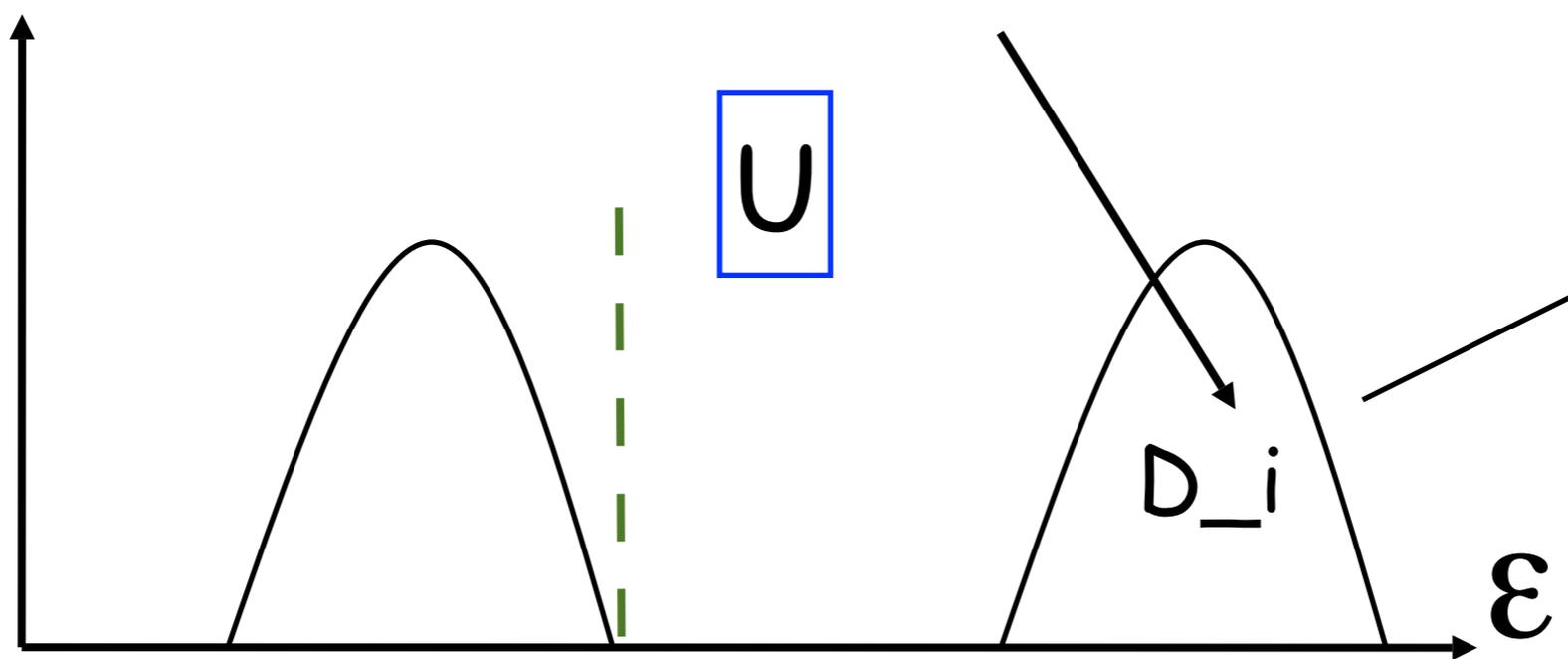


each momentum has SD at two distinct energies

hole-doping?

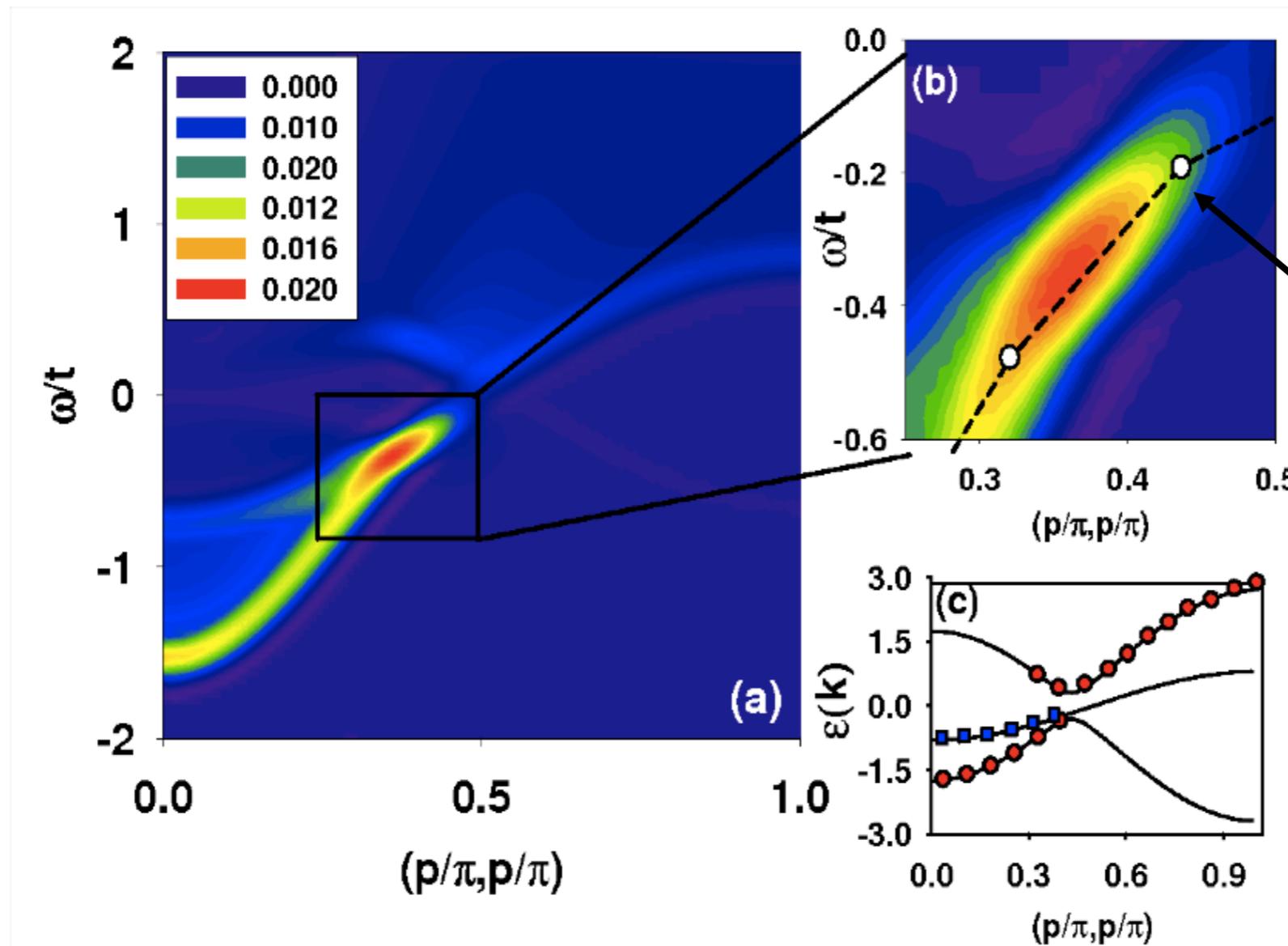
Extend the Hilbert space:  
Associate with U-scale a new  
Fermionic oscillator

$N(\omega)$



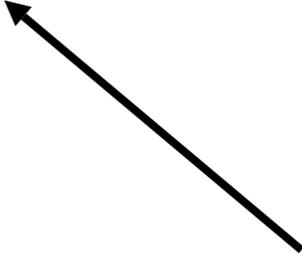
$$U D_i^\dagger D_i$$

# Electron spectral function

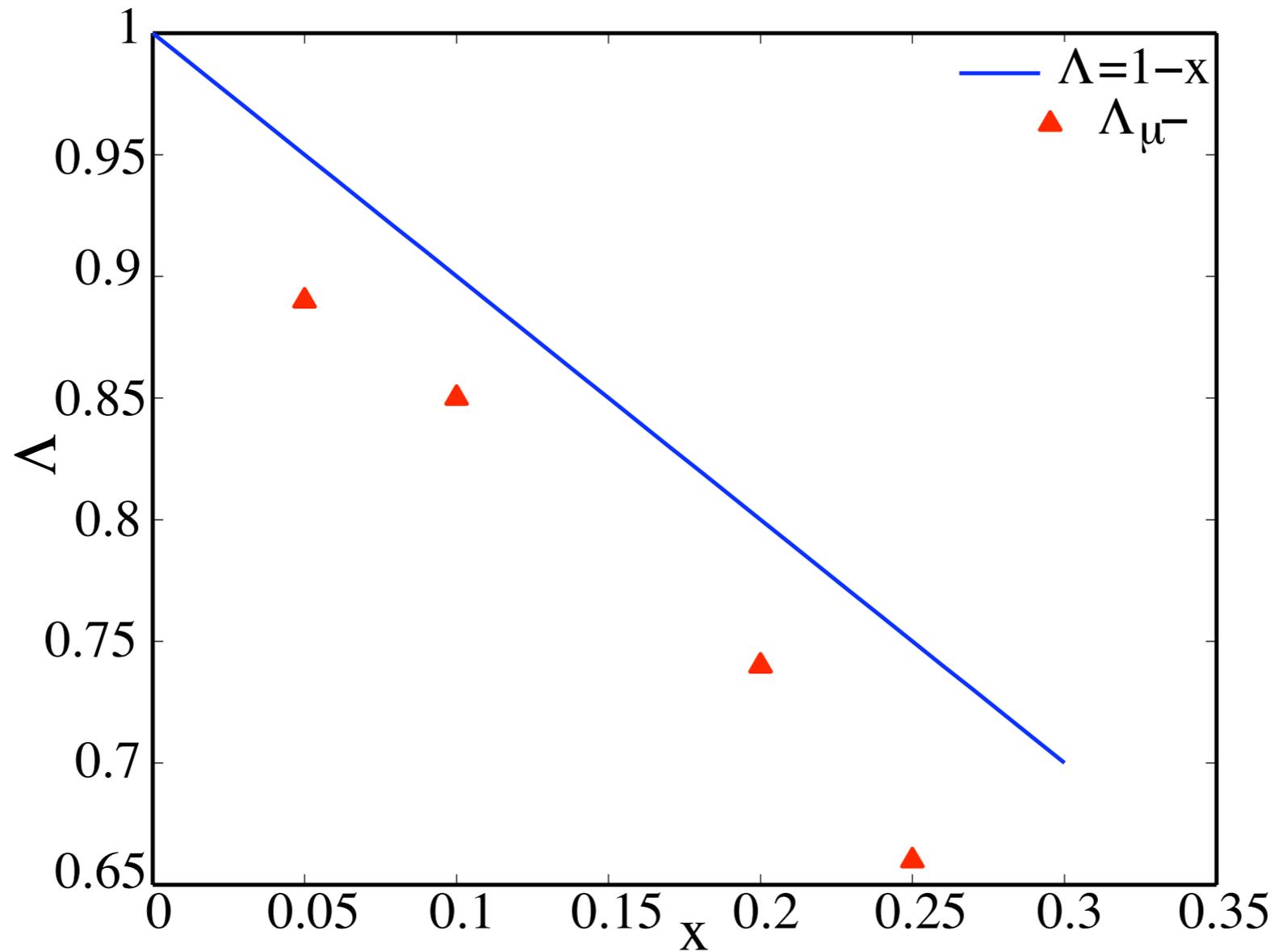


$$t^2/U \sim 60 \text{ meV}$$

# Electron spectral function

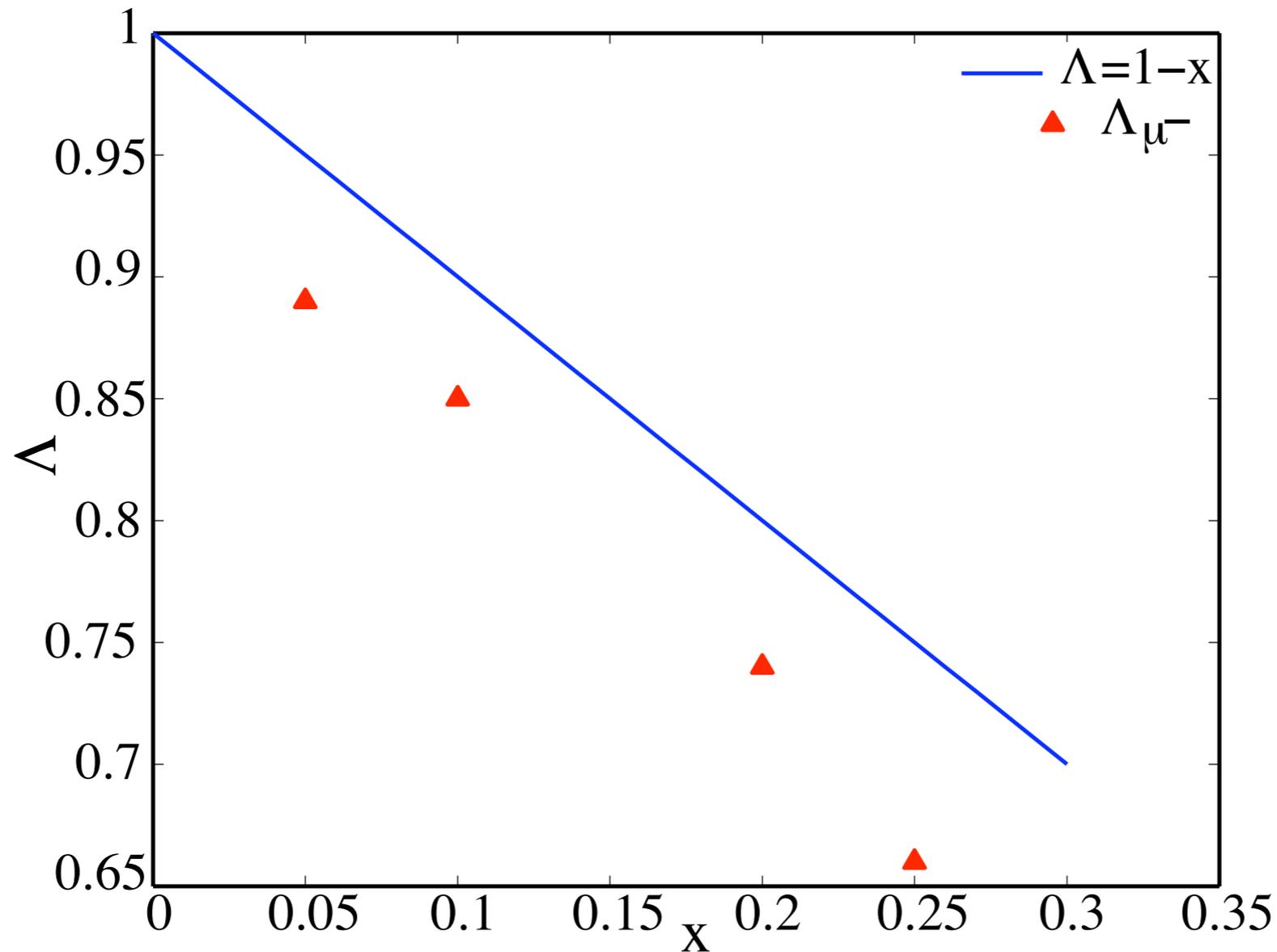

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# Electron spectral function



$\Delta^2/U \sim 60 \text{ meV}$

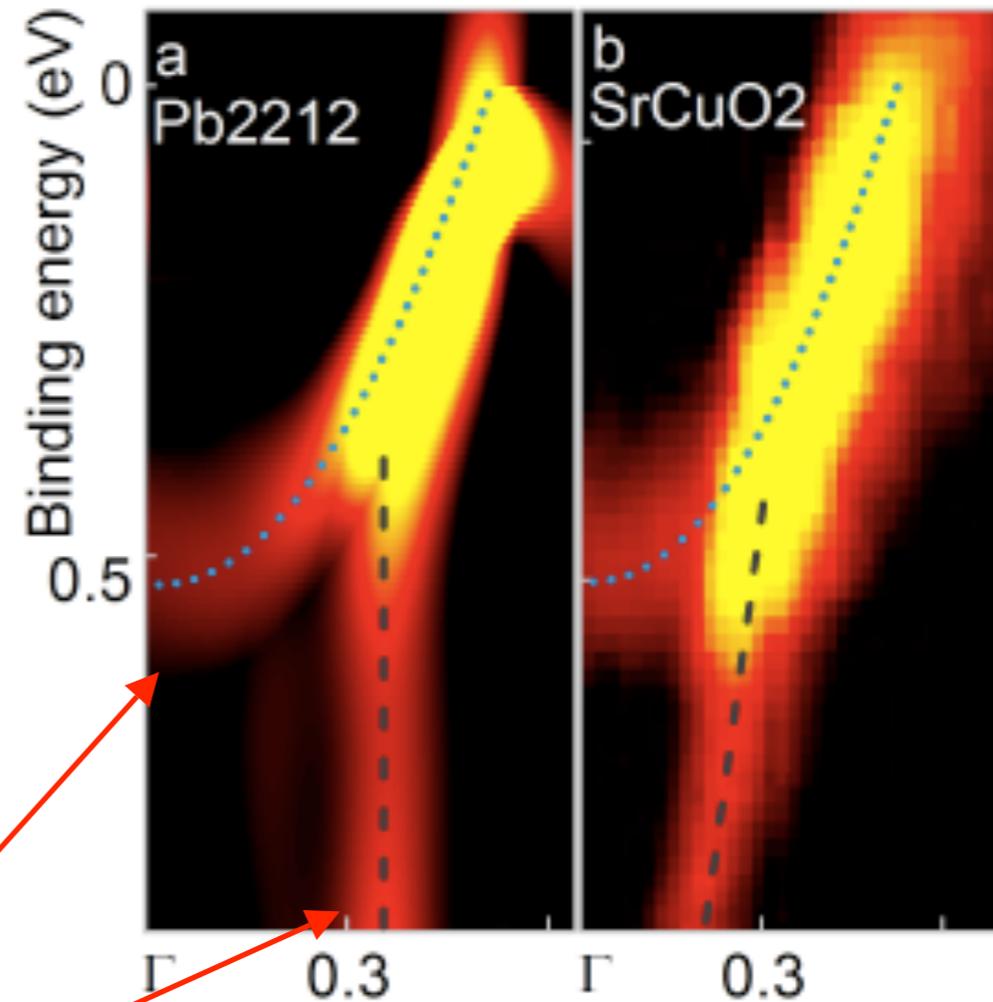
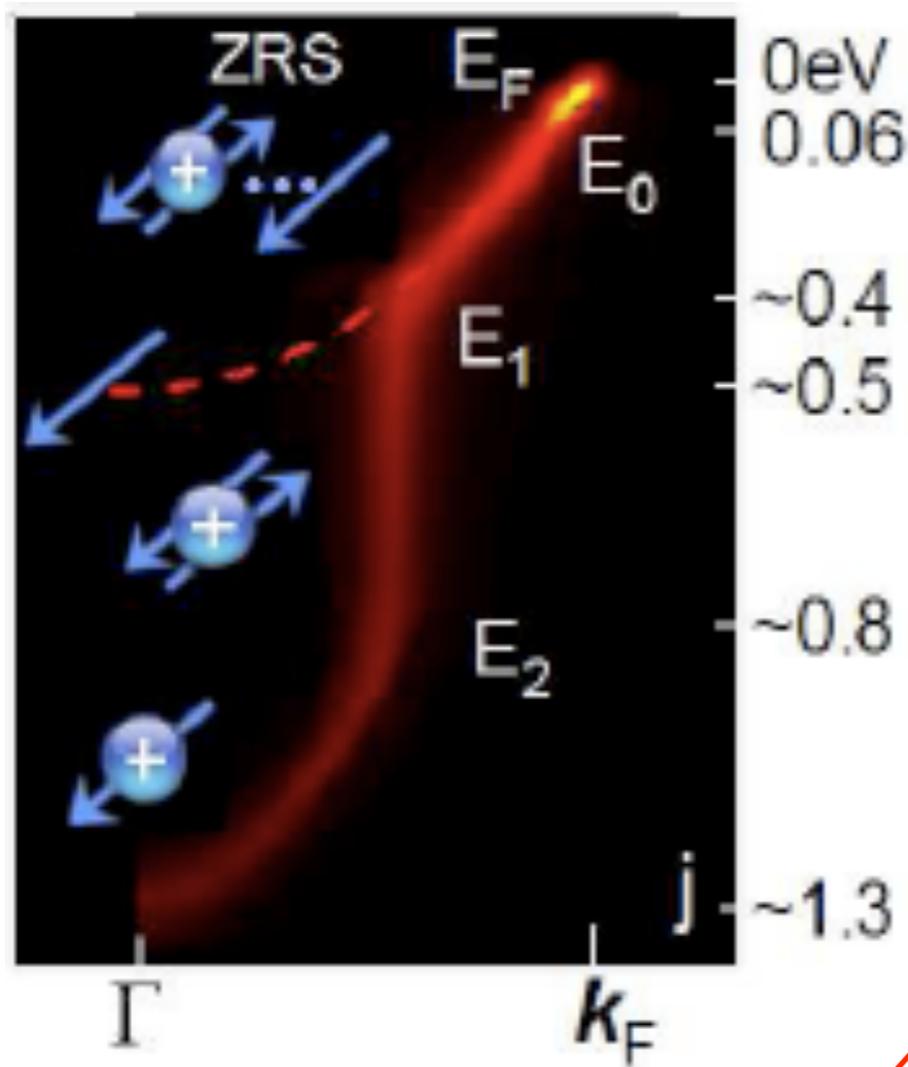
# Electron spectral function



$\Delta^2/U \sim 60 \text{ meV}$

Conserved charge: 
$$Q = \sum_i c_i^\dagger c_i + 2 \sum_i \varphi_i^\dagger \varphi_i$$

Graf, et al. PRL vol. 98, 67004 (2007).



Two bands!!

Spin-charge separation?

# Origin of two bands

Two charge  $e$  excitations

$$c_{i\sigma}$$

$\varphi_i$  is confined (no kinetic energy)

$$\varphi_i^\dagger c_{i\bar{\sigma}}$$

New bound state

Pseudogap

two types of charges

'free'

bound

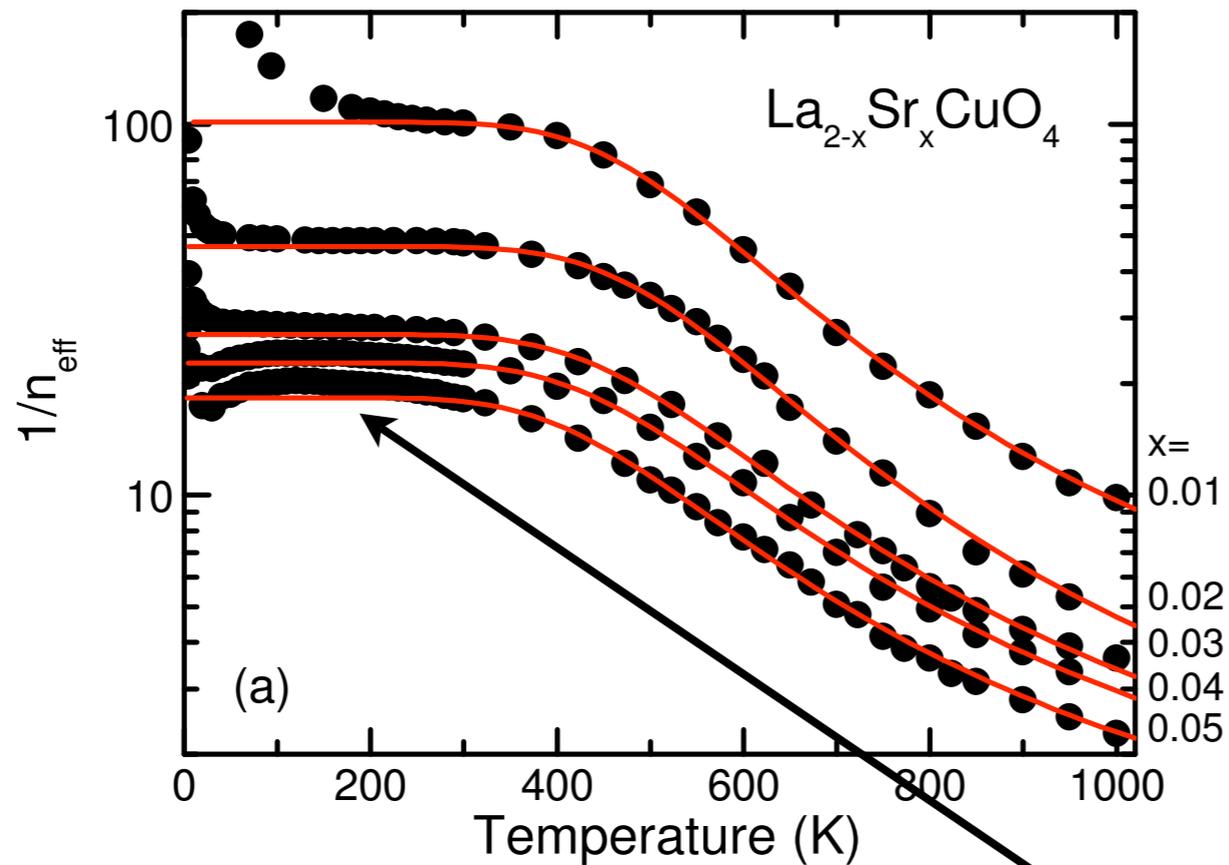
direct evidence

direct evidence

charge carrier density:

direct evidence

charge carrier density:

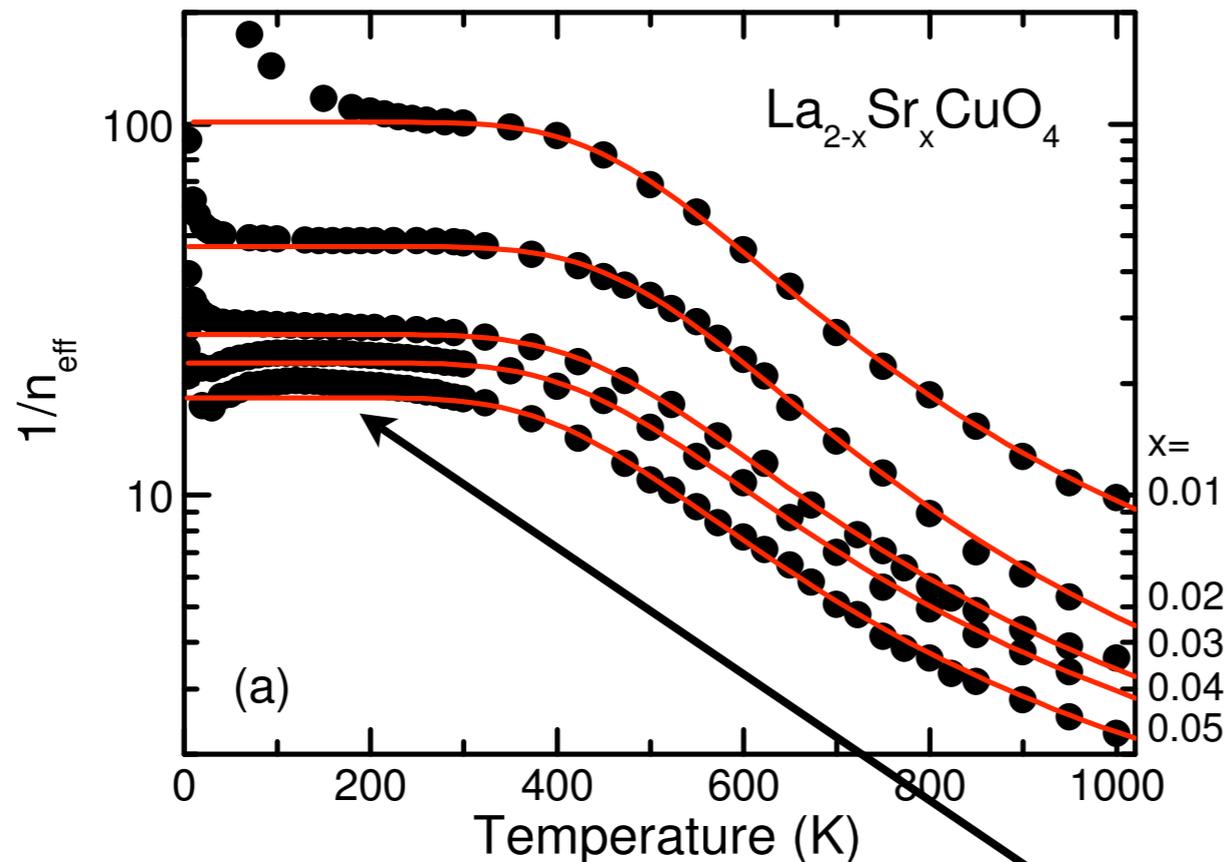


Ono, et al., Phys. Rev. B 75, 024515  
(2007)

$$n_{\text{Hall}}(x, T) = n_0(x) + n_1(x) \exp(-\Delta(x)/T), \quad \text{PRL, vol. 97, 247003 (2006).}$$

direct evidence

charge carrier density:

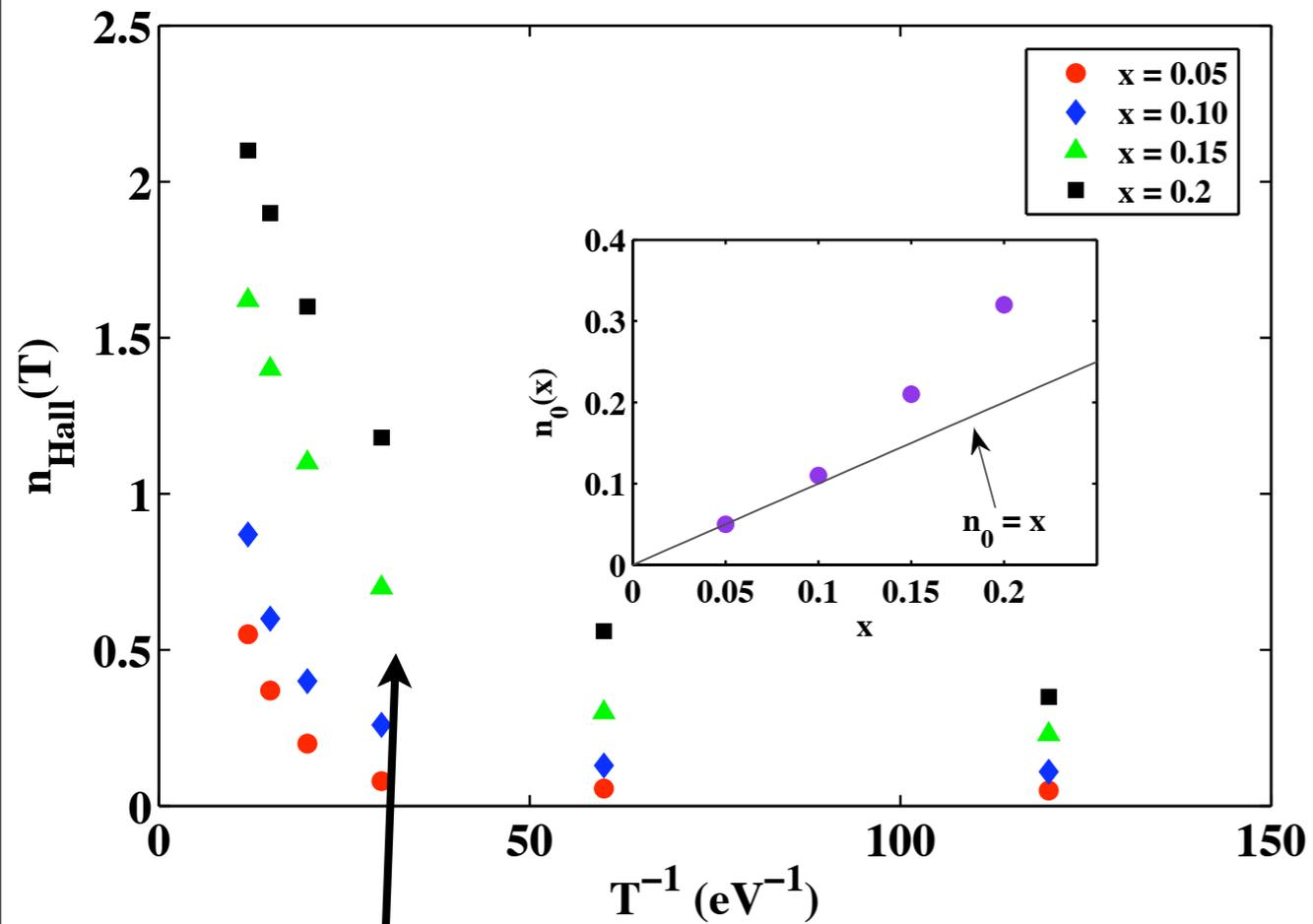


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$$n_{\text{Hall}}(x, T) = n_0(x) + n_1(x) \exp(-\Delta(x)/T), \quad \text{PRL, vol. 97, 247003 (2006).}$$

exponentially suppressed: confinement

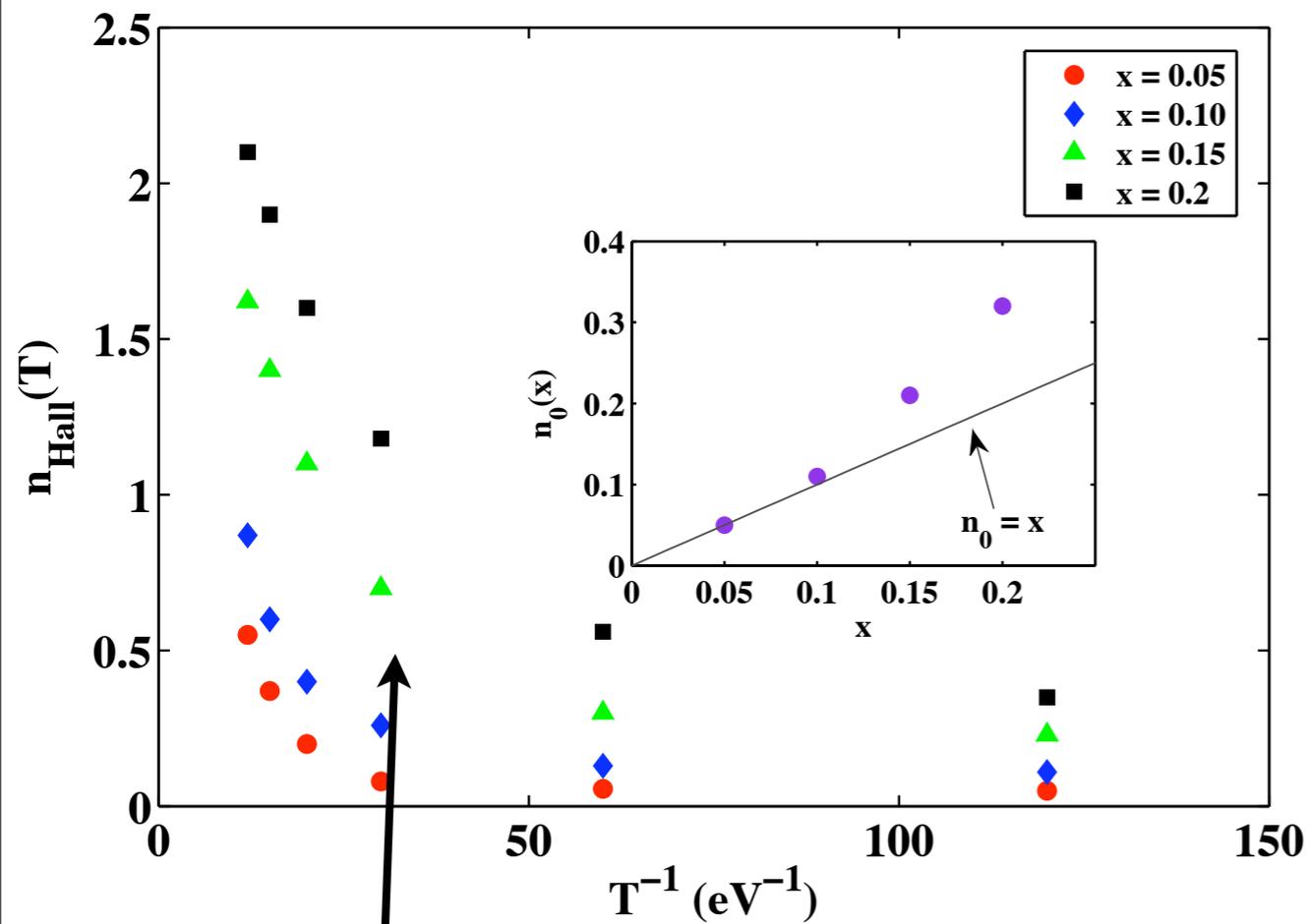
# Our Theory



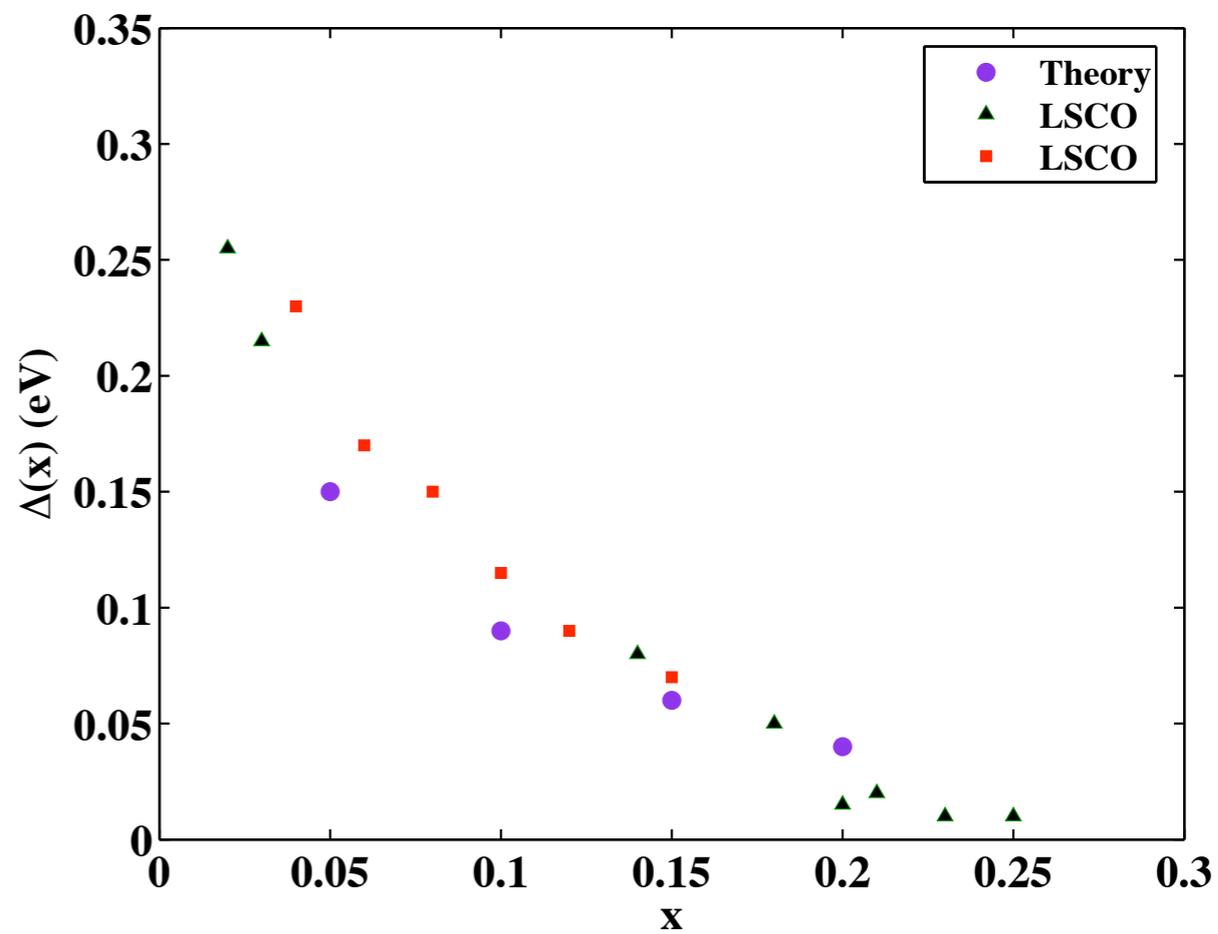
exponential  
T-dependence

# Our Theory

gap

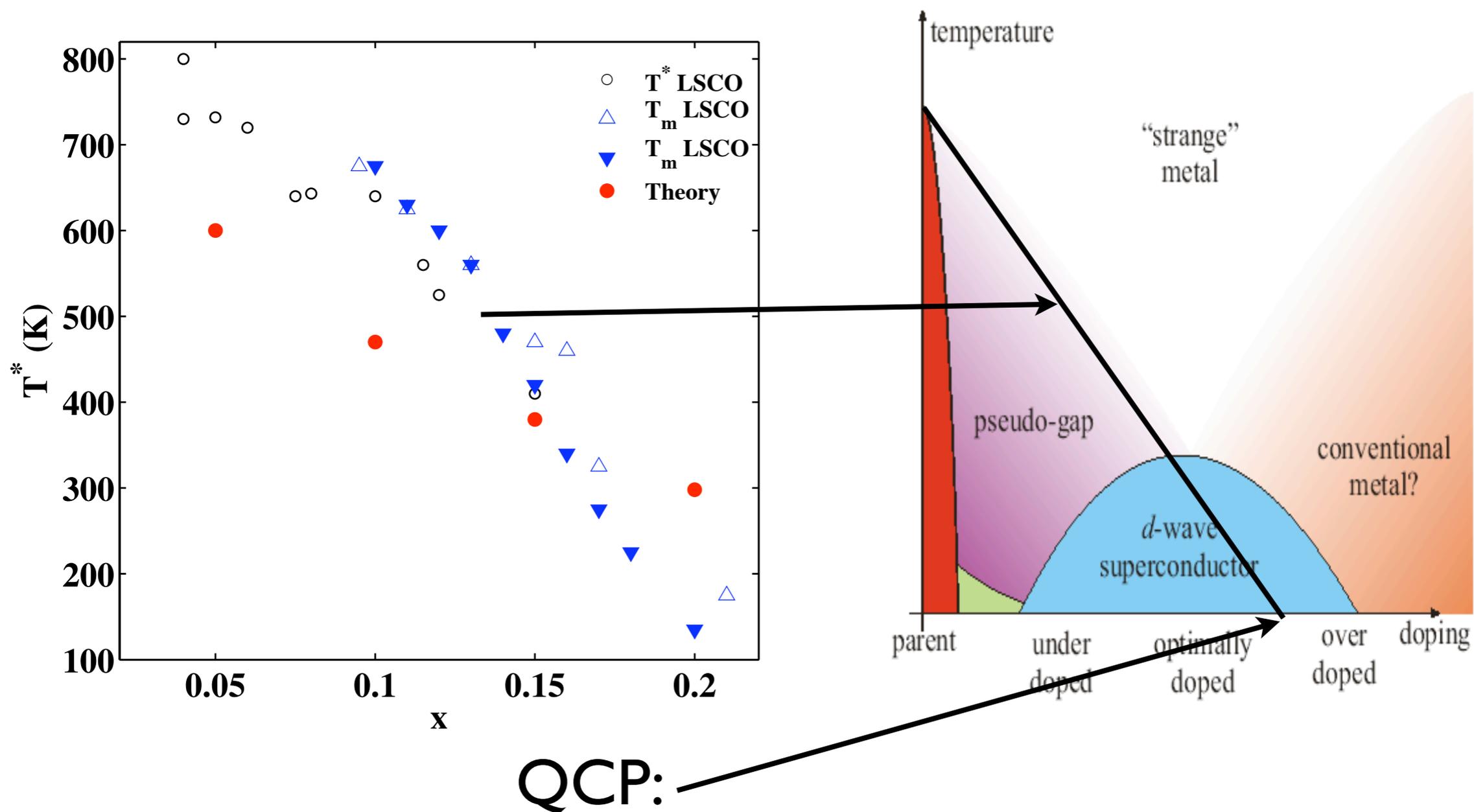


exponential  
T-dependence



no model-dependent  
free parameters: just  
 $t/U$

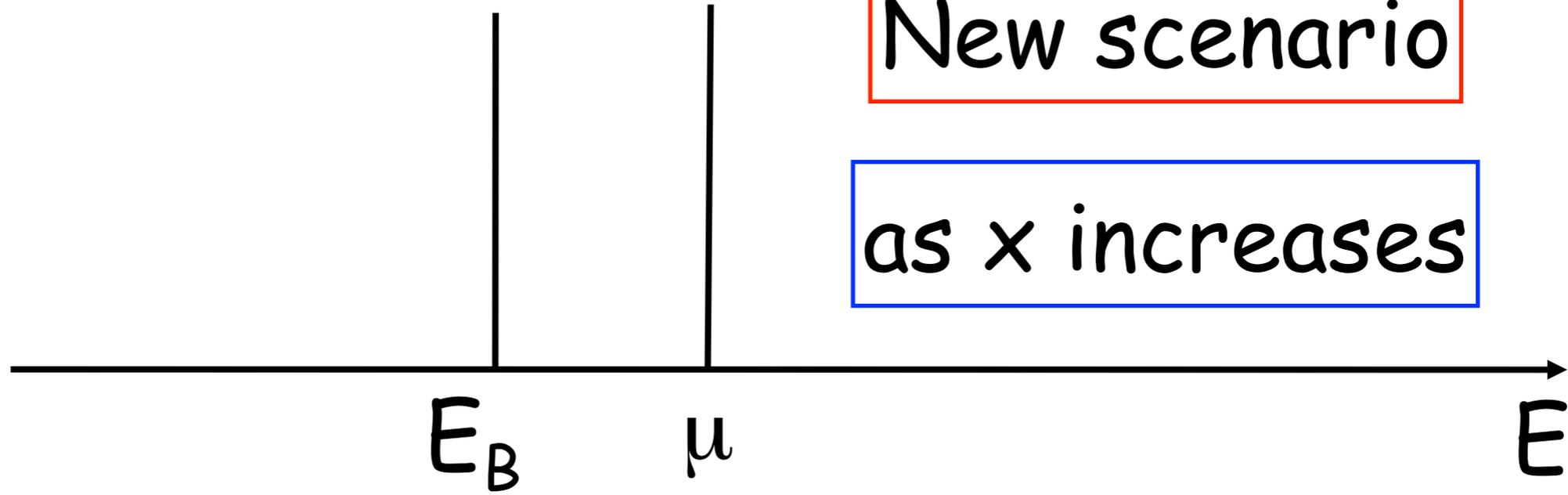
# strange metal: breakup (deconfinement) of bound states



T-linear resistivity

New scenario

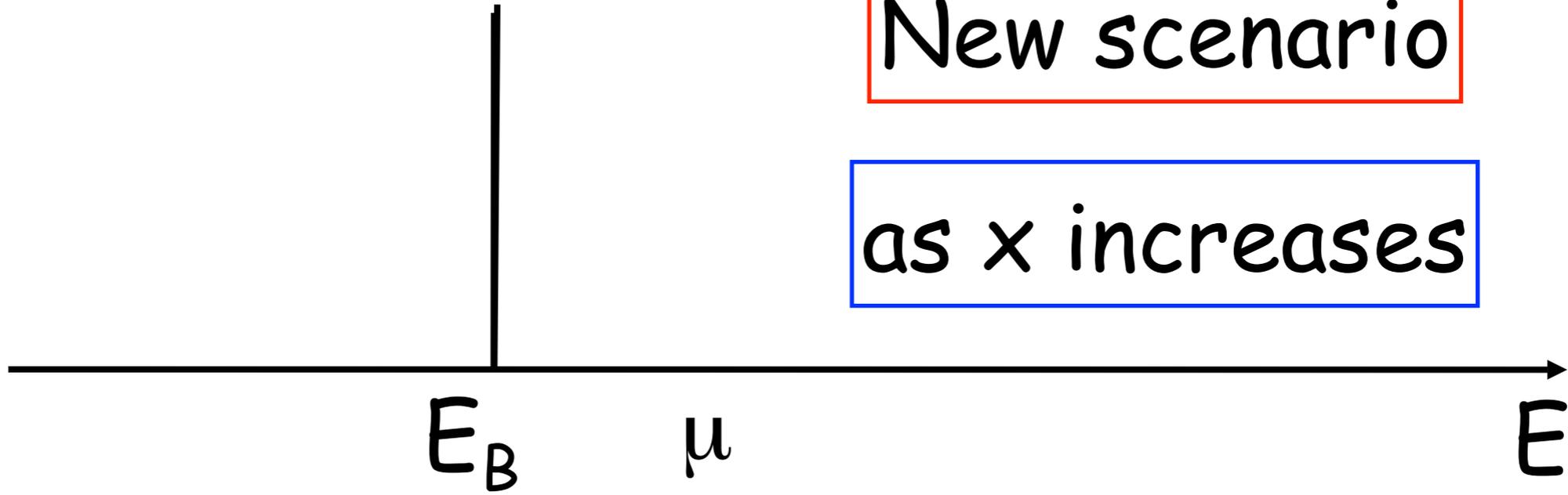
as  $x$  increases



T-linear resistivity

New scenario

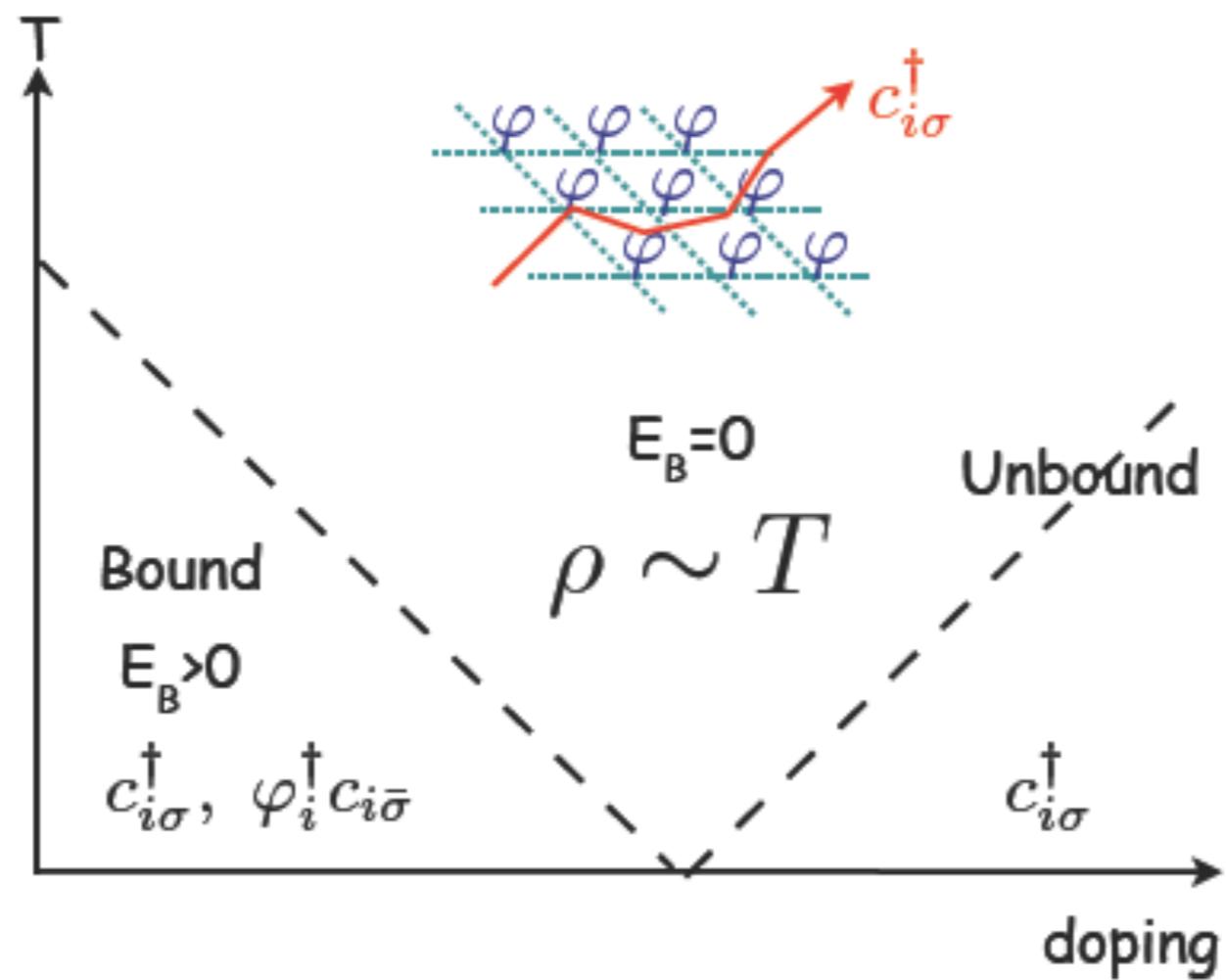
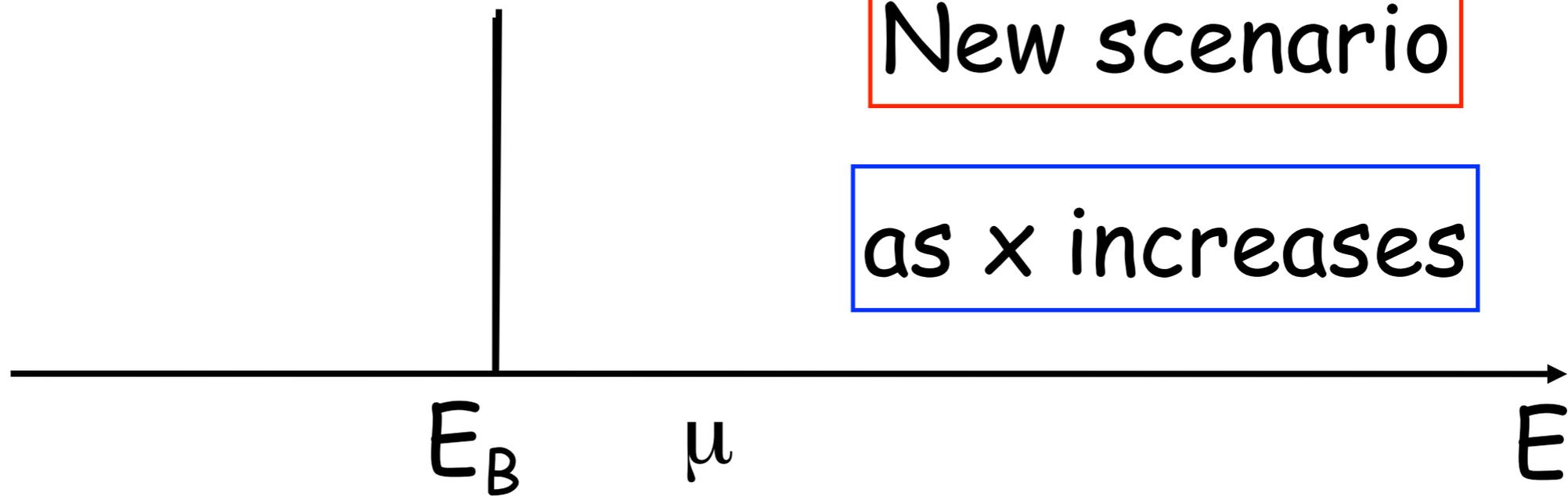
as  $x$  increases

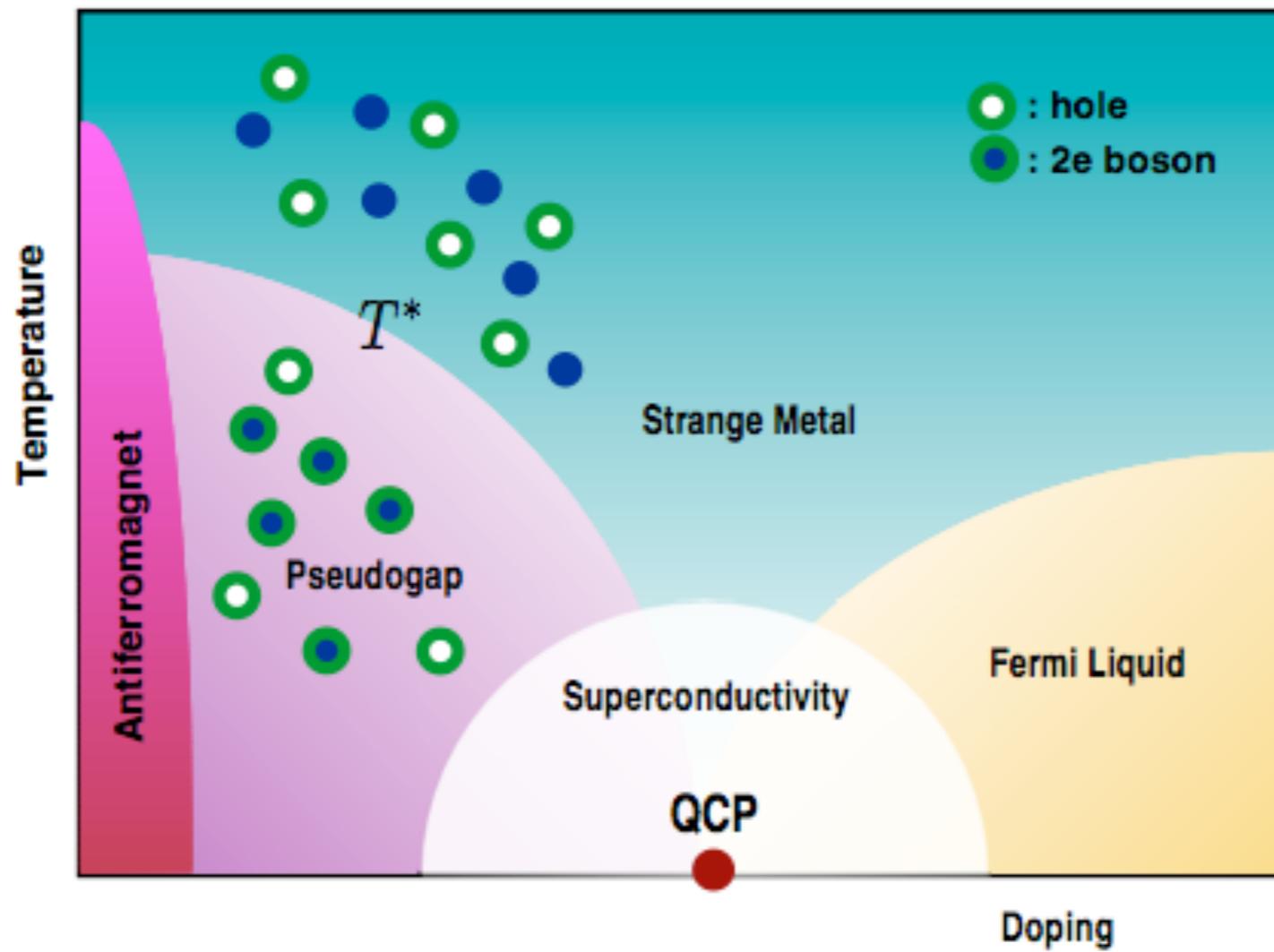


T-linear resistivity

New scenario

as  $x$  increases

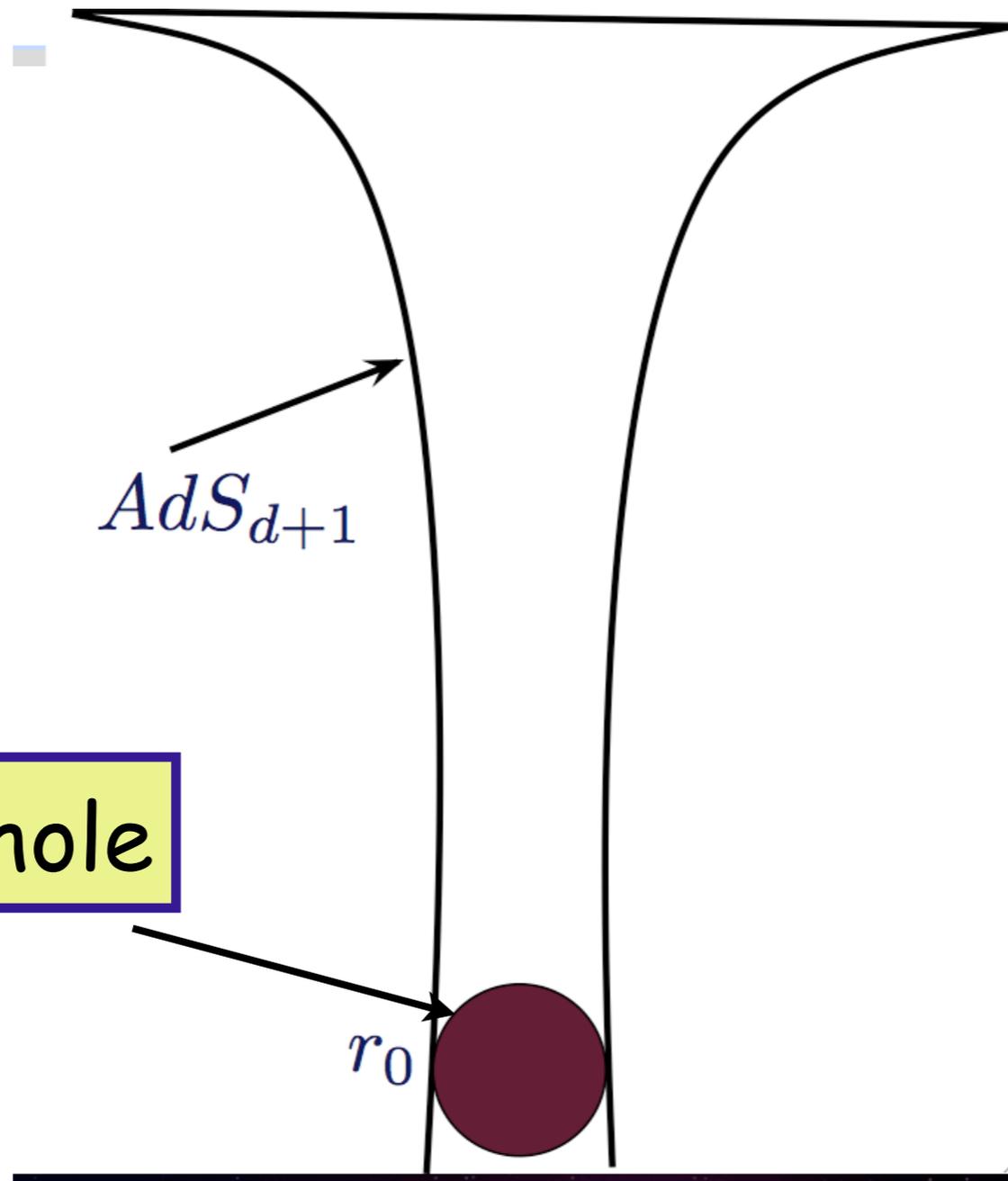




# Mottness from holography

$$H = -t \sum_{i,j} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

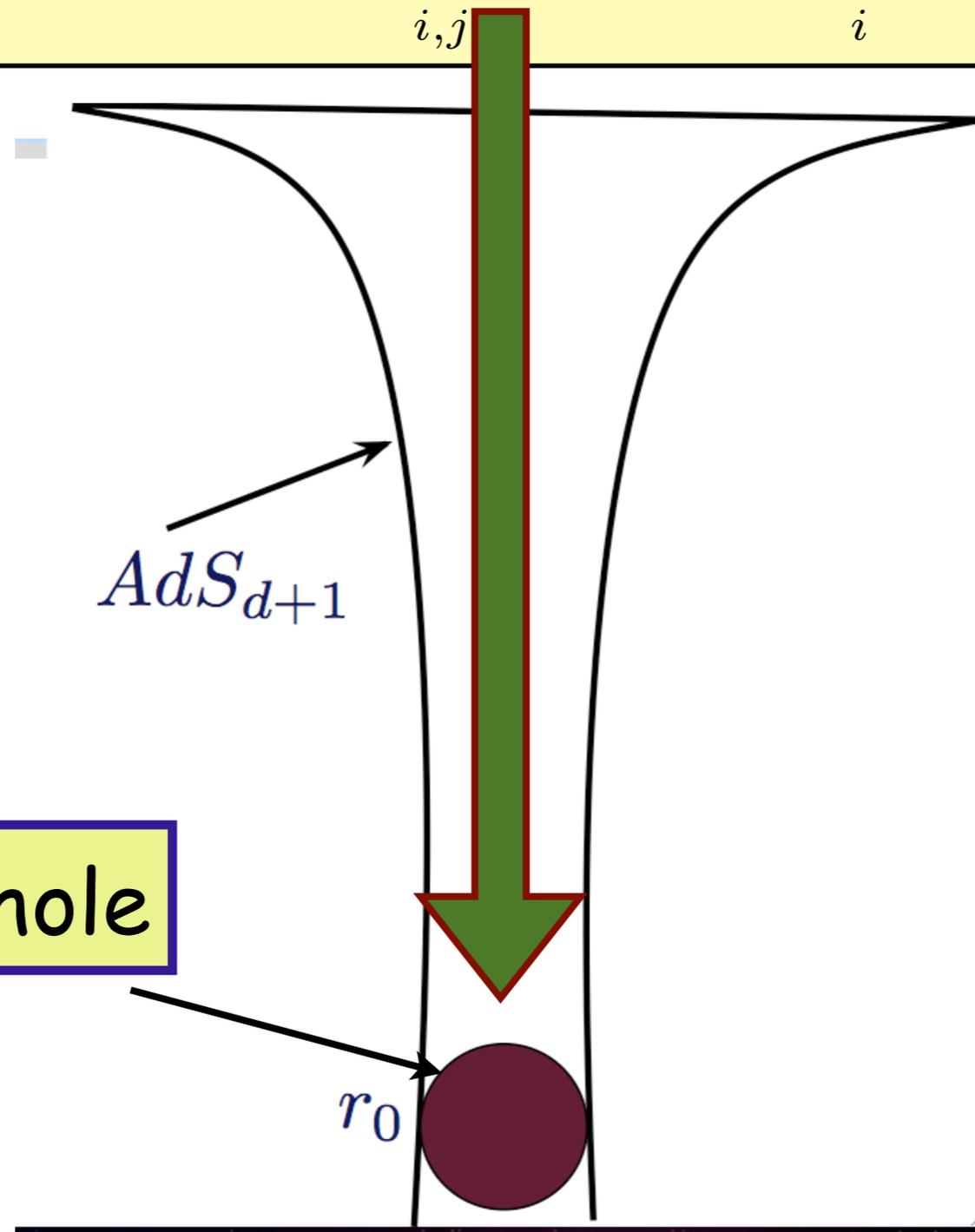
RN black hole



# Motttness from holography

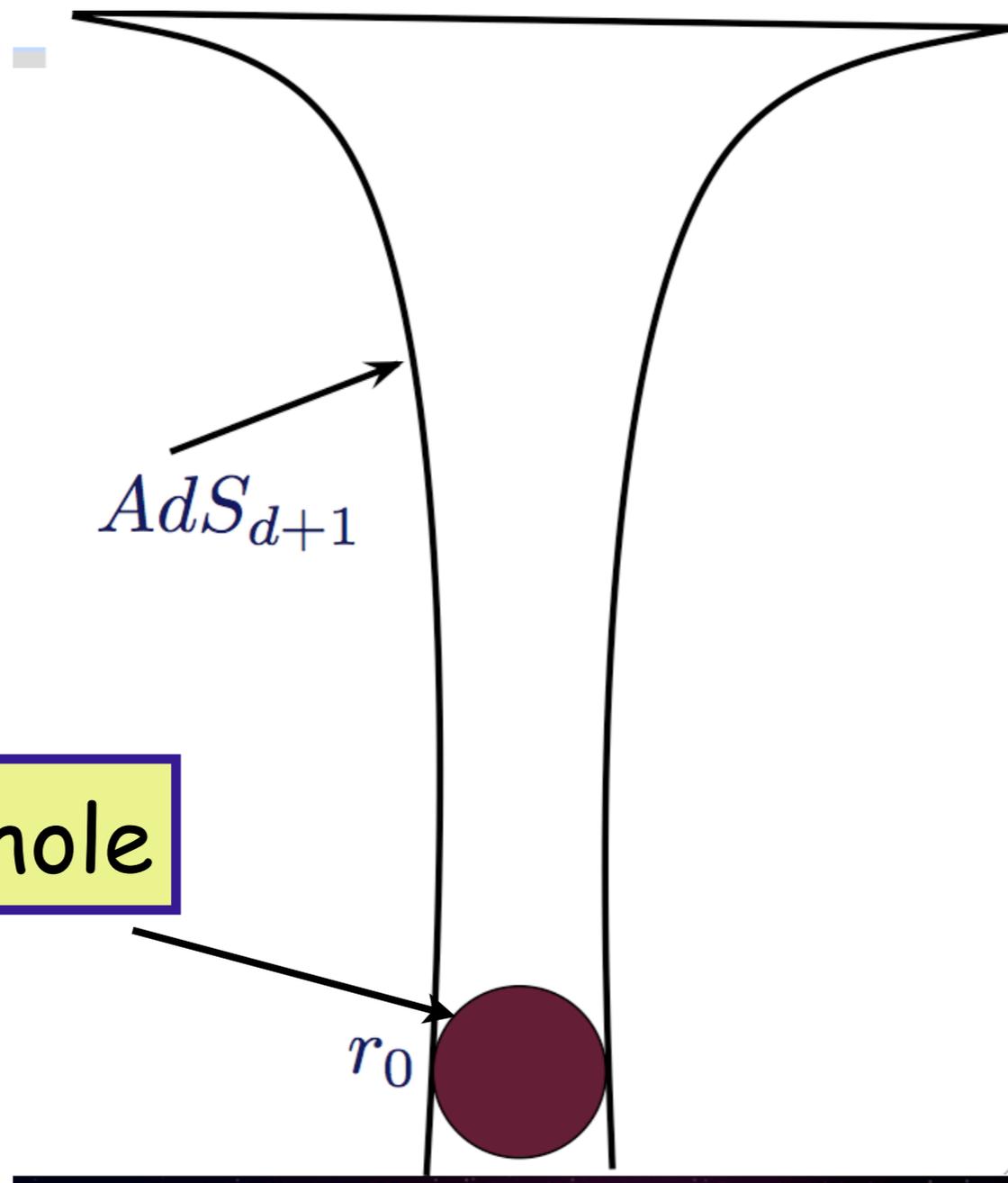
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# Mottness from holography

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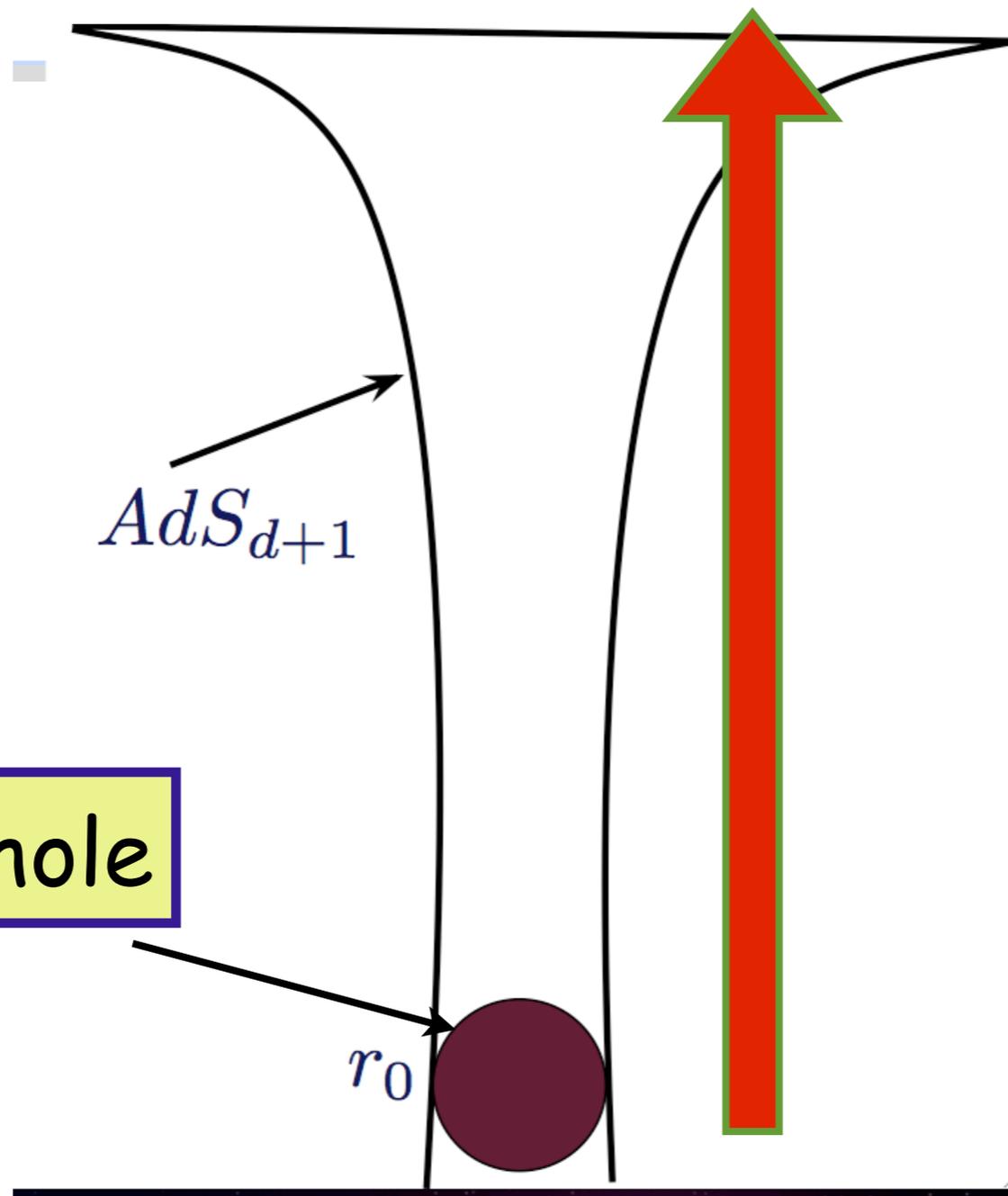


RN black hole

# Motttness from holography

$$H = -t \sum_{i,j} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

RN black hole



bottom-up schemes

$$\sqrt{-g}i\bar{\psi}(D - m)\psi$$

AdS-RN



'non-Fermi liquids'

# bottom-up schemes

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'non-Fermi liquids'

??



Mott Insulator

## bottom-up schemes

$$\sqrt{-g}i\bar{\psi}(D - m)\psi$$

AdS-RN



'non-Fermi liquids'

??



Mott Insulator

consider

$$\sqrt{-g}i\bar{\psi}(\cancel{D} - m - ip\cancel{F})\psi$$

fermions in RN AdS\_{d+1} coupled to a gauge field through a dipole interaction

near horizon

radial Dirac Equation

$$\psi_{I\pm}(\zeta) = \psi_{I\pm}^{(0)}(\zeta) + \omega \psi_{I\pm}^{(1)}(\zeta) + \omega^2 \psi_{I\pm}^{(2)}(\zeta) + \dots$$

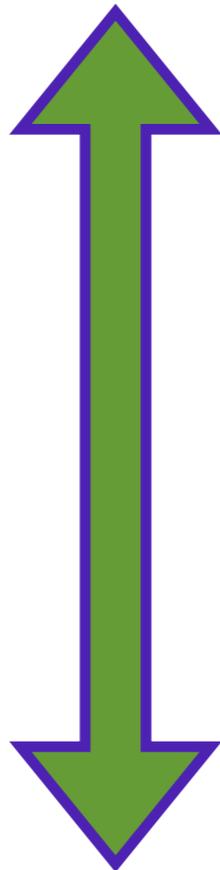
$$-\psi_{I\pm}^{(0)''}(\zeta) = i\sigma_2 \left(1 + \frac{qe_d}{\zeta}\right) - \frac{L_2}{\zeta} \left[ m\sigma_3 + \left( pe_d \pm \frac{kL}{r_0} \right) \sigma_1 \right] \psi_{I\pm}^{(0)}(\zeta),$$

$$e_d = 1/\sqrt{2d(d-1)}$$

$$m_k^2 = m^2 + \left( pe_d \pm \frac{kL}{r_0} \right)^2$$

p shifts momenta up and down through mass coupling

$$\psi_{I_{\pm}}^{(0)}(\zeta)$$



IR CFT

$$\mathcal{O}_{\pm}$$

scaling  
dimension

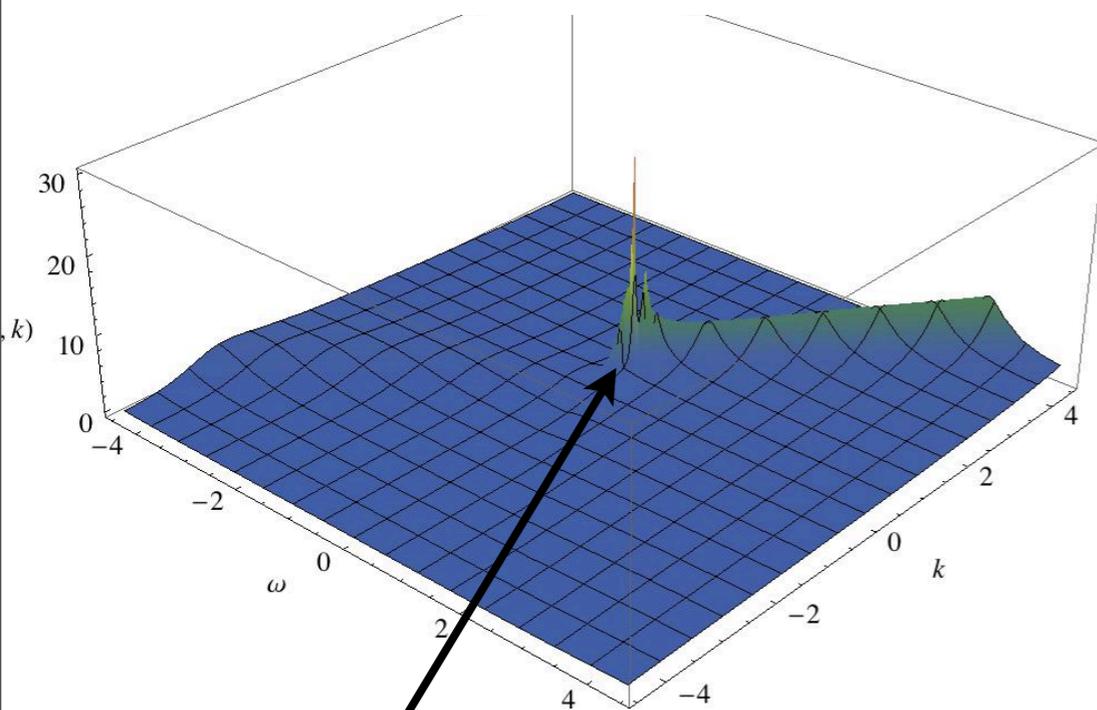
$$\delta_{\pm} = \nu_{\pm} + \frac{1}{2}$$

$$\nu_{\pm} = \sqrt{m_k^2 L_2^2 - q^2 e_d^2 - i\epsilon}$$

How is the spectrum modified?

How is the spectrum modified?

$P=0$

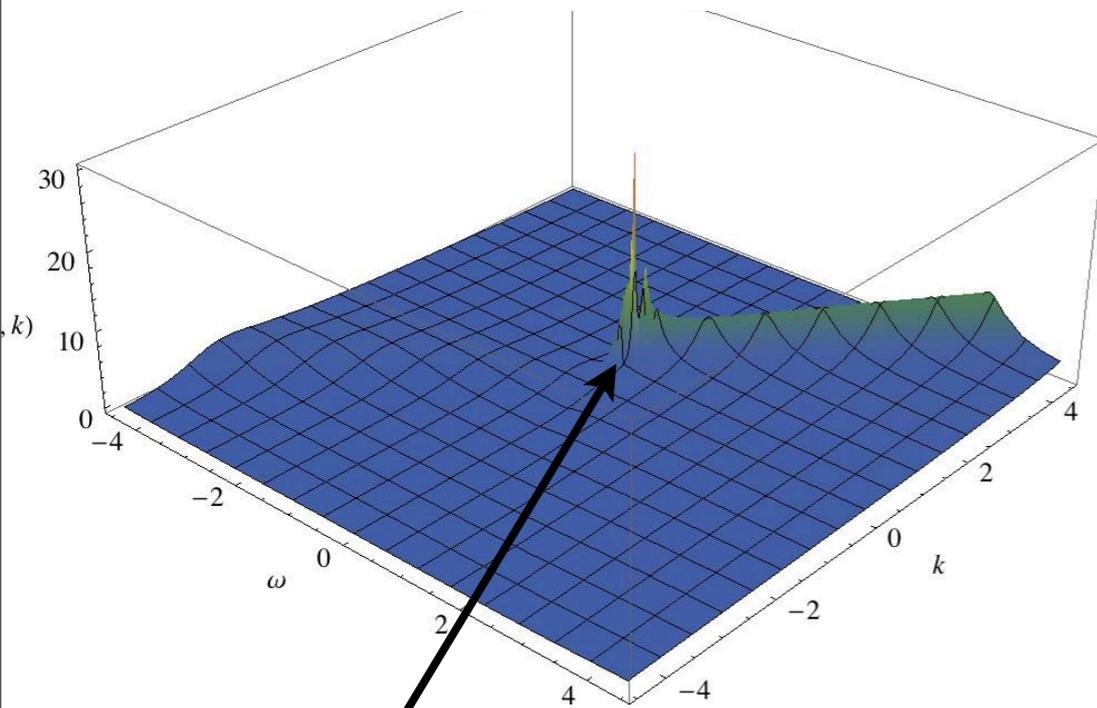


Fermi  
surface  
peak

# How is the spectrum modified?

$P=0$

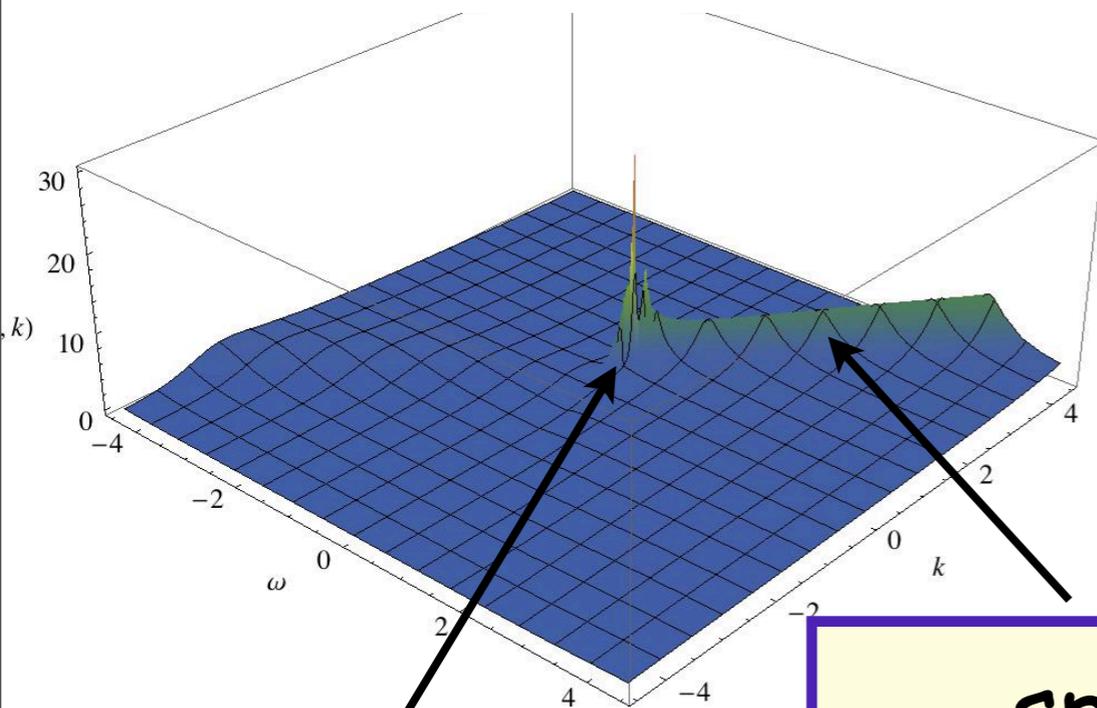
$P > 4.2$



Fermi  
surface  
peak

# How is the spectrum modified?

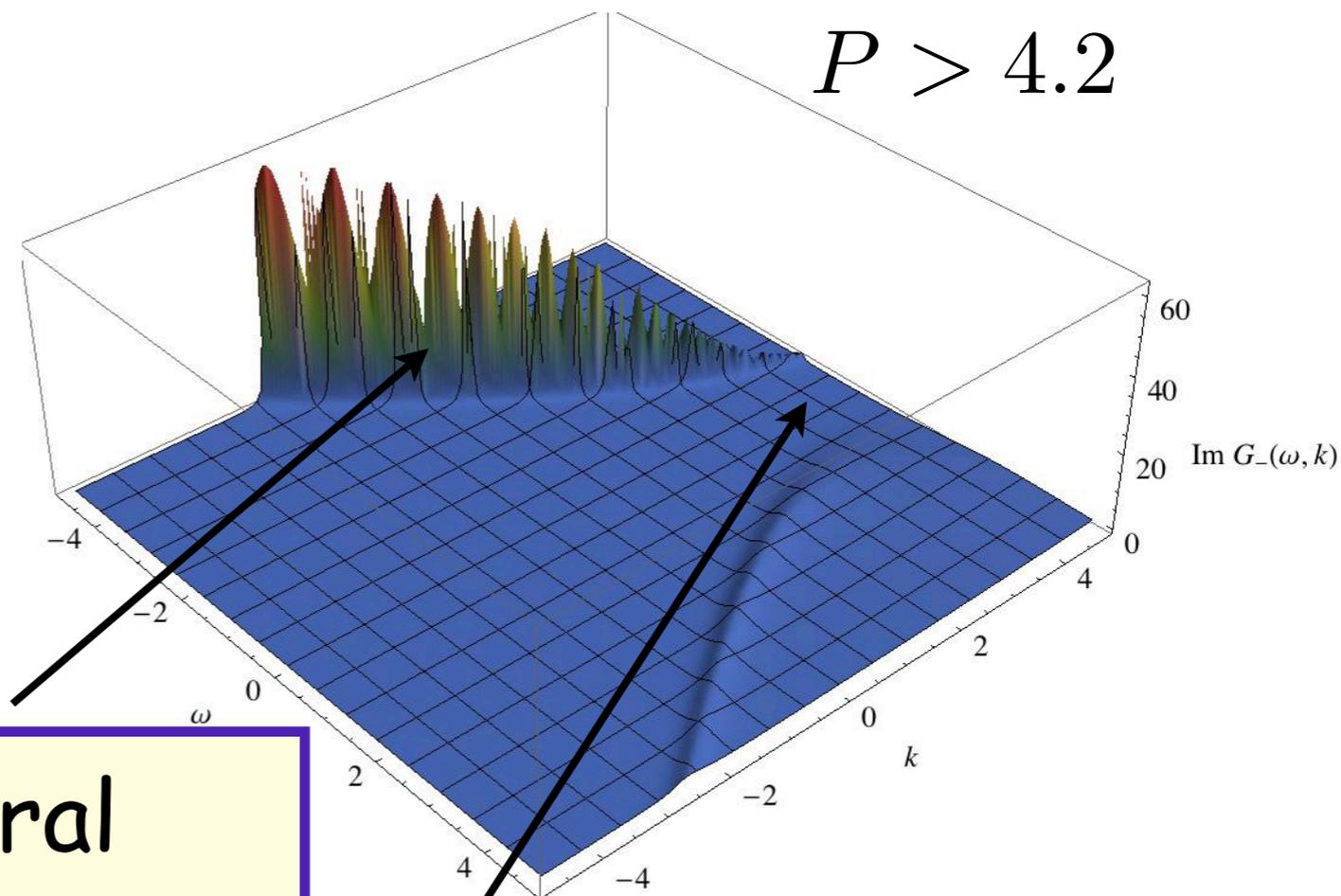
$P=0$



Fermi  
surface  
peak

spectral  
weight transfer

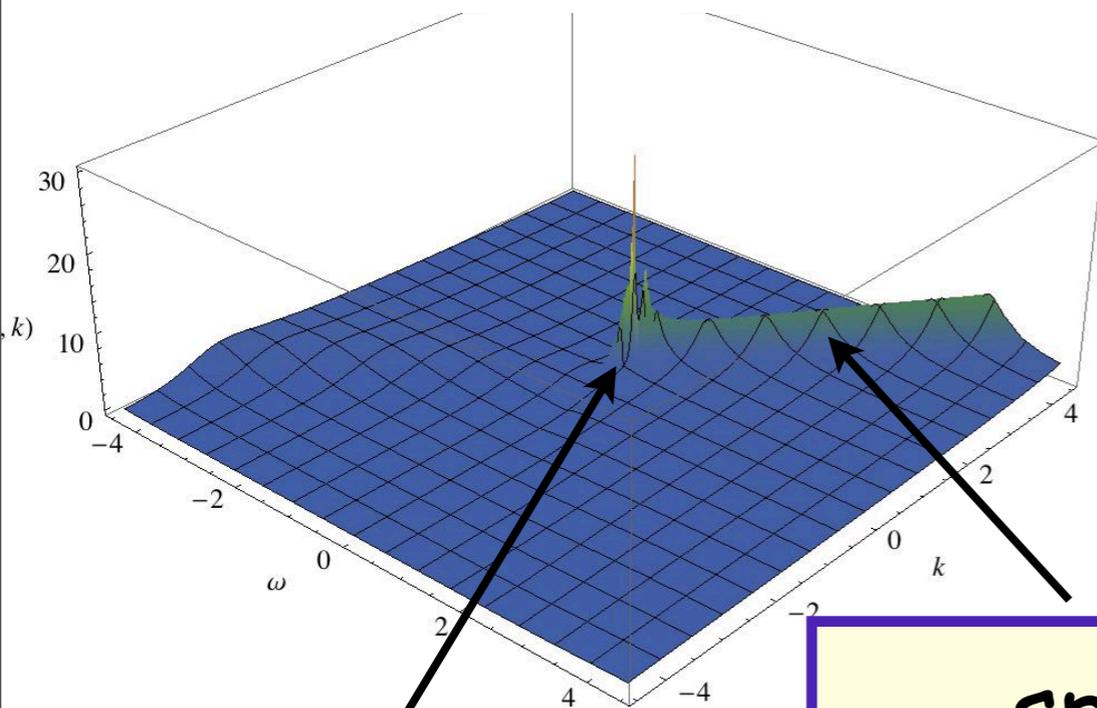
$P > 4.2$



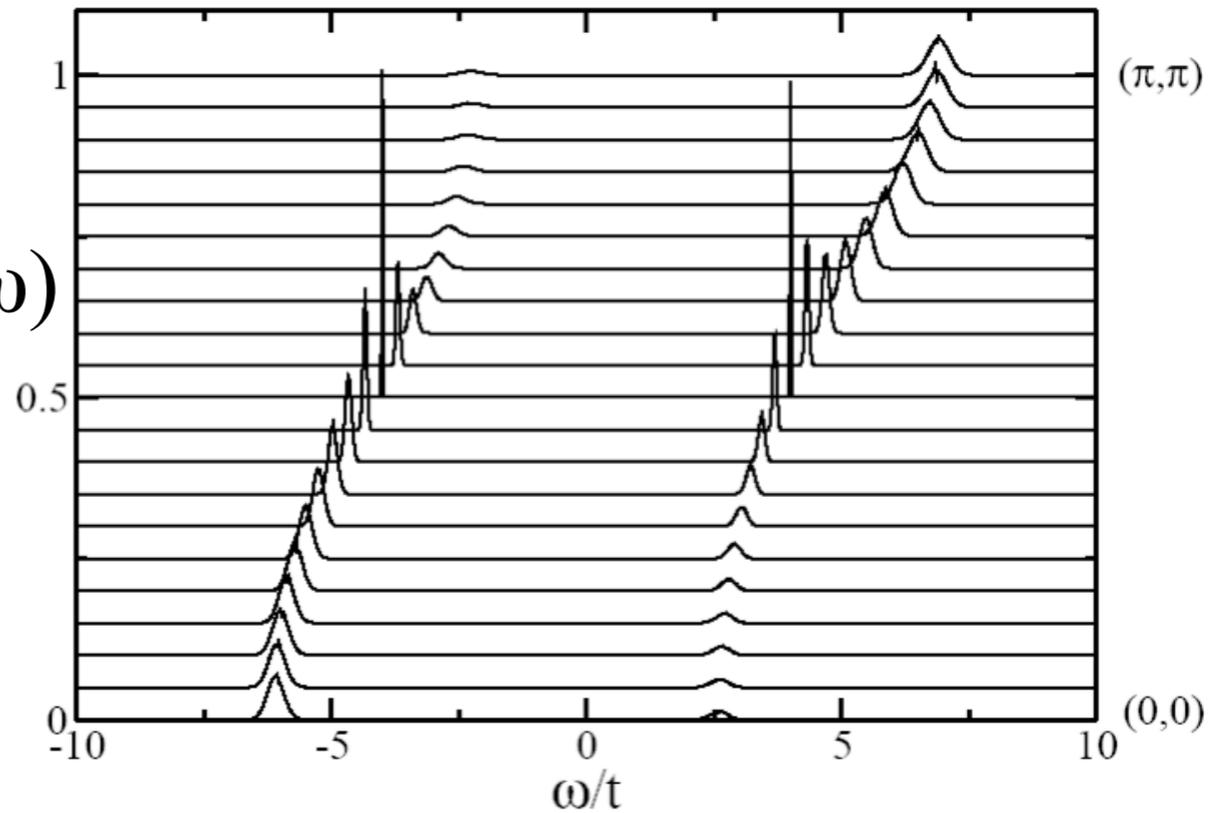
dynamically generated gap:

# How is the spectrum modified?

$P=0$



$A(k, \omega)$



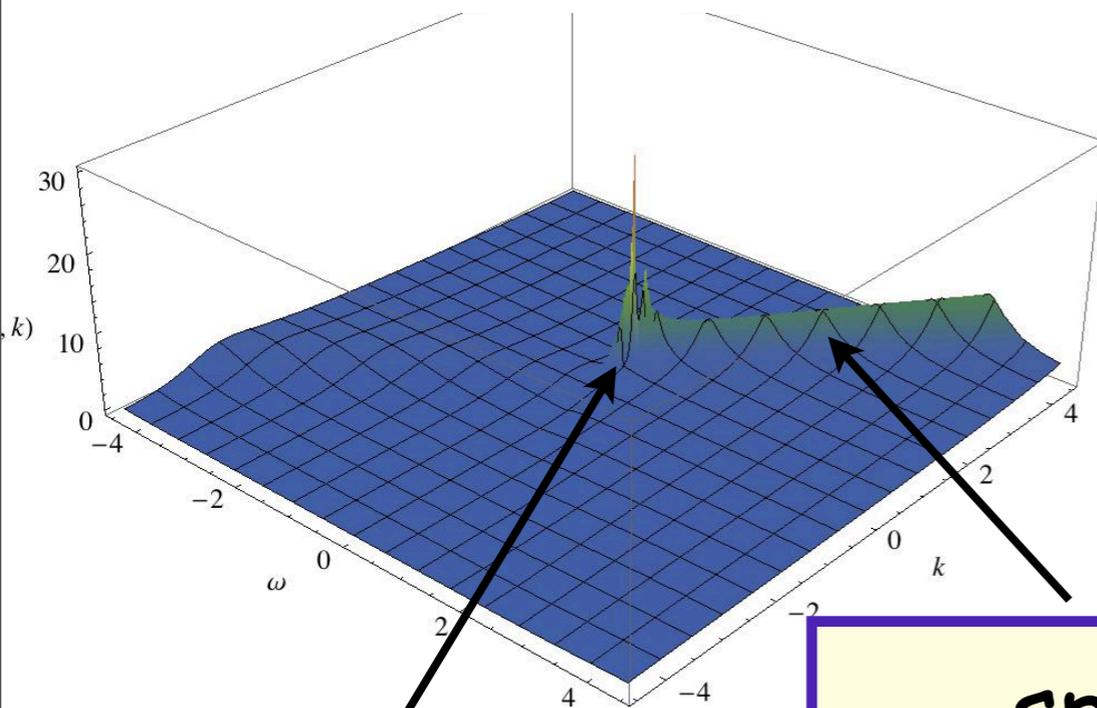
spect  
weight transfer

Fermi  
surface  
peak

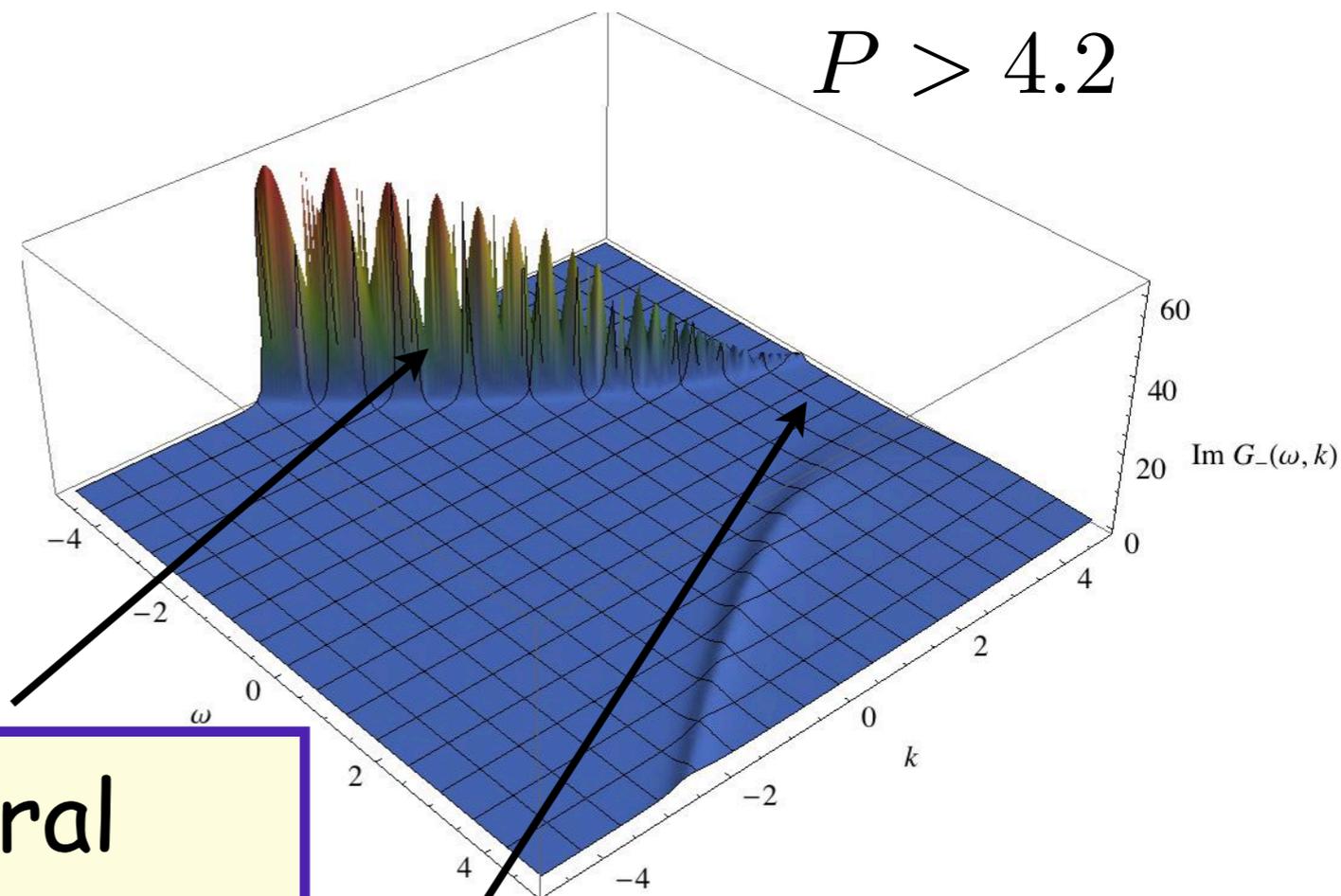
dynamically generated gap:

# How is the spectrum modified?

$P=0$



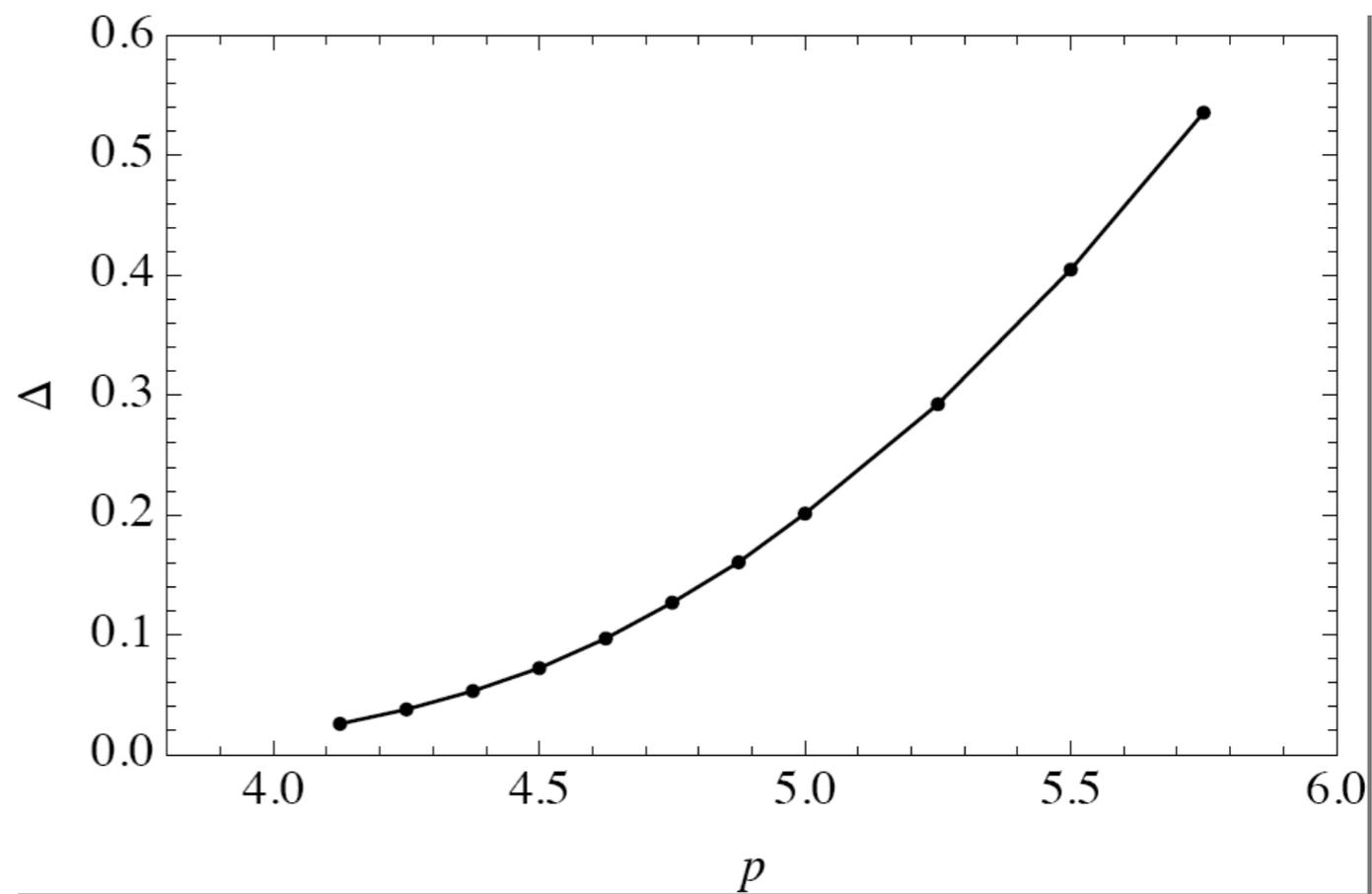
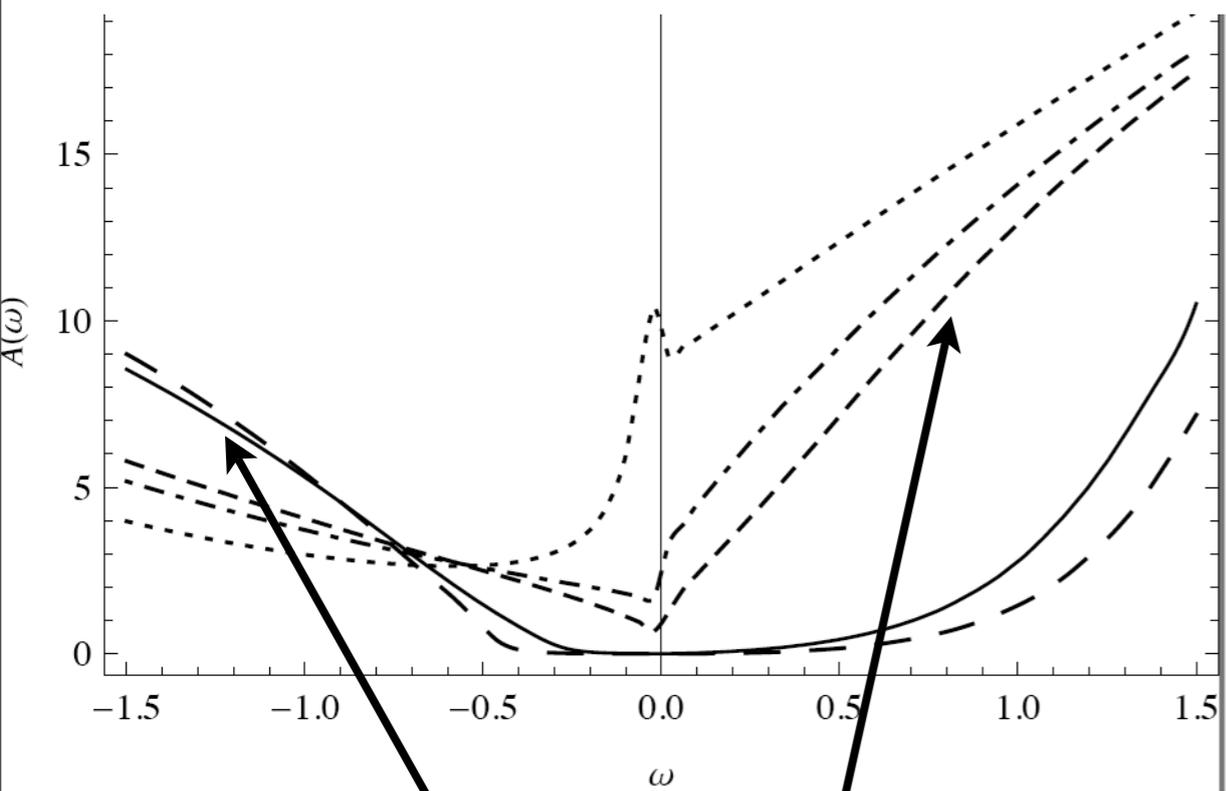
$P > 4.2$



spectral weight transfer

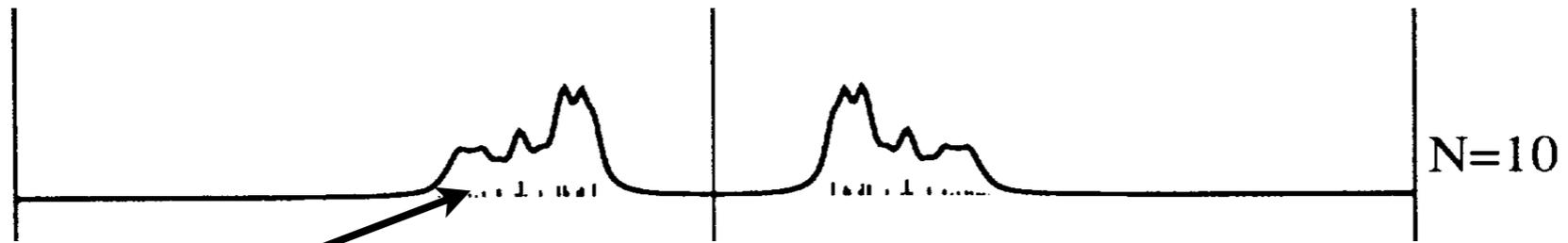
Fermi surface peak

dynamically generated gap:



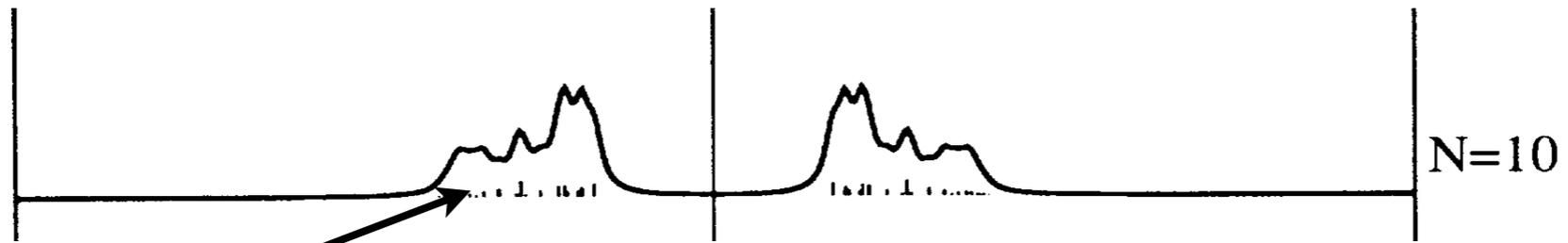
spectral weight  
transfer  
UV-IR mixing

Mott gap



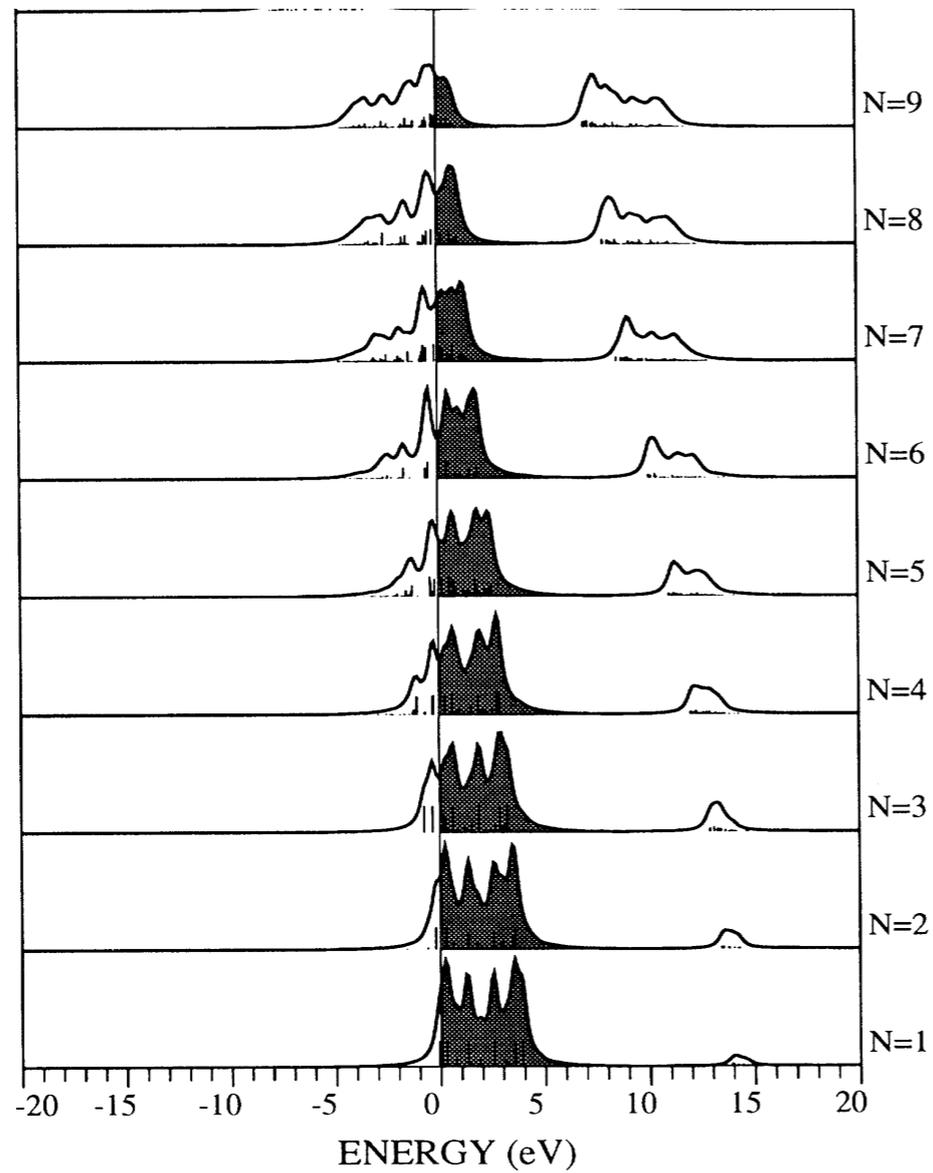
Total weight varies with  $n$

# Mott gap

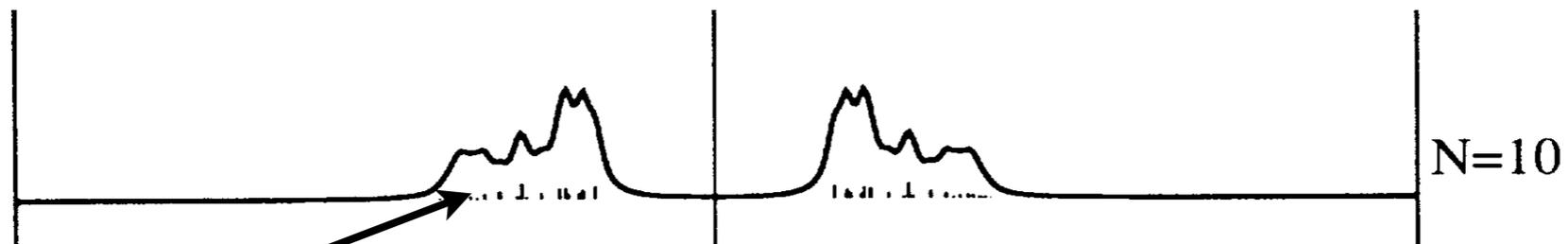


Total weight varies with  $n$

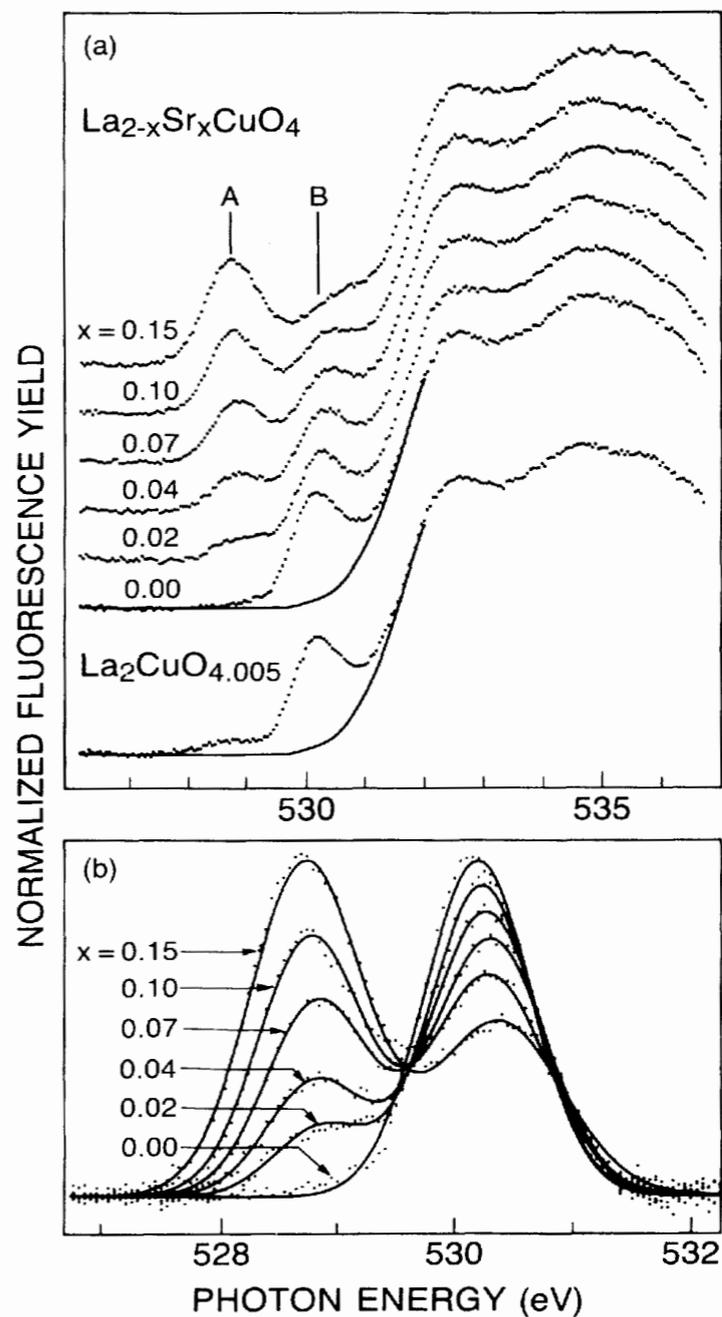
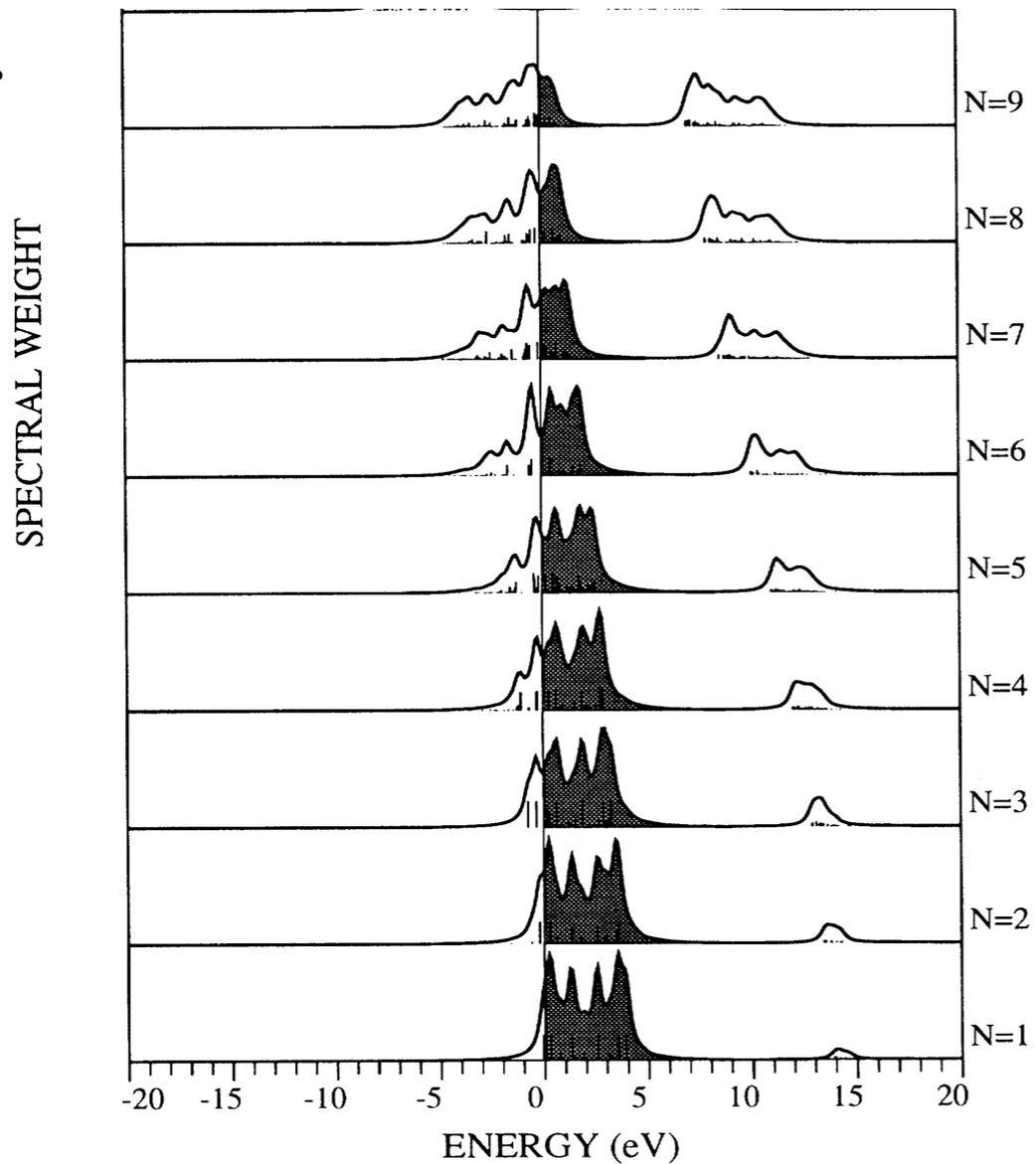
SPECTRAL WEIGHT



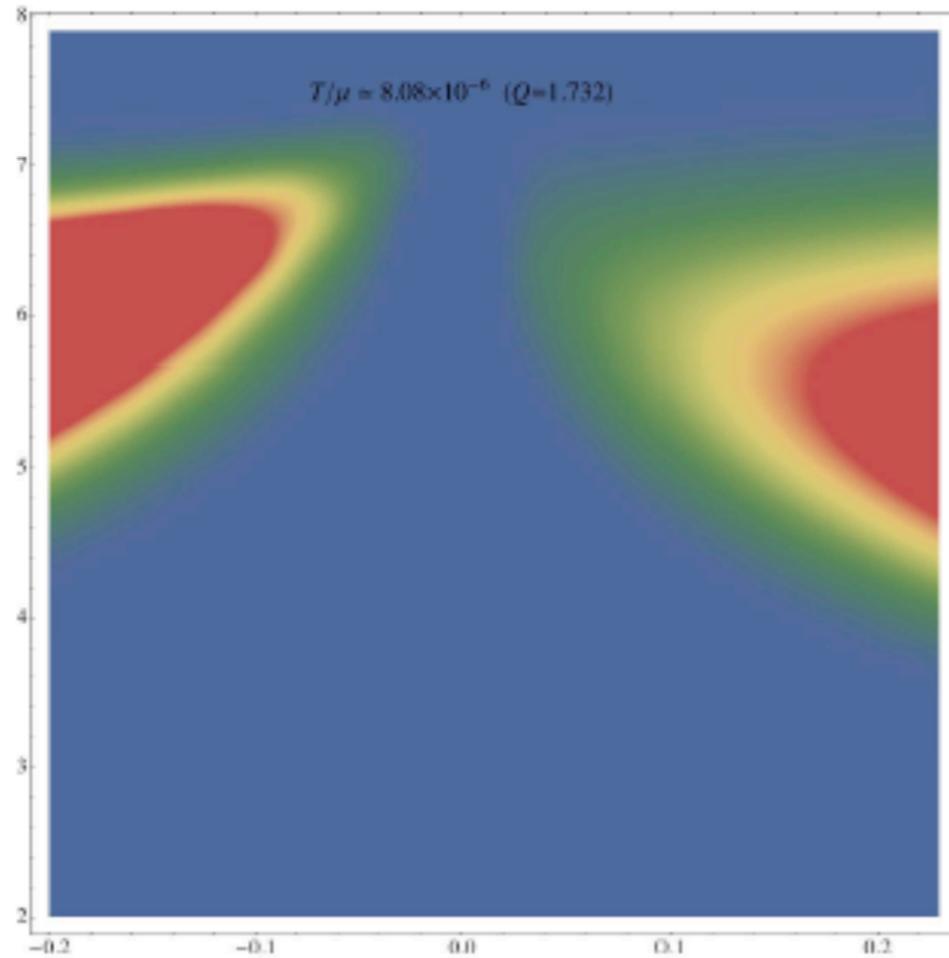
# Mott gap



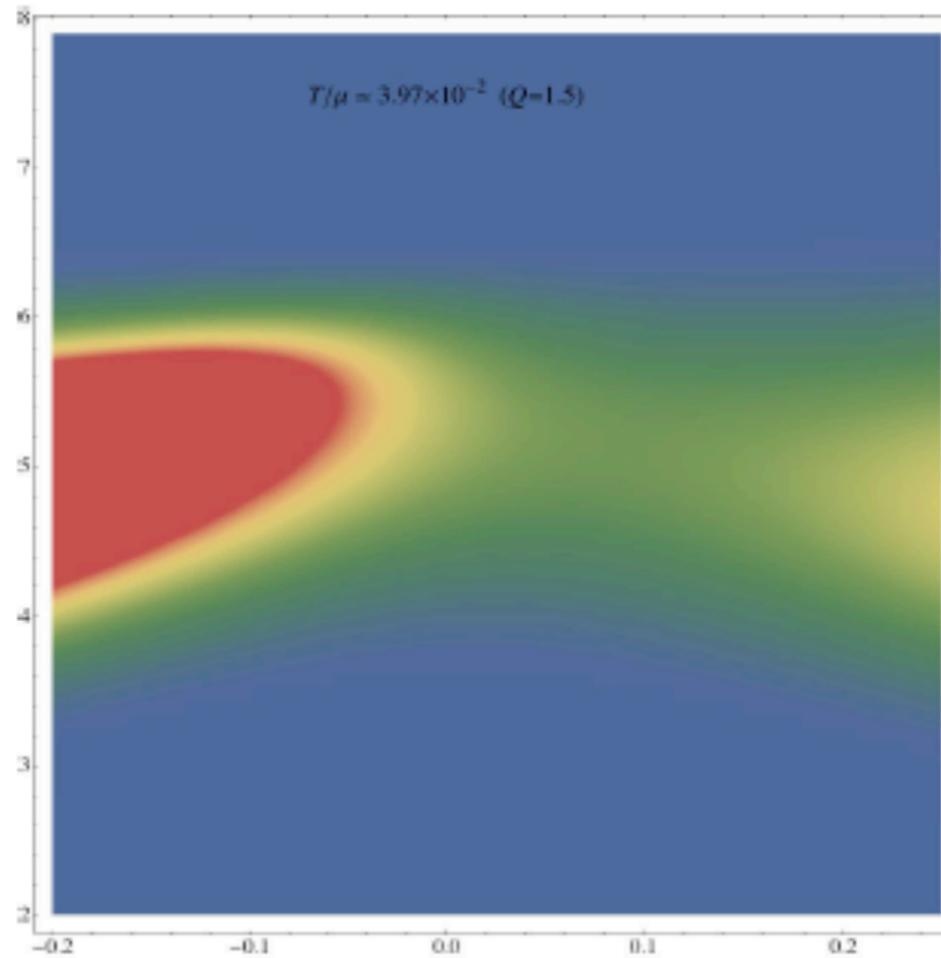
Total weight varies with  $n$



# Finite Temperature Mott transition



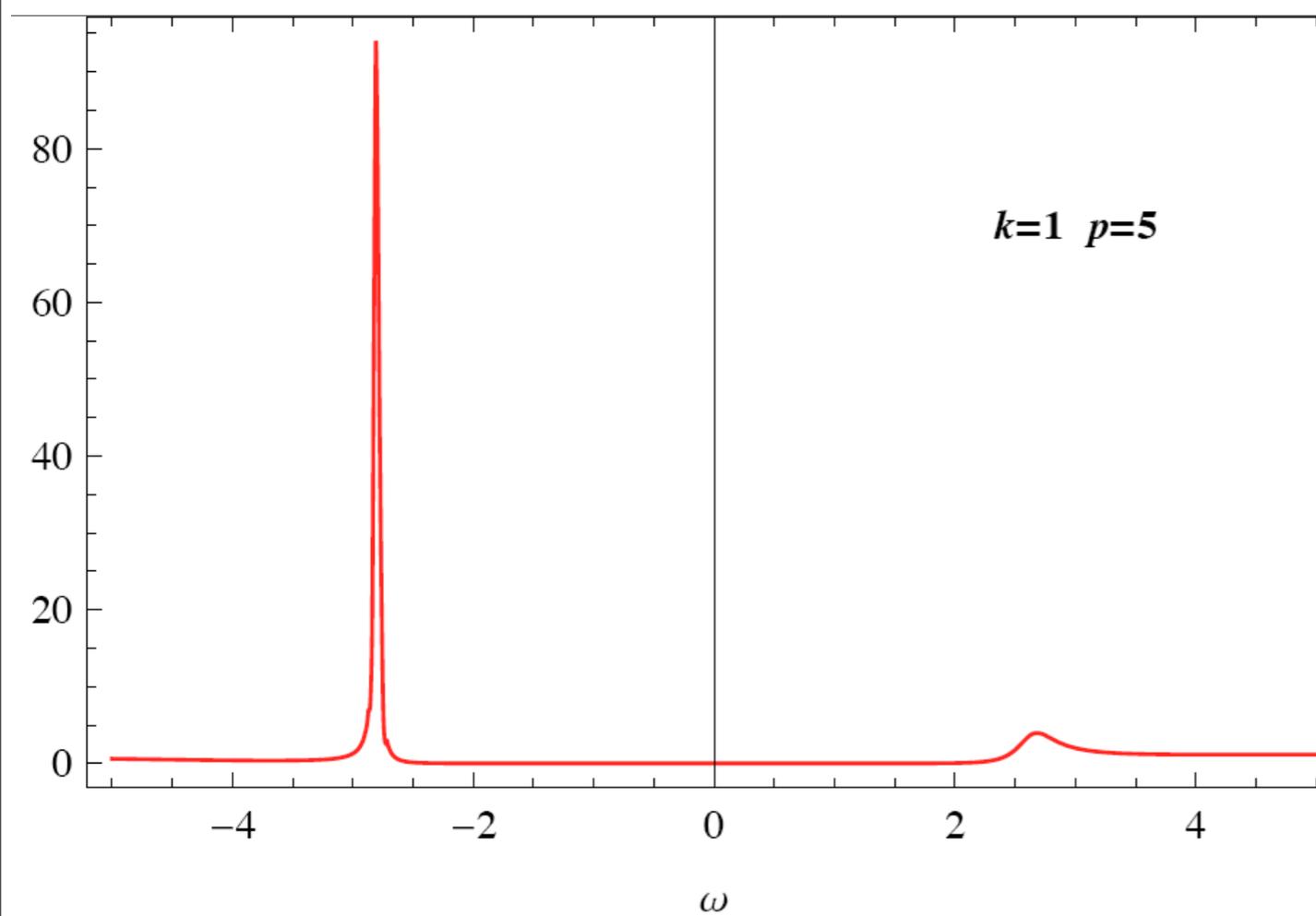
$$\frac{\Delta}{T_{\text{crit}}} \approx 10^3$$



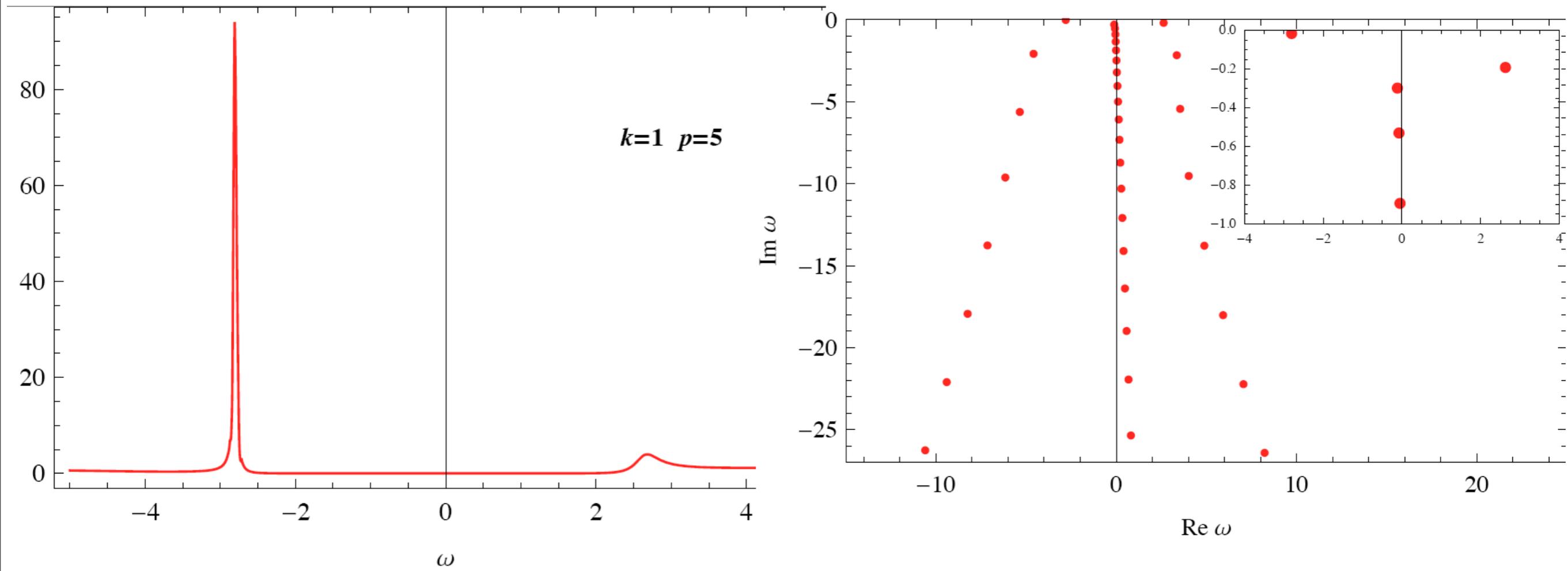
$$\frac{\Delta}{T_{\text{crit}}} \approx 20$$

vanadium oxide

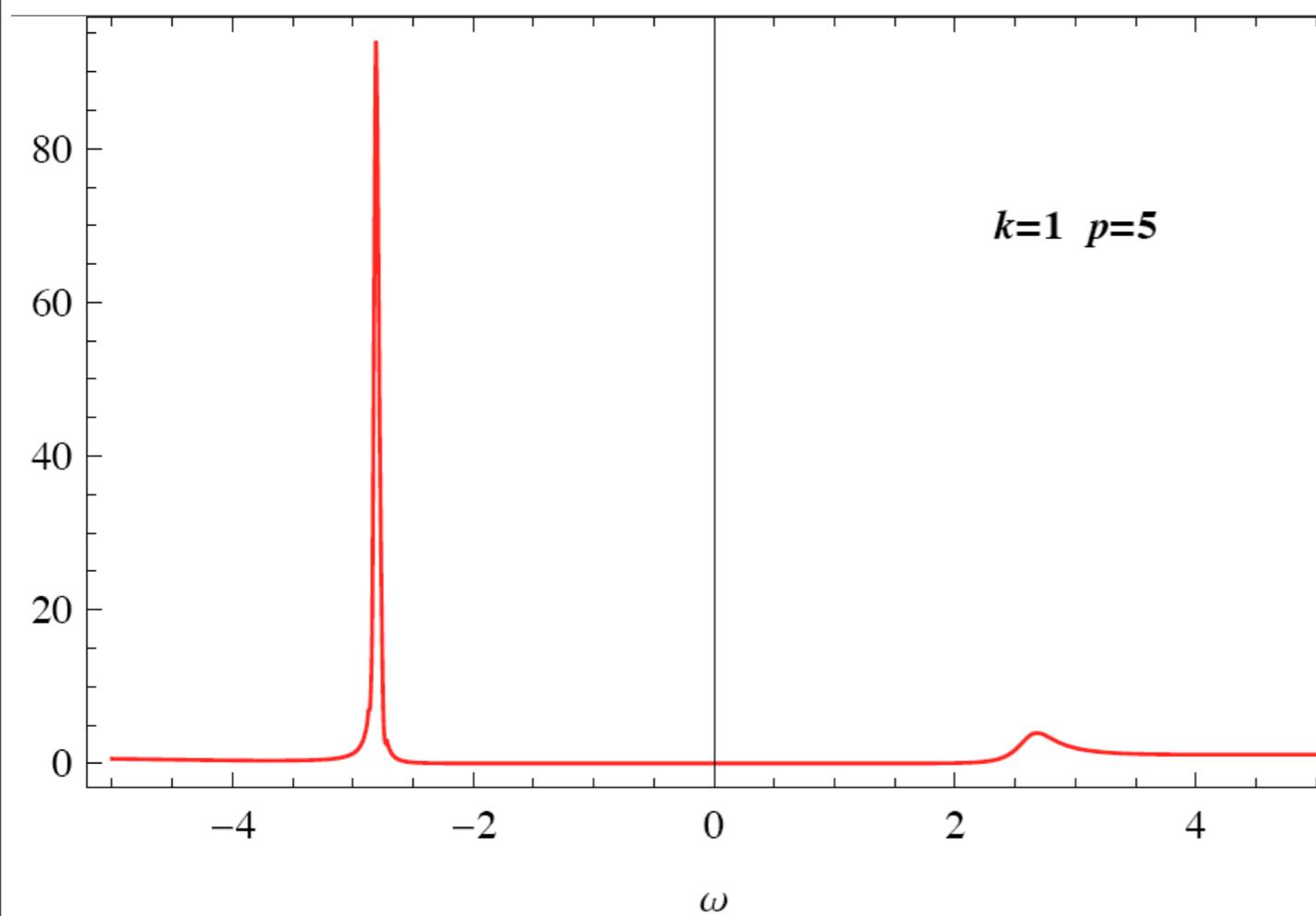
quasi-normal modes: where are the peaks?



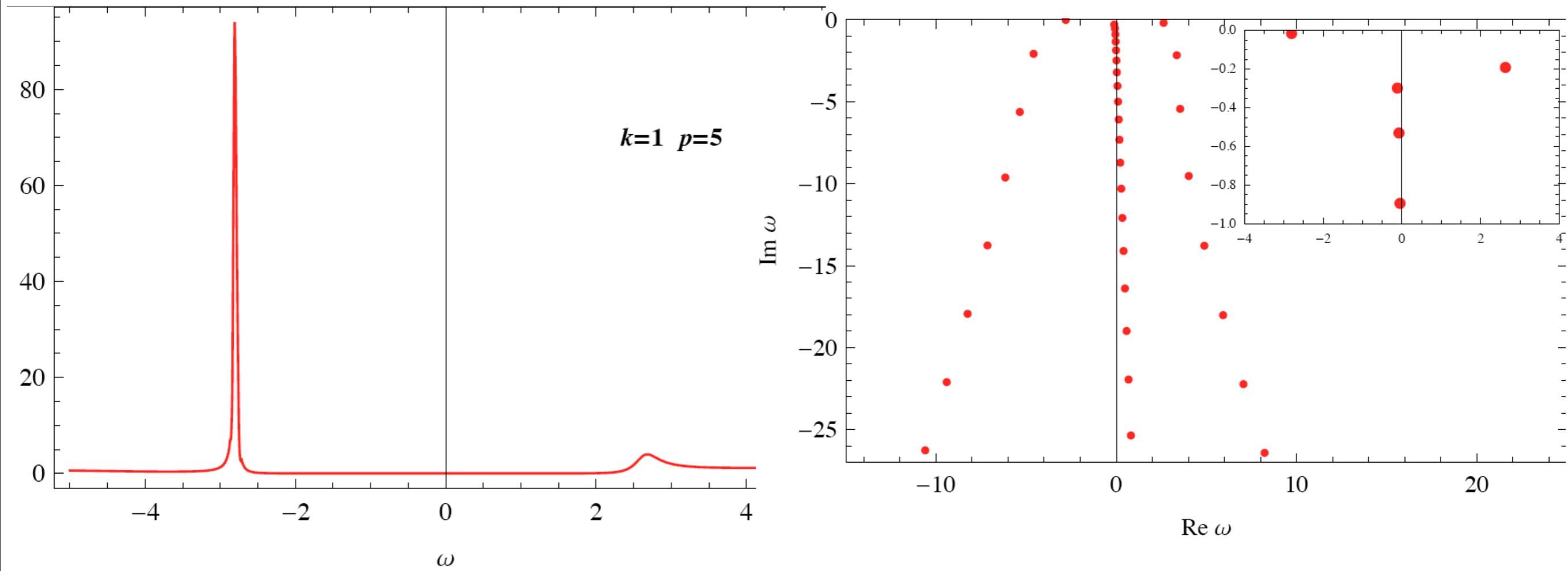
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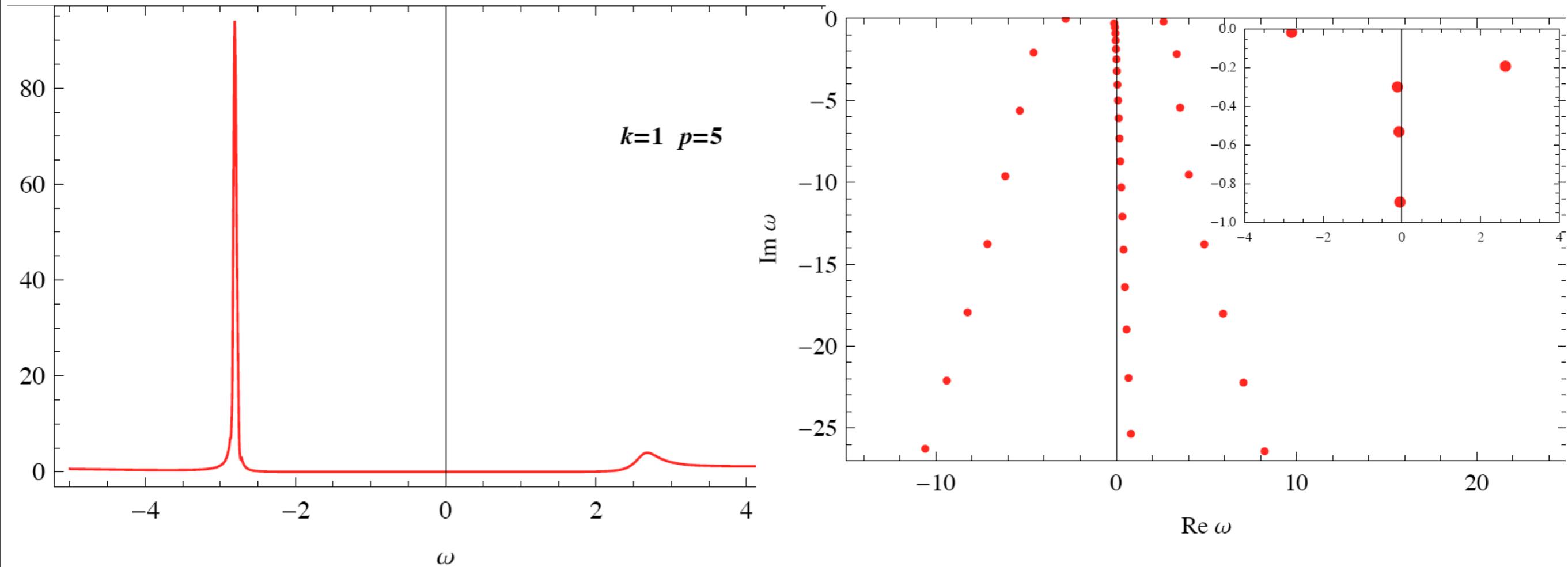
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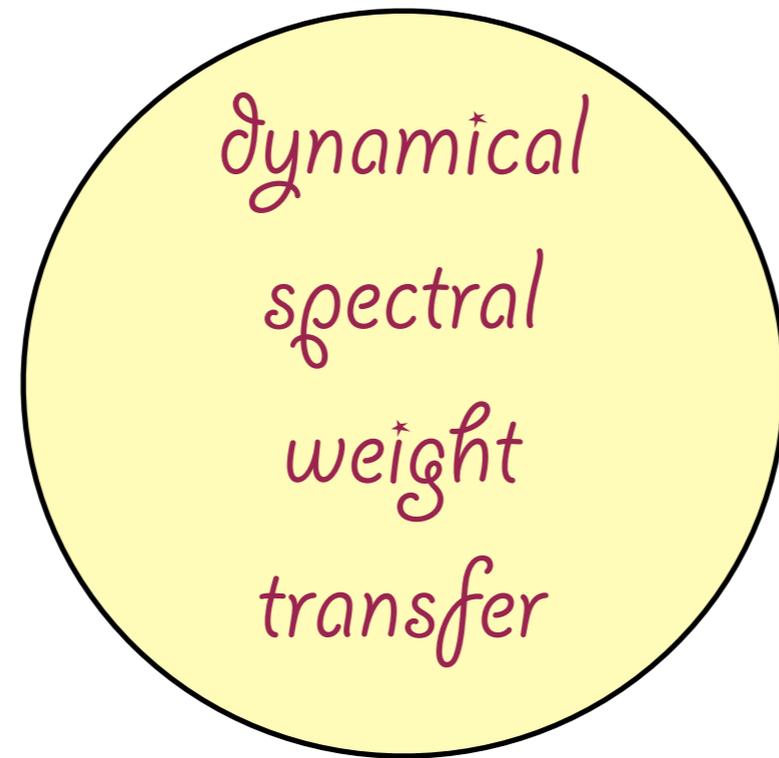
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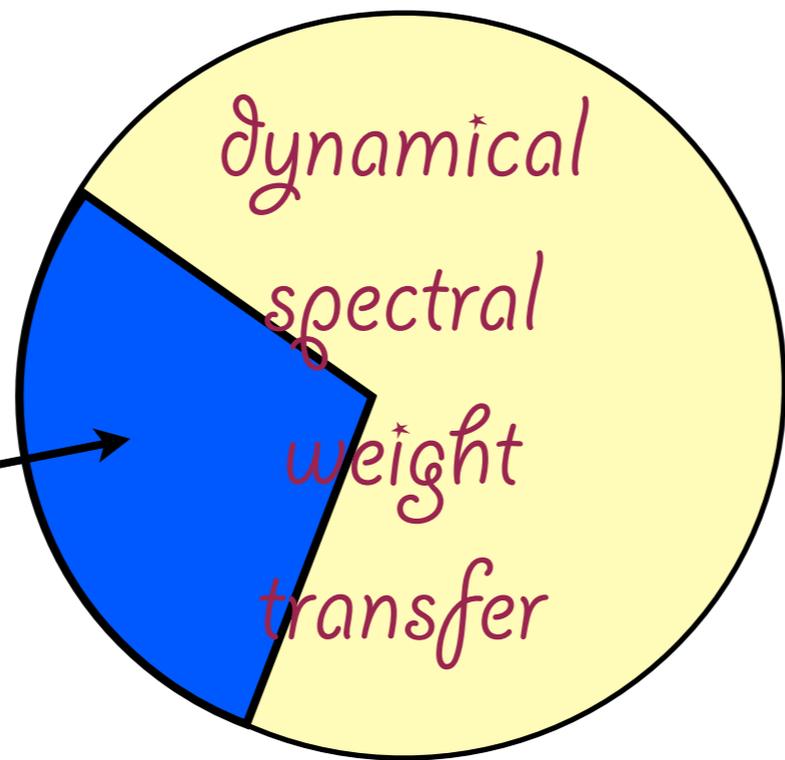


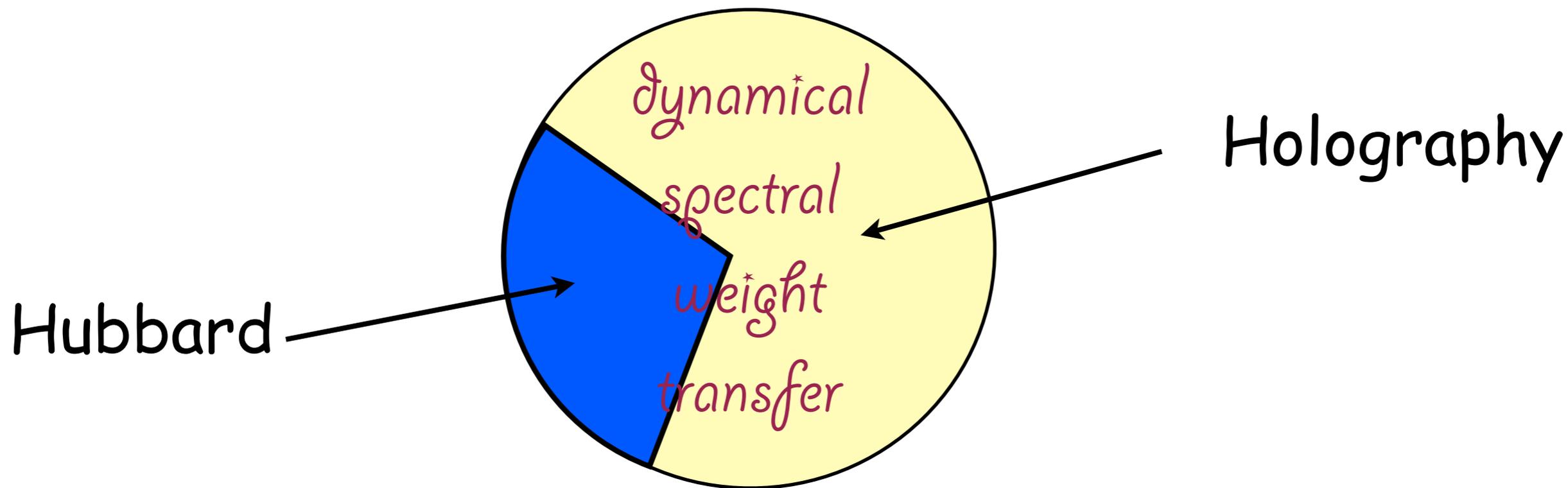
no leaking to  $+\text{Im } \omega$ : no instability



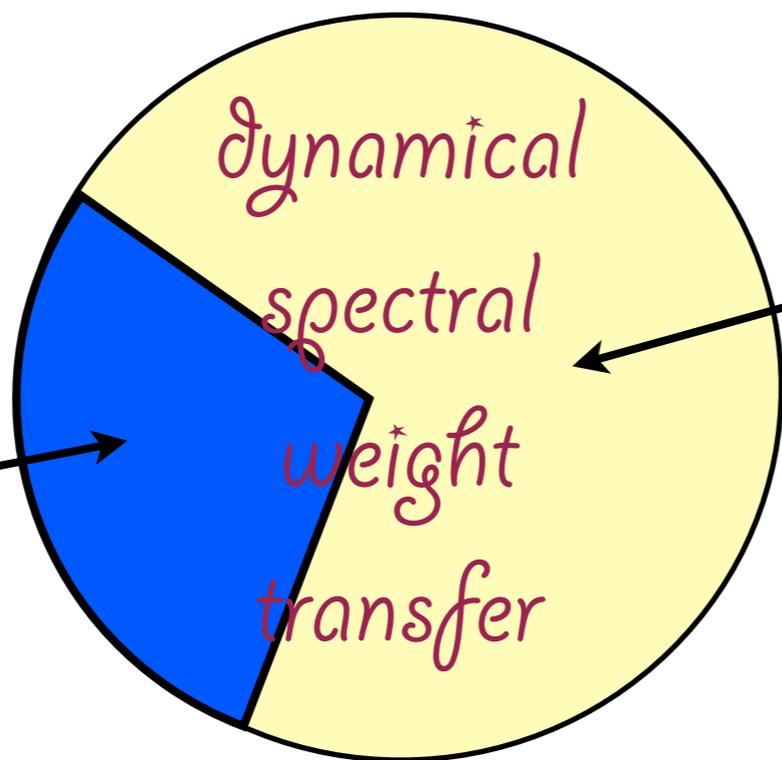
dynamical  
spectral  
weight  
transfer

Hubbard





# Mottness



Hubbard

Holography

*dynamical  
spectral  
weight  
transfer*