Holographic metals and Kondo lattice models

Galileo Galilei Institue, Florence, Nov 4, 2010

Talk online: sachdev.physics.harvard.edu

PHYSICS

HARVARD

Outline

1. Quantum impurities and AdS_2

   Quantum spin coupled to a CFT

2. Phases of the Kondo lattice

   Fermi liquids (FL),
   Fractionalized Fermi liquids (FL),*
   and the Luttinger theorem

3. A mean field theory of a fractionalized Fermi liquid

   A marginal Fermi liquid and $AdS_2 \times R^2$

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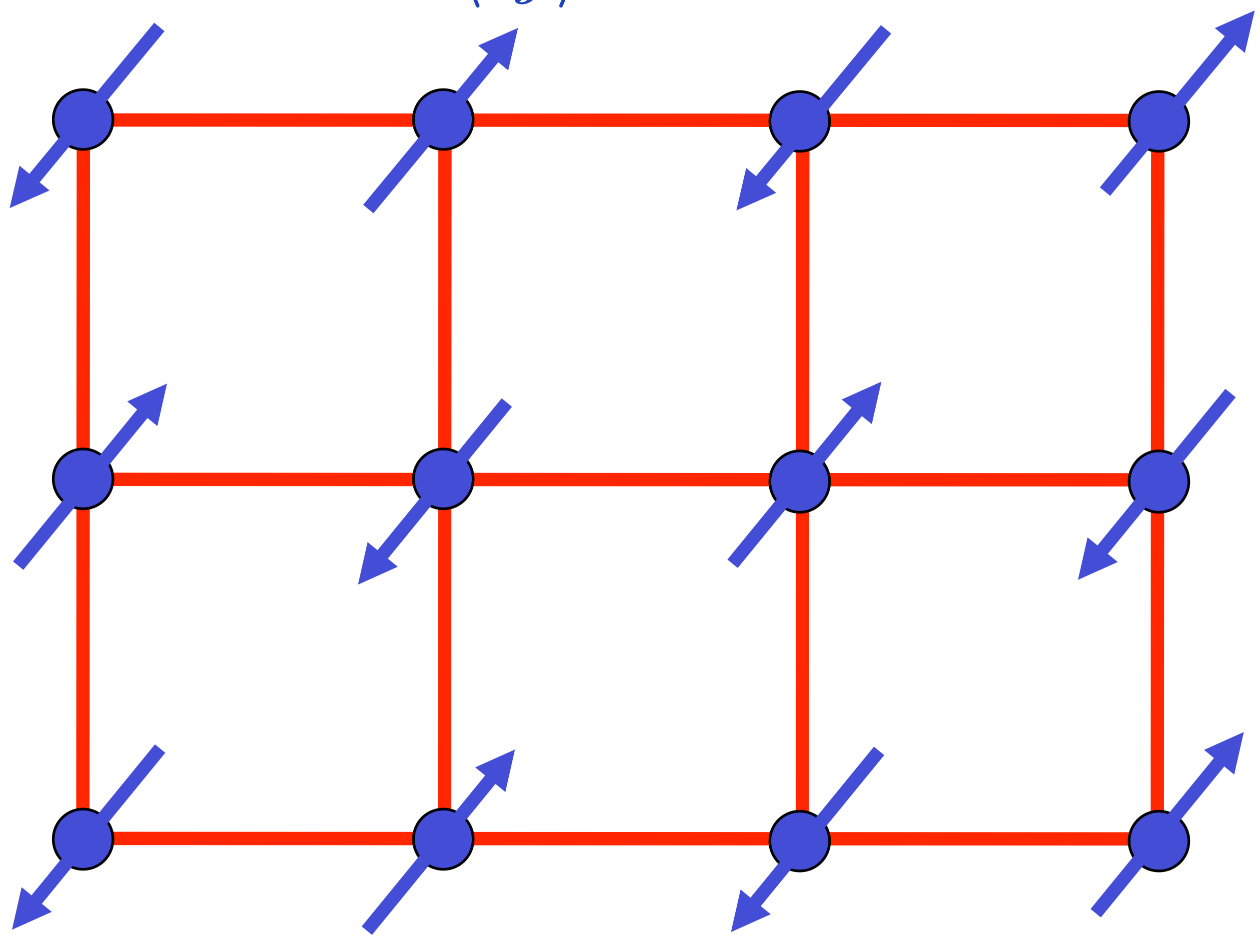
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state has long-range Néel order

Order parameter is a single vector field $\vec{\varphi} = \eta_i \vec{S}_i$

$\eta_i = \pm 1$ on two sublattices

$\langle \vec{\varphi} \rangle \neq 0$ in Néel state.

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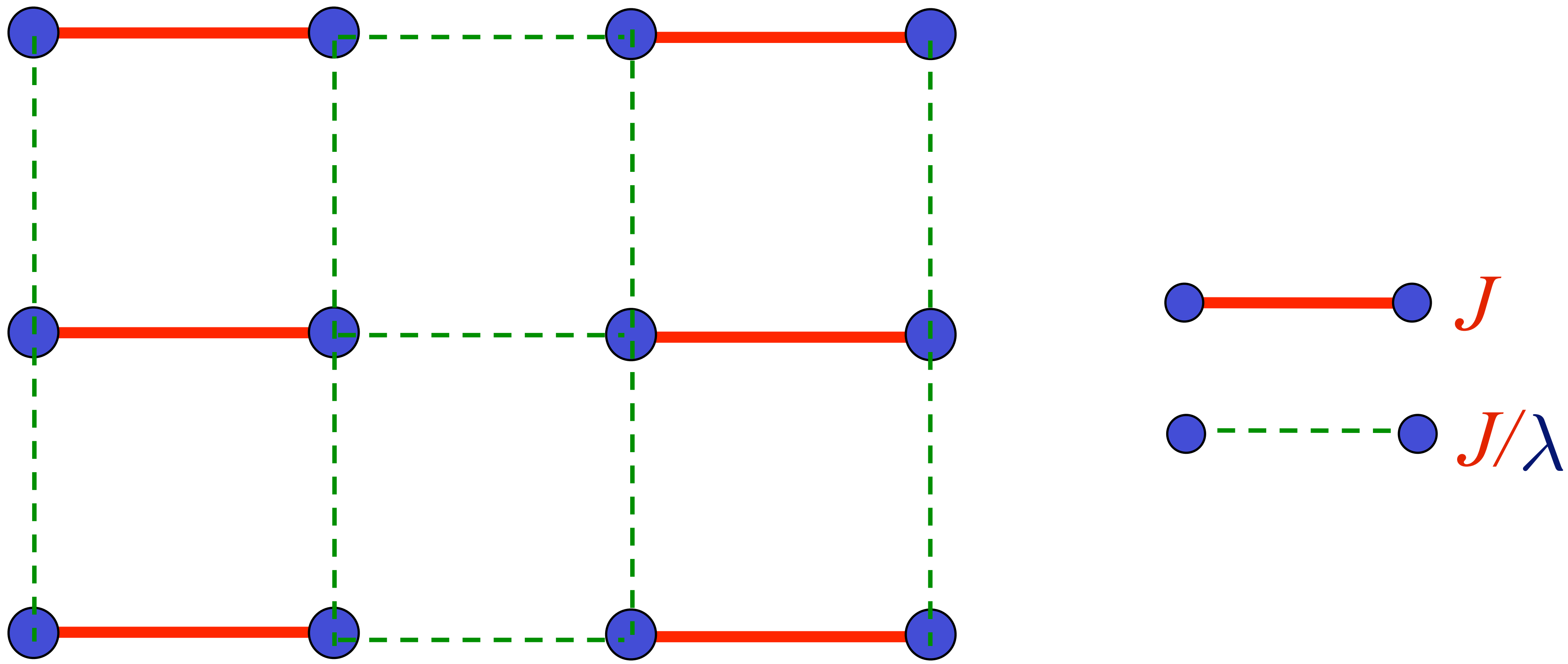
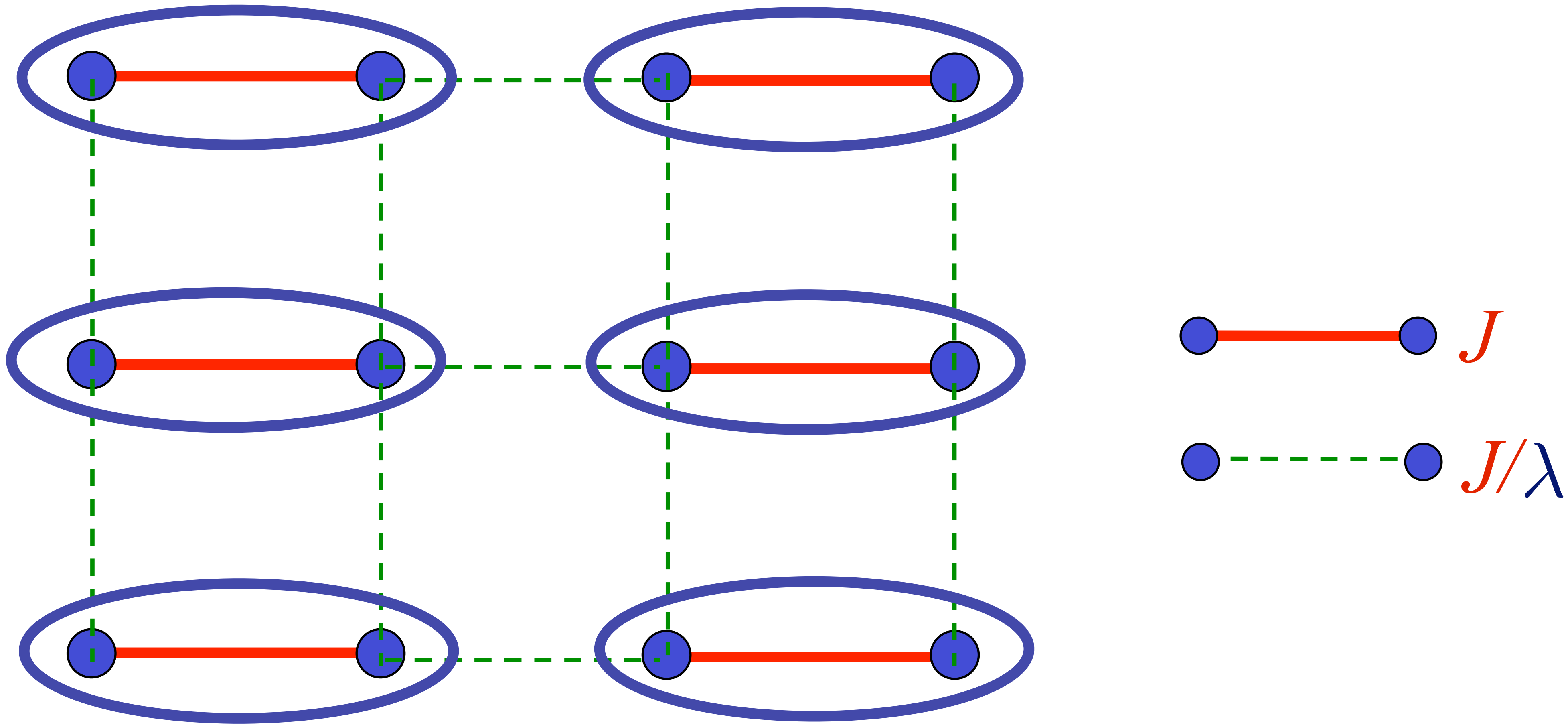


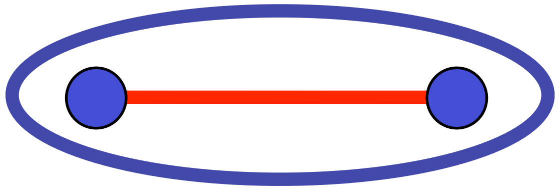
Weaken some bonds to induce spin entanglement in a new quantum phase

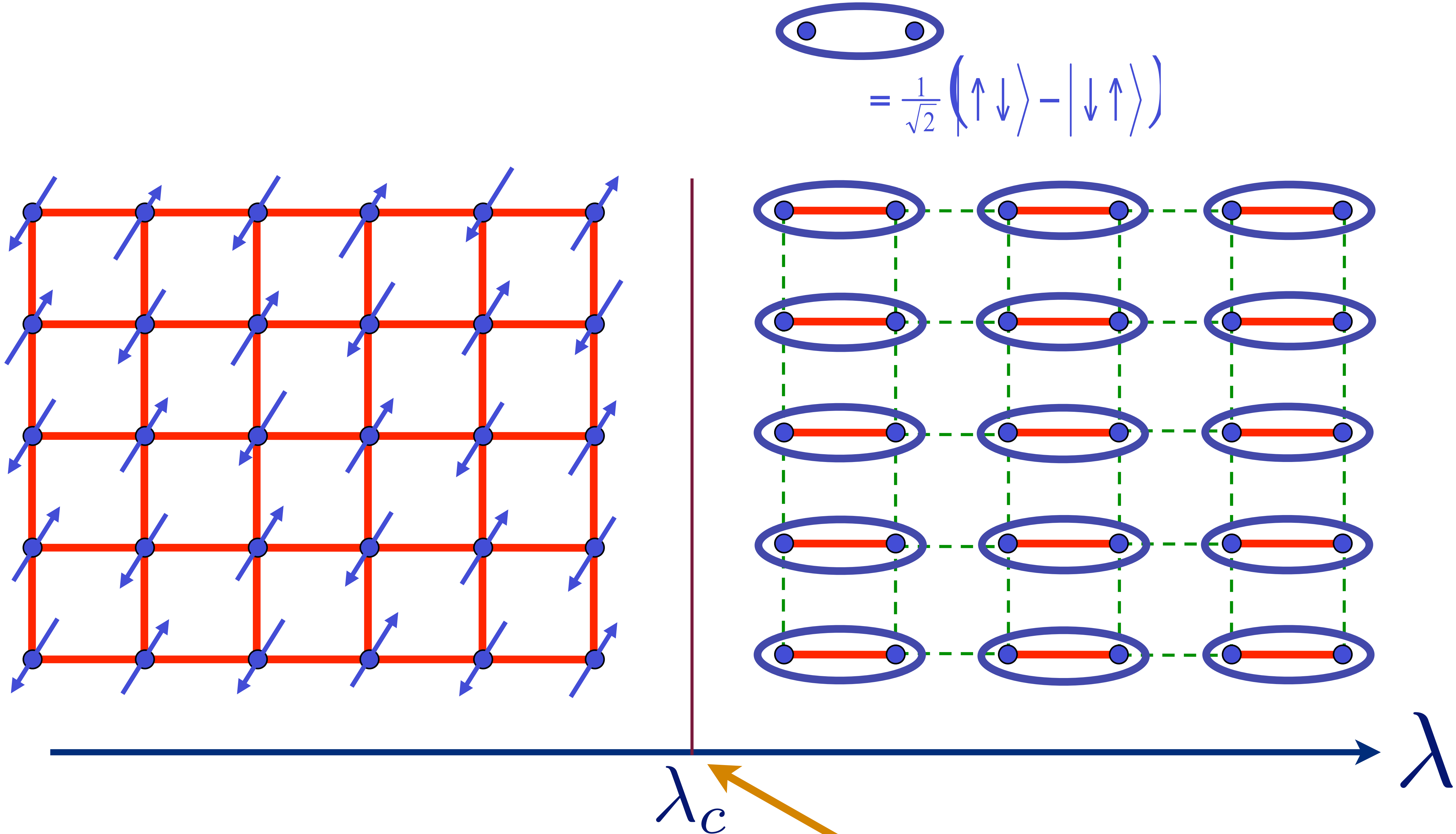
Square lattice antiferromagnet

$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j$$



Ground state is a "quantum paramagnet" with spins locked in valence bond singlets

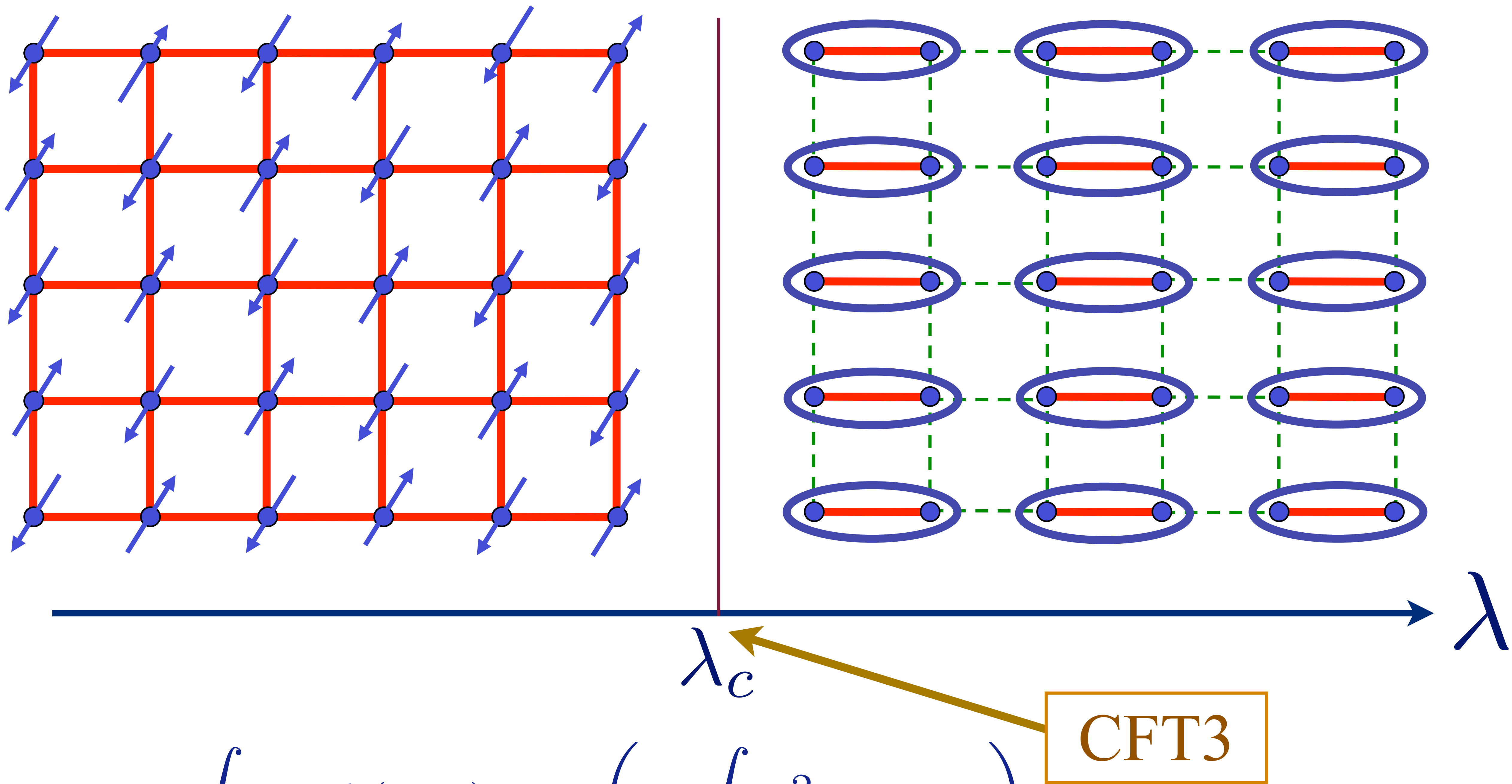
 $= \frac{1}{\sqrt{2}} \left(\left| \uparrow \downarrow \right\rangle - \left| \downarrow \uparrow \right\rangle \right)$



Quantum critical point with non-local entanglement in spin wavefunction

M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, *Phys. Rev.*B **65**, 014407 (2002).

Description using Landau-Ginzburg field theory

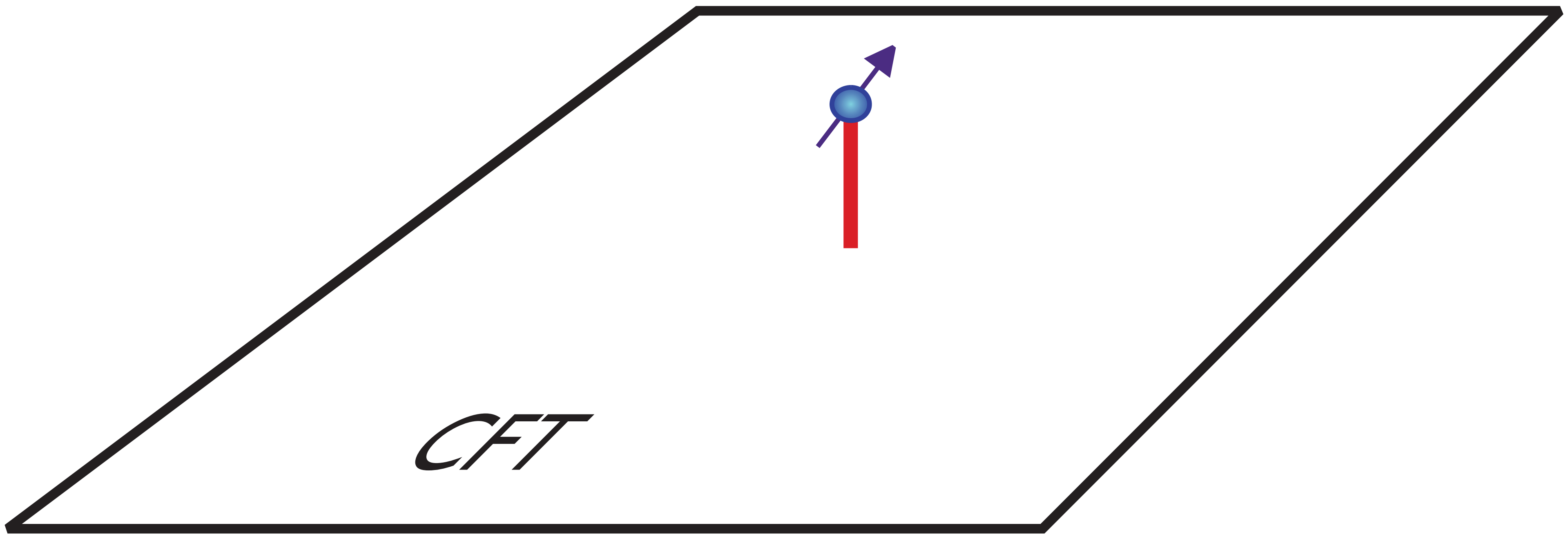


$$\mathcal{Z} = \int \mathcal{D}\varphi^a(r,\tau) \exp\left(- \int d^2 r d\tau \, \mathcal{L}_\varphi \right)$$

$$\mathcal{L}_\varphi = \frac{1}{2}\left[(\partial_\tau \varphi^a)^2 + v^2 (\nabla \varphi^a)^2 + s(\varphi^a)^2 \right] + \frac{u}{4}\left[(\varphi^a)^2 \right]^2$$

$$s \sim \lambda - \lambda_c$$

Quantum impurity coupled to a CFT

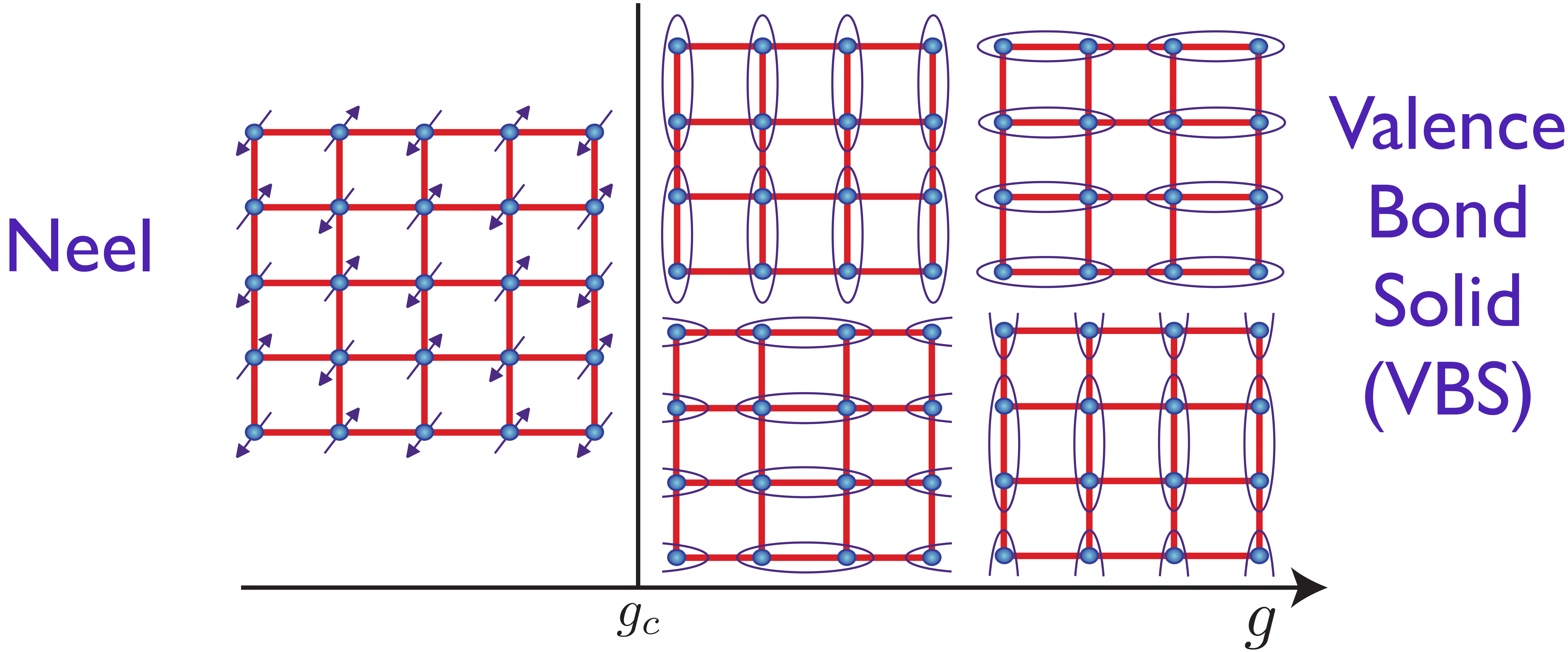


$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}\varphi^a(r,\tau)\mathcal{D}n^a(\tau)\delta([n^a(\tau)]^2-1)\exp\left(-\int d\tau\,\mathcal{L}_{\rm imp} - \int d^2r d\tau\,\mathcal{L}_\varphi\right) \\
\mathcal{L}_{\rm imp} &= \frac{i}{2}\mathcal{A}^a\frac{dn^a}{d\tau} + Jn^a(\tau)\varphi^a(0,\tau)
\end{aligned}
$$

where $\mathcal{A}^a$ is any function of $n^a(\tau)$ obeying $\epsilon^{abc}(\partial\mathcal{A}^b/\partial n^c) = n^a$.

At the critical point $J \Rightarrow J^*$, a universal fixed point

Frustrated antiferromagnet with full square lattice symmetry and one S=1/2 per unit cell



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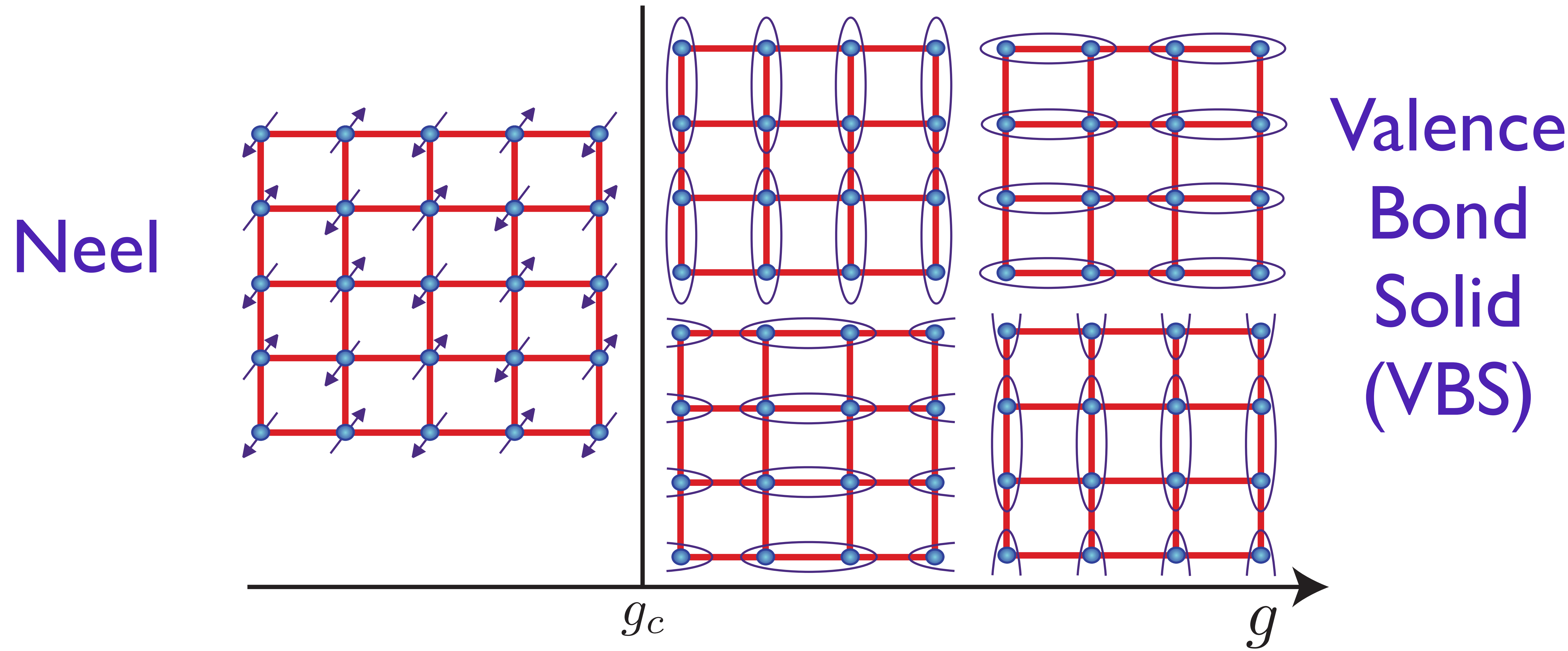
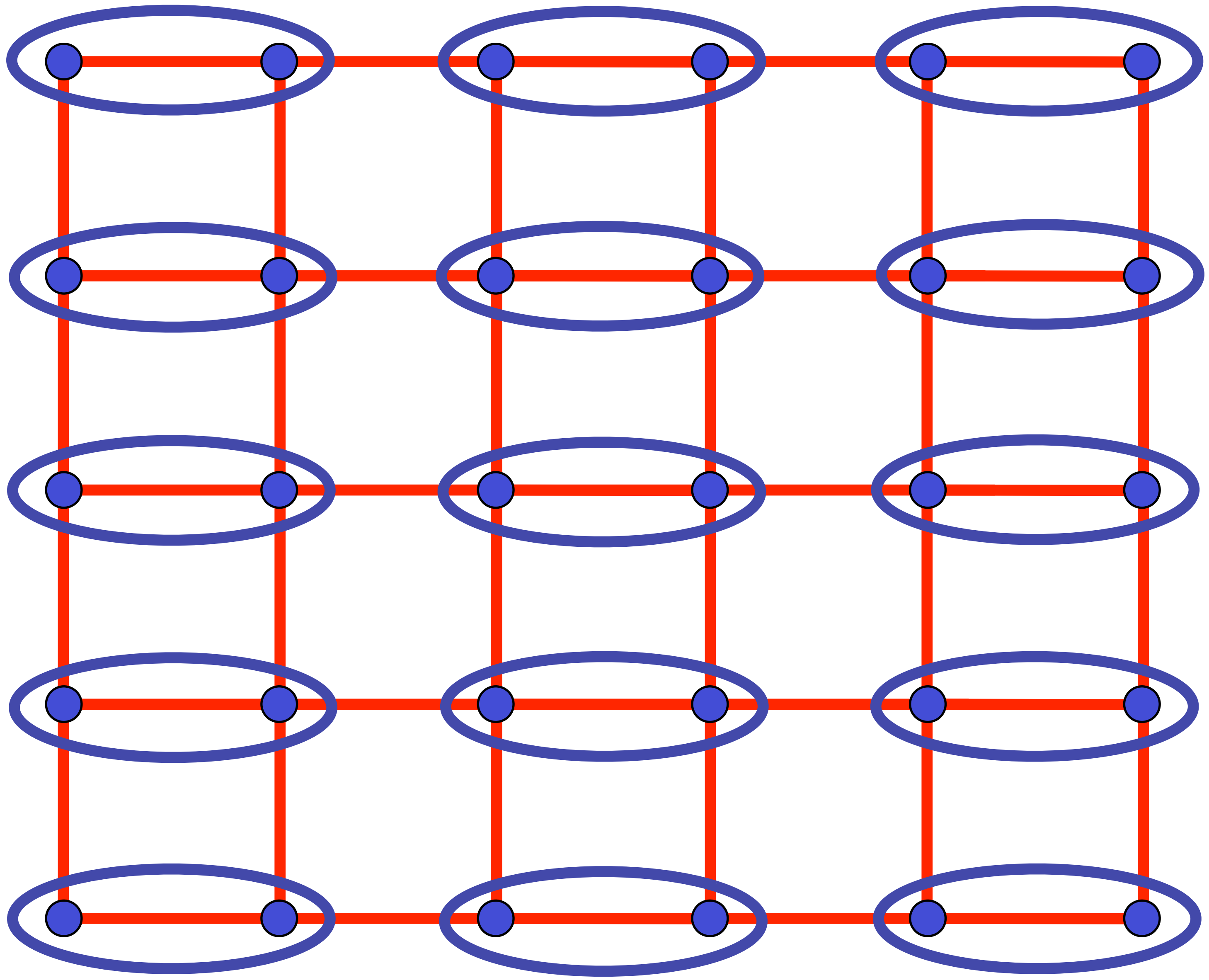
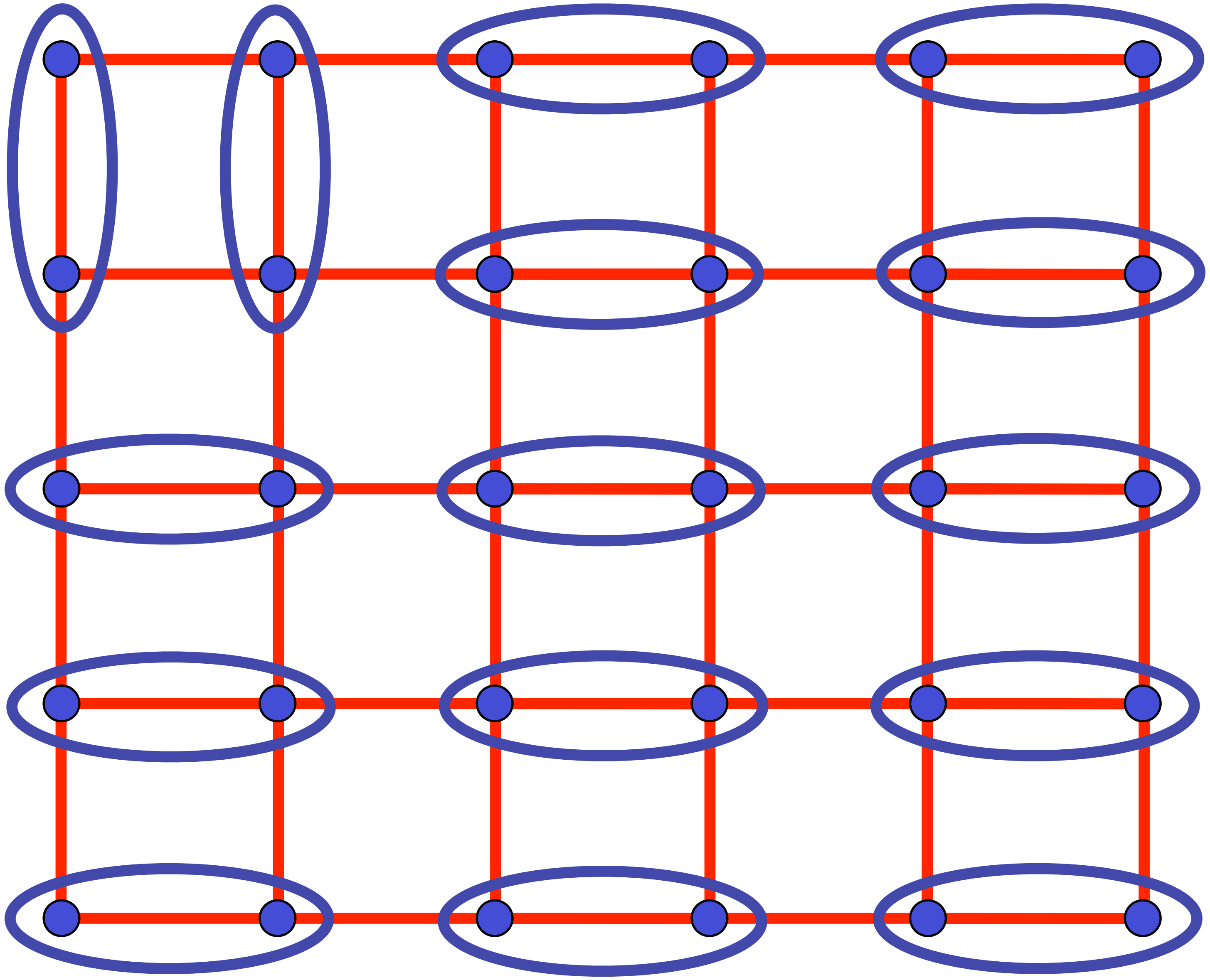
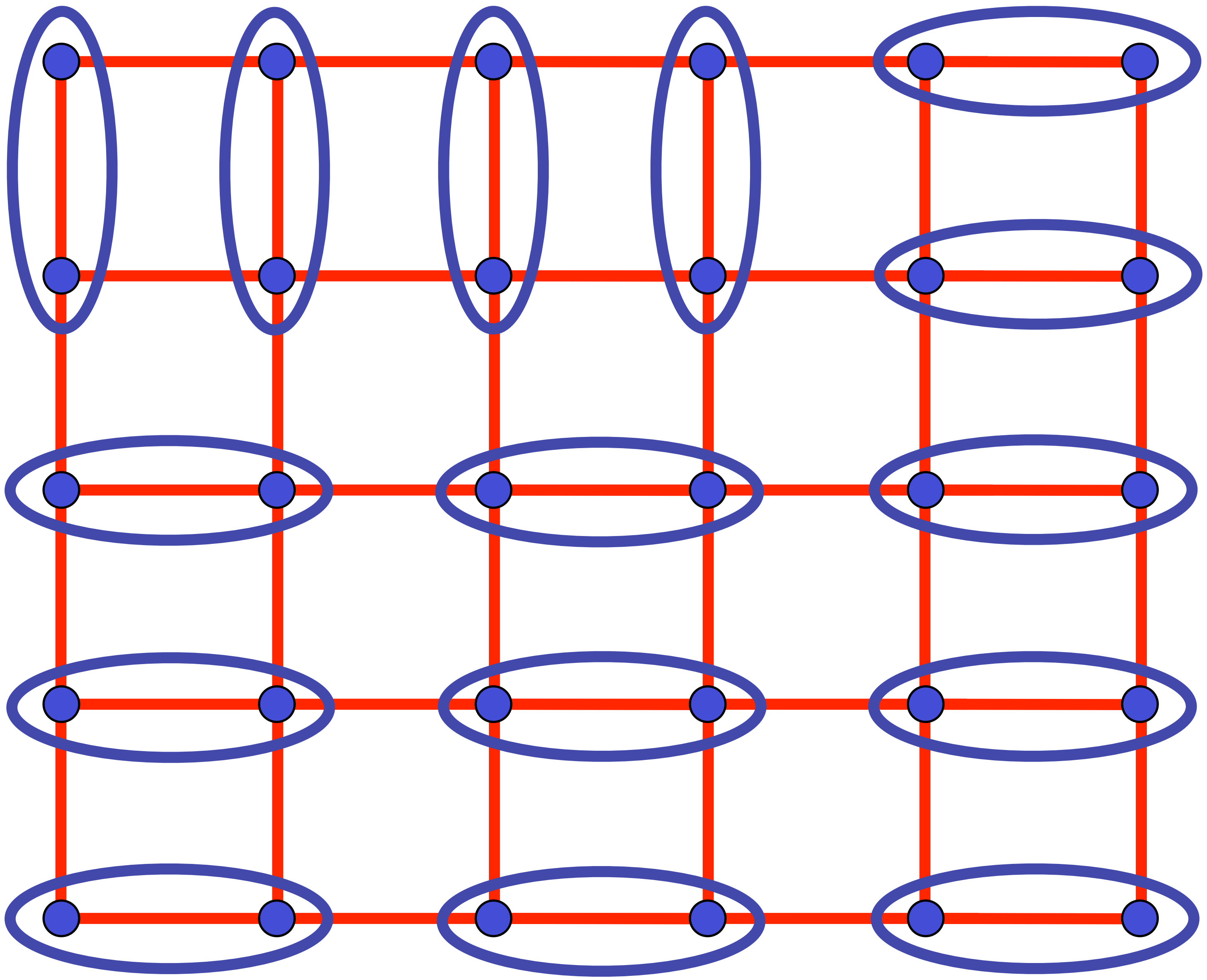


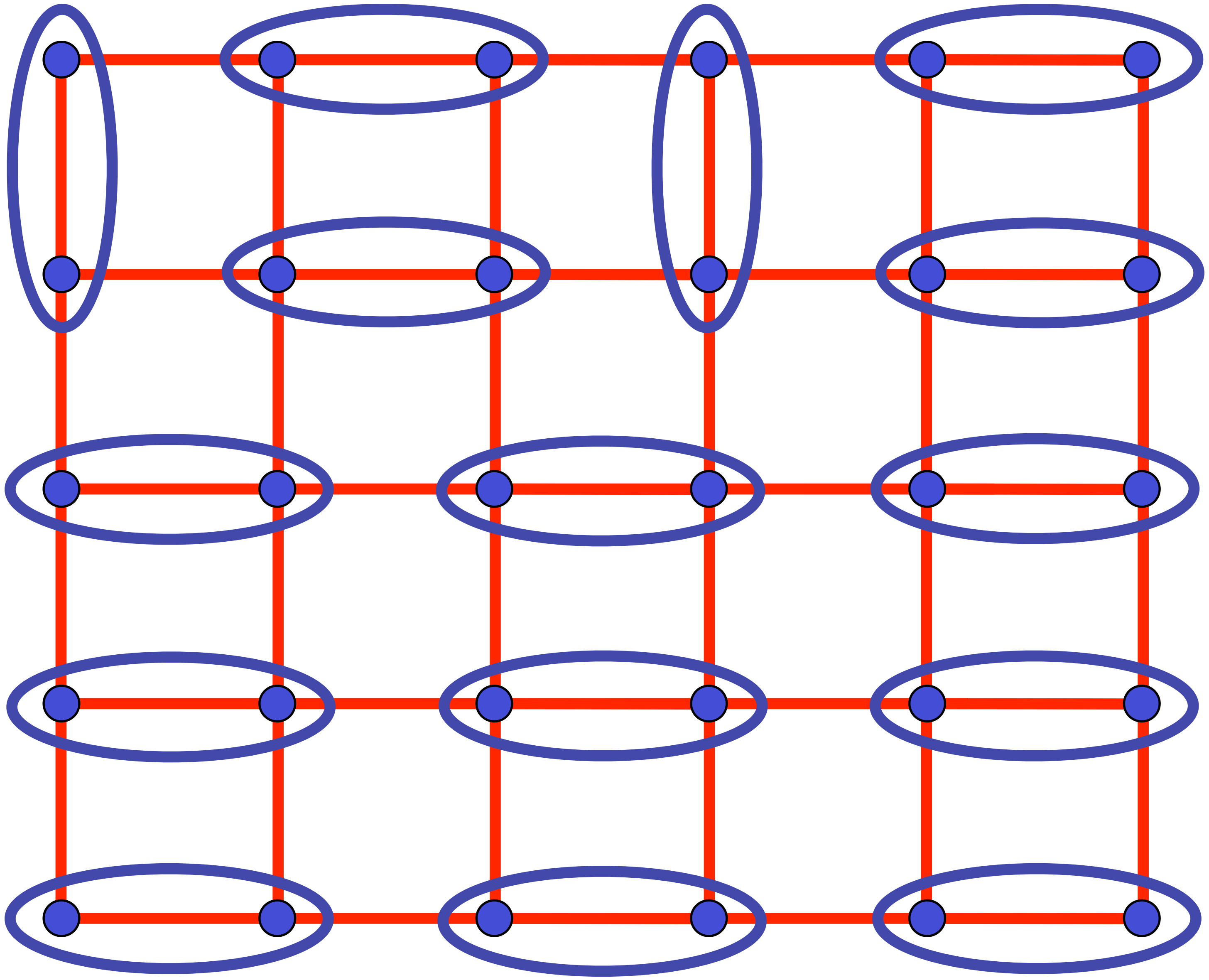
Low energy degrees of freedom:

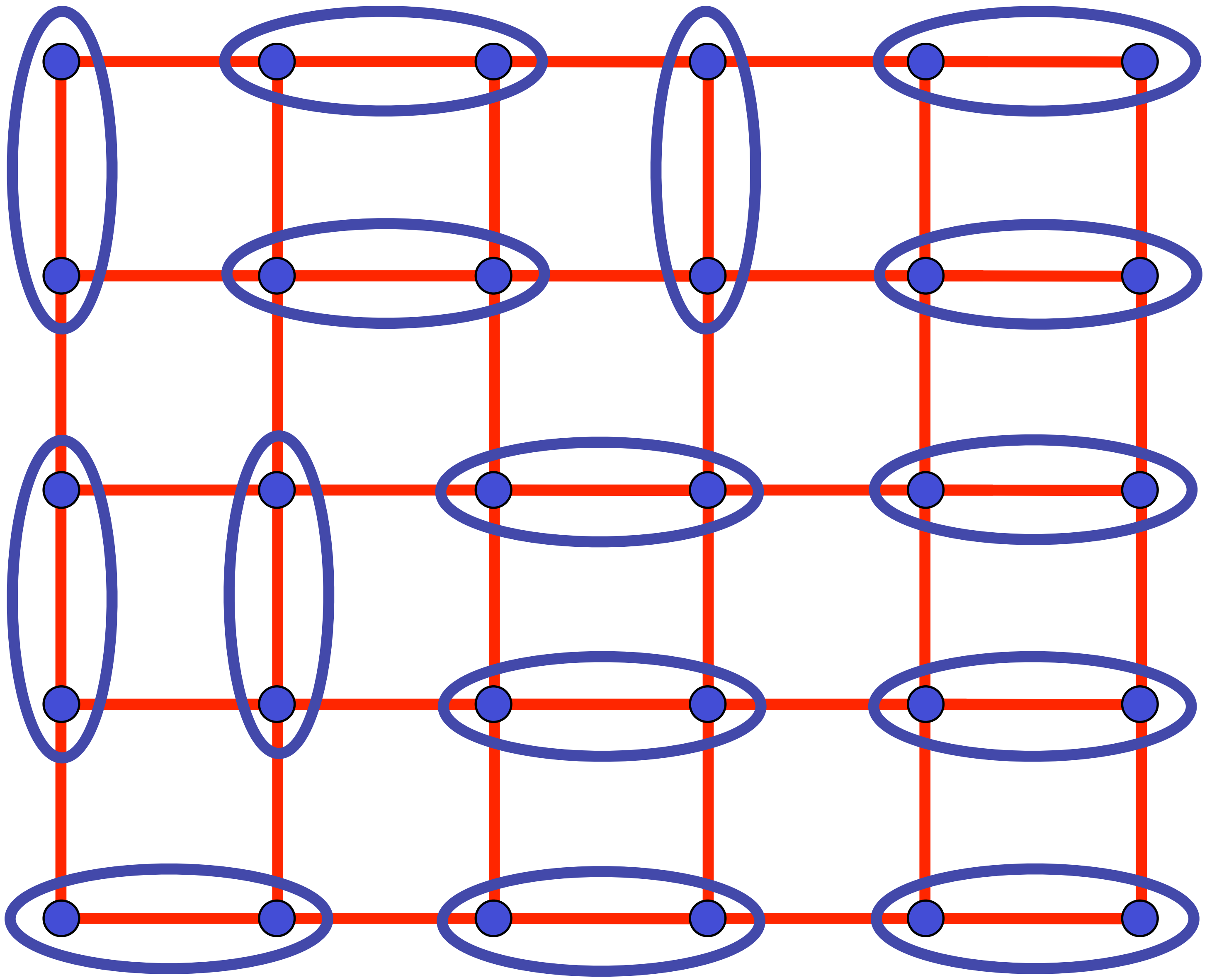
- An electrically neutral complex scalar with spin $S = 1/2$: a 'spinon' z_α
- An emergent U(1) gauge field A_μ

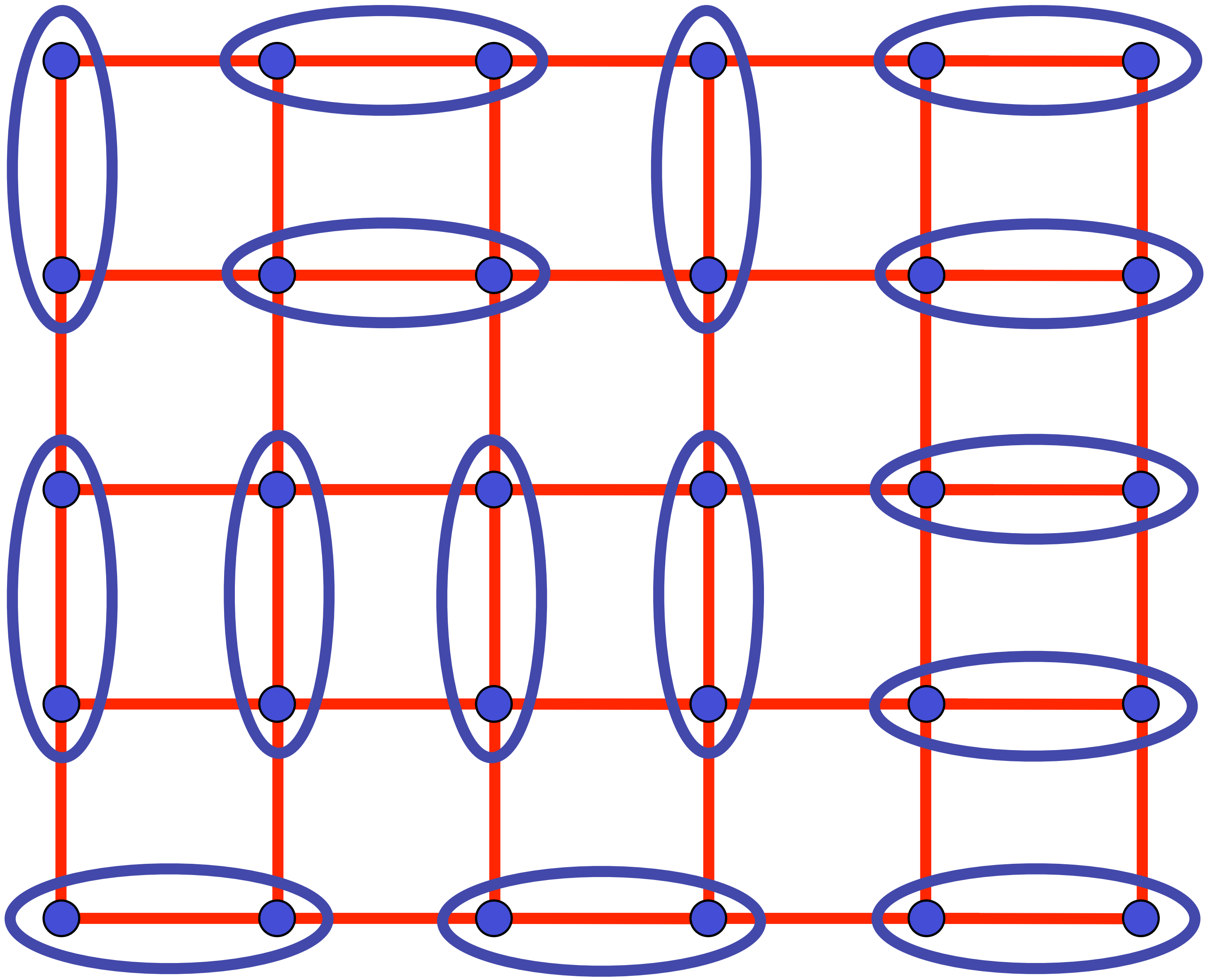


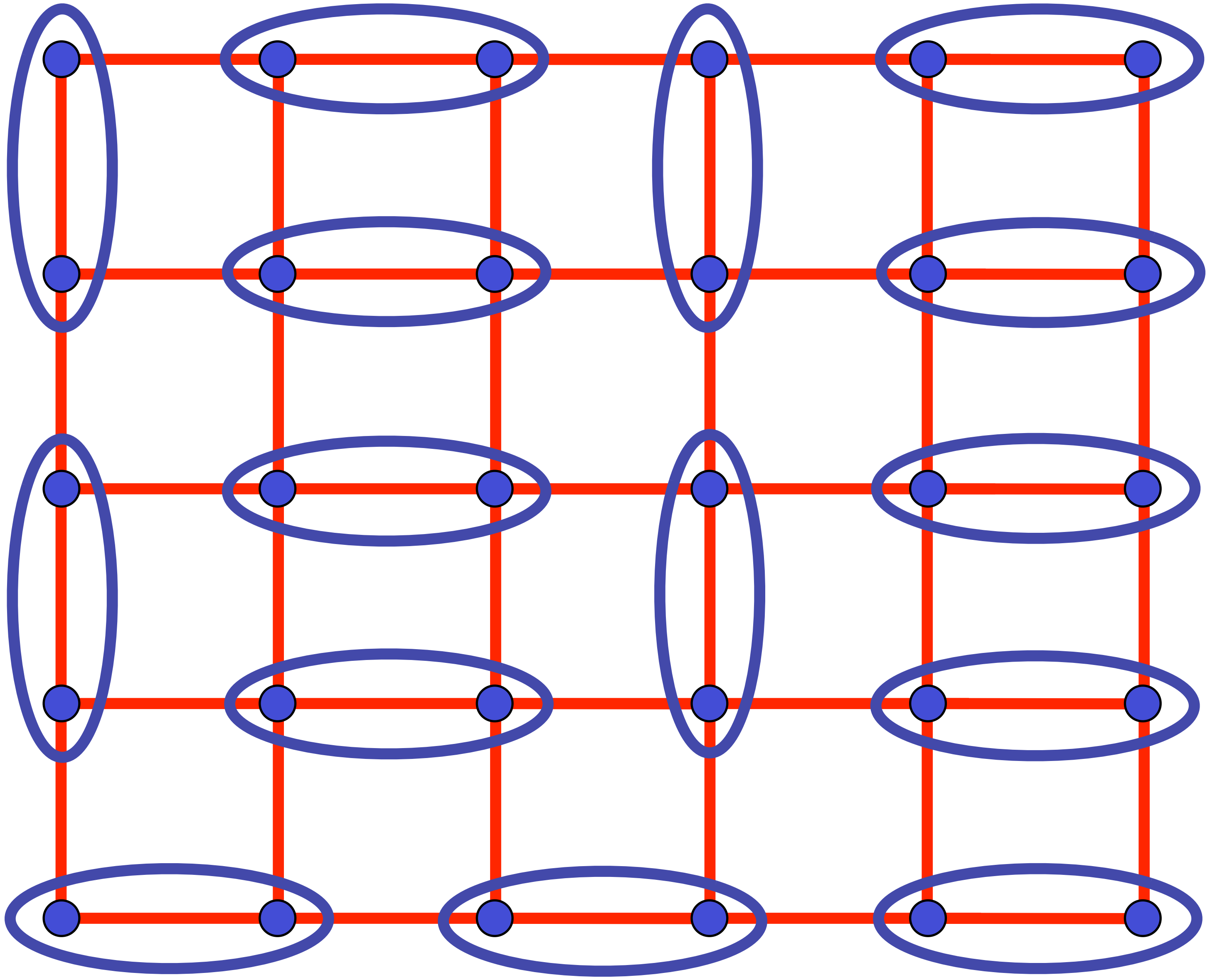


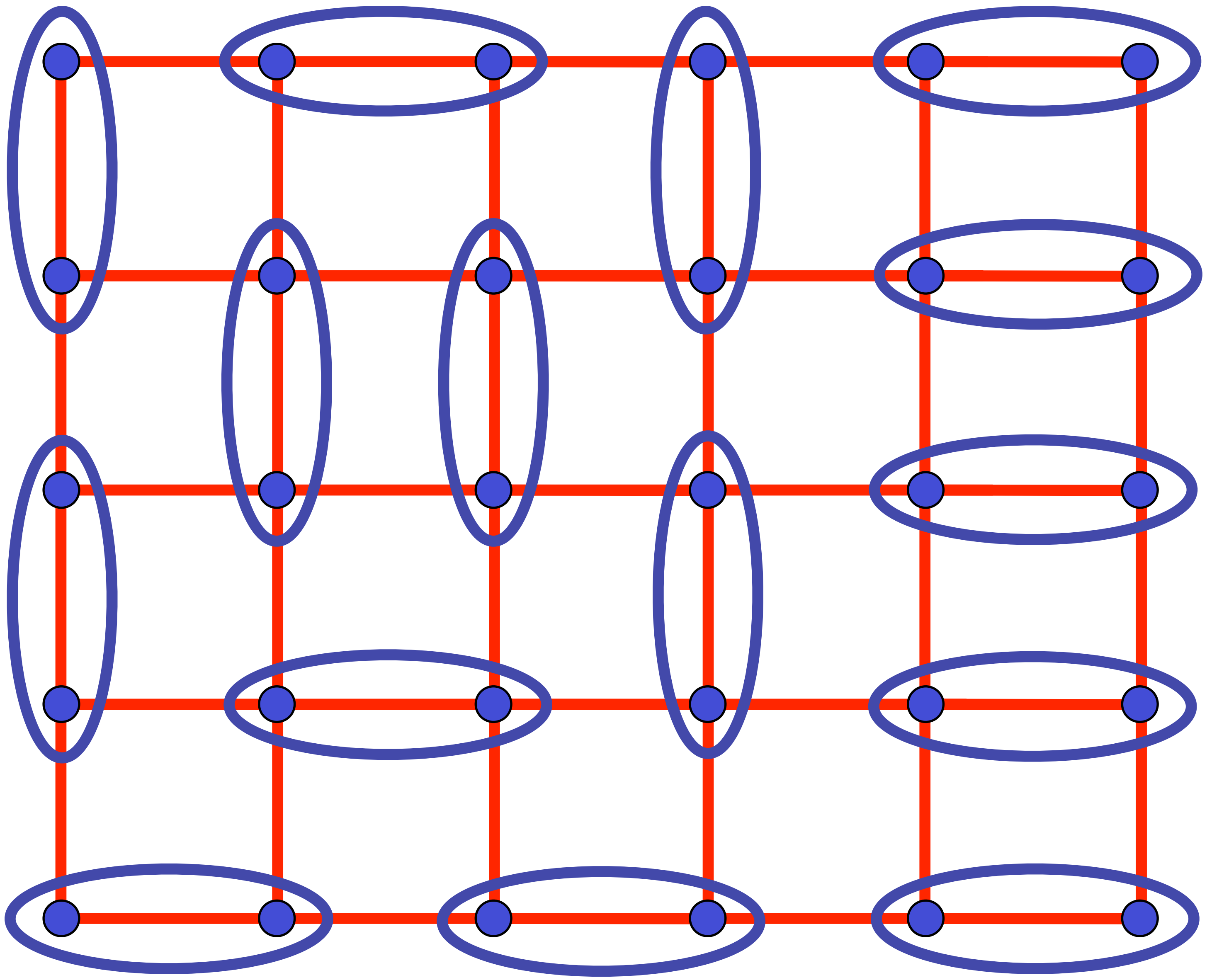




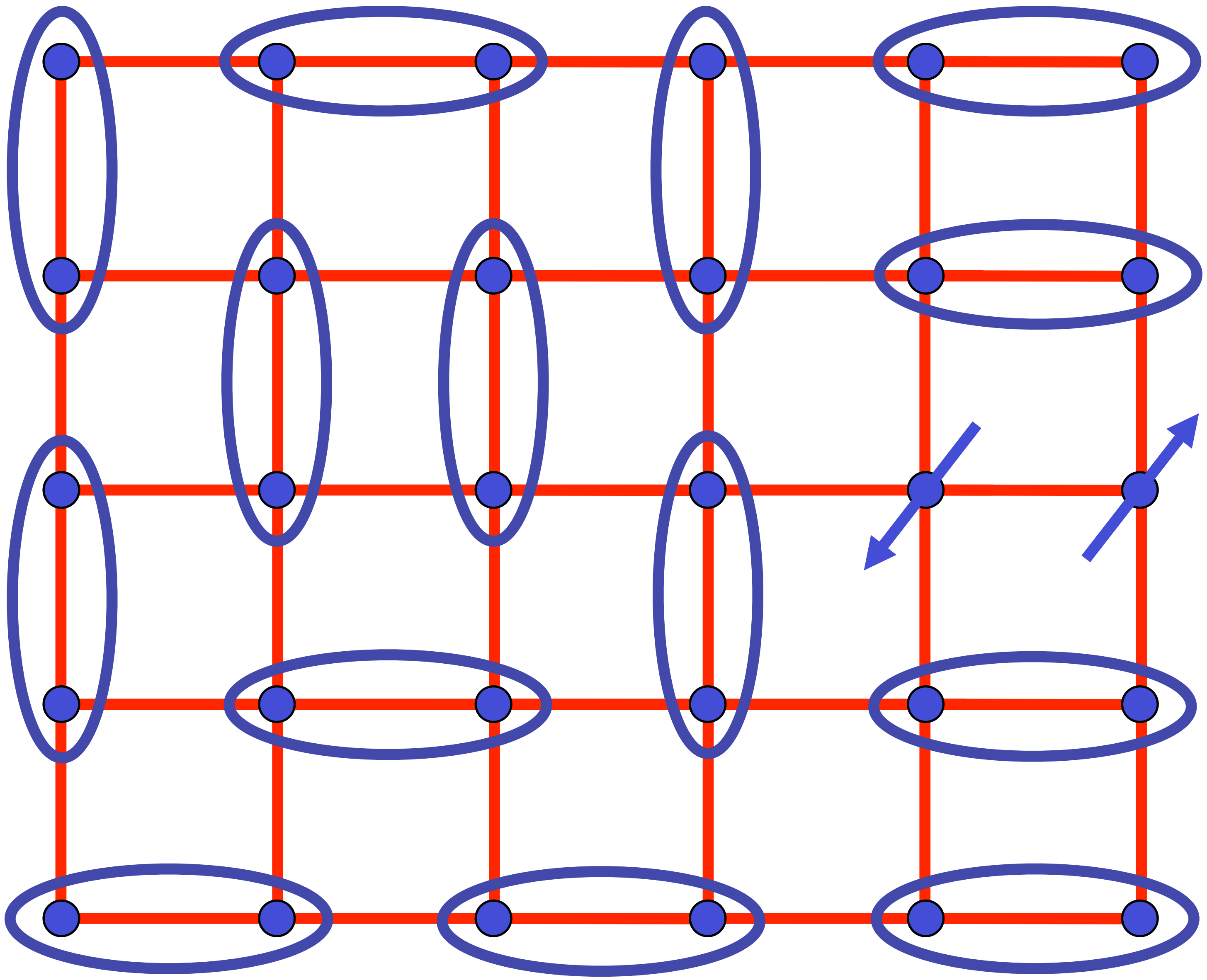


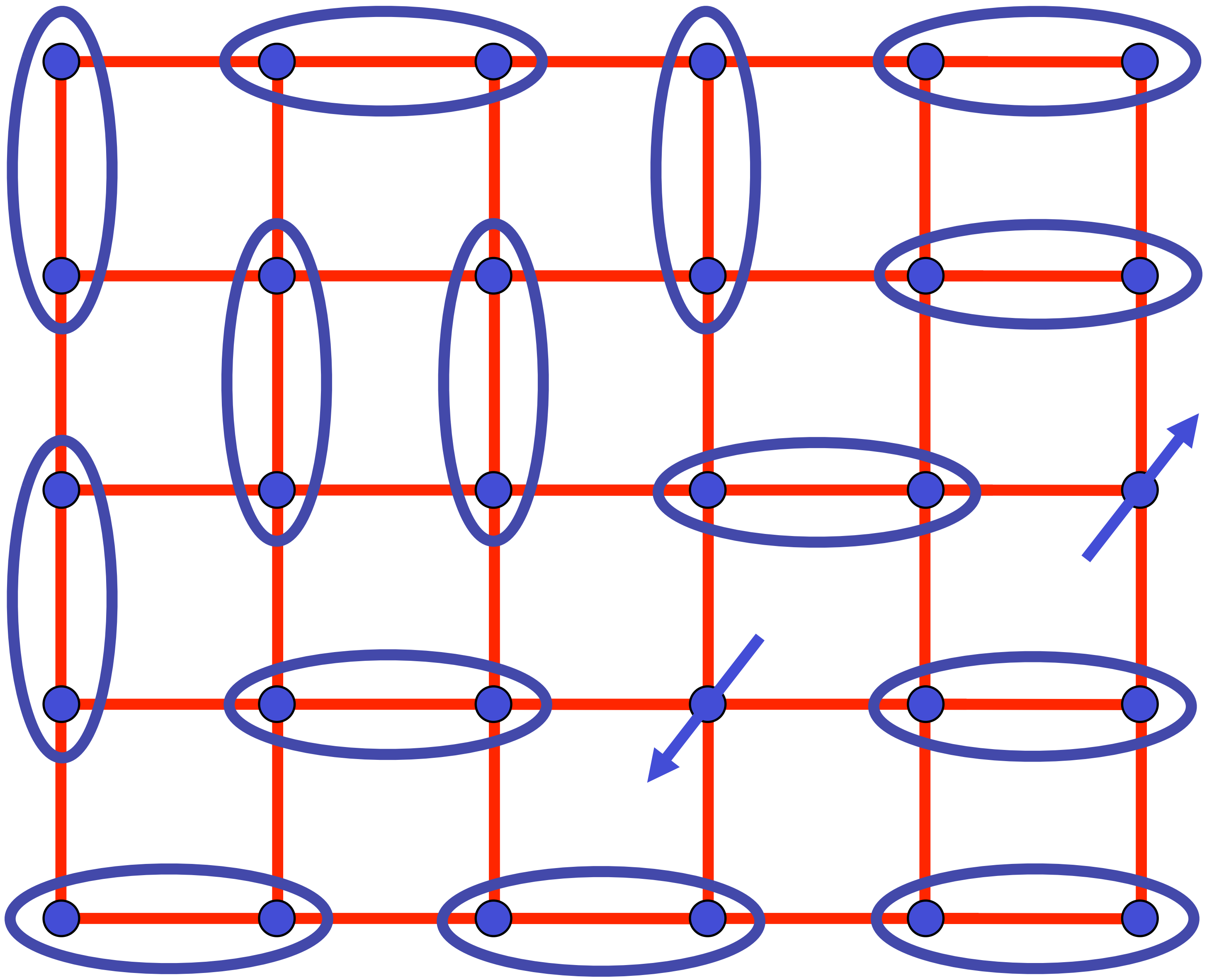


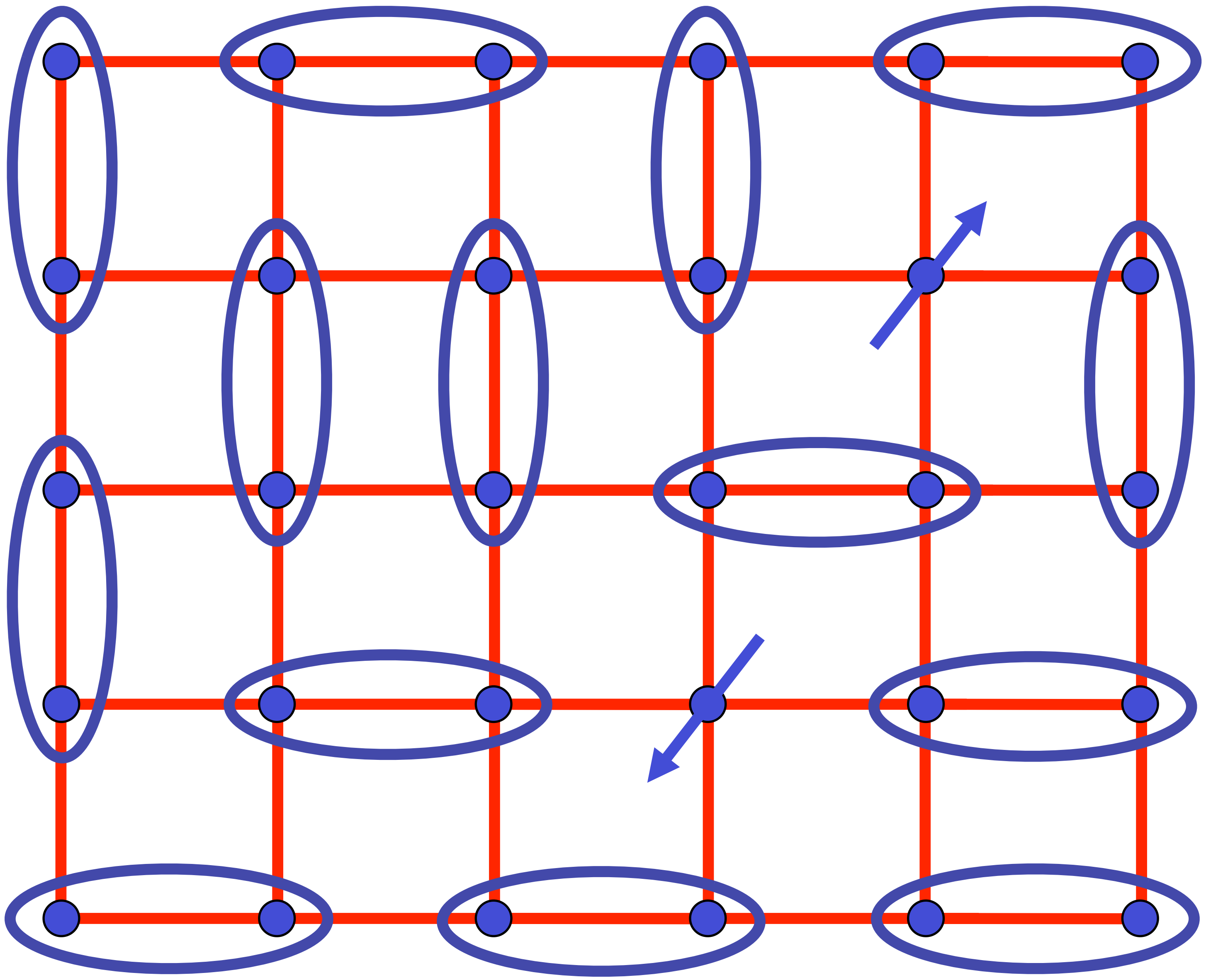


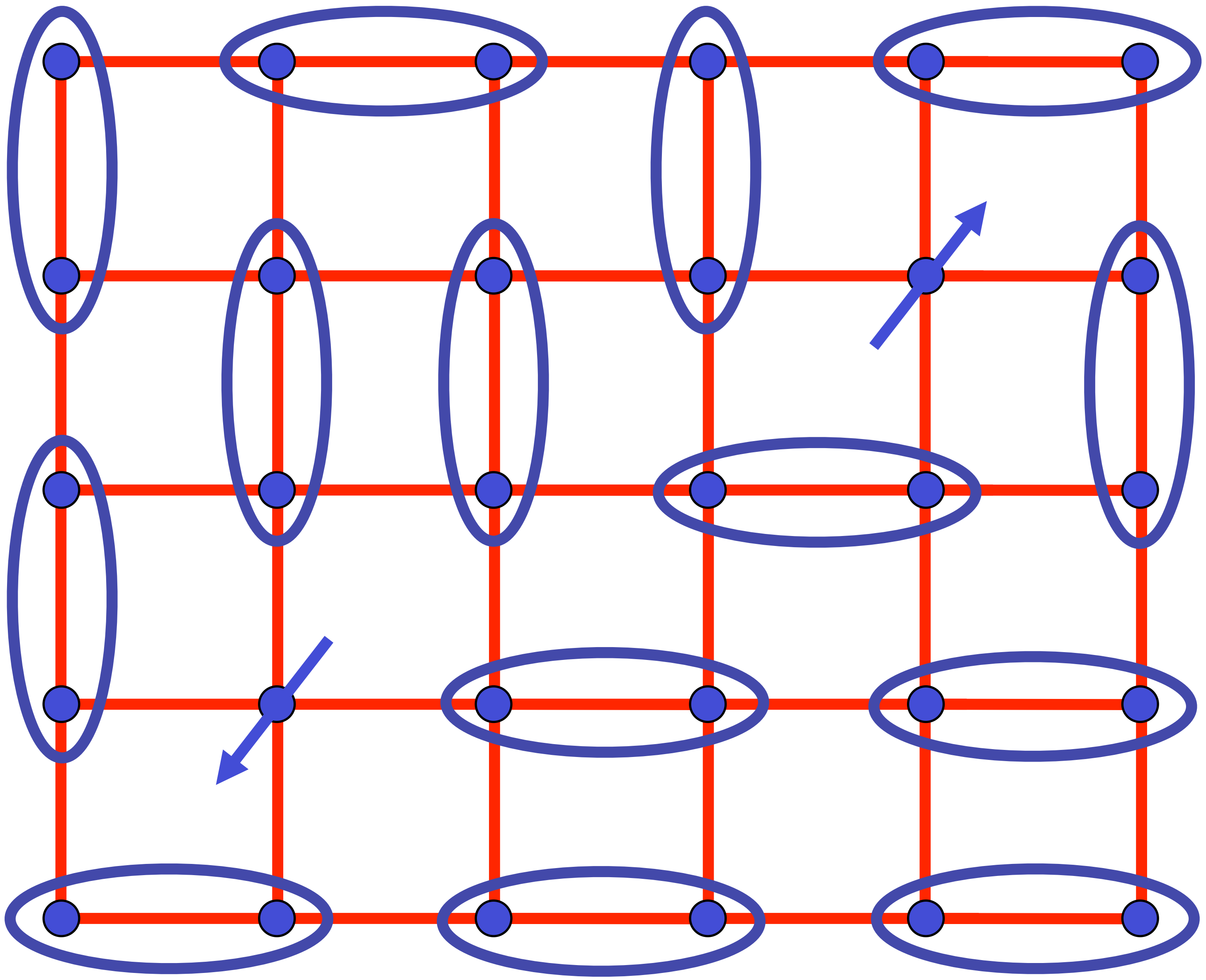


Spinless collective mode: the emergent "photon" A_μ



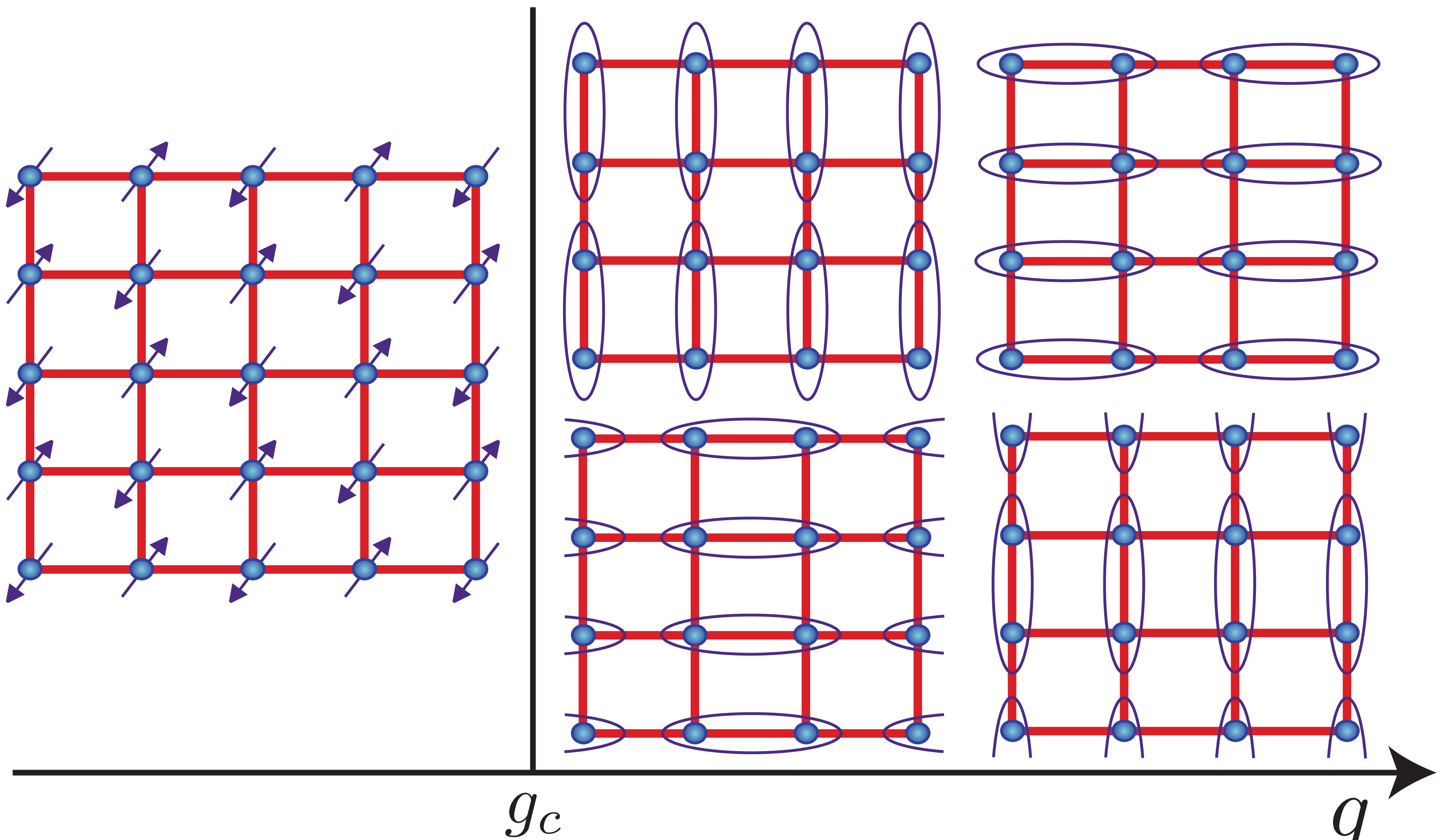






Electrically neutral spinon z_α:
Carries the U(1) charge of the emergent 'photon' A_μ

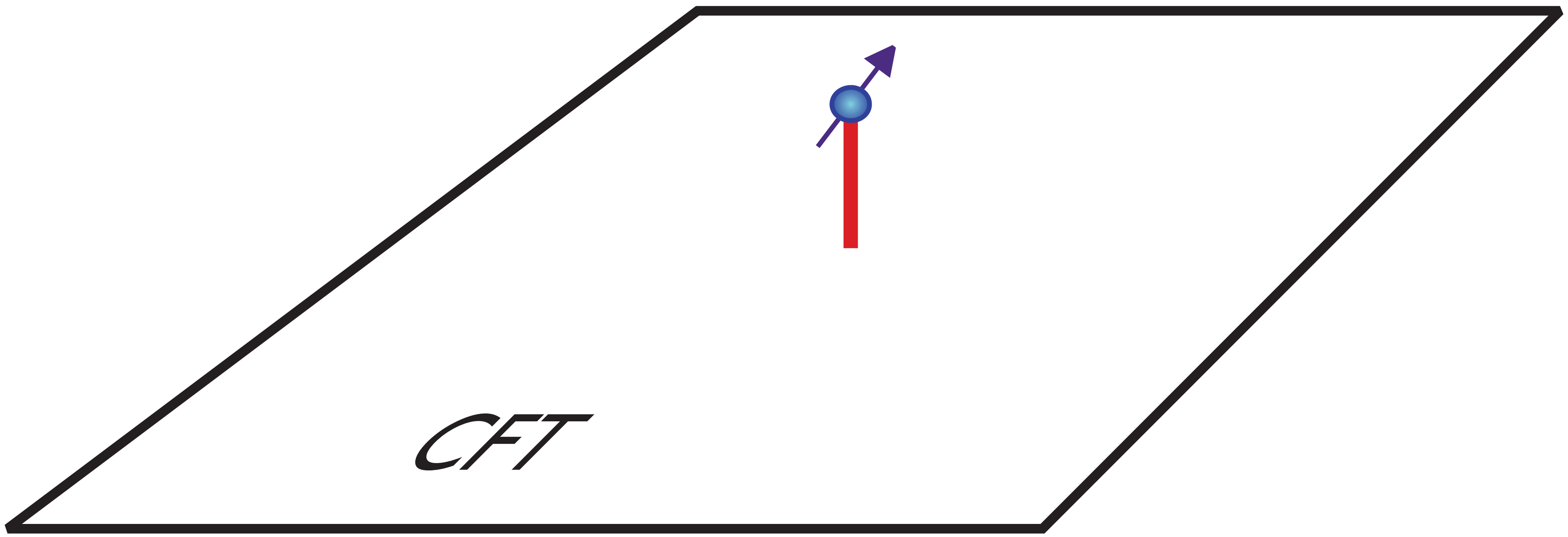
Frustrated antiferromagnet with full square lattice symmetry and one S=1/2 per unit cell



$$\mathcal{Z} = \int \mathcal{D}z^\alpha(r,\tau)\mathcal{D}A_\mu(r,\tau)\exp\left(-\int d^2r d\tau\, \mathcal{L}_z\right)$$

$$\mathcal{L}_z = |(\partial_\mu - iA_\mu)z^\alpha|^2 + s|z^\alpha|^2 + u(|z^\alpha|^2)^2 + \frac{1}{2w^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2$$

Quantum impurity coupled to a CFT

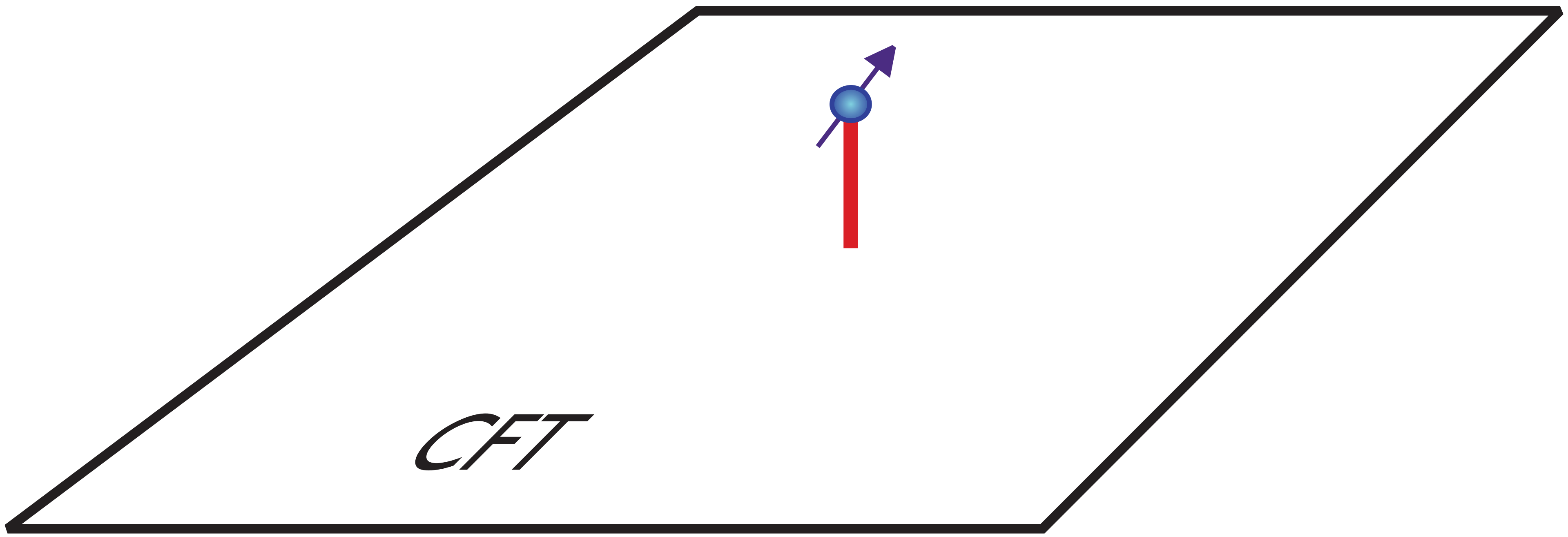


$$\mathcal{Z} = \int \mathcal{D}z^\alpha(r,\tau)\mathcal{D}A_\mu(r,\tau)\mathcal{D}\chi(\tau)\exp\left(-\int d\tau\,\mathcal{L}_{\rm imp} - \int d^2r d\tau\,\mathcal{L}_z\right)$$

$$\mathcal{L}_{\rm imp} = \chi^\dagger\left(\frac{\partial}{\partial\tau} - iA_\tau(0,\tau)\right)\chi$$

χ: spinless localized fermion measuring presence of impurity

Quantum superspin coupled to SYM4



$$\mathcal{S} = \int d^3r d\tau \, \mathcal{L}_{\text{SYM}} + \int d\tau \, \mathcal{L}_{\text{imp}}$$

$$\mathcal{L}_{\text{imp}} = \chi_b^\dagger \frac{\partial \chi^b}{\partial \tau} + i\chi_b^\dagger \left[(A_\tau(0,\tau))_c^b + v^I \left(\phi_I(0,\tau) \right)_c^b \right] \chi^c$$

S. Kachru, A. Karch, and S. Yaida, Phys. Rev. D **81**, 026007 (2010)

Common features

- The correlations of the impurity fermion, or impurity spin, decay with a power-law in time, wtih non-trivial ‘impurity’ exponents.

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- The impurity response to a uniform external field is characterized by an impurity susceptiblity which has a Curie form $\chi_{\rm imp} = \mathcal{C}/T$, where $\mathcal{C}$ is a non-trivial universal number This response is that of an 'irrational' free spin, because $\mathcal{C} \neq S(S+1)/3$, with $2S$ an integer.

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- There is a finite ground state entropy, S_{imp}, at $T = 0$. This entropy is also 'irrational' because $S_{\text{imp}} \neq k_B \ln(\text{an integer})$.

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The SYM case is related in the large *N* limit to a AdS_2 geometry

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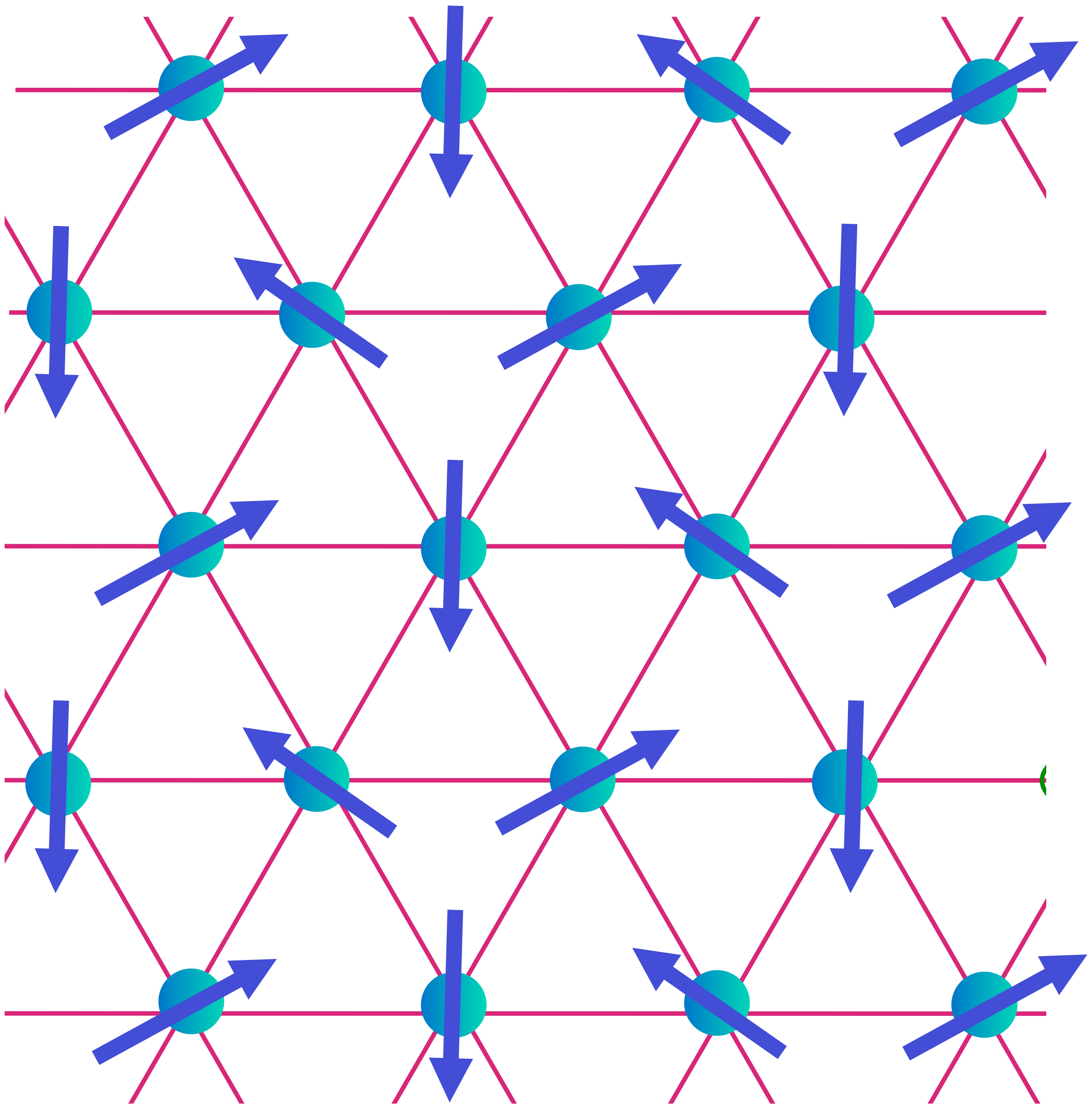
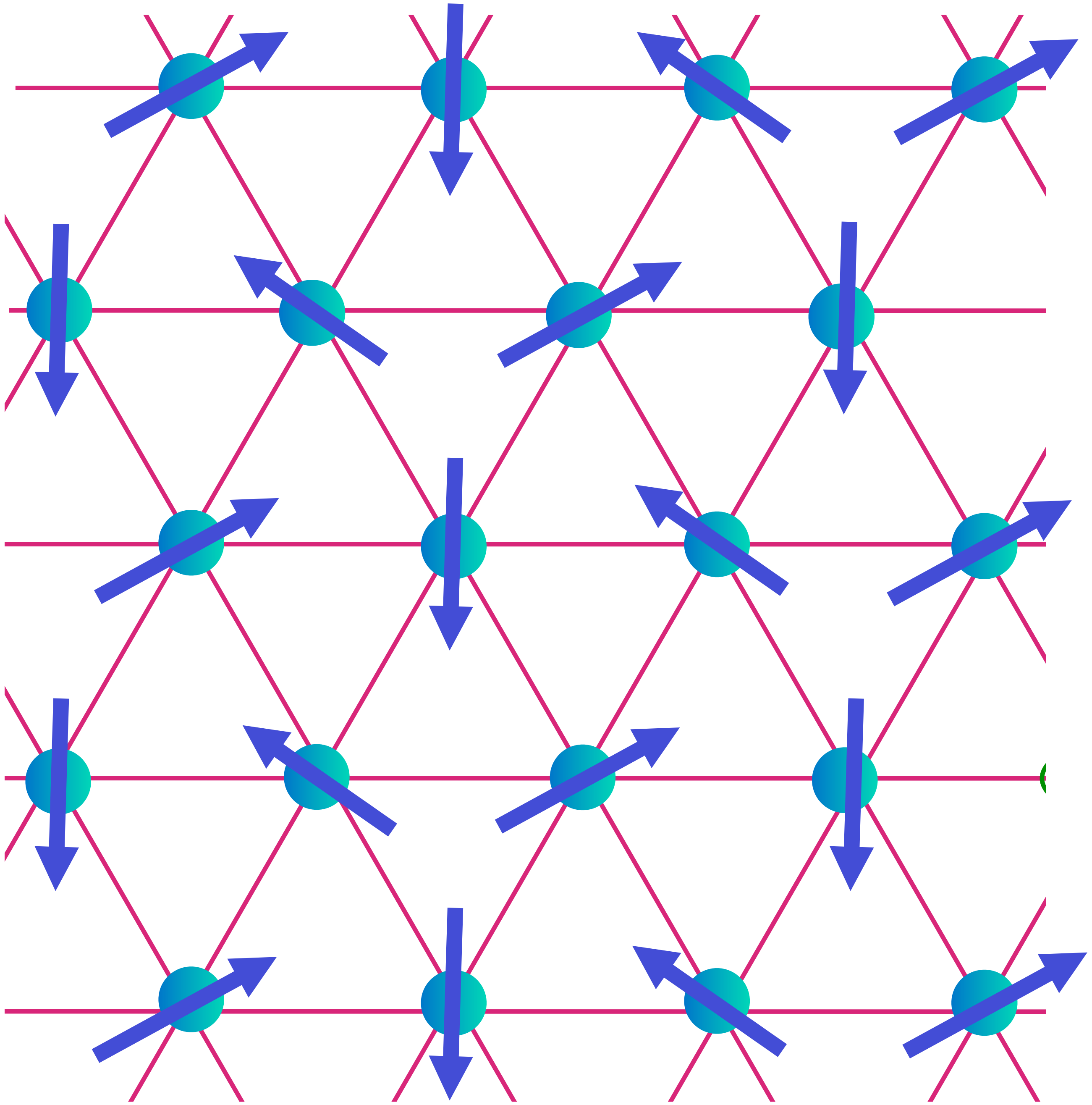
Kondo lattice model



$$\sum_{i<j} J_H(i,j)\vec{S}_i \cdot \vec{S}_j$$

Kondo lattice model



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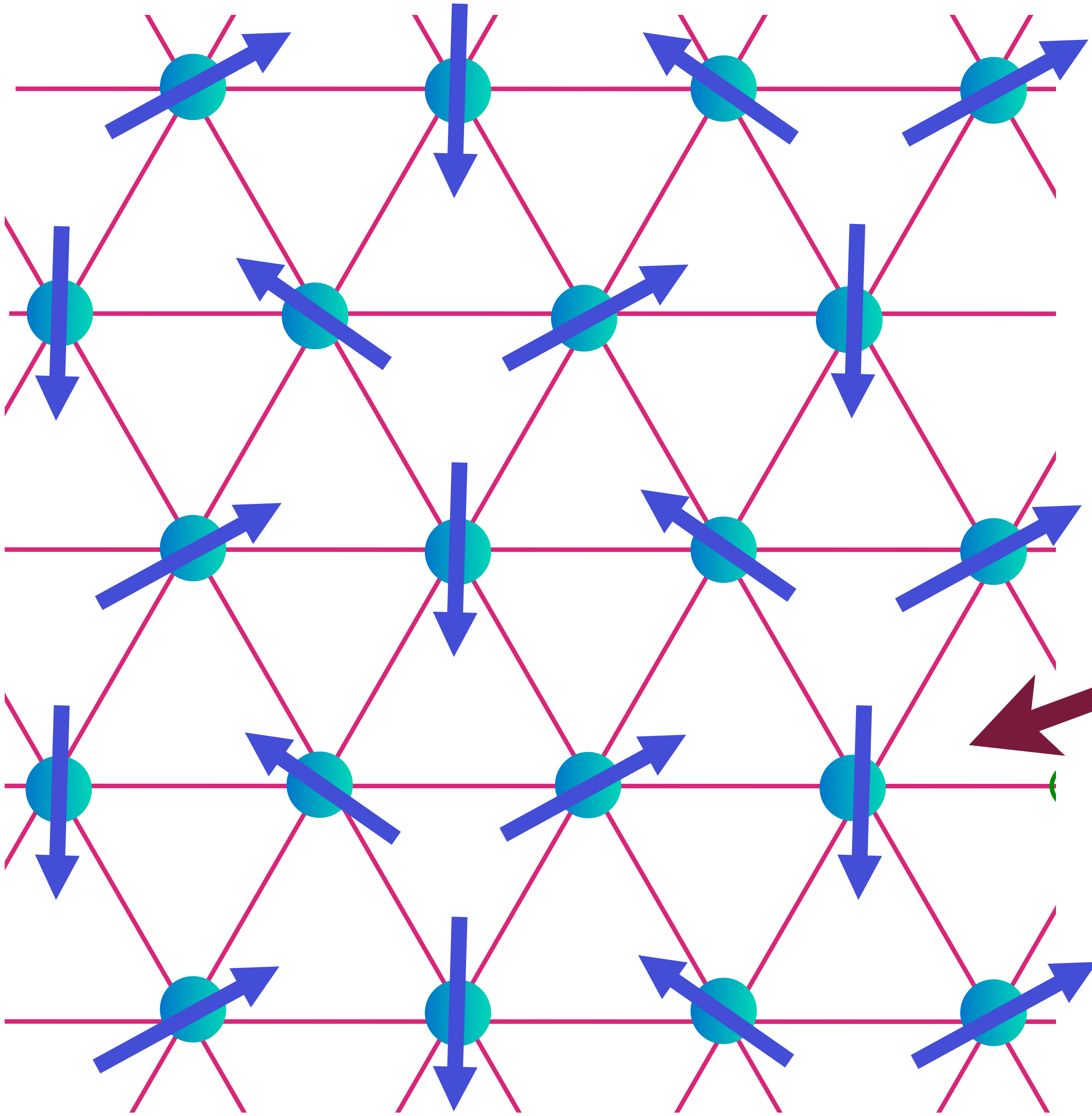
Conduction electrons

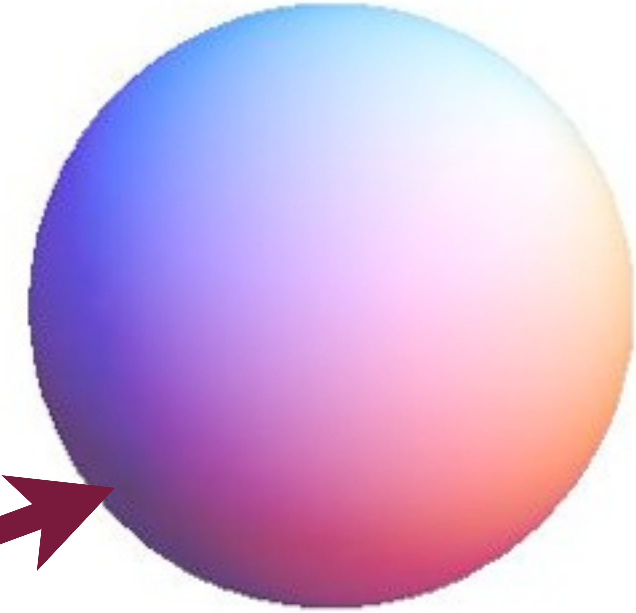
$$\sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k}\alpha} c_{\mathbf{k}\alpha}$$

Kondo lattice model



Kondo exchange

$J_K \sum_i \vec{S}_i \cdot c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}$



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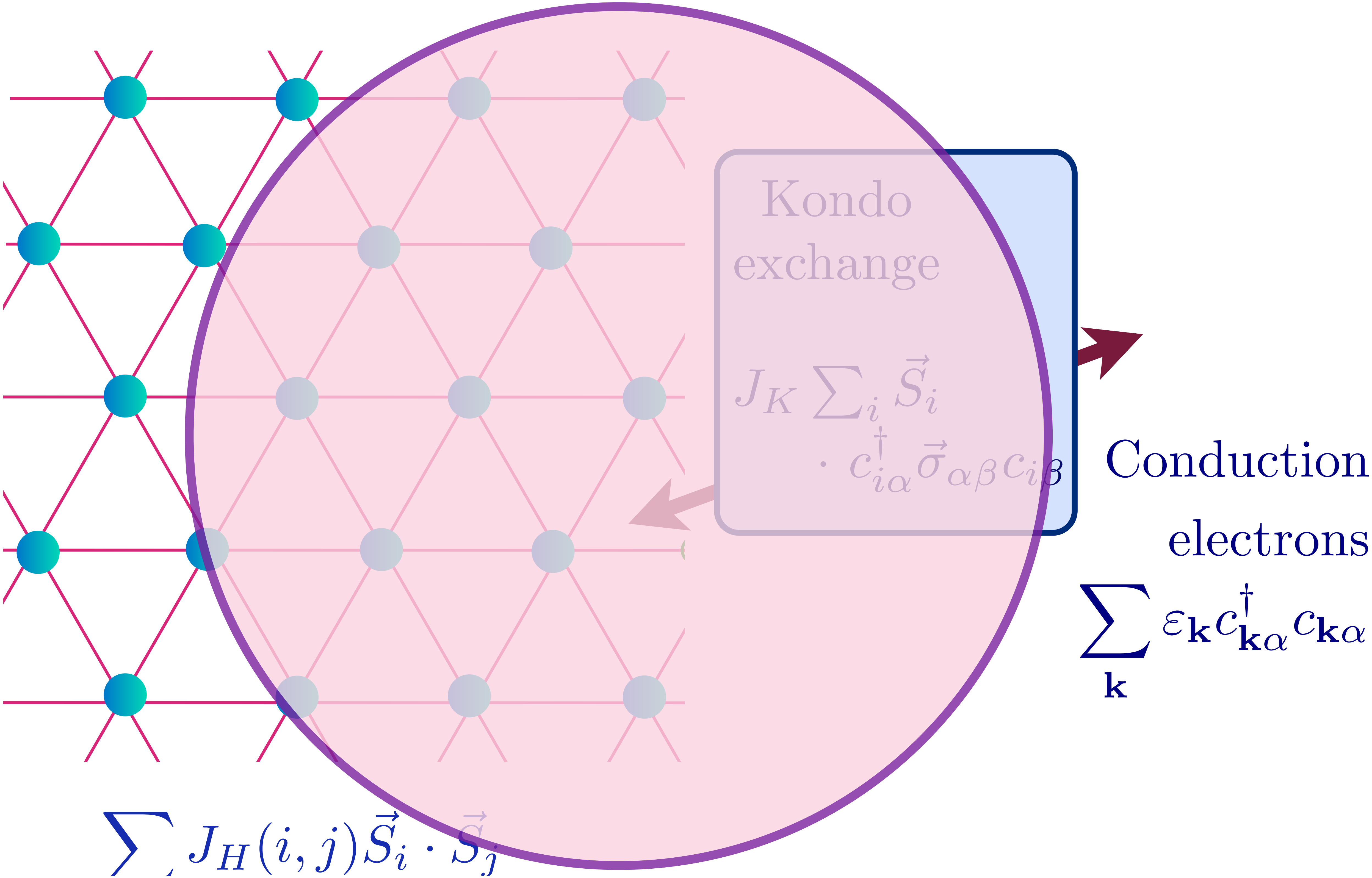
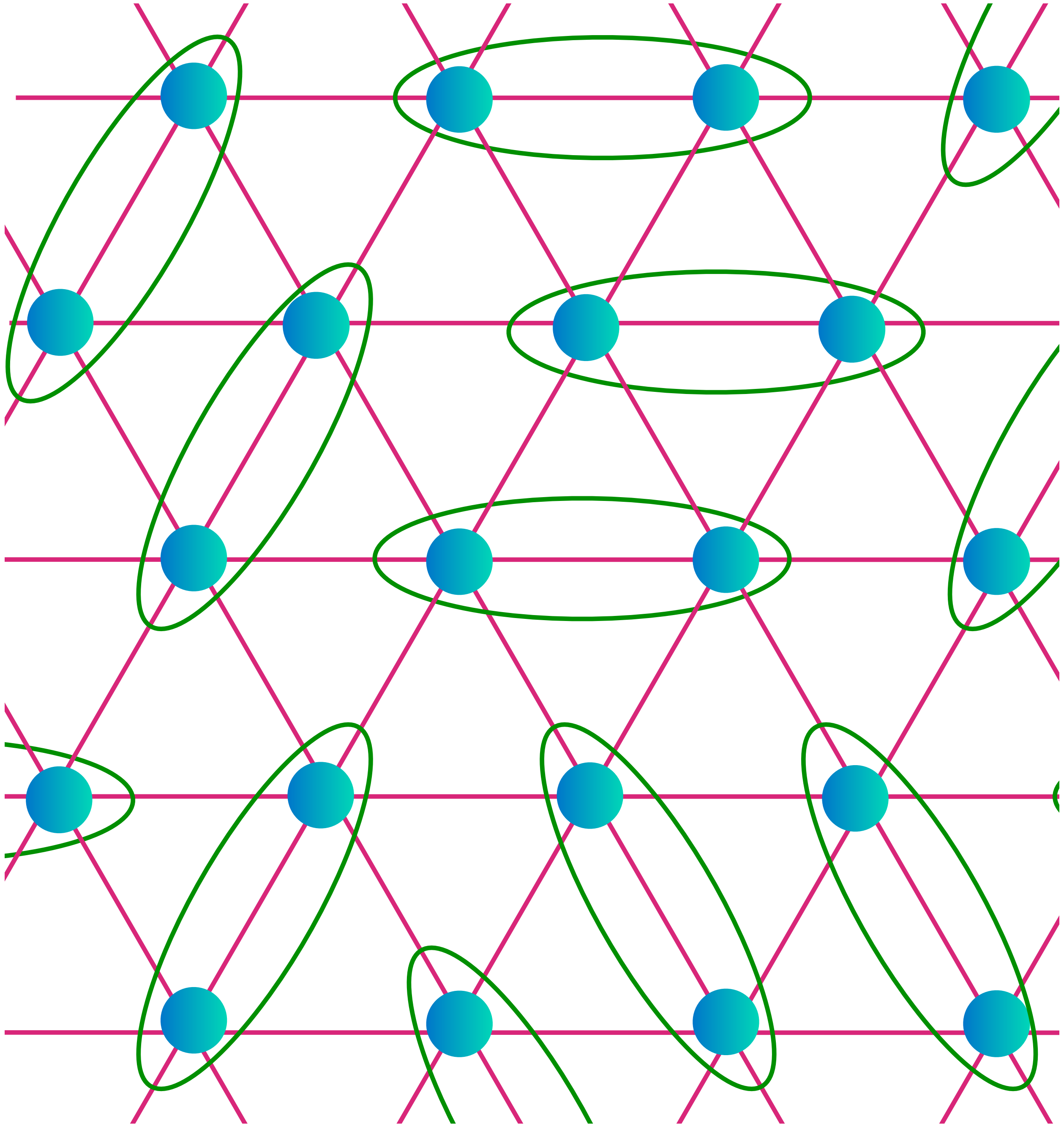


Large Fermi surface Fermi liquid (FL)

Kondo lattice model



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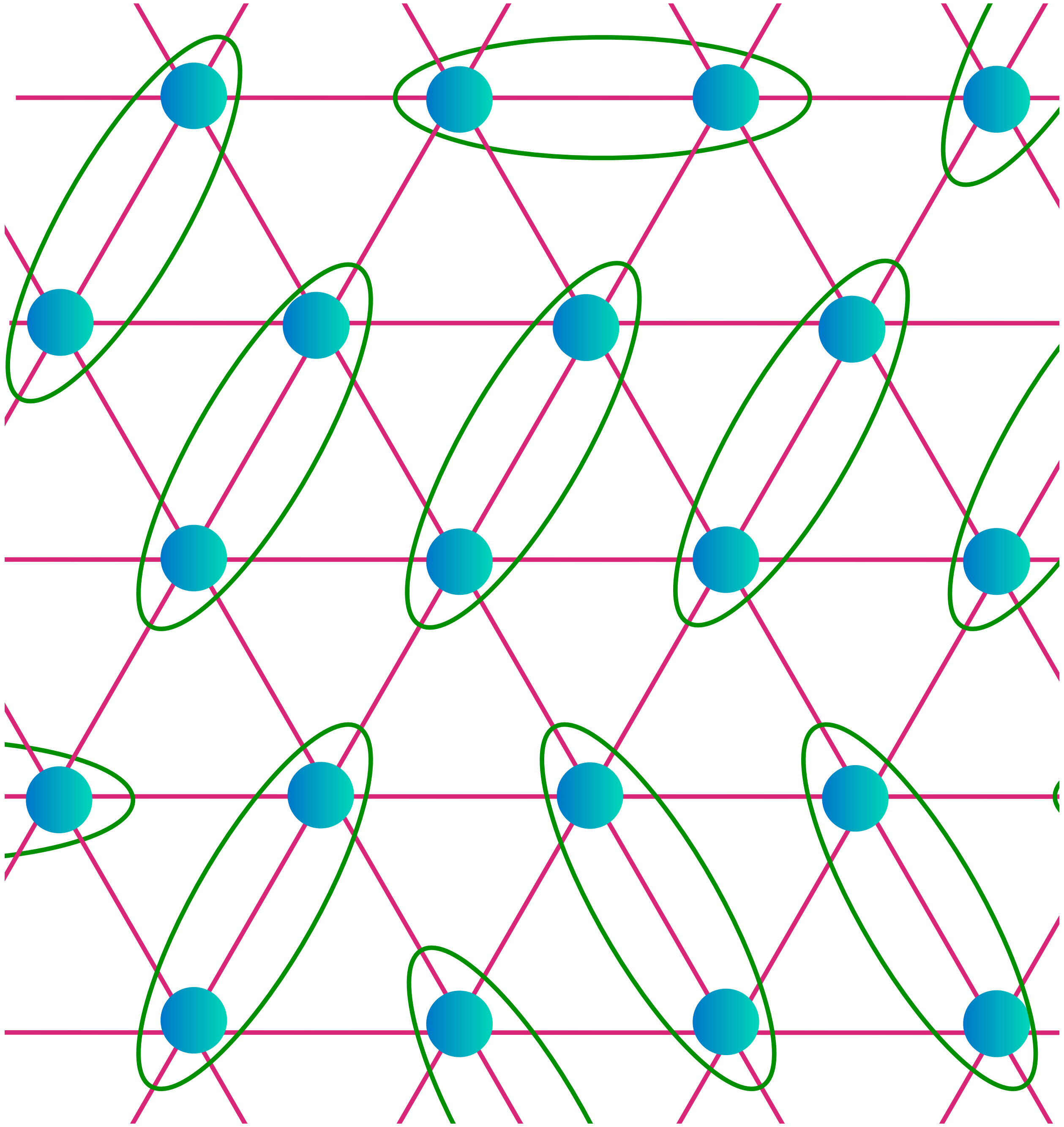
Kondo lattice model



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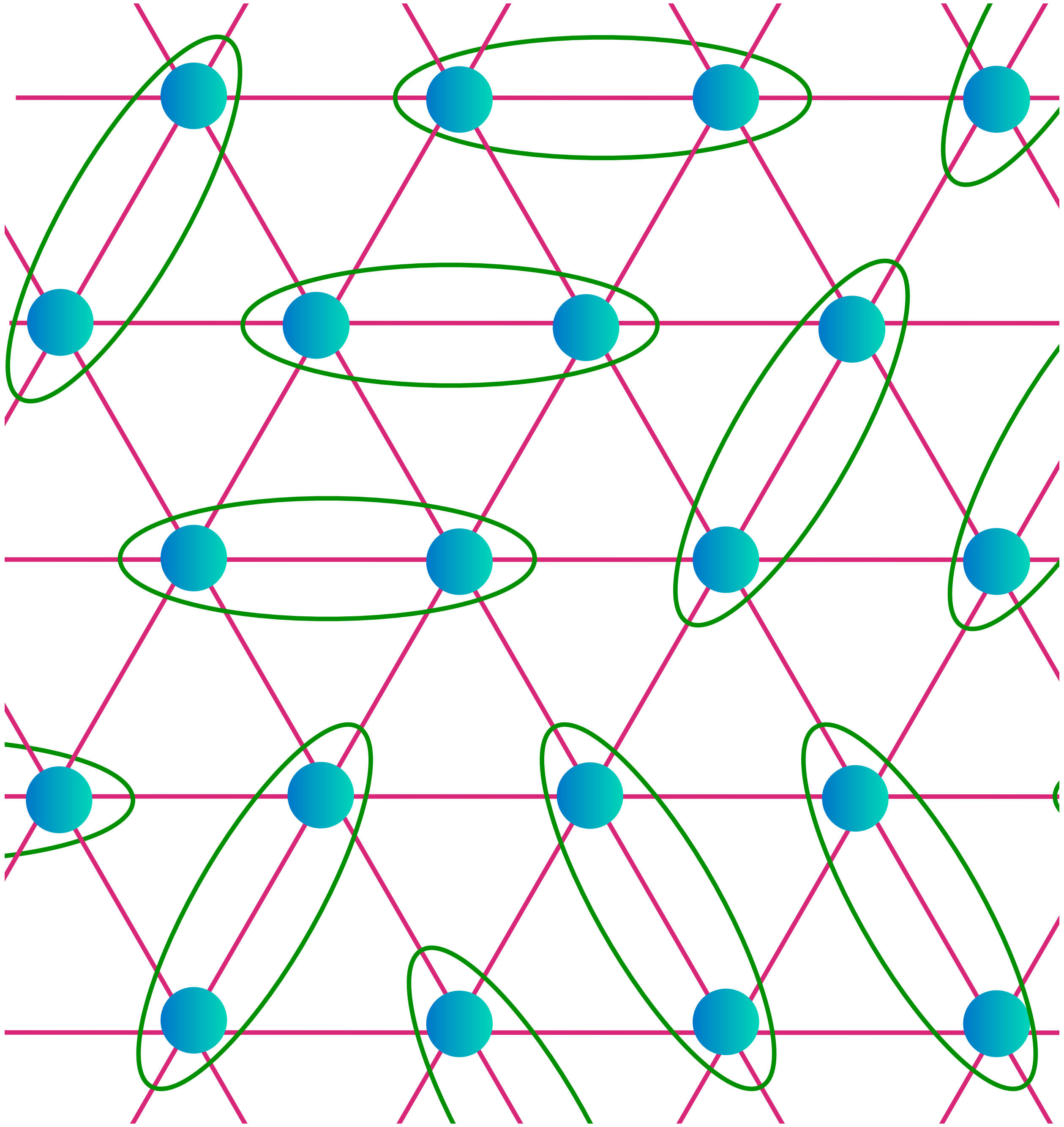
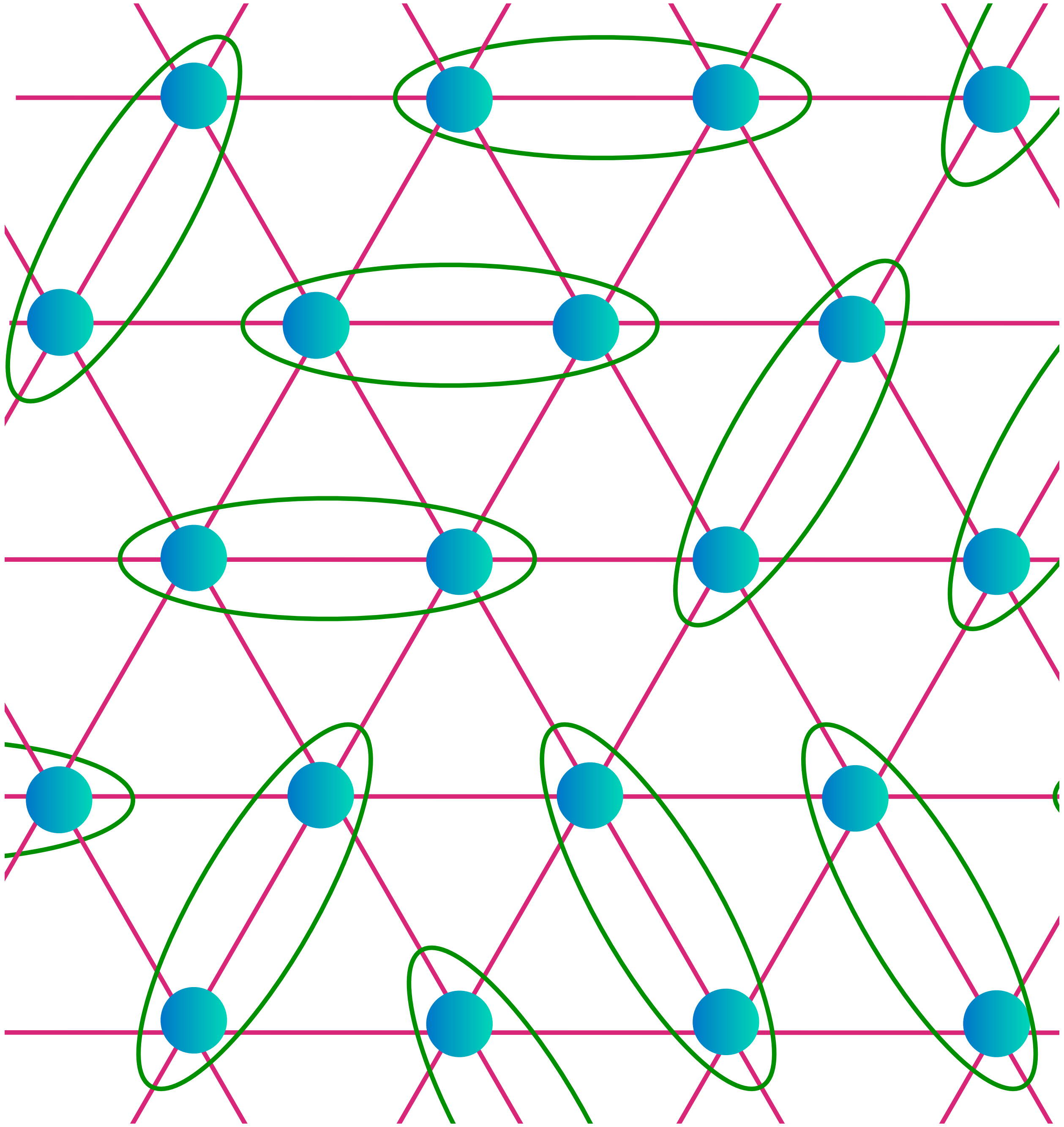


$$\sum_{i<j} J_H(i,j) \vec{S}_i \cdot \vec{S}_j$$

Spin liquid of localized electrons

Kondo lattice model



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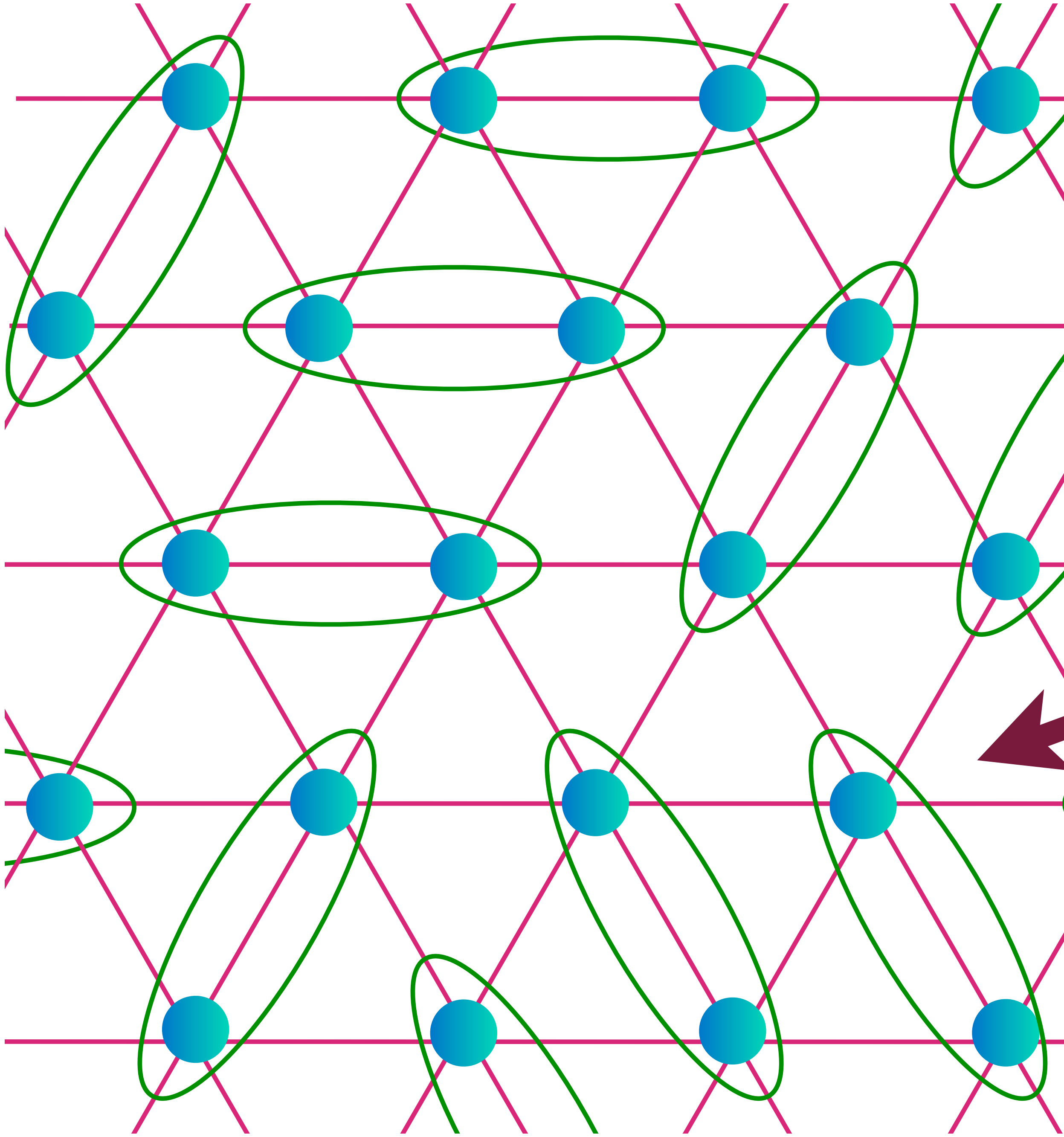
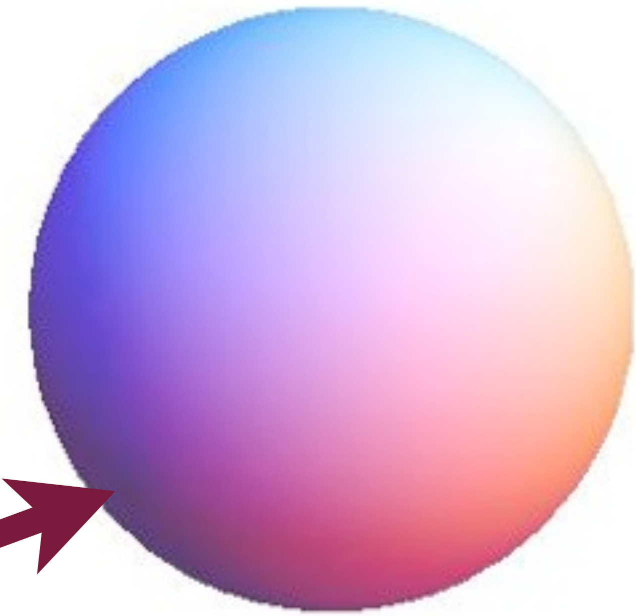


$$\sum_{i<j} J_H(i,j)\vec{S}_i \cdot \vec{S}_j$$

Kondo exchange perturbative



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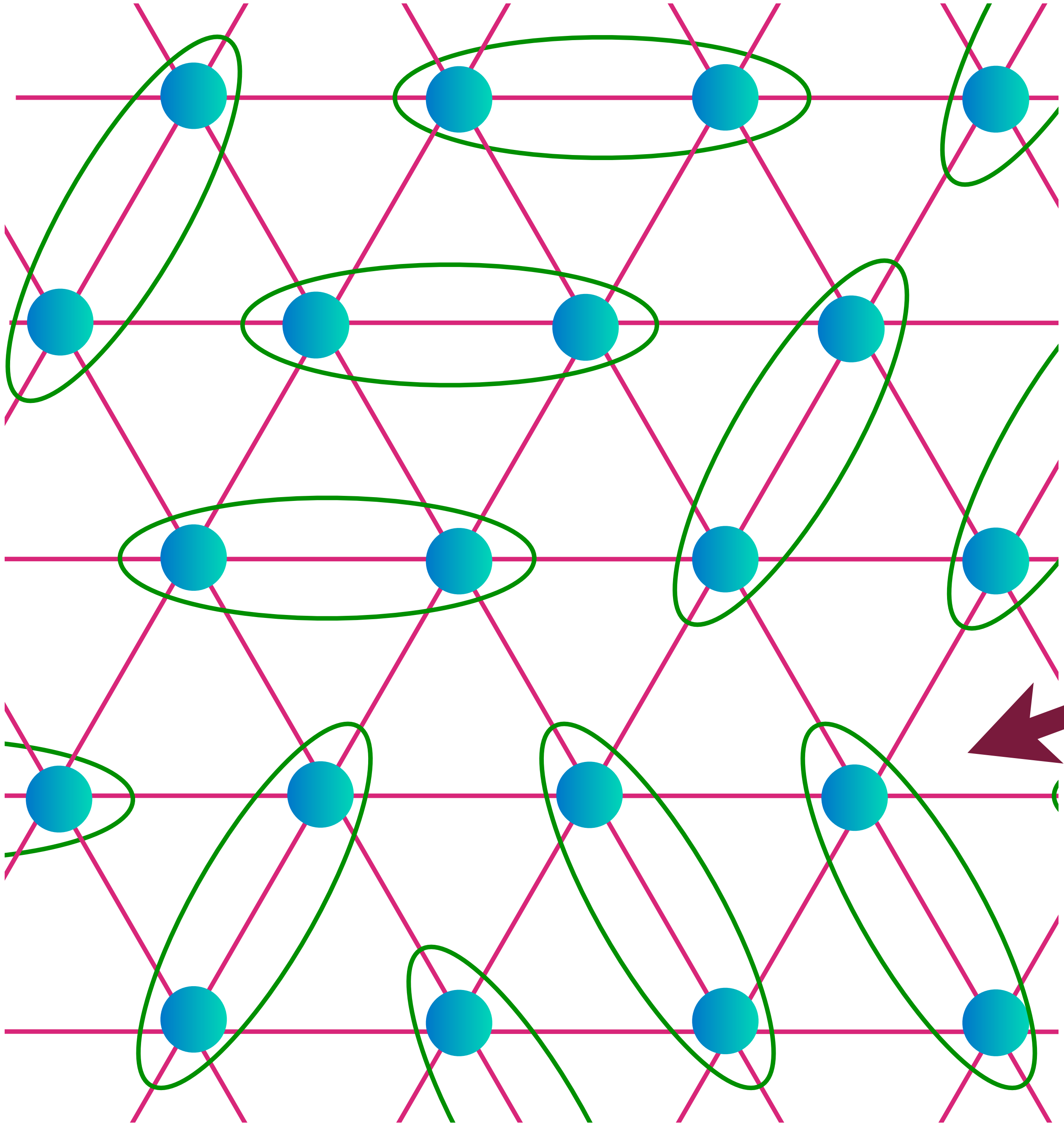

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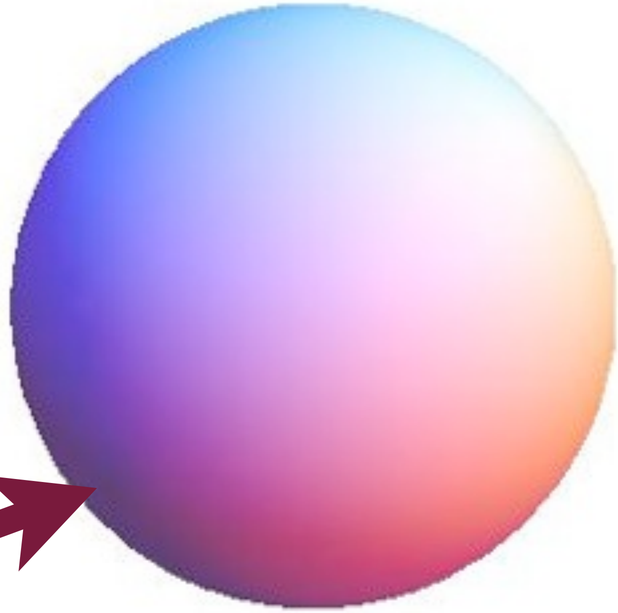

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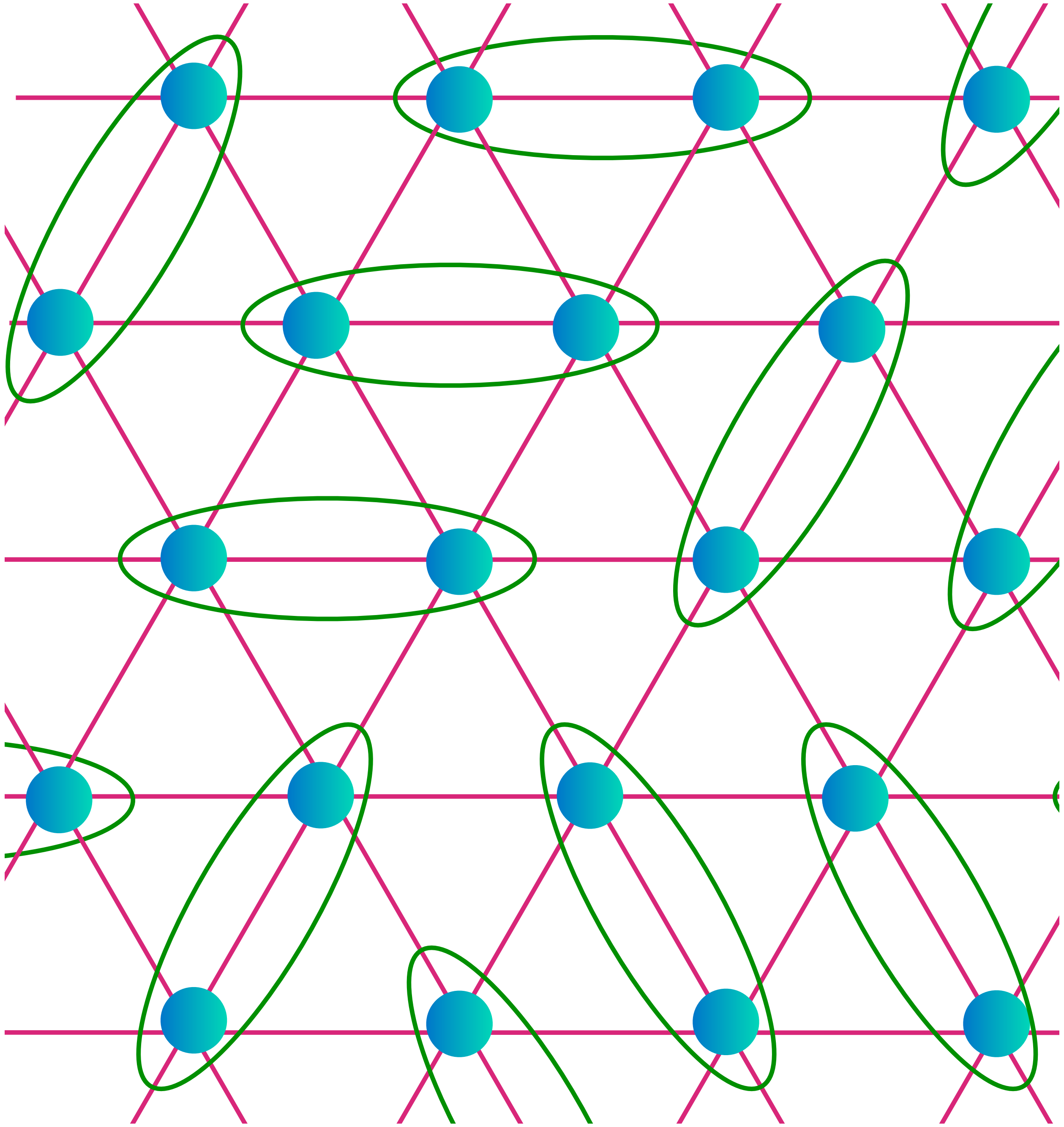
Fractionalized Fermi liquid (FL*)

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003).
T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004).

Kondo lattice model



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Spin liquid of localized electrons

Theory of spin liquid

$$\mathcal{L}_f = f_\alpha^\dagger \left(\frac{\partial}{\partial \tau} - iA_\tau - \varepsilon_f(\mathbf{k} - \mathbf{A}) + \mu_f \right) f_\alpha$$

Electrically neutral spinons f_α coupled to emergent gauge field A_μ

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$$\mathcal{L}_b = \left| \left(\frac{\partial}{\partial \tau} - iA_\tau \right) b \right|^2 + \frac{1}{2m_b} \left| \left(\nabla - i\mathbf{A} - ie\mathbf{A}_{\text{ext}} \right) b \right|^2 + s|b|^2 + u|b|^4$$

Electrically charged bosons b in a Mott-insulating state

Theory of spin liquid

Transport in a spin liquid

$$\begin{aligned} \mathbf{J}_f &= \sigma_f \mathbf{E} \\ \mathbf{J}_b &= \sigma_b \left(\mathbf{E} + \mathbf{E}_{\text{ext}}\right) \end{aligned}$$

Equation of motion of emergent gauge field:

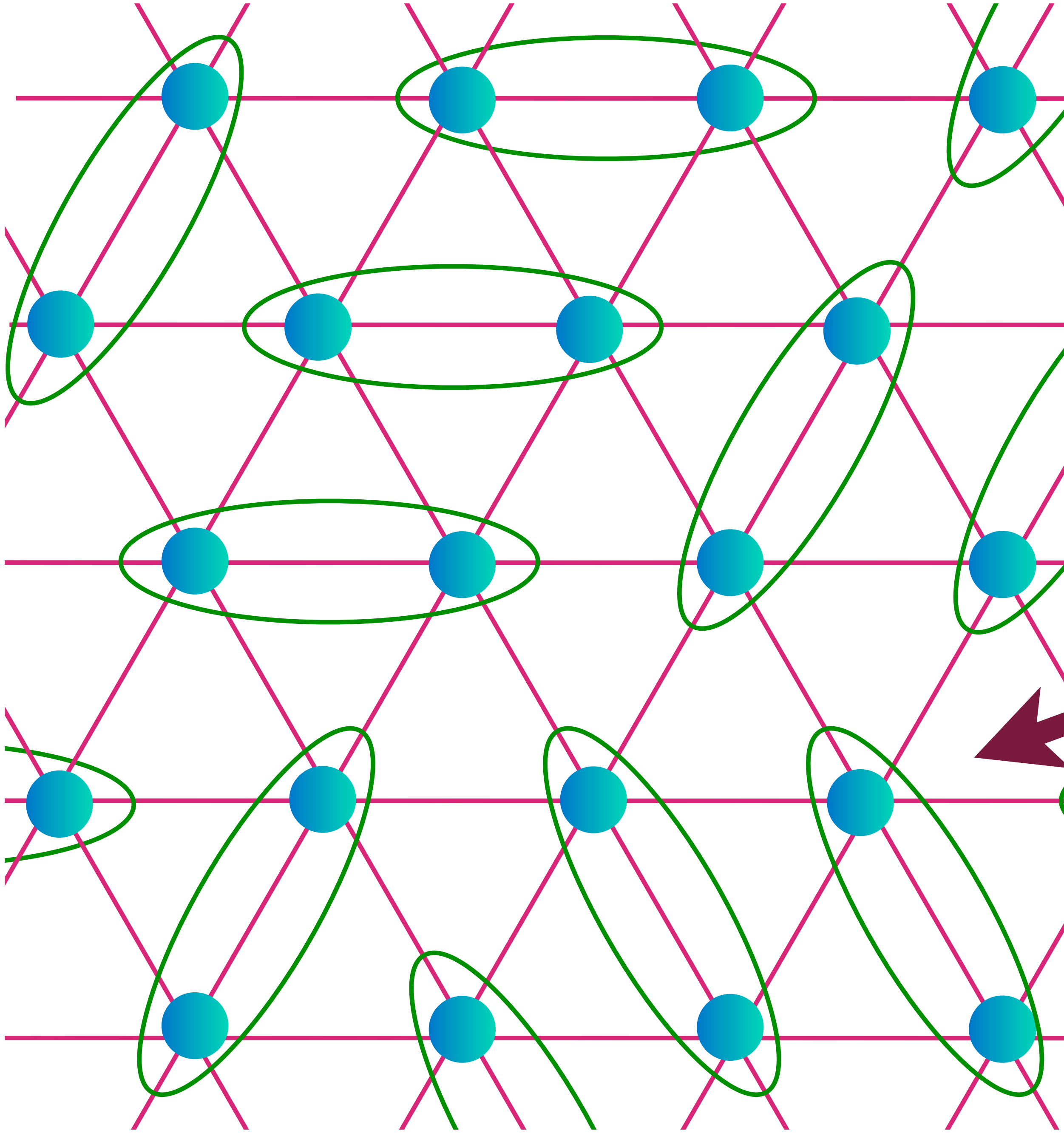
$$\frac{\delta S}{\delta \mathbf{A}} = 0 \quad \Rightarrow \quad \mathbf{J}_f + \mathbf{J}_b = 0 \quad \Rightarrow \quad \mathbf{E} = -\frac{\sigma_b}{\sigma_f + \sigma_b} \mathbf{E}_{\text{ext}}$$

Net electrical current:

$$\mathbf{J}_b = \frac{\sigma_f \sigma_b}{\sigma_f + \sigma_b} \mathbf{E}_{\text{ext}}$$

Because $\sigma_b = 0$ in gapped boson state, we obtain zero conductivity in a finite density state. Note the key role of U(1)×U(1) structure in obtaining this result.
(*cf. Deconstructing holographic liquids*, Dominik Nickel and Dam T. Son, arXiv:1009.3094)

Kondo lattice model



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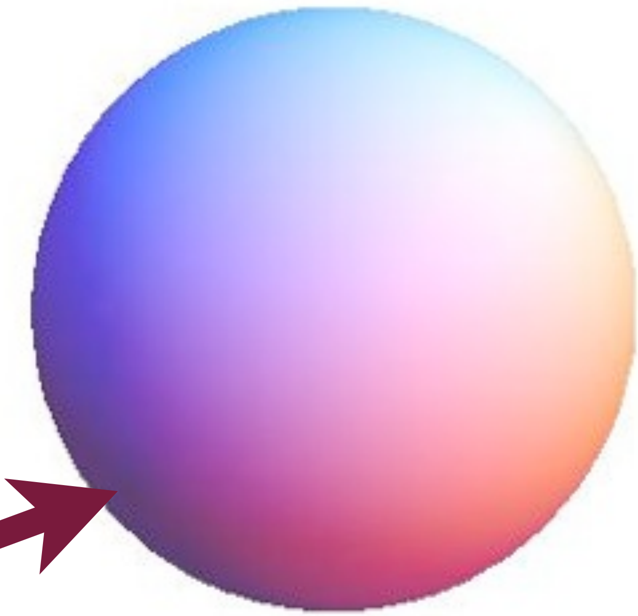
Kondo exchange perturbative



Conduction electrons

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Theory of spin liquid

$$\mathcal{L}_f = f_\alpha^\dagger \left(\frac{\partial}{\partial \tau} - i A_\tau - \varepsilon_f(\mathbf{k} - \mathbf{A}) + \mu_f \right) f_\alpha$$

Electrically neutral spinons f_α coupled to emergent gauge field A_μ

$$\mathcal{L}_b = \left| \left(\frac{\partial}{\partial \tau} - i A_\tau \right) b \right|^2 + \frac{1}{2m_b} \left| \left(\nabla - i\mathbf{A} - ie\mathbf{A}_{\text{ext}} \right) b \right|^2 + s|b|^2 + u|b|^4$$

Electrically charged bosons b in a Mott-insulating state

Theory of FL*

$$\mathcal{L}_f = f_\alpha^\dagger \left(\frac{\partial}{\partial \tau} - i A_\tau - \varepsilon_f(\mathbf{k} - \mathbf{A}) + \mu_f \right) f_\alpha$$

Electrically neutral spinons f_α coupled to emergent gauge field A_μ

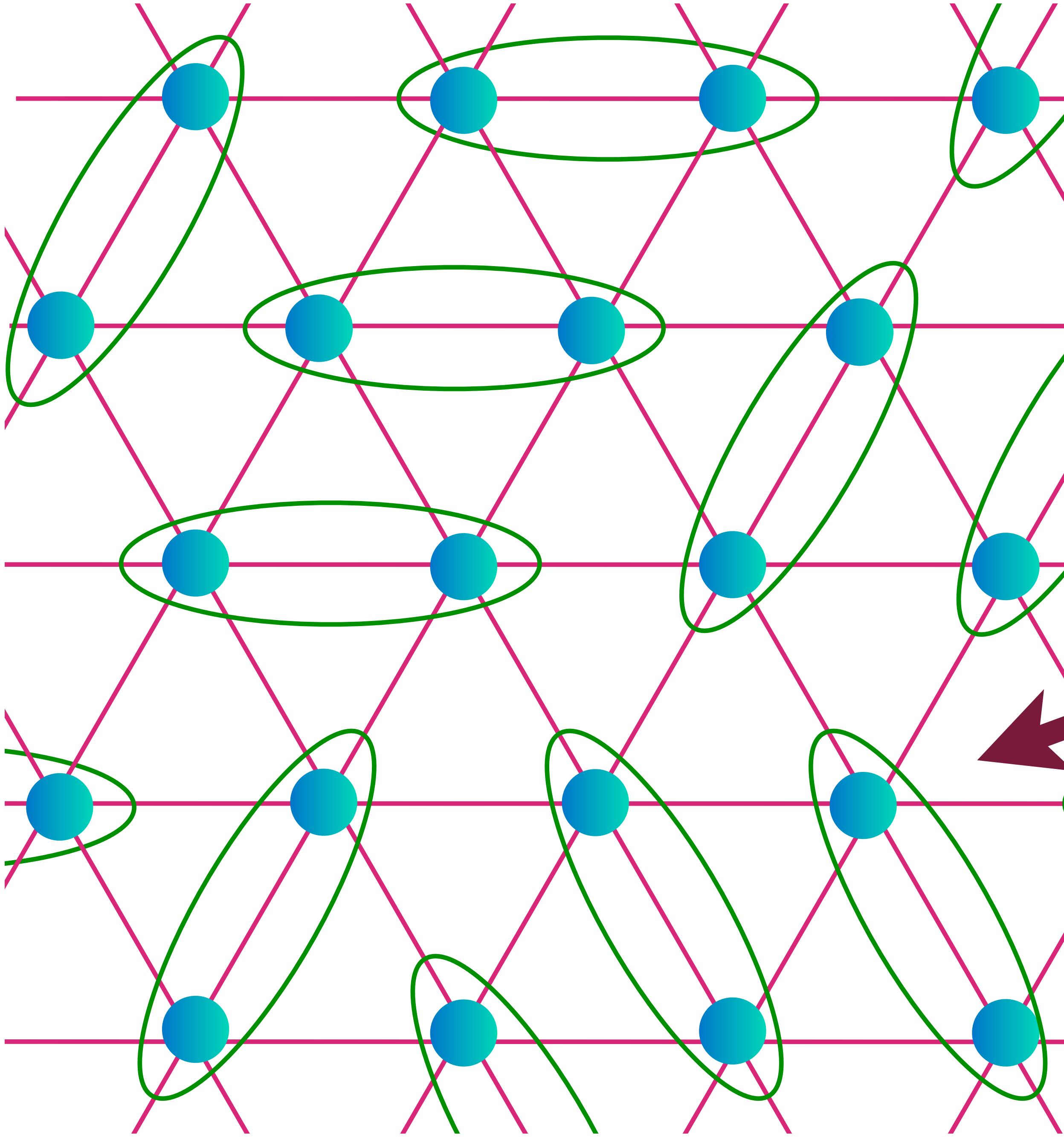
$$\mathcal{L}_b = \left| \left(\frac{\partial}{\partial \tau} - i A_\tau + \mu \right) b \right|^2 + \frac{1}{2m_b} \left| \left(\nabla - i\mathbf{A} - ie\mathbf{A}_{\text{ext}} \right) b \right|^2 + s|b|^2 + u|b|^4$$

Electrically charged bosons b in a Mott-insulating state

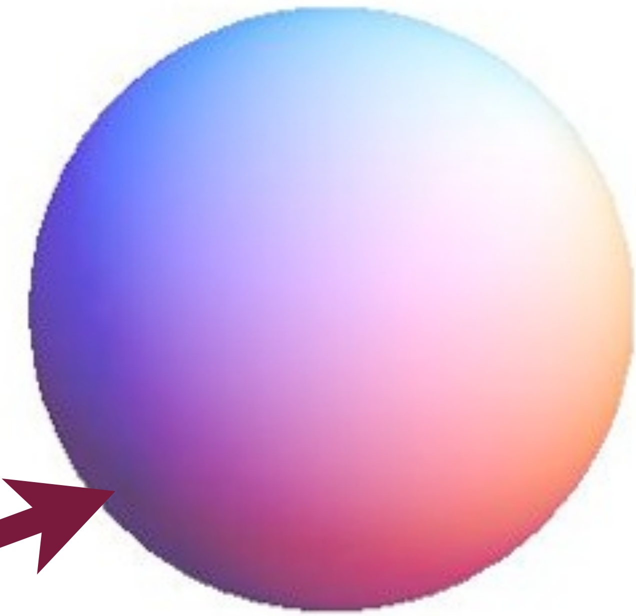
$$\mathcal{L}_c = c_\alpha^\dagger \left(\frac{\partial}{\partial \tau} - \varepsilon_c(\mathbf{k} + e\mathbf{A}_{\text{ext}}) + \mu \right) c_\alpha + \sqrt{J_K} \left(b^\dagger c_\alpha^\dagger f_\alpha + b f_\alpha^\dagger c_\alpha \right)$$

Electrically charged conduction electrons c_α in a small Fermi surface

Kondo lattice model



Kondo exchange perturbative



Conduction electrons

$$\sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} c_{\mathbf{k}\alpha}^{\dagger} c_{\mathbf{k}\alpha}$$

$$\sum_{i<j} J_H(i,j) \vec{S}_i \cdot \vec{S}_j$$

Fractionalized Fermi liquid (FL*)

T. Senthil, S. Sachdev, and M. Vojta, *Phys. Rev. Lett.* **90**, 216403 (2003).

T. Senthil, M. Vojta, and S. Sachdev, *Phys. Rev. B* **69**, 035111 (2004).

Theory of spin liquid

Transport in a spin liquid

$$\mathbf{J}_f = \sigma_f \mathbf{E}$$
$$\mathbf{J}_b = \sigma_b \left(\mathbf{E} + \mathbf{E}_{\text{ext}} \right)$$

Equation of motion of emergent gauge field:

$$\frac{\delta S}{\delta \mathbf{A}} = 0 \quad \Rightarrow \quad \mathbf{J}_f + \mathbf{J}_b = 0 \quad \Rightarrow \quad \mathbf{E} = -\frac{\sigma_b}{\sigma_f + \sigma_b} \mathbf{E}_{\text{ext}}$$

Net electrical current:

$$\mathbf{J}_b = \frac{\sigma_f \sigma_b}{\sigma_f + \sigma_b} \mathbf{E}_{\text{ext}}$$

Because $\sigma_b = 0$ in gapped boson state, we obtain zero conductivity in a finite density state. Note the key role of U(1)×U(1) structure in obtaining this result.
(*cf. Deconstructing holographic liquids*, Dominik Nickel and Dam T. Son, arXiv:1009.3094)

Theory of FL*

Transport in FL*

$$\begin{aligned} \mathbf{J}_f &= \sigma_f \mathbf{E} \\ \mathbf{J}_b &= \sigma_b \left(\mathbf{E} + \mathbf{E}_{\rm ext} \right) \\ \mathbf{J}_c &= \sigma_c \mathbf{E}_{\rm ext} \end{aligned}$$

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Net electrical current:

$$\mathbf{J}_b + \mathbf{J}_c = \left(\frac{\sigma_f \sigma_b}{\sigma_f + \sigma_b} + \sigma_c \right) \mathbf{E}_{\rm ext}$$

Note the key role of U(1)×U(1) structure in obtaining this result

Outline

1. Quantum impurities and AdS_2

   Quantum spin coupled to a CFT

2. Phases of the Kondo lattice

   Fermi liquids (FL),
   Fractionalized Fermi liquids (FL),*
   and the Luttinger theorem

3. A mean field theory of a fractionalized Fermi liquid

   A marginal Fermi liquid and $AdS_2 \times R^2$

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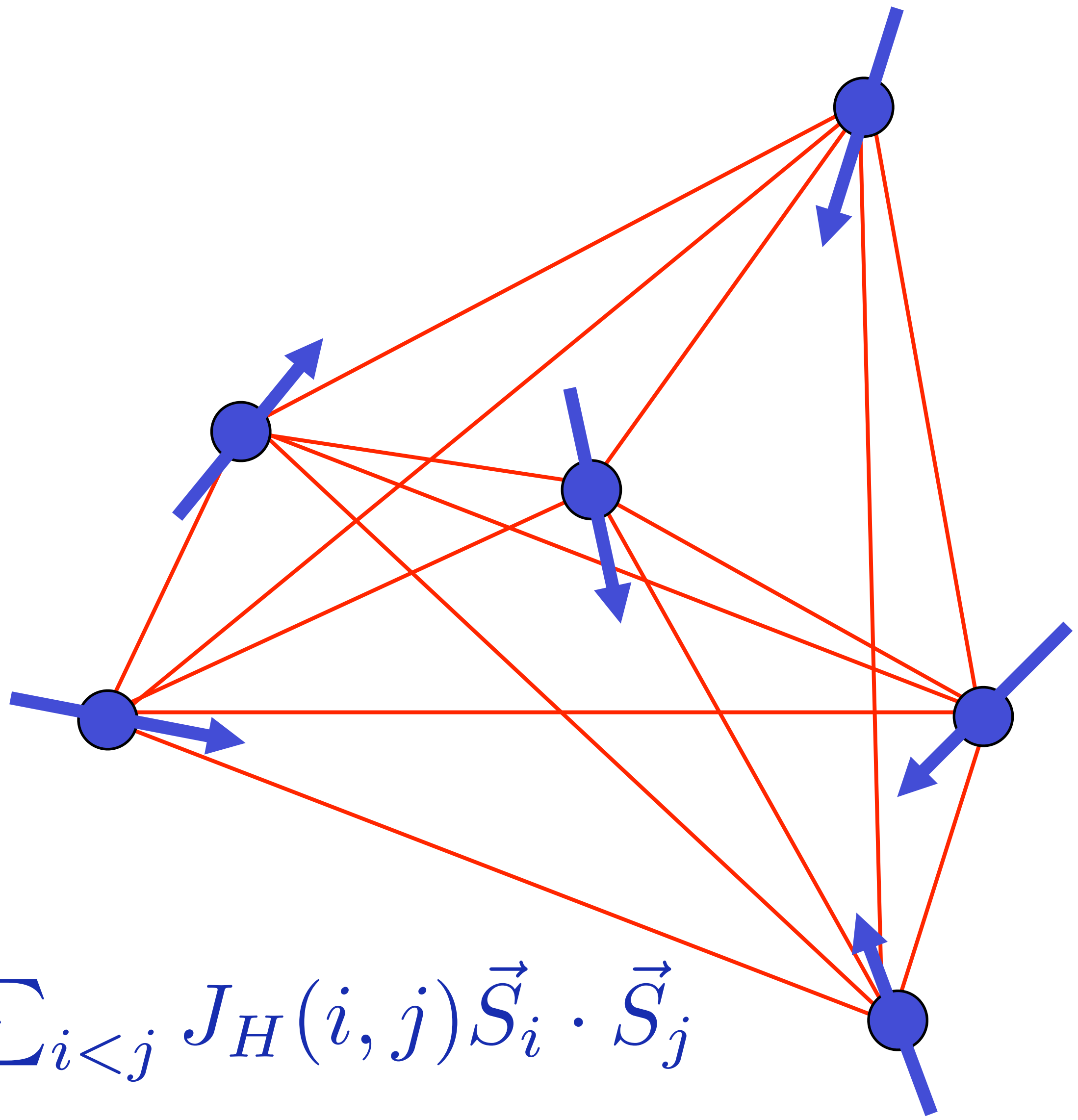
A mean-field theory of a spin liquid



$\sum_{i<j} J_H(i,j) \vec{S}_i \cdot \vec{S}_j$

$J_H(i,j)$ Gaussian random variables.
A quantum Sherrington-Kirkpatrick model of SU(N) spins.

S. Sachdev and J. Ye, Phys. Rev. Lett. **70**, 3339 (1993).
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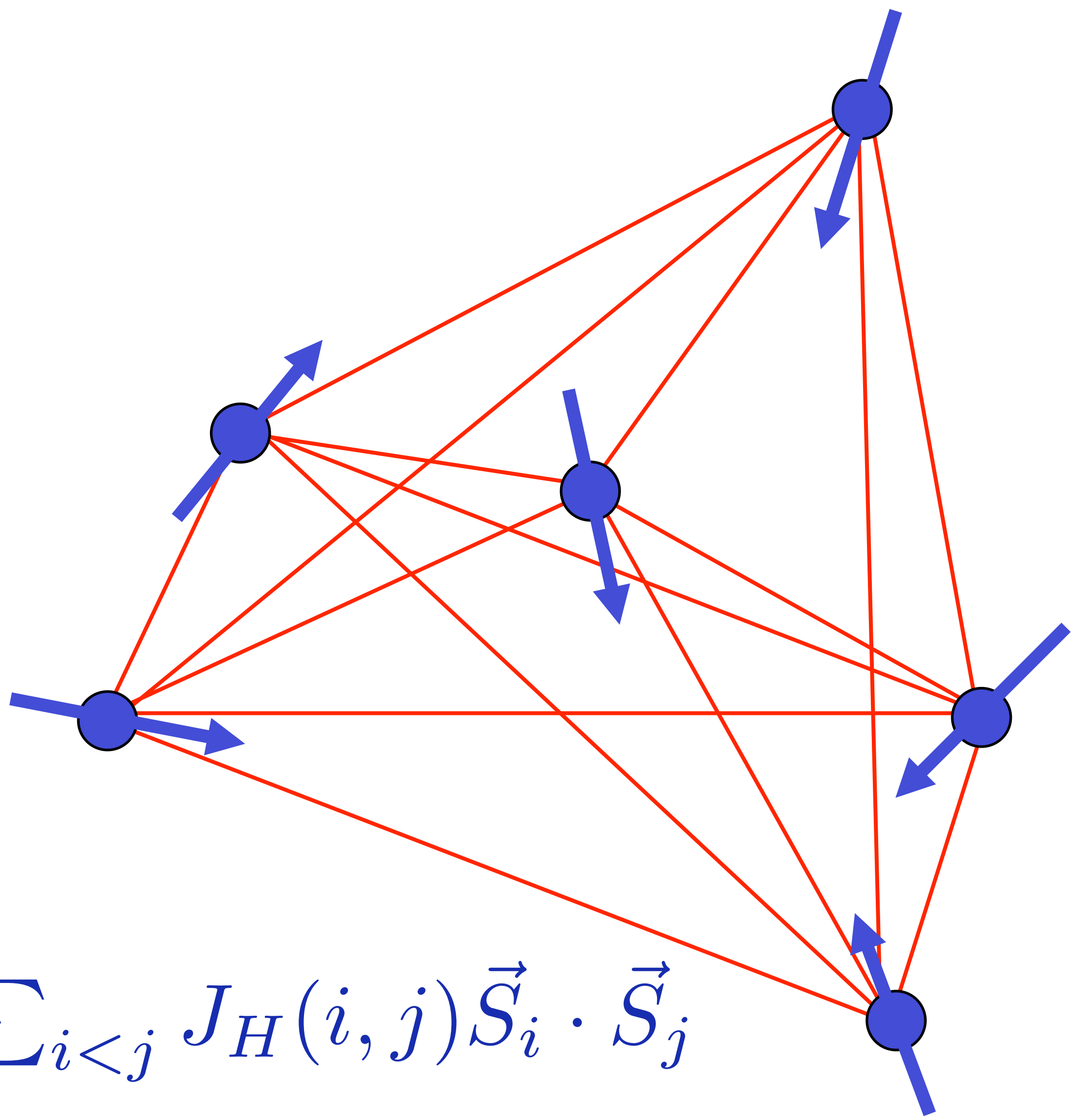

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Described by the quantum mechanics of a spin fluctuating in a self-consistent time-dependent magnetic field: a realization the finite entropy density $AdS_2 \times R^d$ state

S. Sachdev, *Physical Review Letters* **105**, 151602 (2010)

AdS_2 realization in the quantum SK model

Focus on a single $\vec{S}$ spin, and represent its imaginary time fluctuations by a unit vector $\vec{S} = \vec{n}(\tau)/2$ which is controlled by the partition function

$$\mathcal{Z} = \int \mathcal{D}\vec{n}(\tau)\,\delta(\vec{n}^2(\tau)-1)\exp\left(-\mathcal{S}\right)$$

$$\mathcal{S} = \frac{i}{2}\int_0^1 du \int_0^{1/T} d\tau\,\vec{n}\cdot\left(\frac{\partial\vec{n}}{\partial u}\times\frac{\partial\vec{n}}{\partial\tau}\right) - \int_0^{1/T} d\tau\,\vec{h}(\tau)\cdot\vec{n}(\tau)$$

The first term is a Wess-Zumino term, with the "extra dimension" u defined so that $\vec{n}(\tau, u=1) \equiv \vec{n}(\tau)$ and $\vec{n}(\tau, u=0) = (0,0,1)$.

The field $\vec{h}(\tau)$ represents the "environment", which which we take to be a Gaussian random variable with the correlation

$$\left\langle \vec{h}(\tau)\cdot\vec{h}(0)\right\rangle = A\left|\frac{\pi T}{\sin(\pi T\tau)}\right|^{\gamma}$$

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AdS_2 realization in the quantum SK model

Solution of $\mathcal{Z}$ for such an $\vec{h}(\tau)$ yields

$$\langle \vec{n}(\tau) \cdot \vec{n}(0) \rangle = B \left| \frac{\pi T}{\sin(\pi T \tau)} \right|^h$$

with the exponent $h = 2 - \gamma$. The self-consistency condition for the infinite-range model requires that the two-point correlation of $\vec{h}$ is proporionaly to that of $\vec{n}$. This leads to $h = \gamma$, which implies $h = \gamma = 1$.

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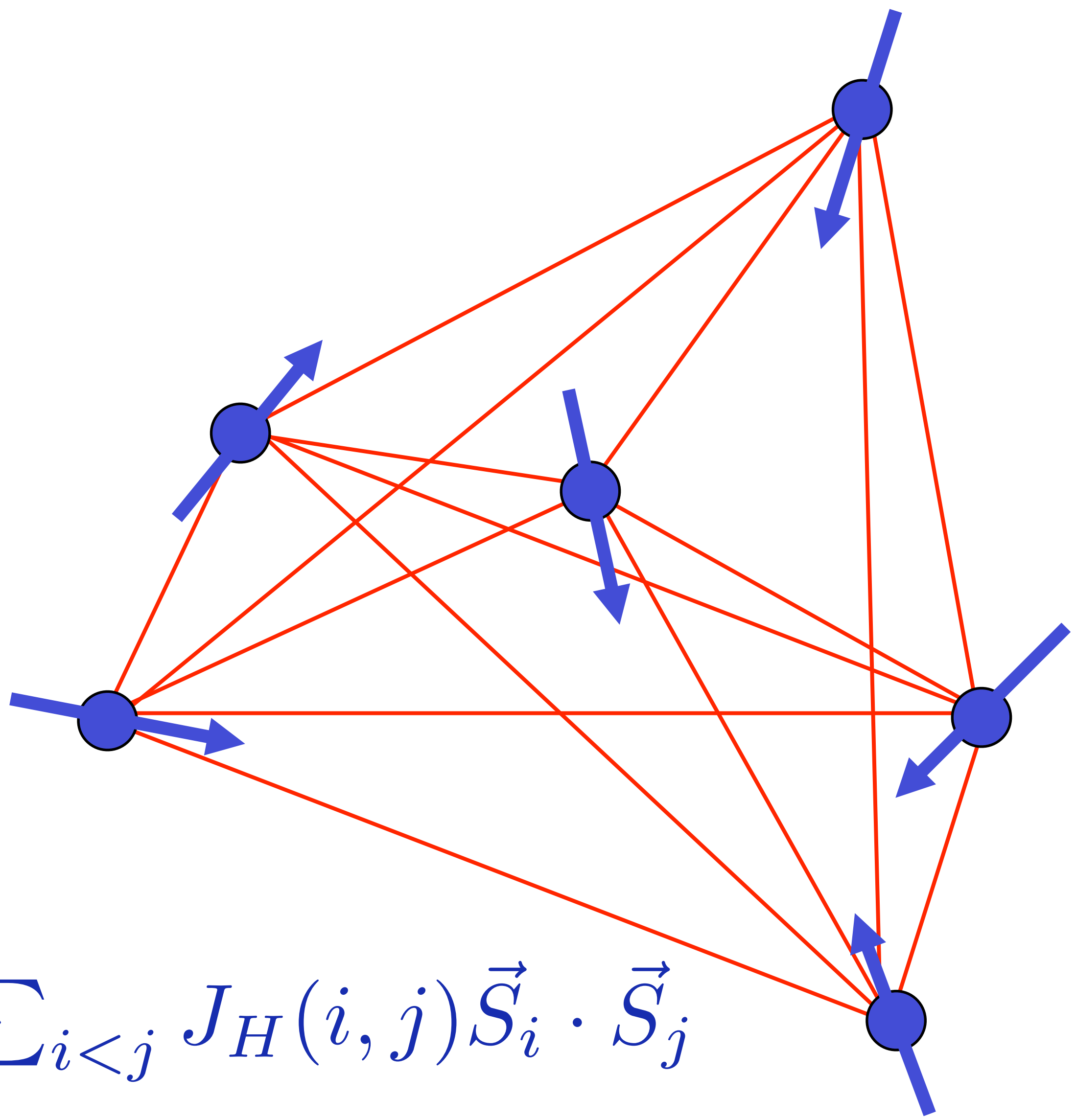

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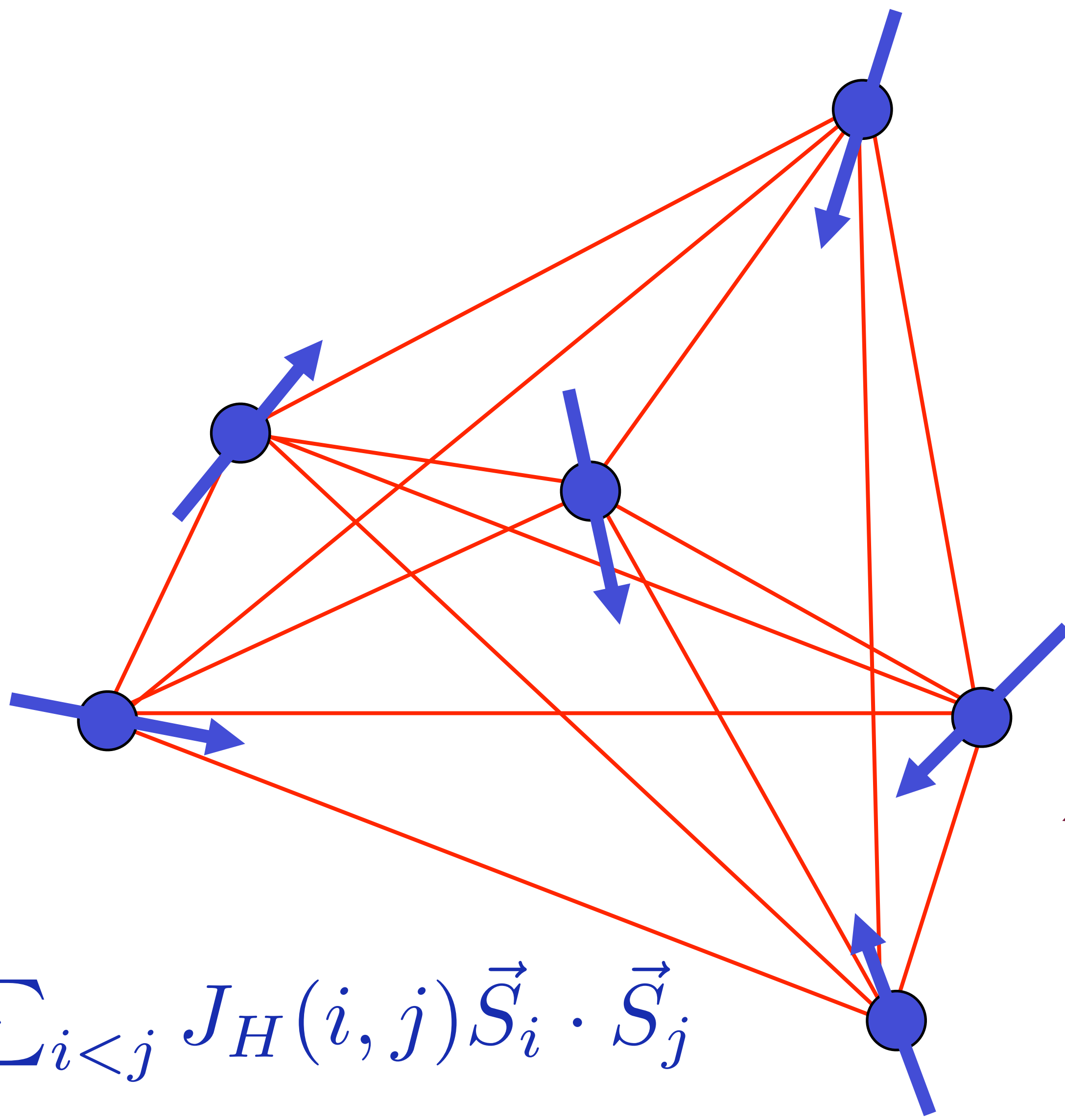
A mean-field theory of FL*



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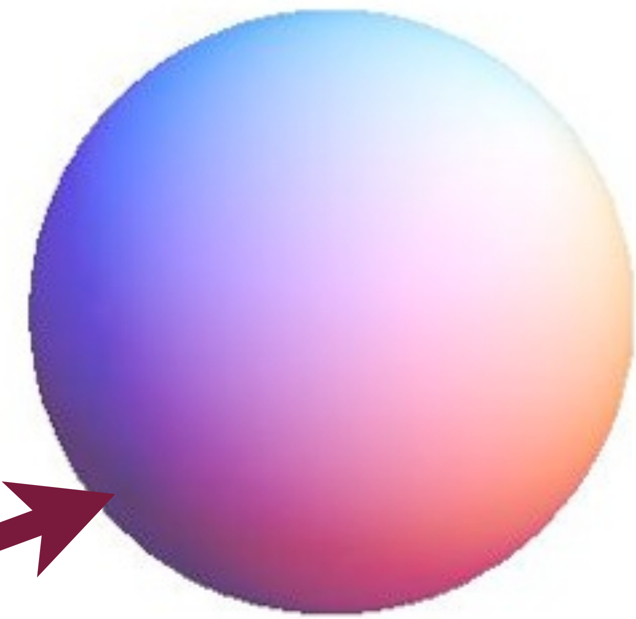

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S. Burdin, D. R. Grempel, and A. Georges, Phys. Rev. B **66**, 045111 (2002)

Effective low energy theory for conduction electrons

The operators acting on the low energy subspace are c_i and $\vec{S}_{fi}$. For the c_i we have the effective theory

$$\mathcal{S}_c = \int \frac{d^d k}{(2\pi)^d} \int d\tau \left[c^\dagger_{\mathbf{k}\sigma} \left(\frac{\partial}{\partial \tau} - \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\sigma} - V F^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} - V c^\dagger_{\mathbf{k}\sigma} F_{\mathbf{k}\sigma} \right]$$

Here the $F_{i\sigma}$ are strongly renormalized operators on the f orbitals, which project onto the low energy theory as

$$F_{i\sigma} \sim \frac{1}{U} \left(\vec{\tau}_{\sigma\sigma'} \cdot \vec{S}_{fi} \right) c_{i\sigma'}$$

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From this we obtain the conduction electron self energy

$$\Sigma_c(\tau) \sim \left[\frac{\pi T}{\sin(\pi T \tau)} \right]^{h+1}$$

This is the marginal Fermi liquid form for $h = 1$.

Connection to holographic metals

- The quantum SK model has $z = \infty$ conformal spin correlations and a finite ground state entropy density: similar to $\mathrm{AdS}_2 \times \mathrm{R}^d$.

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$$F_{i\sigma} \sim \frac{1}{U} \left(\vec{\tau}_{\sigma\sigma'} \cdot \vec{S}_{fi} \right) c_{i\sigma'}$$

- This leads to a 'probe fermion' self energy which is identical to the theory of the holographic metal (T. Faulkner, H. Liu, J. McGreevy and D. Vegh, arXiv:0907.2694.)

Conclusions

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- This correspondence is quite precise in the $z = \infty$ theories of the Sherrington-Kirkpatrick-Kondo model and the extremal Reissner-Nordstrom black hole
- Good prospects for establishing correspondence at finite z