Monte Carlo Tools

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Topics of the lectures

1. Lecture 1: *Tour through Event Generators*
   - Hard physics simulation: Parton Level event generation
   - Dressing the partons: Parton Showers
   - Soft physics simulation: Hadronization
   - Beyond factorization: Underlying Event
2. Lecture 2: *Higher Orders in Monte Carlos*
   - Some nomenclature: Anatomy of HO calculations
   - Merging vs. Matching

Thanks to

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- other MC authors: S.Gieseke, K.Hamilton, L.Lonnblad, F.Maltoni, M.Mangano, P.Richardson, M.Seymour, T.Sjostrand, B.Webber, ...

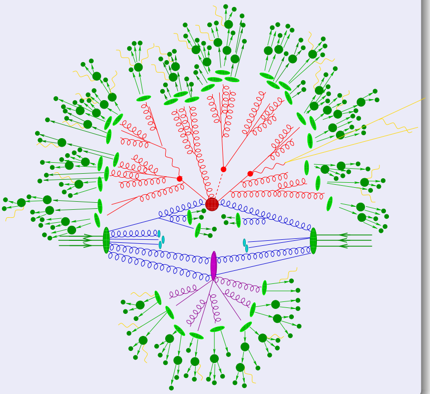
Simulation's paradigm

Basic strategy

Divide event into stages, separated by different scales.

- Signal/background:
  Exact matrix elements.
- QCD-Bremsstrahlung:
  Parton showers (also in initial state).
- Multiple interactions:
  Beyond factorization: Modeling.
- Hadronization:
  Non-perturbative QCD: Modeling.

Sketch of an event



Today's lecture: Event Generation in a Nutshell

- Monte Carlo integration
- Parton level event generation
- Parton showers
- Multiple interactions
- Hadronization

Monte Carlo integration

Convergence of numerical integration

- Consider $I = \int\limits_0^1 \mathrm{d}x^D f(\vec{x})$.
- Convergence behavior crucial for numerical evaluations. For integration (N = number of evaluations of f):
  - Trapezium rule $\simeq 1/N^{2/D}$
  - Simpson's rule $\simeq 1/N^{4/D}$
  - Central limit theorem $\simeq 1/\sqrt{N}$.
- Therefore: Use central limit theorem.

Monte Carlo integration

Monte Carlo integration

- Use random vectors $\vec{x}_i \longrightarrow$:
  Evaluate estimate of the integral $\langle I \rangle$ rather than I.
  $$\langle I(f) \rangle = \frac{1}{N} \sum_{i=1}^{N} f(\vec{x}_i).$$
  (This is the original meaning of Monte Carlo: Use random numbers for integration.)
- Quality of estimate given by error estimator (variance)
  $$\langle E(f) \rangle^2 = \frac{1}{N-1} \left[\langle I^2(f) \rangle - \langle I(f) \rangle^2 \right].$$
- Name of the game: Minimize $\langle E(f) \rangle$.
- Problem: Large fluctuations in integrand f
- Solution: Smart sampling methods

Monte Carlo integration

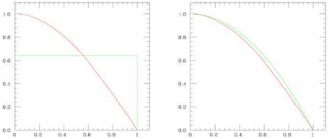
Importance sampling

Basic idea: Put more samples in regions, where f largest

$\Longrightarrow$ improves convergence behavior

(corresponds to a Jacobian transformation).

- Assume a function $g(\vec{x})$ similar to $f(\vec{x})$;
- obviously then, $f(\vec{x})/g(\vec{x})$ is comparably smooth, hence $\langle E(f/g)\rangle$ is small.



$$I = \int_0^1 dx \cos\frac{\pi}{2}x = 0.637 \pm 0.308/\sqrt{N}$$

$$I = \int_0^1 dx(1-x^2)\,\frac{\cos\frac{\pi}{2}x}{1-x^2} = \int d\rho\,\frac{\cos\frac{\pi}{2}x}{1-x^2}[x(\rho)] = 0.637 \pm 0.032/\sqrt{N}$$

Monte Carlo integration

Stratified sampling

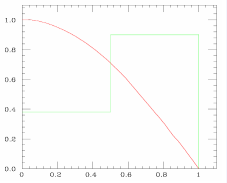
Basic idea: Decompose integral in M sub-integrals

$$\langle I(f)\rangle = \sum_{j=1}^{M}\langle I_j(f)\rangle,\quad \langle E(f)\rangle^2 = \sum_{j=1}^{M}\langle E_j(f)\rangle^2$$

Then: Overall variance smallest, if "equally distributed".

$\Longrightarrow$ Sample, where the fluctuations are.

- Divide interval in bins;
- adjust bin-size or weight per bin such that variance identical in all bins.



$\langle I\rangle = 0.637 \pm 0.147/\sqrt{N}$

Monte Carlo integration

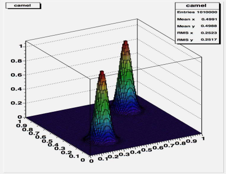
Example for stratified sampling: VEGAS

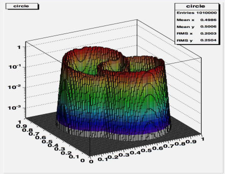
- Assume m bins in each dimension of $\vec{x}$.
- For each bin k in each dimension $\eta \in [1, n]$ assume a **weight (probability)** $\alpha_k^{(\eta)}$ for x_k to be in that bin.

  Condition(s) on the weights:
  $\alpha_k^{(\eta)} \in [0, 1]$, $\sum_{k=1}^{m} \alpha_k^{(\eta)} = 1$.
- For each bin in each dimension calculate $\langle I_k^{(\eta)} \rangle$ and $\langle E_k^{(\eta)} \rangle$.

  Obviously, for all η, $\langle I \rangle = \sum_{k=1}^{m} \langle I_k^{(\eta)} \rangle$, but error estimates different.
- In each dimensions, iterate and update the $\alpha_k^{(\eta)}$; example for updating:

  $$\alpha_k^{(\eta)}(\text{rm new}) \propto \alpha_k^{(\eta)}(\text{rm old}) \left(\frac{E_k^{(\eta)}}{E_{\text{tot.}}(\eta)} \right)^{\kappa}.$$
- Problem with this simple algorithm:
  Gets a hold only on fluctuations $\|$ to binning axes.





Monte Carlo integration

Multichannel sampling

Basic idea: Use a sum of functions $g_i(\vec{x})$ as Jacobian $g(\vec{x})$.

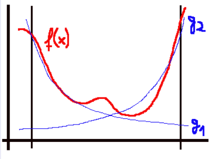
$\Longrightarrow \quad g(\vec{x}) = \sum_{i=1}^{N} \alpha_i g_i(\vec{x})$;

$\Longrightarrow$ condition on weights like stratified sampling;

("Combination" of importance & stratified sampling).

Algorithm for one iteration:

- Select g_i with probability $\alpha_i \longrightarrow \vec{x}_j$.
- Calculate total weight $g(\vec{x}_j)$ and partial weights $g_i(\vec{x}_j)$
- Add $f(\vec{x}_j)/g(\vec{x}_j)$ to total result and $f(\vec{x}_j)/g_i(\vec{x}_j)$ to partial (channel-) results.
- After N sampling steps, update a-priori weights.



This is the method of choice for parton level event generation!

Monte Carlo integration

Selecting after sampling: Unweighting efficiency

Basic idea: Use hit-or-miss method;

Generate $\vec{x}$ with integration method,

compare actual $f(\vec{x})$ with maximal value during sampling;

$\Longrightarrow$ "Unweighted events".

Comments:

- unweighting efficiency, $w_{\rm eff} = \langle f(\vec{x}_j)/f_{\rm max}\rangle$ = number of trials for each event.
- Good measure for integration performance.
- Expect $\log_{10} w_{\rm eff} \approx 3-5$ for good integration of multi-particle final states at tree-level.
- Maybe acceptable to use $f_{\rm max,eff} = K f_{\rm max}$ with $K < 1$.
  Problem: what to do with events where $f(\vec{x}_j)/f_{\rm max,eff} > 1$?
  Answer: Add $\mathrm{int}[f(\vec{x}_j)/f_{\rm max,eff}] = k$ events and perform hit-or-miss on $f(\vec{x}_j)/f_{\rm max,eff} - k$.

Monte Carlo integration

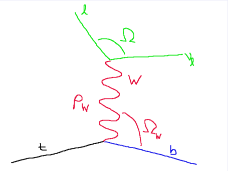
Particle physics example: Evaluation of cross sections

- Simple example: $t \longrightarrow bW^+ \longrightarrow b\bar{l}\nu_l$:

$$|\mathcal{M}|^2 = \frac{1}{2}\left(\frac{8\pi\alpha}{\sin^2\theta_W}\right)^2 \frac{p_t\cdot p_\nu \;\; p_b\cdot p_l}{(p_W^2-M_W^2)^2+\Gamma_W^2 M_W^2}$$

- Phase space integration (5-dim)

$$\Gamma = \frac{1}{2m_t}\frac{1}{128\pi^3}\int \mathrm{d}p_W^2 \frac{\mathrm{d}^2\Omega_W}{4\pi}\frac{\mathrm{d}^2\Omega}{4\pi}\left(1-\frac{p_W^2}{m_t^2}\right)|\mathcal{M}|^2$$



Advantages

- Throw 5 random numbers, construct four-momenta ($\Longrightarrow$ full kinematics, "events")
- Apply smearing and/or arbitrary cuts.
- Simply histogram any quantity of interest - no new calculation for each observable

Parton level simulations

Stating the problem(s)

- Multi-particle final states for signals & backgrounds.
- Need to evaluate $\mathrm{d}\sigma_N$:

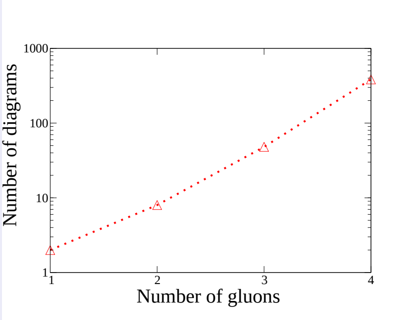
$$\int\limits_{\rm cuts}\left[\prod_{i=1}^{N}\frac{\mathrm{d}^3 q_i}{(2\pi)^3 2E_i}\right]\delta^4\left(p_1+p_2-\sum_i q_i\right)\left|\mathcal{M}_{p_1p_2\to N}\right|^2.$$

- Problem 1: Factorial growth of number of amplitudes.
- Problem 2: Complicated phase-space structure.
- Solutions: Numerical methods.

Parton level simulations

Factorial growth: $e^+e^- \longrightarrow q\bar{q} + ng$

n	$\#_{\text{diags}}$
0	1
1	2
2	8
3	48
4	384



Parton level simulations

Basic ideas of efficient ME calculation

Need to evaluate $|\mathcal{M}|^2 = \left|\sum_i \mathcal{M}_i\right|^2$

- Obvious: Traditional textbook methods (squaring, completeness relations, traces) fail
  $\Longrightarrow$ result in proliferation of terms $(\mathcal{M}_i \mathcal{M}_j^*)$
  $\Longrightarrow$ Better: Amplitudes are complex numbers,
  $\Longrightarrow$ add them before squaring!
- Remember: spinors, gamma matrices have explicit form could be evaluated numerically (brute force)
  But: Rough method, lack of elegance, CPU-expensive

Parton level simulations

Helicity method

- Introduce basic helicity spinors (needs to "gauge"-vectors)
- Write everything as spinor products, e.g.

$$\bar{u}(p_1,\, h_1)u(p_2,\, h_2) = \text{complex numbers.}$$

- Also: $(\not{p} + m) \implies \frac{1}{2}\sum_h \left[\left(1+\frac{m^2}{p^2}\right)\bar{u}(p,\, h)u(p,\, h) + \left(1-\frac{m^2}{p^2}\right)\bar{v}(p,\, h)v(p,\, h)\right]$

(completeness relation)

- Find other genuine expressions:

$$\begin{aligned}
Y(p_1, h_1,\; p_2, h_2) &:= \bar{u}(p_1, h_1)u(p_2, h_2)\\
X(p_1, h_1,\; p_2, h_2,\; p_3) &:= \bar{u}(p_1, h_1)\not{p}_3 u(p_2, h_2)\\
Z(p_1, h_1,\; p_2, h_2;\; p_3, h_3,\; p_4, h_4) &:= \bar{u}(p_1, h_1)\gamma^\mu u(p_2, h_2)\bar{u}(p_3, h_3)\gamma^\mu u(p_4, h_4)\,,
\end{aligned}$$

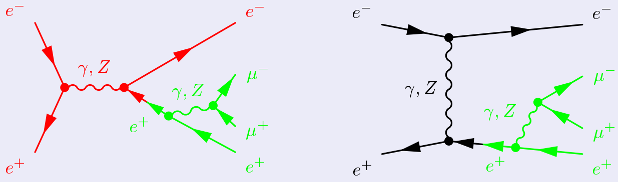
all complex-valued functions of momenta & helicities.

Parton level simulations

Taming the factorial growth in the helicity method

- Reusing pieces: Calculate only once!
- Factoring out: Reduce number of multiplications!

Implemented as a-posteriori manipulations of amplitudes.



Parton level simulations

Recursion methods (off-shell)

Basic idea: Recursively build one-particle off-shell currents (various versions of this: Berends-Giele, Alpha etc.).

"Classical" example: *n*-gluon amplitudes:

- Start with two on-shell gluons, represented by their polarization vectors, hence the currents associated with them are $J^\nu(k) = \varepsilon^\nu(k)$.
- Then the two-gluon current reads (no colors) $J^\mu(k = k_1 + k_2) = \frac{ig_3}{(k_1+k_2)^2} V^{\mu\nu\rho} J_\nu(k_1) J_\rho(k_2)$.
- From this, larger and larger currents can be built recursively.
- For quarks, the currents are given by spinors, and similar reasoning applies for the construction of the one-particle off-shell currents.
- Treatment of color: Color-ordering the amplitudes $\Longrightarrow \mathcal{C}^{(1,\,\dots,\,n)} = \mathrm{Tr}\,[T^{a_1}\,\dots\,T^{a_n}]$, where T^a are color matrices in fundamental representation.
- Problem: Need to sum over all allowed permutations.

Parton level simulations

Integration methods: Multi-channeling

Basic idea: Translate Feynman diagrams into channels

$\Longrightarrow$ decays, s- and t-channel props as building blocks.

R.Kleiss and R.Pittau, Comput. Phys. Commun. **83** (1994) 141

Integration methods: "Democratic" methods

- Rambo/Mambo: Flat & isotropic

  R.Kleiss, W.J.Stirling and S.D.Ellis, Comput. Phys. Commun. **40** (1986) 359;

- HAAG: Follows QCD antenna pattern

  A.van Hameren and C.G.Papadopoulos, Eur. Phys. J. C **25** (2002) 563.

Limitations of parton level simulation

Factorial growth

- . . . persists due to the number of color configurations

  (e.g. $(n-1)!$ permutations for n external gluons).

- Solution: Sampling over colors,
  but correlations with phase space
  $\Longrightarrow$ Best recipe not (yet) found.

- New scheme for color: color dressing

  (C.Duhr, S.Hoche and F.Maltoni,JHEP **0608** (2006) 062)

Limitations of parton level simulation

Factorial growth

- Off-shell vs. on-shell recursion relations:

Final State	BG		BCF		CSW	
	CO	CD	CO	CD	CO	CD
$2g$	0.24	0.28	0.28	0.33	0.31	0.26
$3g$	0.45	0.48	0.42	0.51	0.57	0.55
$4g$	1.20	1.04	0.84	1.32	1.63	1.75
$5g$	3.78	2.69	2.59	7.26	5.95	5.96
$6g$	14.2	7.19	11.9	59.1	27.8	30.6
$7g$	58.5	23.7	73.6	646	146	195
$8g$	276	82.1	597	8690	919	1890
$9g$	1450	270	5900	127000	6310	29700
$10g$	7960	864	64000	-	48900	-

Time [s] for the evaluation of 10^4 phase space points, sampled over helicities & color.

Limitations of parton level simulation

Efficient phase space integration

- Main problem: Adaptive multi-channel sampling translates "Feynman diagrams" into integration channels
  $\Longrightarrow$ hence subject to growth.
- But it is practical only for 1000-10000 channels.
- Therefore: Need better sampling procedures
  $\Longrightarrow$ open question with little activity.

(Private suspicion: Lack of glamour)

Limitations of parton level simulation

General

- Fixed order parton level (LO, NLO, ...) implies fixed multiplicity
- No control over potentially large logs
  (appear when two partons come close to each other).
- Parton level is parton level
  experimental definition of observables relies on hadrons.

Therefore: Need hadron level event generators!

Motivation: Why parton showers?

Some more refined reasons

- Experimental definition of jets based on hadrons.
- But: Hadronization through phenomenological models (need to be tuned to data).
- Wanted: Universality of hadronization parameters (independence of hard process important).
- Link to fragmentation needed: Model softer radiation (inner jet evolution).
- Similar to PDFs (factorization) just the other way around (fragmentation functions at low scale, parton shower connects high with low scale).

Motivation: Why parton showers?

Common wisdom

- Well-known: Accelerated charges radiate
- QED: Electrons (charged) emit photons
  Photons split into electron-positron pairs
- QCD: Quarks (colored) emit gluons
  Gluons split into quark pairs
- Difference: Gluons are colored (photons are not charged)
  Hence: Gluons emit gluons!
- Cascade of emissions: Parton shower

Occurrence of large logarithms

$e^+e^- \longrightarrow$ jets

- Differential cross section:

$$\frac{\mathrm{d}\sigma_{ee\to 3j}}{\mathrm{d}x_1\mathrm{d}x_2} = \sigma_{ee\to 2j}\frac{C_F\alpha_s}{\pi}\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}$$

  Singular for $x_{1,2} \longrightarrow 1$.

- Rewrite with opening angle θ_{qg} and gluon energy fraction $x_3 = 2E_g/E_{\rm c.m.}$:

$$\frac{\mathrm{d}\sigma_{ee\to 3j}}{\mathrm{d}\cos\theta_{qg}\mathrm{d}x_3} = \sigma_{ee\to 2j}\frac{C_F\alpha_s}{\pi}\left[\frac{2}{\sin^2\theta_{qg}}\frac{1+(1-x_3)^2}{x_3} - x_3\right]$$

  Singular for $x_3 \longrightarrow 0$ ("soft"), $\sin\theta_{qg} \longrightarrow 0$ ("collinear").

Occurrence of large logarithms

Collinear singularities

- Use

$$\frac{2\mathrm{d}\cos\theta_{qg}}{\sin^2\theta_{qg}} = \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{qg}}{1+\cos\theta_{qg}} = \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{\bar{q}g}}{1-\cos\theta_{\bar{q}g}} \approx \frac{\mathrm{d}\theta^2_{qg}}{\theta^2_{qg}} + \frac{\mathrm{d}\theta^2_{\bar{q}g}}{\theta^2_{\bar{q}g}}$$

- Independent evolution of two jets (q and $\bar{q}$):

$$\mathrm{d}\sigma_{ee\to 3j} \approx \sigma_{ee\to 2j} \sum_{j\in\{q,\bar{q}\}} \frac{C_F\alpha_s}{2\pi}\frac{\mathrm{d}\theta^2_{jg}}{\theta^2_{jg}} P(z)\,,$$

where $P(z) = \frac{1+(1-z)^2}{z}$ (DGLAP splitting function)

Occurrence of large logarithms

Expressing the collinear variable

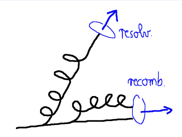
- Same form for any $t \propto \theta^2$:
- Transverse momentum $k_\perp^2 \approx z^2(1-z)^2 E^2\theta^2$
- Invariant mass $q^2 \approx z(1-z)E^2\theta^2$

$$\frac{\mathrm{d}\theta^2}{\theta^2} \approx \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \approx \frac{\mathrm{d}q^2}{q^2}$$

Occurrence of large logarithms

Parton resolution

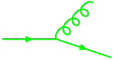
- What is a parton?
  Collinear pair/soft parton recombine!
- Introduce resolution criterion $k_\perp > Q_0$.



- Combine virtual contributions with unresolvable emissions:
  Cancels infrared divergences $\Longrightarrow$ Finite at $\mathcal{O}(\alpha_s)$

  (Kinoshita-Lee-Nauenberg, Bloch-Nordsieck theorems)

- Unitarity: Probabilities add up to one
  $\mathcal{P}(\text{resolved}) + \mathcal{P}(\text{unresolved}) = 1.$

 $+$  $= 1.$

Occurrence of large logarithms

The Sudakov form factor

- Diff. probability for emission between q^2 and $q^2 + \mathrm{d}q^2$:

$$\mathrm{d}\mathcal{P} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}q^2}{q^2}\int\limits_{Q_0^2/q^2}^{1-Q_0^2/q^2} \mathrm{d}z P(z) =: \frac{\mathrm{d}q^2}{q^2}\bar{P}(q^2)\,.$$

- No-emission probability $\Delta(Q^2,\, q^2)$ between Q^2 and q^2.

Evolution equation for Δ: $-\frac{\mathrm{d}\Delta(Q^2,q^2)}{\mathrm{d}q^2} = \Delta(Q^2,\, q^2)\frac{\mathcal{P}}{\mathrm{d}q^2}$.

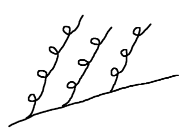
$$\Longrightarrow\quad \Delta(Q^2,\, q^2) = \exp\left[-\int\limits_{q^2}^{Q^2}\frac{dk^2}{k^2}\,\bar{P}(k^2)\right]\,.$$

Occurrence of large logarithms

Many emissions

- Iterate emissions (jets)

Maximal result for $t_1 > t_2 > \dots t_n$:

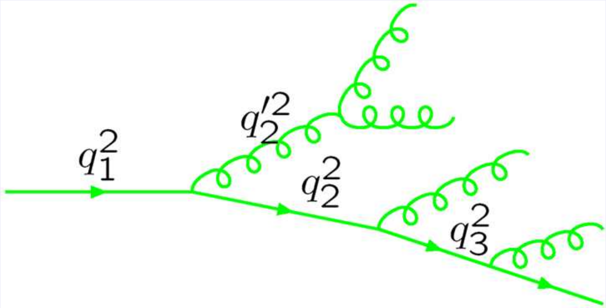


$$d\sigma \propto \sigma_0 \int\limits_{Q_0^2}^{Q^2} \frac{dt_1}{t_1} \int\limits_{Q_0^2}^{t_1} \frac{dt_2}{t_2} \dots \int\limits_{Q_0^2}^{t_{n-1}} \frac{dt_n}{t_n} \propto \log^n \frac{Q^2}{Q_0^2}$$

- How about Q^2? Process-dependent!

Occurrence of large logarithms

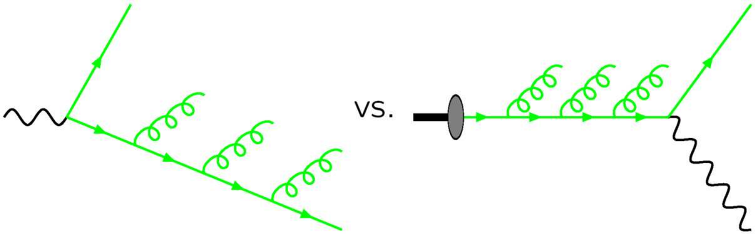
Ordering the emissions : Radiation pattern



$$q_1^2 > q_2^2 > q_3^2,\; q_1^2 > q_2'^2$$

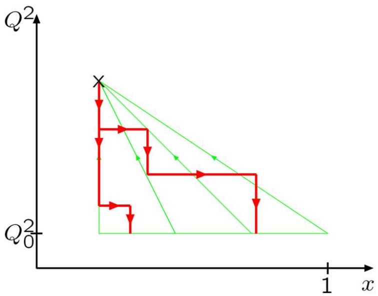
Occurrence of large logarithms

Forward vs. backward evolution: Pictorially



Occurrence of large logarithms

Use of DGLAP evolution



DGLAP evolution:

PDFs at (x, Q^2) as function of PDFs at (x_0, Q_0^2).

Backward evolution:

start from hard scattering at (x, Q^2) and work down in q^2 and up in x.

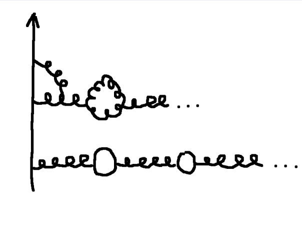
Change in algorithm:

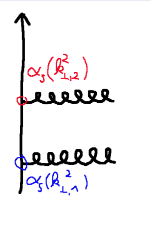
$\Delta_i(q^2) \Longrightarrow \Delta_i(q^2)/f_i(x_i, q^2)$.

Inclusion of quantum effects

Running coupling

- Effect of summing up higher orders (loops): $\alpha_s \longrightarrow \alpha_s(k_\perp^2)$



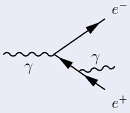


- Much faster parton proliferation, especially for small $k_\perp^2$.
- Must avoid Landau pole: $k_\perp^2 > Q_0^2 \gg \Lambda^2_{\rm QCD}$
  $\Longrightarrow Q_0^2 =$ physical parameter.

Inclusion of quantum effects

Soft logarithms : Angular ordering

- Soft limit for single emission also universal
- Problem: Soft gluons come from all over (not collinear!) Quantum interference? Still independent evolution?
- Answer: Not quite independent.
  - Assume photon into e^+e^- at θ_{ee} and photon off electron at θ
  - Energy imbalance at vertex: $k_\perp^\gamma \sim zp\theta$, hence $\Delta E \sim k_\perp^2/zp \sim zp\theta^2$.
  - Time for photon emission: $\Delta t \sim 1/\Delta E$.
  - ee-separation: $\Delta b \sim \theta_e e \Delta t > \Lambda/\theta \sim 1/(zp\theta)$
  - Thus: $\theta_{ee}/(zp\theta^2) > 1/(zp\theta) \implies \theta_{ee} > \theta$

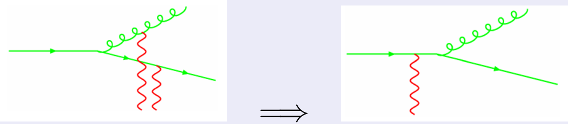


- Thus: Angular ordering takes care of soft limit.

Inclusion of quantum effects

G.Marchesini and B.R.Webber, Nucl. Phys. B **238** (1984) 1.

Soft logarithms : Angular ordering

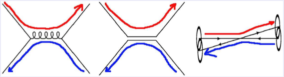


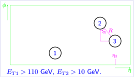
Gluons at large angle from combined color charge!

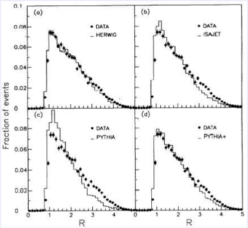
Inclusion of quantum effects

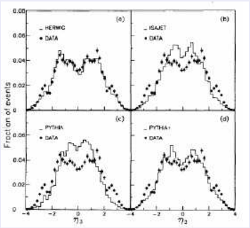
Soft logarithms : Angular ordering

Experimental manifestation:
ΔR of 2nd & 3rd jet in multi-jet events in pp-collisions









Inclusion of quantum effects

Resummed jet rates in $e^+e^- \longrightarrow$ hadrons

S.Catani *et al.* Phys. Lett. **B269** (1991) 432

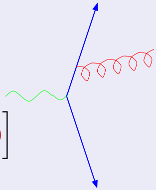
- Use Durham jet measure ($k_\perp$-type):

$$k_{\perp,ij}^2 = 2\min(E_i^2,\, E_j^2)(1-\cos\theta_{ij}) > Q_{\rm jet}^2\,.$$

- Remember prob. interpret. of Sudakov form factor:

$$\mathcal{R}_2(Q_{\rm jet}) = \left[\Delta_q(E_{\rm c.m.},\, Q_{\rm jet})\right]^2$$

$$\mathcal{R}_3(Q_{\rm jet}) = 2\Delta_q(E_{\rm c.m.},\, Q_{\rm jet}) \cdot \int {\rm d}q \left[\alpha_s(q)\bar{P}_q(E_{\rm c.m.},\, q)\frac{\Delta_q(E_{\rm c.m.},\, Q_{\rm jet})}{\Delta_q(q,\, Q_{\rm jet})}\Delta_q(q,\, Q_{\rm jet})\Delta_g(q,\, Q_{\rm jet})\right]$$

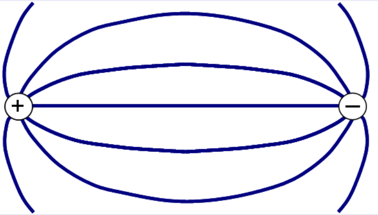


Hadronization

Confinement

- Consider dipoles in QED and QCD

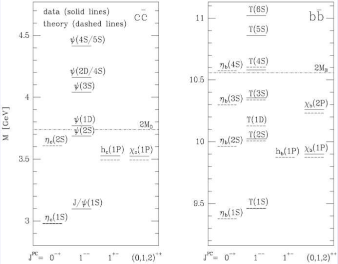
QED:

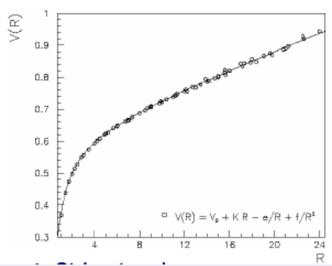


QCD:

Hadronization

Linear QCD potential in quarkonia

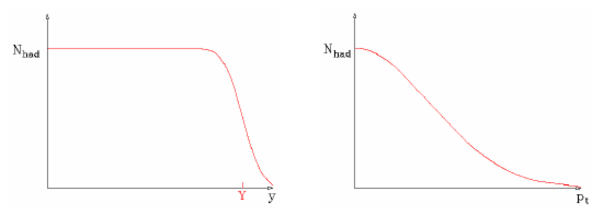




Hadronization

Some experimental facts $\longrightarrow$ naive parameterizations

- In $e^+e^- \longrightarrow$ hadrons: Limits $p_\perp$, flat plateau in y.



- Try "smearing": $\rho(p_\perp^2) \sim \exp(-p_\perp^2/\sigma^2)$

Hadronization

Effect of naive parameterizations

- Use parameterization to "guesstimate" hadronization effects:

$$\begin{aligned} E &= \int_0^Y \mathrm{d}y\mathrm{d}p_\perp^2\,\rho(p_\perp^2)p_\perp\cosh y = \lambda\sinh Y \\ P &= \int_0^Y \mathrm{d}y\mathrm{d}p_\perp^2\,\rho(p_\perp^2)p_\perp\sinh y = \lambda(\cosh Y - 1) \approx E - \lambda \\ \lambda &= \int \mathrm{d}p_\perp^2\,\rho(p_\perp^2)p_\perp = \langle p_\perp\rangle\,. \end{aligned}$$

- Estimate $\lambda \sim 1/R_{\rm had} \approx m_{\rm had}$, with $m_{\rm had}$ 0.1-1 GeV.
- Effect: Jet acquire non-perturbative mass $\sim 2\lambda E$ ($\mathcal{O}(10\rm GeV)$ for jets with energy $\mathcal{O}(100\rm GeV)$).

Hadronization

Implementation of naive parameterizations

- Feynman-Field independent fragmentation.

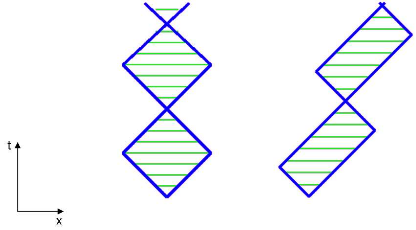
  R.D.Field and R.P.Feynman, Nucl. Phys. B **136** (1978) 1
- Recursively fragment $q \to q' +$ had, where
  - Transverse momentum from (fitted) Gaussian;
  - longitudinal momentum arbitrary (hence from measurements);
  - flavor from symmetry arguments + measurements.
- Problems: frame dependent, “last quark”, infrared safety, no direct link to perturbation theory, ...

Hadronization

Yoyo-strings as model of mesons

B.Andersson, G.Gustafson, G.Ingelman and T.Sjostrand, Phys. Rept. **97** (1983) 31.

- Light quarks connected by string: area law $m^2 \propto$area.
- L=0 mesons only have 'yo-yo' modes:

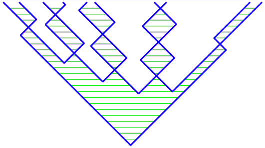


Hadronization

Dynamical strings in $e^+e^- \to q\bar{q}$

B.Andersson, G.Gustafson, G.Ingelman and T.Sjostrand, Phys. Rept. **97** (1983) 31.

- Ignoring gluon radiation: Point-like source of string.
- Intense chromomagnetic field within string: More $q\bar{q}$ pairs created by tunnelling.
- Analogy with QED (Schwinger mechanism): $\mathrm{d}\mathcal{P} \sim \mathrm{d}x\mathrm{d}t \exp\left(-\pi m_q^2/\kappa\right)$, $\kappa =$ "string tension".

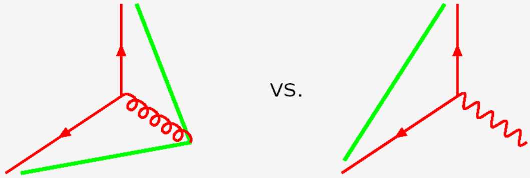


Hadronization

Gluons in strings = kinks

B.Andersson, G.Gustafson, G.Ingelman and T.Sjostrand, Phys. Rept. **97** (1983) 31.

- String model = well motivated model, constraints on fragmentation
  (Lorentz-invariance, left-right symmetry, ...)
- Gluon = kinks on string? Check by "string-effect"

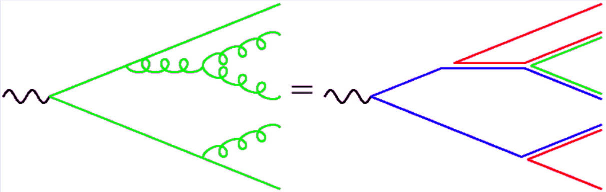


- Infrared-safe, advantage: smooth matching with PS.

Hadronization

Preconfinement

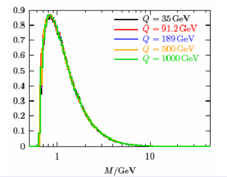
- Underlying: Large N_c-limit (planar graphs).
- Follows evolution of color in parton showers: at the end of shower color singlets close in phase space.
- Mass of singlets: peaked at low scales $\approx Q_0^2$.



Hadronization

Primordial cluster mass distribution

- Starting point: Preconfinement;
- split gluons into $q\bar{q}$-pairs;
- adjacent pairs color connected, form colorless (white) clusters.
- Clusters ("≈ excited hadrons) decay into hadrons



Hadronization

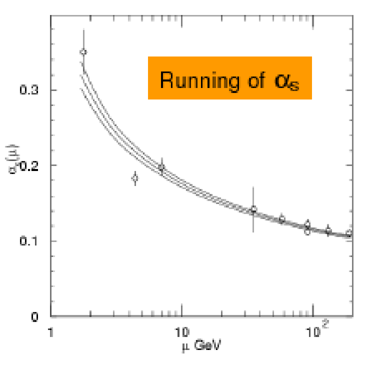
Cluster model

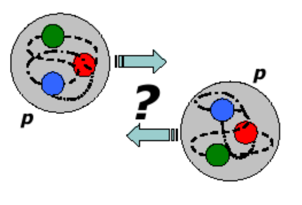
B.R.Webber, Nucl. Phys. B **238** (1984) 492.

- Split gluons into $q\bar{q}$ pairs, form singlet clusters:
  $\Longrightarrow$ continuum of meson resonances.
- Decay heavy clusters into lighter ones;
  (here, many improvements to ensure leading hadron spectrum hard enough, overall effect: cluster model becomes more string-like);
- if light enough, clusters $\to$ hadrons.
- Naively: spin information washed out, decay determined through phase space only $\to$ heavy hadrons suppressed (baryon/strangeness suppression).

Underlying Event

Multiple parton scattering?





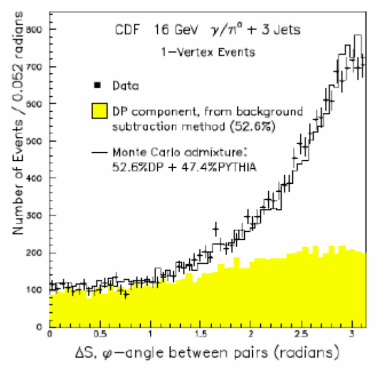
- Hadrons = extended objects!
- No guarantee for one scattering only.
- Running of α_S
  $\Longrightarrow$ preference for soft scattering.

Underlying Event

Evidence for multiple parton scattering

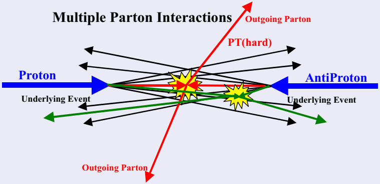
- Events with $\gamma + 3$ jets:
  - Cone jets, $R = 0.7$, $E_T > 5$ GeV; $|\eta_j| < 1.3$;
  - "clean sample": two softest jets with $E_T < 7$ GeV;
- $\sigma_{\rm DPS} = \frac{\sigma_{\gamma j}\sigma_{jj}}{\sigma_{\rm eff}}$, $\sigma_{\rm eff} \approx 14 \pm 4$ mb.

CDF collaboration, Phys. Rev. **D56** (1997) 3811.



Underlying Event

Definition(s)



1. Everything apart from the hard interaction including IS showers, FS showers, remnant hadronization.
2. Remnant-remnant interactions, soft and/or hard.

$\Longrightarrow$ Lesson: hard to define

Underlying event

Model: Multiple parton interactions

- To understand the origin of MPS, realize that

$$\sigma_{\rm hard}(p_{\perp,\rm min}) = \int\limits_{p^2_{\perp,\rm min}}^{s/4} {\rm d}p_\perp^2 \frac{{\rm d}\sigma(p_\perp^2)}{{\rm d}p_\perp^2} > \sigma_{pp,\rm total}$$

for low $p_{\perp,\rm min}$. Here: $\frac{{\rm d}\sigma(p_\perp^2)}{{\rm d}p_\perp^2} = \int\limits_0^1 {\rm d}x_1{\rm d}x_2{\rm d}\hat{t} f(x_1,\,q^2)f(x_2,\,q^2)\frac{{\rm d}\hat{\sigma}_{2\to 2}}{{\rm d}p_\perp^2}\delta\left(1-\frac{\hat{t}\hat{u}}{\hat{s}}\right)$

($f(x,q^2)$ =PDF, $\hat{\sigma}_{2\to 2}$ =parton-parton x-sec)

- $\langle\sigma_{\rm hard}(p_{\perp,\rm min})/\sigma_{pp,\rm total}\rangle \geq 1$
- Depends strongly on cut-off $p_{\perp,\rm min}$ (Energy-dependent)!

Underlying event

Old Pythia model: Algorithm, simplified

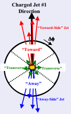
T.Sjostrand and M.van Zijl, Phys. Rev. D **36** (1987) 2019.

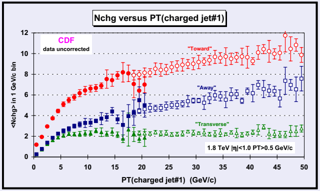
- Start with hard interaction, at scale $Q^2_{\rm hard}$.
- Select a new scale $p^2_\perp$
  (according to $f = \frac{{\rm d}\sigma_{2\to 2}(p^2_\perp)}{{\rm d}p^2_\perp}$ with $p^2_\perp \in [p^2_{\perp,\rm min},\, Q^2]$)
- Rescale proton momentum ("proton-parton = proton with reduced energy").
- Repeat until below $p^2_{\perp,\rm min}$.
- May add impact-parameter dependence, showers, etc..
- Treat intrinsic $k_\perp$ of partons ($\to$ parameter)
- Model proton remnants ($\to$ parameter)

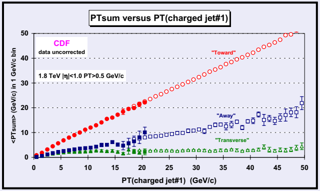
Underlying Event

In the following: Data from CDF, PRD 65 (2002) 092002, plots partially from C.Buttar

Observables

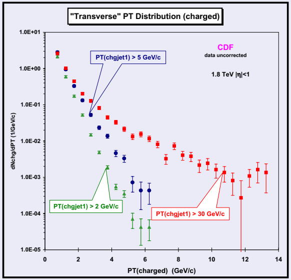


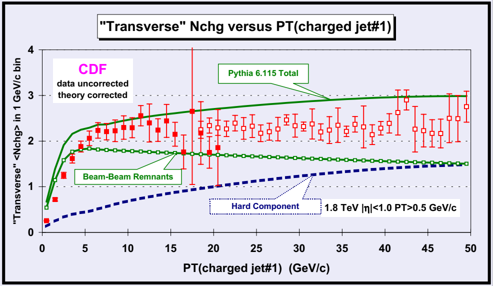




Underlying event

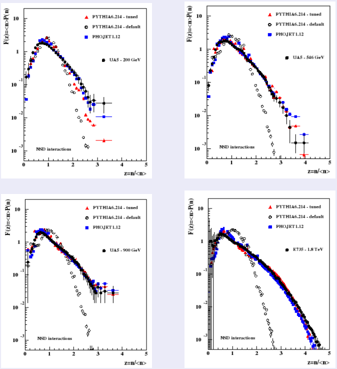
Hard component in transverse region

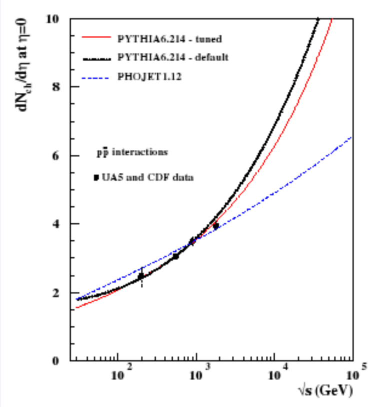




Underlying event

Energy extrapolation





Underlying event

General facts on current models

- No first-principles approach for underlying event:

  Multiple-parton interactions: beyond factorization

  Factorization (simplified) = no process-dependence in use of PDFs.

- Models usually based on xsecs in collinear factorization: $\mathrm{d}\sigma/\mathrm{d}p_\perp \propto p_\perp^{4-8} \implies$ strong dependence on cut-off $p_\perp^{\min}$.
- "Regularization": $\mathrm{d}\sigma/\mathrm{d}p_\perp \propto (p_\perp^2 + p_0^2)^{2-4}$, also in α_S.
- Model for scaling behavior of $p_\perp^{\min}(s) \propto p_\perp^{\min}(s_0)(s/s_0)^\lambda$, $\lambda =$?

  Two Pythia tunes: $\lambda = 0.16$, $\lambda = 0.25$.

- Herwig model similar to old Pythia and SHERPA
- New Pythia model: Correlate parton interactions with showers, more parameters.

Summary so far

1. Hard MEs:
   - Theoretically very well understood, realm of perturbation theory.
   - Fully automated tools at tree-level available, $2 \to 6$ no problem at all.
   - Obstacle(s) for higher multiplicities: factorial growth, phase space integration.
   - NLO calculations much more involved, no fully automated tool, only libraries for specific processes (MCFM, NLOJET++), typically up to $2 \to 3$.
   - NNLO only for a small number of processes.

Summary so far

1. Parton showers:
   - Theoretically well understood, still in realm of perturbation theory, but beyond fixed order.
   - Consistent treatment of leading logs in soft/collinear limit, formally equivalent formulations lead to different results because of non-trivial choices (evolution parameter, etc.).

Summary so far

1. Hadronization
    - Various phenomenological models;
    - different levels of sophistication,
      different number of parameters;
    - tuned to LEP data, overall agreement satisfying;
    - validity for hadron data not quite clear - differences possible (beam remnant fragmentation not in LEP).

Summary so far

1. Underlying event
   - Various definitions for this phenomenon.
   - Theoretically not understood, in fact: beyond theory understanding (breaks factorization);
   - models typically based on collinear factorization and semi-independent multi-parton scattering
     $\Longrightarrow$ very naive;
   - models highly parameter-dependent, leading to large differences in predictions;
   - connection to minimum bias, diffraction etc.?
   - even unclear: good observables to distinguish models.