Monte Carlo Tools

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Topics of the lectures

1. Lecture 1: *Tour through Event Generators*
   - Hard physics simulation: Parton Level event generation
   - Dressing the partons: Parton Showers
   - Soft physics simulation: Hadronization
   - Beyond factorization: Underlying Event
2. Lecture 2: *Higher Orders in Monte Carlos*
   - Some nomenclature: Anatomy of HO calculations
   - Merging vs. Matching

Thanks to

- the other Sherpas: T.Gleisberg, S.Höche, S.Schumann, F.Siegert, M.Schönherr, J.Winter;
- other MC authors: S.Gieseke, K.Hamilton, L.Lonnblad, F.Maltoni, M.Mangano, P.Richardson, M.Seymour, T.Sjostrand, B.Webber, ...

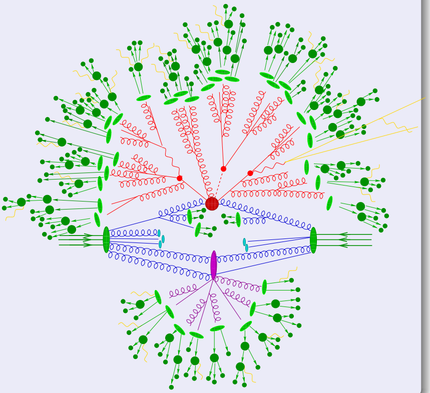
Simulation's paradigm

Basic strategy

Divide event into stages, separated by different scales.

- Signal/background:
  Exact matrix elements.
- QCD-Bremsstrahlung:
  Parton showers (also in initial state).
- Multiple interactions:
  Beyond factorization: Modeling.
- Hadronization:
  Non-perturbative QCD: Modeling.

Sketch of an event

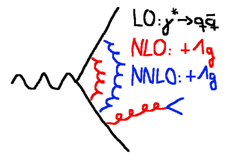


Today's lecture: Higher Orders in Monte Carlos

- Which higher orders? Some anatomy
- First attempts: ME corrections
- Higher orders in rate: MC@NLO
- Higher orders through extra emission: Merging
- A new shower formulation

Nomenclature

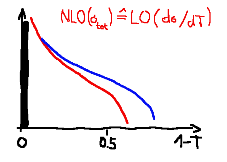
Specifying higher-order corrections: $\gamma^* \longrightarrow$ hadrons



- In general: N^{n}LO $\leftrightarrow$ $\mathcal{O}(\alpha_s^n)$
- But: only for inclusive quantities

(e.g.: total xsecs like γ^* $\longrightarrow$hadrons).

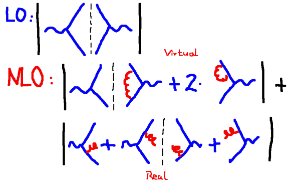
Counter-example: thrust distribution



- In general, distributions are HO.
- Distinguish real & virtual emissions:
  Real emissions $\longrightarrow$ mainly distributions,
  virtual emissions $\longrightarrow$ mainly normalization.

Nomenclature

Anatomy of HO calculations: Virtual and real corrections



NLO corrections: $\mathcal{O}(\alpha_s)$

Virtual corrections	=	extra loops
Real corrections	=	extra legs

- UV-divergences in virtual graphs → renormalization
- But also: IR-divergences in real & virtual contributions
  Must cancel each other, non-trivial to see:
  N vs. $N+1$ particle FS, divergence in PS vs. loop

Nomenclature

Cancelling the IR divergences: Subtraction method

- Total NLO xsec:
  $\sigma_{\rm NLO} = \sigma_{\rm Born} + \int {\rm d}^D k |\mathcal{M}|^2_V + \int {\rm d}^4 k |\mathcal{M}|^2_R$
- IR div. in real piece $\longrightarrow$ regularize:
  $\int {\rm d}^4 k |\mathcal{M}|^2_R \longrightarrow \int {\rm d}^D k |\mathcal{M}|^2_R$
- Construct subtraction term with same IR structure:
  $\int {\rm d}^D k \left(|\mathcal{M}|^2_R - |\mathcal{M}|^2_S\right) = \int {\rm d}^4 k |\mathcal{M}|^2_{RS} =$ finite.
  Possible: $\int {\rm d}^D k |\mathcal{M}|^2_S = \sigma_{\rm Born} \int {\rm d}^D k |\tilde{\mathcal{S}}|^2$, universal $|\tilde{\mathcal{S}}|^2$.
- $\int {\rm d}^D k |\mathcal{M}|^2_V + \sigma_{\rm Born} \int {\rm d}^D k |\tilde{\mathcal{S}}|^2 =$ finite (analytical)

Nomenclature

State-of-the-art NLO calculations: General strategy

- Construct Born + 1st order terms
- Subtraction term: Born term $\times$ (analytical) divergences

  Evaluate loop term analytically - perform cancellation
- Monte Carlo separately over subtracted real emission and virtual+subtraction term

Limitations

- So far only loops with ≤ 5 propagators under full control

  $\Longrightarrow$ in general, only $2 \to 3$ processes at NLO
- Soft/collinear corners maybe still badly described

Nomenclature

Resummation: Basic idea

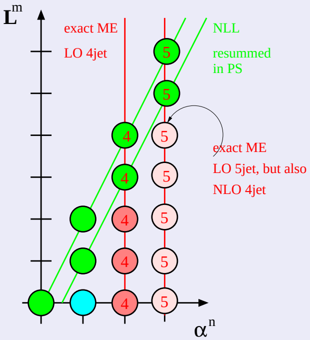
- Observation: Universal soft & collinear divergences @ all orders
  Cutting them produces universal logarithms.
- Universality $\Longrightarrow$ resummation of leading logs @ all orders possible.
  Improves behavior in soft/collinear regions of phase space.
  Example: Thrust distribution.
- Nomenclature: LL, NLL, NNLL, ...
  Limitation due to mixing with finite pieces @ some N^{n}LL.
- Leading logs also in parton shower (=resummation!!)

Orders in ME and PS

ME vs. PS

- Matrix elements good for: hard, large-angle emissions; take care of interferences.
- Parton shower good for: soft, collinear emissions; resums large logarithms.
- Want to combine both! Avoid double-counting.

α_s vs. Log

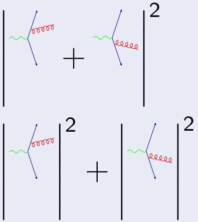


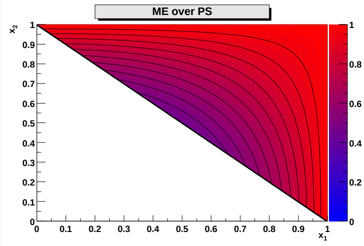
Correcting the parton shower

Example: $e^+e^- \to q\bar{q}g$

ME :

PS :





Correcting the parton shower

Practicalities of ME-corrections

- Obviously, $ME < PS$ is not always fulfilled.
- Could enhance PS expression by a (large) factor. Question: Efficiency of the approach?
- Therefore: realized in few processes only: Best-known: $ee \longrightarrow q\bar{q}$, $q\bar{q} \longrightarrow V$, $t \longrightarrow bW$

MC@NLO

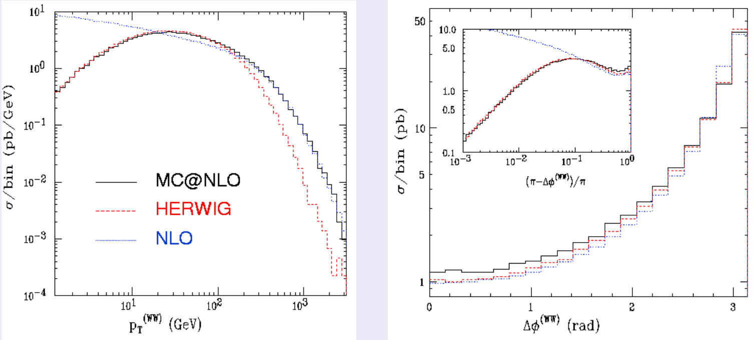
S.Frixione, B.R.Webber, JHEP **0206** (2002) 029
S.Frixione, P.Nason, B.R.Webber, JHEP **0308** (2003) 007

Basic principles

- Want:
  - NLO-Normalization and first (hard) emission correct,
  - Soft emissions correctly resummed in PS.
- Method:
  - Modify subtraction terms for real infrared divergences,
  - use first order parton shower-expression,
  - this is process-dependent!
- In practise much more complicated.
- Implemented for DY, W-pairs, $gg \to H$, Q-pairs.

MC@NLO

Example results: W-pairs @ Tevatron



PowHEG

P. Nason, JHEP **0411** (2004) 040
S. Frixione, P. Nason and C. Oleari, arXiv:0709.2092 [hep-ph]

Basic idea:

- For $k_\perp$-ordered showers, generate hardest emission through

$$\mathrm{d}\sigma = \bar{\mathcal{B}}(\phi_N)\mathrm{d}\phi_N \left[\Delta(\phi_N,\, Q_0) + \Delta(\phi_N,\, k_\perp^{(N+1)})\frac{\mathcal{R}(\phi_{N+1})}{\mathcal{B}(\phi_N)}\mathrm{d}\phi_1\right]$$

with modified Sudakov form factor

$$\Delta(\phi_N,\, p_\perp) = \exp\left\{-\int \mathrm{d}\phi_1 \frac{\mathcal{R}(\phi_{N+1})}{\mathcal{B}(\phi_N)}\vartheta(k_\perp^{(N+1)} - p_\perp)\right\}\,,$$

where $k_\perp$ is the transverse momentum w.r.t. its emitter.

PowHEG, cont'd

P. Nason, JHEP **0411** (2004) 040
S. Frixione, P. Nason and C. Oleari, arXiv:0709.2092 [hep-ph]

Basic idea:

- Get norm correct through

$$\bar{\mathcal{B}}(\phi_N) = \mathcal{B}(\phi_N) + \mathcal{V}_{\rm fin}(\phi_N) + \int \mathrm{d}\phi_1 \left[\mathcal{R}(\phi_{N+1}) - \mathcal{C}(\phi_{N+1}) + \ldots\right]$$

- Advantage: Shower-independent
  caveat: $k_\perp$- vs. angular ordered shower
- Advantage: No extra manipulation of NLO needed.
- Operational mode: Produce parton-level events with 1st emission, hand-over to your preferred shower etc..

Combining MEs & PS

S.Catani, F.K., R.Kuhn and B.R.Webber, JHEP **0111** (2001) 063
F.K., JHEP **0208** (2002) 015

Basic principles

- Want:
  - All jet emissions correct at tree level + LL,
  - Soft emissions correctly resummed in PS
- Method:
  - Separate Jet-production/evolution by $Q_{\rm jet}$ ($k_\perp$ algorithm).
  - Produce jets according to LO matrix elements
  - re-weight with Sudakov form factor + running α_s weights,
  - veto jet production in parton shower.
- Process-independent implementation.

Combining MEs & PS

n-jet rates @ NLL

S.Catani *et al.* Phys. Lett. **B269** (1991) 432

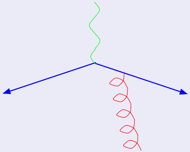
At NLL-Accuracy

$$\mathcal{R}_2(Q_{\rm jet}) = \left[\Delta_q(E_{\rm c.m.}, Q_{\rm jet})\right]^2$$

$$\mathcal{R}_3(Q_{\rm jet}) = 2\Delta_q(E_{\rm c.m.}, Q_{\rm jet}) \cdot \int {\rm d}q \left[\alpha_s(q)\Gamma_q(E_{\rm c.m.}, q)\frac{\Delta_q(E_{\rm c.m.}, Q_{\rm jet})}{\Delta_q(q, Q_{\rm jet})} \Delta_q(q, Q_{\rm jet})\Delta_g(q, Q_{\rm jet})\right]$$

Sudakov weights

Example: $\gamma^* \to q\bar{q}g$



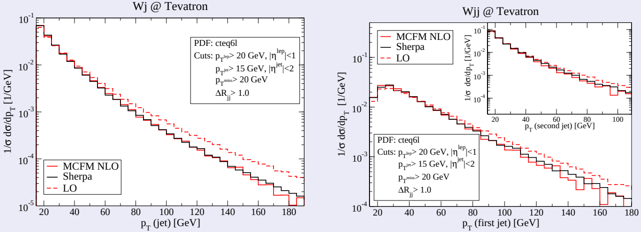
$$\mathcal{W}_{\rm Sud} = \frac{\alpha_s(q)}{\alpha_s(Q_{\rm jet})} \cdot \Delta_q(E_{\rm c.m.}, Q_{\rm jet}) \frac{\Delta_q(E_{\rm c.m.}, Q_{\rm jet})}{\Delta_q(q, Q_{\rm jet})}\Delta_q(q, Q_{\rm jet})\Delta_g(q, Q_{\rm jet})$$

Combining MEs & PS

Algorithm as scale-setting prescription

- Example: $p_\perp$ distribution of jets @ Tevatron
- Consider exclusive $W + 1$- and $W + 2$-jet production

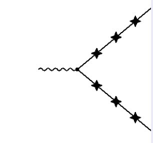
Comparison with MCFM; J.Campbell and R.K.Ellis, Phys. Rev. D **65** (2002) 113007
in : F.K., A.Schälicke, S.Schumann and G.Soff, Phys. Rev. D **70** (2004) 114009



Sherpa = tree-level matrix elements with α_s scales and Sudakov form factors.

Combining MEs & PS

Vetoing the shower



$$\mathcal{W}_{\rm Veto} = \left\{1+\int_{Q_{\rm jet}}^{E_{\rm c.m.}} dq\,\Gamma_q(E_{\rm c.m.},q) + \int_{Q_{\rm jet}}^{E_{\rm c.m.}} dq\,\Gamma_q(E_{\rm c.m.},q)\int_{Q_{\rm jet}}^{q} dq'\,\Gamma_q(E_{\rm c.m.}q') + \dots\right\}^2$$

$$= \left\{\exp\left(\int_{Q_{\rm jet}}^{E_{\rm c.m.}} dq\,\Gamma_q(E_{\rm c.m.},q)\right)\right\}^2 = \Delta_q^{-2}(E_{\rm c.m.},Q_{\rm jet})$$

$\Longrightarrow$ Cancels dependence on $Q_{\rm jet}$.

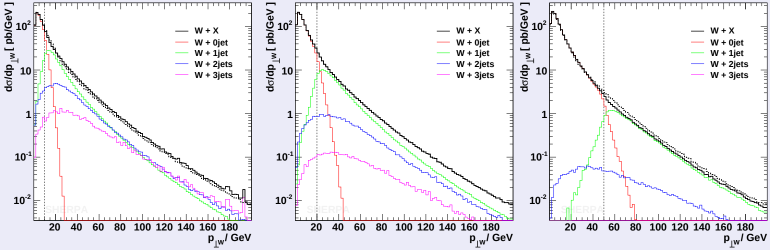
Combining MEs & PS

Independence on $Q_{\rm jet}$

Example: $p_\perp$ of W in $p\bar{p} \to W + X$ @ Tevatron

in F.K., A.Schälicke, S.Schumann and G.Soff, Phys. Rev. D **70** (2004) 114009

$Q_{\rm jet} = 10\,{\rm GeV}$ $Q_{\rm jet} = 30\,{\rm GeV}$ $Q_{\rm jet} = 50\,{\rm GeV}$

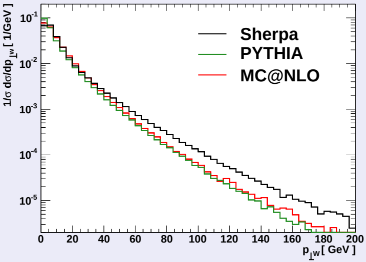


Combining MEs & PS

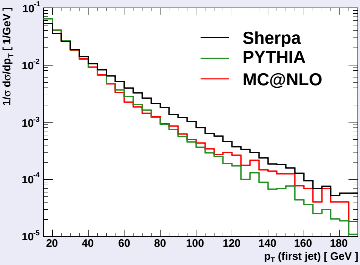
Comparison with other codes

$p_\perp$ of W-bosons & jets in $p\bar{p} \to W + X$ @ Tevatron

$p_\perp^W$



$p_\perp^{\mathrm{1st\,jet}}$

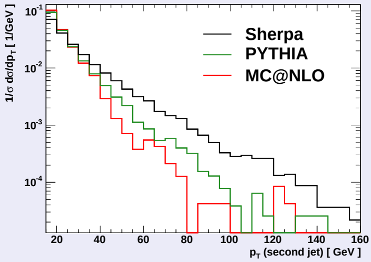


Combining MEs & PS

Comparison with other codes

$p_\perp$ of W-bosons & jets in $p\bar{p} \to W + X$ @ Tevatron

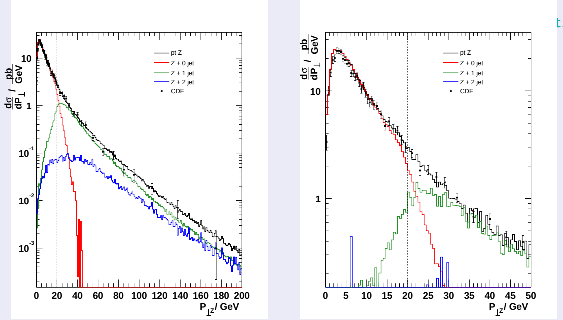
$p_\perp^{\text{2nd jet}}$ $p_\perp^{\text{3rd jet}}$



Combining MEs & PS

Comparison with data from Tevatron

$p_\perp$ of Z-bosons in $p\bar{p} \to Z + X$

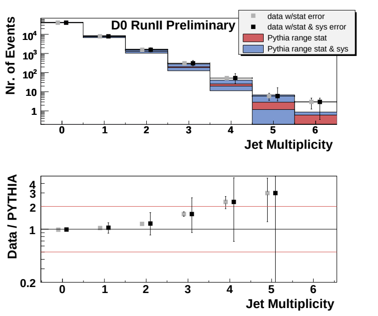


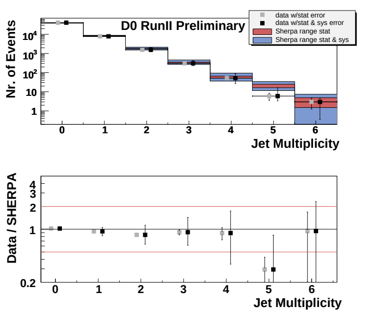
t. 84 (2000) 845

Combining MEs & PS

Comparison with data from Tevatron

Jet rates in $p\bar{p} \to Z + X$ (D0-Note 5066)

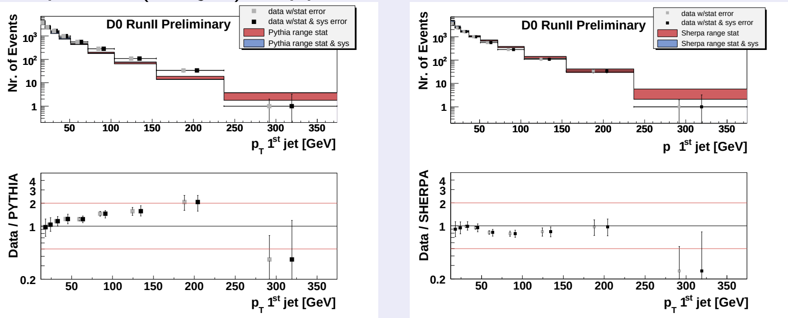




Combining MEs & PS

Comparison with data from Tevatron

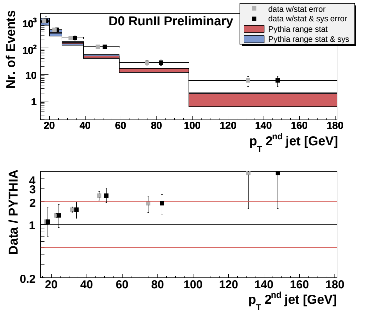
Jet spectra (1st jet) in $p\bar{p} \to Z + X$ (D0-Note 5066)

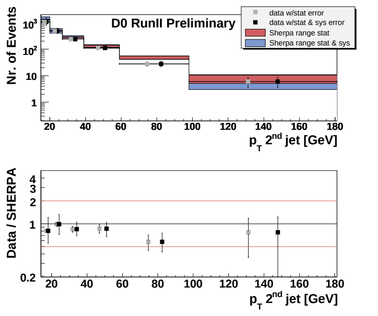


Combining MEs & PS

Comparison with data from Tevatron

Jet spectra (2nd jet) in $p\bar{p} \to Z + X$ (D0-Note 5066)

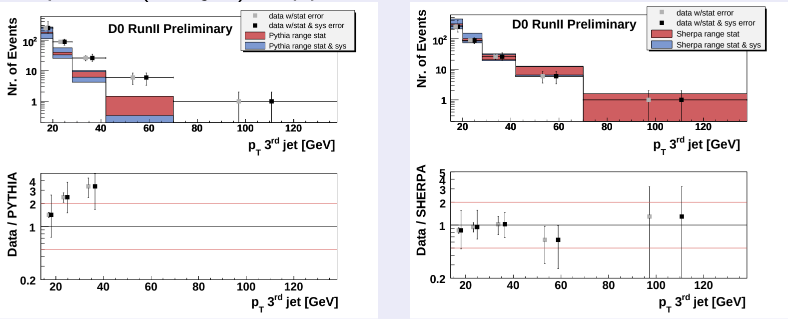




Combining MEs & PS

Comparison with data from Tevatron

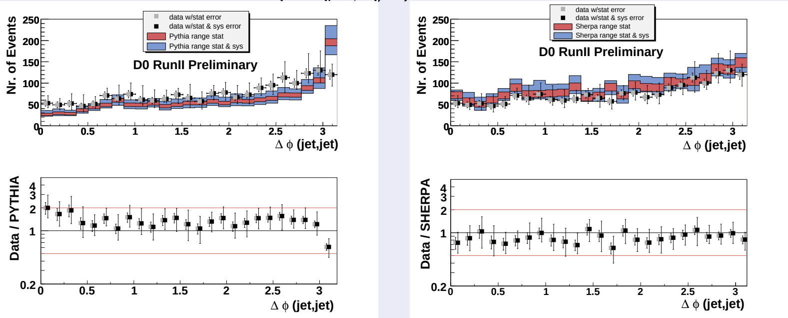
Jet spectra (3rd jet) in $p\bar{p} \to Z + X$ (D0-Note 5066)



Combining MEs & PS

Comparison with data from Tevatron

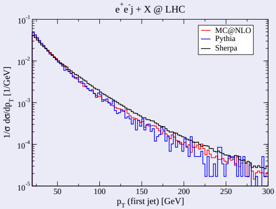
Azimuthal correlation ($\angle_{1.\text{jet},2.\text{jet}}$) in $p\bar{p} \to Z + X$ (D0-Note 5066)

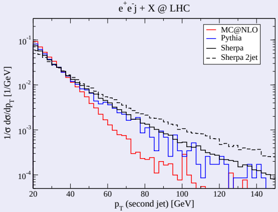


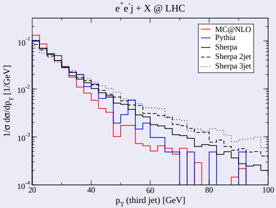
Combining MEs & PS

Extrapolation to LHC: Jets

- $p_\perp$ of jets in inclusive Z+jets
- Influence of more jets.
- Displayed here: x-sections.
- Difference in shape & x-sec.



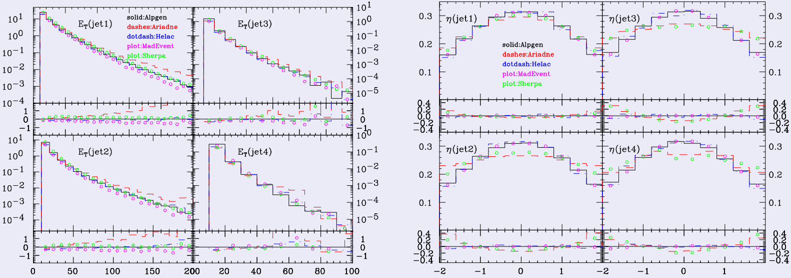




Combining MEs & PS

Comparison with other merging algorithms: MLM

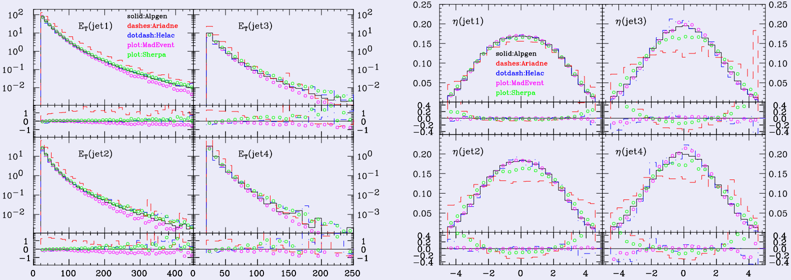
$p_\perp$ of jets in inclusive W+jets at Tevatron



Combining MEs & PS

Comparison with other merging algorithms: MLM

$p_\perp$ of jets in inclusive W+jets at LHC



Further developments of parton showers

Shower based on Catani-Seymour splitting kernels

First discussed in: Z.Nagy and D.E.Soper, JHEP **0510** (2005) 024.

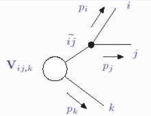
- Catani-Seymour dipole subtraction terms as universal framework for QCD NLO calculations.
- Factorization formulae for real emission process:
- Full phase space coverage & good approx. to ME.
- Currently implemented into SHERPA by S.Schumann.

Example: final-state final-state dipoles

splitting: $\tilde{p}_{ij} + \tilde{p}_k \longrightarrow p_i + p_j + p_k$

variables: $y_{ij,k} = \frac{p_i p_j}{p_i p_j + p_i p_k + p_j p_k}$, $z_i = \frac{p_i p_k}{p_i p_k + p_j p_k}$

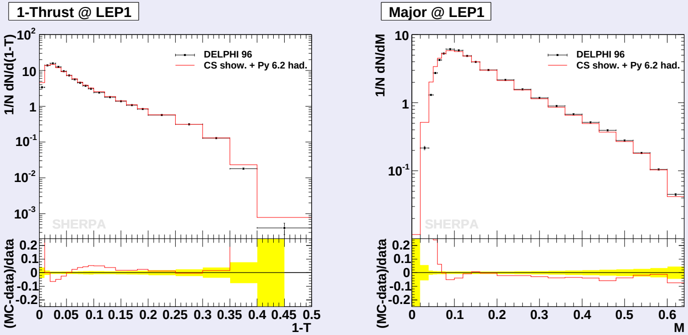
consider $\mathrm{q}_{ij} \to \mathrm{q}_i \mathrm{g}_j$: $\langle V_{\mathrm{q}_i \mathrm{g}_j,k}(\tilde{z}_i, y_{ij,k})\rangle = C_F \left\{\frac{2}{1-\tilde{z}_i+\tilde{z}_i y_{ij,k}} - (1+\tilde{z}_i)\right\}$



Further developments of parton showers

Shower based on Catani-Seymour splitting kernels

Results for $e^+e^- \longrightarrow$ hadrons



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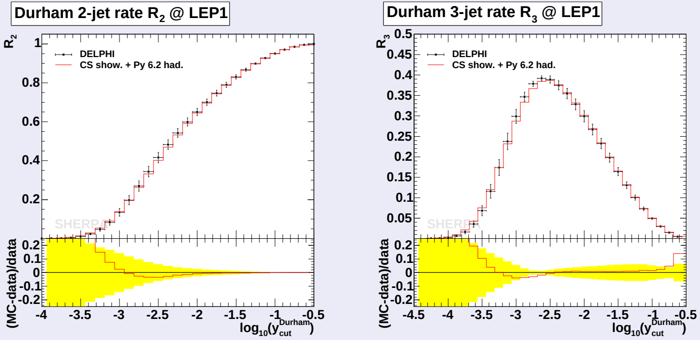
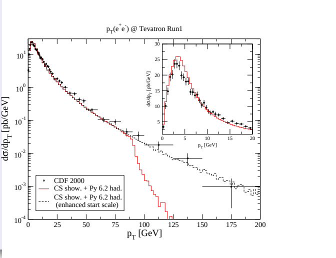


Further developments of parton showers

Shower based on Catani-Seymour splitting kernels

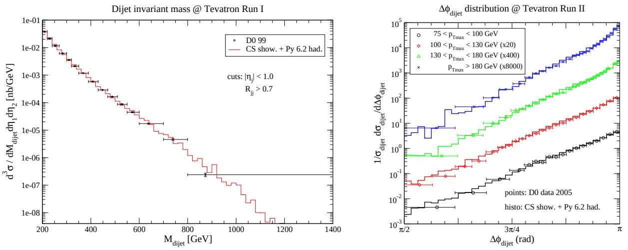
Results for $p\bar{p} \longrightarrow \ell^+\ell^-$



Further developments of parton showers

Shower based on Catani-Seymour splitting kernels

Results for $p\bar{p} \longrightarrow$ jets



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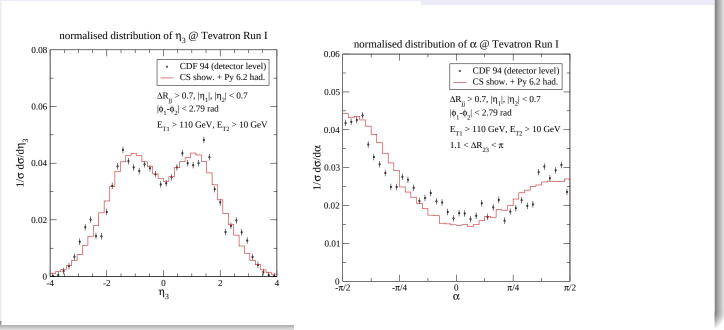
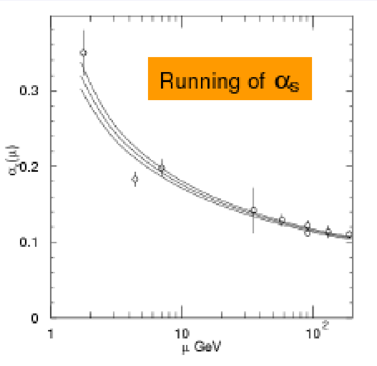
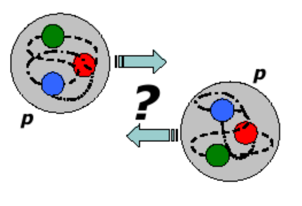


Underlying Event

Multiple parton scattering?





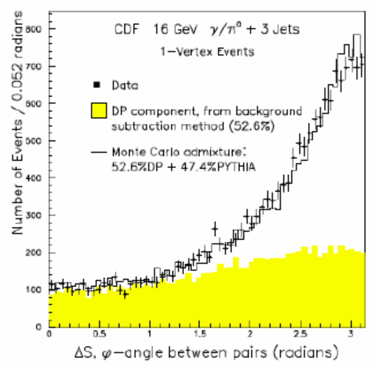
- Hadrons = extended objects!
- No guarantee for one scattering only.
- Running of α_S
  $\Longrightarrow$ preference for soft scattering.

Underlying Event

Evidence for multiple parton scattering

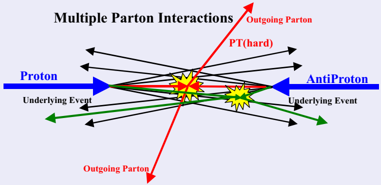
- Events with $\gamma + 3$ jets:
  - Cone jets, $R = 0.7$, $E_T > 5$ GeV; $|\eta_j| < 1.3$;
  - "clean sample": two softest jets with $E_T < 7$ GeV;
- $\sigma_{\rm DPS} = \frac{\sigma_{\gamma j}\sigma_{jj}}{\sigma_{\rm eff}}$, $\sigma_{\rm eff} \approx 14 \pm 4$ mb.

CDF collaboration, Phys. Rev. **D56** (1997) 3811.



Underlying Event

Definition(s)



1. Everything apart from the hard interaction including IS showers, FS showers, remnant hadronization.
2. Remnant-remnant interactions, soft and/or hard.

$\Longrightarrow$ Lesson: hard to define

Underlying event

Model: Multiple parton interactions

- To understand the origin of MPS, realize that

$$\sigma_{\rm hard}(p_{\perp,\rm min}) = \int\limits_{p^2_{\perp,\rm min}}^{s/4} {\rm d}p_\perp^2 \frac{{\rm d}\sigma(p_\perp^2)}{{\rm d}p_\perp^2} > \sigma_{pp,\rm total}$$

for low $p_{\perp,\rm min}$. Here: $\frac{{\rm d}\sigma(p_\perp^2)}{{\rm d}p_\perp^2} = \int\limits_0^1 {\rm d}x_1{\rm d}x_2{\rm d}\hat{t} f(x_1,\, q^2) f(x_2,\, q^2) \frac{{\rm d}\hat{\sigma}_{2\to 2}}{{\rm d}p_\perp^2} \delta\left(1 - \frac{\hat{t}\hat{u}}{\hat{s}}\right)$

($f(x, q^2)$ =PDF, $\hat{\sigma}_{2\to 2}$ =parton-parton x-sec)

- $\langle \sigma_{\rm hard}(p_{\perp,\rm min})/\sigma_{pp,\rm total}\rangle \geq 1$
- Depends strongly on cut-off $p_{\perp,\rm min}$ (Energy-dependent)!

Underlying event

Old Pythia model: Algorithm, simplified

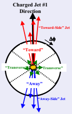
T.Sjostrand and M.van Zijl, Phys. Rev. D **36** (1987) 2019.

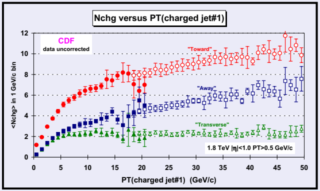
- Start with hard interaction, at scale $Q^2_{\rm hard}$.
- Select a new scale $p_\perp^2$
  (according to $f = \frac{d\sigma_{2\to 2}(p_\perp^2)}{dp_\perp^2}$ with $p_\perp^2 \in [p^2_{\perp,\rm min},\, Q^2]$)
- Rescale proton momentum ("proton-parton = proton with reduced energy").
- Repeat until below $p^2_{\perp,\rm min}$.
- May add impact-parameter dependence, showers, etc..
- Treat intrinsic $k_\perp$ of partons ($\rightarrow$ parameter)
- Model proton remnants ($\rightarrow$ parameter)

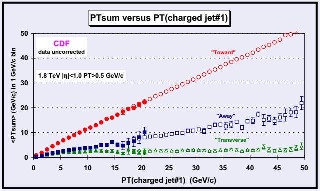
Underlying Event

In the following: Data from CDF, PRD 65 (2002) 092002, plots partially from C.Buttar

Observables

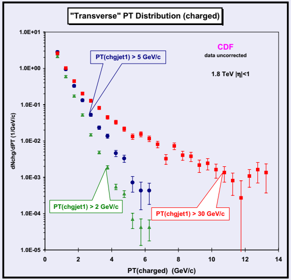


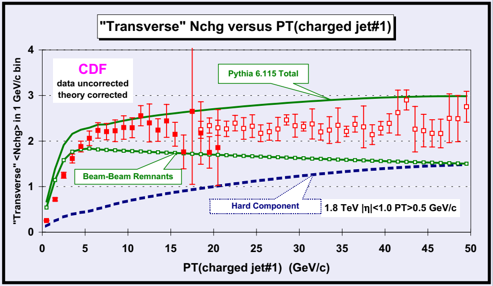




Underlying event

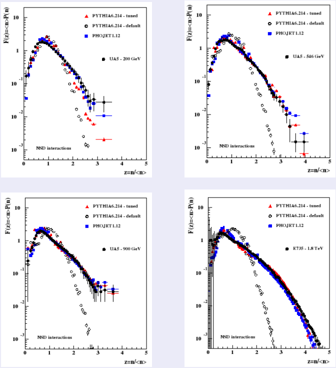
Hard component in transverse region

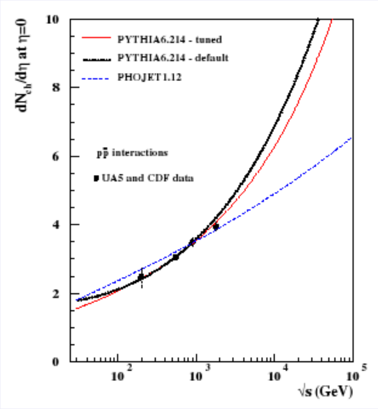




Underlying event

Energy extrapolation





Underlying event

General facts on current models

- No first-principles approach for underlying event:

  Multiple-parton interactions: beyond factorization

  Factorization (simplified) = no process-dependence in use of PDFs.

- Models usually based on xsecs in collinear factorization: $\mathrm{d}\sigma/\mathrm{d}p_\perp \propto p_\perp^{4-8} \implies$ strong dependence on cut-off $p_\perp^{\min}$.
- "Regularization": $\mathrm{d}\sigma/\mathrm{d}p_\perp \propto (p_\perp^2 + p_0^2)^{2-4}$, also in α_S.
- Model for scaling behavior of $p_\perp^{\min}(s) \propto p_\perp^{\min}(s_0)(s/s_0)^\lambda$, $\lambda =$?

  Two Pythia tunes: $\lambda = 0.16$, $\lambda = 0.25$.

- Herwig model similar to old Pythia and SHERPA
- New Pythia model: Correlate parton interactions with showers, more parameters.

Summary & outlook

Summary: QCD & simulation tools

- Many interesting signals at LHC "spoiled" by QCD.
- Need to understand & describe QCD to high precision.
- Time to improve & validate essential tools is now!
- New methods of merging of ME& PS extremely powerful.
- Different, complementary aspects w.r.t. MC@NLO.
- Important: educated choice which tool to use!
- Important: know your Monte Carlo!
- Important: know the assumptions!