Preheating

Juan García-Bellido
Física Teórica UAM
6th September 2006

Galileo Galilei Institute
Florence, 2006

Outline

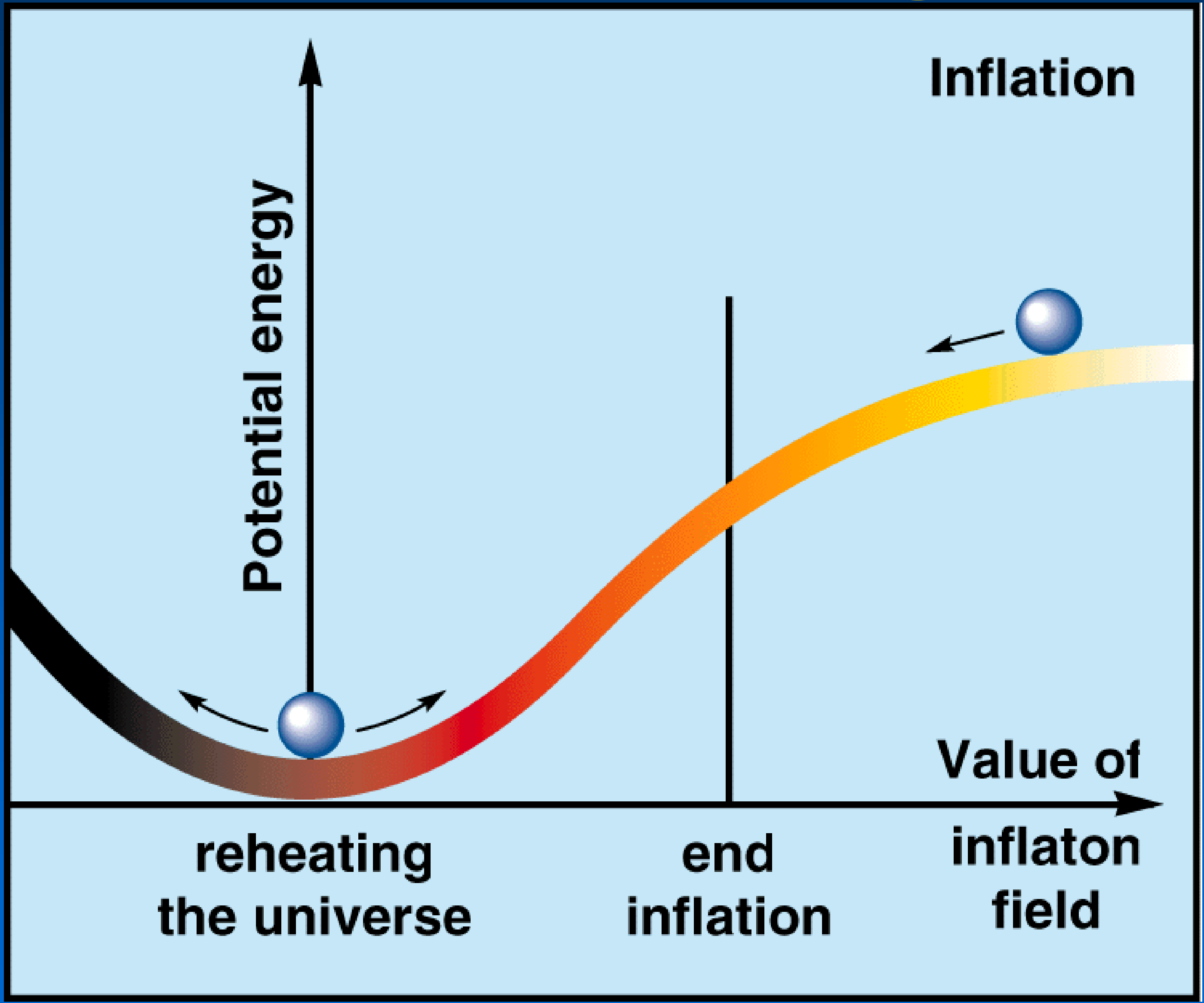
Reheating: Standard perturbative decay

- Oscillating inflaton field
- Perturbative decay rates
- Reheating temperature

Preheating: Very rich phenomenology

- Parametric resonance and tachyonic inst.
- Production massive part. + top. defects
- EW baryogenesis & leptogenesis
- Stochastic background gravitational waves
- Primordial magnetic fields

Reheating



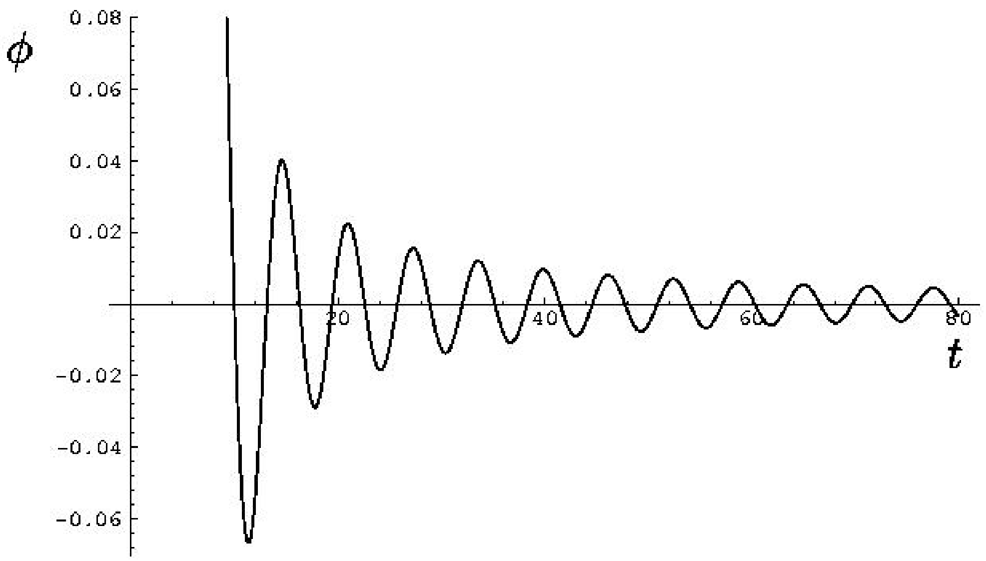
Inflation
Potential energy
Value of
inflaton
field
reheating
the universe
end
inflation

Inflaton oscillating at end of inflation

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0 \quad \Rightarrow \quad \phi(t) = \Phi(t)\sin mt$$

$$\langle \rho \rangle = \frac{1}{2}m^2\Phi^2(t)\left(\langle \cos^2 mt \rangle + \langle \sin^2 mt \rangle\right) = \frac{1}{2}m^2\Phi^2(t)$$

$$\langle p \rangle = \frac{1}{2}m^2\Phi^2(t)\left(\langle \cos^2 mt \rangle - \langle \sin^2 mt \rangle\right) = 0$$



like matter

$$\rho_\phi(t) \sim a^{-3}(t)$$

$$n_\phi(t) = m\Phi^2 \sim a^{-3}(t)$$

$$\Phi(t) \sim t^{-1}$$

Inflaton coupled to rest of the universe

$$L = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\xi\chi^2 R$$

$$+\overline{\psi}(i\gamma^\mu\partial_\mu + m_\psi)\psi - h\phi\overline{\psi}\psi - \frac{1}{2}g^2\chi^2\phi^2 - g^2 v\phi\chi^2$$

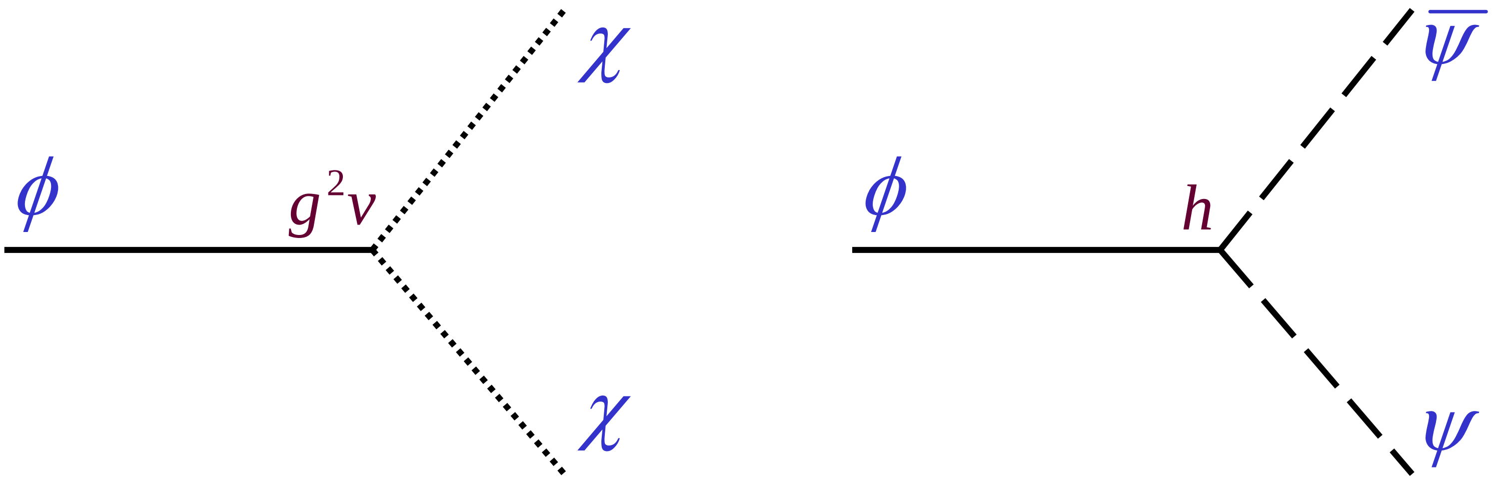
$$\ddot{\phi} + 3H\dot{\phi} + (m^2 + \Pi(w))\phi = 0$$

$$\mathbf{Im}\,\Pi(m) = m\,\Gamma_\phi$$ optical theorem

$$\ddot{\phi} + 3H(t)\dot{\phi} + \Gamma_\phi\dot{\phi} + m^2\phi = 0$$ phenom.

$$\phi(t) = \frac{\Phi_0}{t} e^{-\frac{1}{2}\Gamma_\phi t}\,\mathbf{sin}\,mt \quad \Rightarrow \quad \frac{d}{dt}\left(\rho_\phi a^3\right) = -\Gamma_\phi\,\rho_\phi a^3$$

Perturbative decay of inflaton



$$\Gamma_\phi = \sum_i \Gamma(\phi \to \chi_i \chi_i) + \sum_i \Gamma(\phi \to \overline{\psi}_i \psi_i)$$

$$\Gamma(\phi \to \chi_i \chi_i) = \frac{g_i^4 v^2}{8\pi m} \qquad \Gamma(\phi \to \overline{\psi}_i \psi_i) = \frac{h_i^2 m}{8\pi}$$

$$\Gamma_\phi \equiv \frac{h_{\text{eff}}^2 m}{8\pi} << m, \quad h_{\text{eff}}^2 = \sum \left(h_i^2 + \frac{g_i^4 v^2}{m^2} \right) << 10^{-6}$$

Perturbative reheating

$$\Gamma_\phi \ll \frac{2}{t} = 3H \ll m \quad \text{initially}$$

inflaton lifetime $\tau_\phi = \Gamma_\phi^{-1} \ll t_U = H^{-1}$ age universe

$$H = \Gamma_\phi \;\Rightarrow\; \rho(t_{reh}) = \frac{3\Gamma_\phi^2 M_P^2}{8\pi} \equiv \frac{\pi^2}{30} g(T_{reh}) T_{reh}^4$$

$$T_{reh} \simeq 0.1\sqrt{\Gamma_\phi M_P} = 2\times 10^{14} h_{\text{eff}}\ \text{GeV} \le 10^{11}\ \text{GeV}$$

$$\Gamma_{grav} \sim \frac{m^3}{M_P^2} \;\Rightarrow\; T_{reh} \sim 10^9\ \text{GeV}$$

Non-perturbative decay of inflaton

$$H = \frac{1}{2}\Pi_\chi^2 + \frac{1}{2}(\nabla\chi)^2 + \frac{1}{2}m_\chi^2(t)\chi^2$$

$$m_\chi^2(t) = m_\chi^2 + g^2\phi^2(t)$$

$$\hat{\chi}(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}}\left[f_k(t)\hat{a}_{\vec{k}}\, e^{i\vec{k}\vec{x}} + h.c.\right]$$ free quantum field

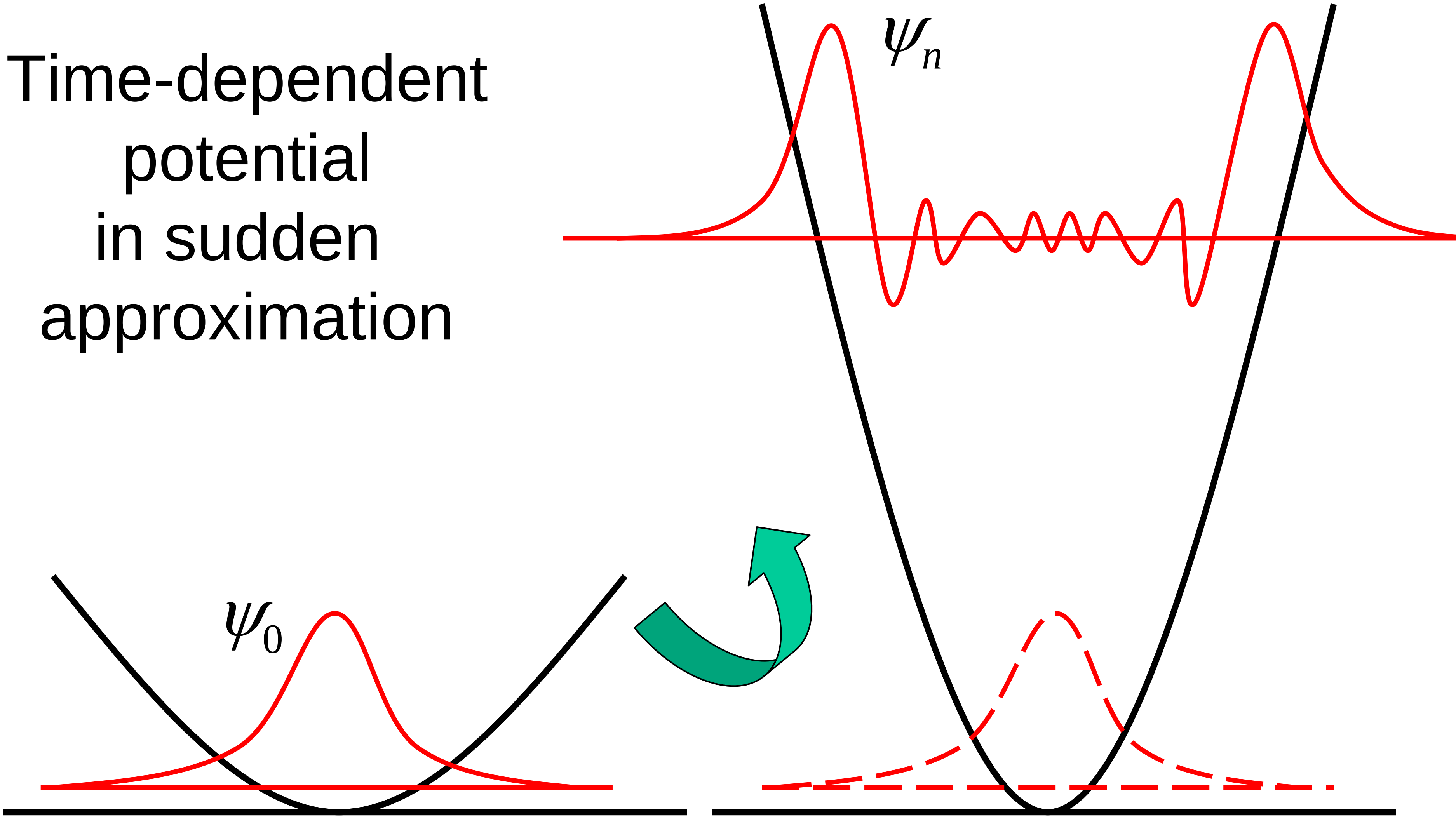
$$\left[\hat{\chi}(t,\vec{x}),\hat{\Pi}_\chi(t,\vec{x}')\right] = i\hbar\delta^3(\vec{x}-\vec{x}') \Rightarrow \left[\hat{a}_{\vec{k}},\hat{a}^+_{\vec{k}'}\right] = \delta^3(\vec{k}-\vec{k}')$$

$$\ddot{f}_k + \omega_k^2(t) f_k = 0, \quad \omega_k^2(t) = k^2 + m_\chi^2(t)$$ time dep

$$g_k = i\dot{f}_k, \quad \mathbf{Re}(f_k^* g_k) = \frac{1}{2}$$ Wronskian

Particle production (Schrödinger)

Time-dependent potential in sudden approximation



Occupation number of χ field

$$n(t) = \frac{1}{V}\langle 0 | N | 0 \rangle = \int \frac{d^3k}{(2\pi)^3} n_k(t)$$

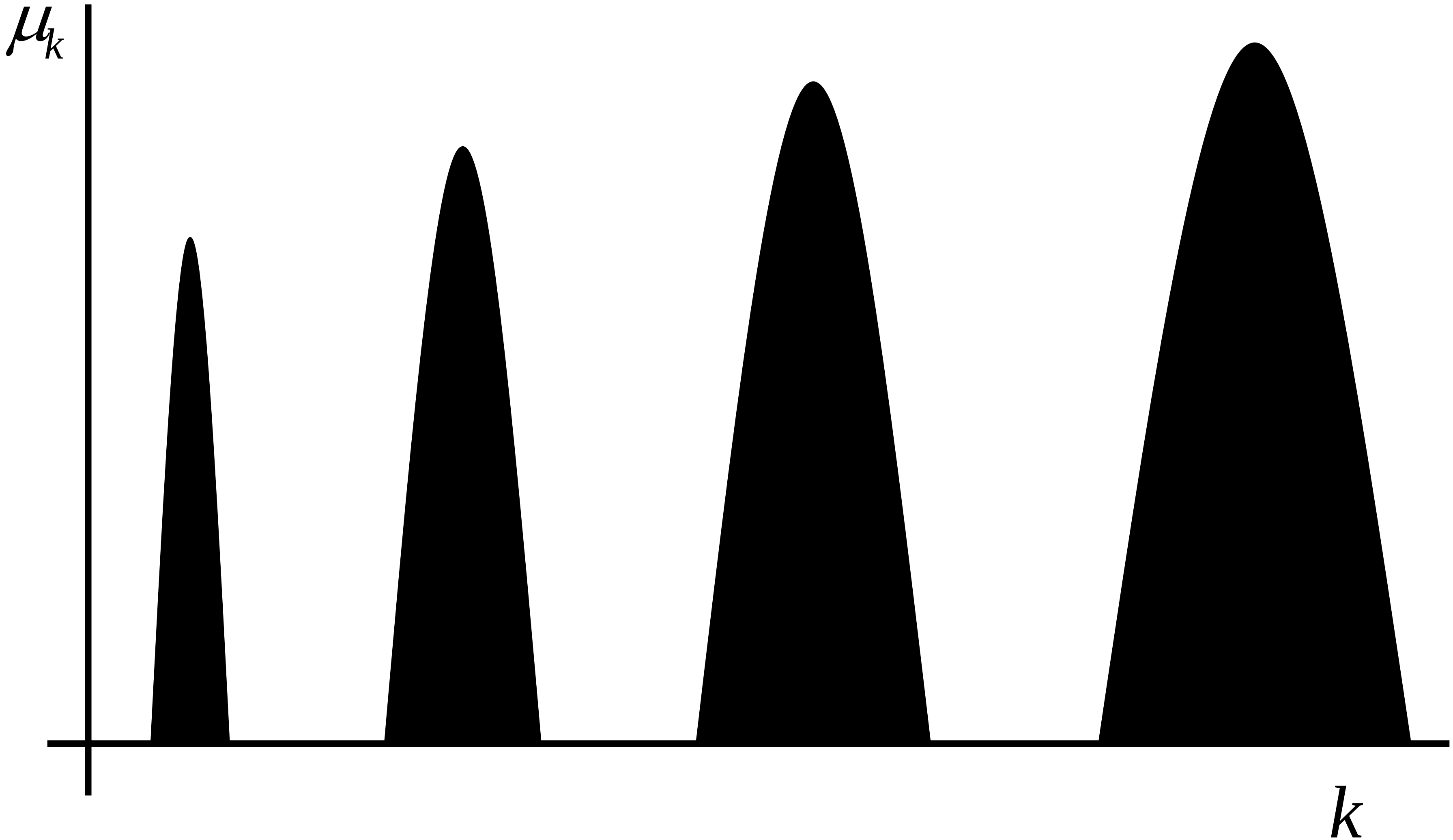
$$n_k(t) = \frac{\omega_k}{2} | f_k |^2 + \frac{1}{2\omega_k} | g_k |^2 - \frac{1}{2}$$

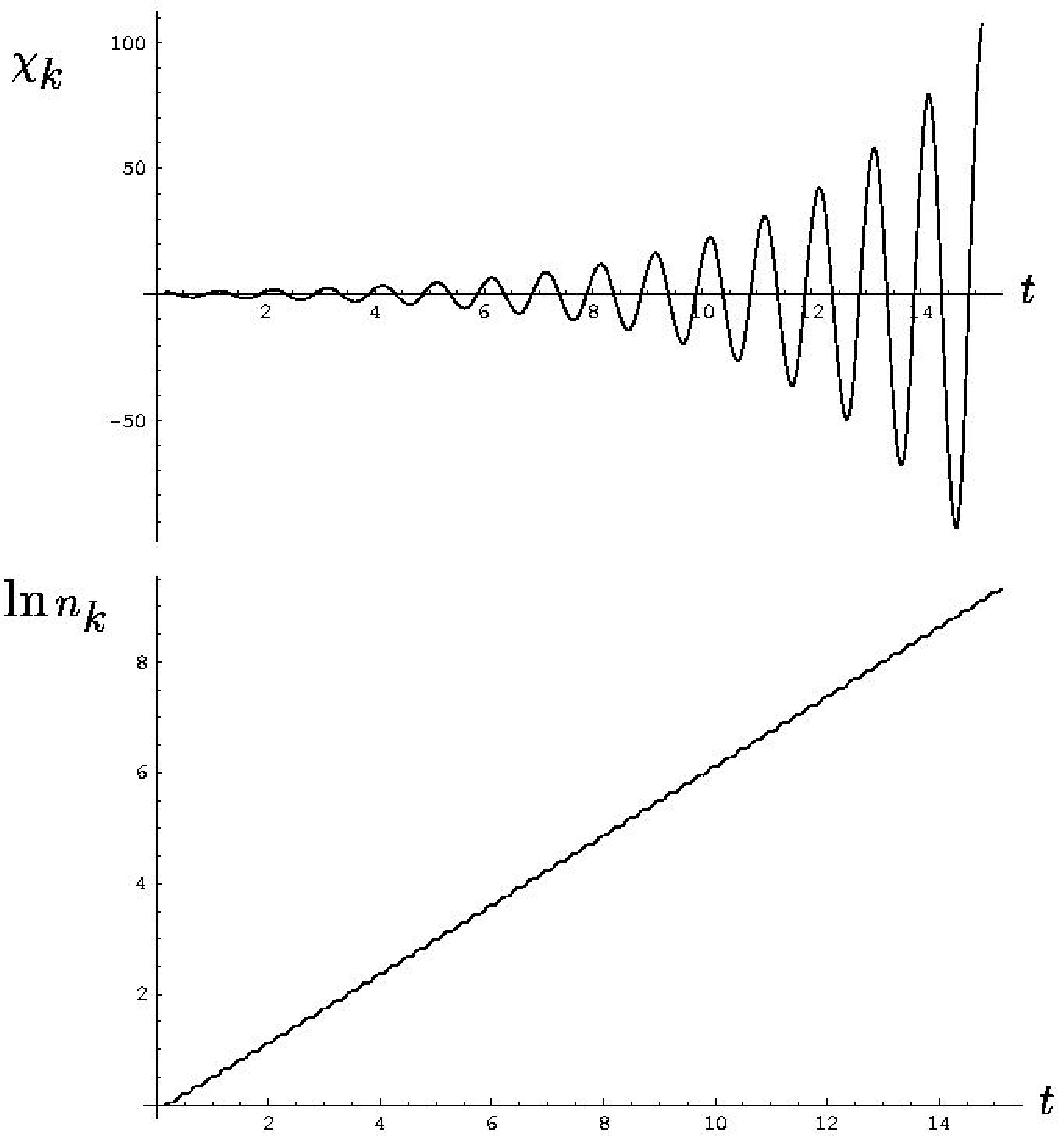
$$f_k'' + (A_k - 2q \cos 2z) f_k = 0$$ Mathieu equatior

$$A_k = \frac{k^2 + m_\chi^2}{m^2} + 2q, \quad q = \frac{g^2 \Phi^2(t)}{4m^2}$$

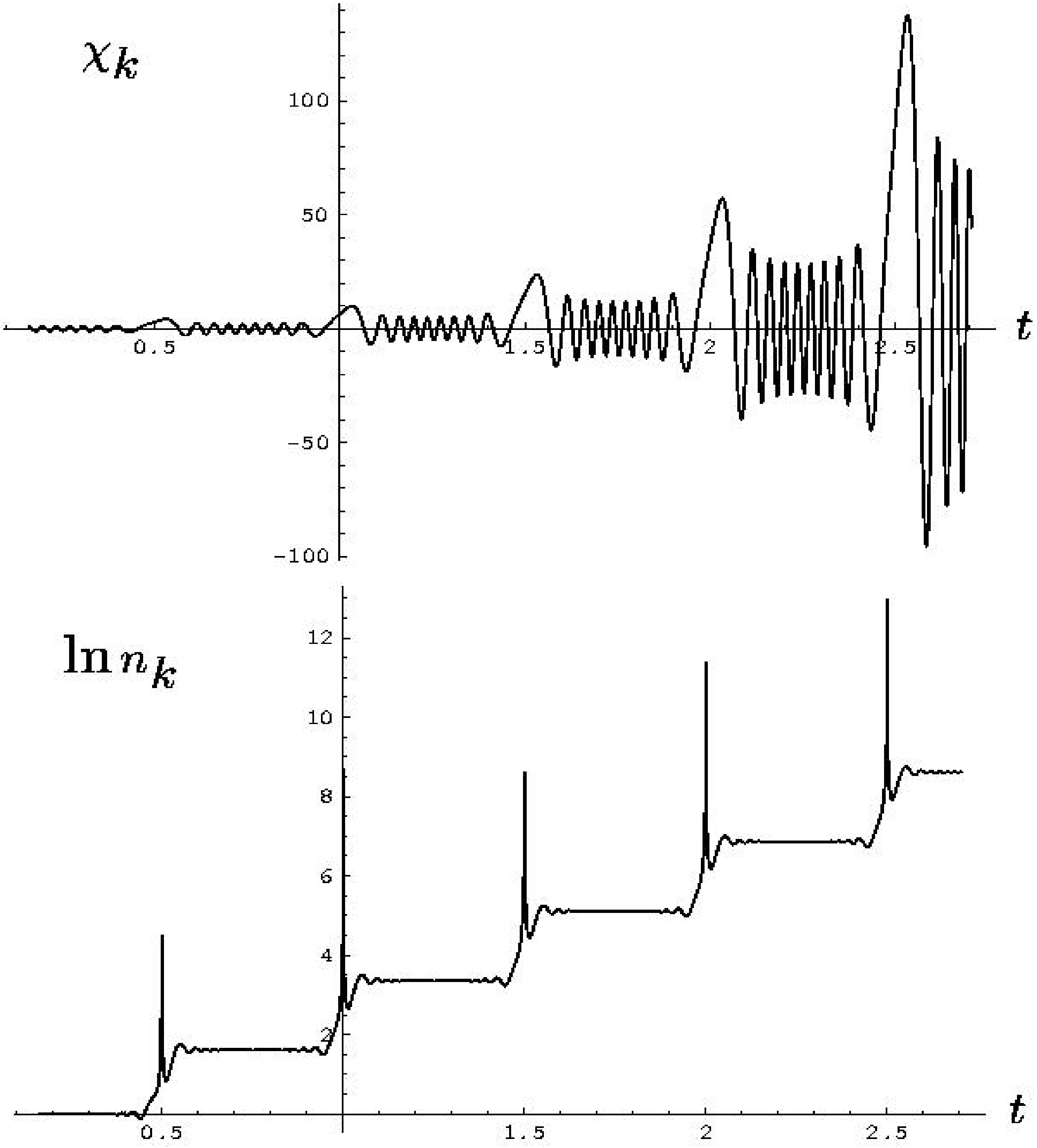
solution $f_k(t) = e^{\mu_k mt} p(z) \quad \Rightarrow \quad n_k(t) \sim e^{2\mu_k mt} \gg 1$

Band structure

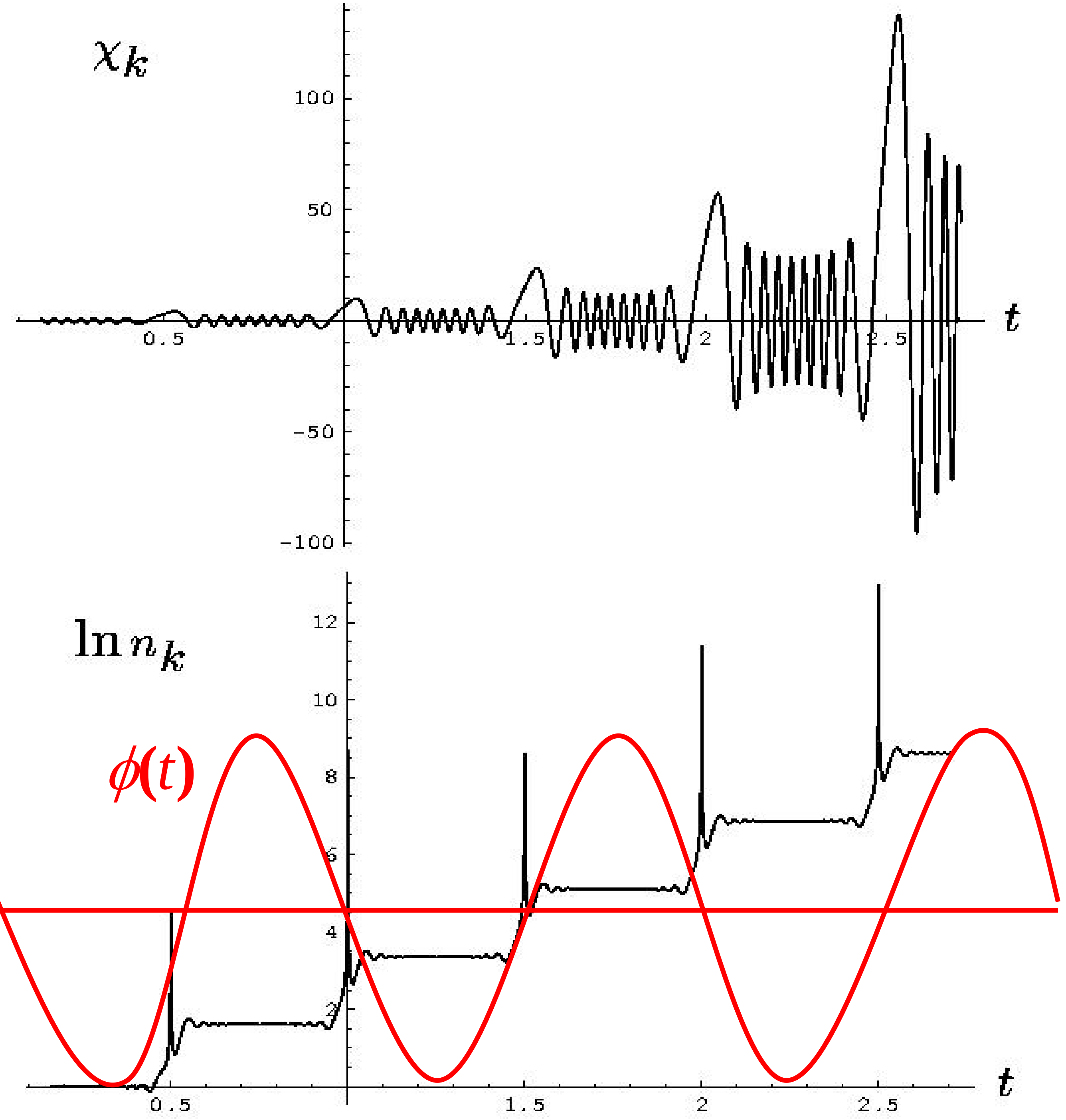




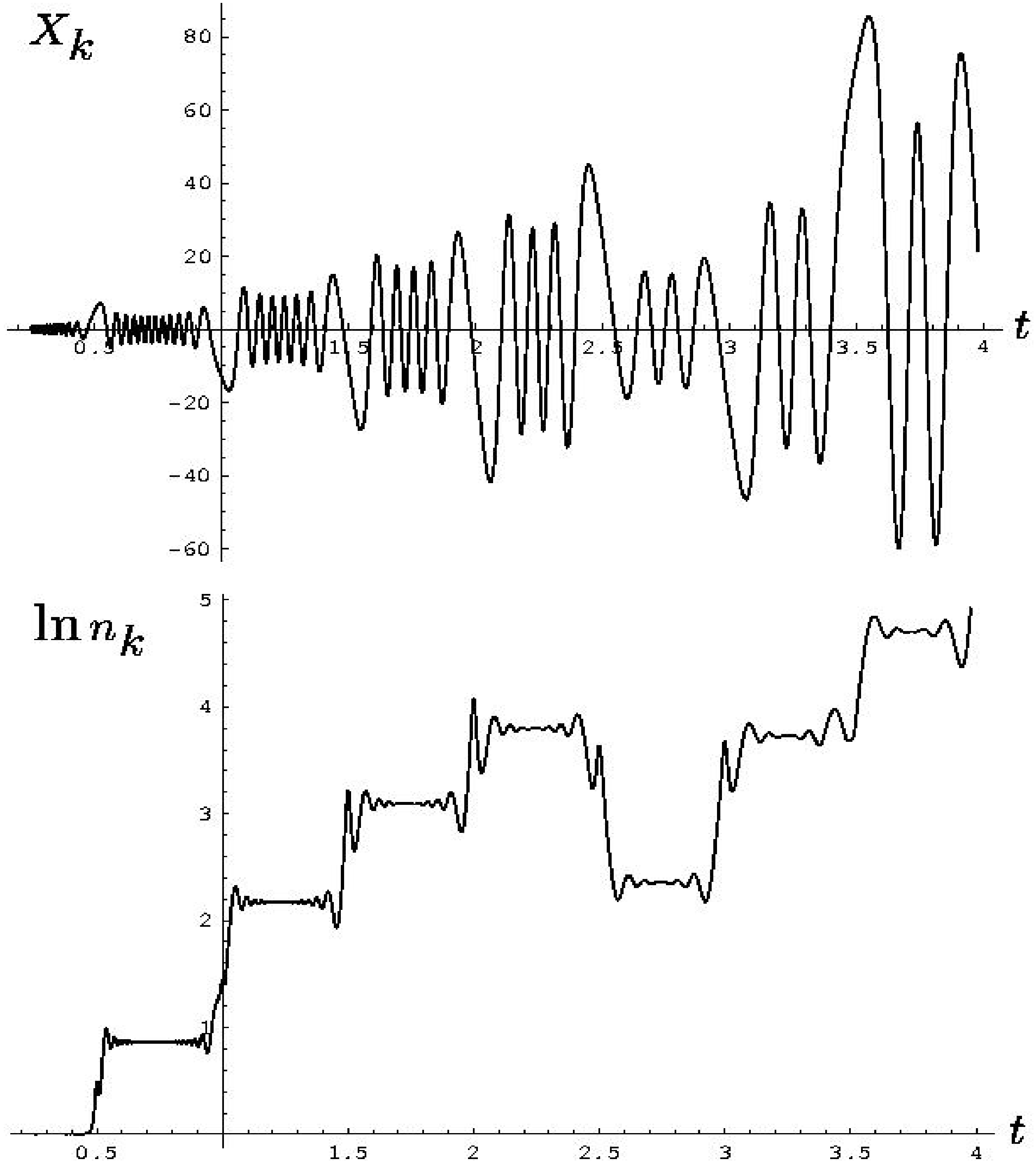
Narrow resonance



Broad resonance

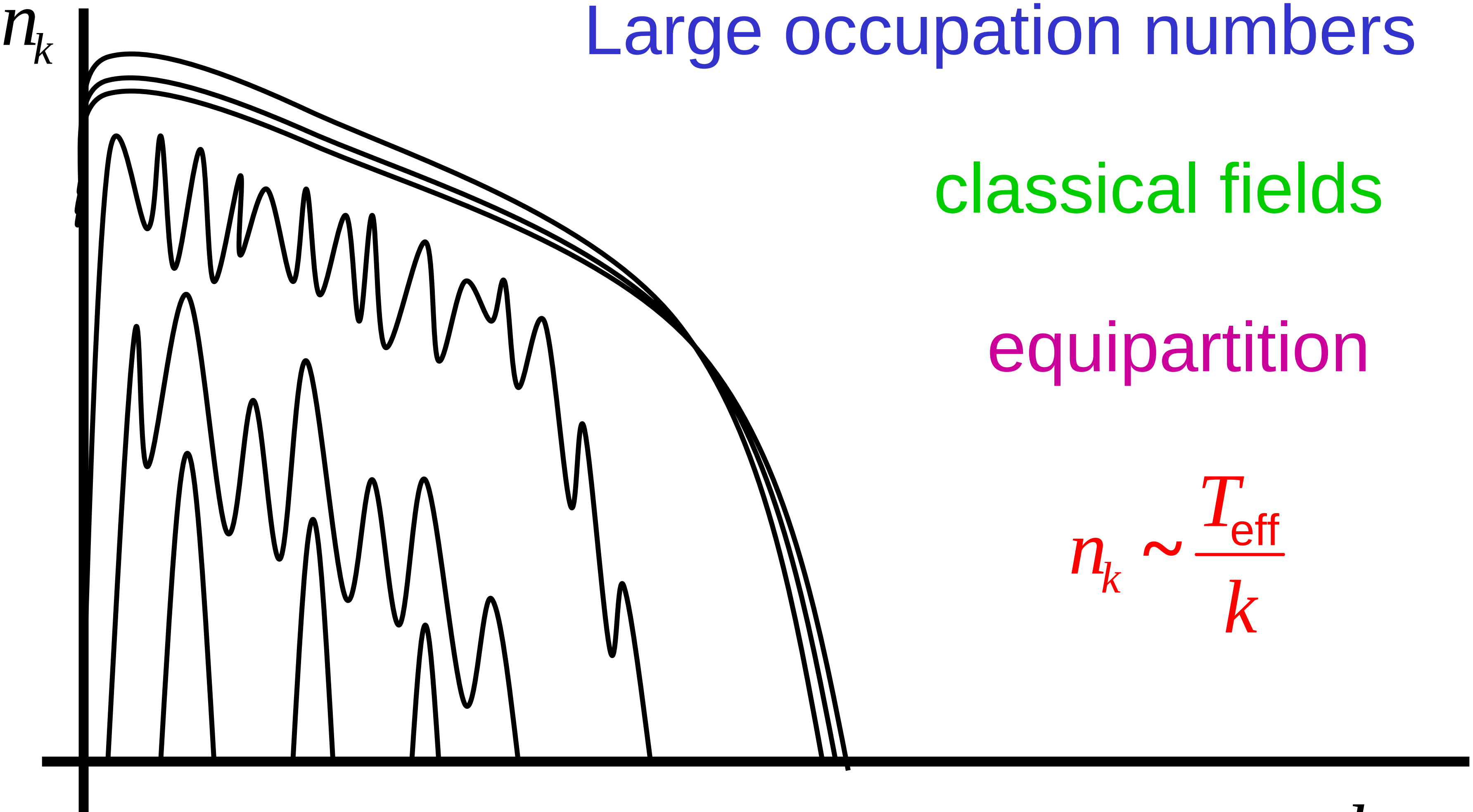


Broad resonance



Expanding universe

Lattice simulations

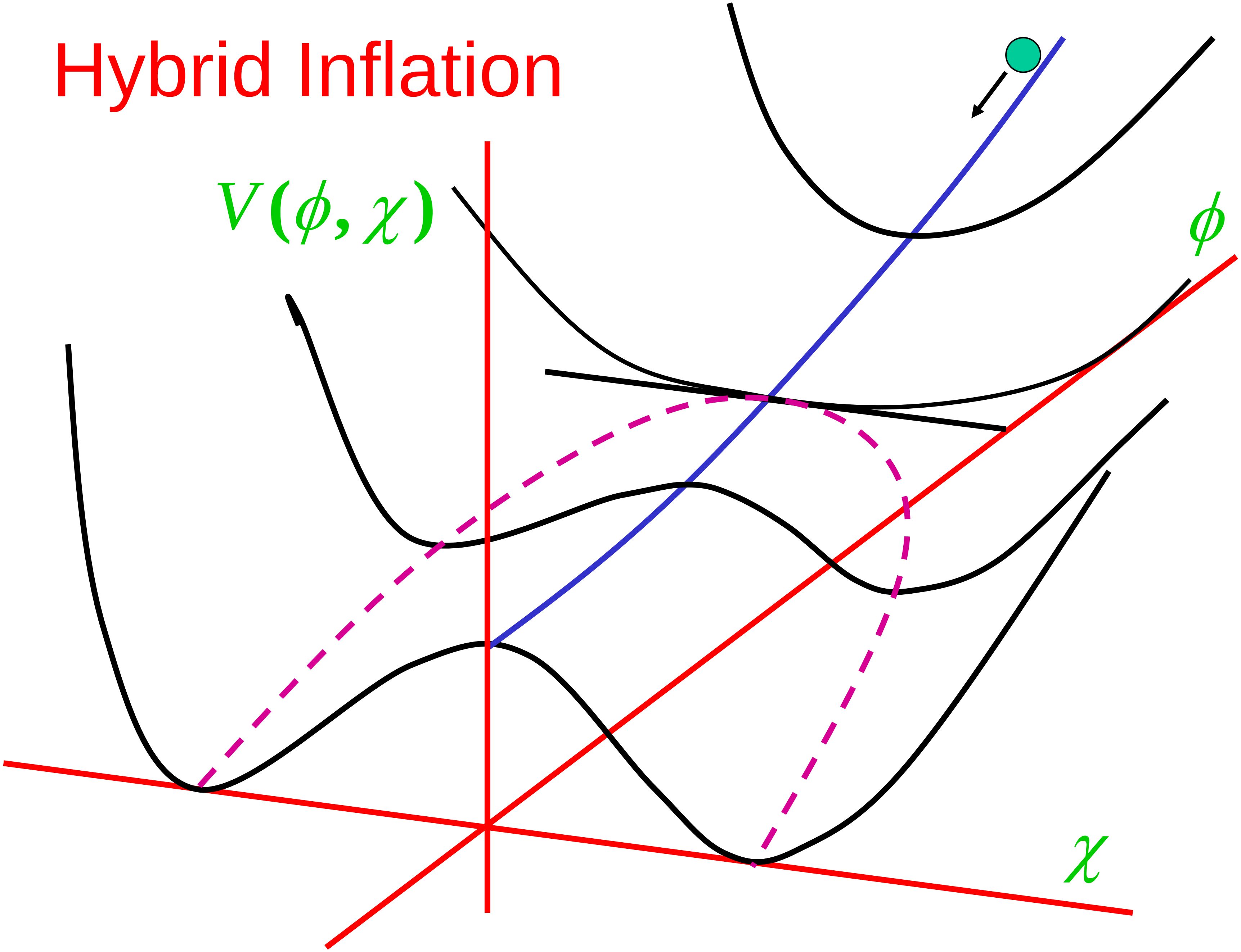


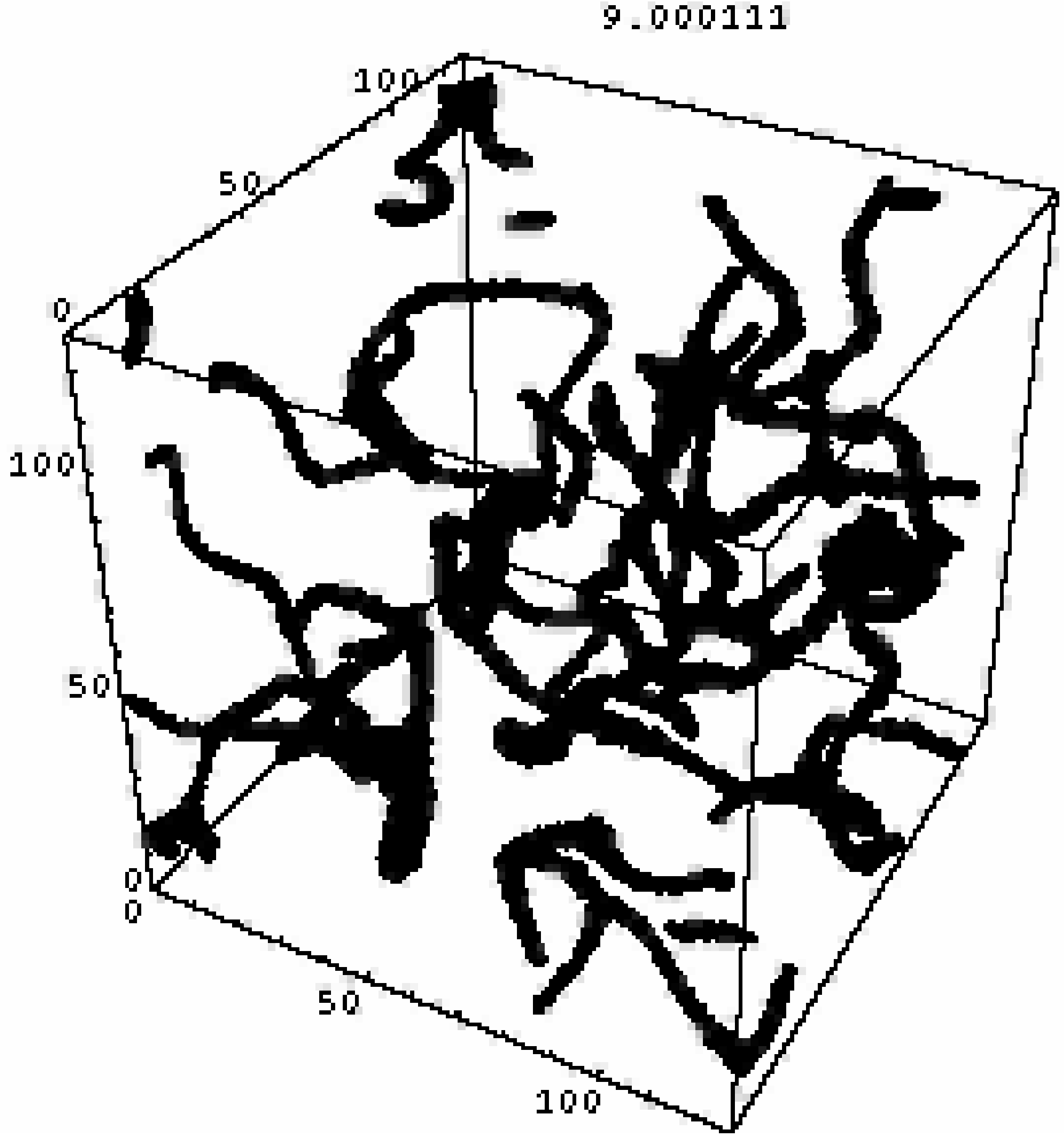
Preheating

Very rich phenomenology after inflation

- Non-thermal production of particles (CDM)
- Production of topological defects
- EW baryogenesis & leptogenesis
- Production of gravitational waves
- Production of primordial magnetic fields
- etc.

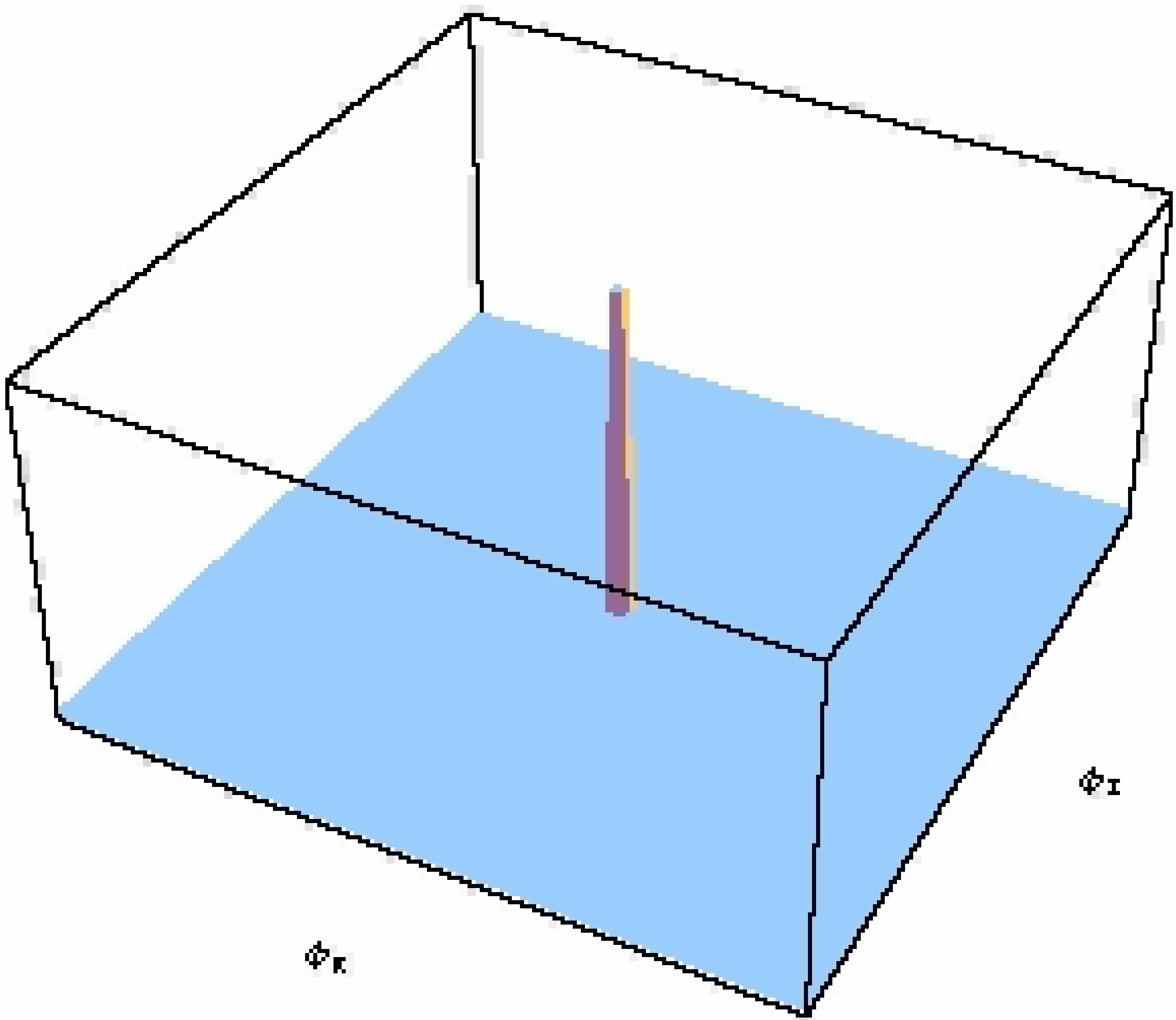
Hybrid Inflation





$\chi \in U(1)$

String production @ end inflation



ϕ_R
ϕ_I

Tachyonic preheating

JGB, Linde	PRD57, 6075 (1998)
Felder, JGB, Kofman, Linde, Tkachev	PRL87, 011601 (2001) PRD64, 123517 (2001)
JGB, Garcia-Perez, Gonzalez-Arroyo	PRD67, 103501 (2003)

Tachyonic Preheating

Spinodal growth of long wave Higgs modes

- At the end of Hybrid Inflation
- Higgs couples to gauge fields
- Strong production of fermions

The Higgs Evolution

$$m_\phi^2 = m^2\left(\frac{\chi^2}{\chi_c^2} - 1\right) \approx -2Vm^3(t - t_c)$$

$$= -M^3(t - t_c) = -M^2\tau$$

$$H = \frac{1}{2}\int d^3k\left[p_k(\tau)\,p_k^+(\tau) + (k^2 - \tau)\,y_k(\tau)\,y_k^+(\tau)\right]$$

$$\left[y_k(\tau), p_{k'}(\tau)\right] = i\hbar\,\delta^3(k - k')$$

Higgs Quantum Field

$$y_k(\tau) = f_k(\tau)a_k(\tau_0) + f_k^*(\tau)a_{-k}^+(\tau_0)$$

$$p_k(\tau) = -i\left[g_k(\tau)a_k(\tau_0) - g_k^*(\tau)a_{-k}^+(\tau_0)\right]$$

$$f_k'' + (k^2 - \tau) f_k = 0 \qquad g_k = i f_k'$$

$$\Omega_k(\tau) = \frac{g_k^*(\tau)}{f_k^*(\tau)} = \frac{1 - 2iF_k(\tau)}{2|f_k(\tau)|^2}$$

$$F_k(\tau) = \mathbf{Im}(f_k^* g_k)$$

Quantum Initial Conditions

$$\forall k \quad a_k(\tau_0)\left|0,\tau_0\right\rangle = 0 \;\Rightarrow\; \Psi_0(\tau_0) = N_0 e^{-k|y_k^0|^2}$$

Unitary Evolution

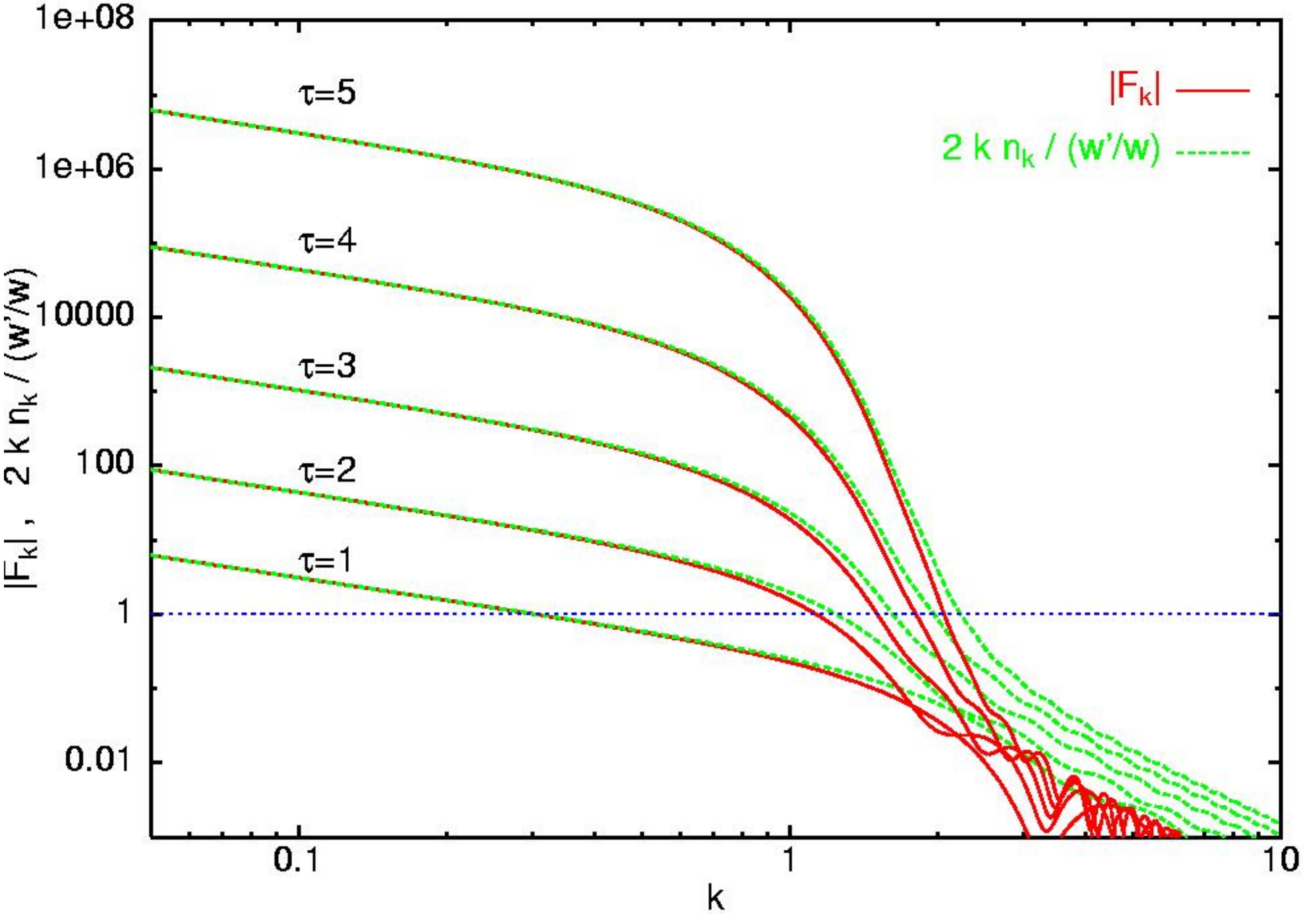
$$\left|0,\tau\right\rangle = U\left|0,\tau_0\right\rangle \;\Rightarrow\; \Psi_0(\tau) = \frac{1}{\sqrt{\pi}\,|\,f_k\,|} e^{-\Omega_k(\tau)|y_k^0|^2}$$

Occupation number of mode k

$$n_k(\tau) = \left\langle 0,\tau\right| N_k(\tau_0) \left|0,\tau\right\rangle = \frac{1}{2k}|\,g_k\,|^2 + \frac{k}{2}|\,f_k\,|^2 - \frac{1}{2}$$

Quantum to Classical Transition

$$\langle 0,\tau | G(\hat{y},\hat{p}) | 0,\tau \rangle \approx \langle G_0(y,p) \rangle_{gaussian}$$



$$|F_k(\tau)| \gg 1$$

Quantum to Classical Transition

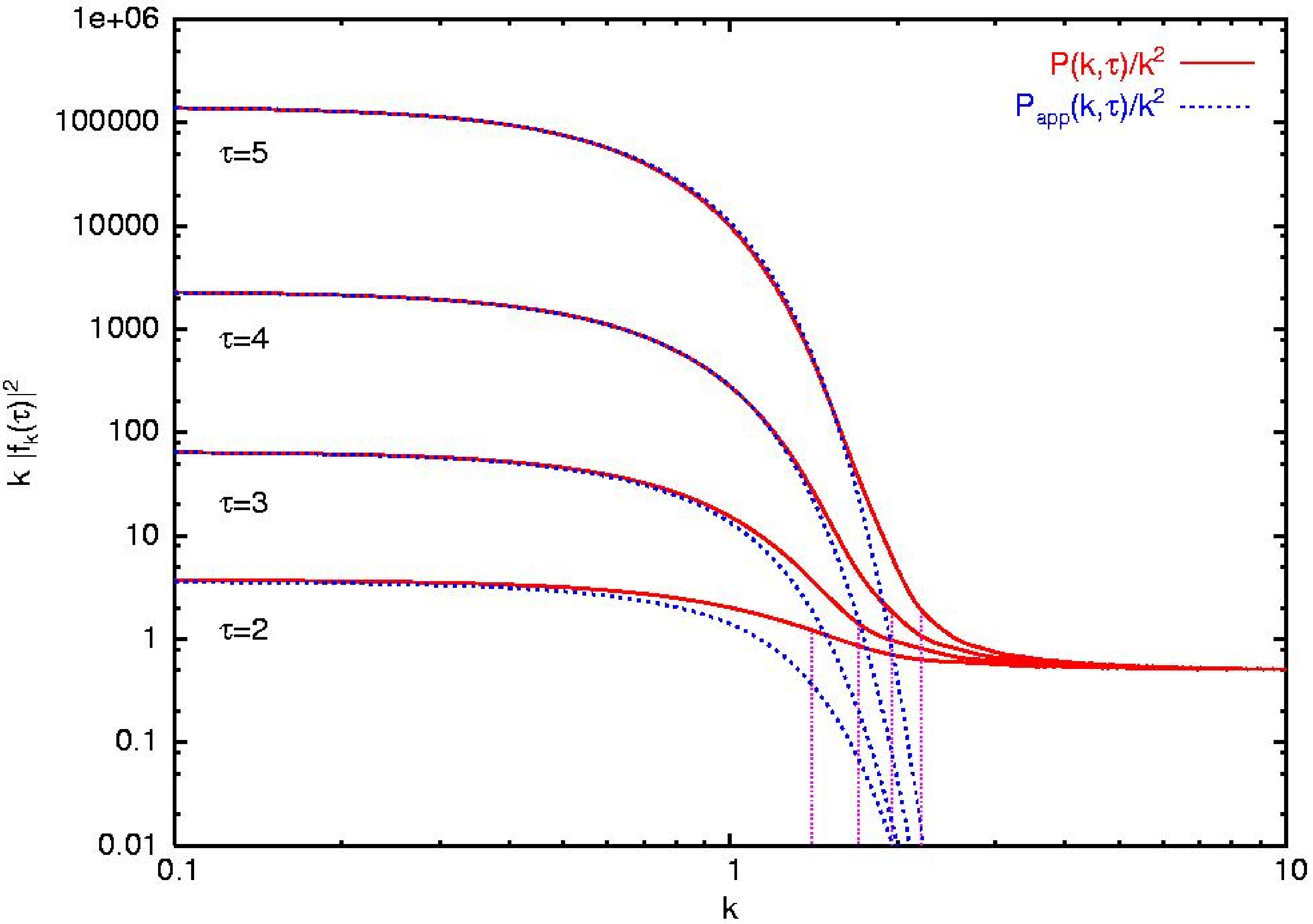
For $k < \sqrt{\tau}$ longwave modes

$$P_{app}(k,\tau) = k^3 |f_k(\tau)|^2 = A(\tau) k^2 e^{-B(\tau)k^2}$$

$$A(\tau) = A_0 Bi^2(\tau) \approx \frac{A_0}{\pi\sqrt{\tau}} e^{\frac{4}{3}\tau^{3/2}}$$

$$B(\tau) = 2\sqrt{\tau}$$

Power spectrum of longwave modes



Lattice Simulations

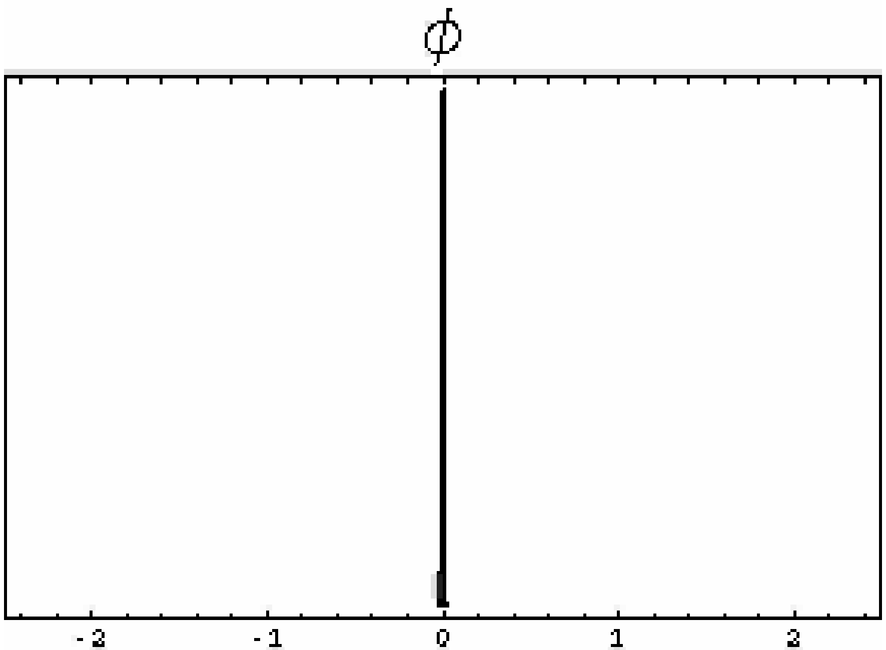
Quantum averages = Ensemble averages

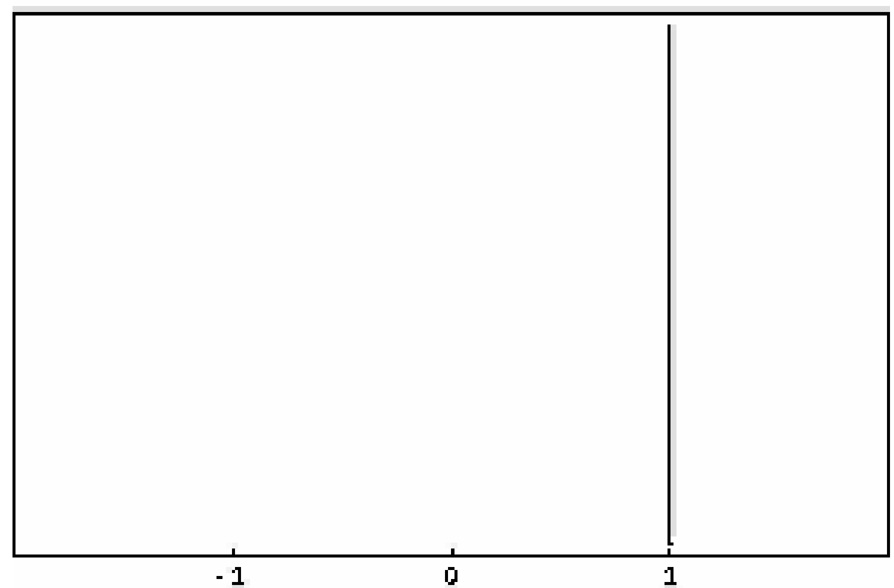
Initial conditions: Highly occupied modes

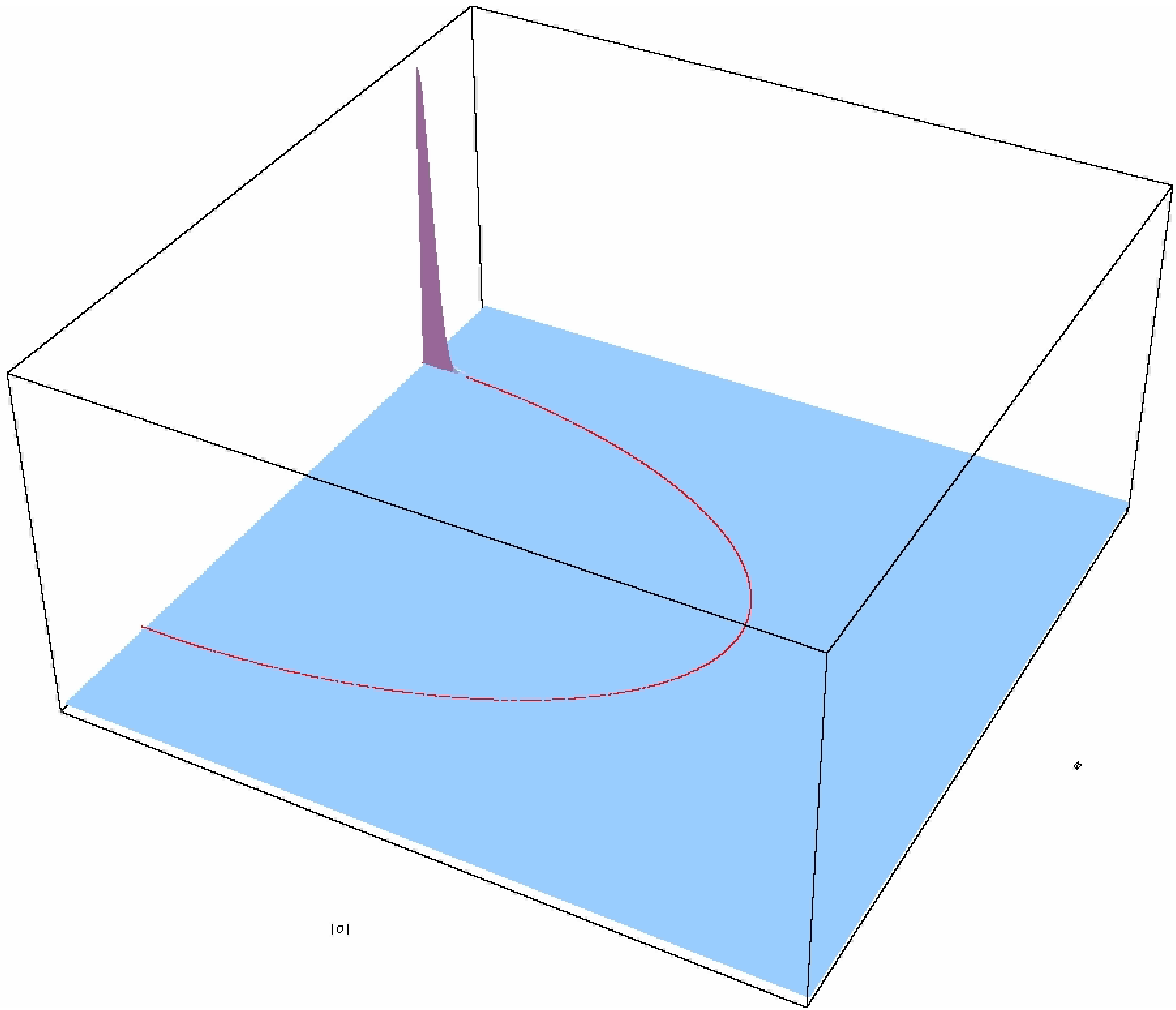
$$\left|0,\tau\right\rangle = U\left|0,\tau_0\right\rangle \;\Rightarrow\; \Psi_0(\tau) = \frac{1}{\sqrt{\pi}\,|\,f_k\,|}\,e^{-\Omega_k(\tau)|y_k^0|^2}$$

$$P_\Psi(|\,\phi_k\,|)\,d\,|\,\phi_k\,|\,d\theta_k = e^{-\frac{|\phi_k|^2}{|f_k|^2}}\,\frac{d\,|\,\phi_k\,|^2}{|\,f_k\,|^2}\,\frac{d\theta_k}{2\pi}$$

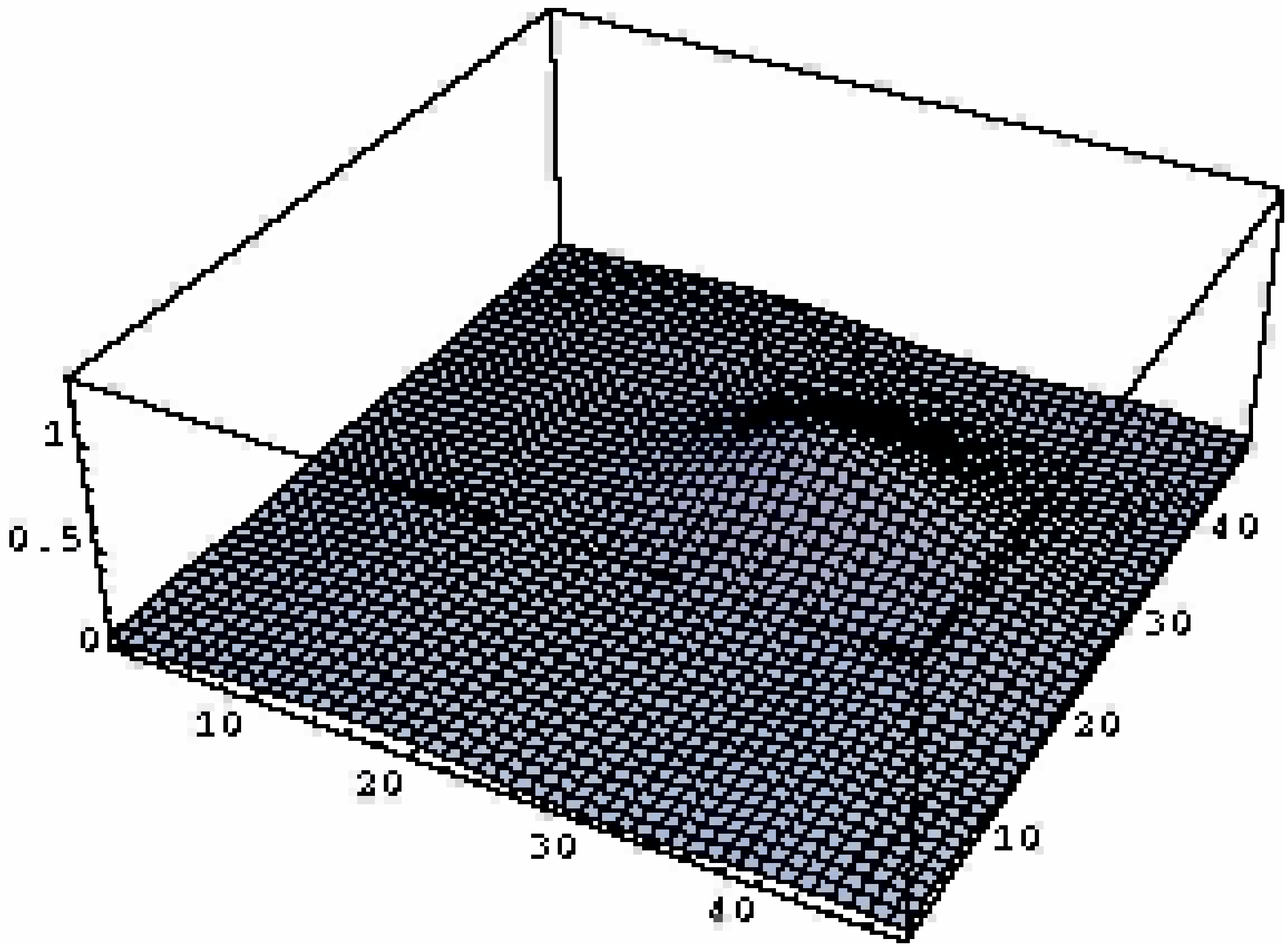
Histograms of
Higgs field and Inflaton field



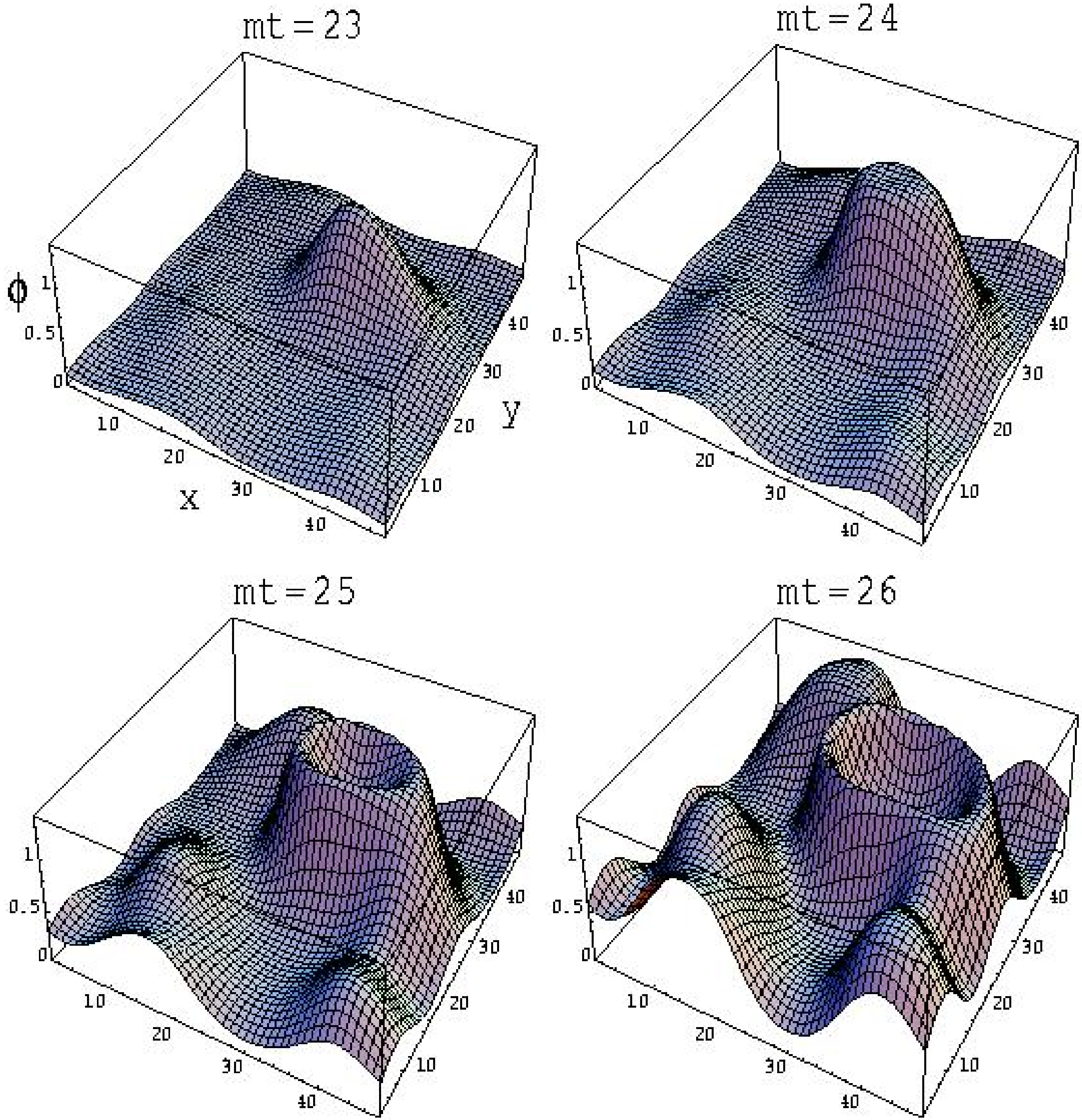


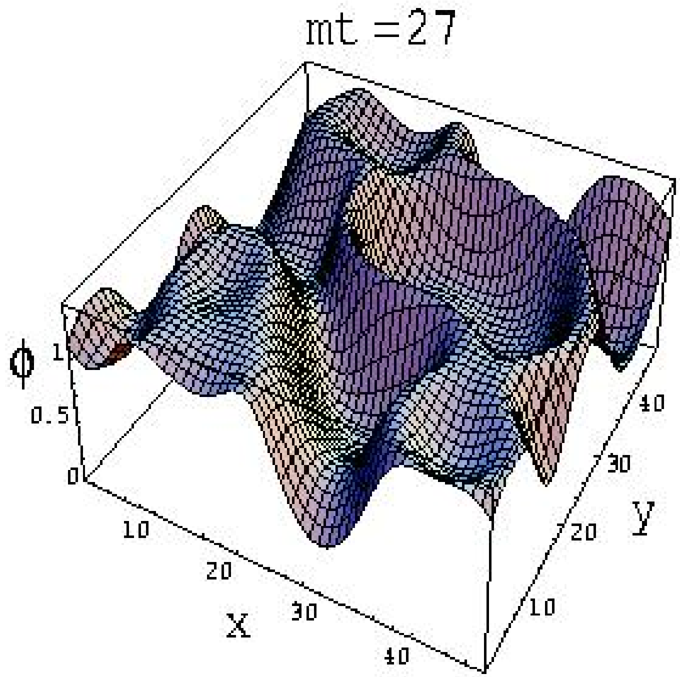


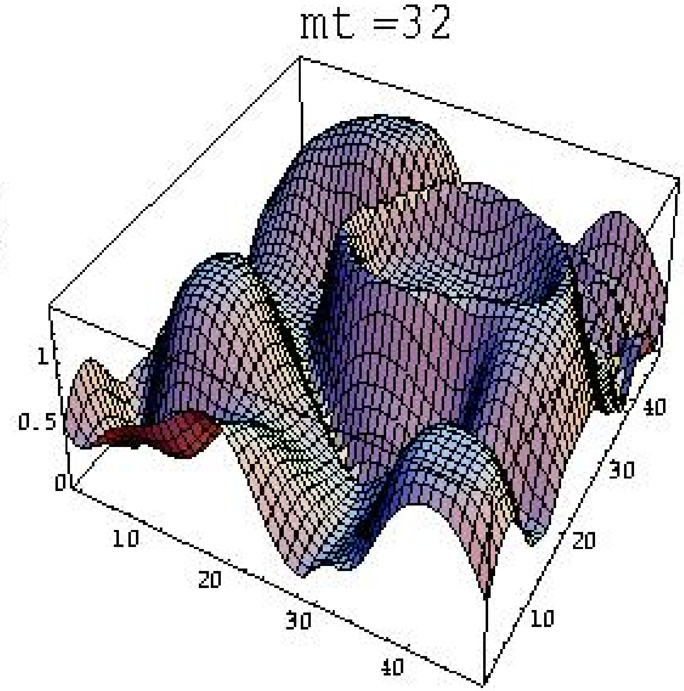
|σ|
φ

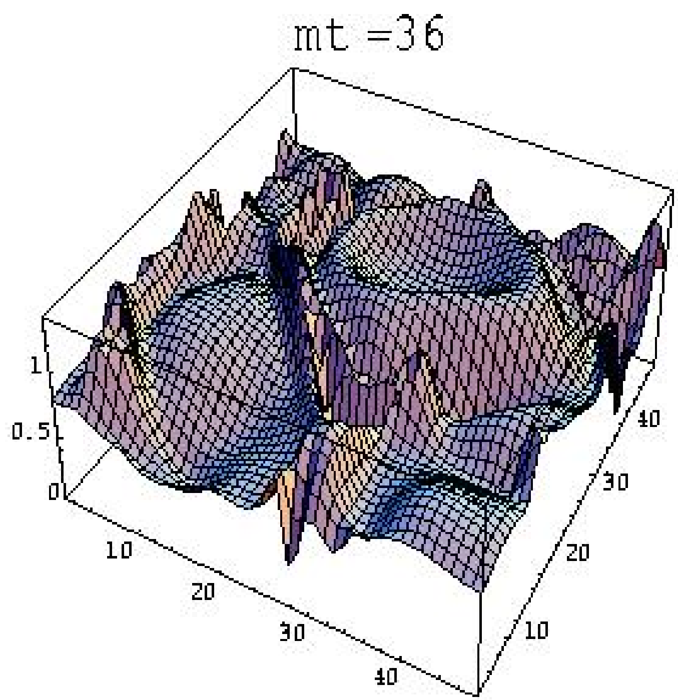


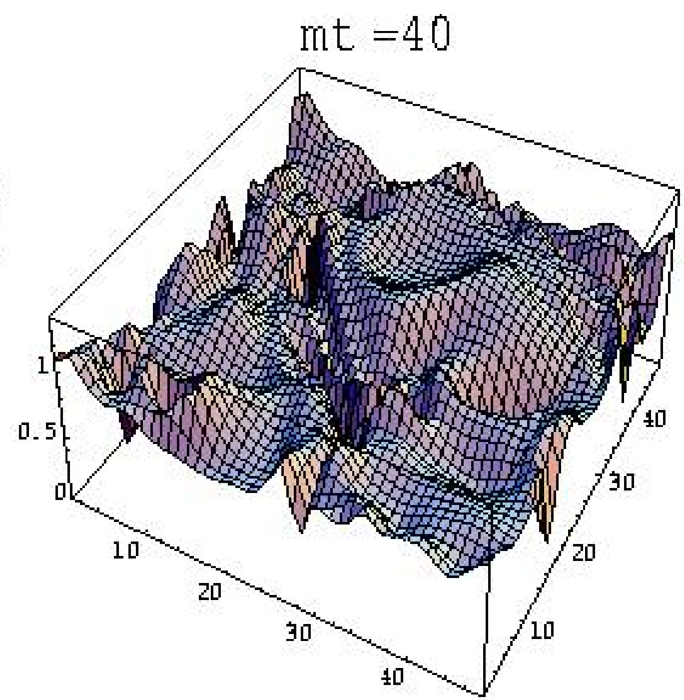
1
0.5
0
10
20
30
40
10
20
30
40


mt = 23
mt = 24
mt = 25
mt = 26
ϕ
x
y
0
0.5
1
10
20
30
40

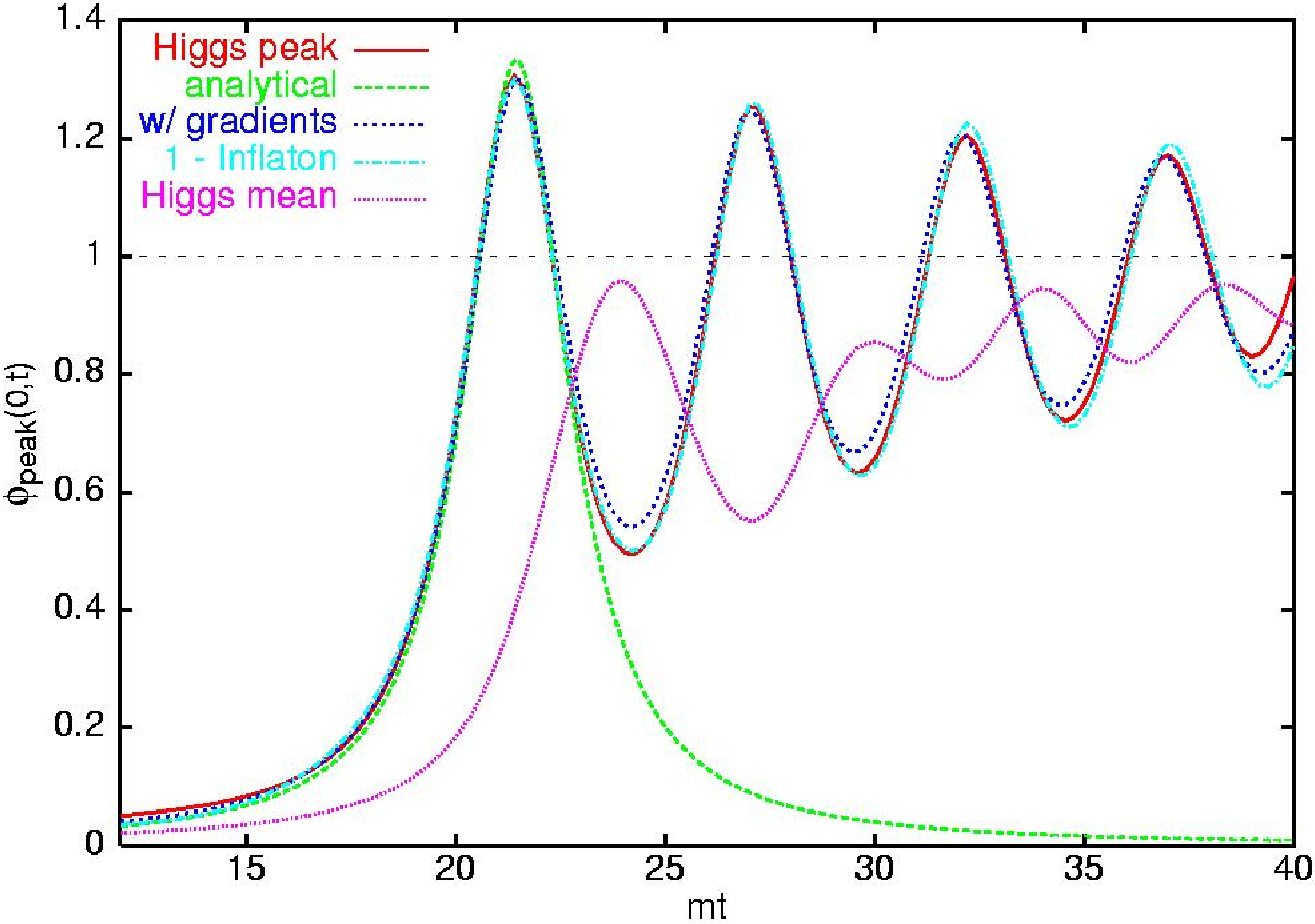

mt =27
φ
x
y

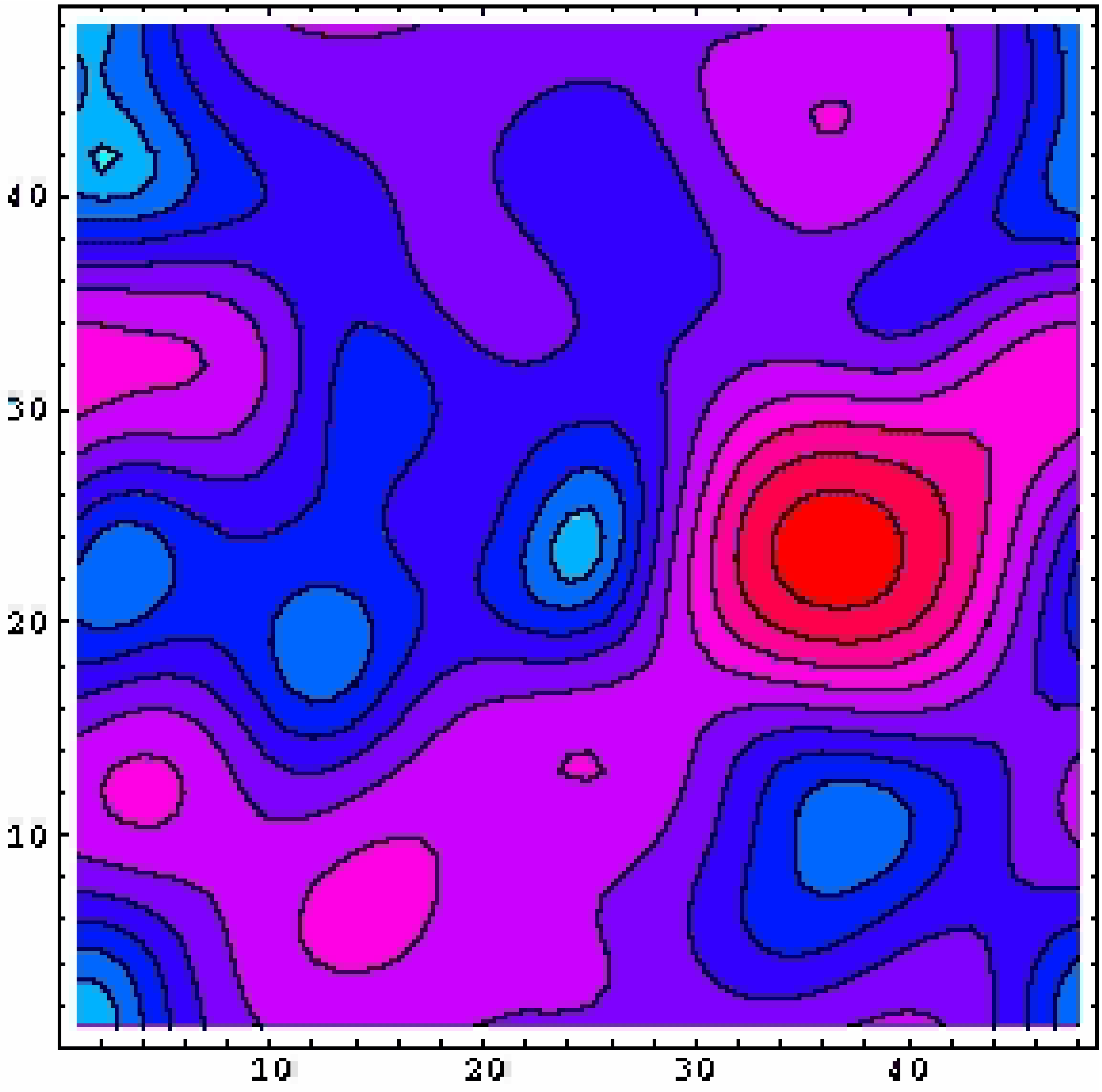

mt =32


mt =36


mt =40

High peaks and mean of Higgs field




40
30
20
10
10
20
30
40

Cold EW Baryogenesis

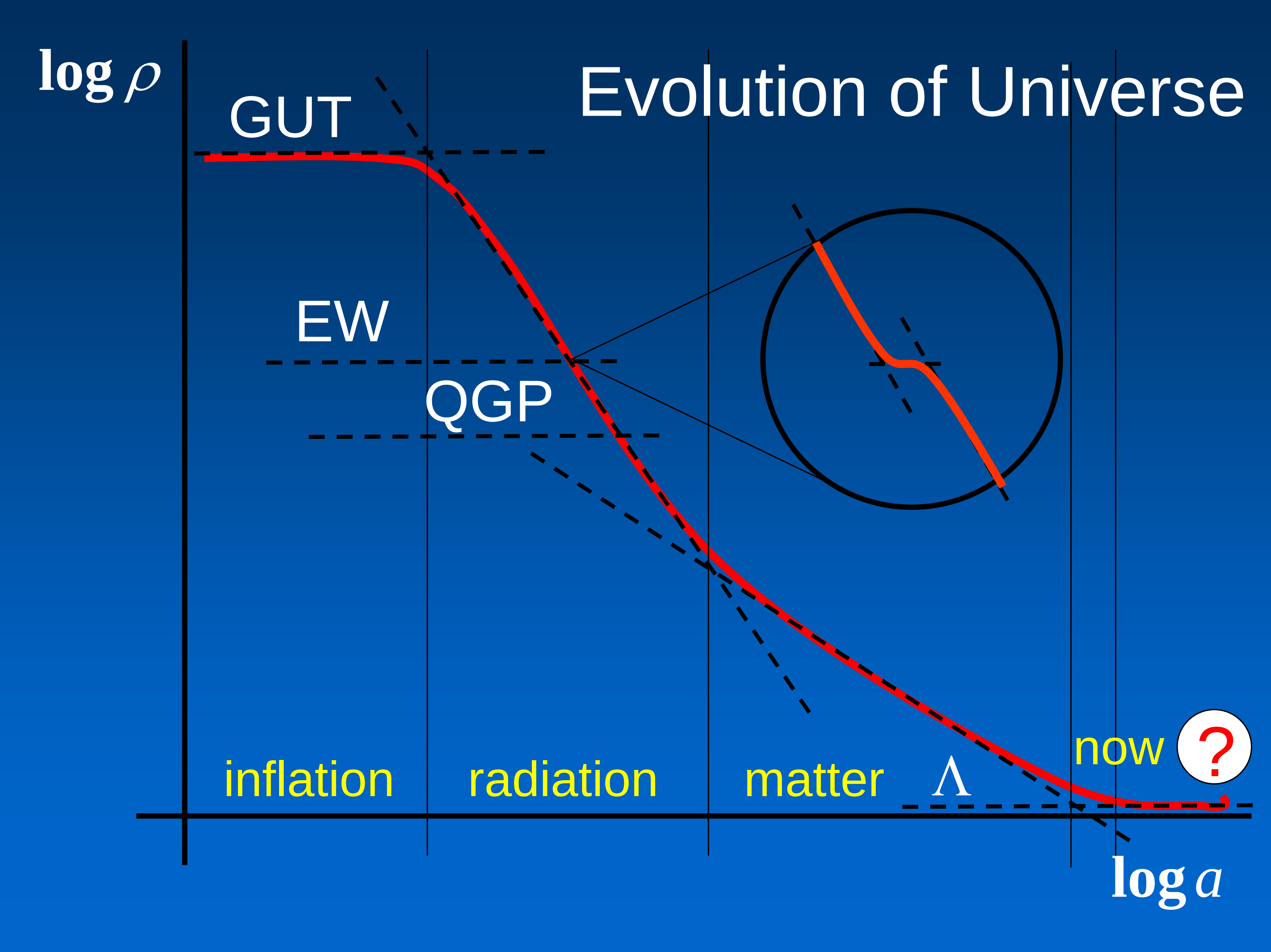
J. G.-B.
Dmitri Grigoriev
Alex Kusenko
Misha Shaposhnikov

PRD60,123504(1999)

GGI 2006, Florence
6th September, 2006

Sakharov conditions

- B violation
- C and CP violation
- Out of equilibrium



Evolution of Universe
log ρ
GUT
EW
QGP
inflation
radiation
matter
Λ
now
?
log a

The SU(2) Higgs-Inflaton model

$$L = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + Tr[(D_\mu \Phi)^+ D^\mu \Phi] + \frac{1}{2}(\partial_\mu \chi)^2 - V(\Phi, \chi)$$

$$D_\mu = \partial_\mu - \frac{i}{2} g_w A^a_\mu \tau_a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_w \varepsilon^{abc} A^b_\mu A^c_\nu$$

$$Tr[\Phi^+ \Phi] = \frac{1}{2}(\phi_0^2 + \phi^a \phi_a) \equiv \frac{1}{2}\phi^2$$

$$V(\phi, \chi) = \frac{\lambda}{4}(\phi^2 - v^2) + \frac{g^2}{2}\phi^2 \chi^2 + \frac{1}{2} m^2 \chi^2$$

Chern-Simons Number

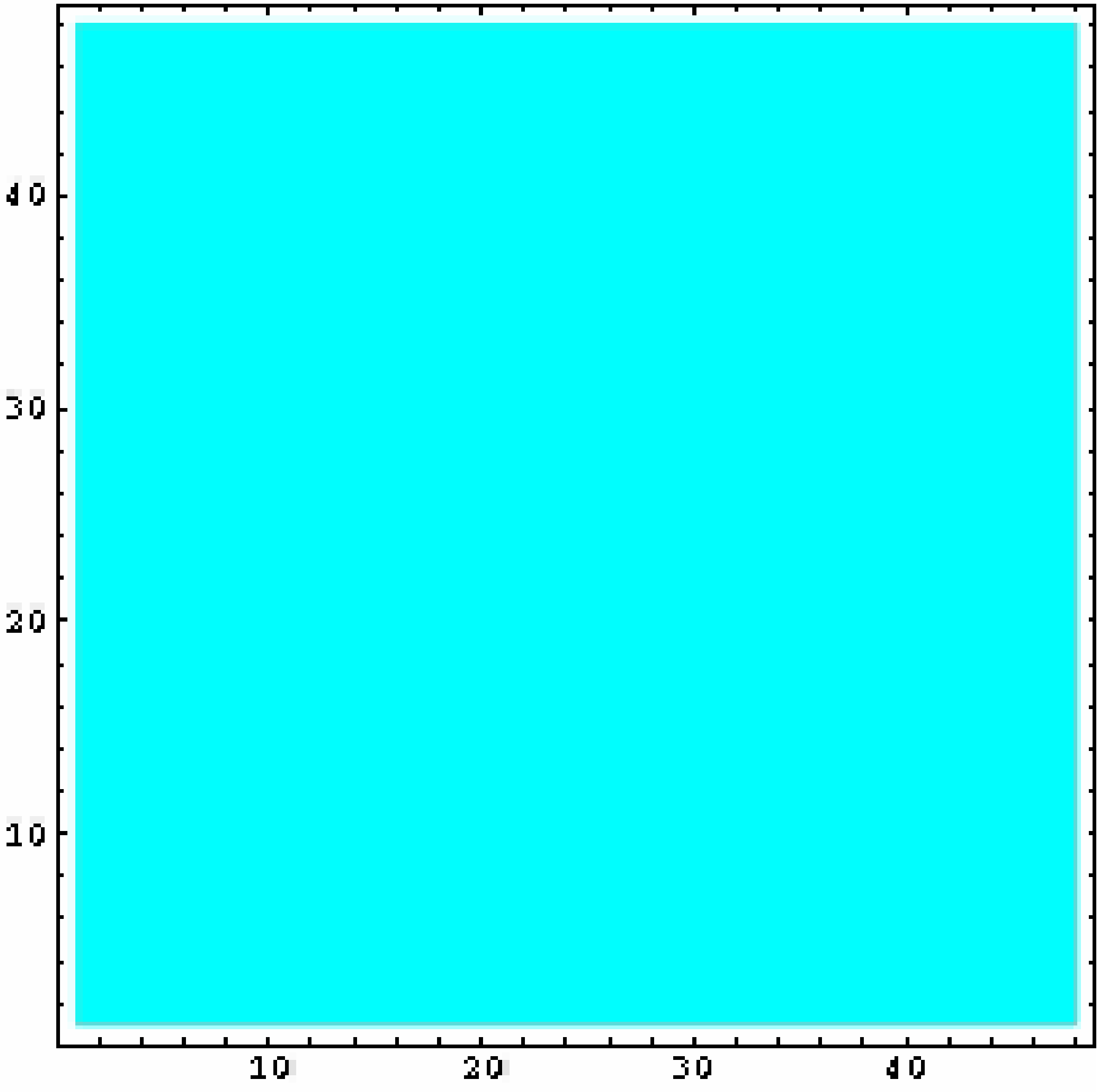
$$\Delta N_{CS} = \frac{g_w^2}{16\pi^2} \int_{t_i}^{t} dt \int d^3x \, Tr[F_{\mu\nu} \tilde{F}^{\mu\nu}]$$

$$\equiv \frac{1}{16\pi^2} \int_{t_i}^{t} dt \int d^3x \, Q(x,t)$$

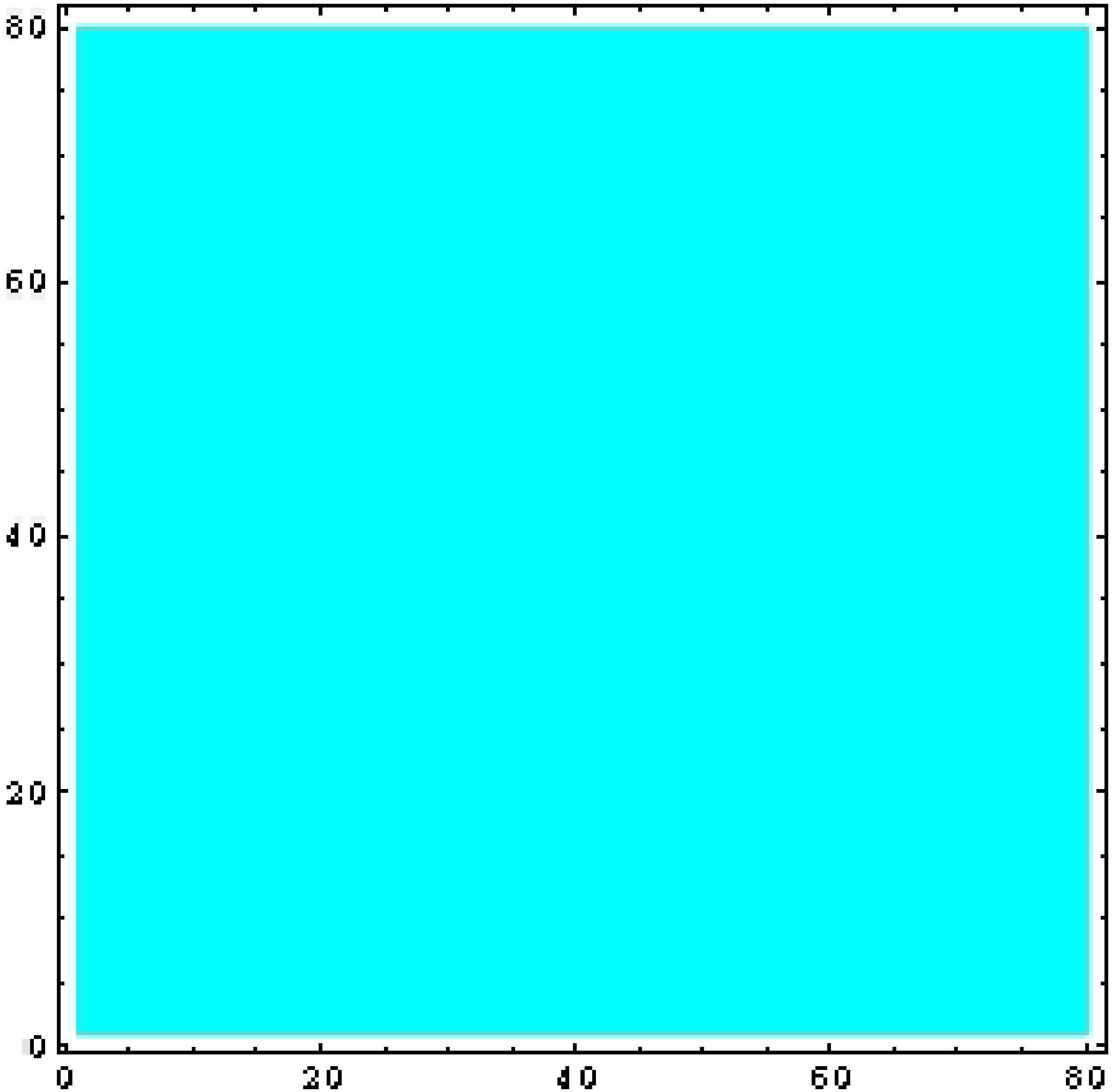
$$\Gamma(t) \equiv \frac{1}{Vm^4} \frac{d}{dt} \left\langle \Delta N_{CS}^2(t) \right\rangle \qquad I(mt) = \int_{t_i}^{t} mdt \, \Gamma(t)$$

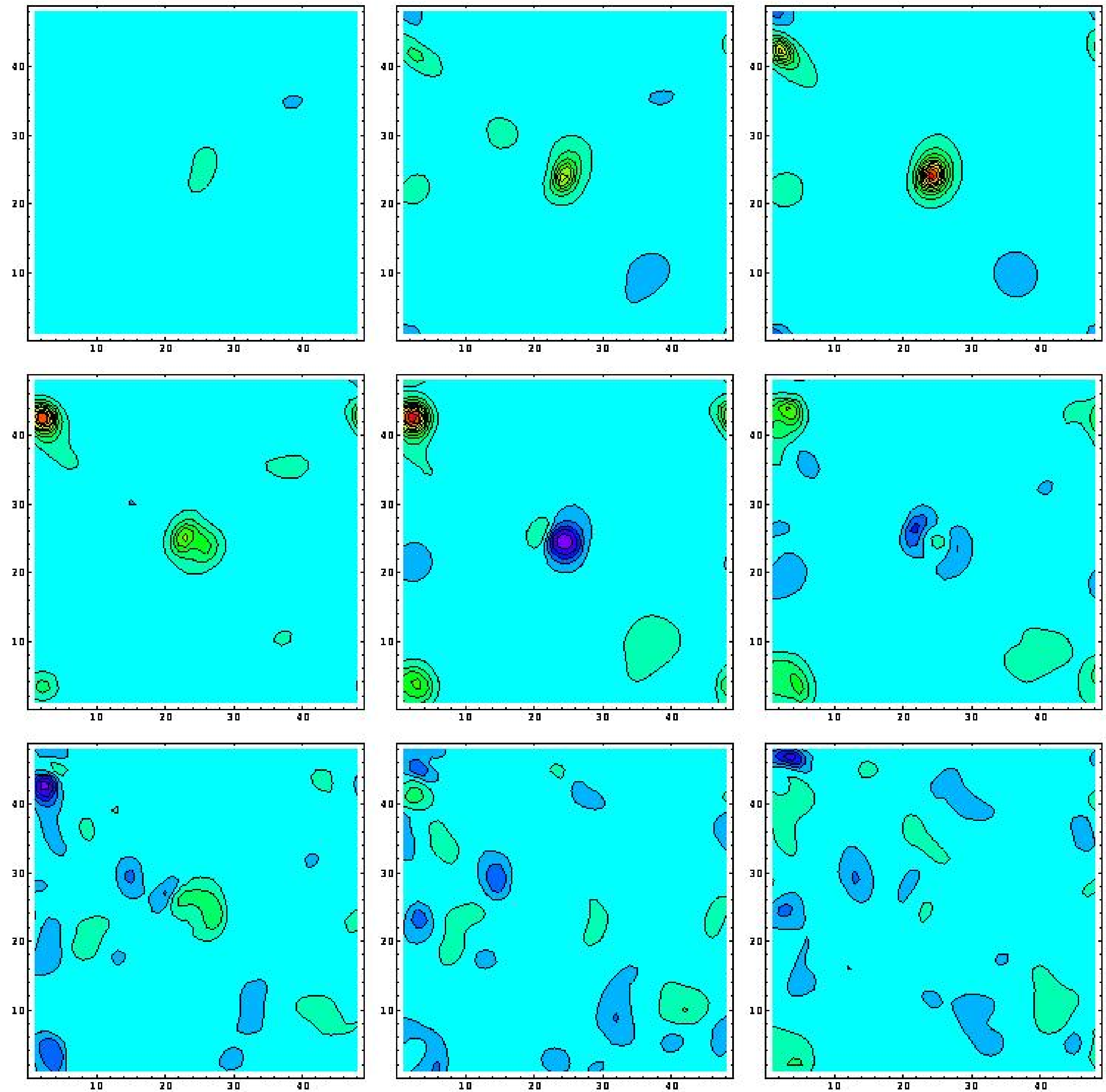
$$I(45) = (12.07 \pm 0.64) \cdot 10^{-5}$$

Chern-Simons Charge $Q(x,t)$

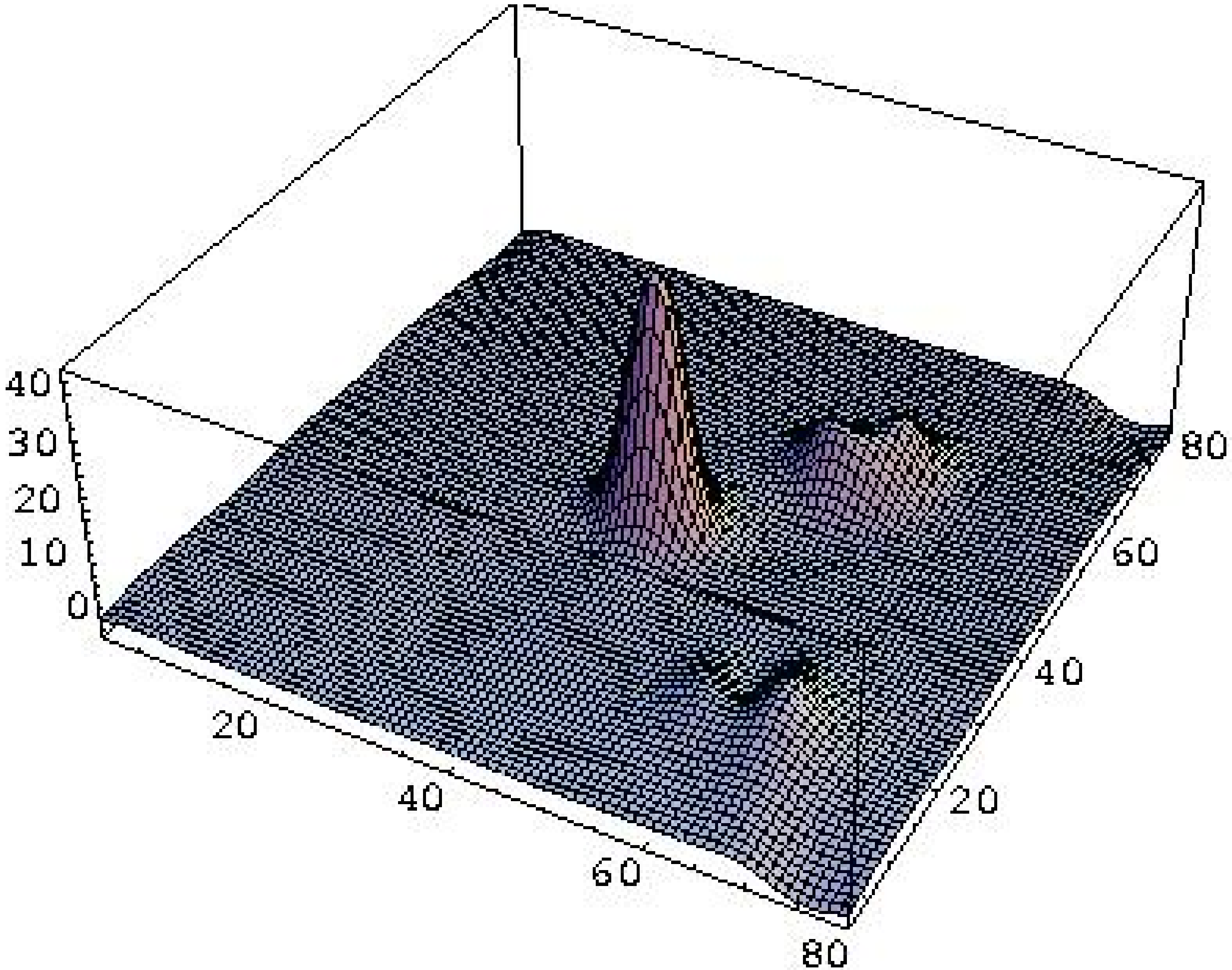


Chern-Simons Charge $Q(x,t)$

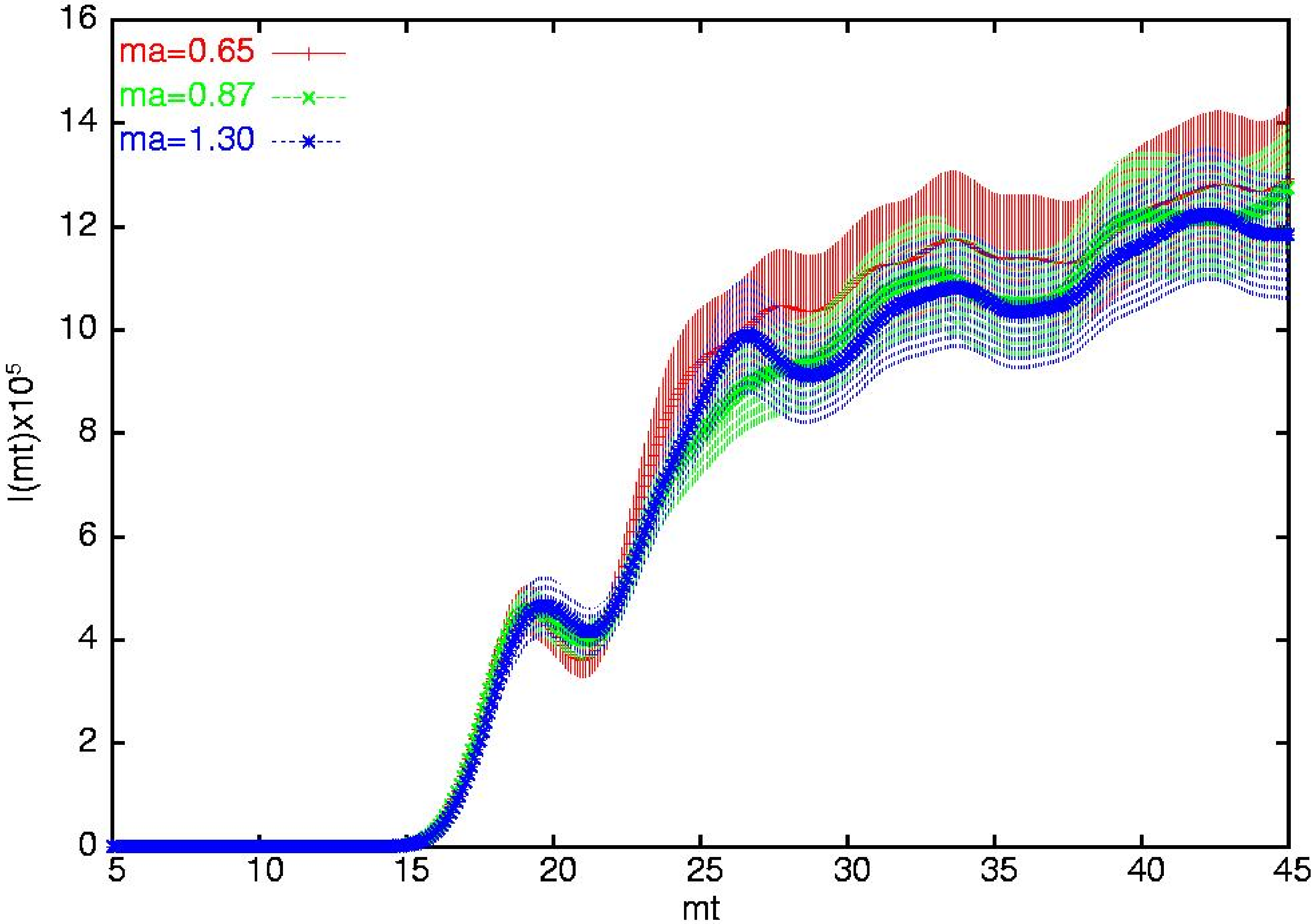



10
20
30
40

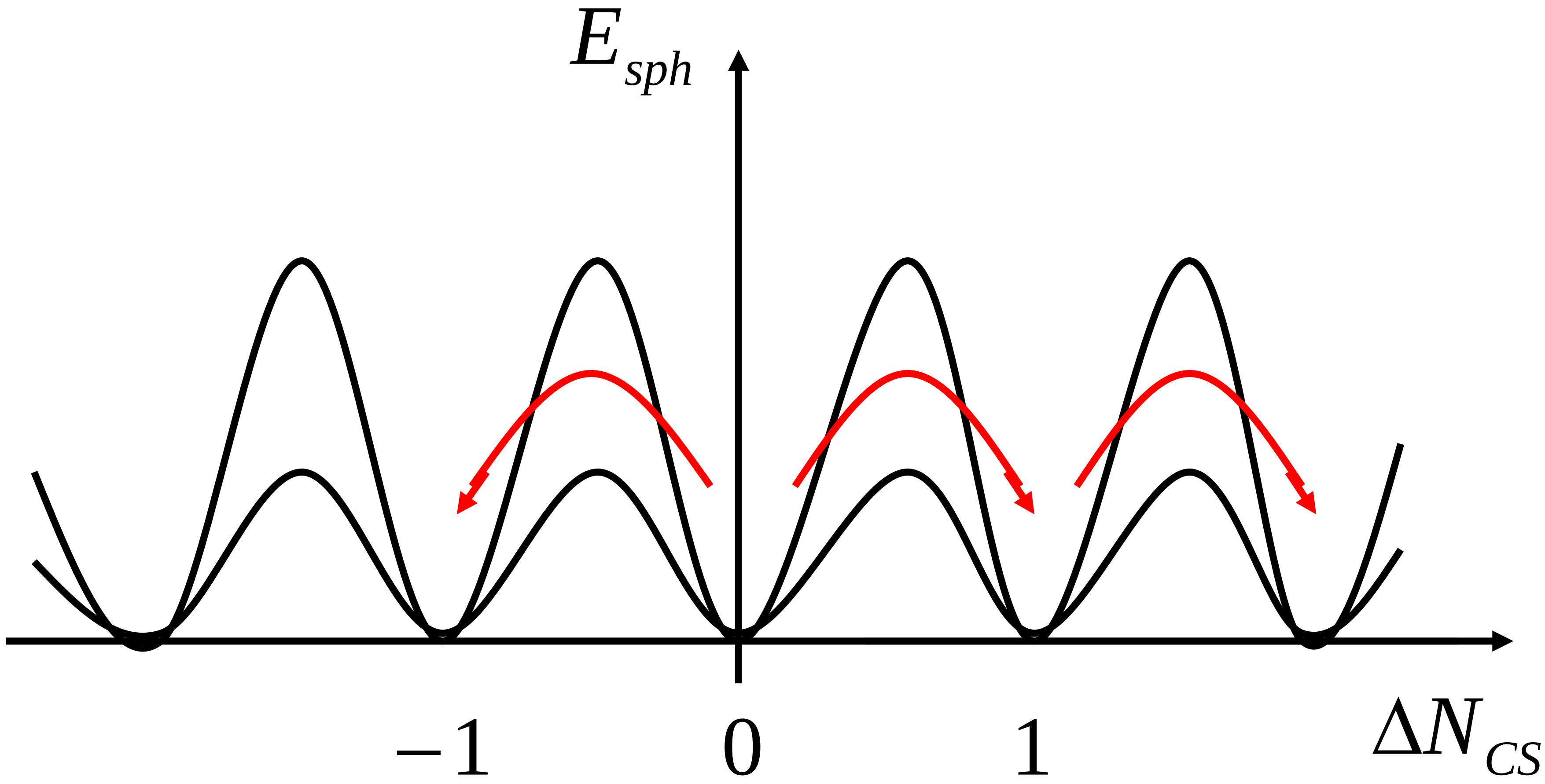
Peak in Chern-Simons Charge



Sphaleron Production $\left\langle \Delta N_{CS}^{2} \right\rangle$



Sphaleron Production $\langle \Delta N_{CS}^2 \rangle$



Cold EW Baryogenesis (I)

Baryonic current

$$j^{\mu} = \overline{\psi}_L \gamma^{\mu} \psi_L$$

Chiral anomaly

$$\partial_{\mu} j_B^{\mu} = \partial_{\mu} j_L^{\mu} \equiv \frac{3}{16\pi^2} Q(x,t)$$

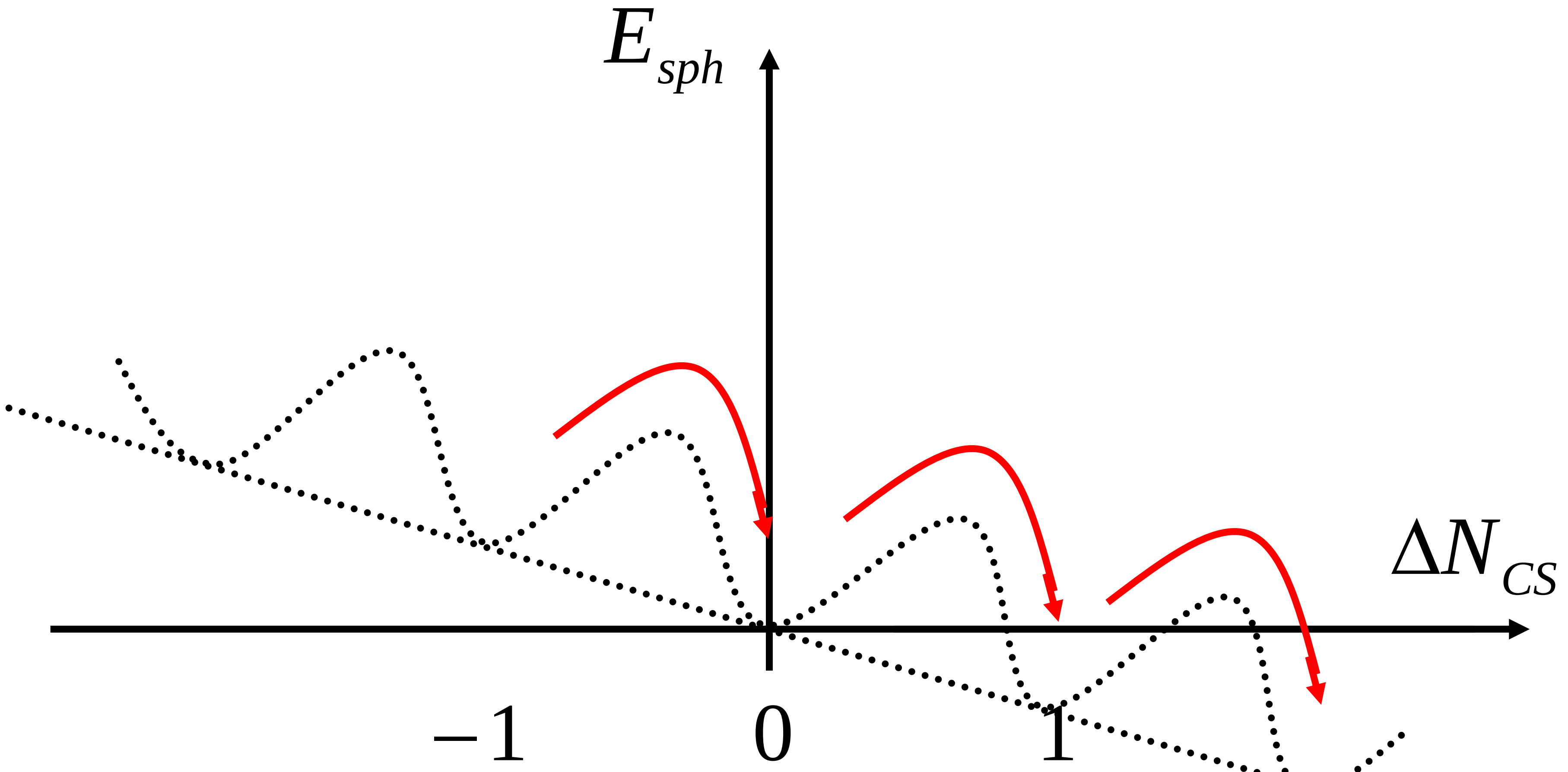
$$\Rightarrow \Delta B = \Delta L = 3\Delta N_{CS}$$

CP violation

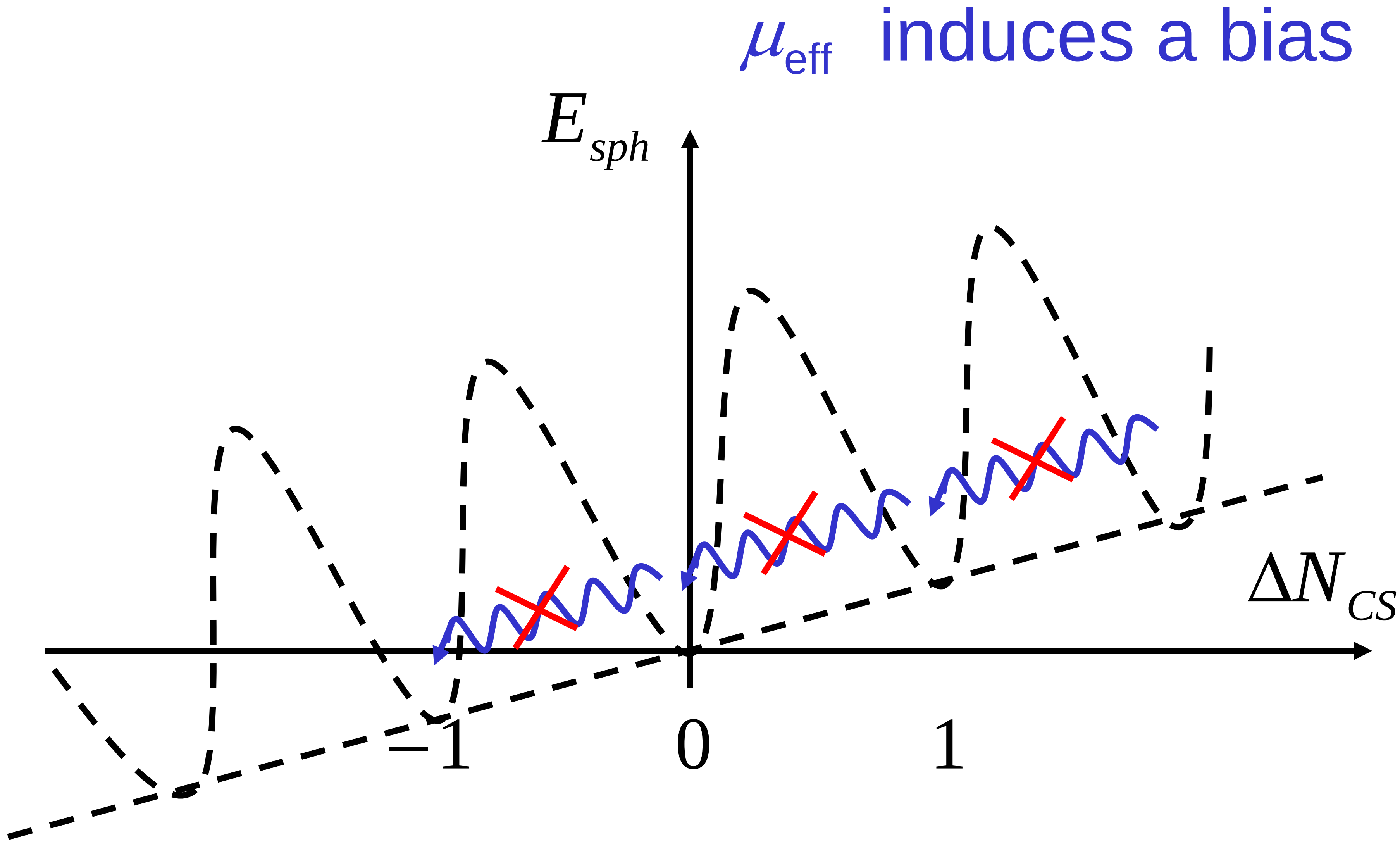
$$\mathrm{L}_{CP} = \delta_{CP} \frac{\Phi^{+}\Phi}{M_{\text{new}}^2} \frac{3g_w^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

CP violation

μ_{eff} induces a bias



CP violation



Cold EW Baryogenesis (II)

Effective potential $$\mu_{\text{eff}} = \frac{\delta_{CP}}{M_{\text{new}}^2} \frac{d}{dt} \left\langle \Phi^+ \Phi \right\rangle$$

Boltzman equation $$\frac{d}{dt} n_B = \mu_{\text{eff}} \frac{\Gamma_{\text{sph}}}{T_{\text{eff}}} - \Gamma_B n_B$$

$$\Rightarrow \quad \frac{n_B}{s} = 2 \times 10^{-6} \, \delta_{CP} \frac{v^2}{M_{\text{new}}^2} = 10^{-8} \, \delta_{CP}$$

EW Symmetry Breaking can lead to the production of baryons via sphaleron production at tachyonic preheating after hybrid inflation

The right amount of baryons depends on CP violation param.

Primordial Magnetic Fields

J. G.-B.
Andres Diaz-Gil
Margarita Garcia-Perez
Antonio Gonzalez-Arroyo

hep-lat/0509094

GGI 2006, Florence
6th September, 2006

EW Tachyonic Preheating

Spinodal growth of long wave Higgs modes

- At the end of EW Hybrid Inflation
- Inflaton couples to Higgs
- Higgs couples to SM fields
- Strong production of fermions and gauge fields

The SU(2)xU(1) Higgs-Inflaton model

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu} + Tr[(D_\mu\Phi)^+ D^\mu\Phi] + \frac{1}{2}(\partial_\mu\chi)^2 - V(\Phi,\chi)$$

$$D_\mu = \partial_\mu - \frac{i}{2}g_w A^a_\mu \tau_a - \frac{i}{2}g_Y B_\mu \tau_3$$

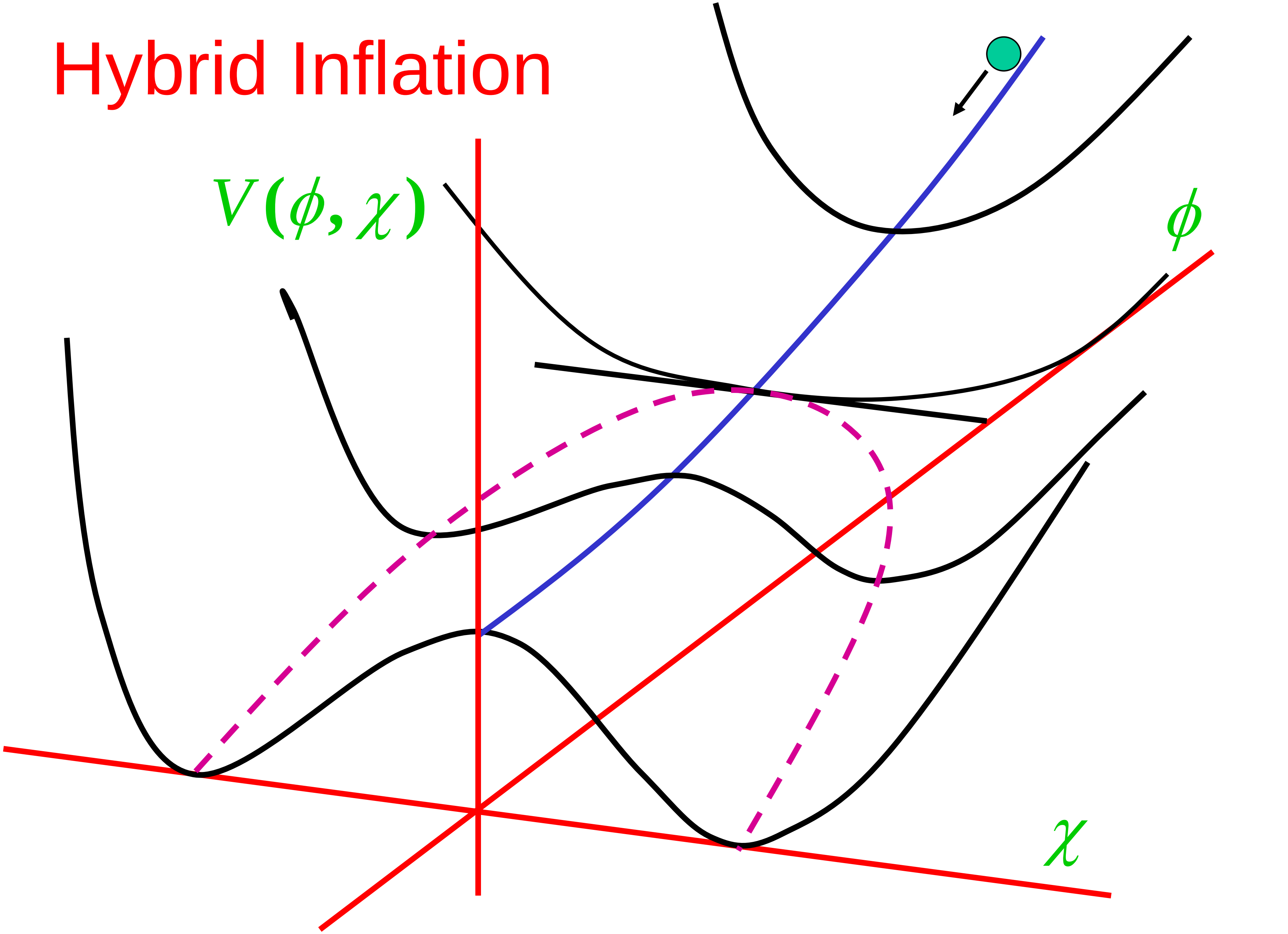
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_w \varepsilon^{abc} A^b_\mu A^c_\nu$$

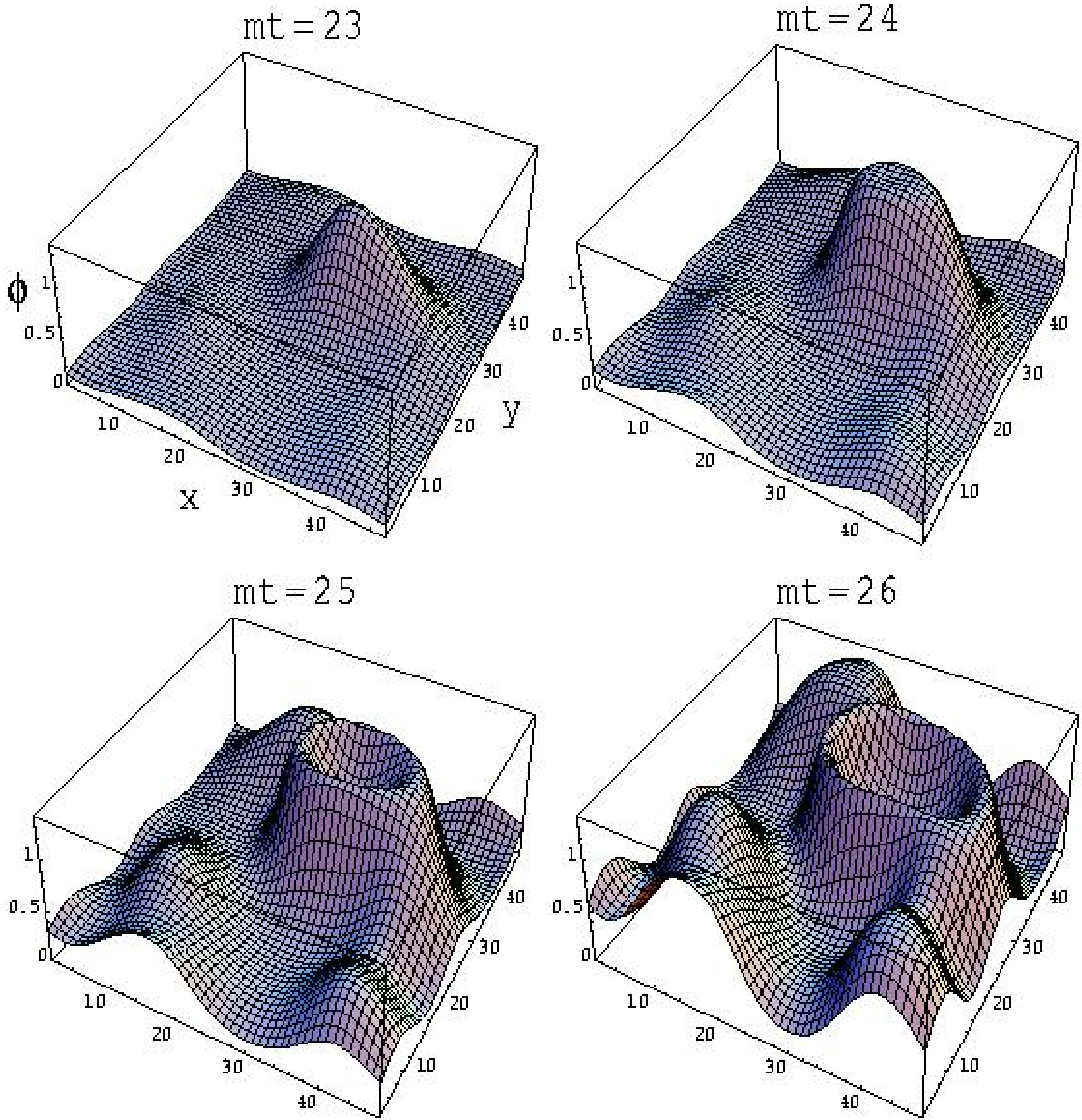
$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$Tr[\Phi^+\Phi] = \frac{1}{2}(\phi_0^2 + \phi^a\phi_a) \equiv \frac{1}{2}\phi^2$$

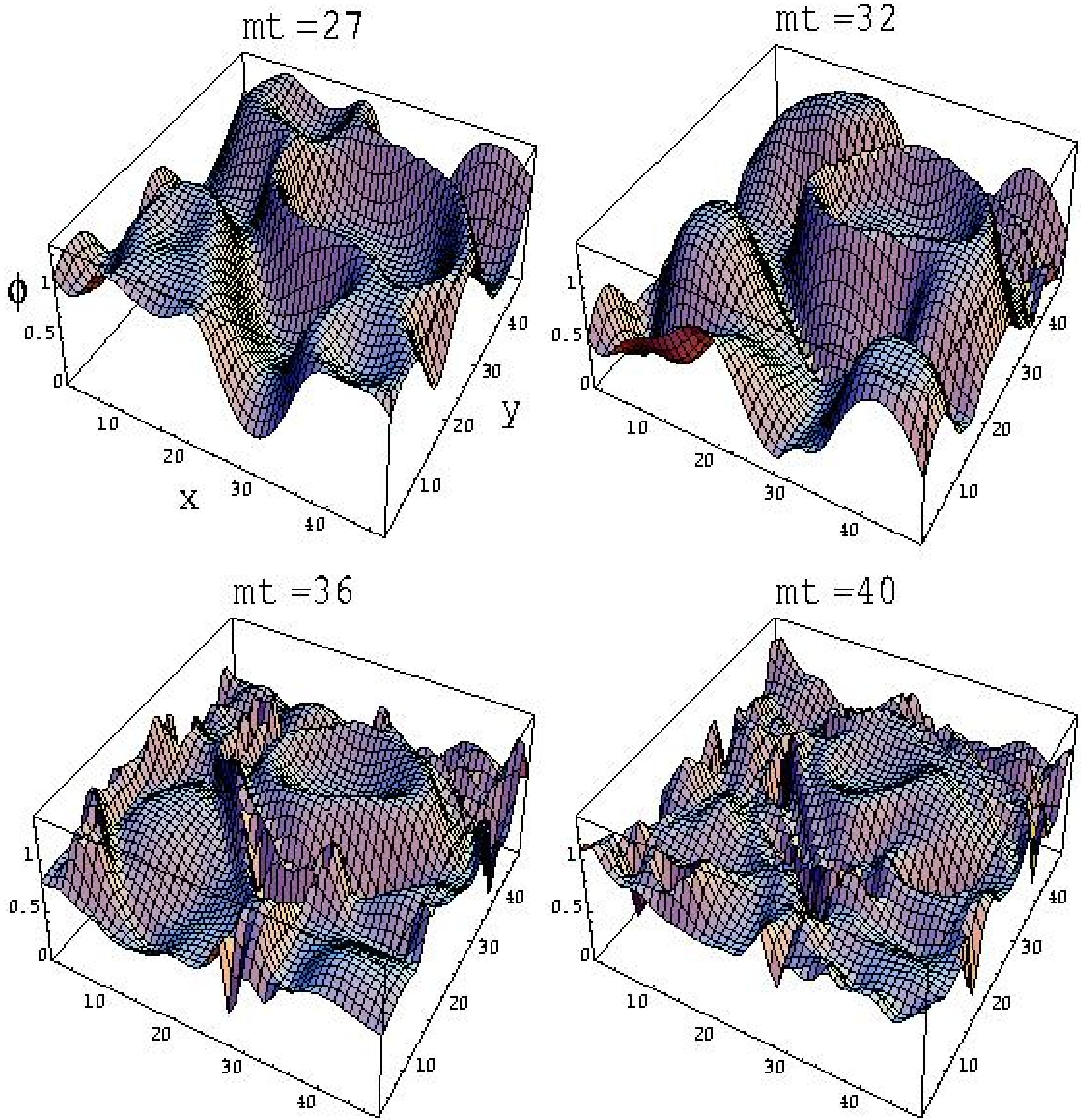
$$V(\phi,\chi) = \frac{\lambda}{4}(\phi^2 - v^2) + \frac{g^2}{2}\phi^2\chi^2 + \frac{1}{2}m^2\chi^2$$

Hybrid Inflation



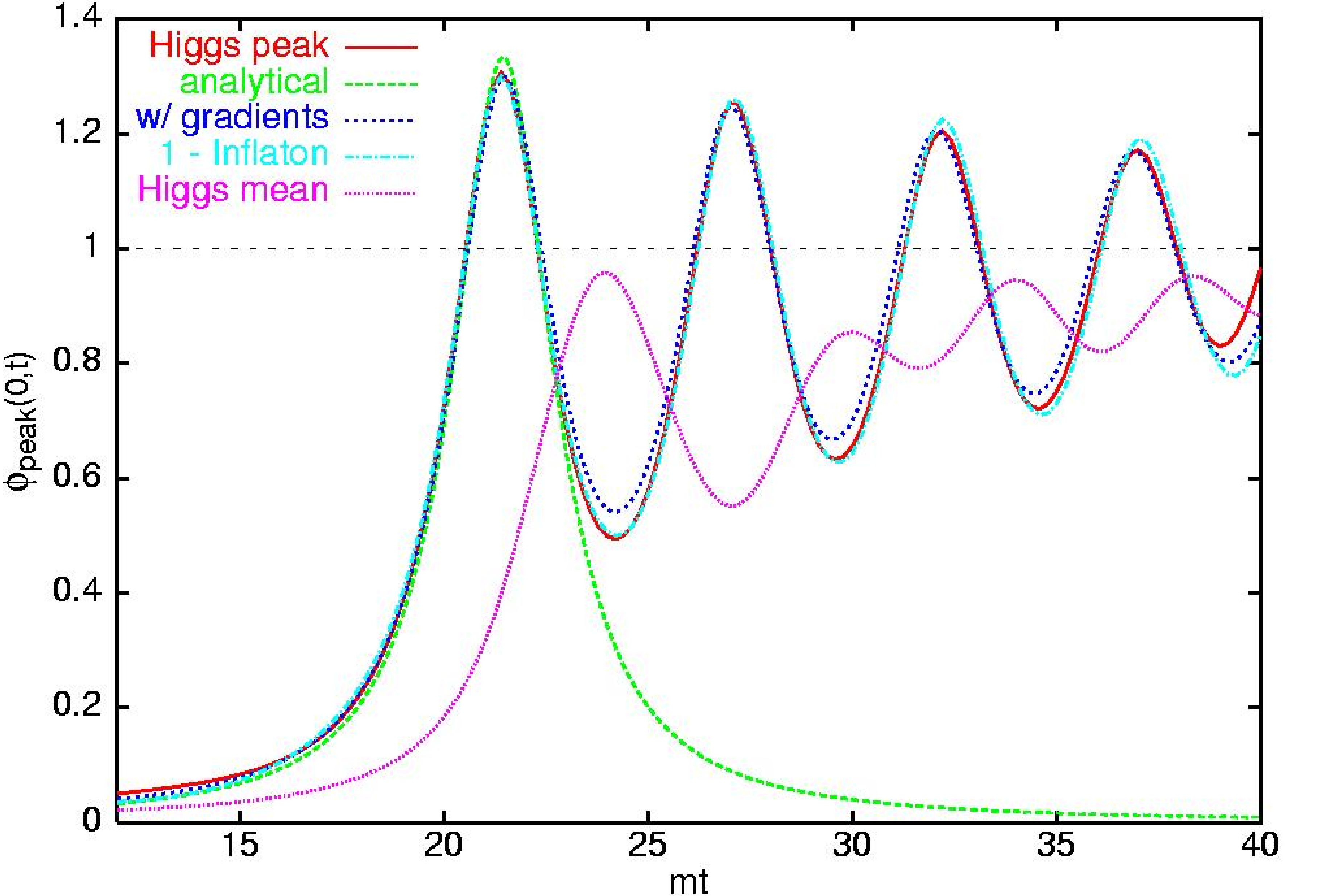


mt = 23
mt = 24
mt = 25
mt = 26
φ
x
y
0
0.5
1
10
20
30
40

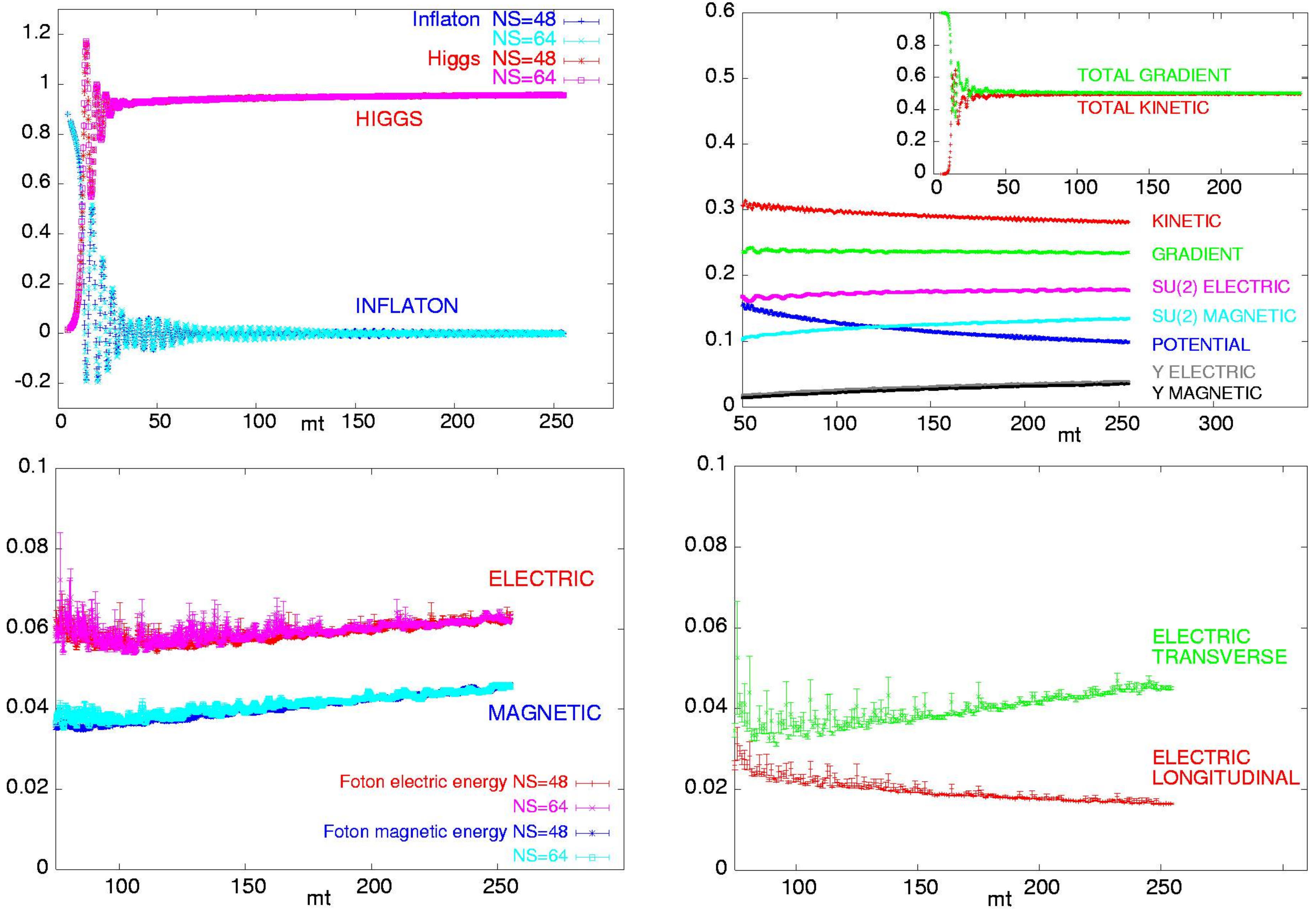


mt =27
mt =32
mt =36
mt =40
ϕ
x
y

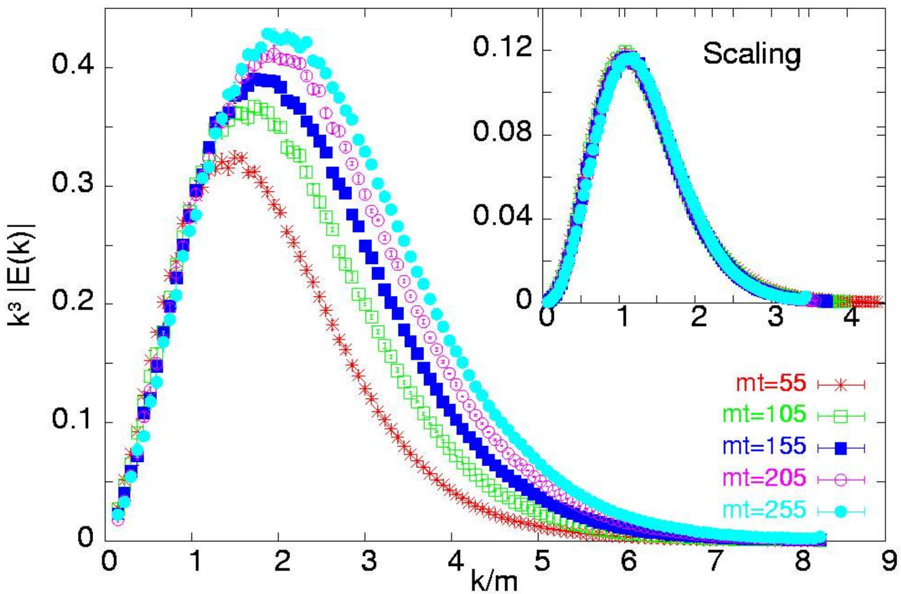
High peaks and mean of Higgs field



Evolution after EWSB



Kinetic Turbulence & Scaling



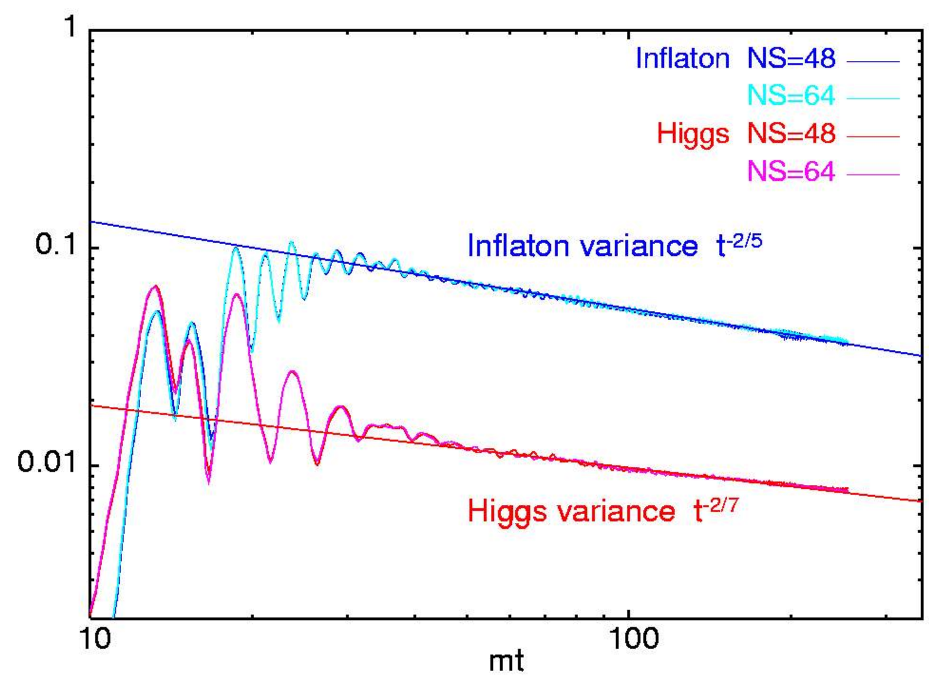
$$n(k,t) = t^{-q} n_0(kt^{-p})$$

$$q = 3.5p$$

$$p = \frac{1}{2m-1}$$

$$\Delta\phi^2 = \langle\phi^2\rangle - \langle\phi\rangle^2 \propto t^{-\nu}$$

$$\nu = \frac{2}{2m-1}$$



The amplitude of magnetic fields

$$\rho_{mag} \leq 10^{-4} V_0 \approx 10^{-4} m_H^2 v^2 = (10\, GeV)^4$$

$$\rho_{mag}^{(0)} = \left(\frac{a_{rh}}{a_0} \right)^4 \rho_{mag} \approx (0.3\, \mu G)^2$$

$$\frac{1}{8\pi} Gauss^2 = 1.39 \times 10^{-42} GeV^4$$ Conversion factor

The coherence scale of magnetic fields

$\xi \propto t$ During kinetic turbulence

$\xi \propto a(t)$ After e+e- annihilation

$$\xi(today) \approx 10 - 100\, kpc$$

EW Symmetry Breaking can lead to the production of primordial magnetic fields at tachyonic preheating after hybrid inflation

The right amplitude and scale of magnetic fields depends on the extent of kinetic turbulence

Initial conditions for magneto-Hydrodynamic simulations