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Large-N expansions and equivalences

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(To Giuliano Toraldo di Francia, remarkable man and teacher)

From my talk here June 2008

Is the time ripe for a large-N workshop at the GGI?

Outline

* Prehistory (= pre 't Hooft)
* History
* Orientifold planar equivalence
    * Arguments, counter-arguments, dust-settling
    * SUSY relics in QCD?
* Large-N volume independence (EK-reduction)
    * KUY's proposal
    * Further developments
* A 2D toy model
* Outlook

w/ apologies for so many topics left out...

Prehistory (1970-'74)

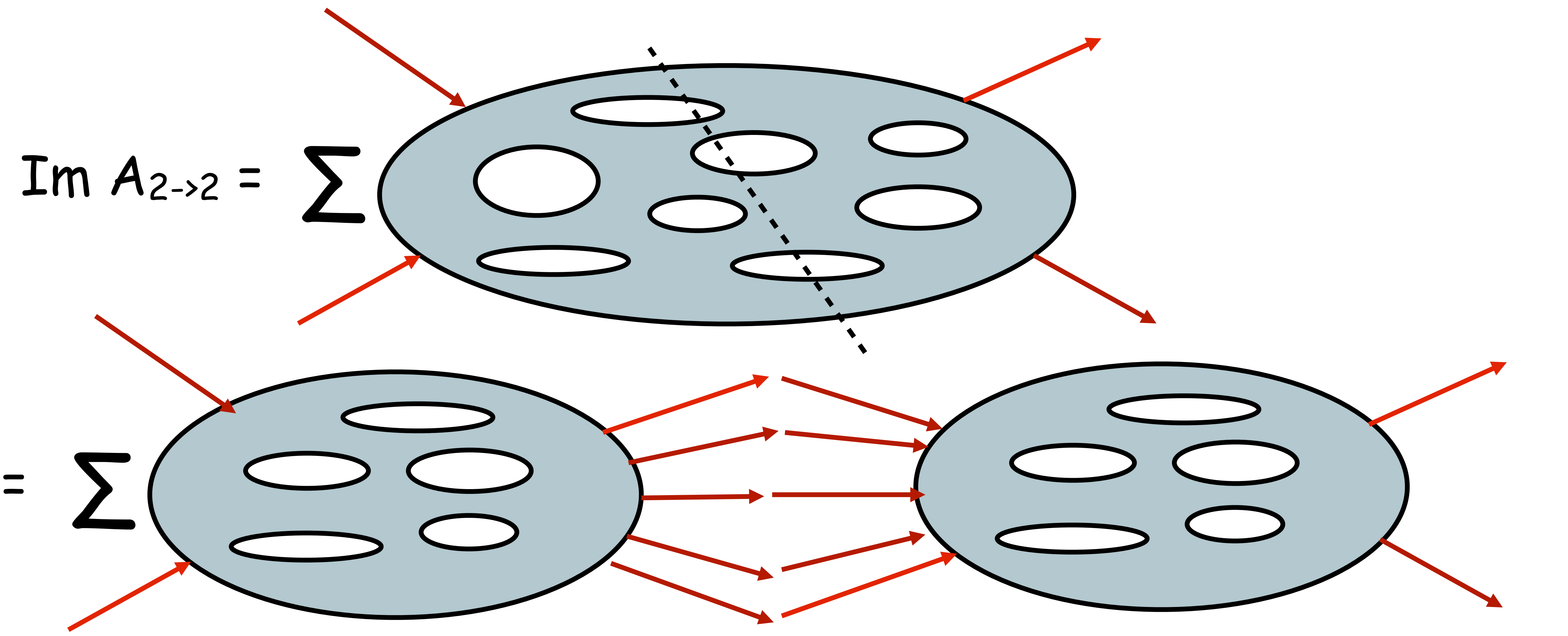
Di Giacomo, Fubini, Sertorio, GV (1970)

- Implementing unitarity in Dual Resonance Models (DRM) looked like "mission impossible".
- In QFT resonances get a width only after resumming an infinite number of bubble diagrams. But in DRM bubble and non-bubble diagrams come together.
- Idea: isolate a minimal subclass of diagrams giving finite widths while preserving some simple topology.
- Then add more complicated topologies.

Simplest topology: planar diagrams

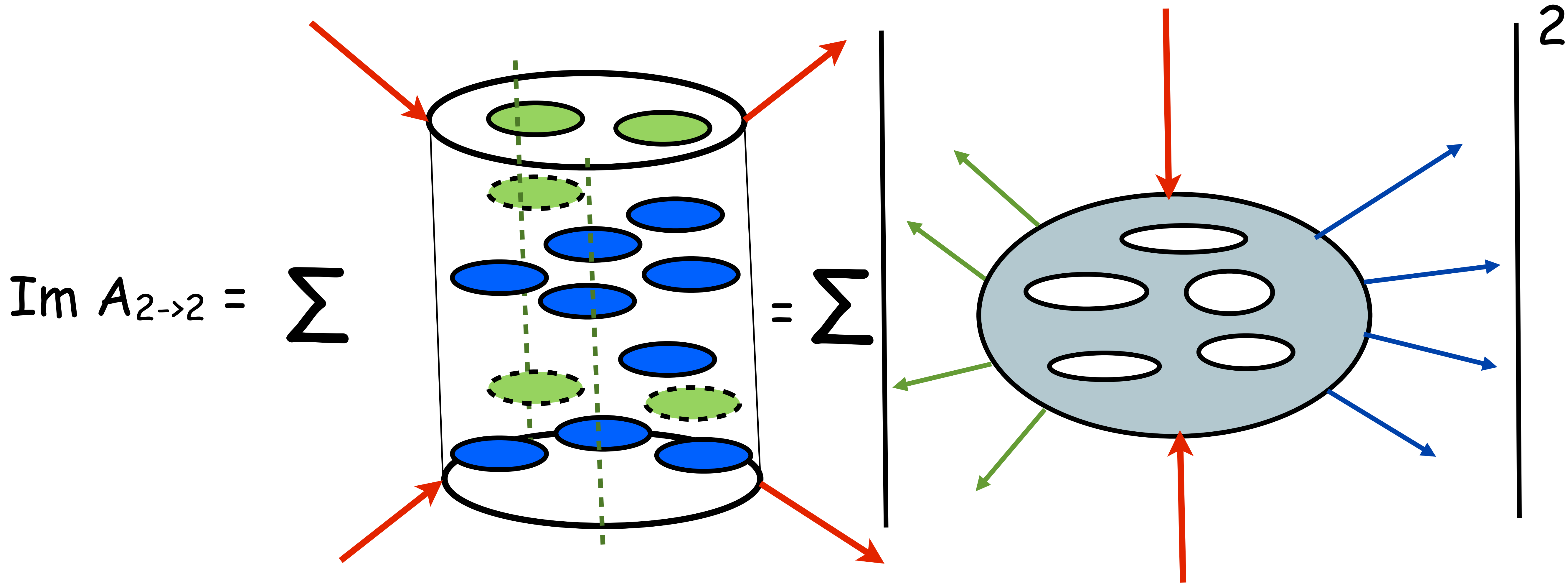
=> Planar unitarity (for $q\bar{q}$ mesons)

=> Regge trajectory, with $\alpha_R(0) \sim 1 - d\langle n\rangle/dy < 1$



Next to simplest topology: cylinder diagrams

- Cylinder topology => (bare, soft) Pomeron with $\alpha_P(0) = 1$ up to correlations (Huan Lee, GV ~1973)



Higher topologies => Gribov's RFT

(Ciafaloni, Marchesini & GV, 1974)

- Hard to sell...
- Then came 't Hooft's $1/N_c$ ...
- Topological reasoning in DRM reinterpreted in terms of a $1/N_f$ expansion (GV 1974).
- ... but then I basically gave up on hadronic string and switched to QCD...

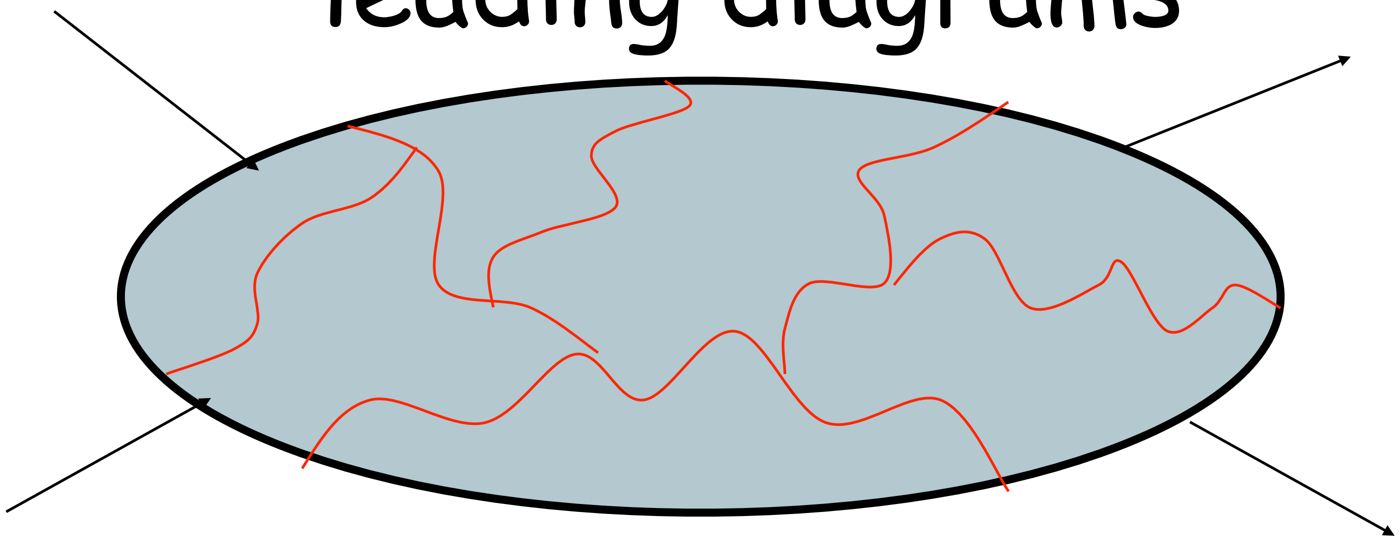
History

Large-N expansions in QCD

Planar & quenched limit ('t Hooft, 1974)

$1/N_c$ expansion @ fixed $\lambda = g^2 N_c$ and N_f

leading diagrams



Corrections: $O(N_f/N_c)$ from q-loops,

$O(1/N_c^2)$ from higher-genus diagrams

Properties at leading order

1. Resonances have zero width
2. U(1) problem not solved, WV @ NLO
3. Multiparticle production not allowed

Theoretically appealing: should give the **tree level** of **some** kind of string theory

Proven hard to solve, except in D=2....

Planar limit = Topological Expansion (GV, 1976)

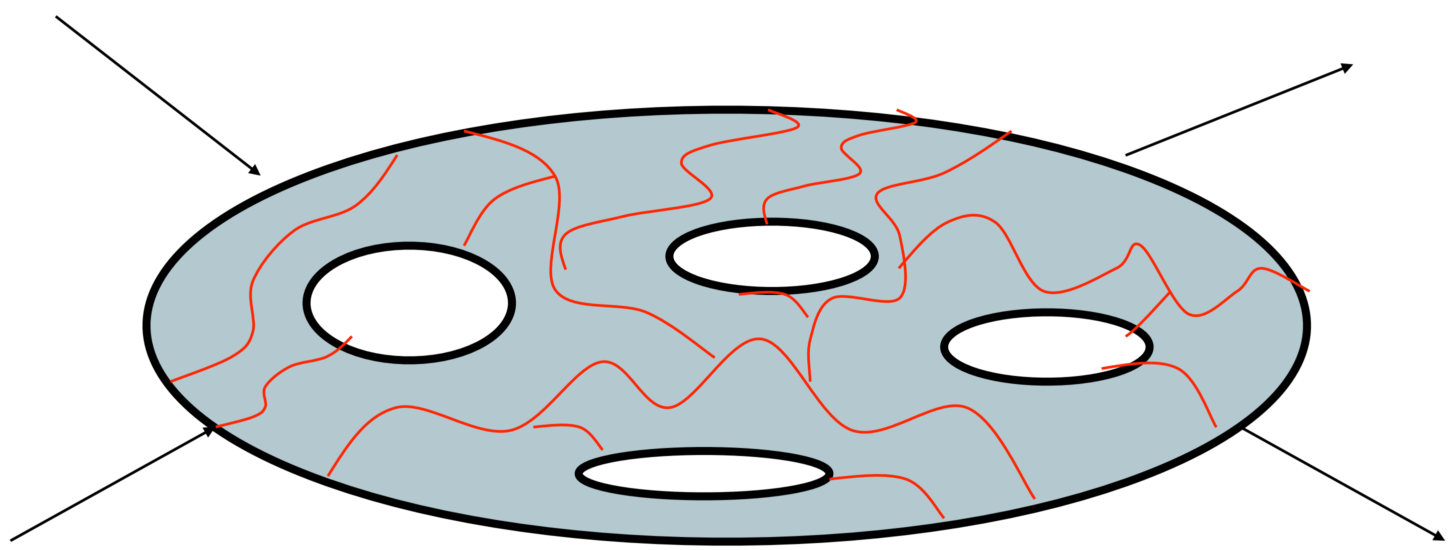
= 1/N expansion at fixed g^2N <u>and</u> N_f/N_c

Leading diagrams planar but include "empty" q-loops

Corrections: $O(1/N^2)$ from non-planar diagrams

First discussion of necessity & properties*) of glueballs @ large N?

*) e.g. mixing of glueballs and mesons



Properties at leading order

1. Widths are O(1)
2. U(1) problem solved to leading order, no reason for WV to be good (small N_f/N_c?)
3. Multiparticle production allowed

   => Bare Pomeron & Gribov's RFT
4. Phase transitions at critical values of N_f/N_c (conformal windows, loss of AF)

Perhaps phenomenologically more appealing than 't Hooft's but even harder to solve...

But there is a third possibility…

Generalize QCD to $N \neq 3$ ($N = N_c$ hereafter) in other ways by **playing with matter rep**. The conventional way, QCD_F, is to keep the quarks in $(N + N^*)$ rep.

Another possibility, called for stringy reasons QCD_{OR}, is to assign quarks to the 2-index-antisymm. rep. of SU(N) (+ its c.c.).

As in 't Hooft's exp. (and unlike in TE), N_f is kept fixed ($N_f < 6$, or else AF lost at large N).

NB: For **N = 3** this is still **good old QCD!**

Leading diagrams are planar, include "filled" q-loops since there are $O(N^2)$ quarks

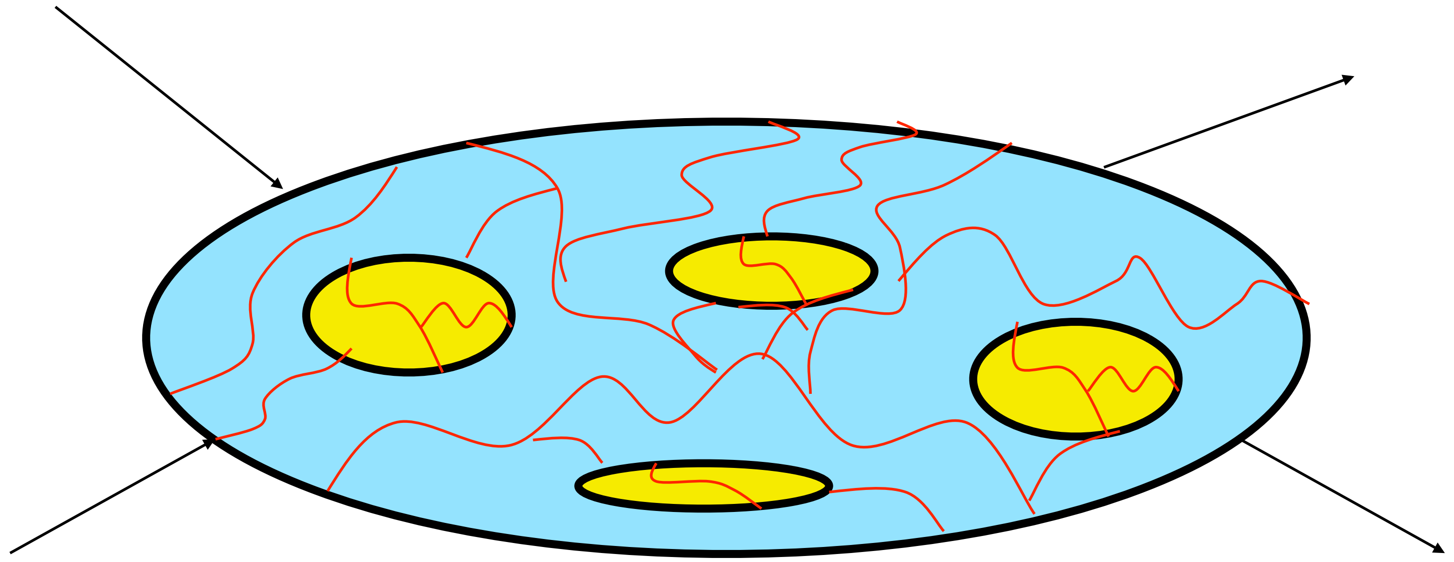
Widths are zero, U(1) problem solved, no p.pr.

Phenomenologically interesting?

Better manageable? In some cases, I will claim…

QCD_{OR} as an interpolating theory:

1. Coincides with pure YM (AS fermions decouple) @ N=2
2. Coincides with QCD @ N=3
3. … and at large N?



Armoni-Shifman-GV claim (2003)

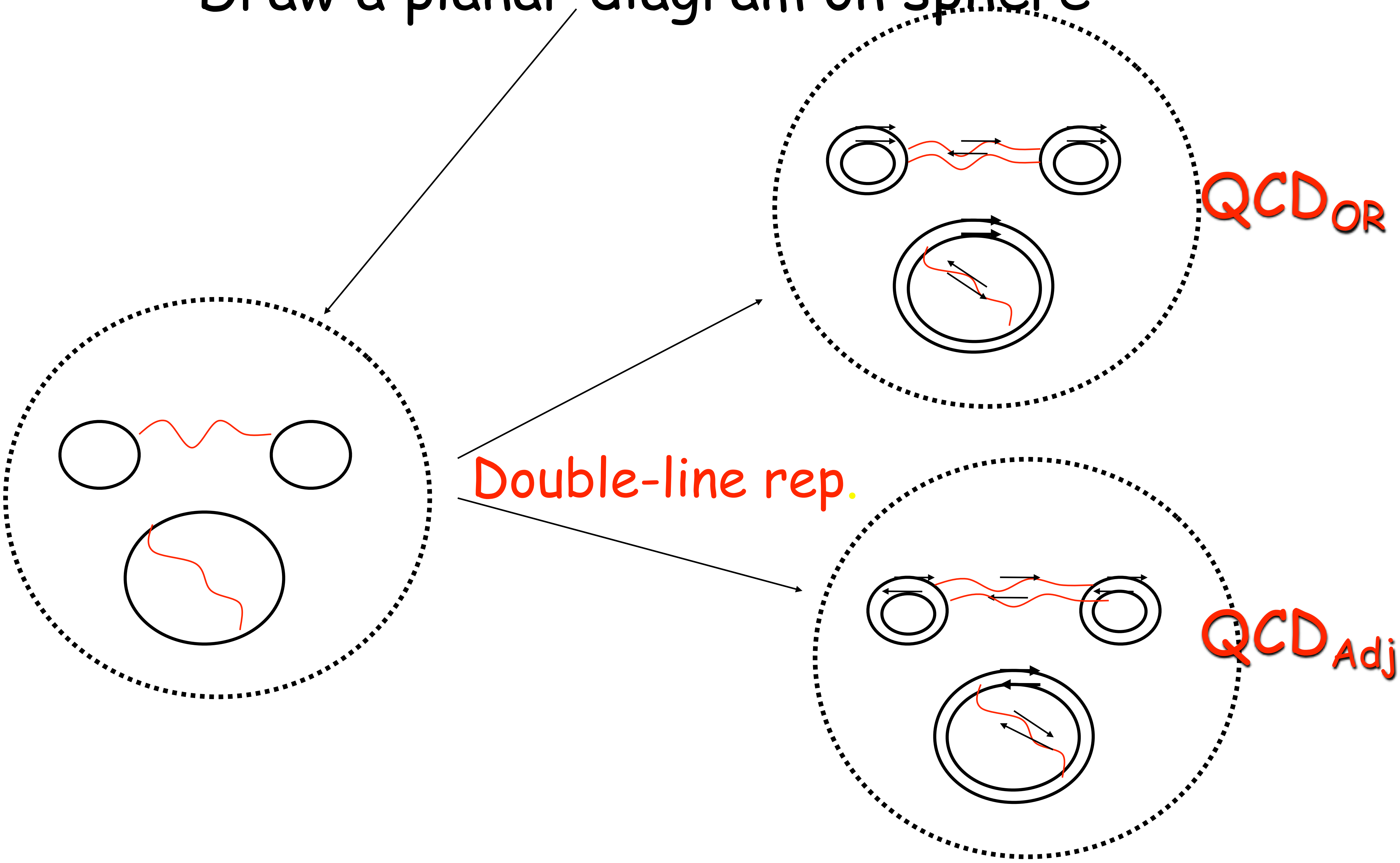
At large-N a bosonic sector of QCD_{OR} is equivalent to a corresponding sector of QCD_{Adj} i.e. of QCD with N_f Majorana fermions in the adjoint representation.

NB: Expected accuracy 1/N hopefully improved by interpolation w/ N=2 case (Cf. N_f/N_c of 'tH!)

ASV gave both perturbative and NP arguments

Perturbative arguments, checks

Draw a planar diagram on sphere



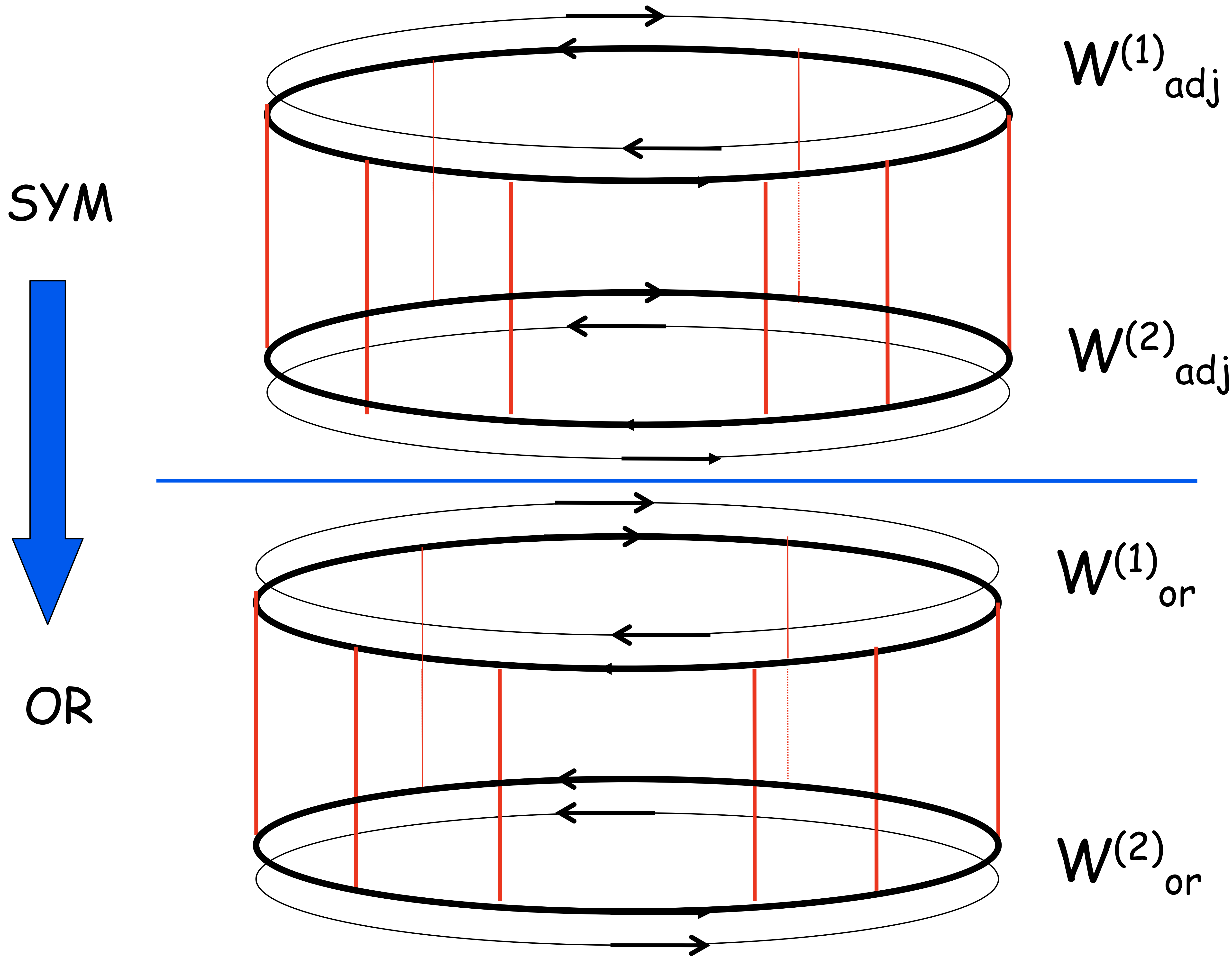
Differ by an even number of - signs...

Sketch of non-perturbative argument

(ASV '04, A. Patella, '05 + thesis '08)

- Integrate out fermions (after having included masses, bilinear sources)
- Express Trlog($\not{D}$+m+J) in terms of Wilson-loops using world-line formulation (expansion convergent?)
- Use large-N to write adjoint and AS Wilson loop as products of fundamental and/or antifundamental Wilson loops (e.g. $W_{adj} = W_F \times W_{F^*} + O(1/N^2)$)
- Use symmetry relations between F and F* Wilson loops and their connected correlators

An example: $\langle W^{(1)}\, W^{(2)} \rangle_{conn}$



SYM
OR
$W^{(1)}_{adj}$
$W^{(2)}_{adj}$
$W^{(1)}_{or}$
$W^{(2)}_{or}$

Key ingredient is C!

- Clear from our NP proof that C-invariance is necessary. Kovtun, Unsal and Yaffe have argued that it is also sufficient
- U&Y (see also Barbon & Hoyos) have also shown that C is spontaneously broken if the theory is put on $R^3 x S^1$ w/ small enough S^1. PE doesn't (was never claimed to) hold in that case
- Numerical calculations (De Grand and Hoffmann) have confirmed this, but also shown that C is restored for large radii and in particular on R^4
- Lucini, Patella & Pica have shown (analyt.lly & numer.lly) that SB of C is also related to a non-vanishing Lorentz-breaking F#-current generated at small R but disappearing as well as R is increased

Uncontroversial formulation of PE?

Provided that C is not spontaneously broken, the C-even bosonic sector of QCD_{OR} (with N_f Dirac fermions in the 2-index AS or S representation) is planar-equivalent to the corresponding sector of QCD_{Adj} i.e. of QCD with N_f Majorana fermions in the adjoint representation.

Corollary: for $N_f = 1$ and $m = 0$, QCD_{OR} is planar-equivalent to supersymmetric Yang-Mills (SYM)theory

Some properties of the latter should show up in one-flavour QCD… if $N = 3$ is large enough.

But PE has even more interesting consequences…

Incidentally...

Irrespectively of PE, it would be interesting to study (unquenched) QCD_{adj} for its own sake, e.g.

- As one varies N_f, the singlet PS mass should grow like N_f & coincide with the singlet S mass at N_f=1, m=0
- For N_f=1, m≠0 one should recover the behaviour of SYM when SUSY and Z_{2N} are softly broken (degeneracy of N-vacua is lifted, multiplets split etc.)
- Last but not least: to check volume independence (see below)

SUSY relics in one-flavour QCD

① Approximate bosonic parity doublets:

$m_S = m_P = m_F$ in SYM => $m_S \sim m_P$ in QCD

Looks ~ OK if can we make use of:

i) WV for m_P ($m_P \sim \sqrt{2}(180)^2/95$ MeV ~ 480 MeV),

ii) Experiments for m_S (σ @ 600MeV w/ quark masses)

Lattice work by Keith-Hynes & Thacker also support this approximate degeneracy

② Approximate absence of "activity" in certain chiral correlators

In SYM, a well-known WI gives

$$\langle\lambda\lambda(x)\lambda\lambda(y)\rangle = const.\,, \quad \langle\lambda\lambda(x)\bar{\lambda}\bar{\lambda}(y)\rangle \neq const.$$

PE then implies that, in the large-N limit:

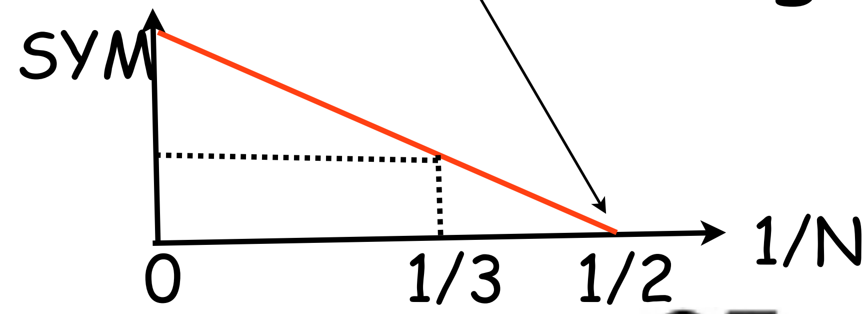
$$\langle\bar{\Psi}_R\Psi_L(x)\bar{\Psi}_R\Psi_L(y)\rangle = const.\,, \quad \langle\bar{\Psi}_R\Psi_L(x)\bar{\Psi}_L\Psi_R(y)\rangle \neq const.$$

Of course the constancy of the former is due to an exact cancellation between intermediate scalar and pseudoscalar states.

The quark condensate in N_f=1 QCD

Using $\langle\bar{\lambda}\lambda\rangle_\mu = -\frac{9}{2\pi^2}\mu^3\lambda_\mu^{-2} exp\left(-\frac{1}{\lambda_\mu}\right)$ $\lambda_\mu = \alpha_s(\mu)N/2\pi$

and vanishing of quark cond. at N=2, we get



$$\langle\bar{\psi}\psi\rangle_\mu = -\frac{3}{2\pi^2}\mu^3\lambda_\mu^{-1578/961} exp\left(-\frac{27}{31\lambda_\mu}\right) k(1/3)$$

1±0.3?

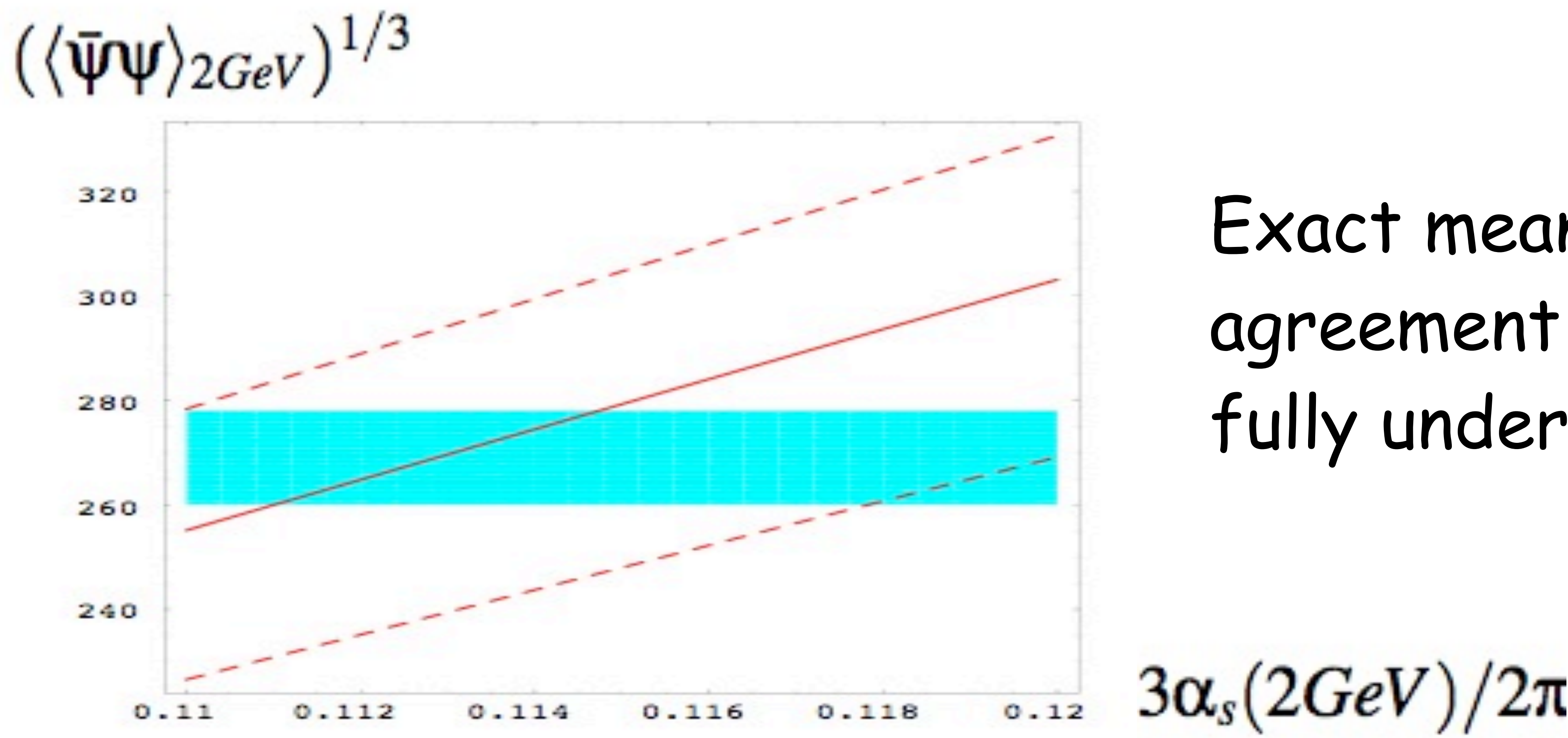
$$< (g^2)^{12/31}\bar{\psi}\psi > = -1.1k(1/3)\Lambda_{st}^3$$

$$\Lambda_{st} = \mu\ exp\left[-\frac{N}{\beta_0\lambda_\mu}\right]\left(\frac{2N}{\beta_0\lambda_\mu}\right)^{\beta_1/\beta_0^2}$$

N_f=1 condensate "measured"?

DeGrand, Hoffmann, Schaefer & Liu, hep-th/0605147

(using dynamical overlap fermions and distribution of low-lying eigenmodes)



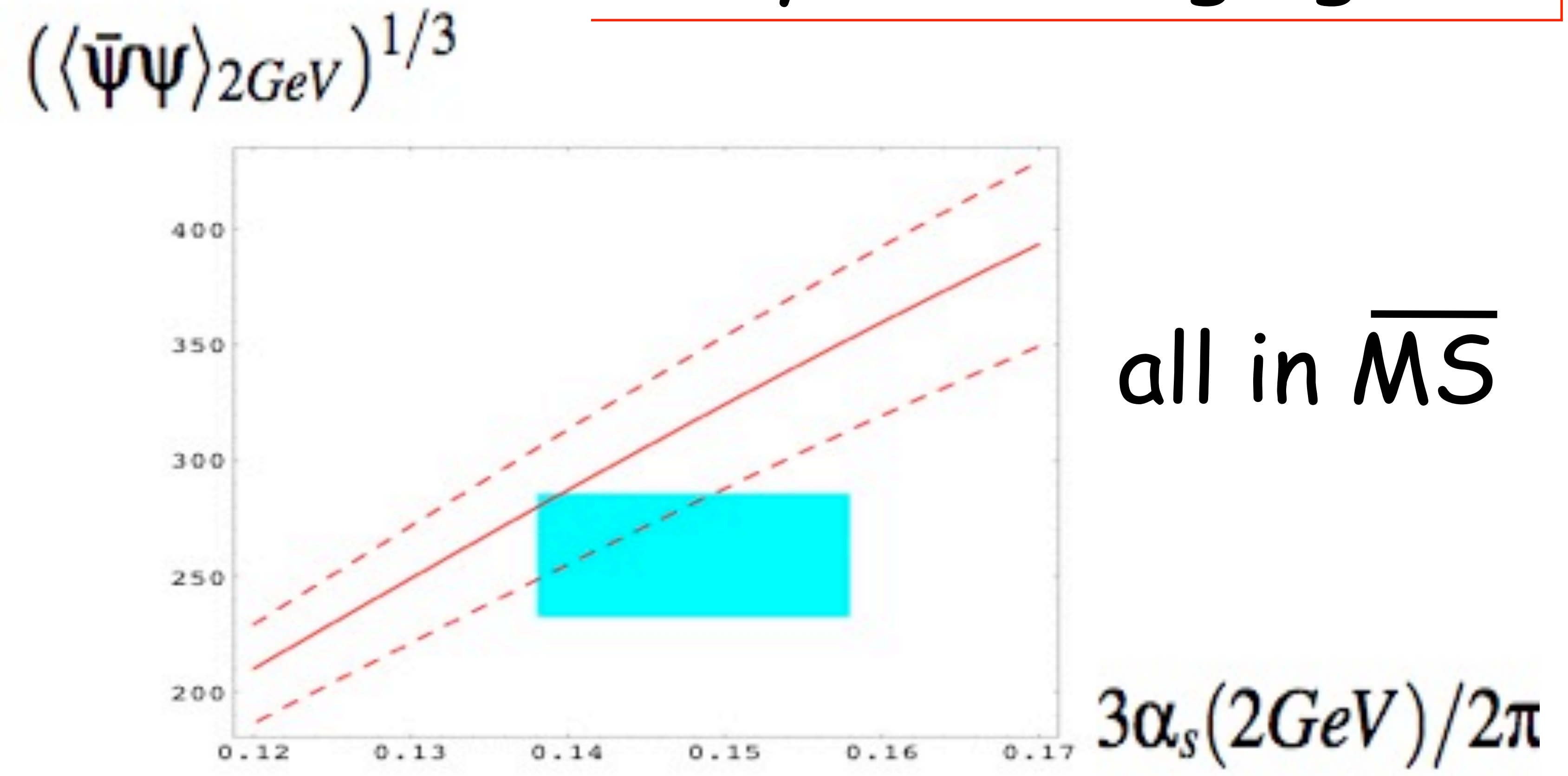
Exact meaning of agreement still to be fully understood

Extension to $N_f > 1$
(Armoni, G. Shore and GV, '05)

- Take OR theory and add to it n_f flavours in N+N* .
- At N=2 it's n_f-QCD, @ N=3 it's $N_f(= n_f+1)$-QCD.
- At large N cannot be distinguished from OR (fits SYM β-functions even better at $n_f = 2$: e.g. same β_0)
- Vacuum manifold, NG bosons etc. are different!
- Some correlators should still coincide in large-N limit. In above paper it was argued how to do it for the quark condensate

Quark condensate (ren. @ 2 GeV) vs α_s(2GeV) for N_f=3

Very encouraging!



all in $\overline{MS}$

$$\langle\bar{\psi}\psi\rangle_\mu = -\frac{3}{2\pi^2}\mu^3\lambda_\mu^{-\frac{44}{27}} exp\left(-\frac{1}{\lambda_\mu}\right)$$

Cf. $$\langle\bar{\lambda}\lambda\rangle_\mu = -\frac{9}{2\pi^2}\mu^3\lambda_\mu^{-2} exp\left(-\frac{1}{\lambda_\mu}\right)$$

Volume Independence (KUY 0702.021)

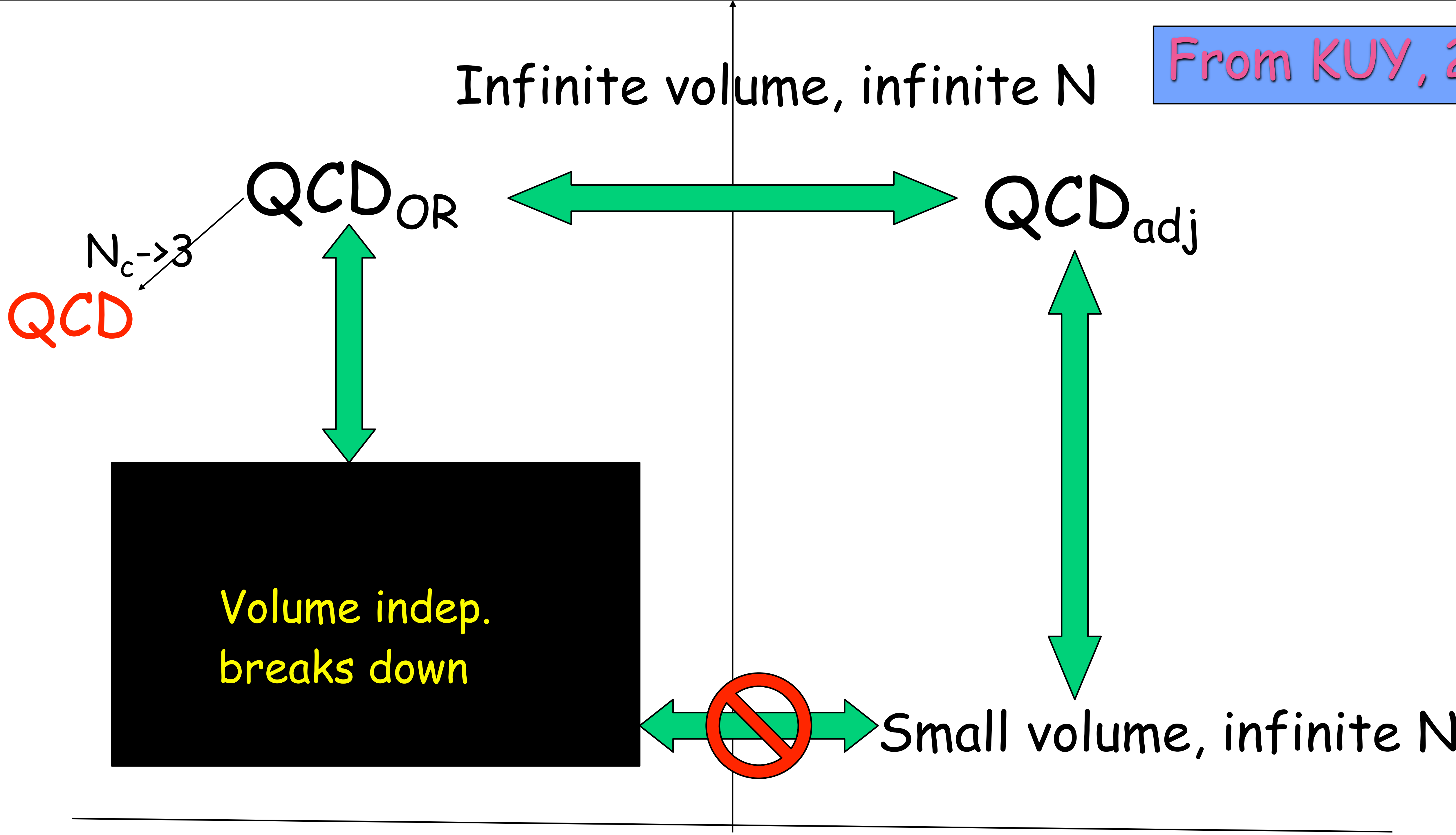
Kovtun, Unsal & Yaffe have made the interesting claim that QCD_{adj}, at small enough mass and unlike YM, QCD_F or QCD_{OR}, suffers no phase transition as an Eguchi-Kawai volume-reduction is performed at large-N.

Reason: adjoint (periodic) fermions help keeping the center symmetry unbroken.

If so, we can get properties of QCD_{adj} at small volume by numerical methods and use them at large volume where the connection to QCD_{OR} can be established (C being unbroken there).

Finally, one would make semi-quantitative predictions for QCD itself (in an interesting range for N_f and quark masses) by extra(inter)polating to N=3.

From KUY, 2007



Bottom line combining PE and VI

Solving QCD$_{adj}$ at infinite N and small volume should provide an $O(1/N_c)$ approximation to QCD with < 6 light flavours

(Some) further developments

Bringoltz & Sharpe (0906.3538)

Found supporting evidence for the KUYconjecture by numerical simulations of QCD on T^4 with a single Dirac (i.e. 2 Majorana) adjoint fermion(s)

$N = 8, \ldots 15$, $\kappa = 0.05, \ldots 0.2$ (i.e. around $\kappa = 1/8$)

Center symmetry looks OK for a single site lattice!

see also analytic lattice arguments by:

Poppitz & Unsal, 0911.0358

and a review of previous numerical work:

Narayanan & Neuberger, 0710.0098

Unsal & Yaffe (1006.2102)

- Analytic considerations on large-N volume independence of both conformal and confining theories.
- QCD compactified on $R^{4-k} \times (S^1)^k$ for k = 1,2,3,4 and with a number N_F of adjoint Majorana fermions.
- In particular: for k = 1 (or 2) discuss center symmetry Z_N (or Z_N^2) and its possible breaking via Wilson lines as a function of the fermion mass m and the size(s) L (L_1, L_2) of the circle(s).
- In confining case with scale Λ: at $\Lambda L \ll 1$ and m=0 volume independence holds for all N_F including SYM (N_F = 1).

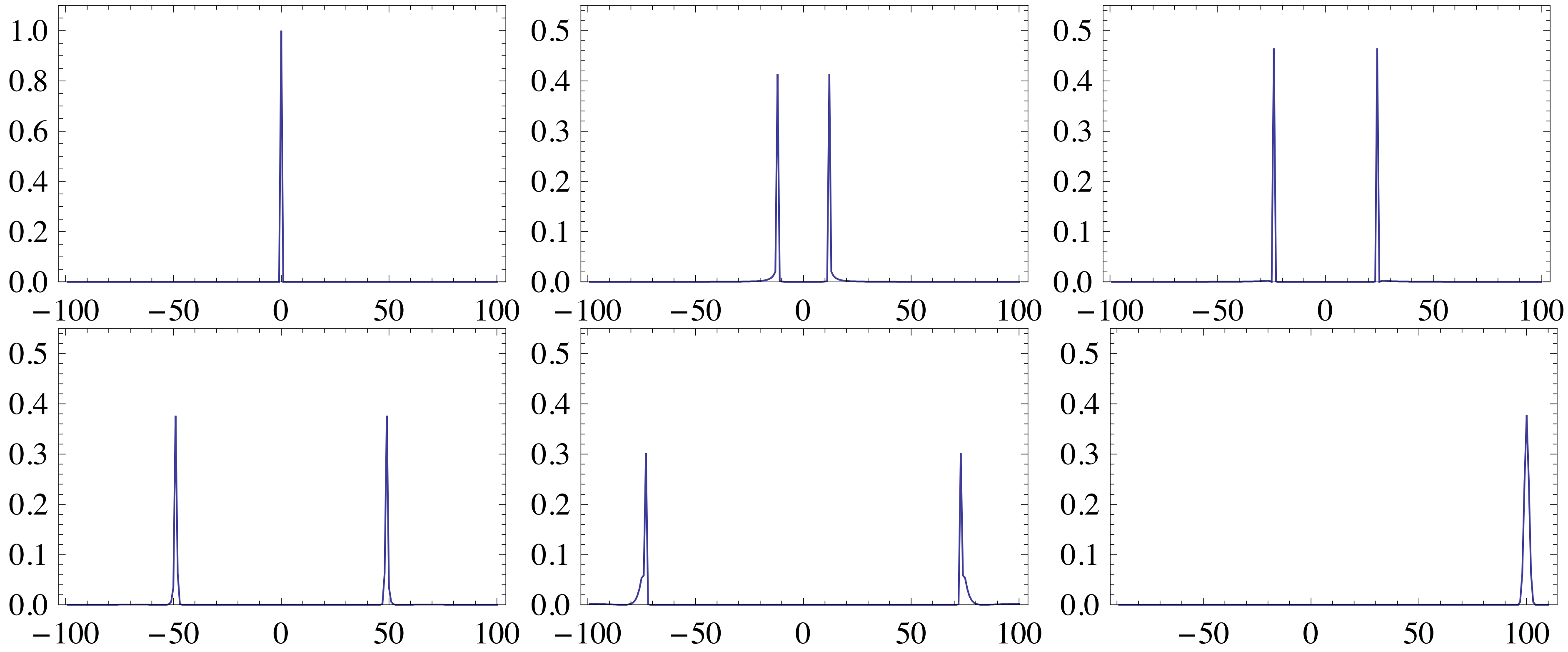
- Interesting observation: in unbroken-Z_N phase the KK modes have spacing $1/NL$ (instead of usual $1/L$) since the non-trivial Wilson lines shift masses in multiples of $1/NL$.
- Some KK modes could be necessary at large N (even for very small ΛL) in order to recover the infinite-V theory.
- @ $L \ll 1/N\Lambda$ semiclassical, abelian conf., no VI
- @ $L \gg 1/N\Lambda$ quantum, non-abelian conf., VI
- Semiclassical analysis sometimes unreliable. E.g. k=2 with frozen values of the Wilson lines one would break Z_N^2 but quantum effects can lead to tunneling among the different center-symmetry-related semiclassical vacua and fully restore the symmetry.
- Q: What happens to KK spacing in that case?

A 2D toy model (see also J.Wosiek's talk)

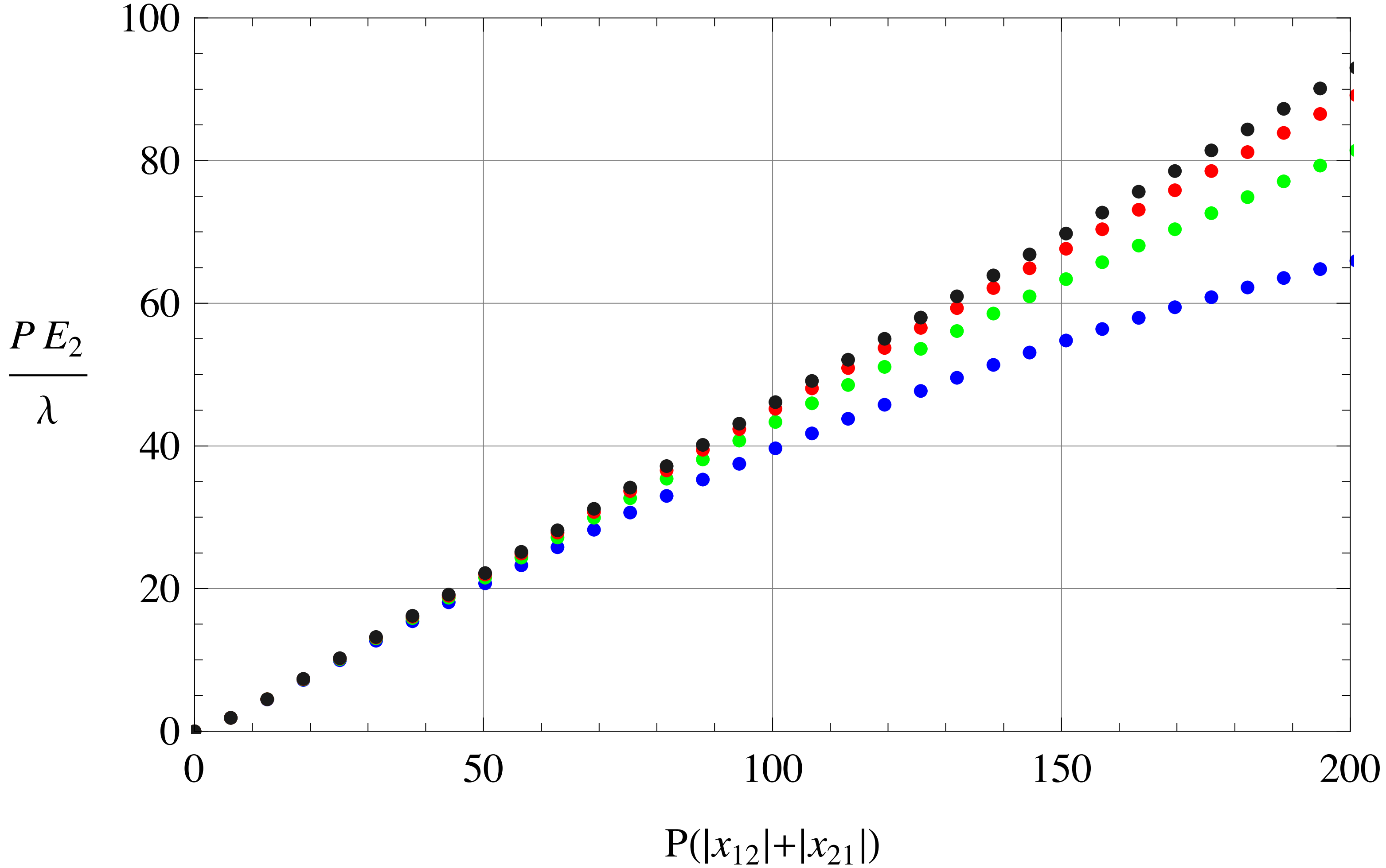
- D. Dorigoni, J. Wosiek and myself have considered (1010.1200) the dimensional reduction of D=4, N=1 SYM theory down to 2D with the hope that, in the large-N limit, it could reproduce the uncompactified theory.
- Besides, the model could be interesting per-se and actually several variations of it have been considered in the past.
- In the case at hand the model has a (2,2) SUSY with 2 bosons and 2 fermions all in the adjoint of SU(N).
- We have performed a LC gauge-LC quantization of the system and compactified LC space on a circle while preserving the full SUSY.

- The system can be studied numerically by diagonalizing the finite-dimensional LC Hamiltonian, finding eigenvalues and eigenvectors, and by taking eventually the large L limit checking convergence to finite values.
- In principle this can be done for the full Hamiltonian but, for the moment, it was only carried out for a truncated version where only the leading IR terms are kept.
- These neatly cancel for colour singlets and lead to a nice string picture for the multiparton bound states with the expected string tension between pairs of neighbour partons along the single-trace of the large-N limit.
- In this approximation the parton number p is conserved.

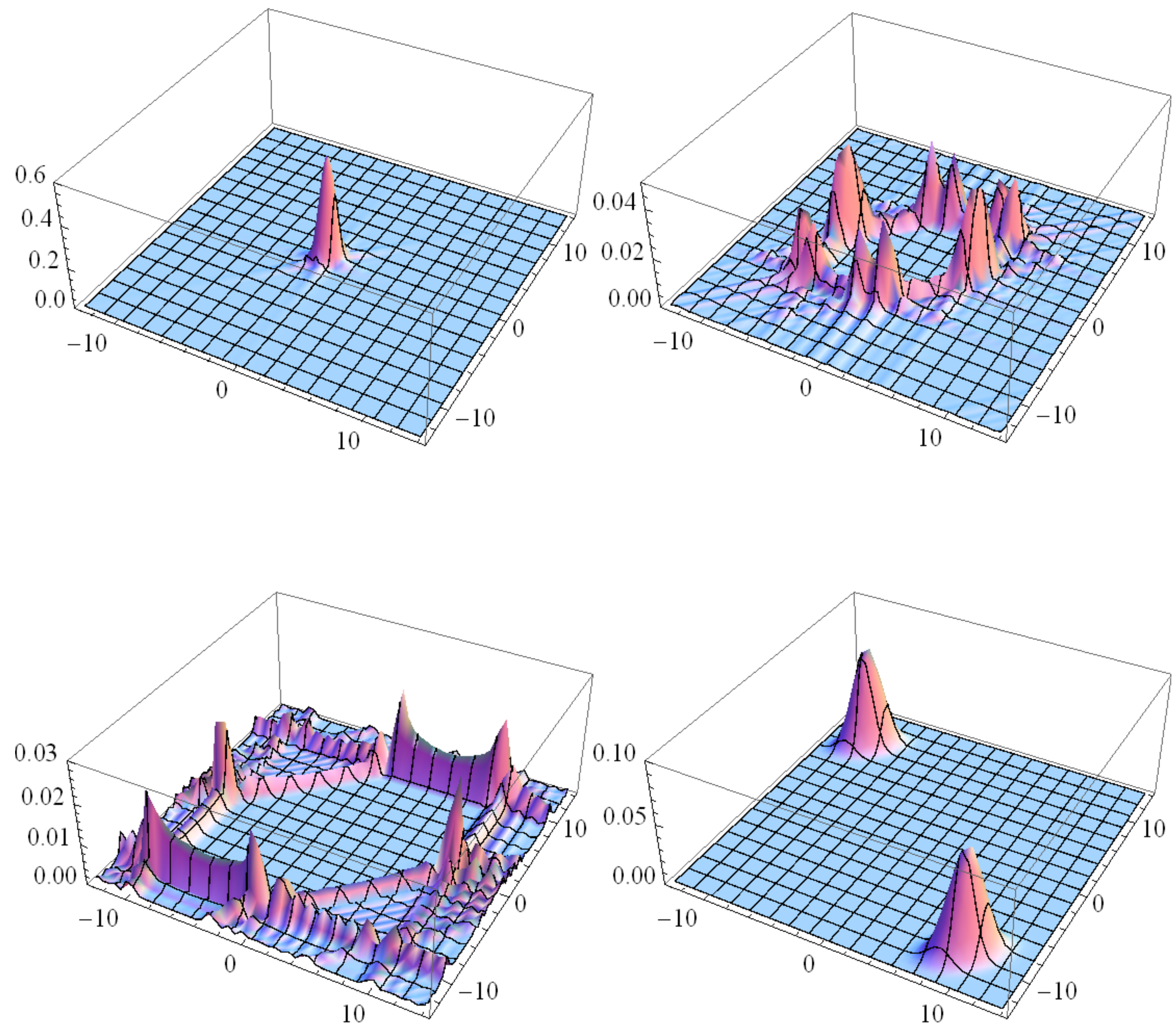
p=2 wave-functions in LC position space



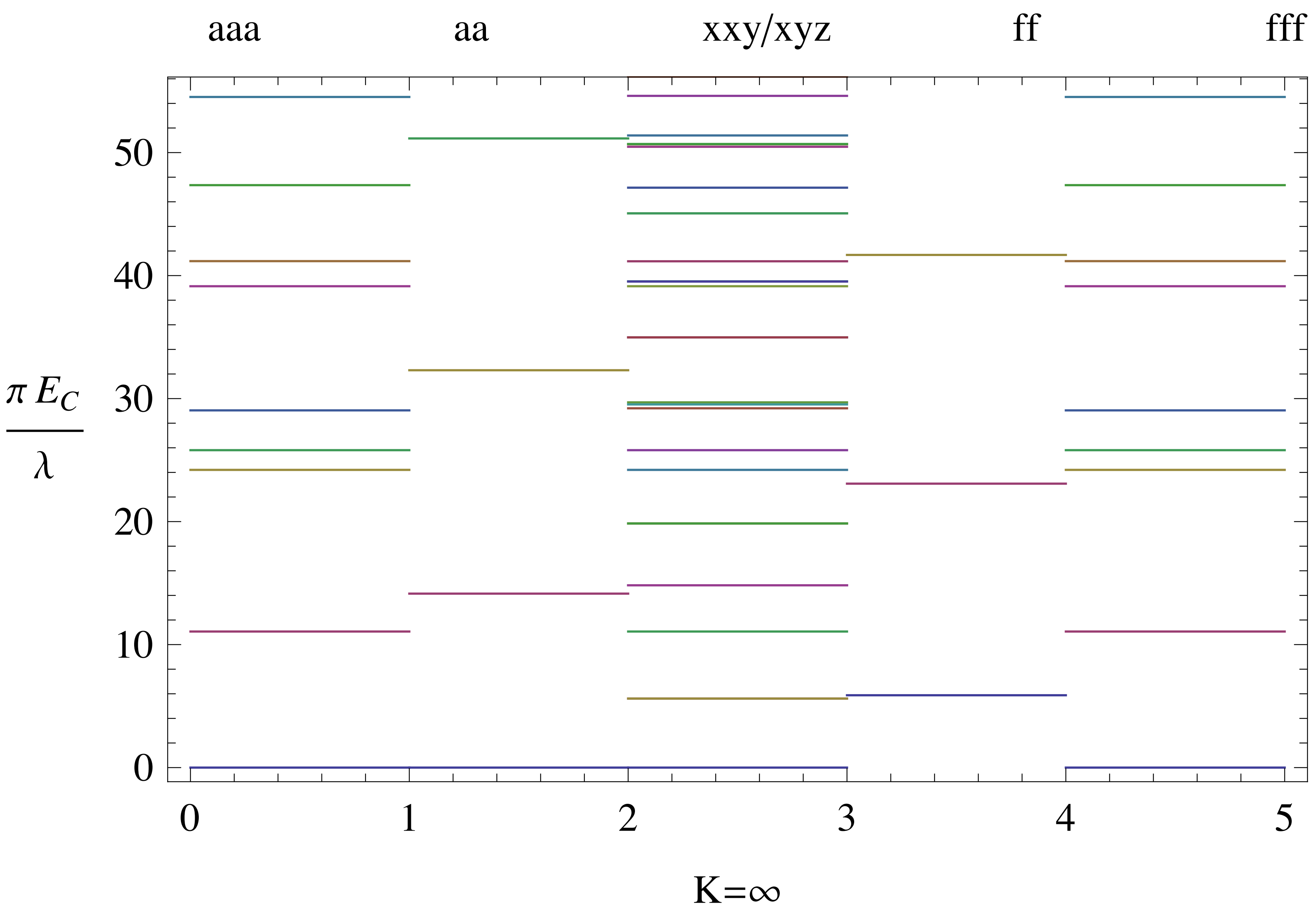
Energy eigenvalues versus average distance



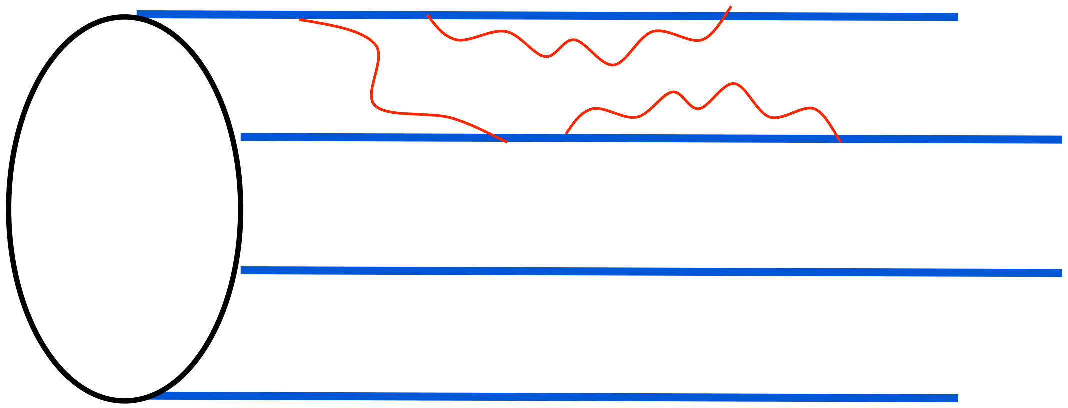
p=3 wave-functions in LC position space



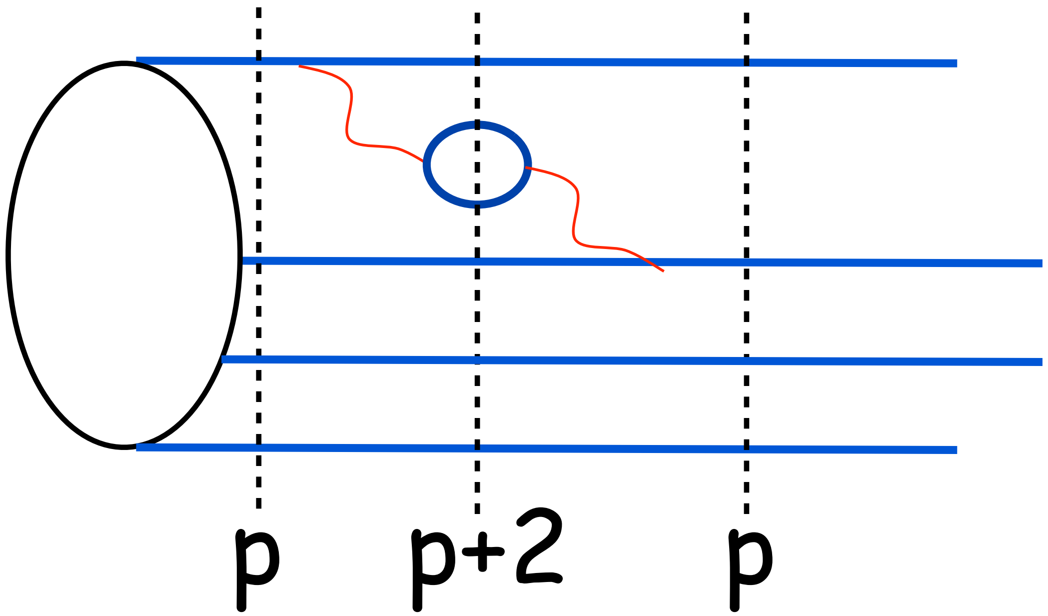
Approximate (2,2) SUSY between p=2 and p=3 spectra



- All this looks very encouraging but then people (e.g. Armoni) asked whether these features survive when we consider the full Hamiltonian, in particular terms that mix different numbers of partons.
- Naively one might expect small mixing effects but there is a definte chance that, instead, massless fermionic and bosonic loops completely change the IR dynamics e.g. by screening the confining potential (cf. topological expansion at large N_F).
- Here the phase transition may occur at a small value of the parton masses.
- Large-N Feynman diagrams can be interpreted either way.



pairwise confining potential



mixing or screening?

NB: wiggly lines are NOT physical partons

Conclusions

- Almost 40 years after the idea of large-N came into particle physics we are still in search of the true string theory it should correspond to for confining gauge theories (we know the fake one it led to!).
- Much more progress has been made instead in connection with non-confining SUSY gauge theories such as in the AdS/CFT correspondence (with possible applications to the QCD QG-plasma in the strong-coupling regime).

- In the last decade new ideas have emerged that might eventually provide a practical implementation of the large-N program.
- In particular, the combination of ASV's planar equivalence and KUY's volume independence may allow to compute QCD properties (modulo 1/N corrections) in a physically interesting range of N_F and quark masses using small-lattice computations at large-N.
- Estimating the size of 1/N corrections (both for VI and for PE) should come next.

A question from my 2008 talk

How come that lattice calculations become more and more complicated as we increase N when the actual dynamics should become simpler?
There should be some way to approach directly the large-N limit even numerically...

...but I'm probably too naive!