Rapid thermalization in the BMN matrix model

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Based on: arXiv:1011.2749 (w. D. Trancanelli)
arxiv:1104.5469 (w. C. T. Asplund and D. Trancanelli)

Motivation

- AdS/CFT correspondence suggests that gravity (geometry) should be interpreted as an emergent phenomenon for strongly coupled field theory.
- Black hole information paradox is solved only IN PRINCIPLE. We would like to have more details about what black holes are made of, how they form and how they evaporate: want dual picture

More motivation

- Understand large N better: both classically and quantum
- Address thermalization processes and check fast scrambling conjectures.

Plan of talk

- Small and large AdS black holes
- Dual picture
- Matrix quantum mechanical models
- Initial conditions and thermalization trajectories.

SMALL ADS BLACK HOLES

AdS black holes were studied by Hawking and Page '82

Assume spherical symmetry on AdS global coordinates:
5- sphere is not touched.

Two varieties: small and large.

Large black holes have positive specific heat: eternal.

Large AdS black holes are dual to thermal state in CFT, Witten

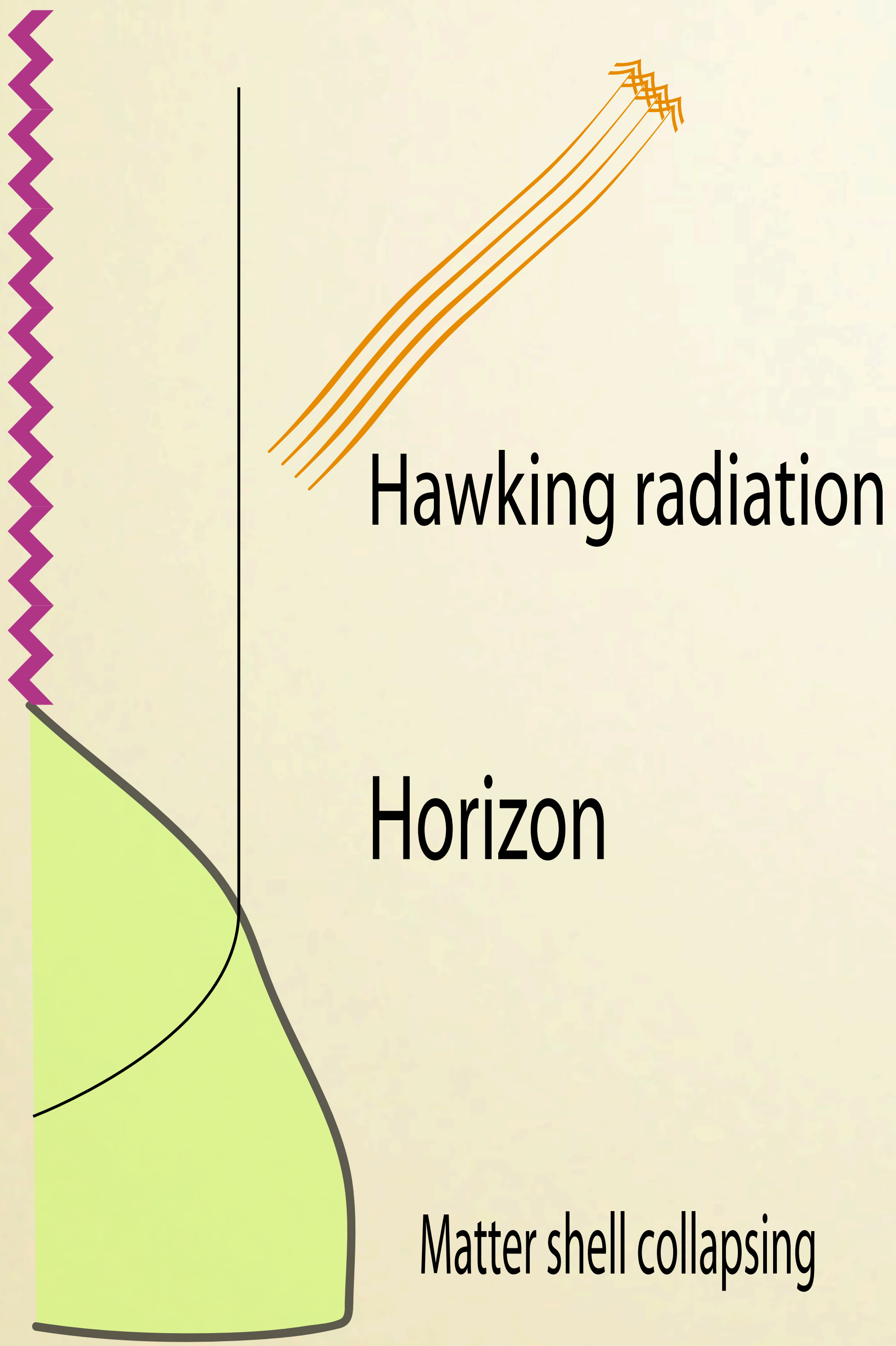
- Small AdS black holes are like Schwarzschild
- Negative specific heat.
- Unstable to Gregory-Laflamme instability along S^5

Second class of small black holes:

10D Schwarzschild

Localized on S^5

Stable, negative specific heat, evaporate via Hawking radiation; not eternal



Need to reproduce history of formation and evaporation of black hole on dual

Dual picture

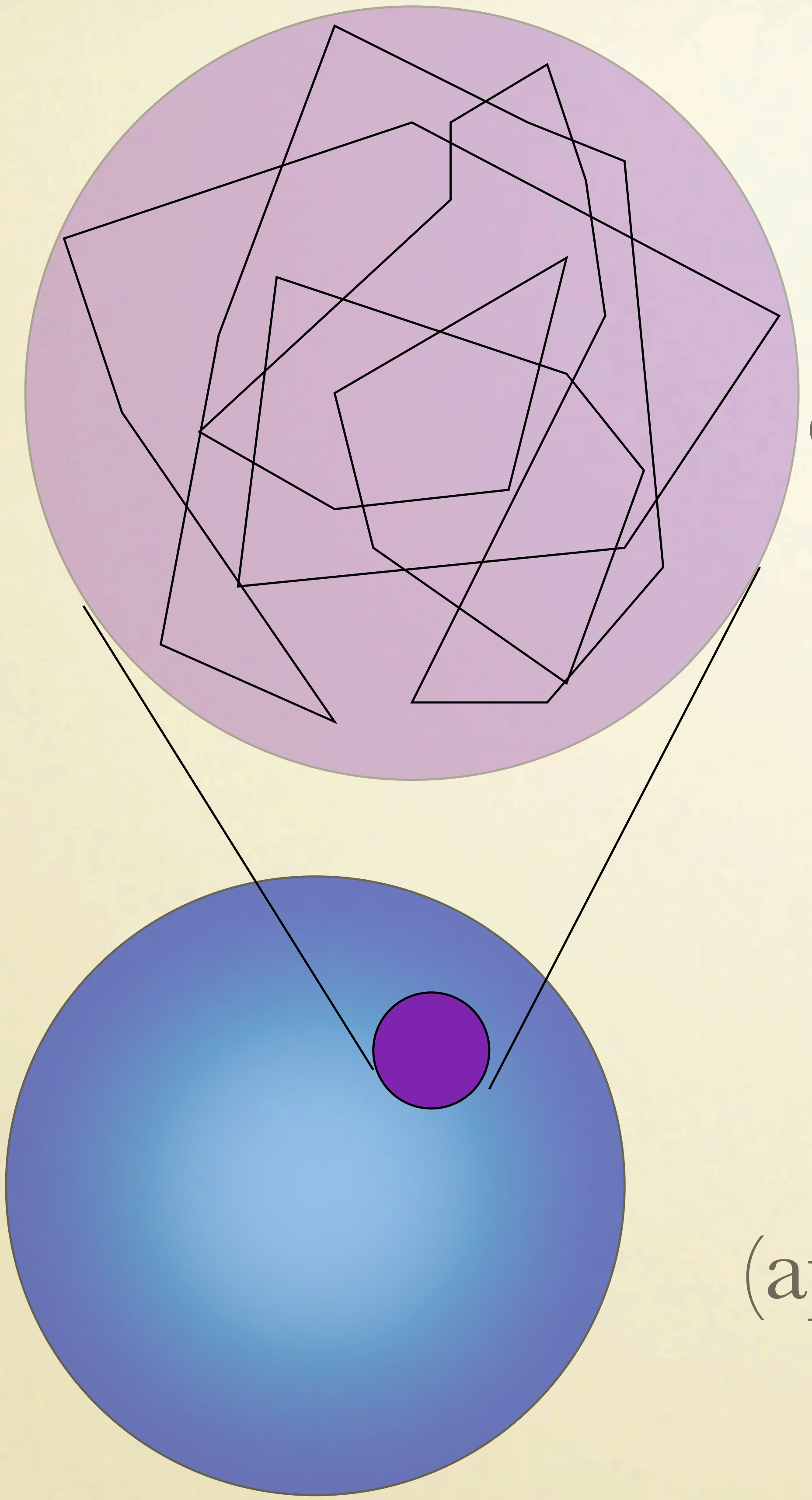
Based on:arxiv0809.0712,
w. C. Asplund

Read the following as a script for what to do

- Prepare states.
- Keep approximations for as long as we can.
- See system collapse and thermalize.
- Argue for thermality, entropy, and eventual evaporation.
- Finally answer: What are black holes?

What one can argue

- In classic N=4 SYM one can argue for a geometric picture of the 5-sphere purely from gauge theory.
- Picture is suggestive of how to setup initial conditions geometrically: in a nutshell, eigenvalues dominate.
- Classical motion of eigenvalues leads to particle production on off-diagonal modes.
- Entropy is generated and it is possible to argue that nonabelian configurations become important.



Prepare a dense gas of off-diagonal modes on a small region of sphere.

Treat the diagonal modes as slow degrees of freedom

Treat off-diagonal modes as (approximately free) fast degrees of freedom for a while.

Energy of off-diagonal modes grows with distance.

There is an extra force on eigenvalues in small region from stretched string bits that makes them compress.

So long as eigenvalue motion is slow the off-diagonal modes are adiabatic (no creation annihilation of off-diagonal modes)

$$\eta = \frac{\dot{\omega}}{\omega^2} << 1$$

Eventually gas of eigenvalue is so compressed that adiabatic approximation breaks down and there is (heavy) production of off-diagonal modes (like in Douglas, Kabat, Pouliot, Shenker)

This generates 'entropy'. There are more configurations like this than initial conditions of the type we study.
(Moduli trapping mechanism, Kofman et al. hep-th/0403001)

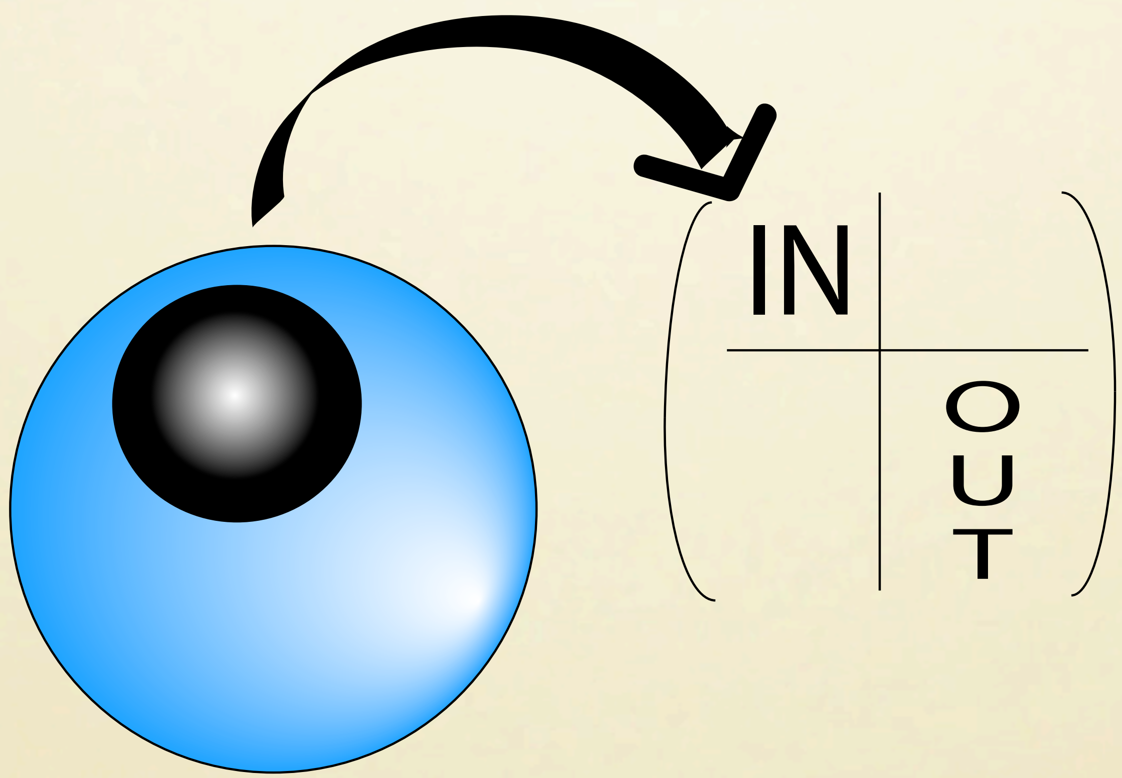
Also, one can see that the no-interacting approximation also breaks down, because it depends on inverse masses that are becoming light.

If the system is chaotic, one gets lost in these configurations.

The time scale is expected to be fast (this is a fast scrambler)

Y. Sekino, L. Susskind arXiv:0808.2096

One finds that the small black hole should be thermal on M<<N eigenvalues that have approximately decoupled from the outside.



What can't be done yet

- Study the details of the thermalization dynamics of N=4 SYM at strong coupling. Requires very large N real time dynamics with particle production.
- Some classical dynamics, but a lot of quantum on top of it: UV catastrophe reasoning.
- Details of vacua and approximate dynamics still poorly understood.
- Evaporation requires deep understanding of out of equilibrium system.

General ingredients for toy model

- Approximately commuting matrices for initial states.
- Modes extended between `eigenvalues' become light and there is particle production.
- For numerics, finitely many degrees of freedom

Matrix QM models

MODELS

- BFSS matrix model, scatter gravitons at high energy, expect to form matrix black holes
- Drawbacks: need details of wave functions making graviton (bound states at threshold)
- Another option: BMN matrix model.

BMN matrix model '02

- Massive deformation of BFSS matrix model
- Dual to M-theory on plane wave.
- Also obtained from truncation of N=4 SYM to SU(2) invariant states on sphere (Kim, Klose, Plefka)
- Better for lattice analysis anyhow (Caterall, Wiseman, van Anders)
- Lose 'scattering problem'.

$$S_{BFSS} = \frac{1}{2g^2}\int dt\,\left((D_t X^I)^2 + \frac{1}{2}[X^I, X^J]^2\right) + \text{fermions}$$

Split 9X into 3 X +6 Y

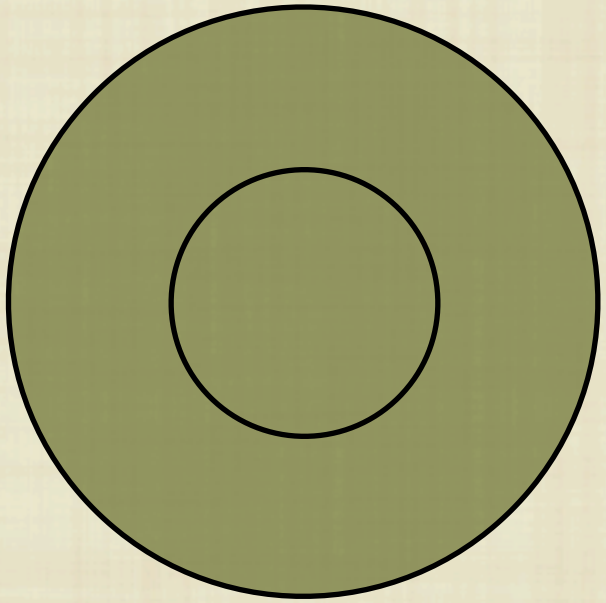
$$S_{BMN} = S_{BFSS} - \frac{1}{2g^2}\int dt\,\left(\mu^2 (X^i)^2 + \frac{\mu^2}{4}(Y^a)^2 + 2\mu i\,\epsilon_{\ell jk} X^\ell X^j X^k\right)$$

$$+\text{fermions}$$

Work with D. trancanelli

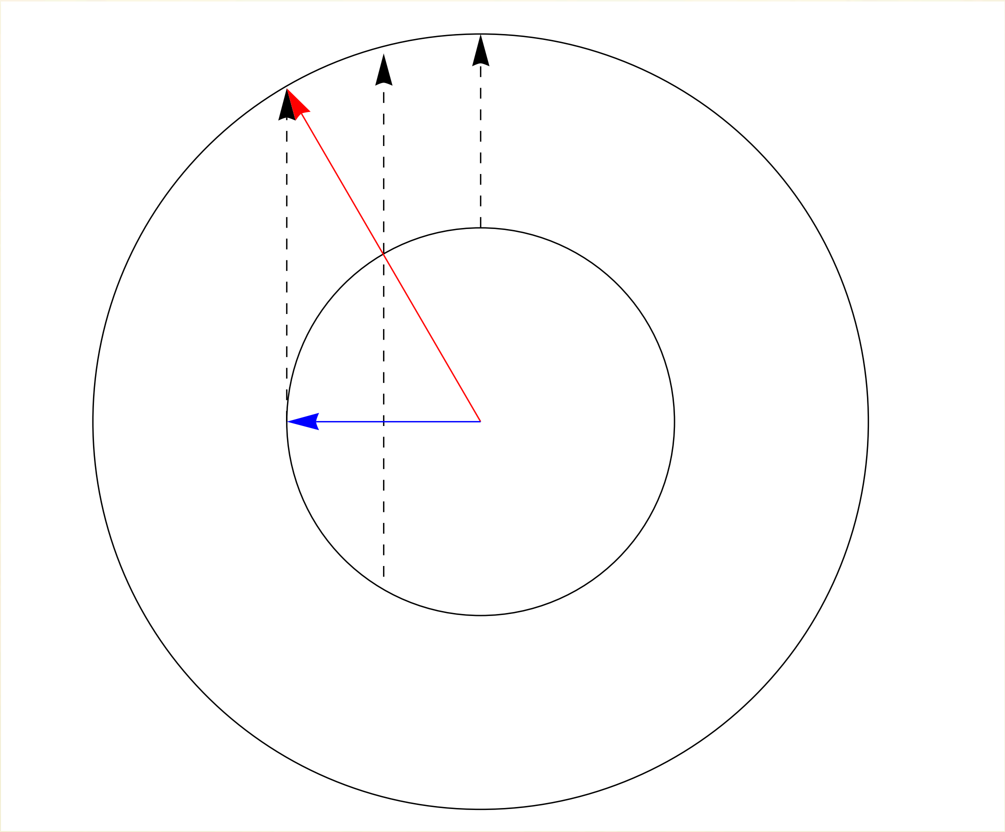
arXiv:1011.2749

Ground state is made of concentric fuzzy spheres.



Spheres can be kicked independently

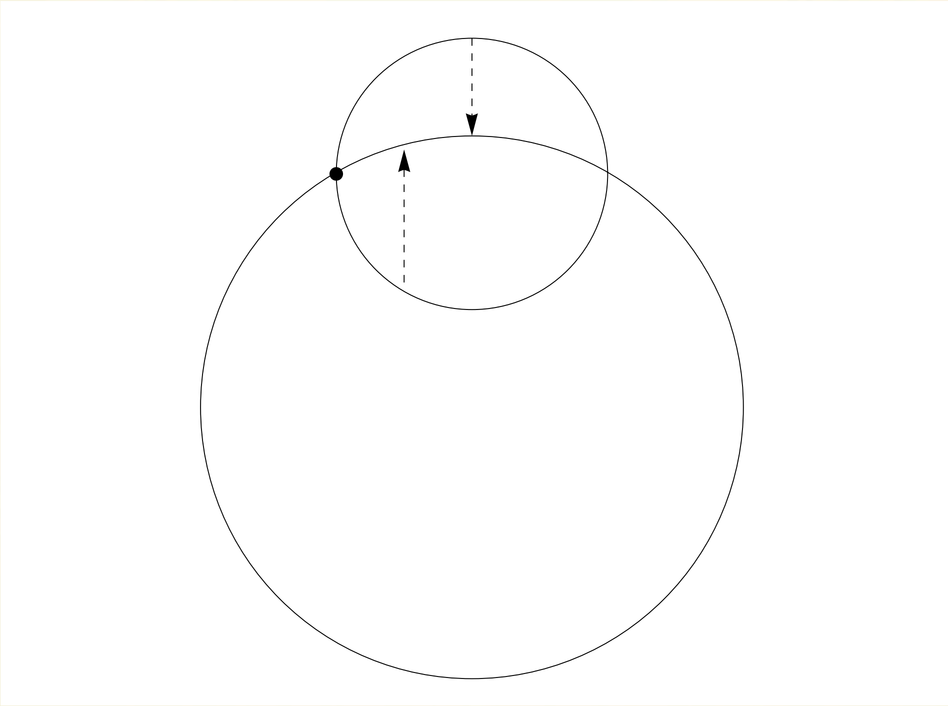
Picture as D2-branes with D0 charge (magnetic flux)



String ends carry angular momentum (highest weight states depicted)

Constant density of possible string ends on each sphere

Mass of off-diagonal modes proportional to distance.



Length of strings change
as we displace fuzzy spheres
Where spheres intersect we get classical
tachyon (great for simulations): these
also carry angular momentum

- At high T, fermions stop contributing to entropy, etc.
- Classical tachyons means that any small fluctuation (quantum seed) grows classically.
- Easy to program (classical leap frog evolver with ‘quantum fluctuation’ seeds).
- All the problem is figuring out the right initial conditions.

Details of tachyon computation

$$V_{BMN}^{(X)} = \frac{1}{2g^2}\mathrm{tr}\left[\left(i[X^2,X^3]+\mu X^1\right)^2+\left(i[X^3,X^1]+\mu X^2\right)^2+\left(i[X^1,X^2]+\mu X^3\right)^2\right] .$$

$$\ddot{X}^i = -\mu^2 X^i - 3i\mu\,\epsilon^{ijk}X^jX^k - \left[\left[X^i,X^I\right],X^I\right] ,$$

Some classical solutions

$$\langle X^i \rangle = \begin{pmatrix} L^i_{(n_1)} + \Re e(b^i_1 * \mathbf{1}_{(n_1)} \exp(it)) & 0 & \dots \\ 0 & L^i_{(n_2)} + \Re e(b^i_2 * \mathbf{1}_{(n_2)} \exp(it)) & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

These are the spheres oscillating about zero with different amplitudes.

Expand in fluctuations: linearized analysis

$$X^3 \simeq L^3 + b\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix} + \sum_{\ell,m} \delta x^3_{\ell m} Y_{\ell m} + (\delta x^3_{\ell m})^* Y^\dagger_{\ell m}\,,$$

$$X^+ \simeq L^+ + \sum_{\ell,m} \delta x^+_{\ell m-1} Y_{\ell m} + (\delta x^-_{\ell m+1})^* Y^\dagger_{\ell m}\,,$$

We use a basis of 'fuzzy monopole spherical harmonics'

IMPORTANT: l, m labels don't mix.

Need kinetic terms

$$\mathcal{L}_{kin}^{(X)} = \frac{1}{2}\left[\sum_{\ell,m} |\delta\dot{x}_{\ell m}^{3}|^2 + \frac{1}{2}|\delta\dot{x}_{\ell m-1}^{+}|^2 + \frac{1}{2}|\delta\dot{x}_{\ell m+1}^{-}|^2\right]$$

Need to project orthogonal to gauge transformations.

$$\delta_\theta X^3 = i[\delta\theta_{\ell m} Y_{\ell m} + h.c., X^3] = -i(m-b)\delta\theta_{\ell m} Y_{\ell m} + i(m-b)\delta\theta^*_{\ell m} Y^\dagger_{\ell m} \,.$$

And similar for other modes.

End result

$$\omega^2_{\ell m} = \begin{pmatrix} 1+\ell+\ell^2-m^2 & (b-m+1)\Lambda_- & (b-m-1)\Lambda_+ \\ (b-m+1)\Lambda_- & b+(b-m)^2+\Lambda_-^2 & -\Lambda_+\Lambda_- \\ (b-m-1)\Lambda_+ & -\Lambda_+\Lambda_- & -b+(b-m)^2+\Lambda_+^2 \end{pmatrix},$$

$$\Lambda_\pm \equiv \sqrt{\frac{(\ell \pm m)(\ell \mp m + 1)}{2}}\,.$$

+ projection.

Special case of no mixing:

$$(\omega^-_{\ell,\ell+1})^2 = -b + (b - \ell - 1)^2\,,$$
$$(\omega^+_{\ell,-\ell-1})^2 = b + (b + \ell + 1)^2\,.$$

These are modes of maximum angular momentum.

Tachyonic for some values of b.

Same instability as Nielsen-Olesen:
charged gluons in constant chromomagnetic field become tachyonic due to large magnetic moment.

`Floquet' or 'Bloch' analysis of time dependence.

$$\ddot{q}_\ell(t) + (m_\ell^\pm(t))^2 q(t) = 0\,.$$

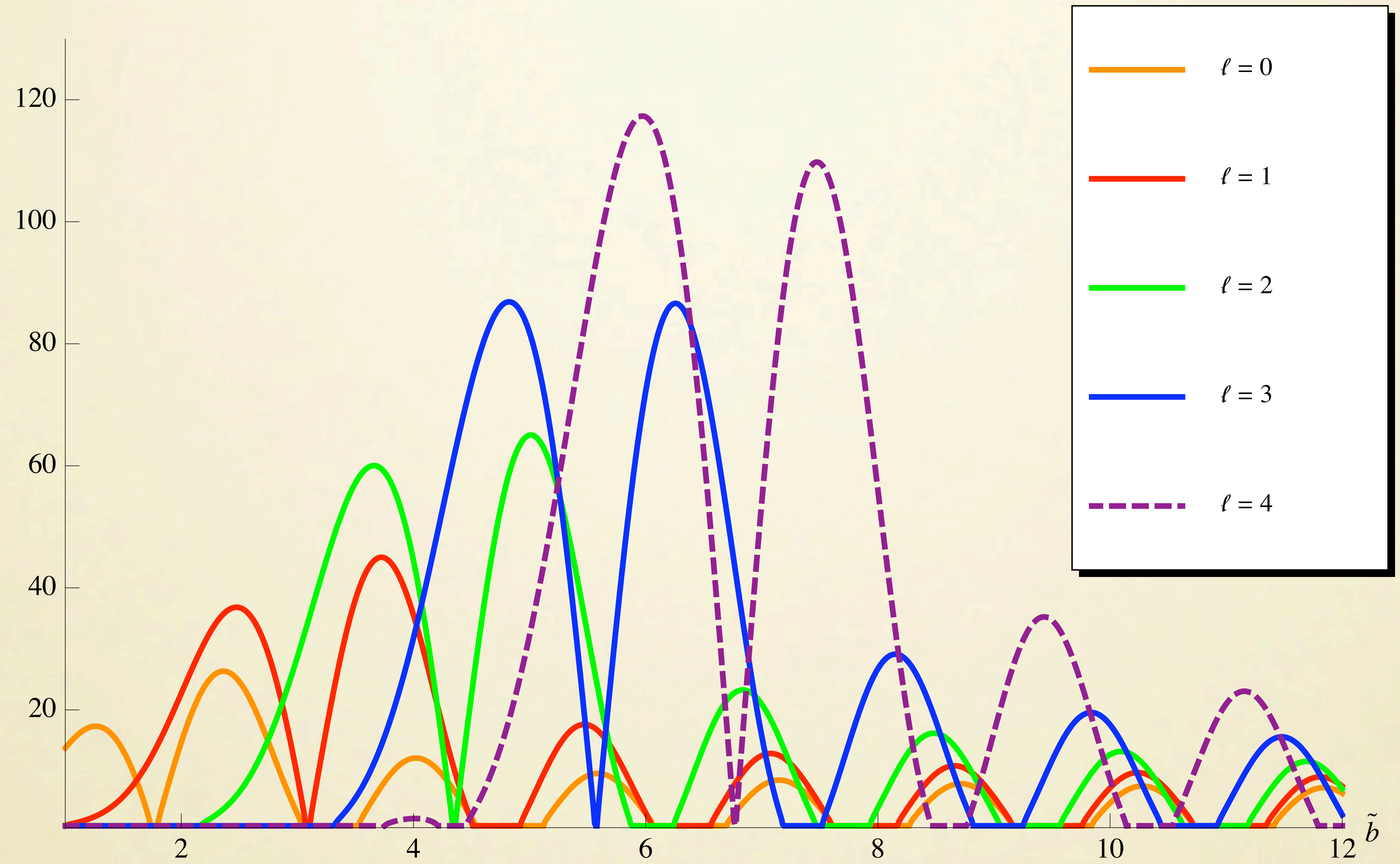
$$b(t) = \tilde{b}\sin(t)$$

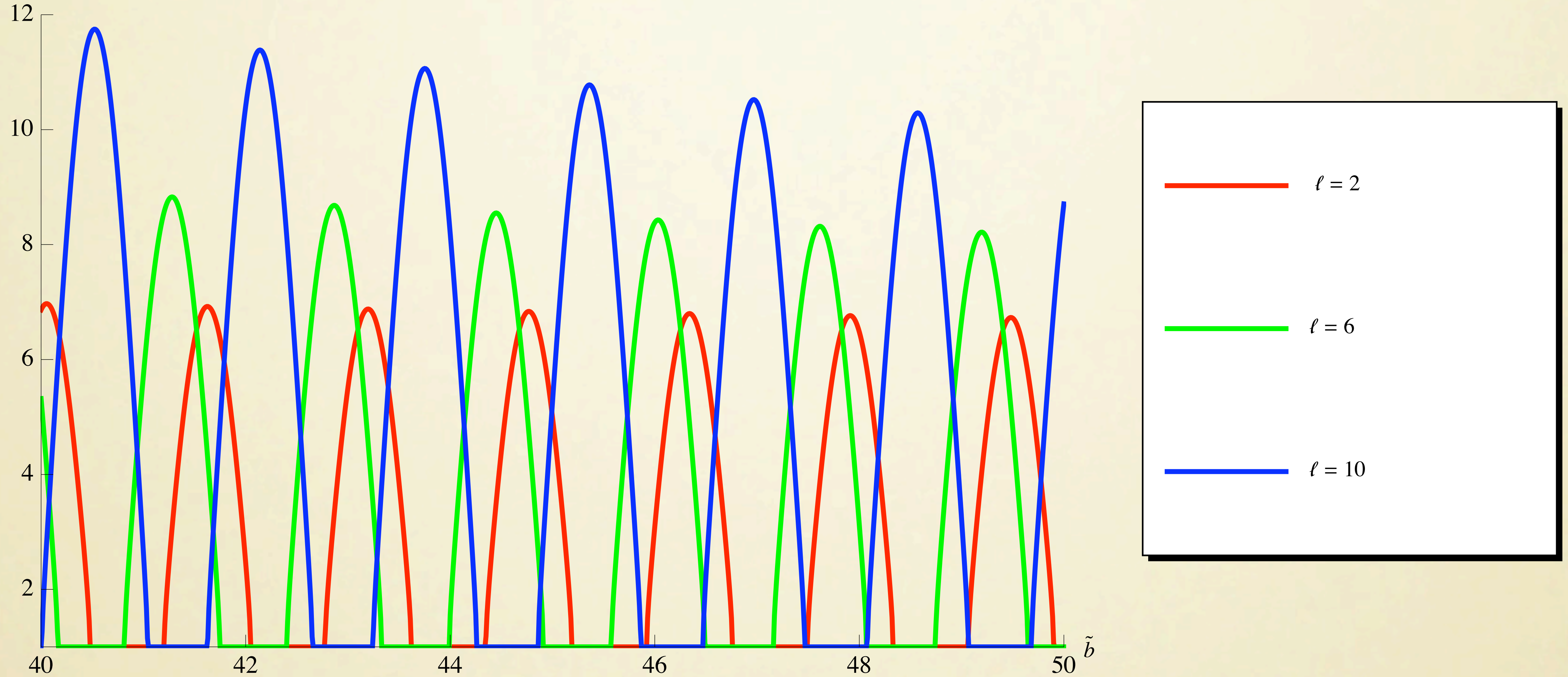
Like a Schrodinger problem in a 1-d periodic potential: leads to energy band analysis.

$$\begin{pmatrix} q_1(t+2\pi) \\ q_2(t+2\pi) \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} q_1(t) \\ q_2(t) \end{pmatrix}$$

Eigenvalues of matrix determine if stable (eigenvalue unitary = in energy band), or unstable (eigenvalues real = outside bands).

Most unstable mode typically has highest l





12
10
8
6
4
2
40
42
44
46
48
50
$\tilde{b}$
ℓ = 2
ℓ = 6
ℓ = 10

Full Simulations:

classical with semi-classical initial conditions

arxiv: 1104.5469+ work in progress.

C. Asplund, D. Trancanelli

Take same classical configurations as before.

Add quantum fluctuation seeds: generate randomly from gaussian distribution normalized to harmonic oscillator wave functions.

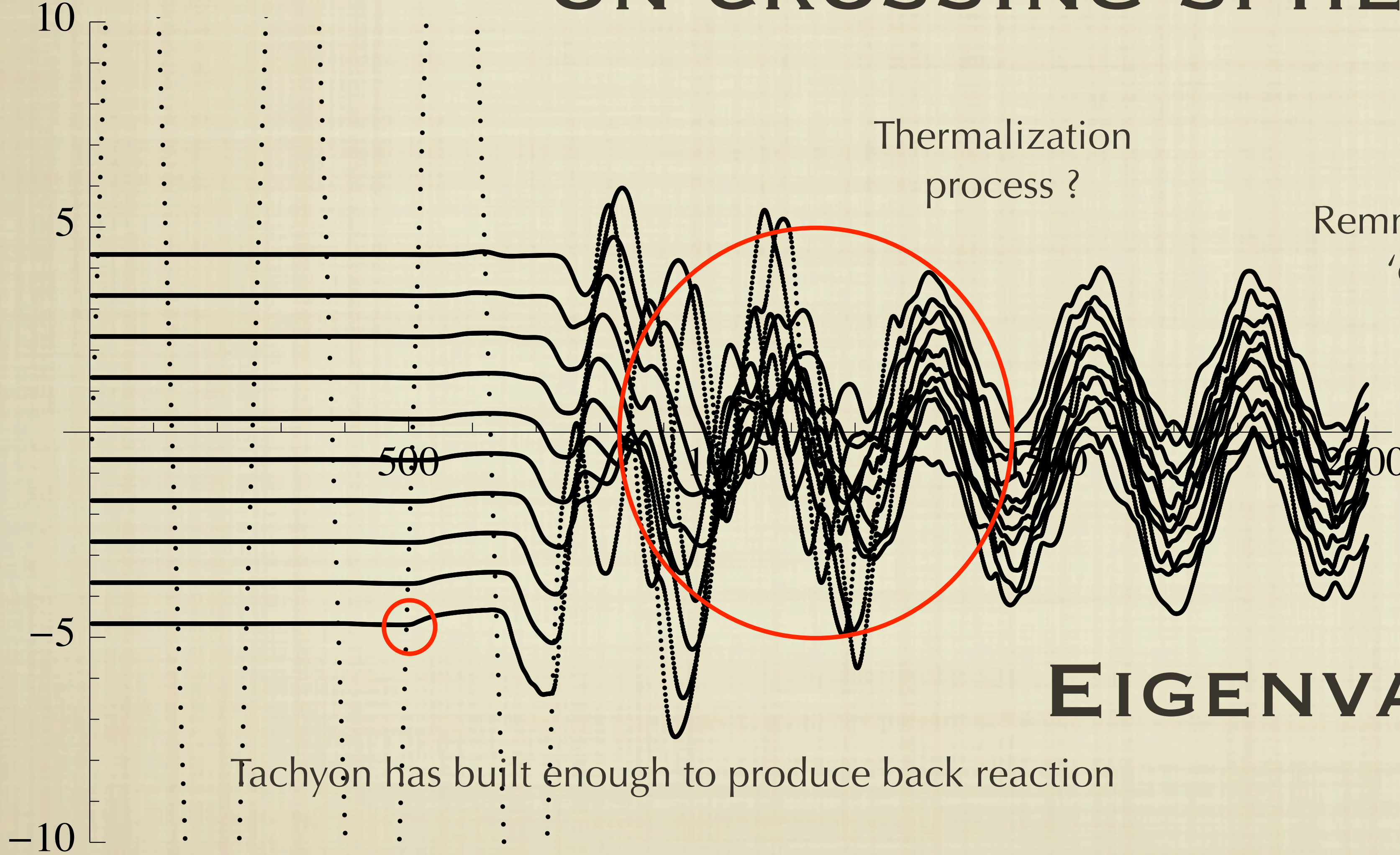
$$X^0 = \begin{pmatrix} L_n^0 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^1 = \begin{pmatrix} L_n^1 & \delta x_1 \\ \delta x_1^\dagger & 0 \end{pmatrix}, \quad X^2 = \begin{pmatrix} L_n^2 & \delta x_2 \\ \delta x_2^\dagger & 0 \end{pmatrix},$$

$$P^0 = \begin{pmatrix} 0 & 0 \\ 0 & v \end{pmatrix}, \quad P^{1,2} = 0 = Q^{1,\ldots,6}, \quad Y^a = \delta y^a.$$

$$\delta x, \delta y \simeq \sqrt{\hbar/n}$$

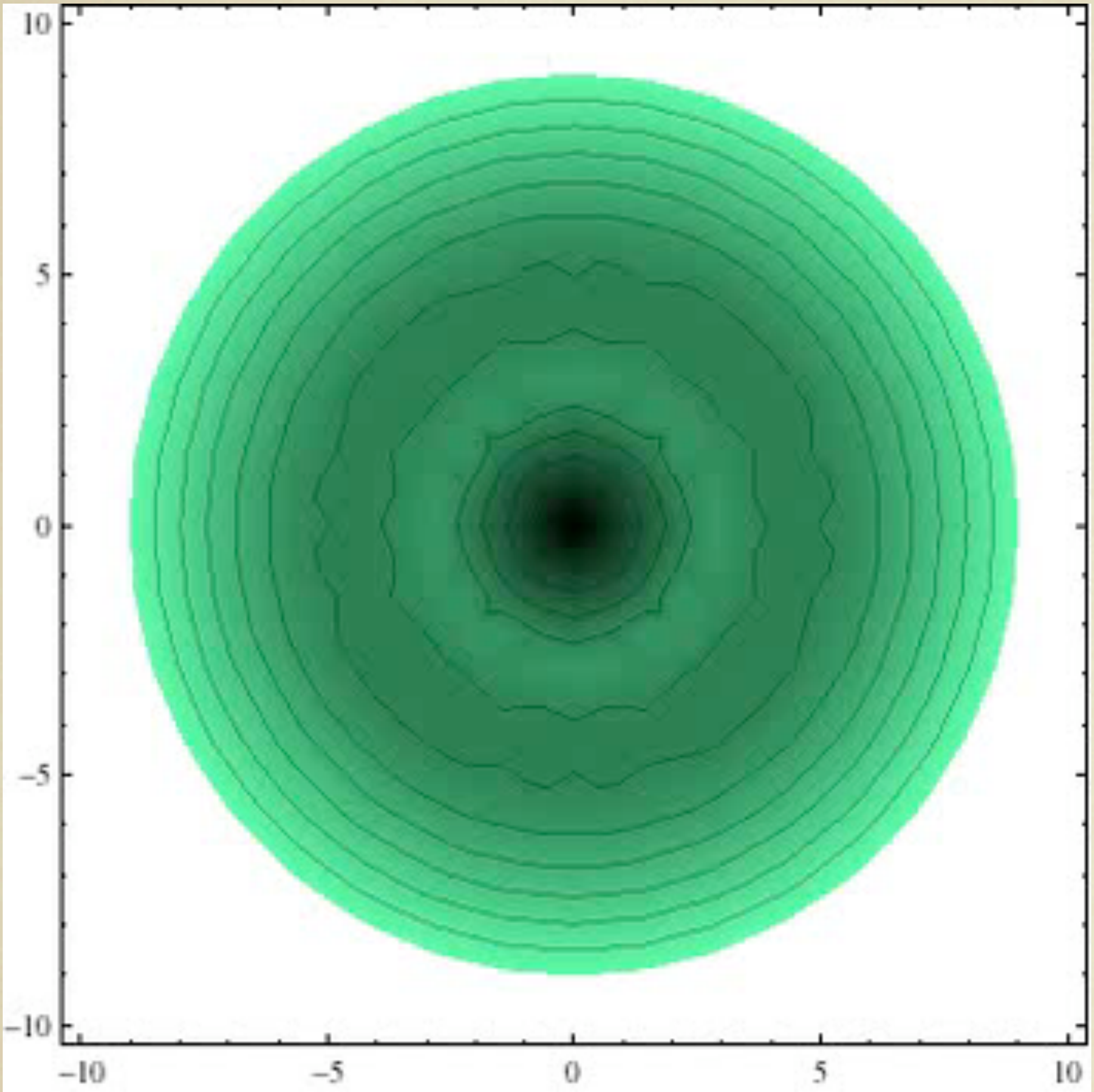
Results:

Kick single eigenvalue: tachyon still forms for short time on crossing sphere



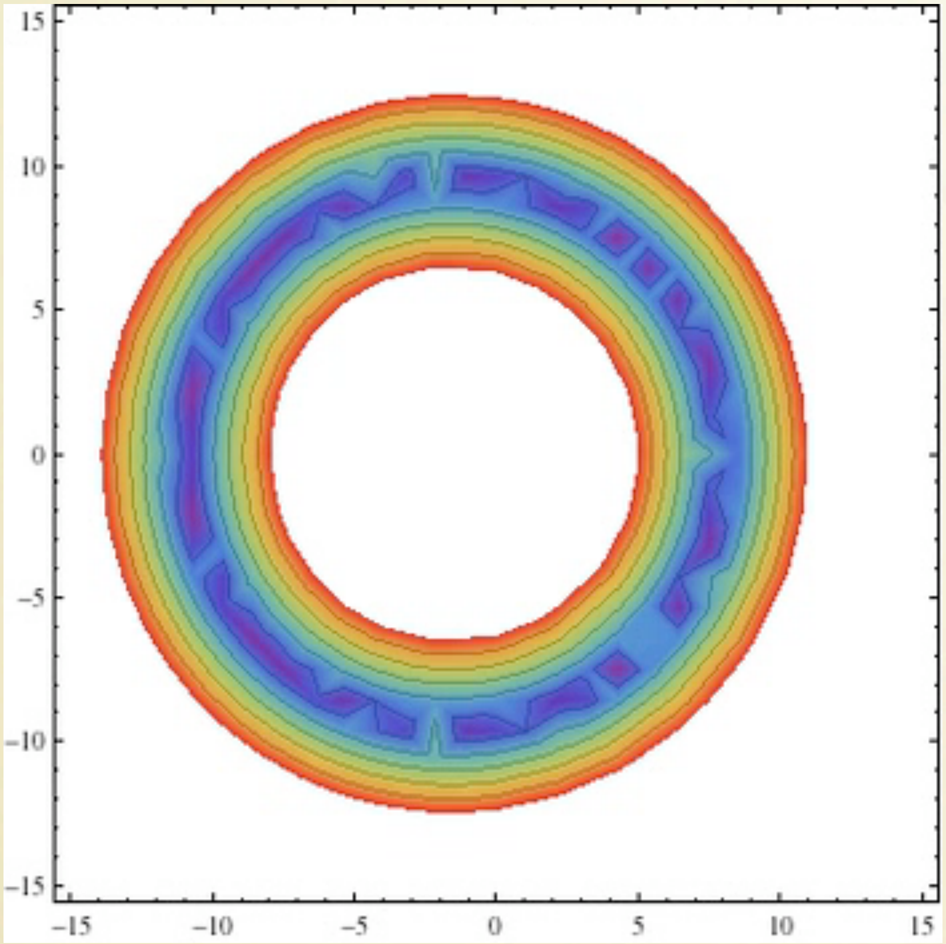
Eigenvalue spectrum

Movies



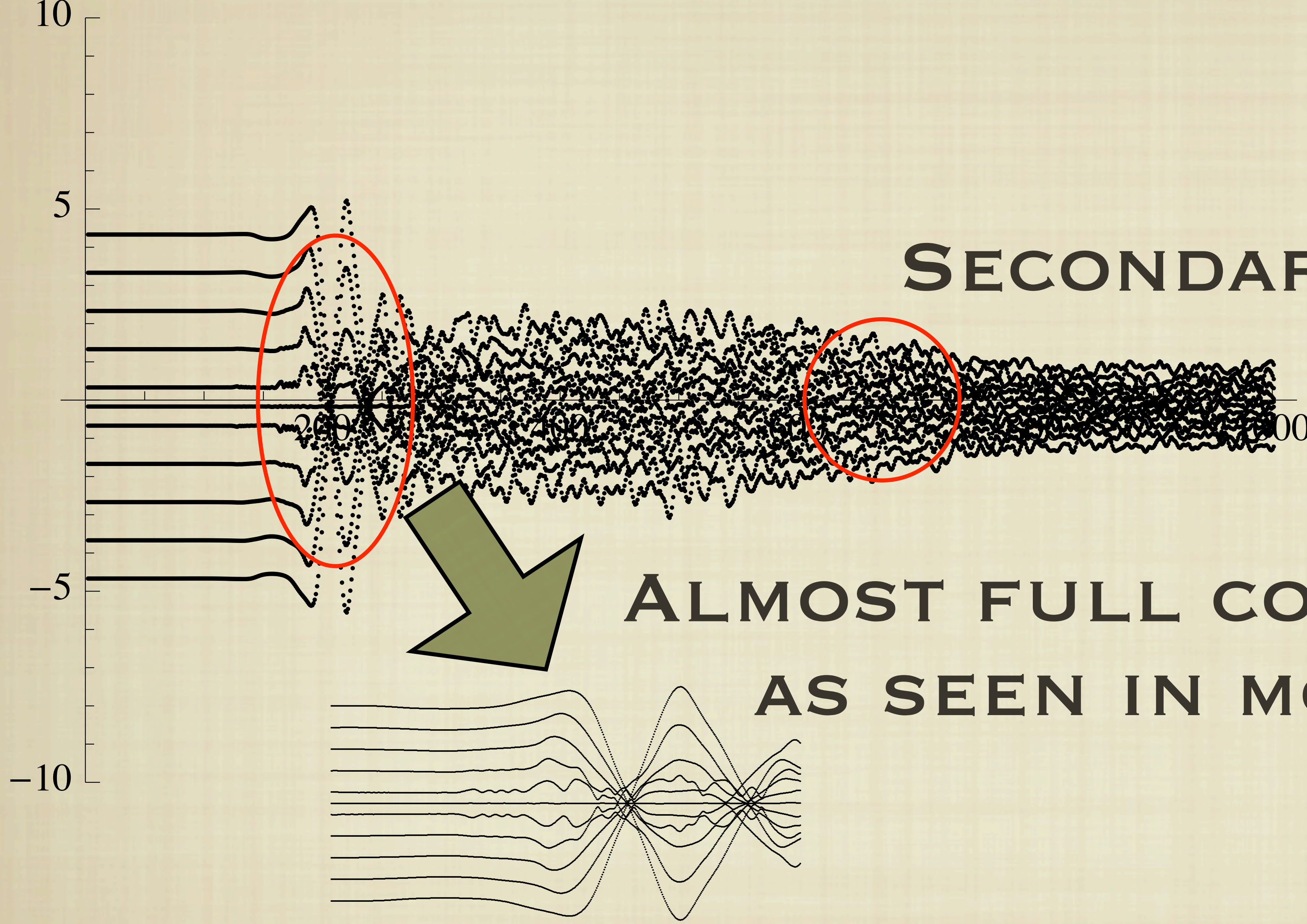
2d cross section of geometry as seen from a probe eigenvalue point of view: mass of lightest Y mode connecting to eigenvalue distribution.

As seen from the fermion masses point of view



Fermion masses seem
to track
matter distribution better.

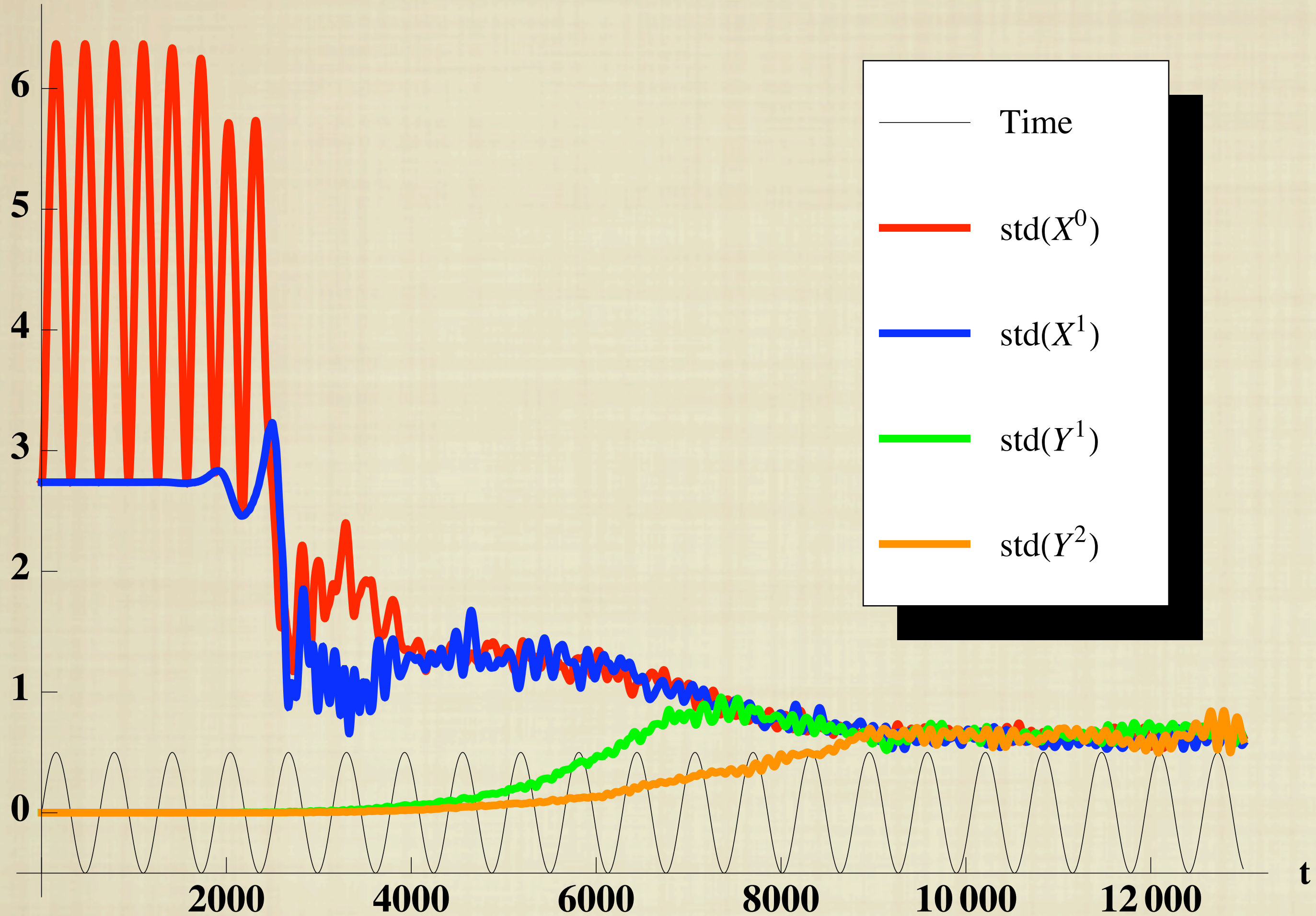
Another axis of sphere



Secondary shrinkage?

Almost full collapse as seen in movie

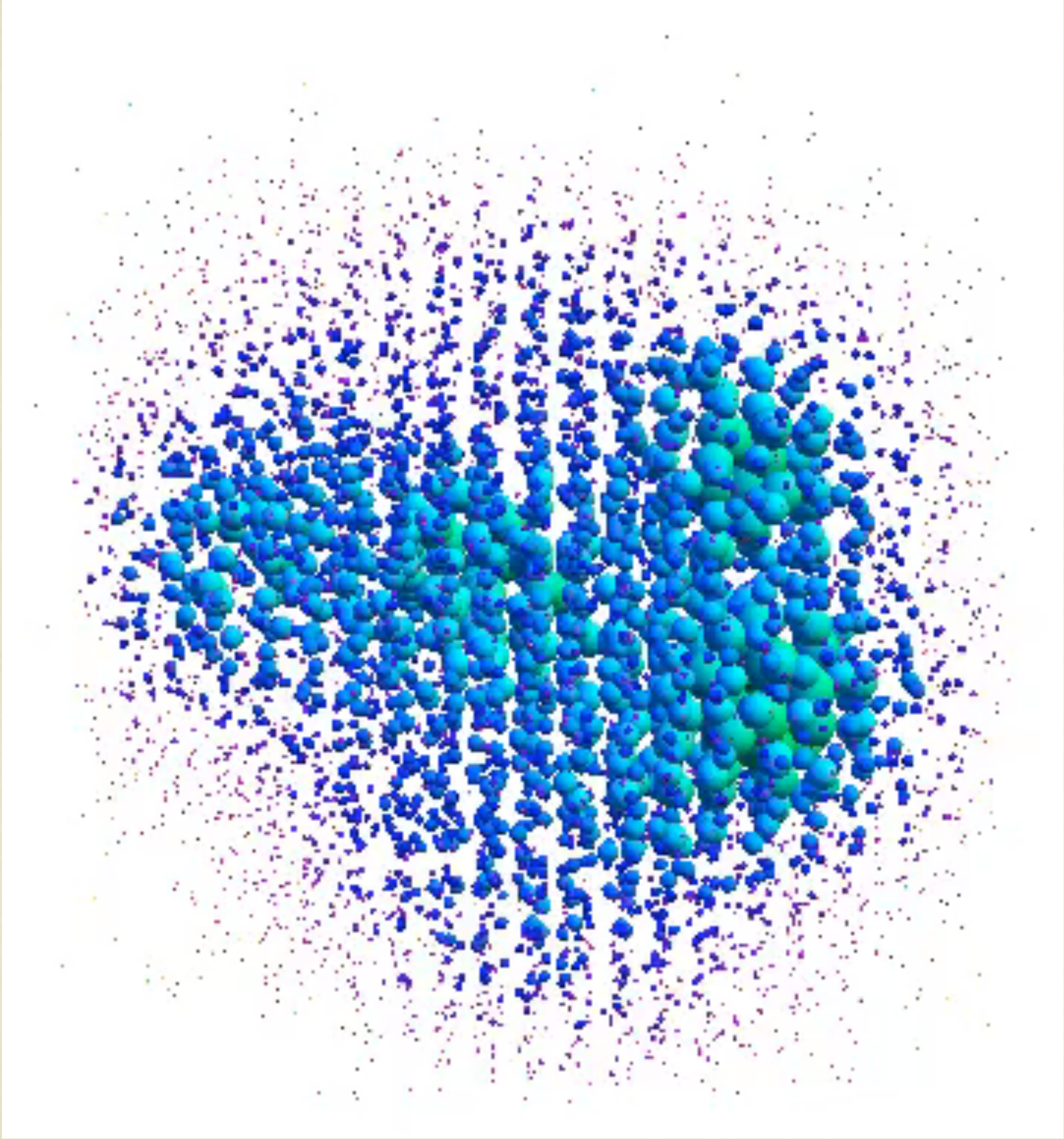
Approximately Converges to Spherical configuration



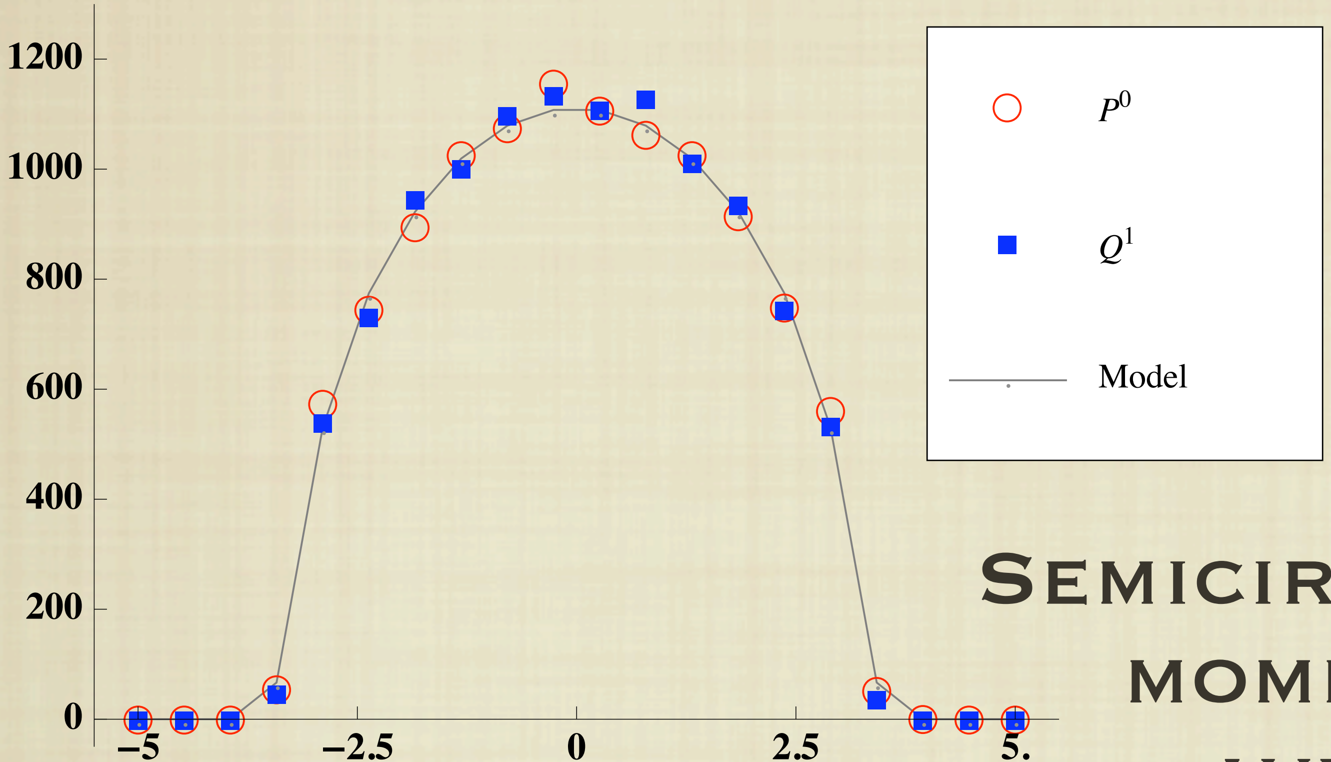
Trace of X,Y decoupled: serves as physical clock.

Secondary shrinkage is from growth of Y matrices (parametric resonance)

After thermalization IN 3D



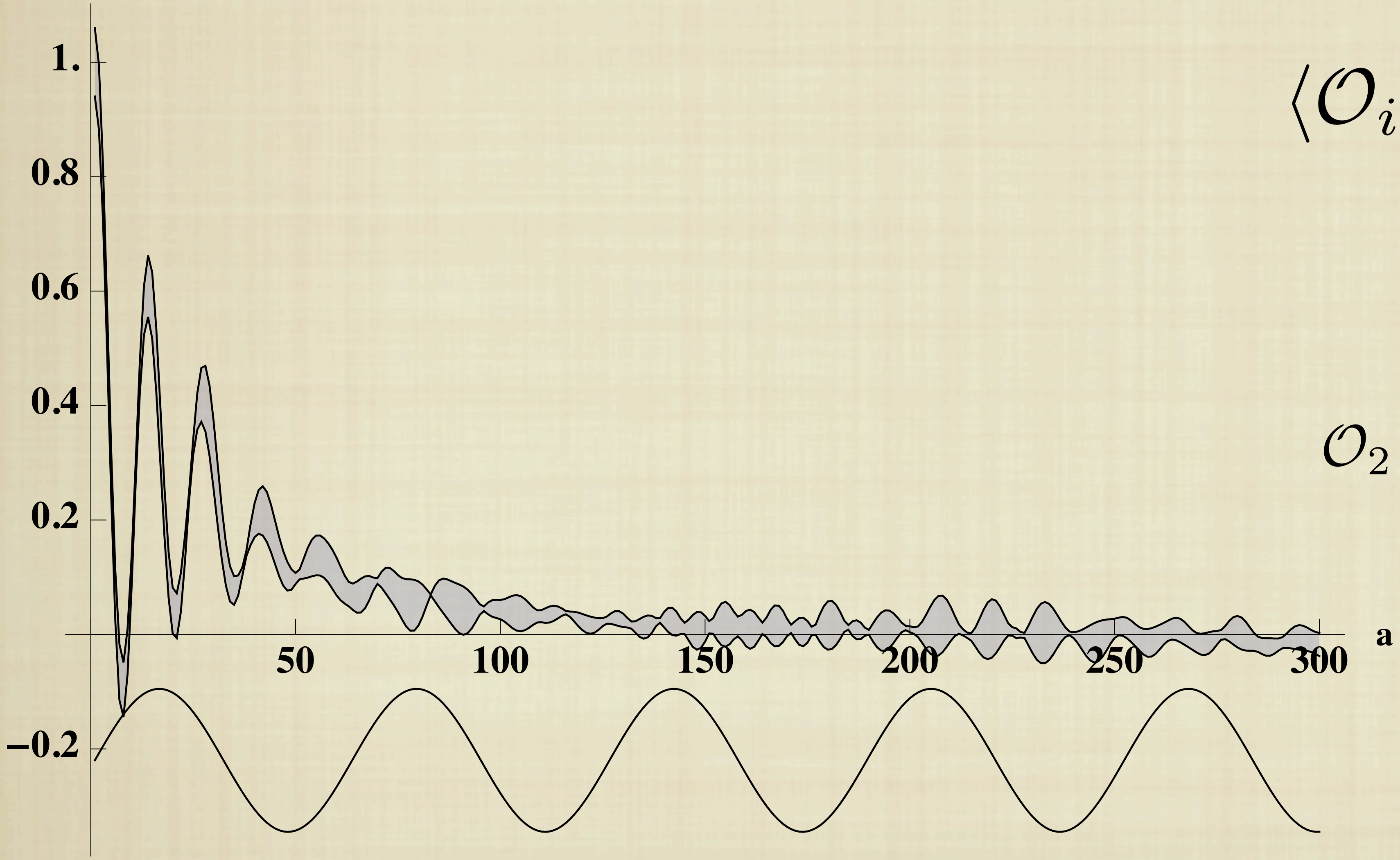
Tests of thermality



Semicircle distribution for momenta eigenvalues: average over time.

Fast thermalization?

test via Normalized Autocorrelation functions

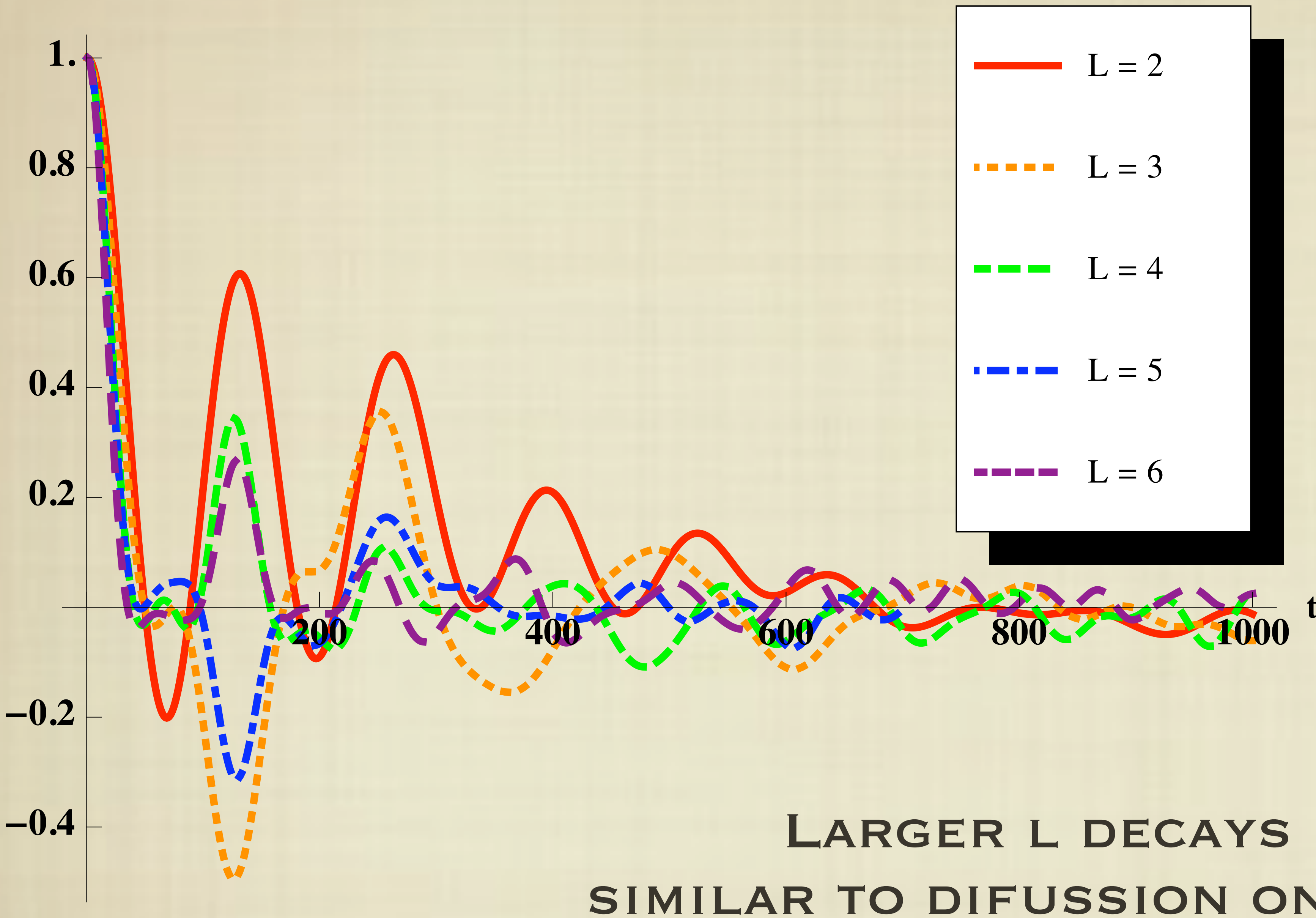


$$\langle \mathcal{O}_i(t)\mathcal{O}_i^\dagger(t+a)\rangle$$

$$\mathcal{O}_2 = \mathrm{tr}[(X^1 + iX^2)^2]$$

More

$$\mathcal{O}_L = \mathrm{tr}[(X^1 + iX^2)^L]$$



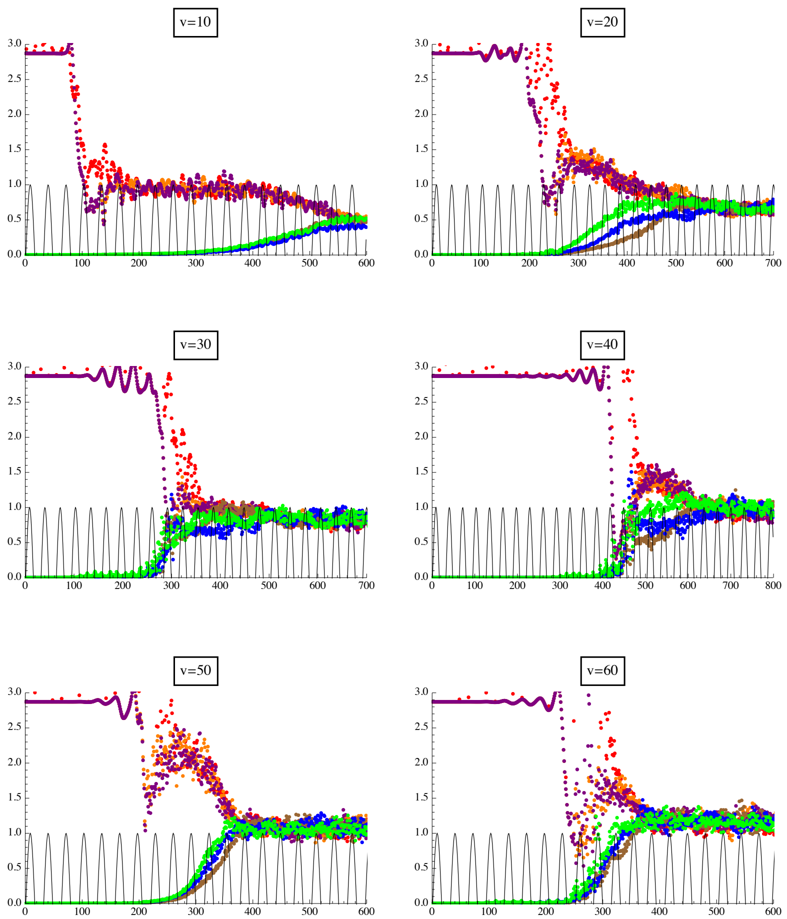
Larger L decays faster:
similar to difussion on BH horizon.

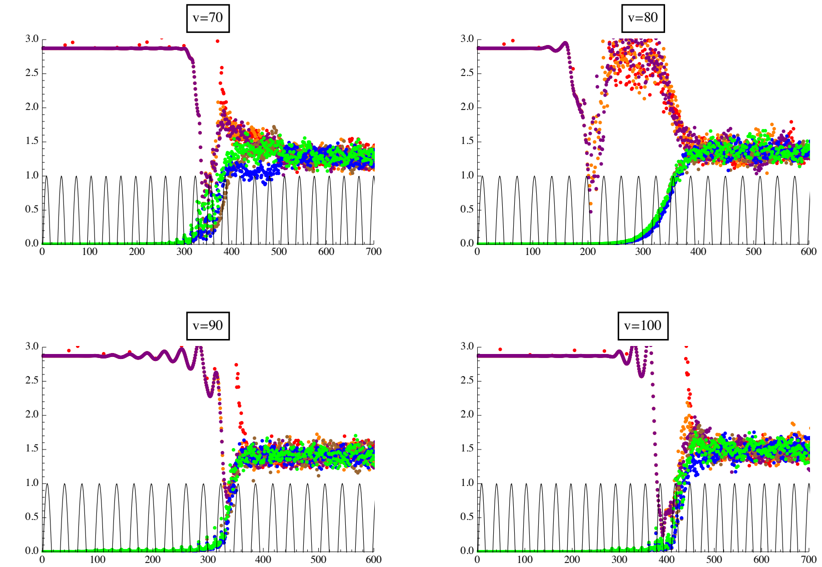
More Preliminary results

Standard deviations as function of the initial velocity

initial data: n1=10, n2=1, delta=0.008, Ncycle=250, hbar=10^-6

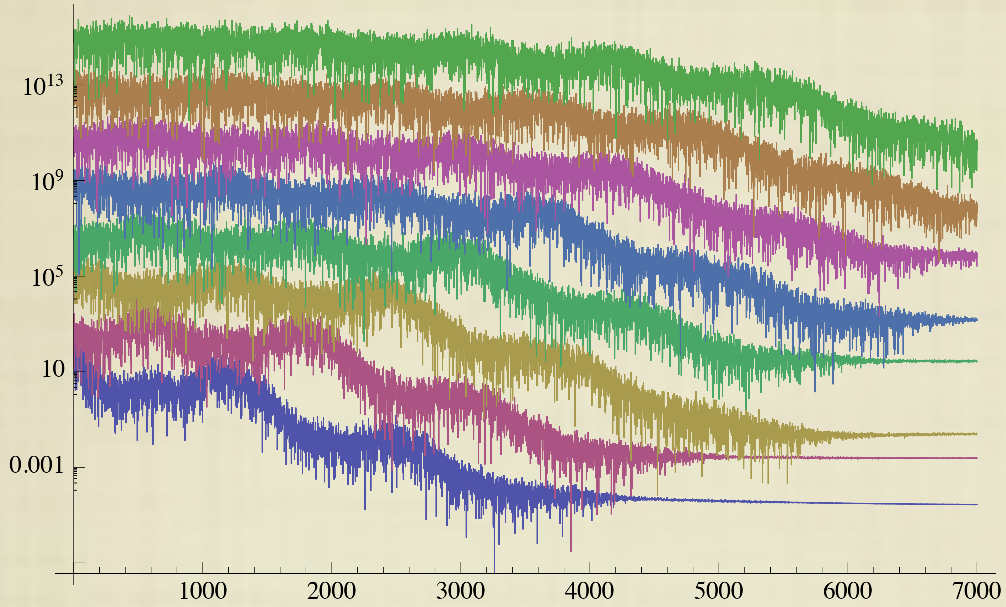
color code:
(red,X0) (orange,X1) (purple,X2) (brown,Y0) (blue,Y1) (green,Y2) (black,TrX0)



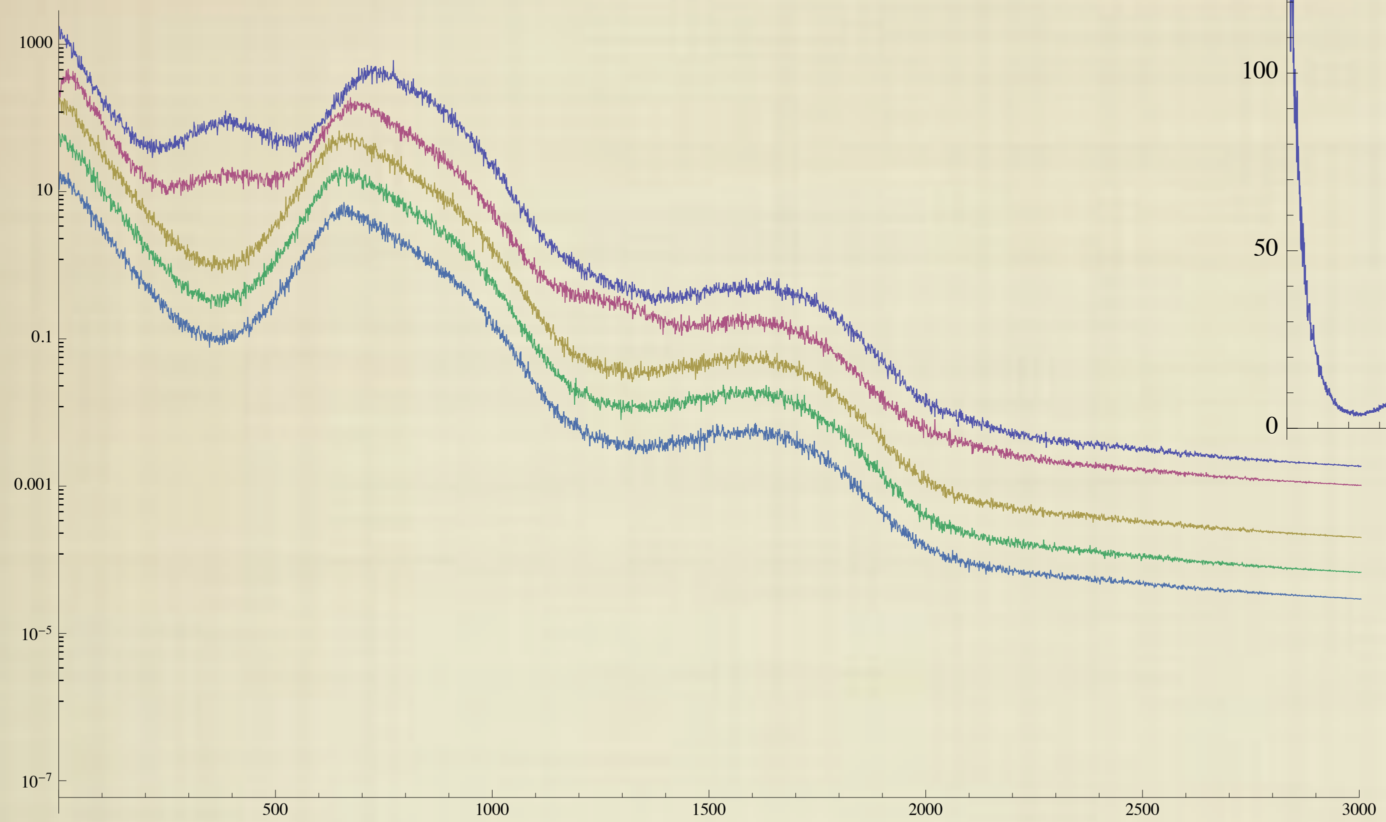


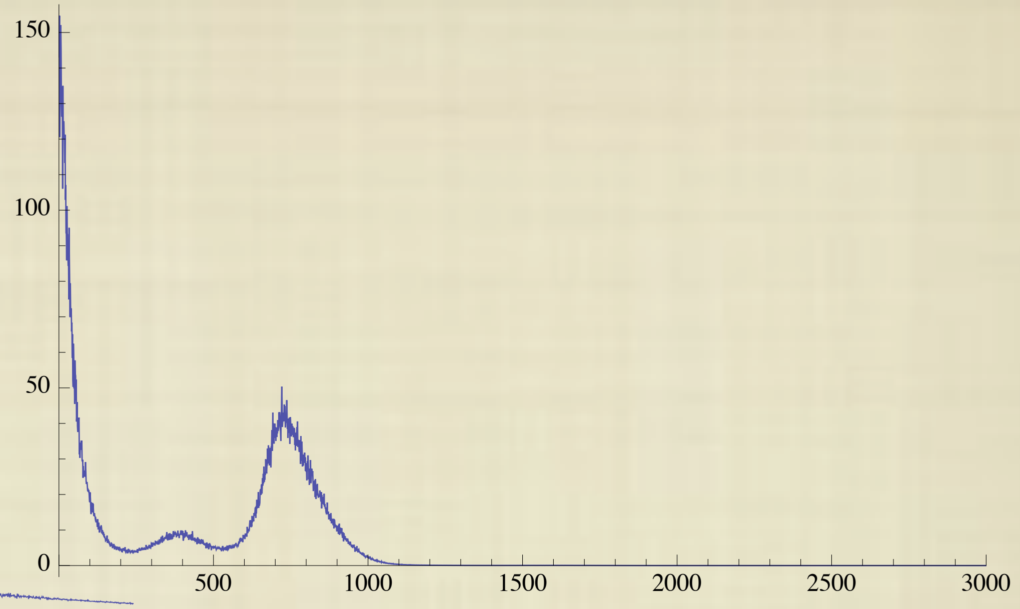
Thermalization seems to happen always at around ~14 oscillations.

Power spectra

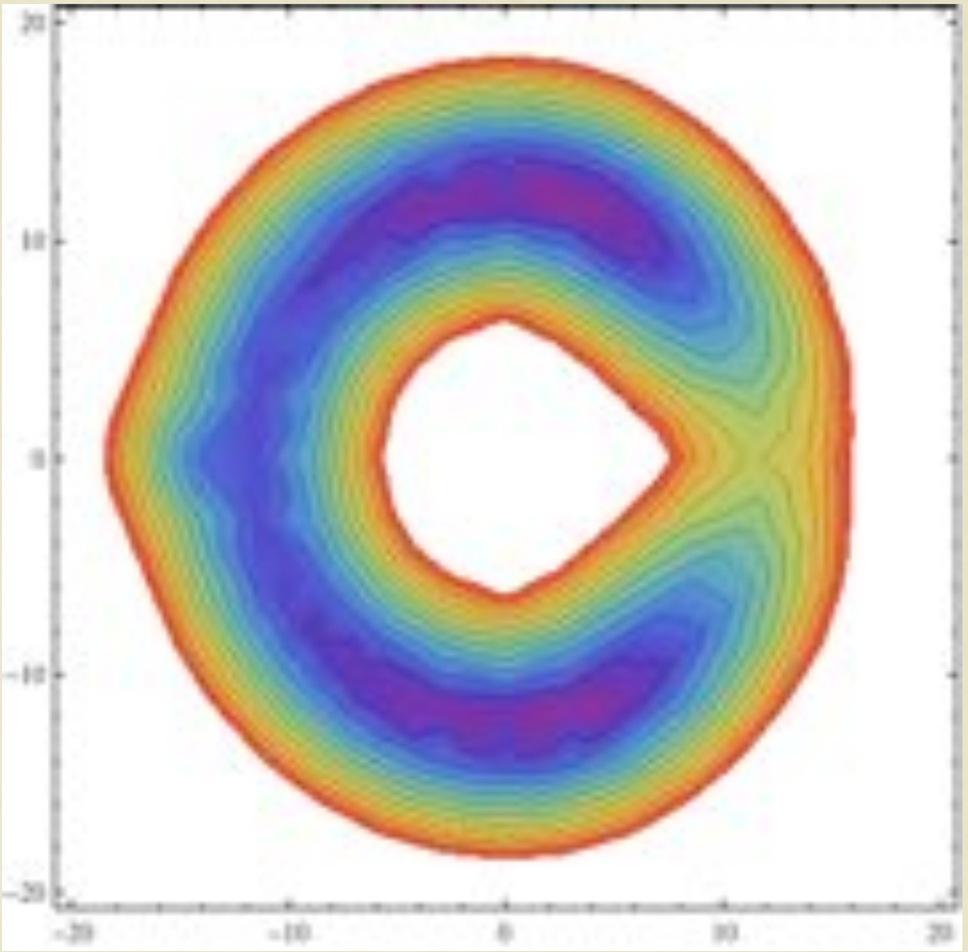


HIGH QUALITY L=2.





Doesn't thermalize always



Conclusion

- Dual black hole dynamics on AdS/CFT can be accessed.
- Partial understanding of thermalization in large N QFT
- Simulations in real time BMN matrix model seem to indicate fast thermalization.

To do

- Explore phase space: angular momentum, topology changes, configurations that don't thermalize.
- Add fermions: in BFSS they might help black hole evaporate.
- Vary N and test fast scrambler conjectures.
- Is there a hydrodynamic way of thinking hiding in these models?