Aspects of large-N gauge theories from the lattice

Michael Teper (Oxford) - GGI, Florence 2011

- Large N :

  $T = 0$ and $T \neq 0$, $D = 2 + 1$ and $D = 3 + 1$: a lightning survey
  – but avoiding what other speakers might talk about e.g. small V, k-strings, ...

- Flux tubes and string theory :

  effective string theories - recent progress

  fundamental flux tubes in D=2+1

  fundamental flux tubes in D=3+1

- Concluding remarks

Basic questions for the large-N limit

- Large-N scaling?

- Is $N = \infty$ confining?

- Is $N = 3$ close to $N = \infty$?

– glueball masses

– meson masses

large-N scaling:

- $g^2 N$ fixed as $N \to \infty$?
- leading correction $1/N^2$?

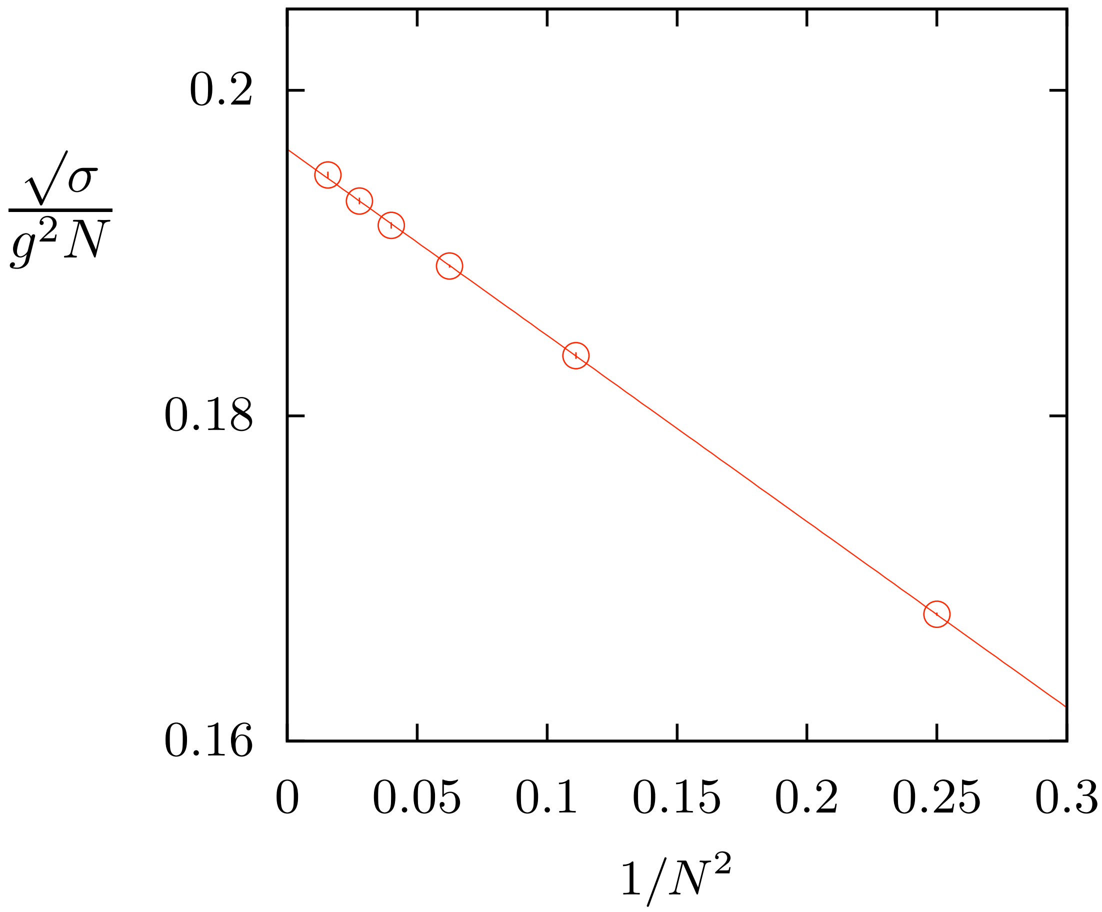
$\Longrightarrow$ use accurate calculations of $\sqrt{\sigma}$:

$D = 2+1:$ $\quad \lim_{a\to 0} \frac{\sqrt{\sigma}}{g^2 N}$ versus $\frac{1}{N^2}$

$D = 3+1:$ $\quad g^2(\mu)N$ versus μ for various N

Continuum limit of $N \in [2, 8]$ in $D = 2 + 1$

Bringoltz,MT hep-th/0611286



conventional fit: $\frac{\sqrt{\sigma}}{g^2 N} = 0.19638(9) - \frac{0.1144(8)}{N^2}$

Athenodorou,Bringoltz,MT ArXiv:1103.5854

fits $\Longrightarrow$

$\frac{\sqrt{\sigma}}{g^2 N} = c_0 + \frac{c_1}{N^\gamma} \quad \Rightarrow \quad \gamma = 1.97 \pm 0.10$

$\frac{\sqrt{\sigma}}{g^2 N^\alpha} = c_0 + \frac{c_1}{N^2} \quad \Rightarrow \quad \alpha = 1.002 \pm 0.004$

$\frac{\sqrt{\sigma}}{g^2 N^\alpha} = c_0 + \frac{c_1}{N^\gamma} \quad \Rightarrow \quad \alpha = 1.008 \pm 0.015, \ \gamma = 2.18 \pm 0.40$

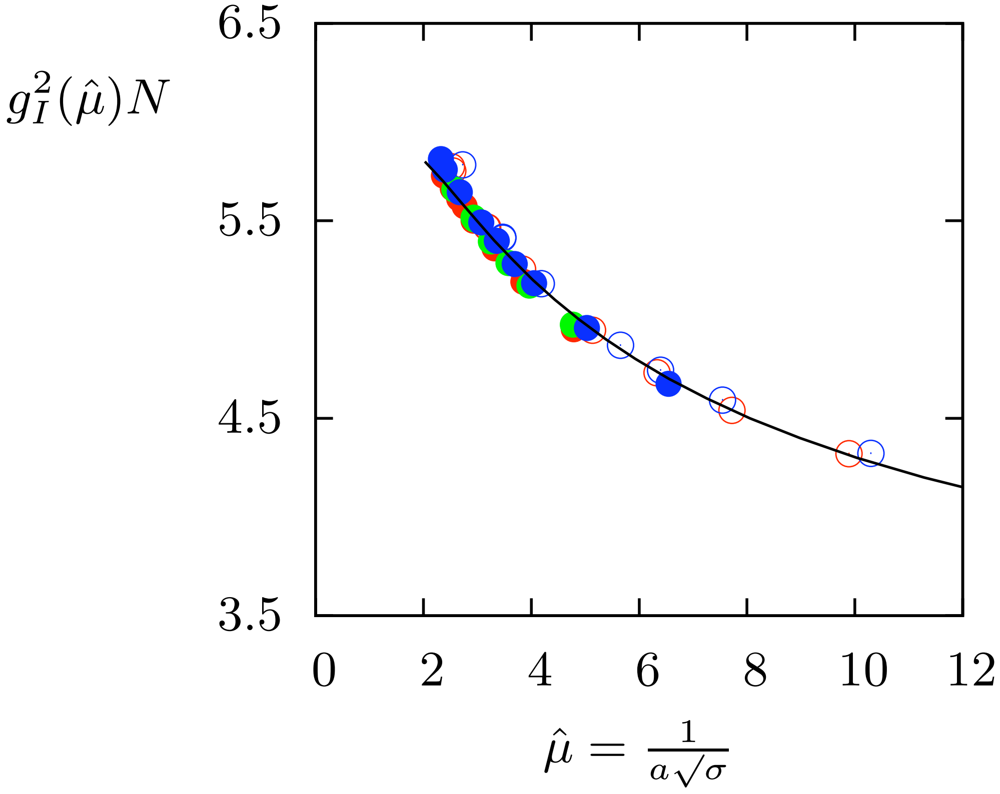
$\Longrightarrow$

strong support for non-perturbative validity of usual large-N counting

i.e.

$$\frac{\sqrt{\sigma}}{g^2 N} = c_0 + \frac{c_1}{N^2} + \cdots$$

$D = 3 + 1$: (lattice) running coupling for $N \in [2, 8]$

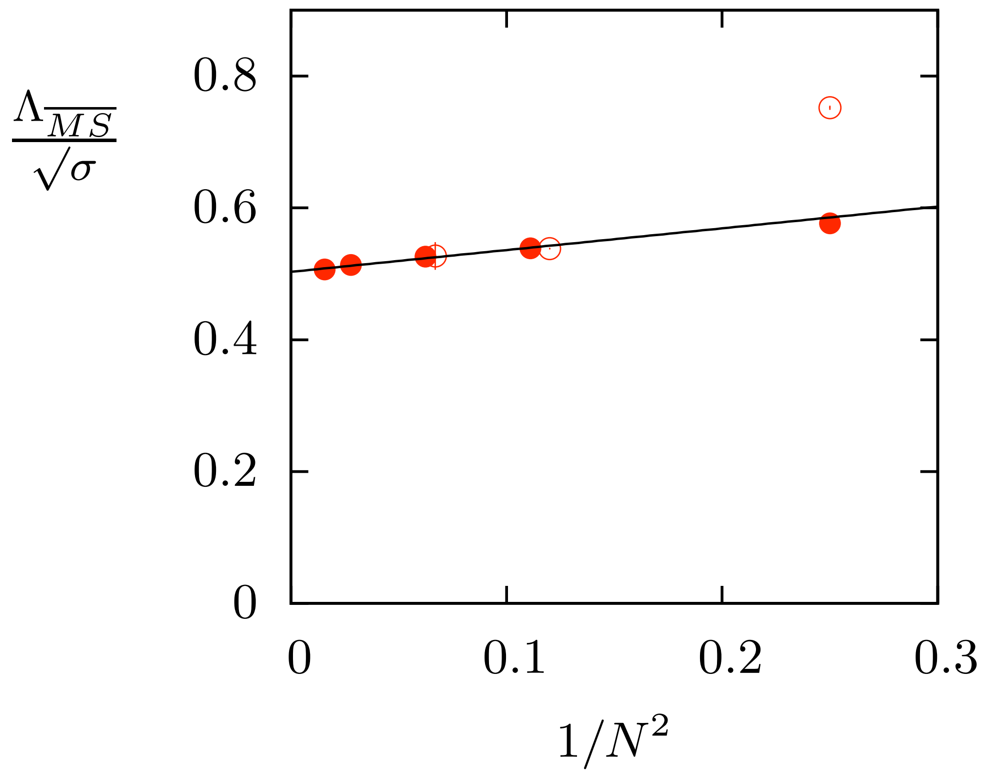


SU(2), ○; SU(3), ○; SU(4), ●; SU(6), ●; SU(8), ●.

Continuum running: SF coupling, *Alpha* collaboration + Lucini and Moriatis, 0805.2913.

Fitting the coupling at various N one finds:

●, Allton, MT, Trivini, arXiv:0803.1092; ○ Lucini, Moraitis, arXiv:0805.2913

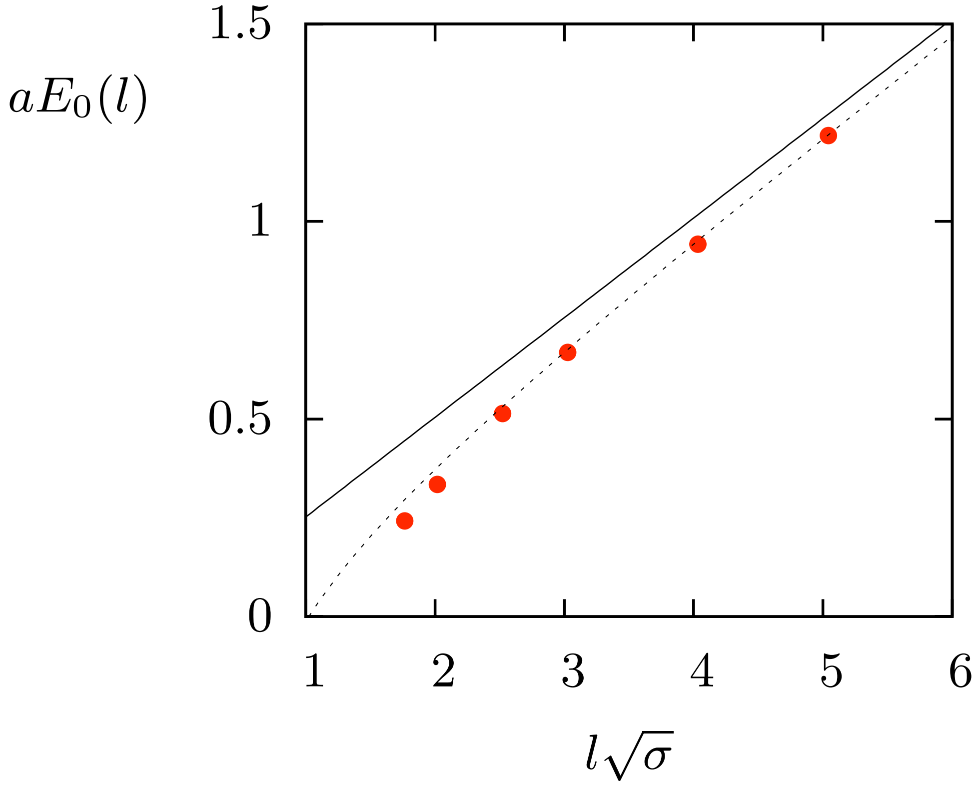


only show statistical errors ... fit: $\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.503(2)(40) + \frac{0.33(3)(3)}{N^2}$

i.e. SU(3) close to SU(∞)

D=3+1, SU(6) : energy of flux loop closed around a spatial torus

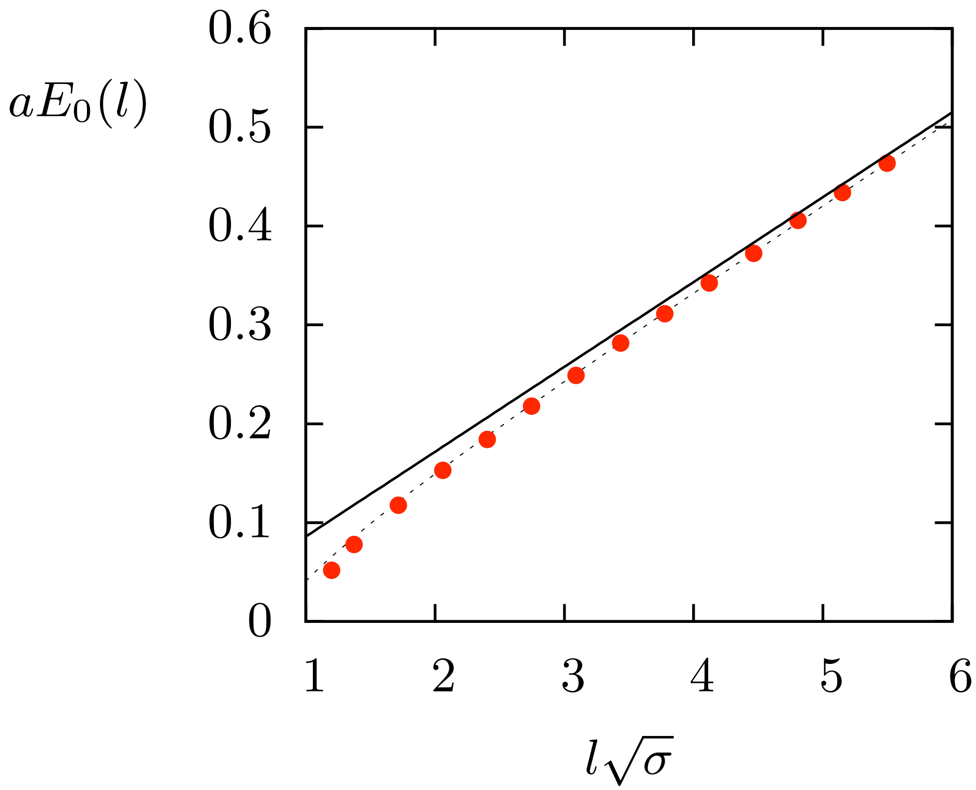
H. Meyer, M. Teper: hep-lat/0411039 $a\sqrt{\sigma} \simeq 0.252$



$\longrightarrow$ linear confinement: $E_0(l) \simeq \sigma l - \frac{\pi}{3l} + \cdots$ at large N

D=2+1, SU(6) : energy of flux loop closed around a spatial torus

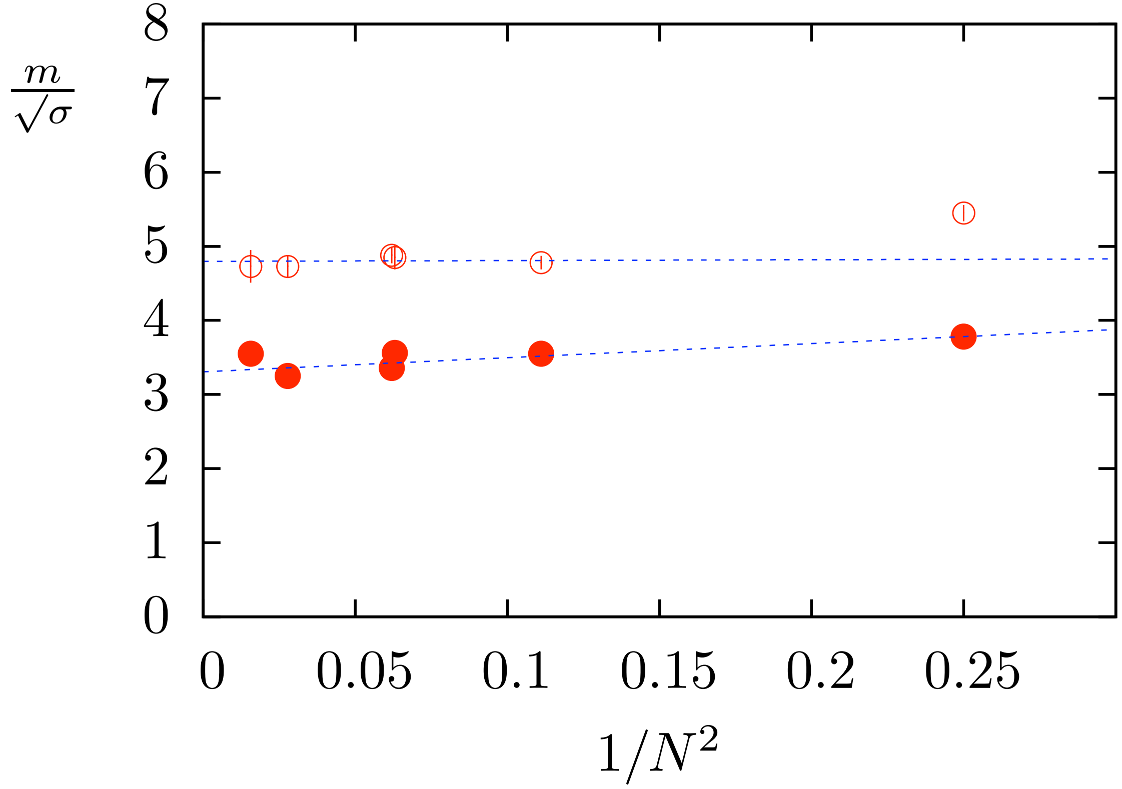
Athenodorou, Bringoltz, MT: arXiv:1103.5854 $a\sqrt{\sigma} \simeq 0.086$



$\longrightarrow$ linear confinement: $E_0(l) \simeq \sigma l - \frac{\pi}{6l} + \cdots$ at large N

Glueball mass spectrum: large-N limit

B.Lucini, MT, U.Wenger: hep-lat/0404008



(•) 0^{++}; (∘) 2^{++} $\longrightarrow$ SU(3) is 'close to' SU(∞) for lightest masses

$\longrightarrow$

- $N = \infty$ gauge theories are linearly confining
- SU(3) is 'close to' SU(∞) for some basic physical quantities

Is QCD_3 close to QCD_∞?

- Del Debbio, Lucini, Patella and Pica, arXiv:0712:3036.
- Bali and Bursa, arXiv:0806:2278; arXiv:0708:3427.

Hietanen, Narayanan, Patel and Prays, arXiv:0901:3752.

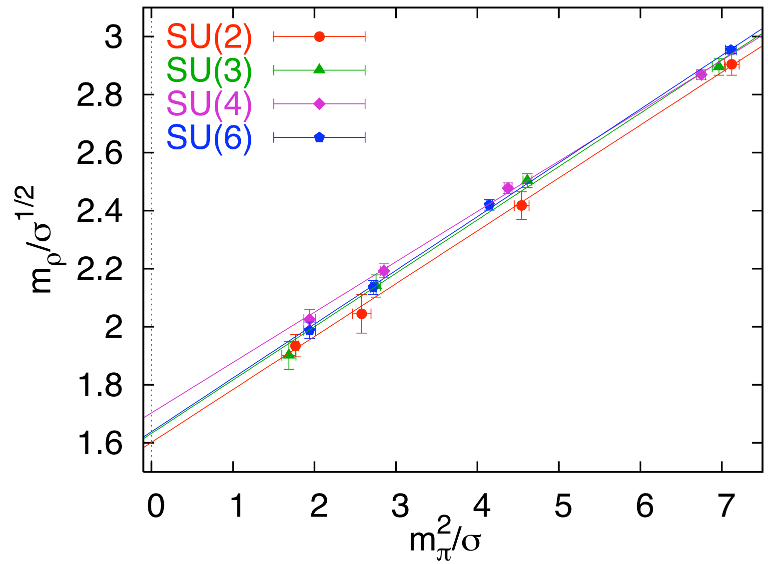
Strategy:

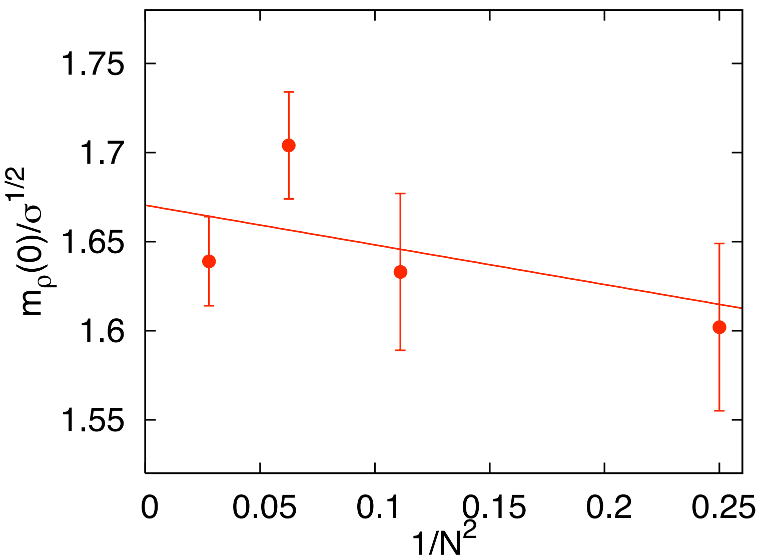
$$\text{quenched } QCD_N \stackrel{N\to\infty}{\longrightarrow} \text{full } QCD_{N=\infty}$$

- perform quenched QCD calculations at various N at a common value of a and various common values of m
- extrapolate at each fixed m to $N = \infty$, with $O(1/N^2)$ corrections
- now do conventional (full QCD) chiral extrapolation
- repeat for various a and extrapolate to continuum

- now compare to full QCD (or expt!) with SU(3)

$N = 2, 3, 4, 6$ for $a\sqrt{\sigma} \simeq 0.21$ Bali, Bursa





also for $a\sqrt{\sigma} \simeq 0.33$ Del Debbio et al

DD $\xrightarrow{\text{etal, }}$ BB

$$\lim_{N\to\infty,a\to 0} \frac{m_\rho}{\sqrt{\sigma}} = 1.79(5)$$

versus, in the real world :

$$\frac{m_\rho}{\sqrt{\sigma}} \simeq \frac{770\text{MeV}}{440\text{MeV}} \simeq 1.75$$

$\Longrightarrow$

$$N = 3 \text{ is 'close to' } N = \infty \text{ for full QCD} \cdots$$

BUT: Hietanen et al $\Rightarrow m_\rho \overset{N=19}{\sim} 2m_\rho^{QCD_3}$!

Some questions for QCD_∞:

Scalar mesons as $N \to \infty$: do the $\leq$ 1GeV states disappear?

The scalar nonet and the place of lightest scalar glueball?

Flavour singlet tensor and pseudoscalar mesons and glueballs?

Excited states stable $\longrightarrow$ Regge trajectories?

Excited states stable $\longrightarrow$ clean meson excitation spectrum.

SU($2n_f$) baryon (Dashen-Manohar) symmetry as $N \to 3$.

Physics at finite temperature:

- RHIC and LHC experiments

- area of choice for AdS/CFT applications:

SUSY $\sim$ gauge theory : $T_c < T < \text{few} \times T_c$

since :

adjoint fermions acquire a Matsubara mass

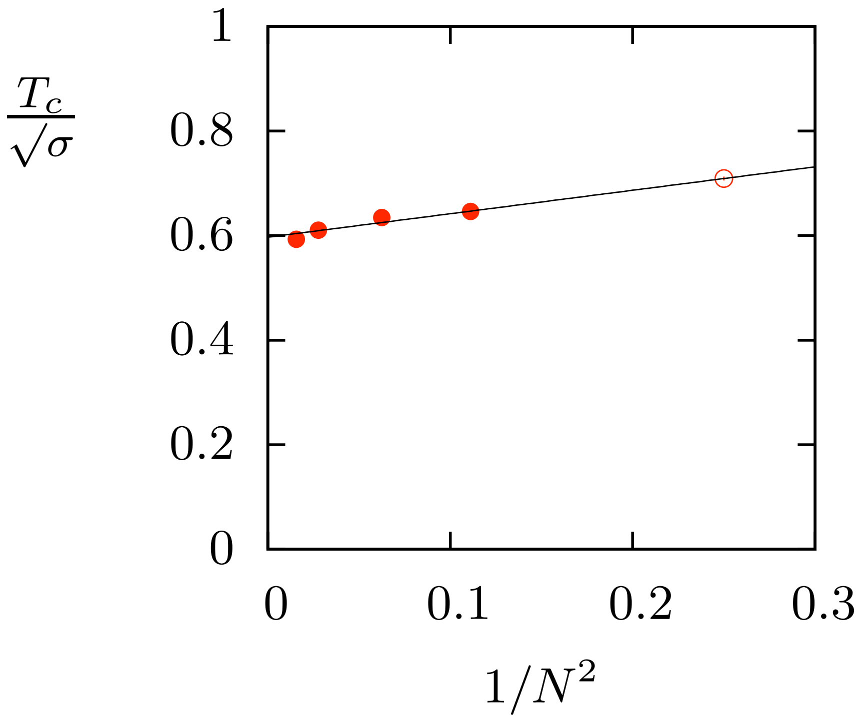
scalars then unprotected and acquire mass

also :

SQGP – strongly coupled quark-gluon plasma paradigm

- AdS/CFT is at large N, so important to check what features of QCD at $T \geq T_c$ have small finite N corrections.

Deconfining temperature in $D = 3 + 1$ Lucini, MT, Wenger: hep-lat/0307017,0502003



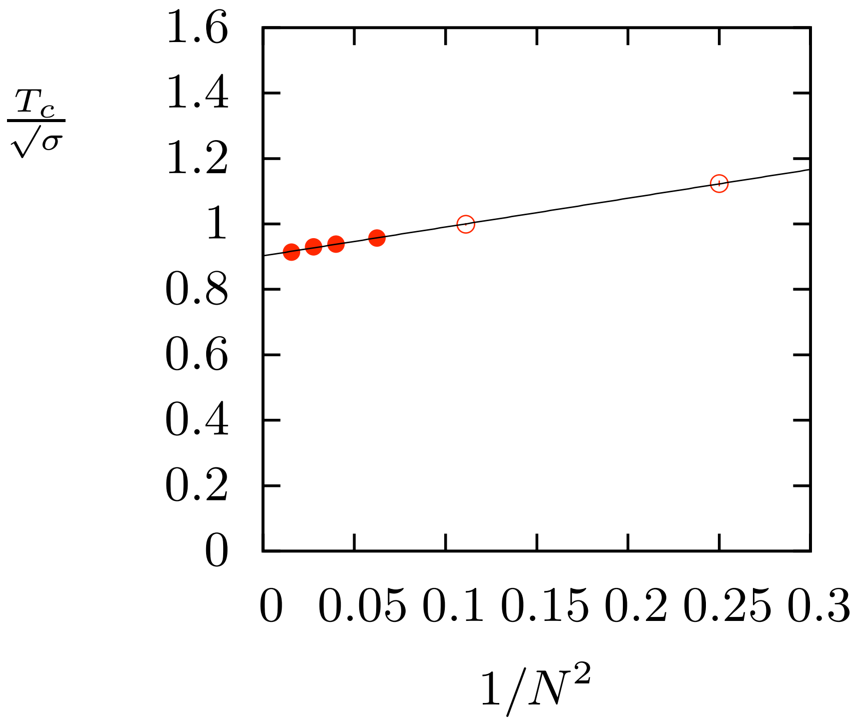
2nd order ○ ; 1st order ●

$\Rightarrow$ $\frac{T_c}{\sqrt{\sigma}} = 0.597(4) + \frac{0.45(3)}{N^2}$ and $E_{vac} \sim -O(N^2) \sim \textit{gluon condensate}$

T_c in $D = 2+1$

J. Liddle, MT : arXiv:0803.2128

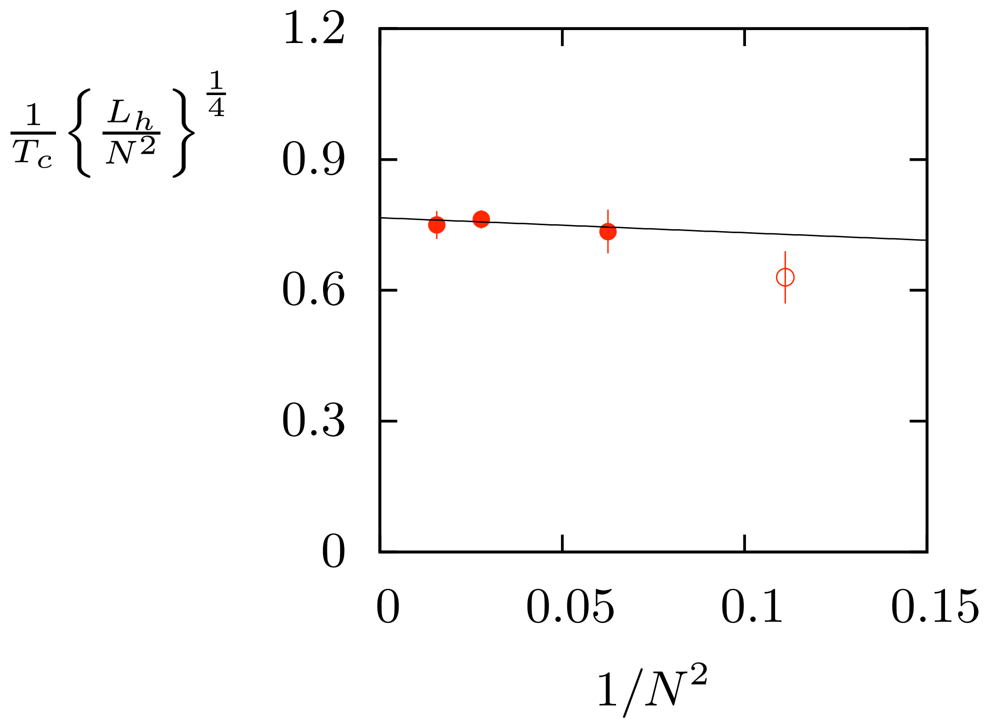
Holland, Pepe, Wiese : arXiv:0712.1216



$\Rightarrow$ $\frac{T_c}{\sqrt{\sigma}} = 0.903(3) + \frac{0.88(5)}{N^2}$ and $E_{vac} \sim -O(N^2) \sim gluon\, condensate$

Confinement-deconfinement latent heat in $D = 3 + 1$

Lucini, MT, Wenger: hep-lat/0307017,0502003

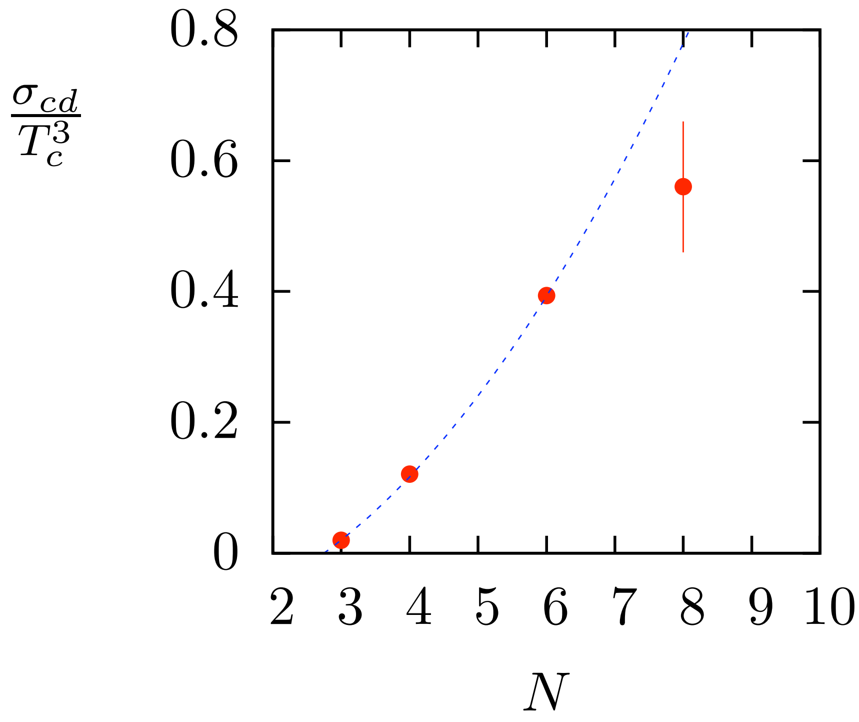


$\Rightarrow$

large-N deconfinement is 'normal' first order

Confinement-deconfinement interface tension

$a = 1/5T_c$ Lucini, MT, Wenger: hep-lat/0502003

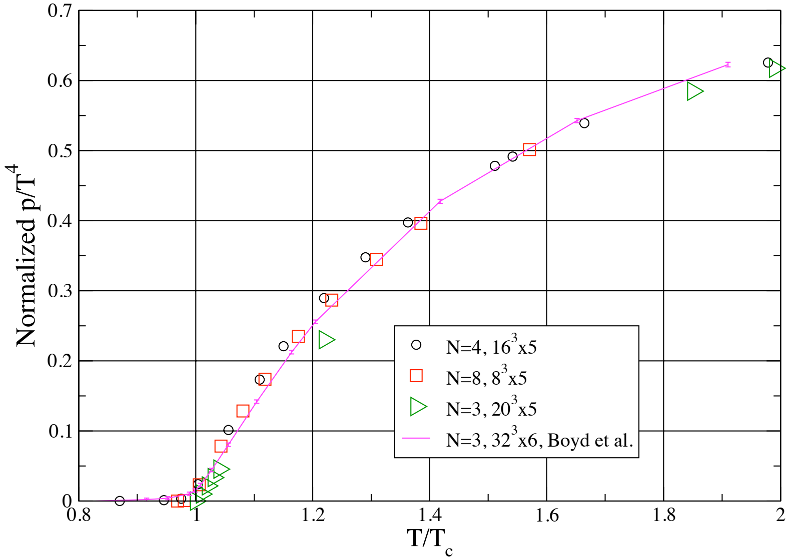


fit : $\frac{\sigma_{cd}}{T_c^3} = 0.0138N^2 - 0.104 = 0.0138N^2\left(1 - \frac{7.53}{N^2}\right)$

$\Rightarrow$ interface tension small and $O(1/N^2)$ corrections are large

Strongly Coupled Gluon Plasma - at large N?

Bringoltz, MT, hep-lat/0506034; Panero, arXiv:0907.3719; Datta, Gupta,arXiv:1006.0938

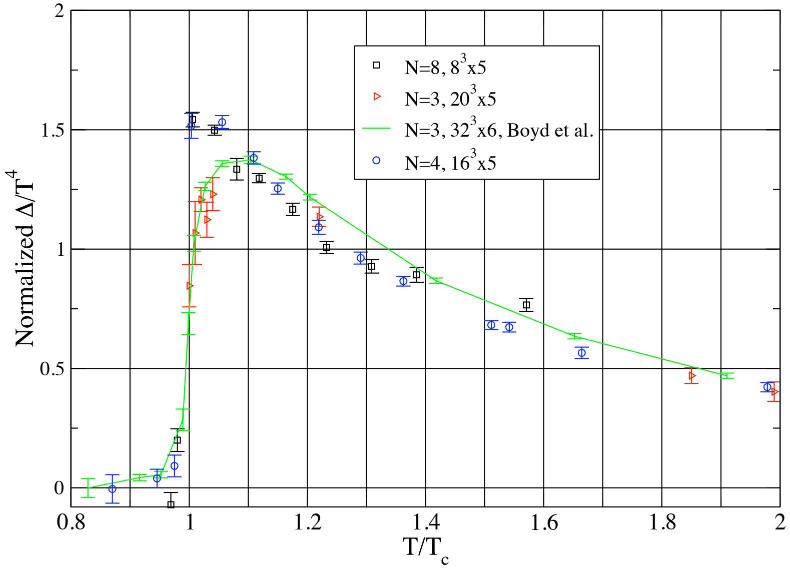


$T_c < T <$ few $\times T_c$ pressure anomaly is 'independent' of N

$$\Delta \equiv \epsilon - 3p$$

Bringoltz, MT : hep-lat/0506034

[$\Delta = 0$ in Stefan-Boltzman gas and SUSY]



Some conclusions from this (highly incomplete) survey

- large-N counting as described by 't Hooft

- $SU(\infty)$ linearly confining

- $N = 3$ close to $N = \infty$ for many basic physical quantities, both at $T = 0$ and $T \neq 0$

- ... but not always, e.g. interface tension

- and mesons in QCD_N need to be clarified

- SGP is $\sim$ 'independent' of N

Flux tubes and string theory

$\leftrightarrow$

Confining flux tubes in SU(N) gauge theories and their effective string theory description

- Veneziano amplitude
- 't Hooft large-N – genus diagram expansion
- Polyakov action
- Maldacena ... AdS/CFT/QCD ...

at large N, flux tubes and perhaps the whole gauge theory can be described by a weakly-coupled string theory

focus on the spectrum of flux tubes
closed around a spatial torus of length l
— both D=2+1 and D=3+1

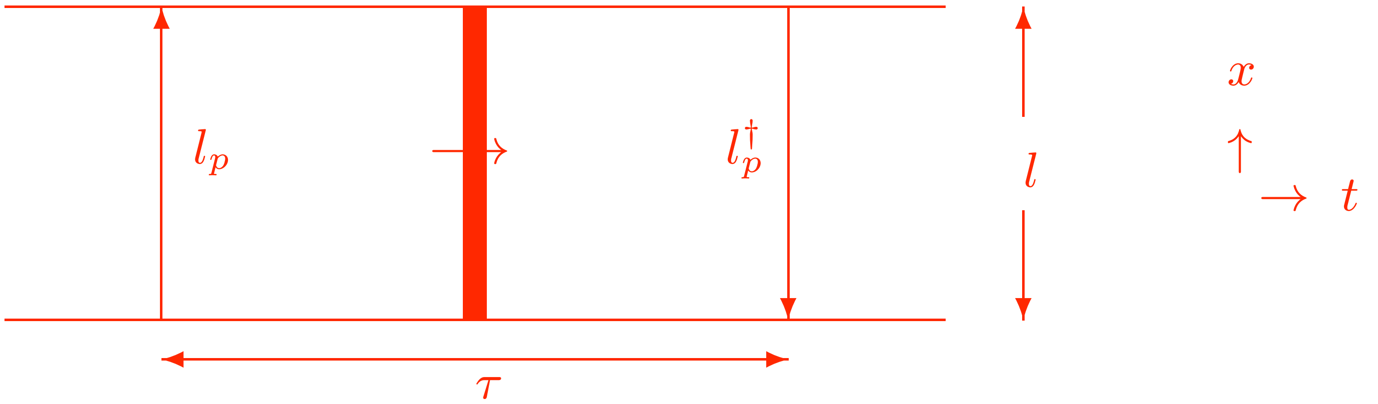
- flux localised in 'tubes'; long flux tubes, $l\sqrt{\sigma} \gg 1$ look like 'thin strings'
- at $l = l_c = 1/T_c$ there is a 'deconfining' phase transition: 1st order for $N \geq 3$ in $D = 4$ and for $N \geq 4$ in $D = 3$
- so may have a simple string description of the closed string spectrum for all $l \geq l_c$
- most plausible at $N \to \infty$ where scattering, mixing and decay, e.g string $\to$ string + glueball, go away

Note: the static potential $V(r)$ describes the transition in r between UV (Coulomb potential) and IF (flux tubes) physics; potentially of great interest as $N \to \infty$.

calculate the ground state energy of a confining flux tube winding around a spatial torus of length l, using correlators of Polyakov loops:

$$\langle l_p^\dagger(\tau) l_p(0)\rangle = \sum_{n,p} c_n(p,l) e^{-E_n(p,l)\tau} \overset{\tau\to\infty}{\propto} \exp\{-E_0(l)\tau\}$$

in pictures

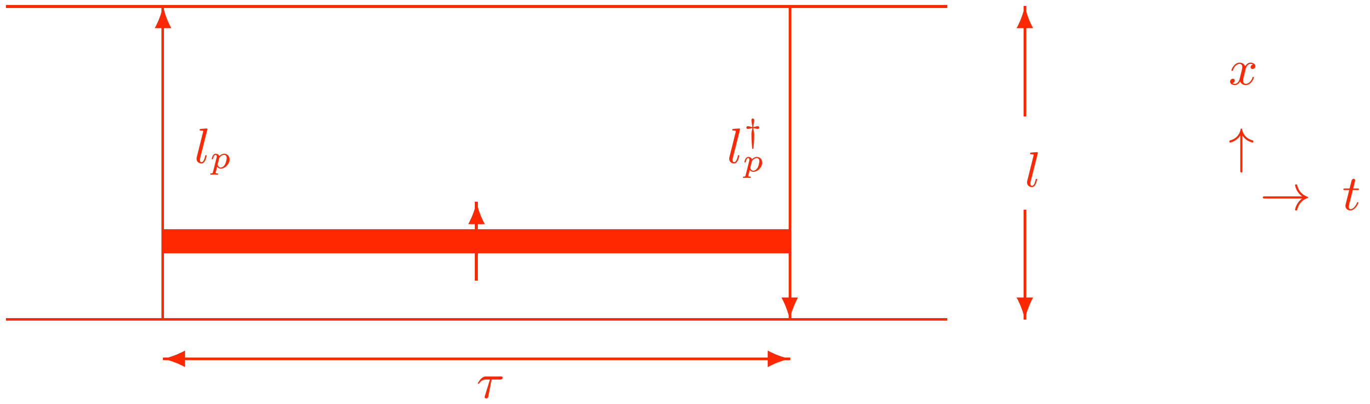


a flux tube sweeps out a cylindrical $l \times \tau$ surface S $\cdots$ integrate over these world sheets with an effective string action $\propto \int_{cyl=l\times\tau} dS e^{-S_{eff}[S]}$

but also a flux tube attached to the static sources propagating in the x-direction:

$$\langle l_p^\dagger(\tau) l_p(0)\rangle = \sum_n e^{-\hat{E}_n(\tau)l} \overset{l\to\infty}{\propto} \exp\{-\hat{E}_0(\tau)l\}$$

in pictures



this is an example of an open-closed string 'duality'

$\Rightarrow$

$$\sum_{n,p} c_n(p,l)e^{-E_n(p,l)\tau} = \sum_n e^{-\hat{E}_n(\tau)l} = \int_{cyl=l\times\tau} dSe^{-S_{eff}[S]}$$

where $S_{eff}[S]$ is the effective string action for the surface S

$\Rightarrow$

the string partition function will predict the spectrum $\hat{E}_n(\tau)$ – just a Laplace transform – but will be constrained by the Lorentz invariance encoded in $E_n(p,l)$

Parameterising S (static gauge):

• $h(x, t)$ is transverse displacement (vector in $D = 3 + 1$) from minimal surface $x \in [0, l]$ and $t \in [0, \tau]$, i.e.

$$S_{eff}[S] \longrightarrow S_{eff}[h]$$

and we integrate over the field $h(x, t)$

• translation invariance $\Rightarrow S_{eff}[h]$ cannot depend on position but only on $\partial_\alpha h$, with $\alpha = x, t$, $\quad \Rightarrow \quad$ we can do a derivative expansion:

$$S_{eff} \sim \sigma l \tau + \int_0^\tau dt \int_0^l dx \frac{1}{2} \partial h \partial h + \sum c_{n,i} \int_0^\tau dt \int_0^l dx \partial^{n+i} h^n$$

• $\Rightarrow$ an expansion of $E_n(l)$ in powers of $1/\sigma l^2$

• open-closed duality constrains some of these coefficients $\Rightarrow$ some correction terms in $E(l) = \sigma l + \cdots$ are 'universal'

$\Rightarrow$

e.g. any $S_{eff} \Rightarrow$

$$E_0(l) \stackrel{l\to\infty}{=} \sigma l - \frac{\pi(D-2)}{6l} - \frac{\{\pi(D-2)\}^2}{72}\frac{1}{\sigma l^3} - \frac{\{\pi(D-2)\}^3}{432}\frac{1}{\sigma^2 l^5} + O\left(\frac{1}{l^7}\right)$$

universal terms:

- $O\left(\frac{1}{l}\right)$ Luscher correction, $\sim$ 1980
- $O\left(\frac{1}{l^3}\right)$ Luscher, Weisz; Drummond, $\sim$ 2004
- $O\left(\frac{1}{l^5}\right)$ Aharony et al, $\sim$ 2009-10

also for $E_n(l)$

$\longrightarrow$ simplest free string theory : Nambu-Goto in flat space-time $\cdots$

Nambu-Goto free string theory

$$\int \mathcal{D}S e^{-\kappa A[S]}$$

Spectrum:

$$E^2(l) = (\sigma\, l)^2 + 8\pi\sigma \left(\frac{N_L + N_R}{2} - \frac{D-2}{24} \right) + \left(\frac{2\pi q}{l} \right)^2 .$$

$p = 2\pi q / l =$ total momentum along string;

$N_L, N_R =$ sum left and right 'phonon' momentum:

$$N_L = \sum_{k>0} n_L(k)\, k, \quad N_R = \sum_{k>0} n_R(k)\, k, \quad N_L - N_R = q$$

where

$$\text{state} = \prod_{k>0} a_k^{n_L(k)} a_{-k}^{n_R(k)} |0\rangle$$

Nambu-Goto $\Rightarrow$

$$
\begin{aligned}
E_0(l) &= \sigma l \left(1 - \frac{\pi(D-2)}{3\sigma l^2}\right)^{\frac{1}{2}} \\
&= \sigma l - \frac{\pi(D-2)}{6l} - \frac{\{\pi(D-2)\}^2}{72}\frac{1}{\sigma l^3} - \frac{\{\pi(D-2)\}^3}{432}\frac{1}{\sigma^2 l^5} + O\left(\frac{1}{l^7}\right)
\end{aligned}
$$

– same universal correction terms to ground state

– also for excited states, e.g.

$$
E_n(l) = \sigma l \left(1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{D-2}{24}\right)\right)^{\frac{1}{2}} \stackrel{l > l_n}{=} \sigma l + \sum_{n=0} \frac{c_n}{\sigma^n l^{2n+1}}
$$

where $l_n\sqrt{\sigma} \sim \sqrt{8\pi n}$

What does one find numerically?

results here are from:

- $D = 2 + 1$ Athenodorou, Bringoltz, MT arXiv:1103.5854
- $D = 3 + 1$ Athenodorou, Bringoltz, MT arXiv:1007.4720

also

Cracow School Lectures, MT arXiv:0912.3339

also see for references to other work

for recent analytic work see:

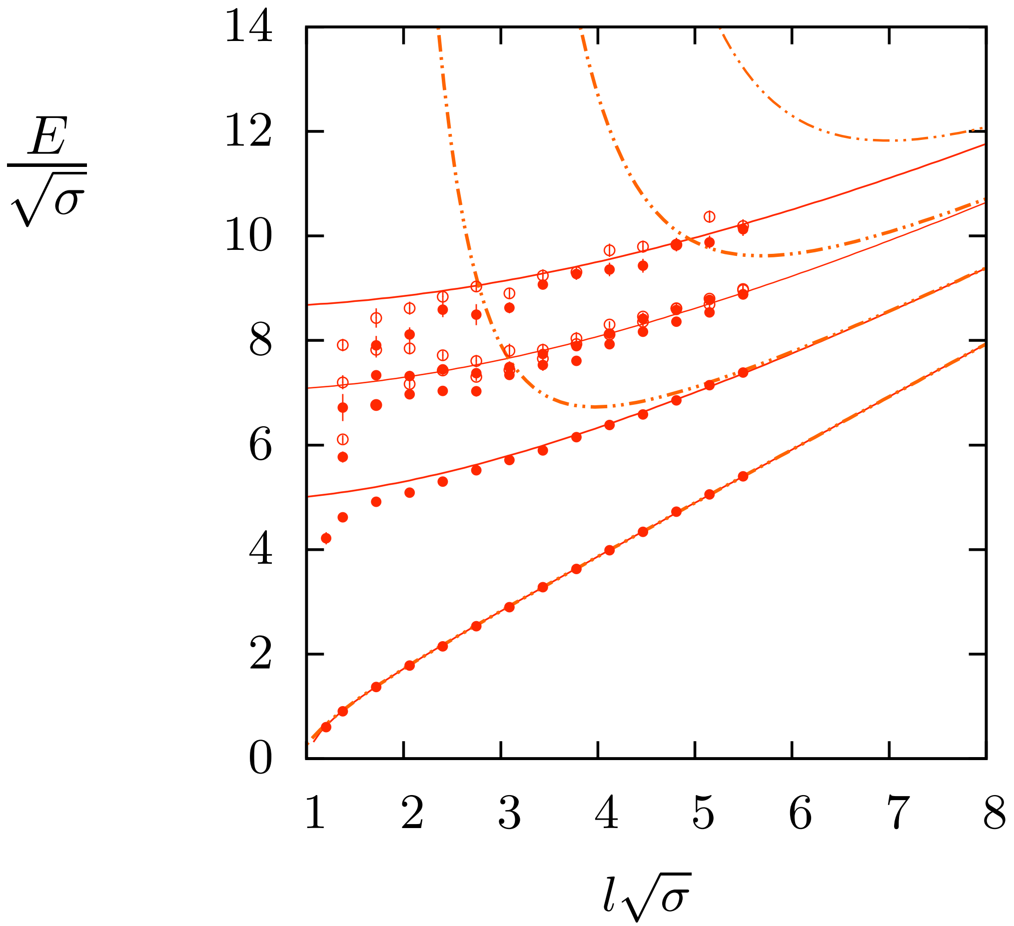
O. Aharony and E. Karzbrun, arXiv:0903.1927

O. Aharony amd M. Field, arXiv:1008.2636

O. Aharony and N. Klinghoffer, arXiv:1008.2648

O. Aharony, Talk at *Confining Flux Tubes and Strings* (ECT, July 2010).

lightest 8 states with $p = 0$ $D = 2 + 1$ SU(6), small a



Nambu-Goto : solid lines universal terms: dashed lines

States:

$|0\rangle$

$a_1 a_{-1}|0\rangle$

$a_2 a_{-2}|0\rangle,\ a_2 a_{-1} a_{-1}|0\rangle,\ a_1 a_1 a_{-2}|0\rangle,\ a_1 a_1 a_{-1} a_{-1}|0\rangle$

$\ldots$

$P = (-1)^{number\ phonons} = +, -$ for $\bullet,\ \circ$

$\Rightarrow$

observe stringy degeneracies

note: Nambu-Goto predictions for excited states have no free parameters

$\Longrightarrow$

• NG very good down to $l\sqrt{\sigma} \sim 2$, i.e energy

fat short flux 'tube' $\sim$ ideal thin string

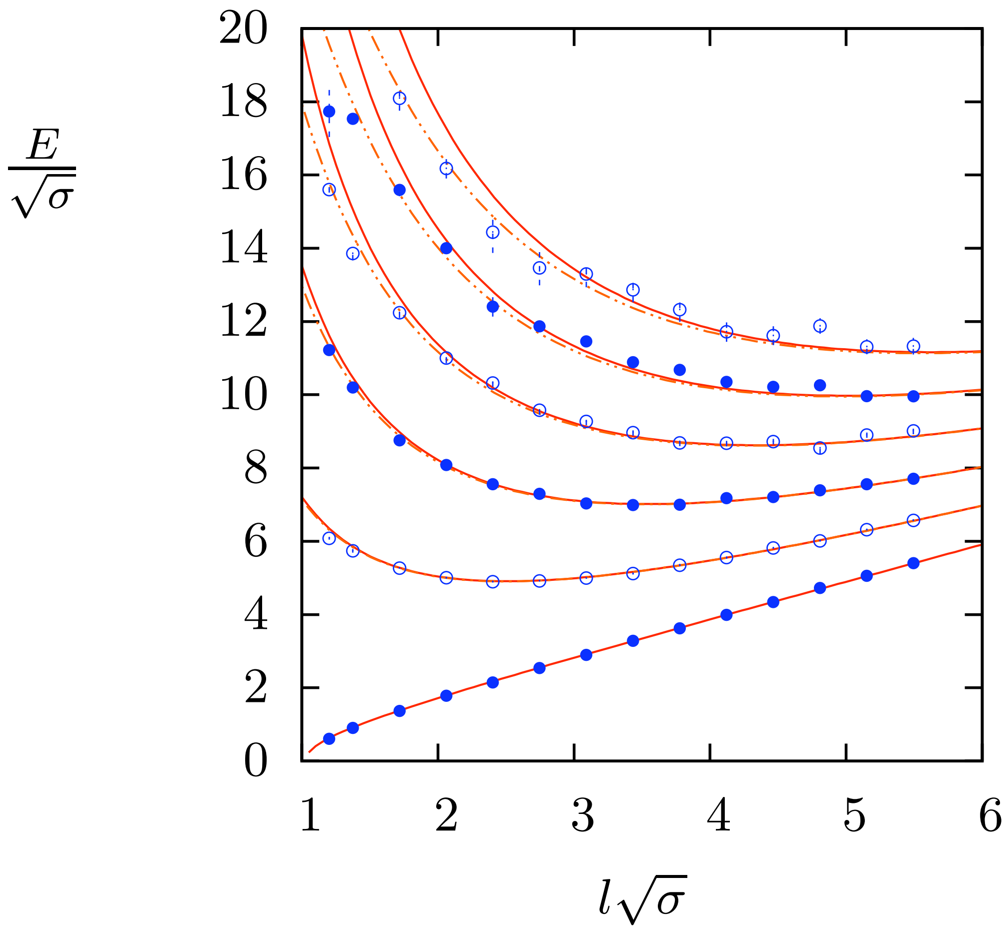
• NG very good far below value of $l\sqrt{\sigma}$ where the power series expansion diverges, i.e. where all orders are important $\Rightarrow$

universal terms not enough to explain this agreeement ...

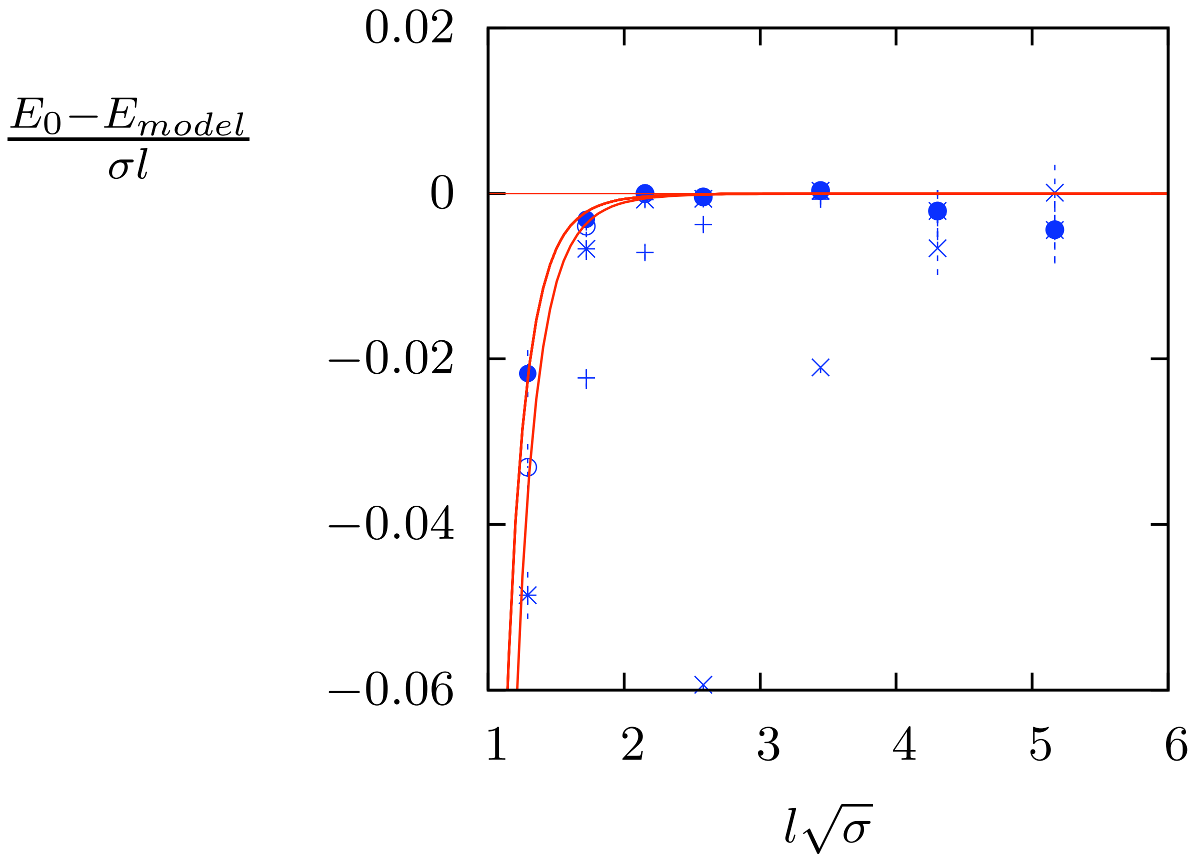
• no sign of any non-stringy modes, e.g.

$E(l) \simeq E_0(l) + \mu$ where e.g. $\mu \sim M_G/2 \sim 2\sqrt{\sigma}$

lightest $P = -$ states with $p = 2\pi q/l$: $q = 0, 1, 2, 3, 4, 5$ $\qquad a_q|0\rangle$



Nambu-Goto : solid lines $\qquad (ap)^2 \rightarrow 2 - 2\cos(ap)$: dashed lines ground state deviation from various 'models' $D = 2 + 1$



model = Nambu-Goto, $\bullet$, universal to $1/l^5$, $\circ$, to $1/l^3$, $\star$, to $1/l$, $+$, just σl, $\times$

lines = plus $O(1/l^7)$ correction

$\Longrightarrow$

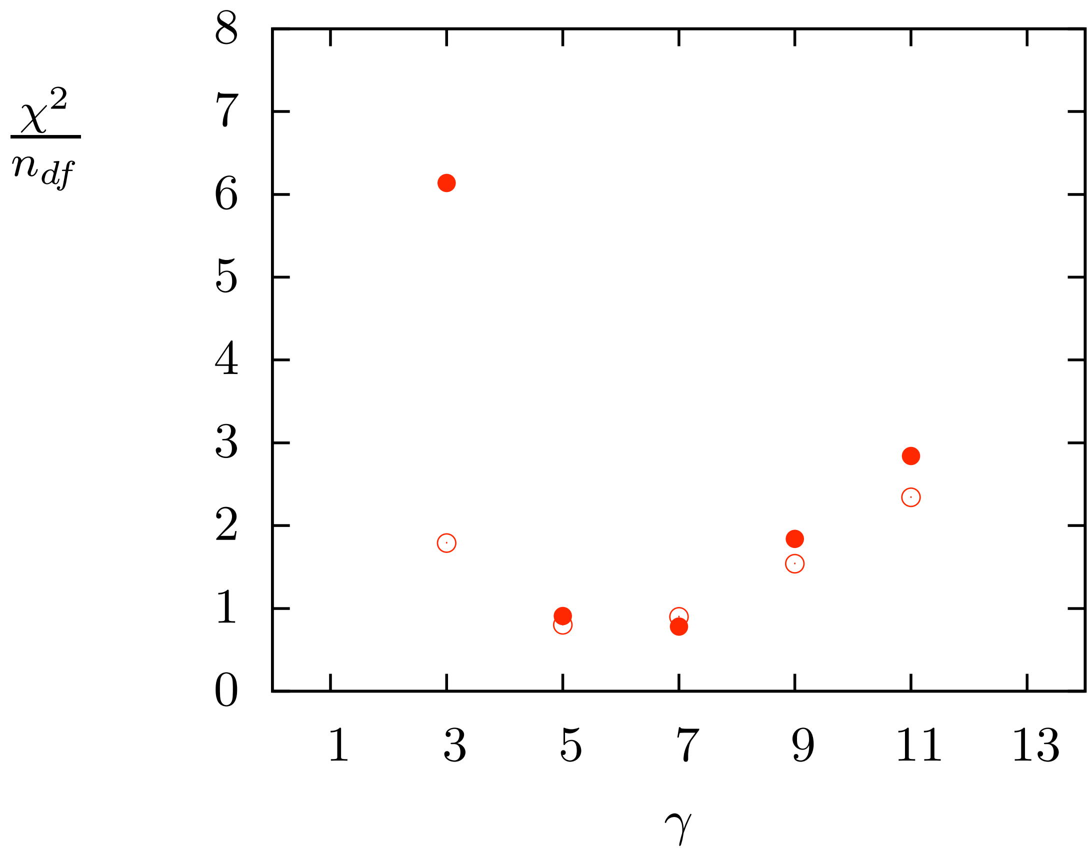
$\circ$ for $l\sqrt{\sigma} \gtrsim 2$ agreement with NG to $\lesssim 1/1000$

moreover

$\circ$ for $l\sqrt{\sigma} \sim 2$ contribution of NG to deviation from σl is $\gtrsim 99\%$

despite flux tube being short and fat

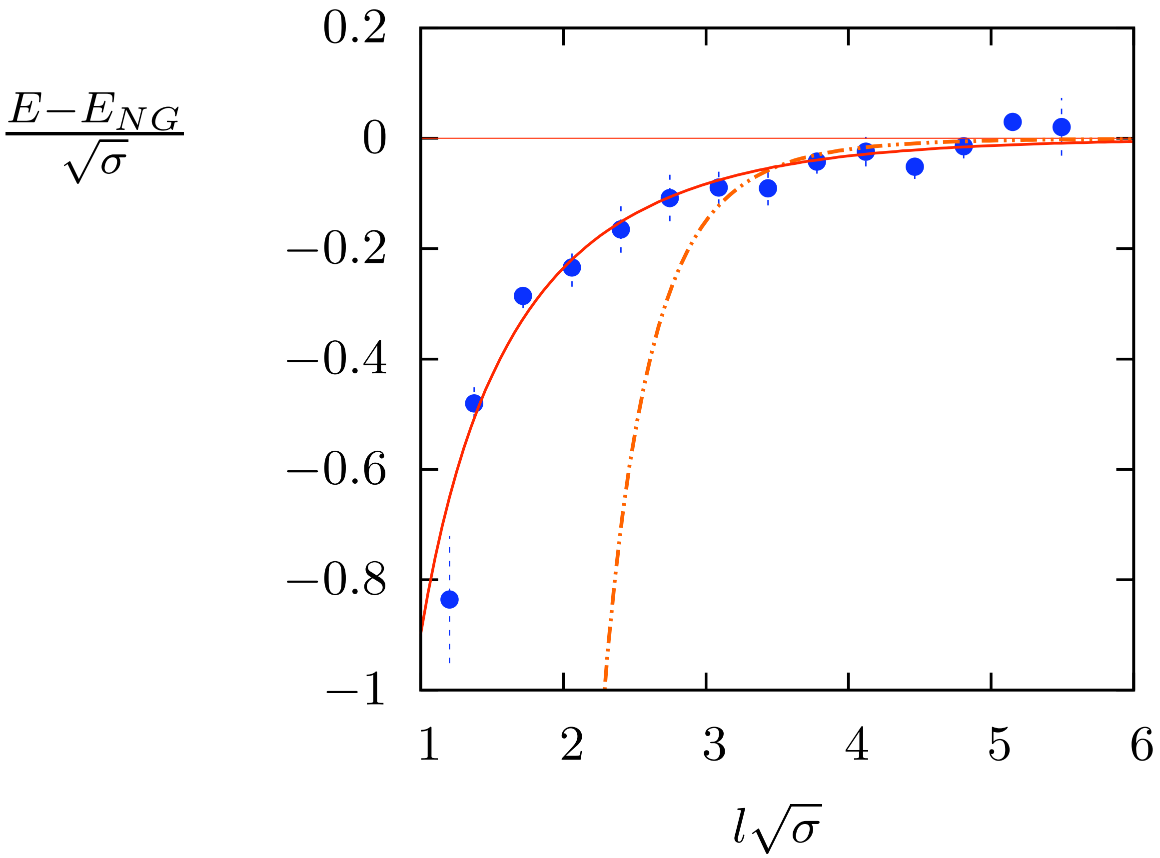
$\circ$ and leading correction to NG consistent with $\propto 1/l^7$ as expected from current universality results



χ^2 per degree of freedom for the best fit

$$E_0(l) = E_0^{NG}(l) + \frac{c}{l^\gamma}$$

first excited $q = 0$, $P = +$ state $\qquad$ $D = 2 + 1$



fits:

$\frac{c}{(l\sqrt{\sigma})^7}$ - dotted curve; $\qquad$ $\frac{c}{(l\sqrt{\sigma})^7}\left(1 + \frac{25.0}{l^2\sigma}\right)^{-2.75}$ - solid curve

$\Longrightarrow$ if we write

$$\frac{1}{\sqrt{\sigma}}E_n(l) = \frac{1}{\sqrt{\sigma}}E_n^{NG}(l) + \frac{1}{\sqrt{\sigma}}\Delta E_n(l) \tag{1}$$
$$\stackrel{l\to\infty}{=} \frac{1}{\sqrt{\sigma}}E_n^{NG}(l) + \frac{c}{(l\sqrt{\sigma})^7}\left\{1 + \frac{c_1}{l^2\sigma} + \frac{c_2}{(l^2\sigma)^2} + \cdots\right\}$$

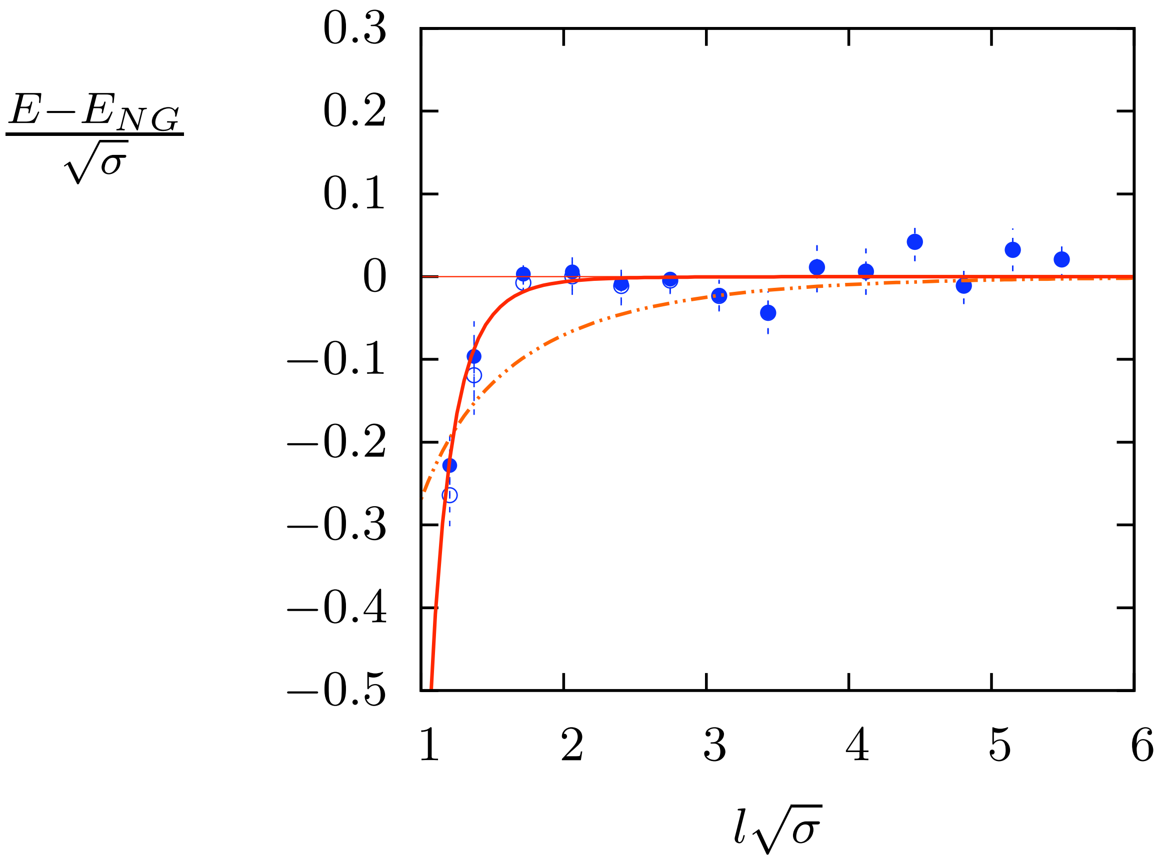
then correction to NG resums, just like NG,

$$\frac{1}{\sqrt{\sigma}}\Delta E_n(l) = \frac{c}{(l\sqrt{\sigma})^7}\left(1 + \frac{c'}{l^2\sigma}\right)^{-\gamma} \simeq \begin{cases} \frac{c}{(l\sqrt{\sigma})^7} & l \gg l_d \\ \frac{cc'^{-\gamma}}{(l\sqrt{\sigma})^{7-2\gamma}} & l \ll l_d \end{cases}$$

and with our fit we find $c \sim 0.6 \times c_7^{NG}$

for most but not all light excited states:

$q = 1, P = -$ ground state $\qquad SU(6), \;\; D = 2+1$



fits:

$\frac{c}{(l\sqrt{\sigma})^7}$ solid curve; $\qquad \frac{c}{(l\sqrt{\sigma})^7}\left(1+\frac{25.0}{l^2\sigma}\right)^{-2.75}$: dashed curve

$$D = 2 + 1 \qquad \text{to} \qquad D = 3 + 1$$

- additional rotational quantum number: phonon carries spin 1

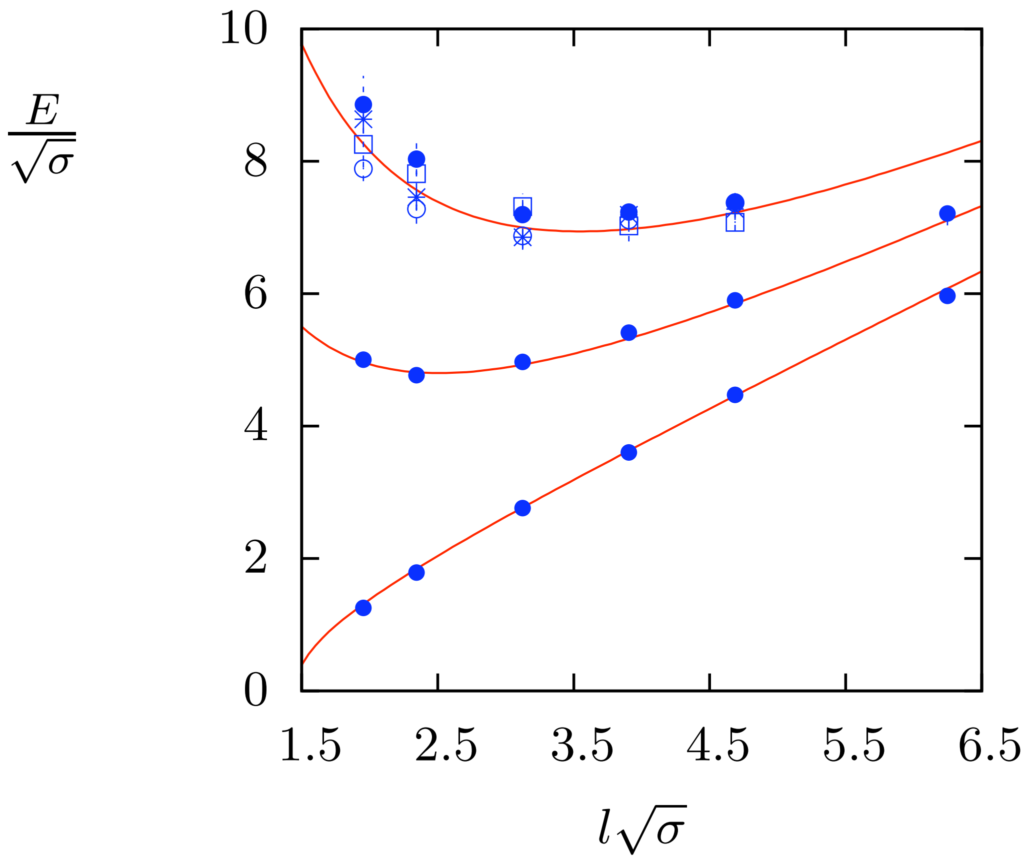
- Nambu-Goto again remarkably good for most states

- BUT now there are some candidates for non-stringy (massive?) mode excitations ...

however in general results are considerably less accurate

$p = 2\pi q/l$ for $q = 0, 1, 2$

$D = 3+1,\ SU(3),\ l_c\sqrt{\sigma} \sim 1.5$

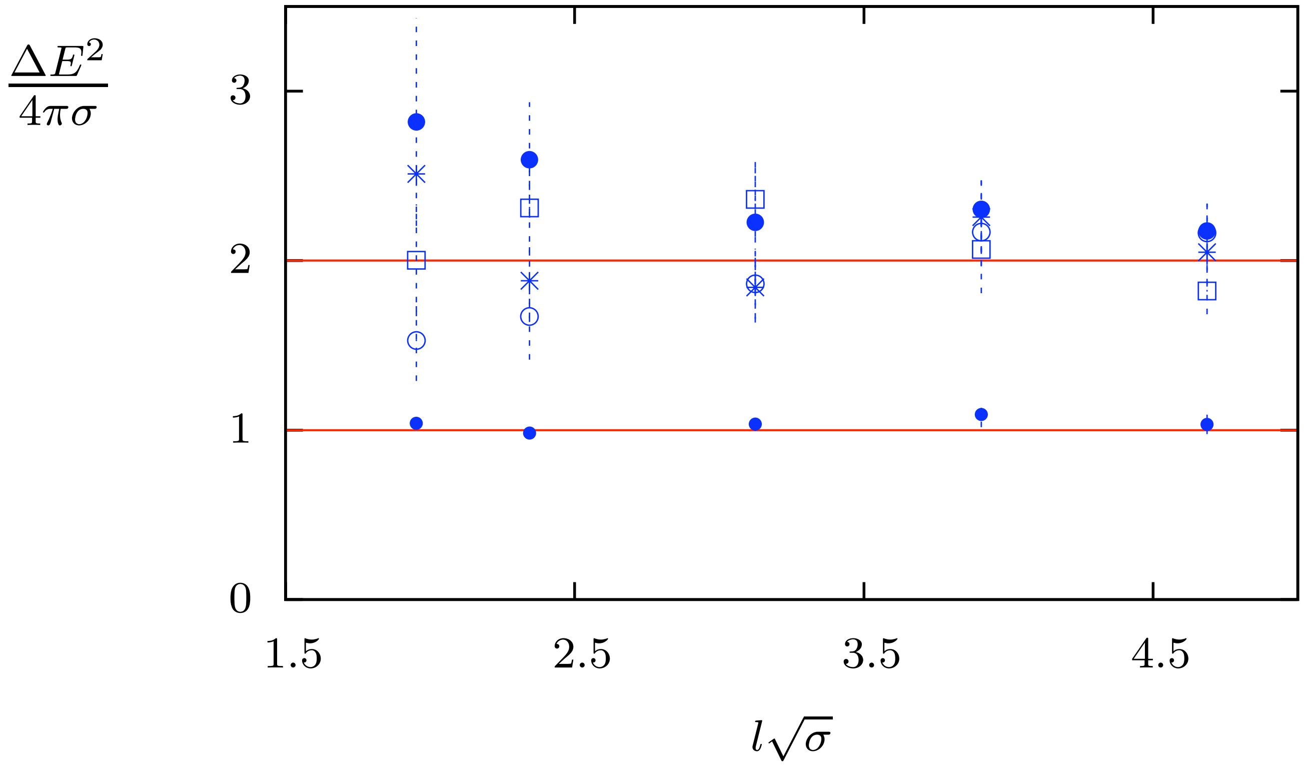


The four $q = 2$ states are: $J^{P_t} = 0^{+}(\star),\ 1^{\pm}(\circ),\ 2^{+}(\Box),\ 2^{-}(\bullet)$.
Lines are Nambu-Goto predictions.

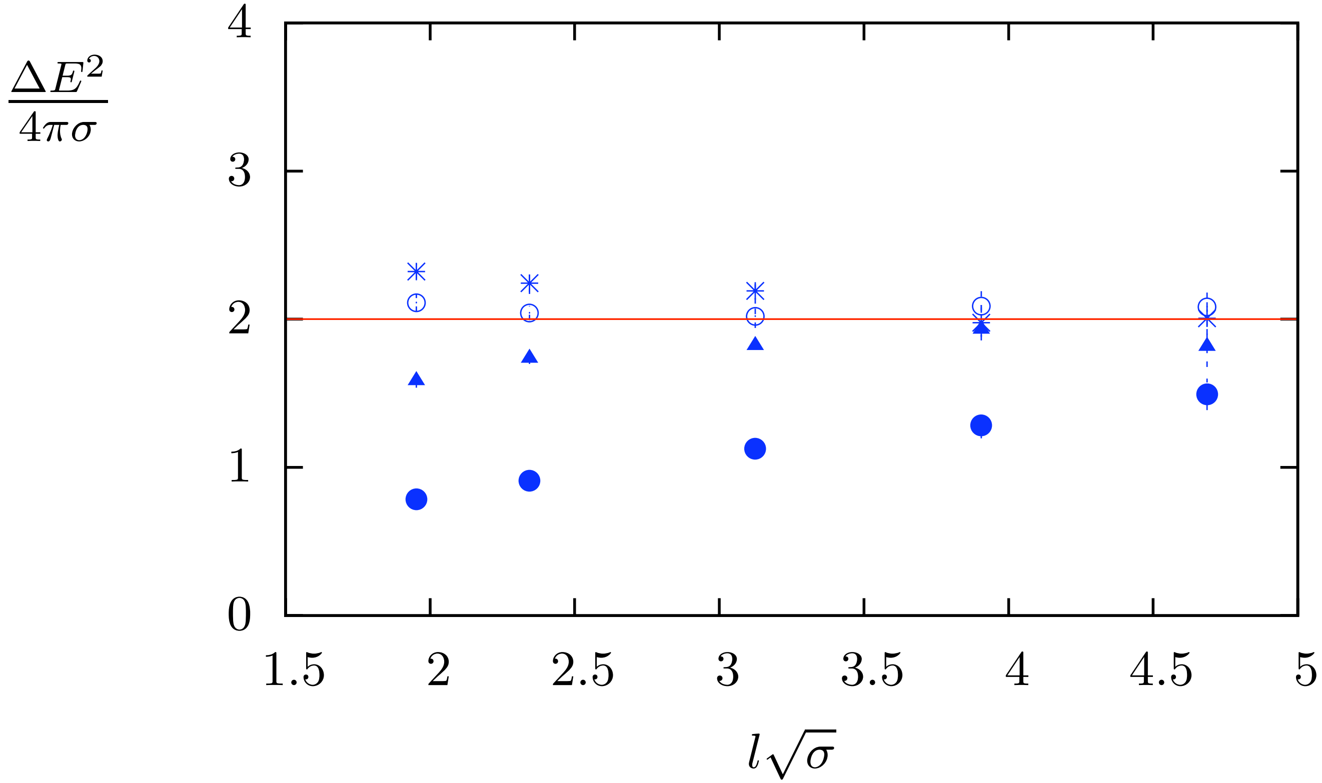
for a precise comparison with Nambu-Goto, define:

$$\Delta E^2(q,l) = E^2(q;l) - E_0^2(l) - \left(\frac{2\pi q}{l}\right)^2 \stackrel{NG}{=} 4\pi\sigma(N_L + N_R)$$

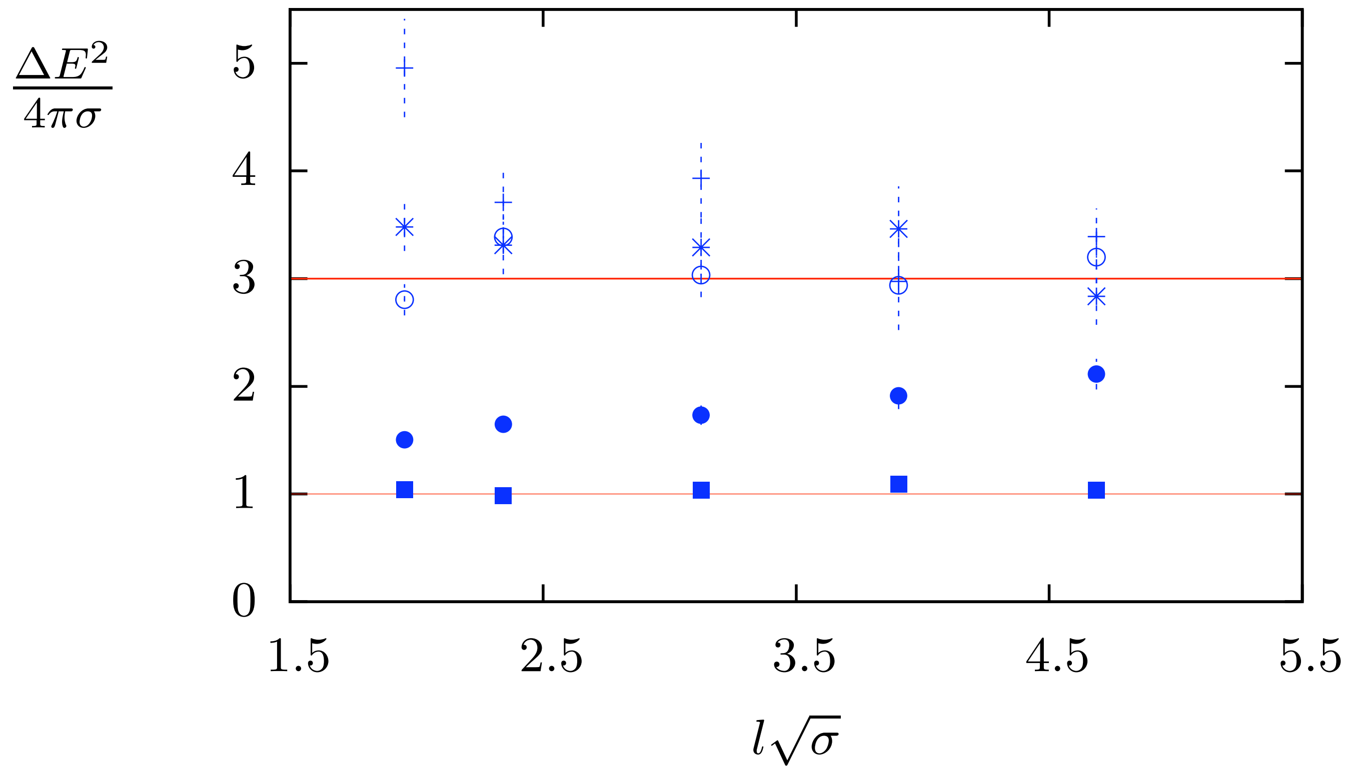
$\Longrightarrow$ lightest $q = 1, 2$ states:



lightest few $p = 0$ states



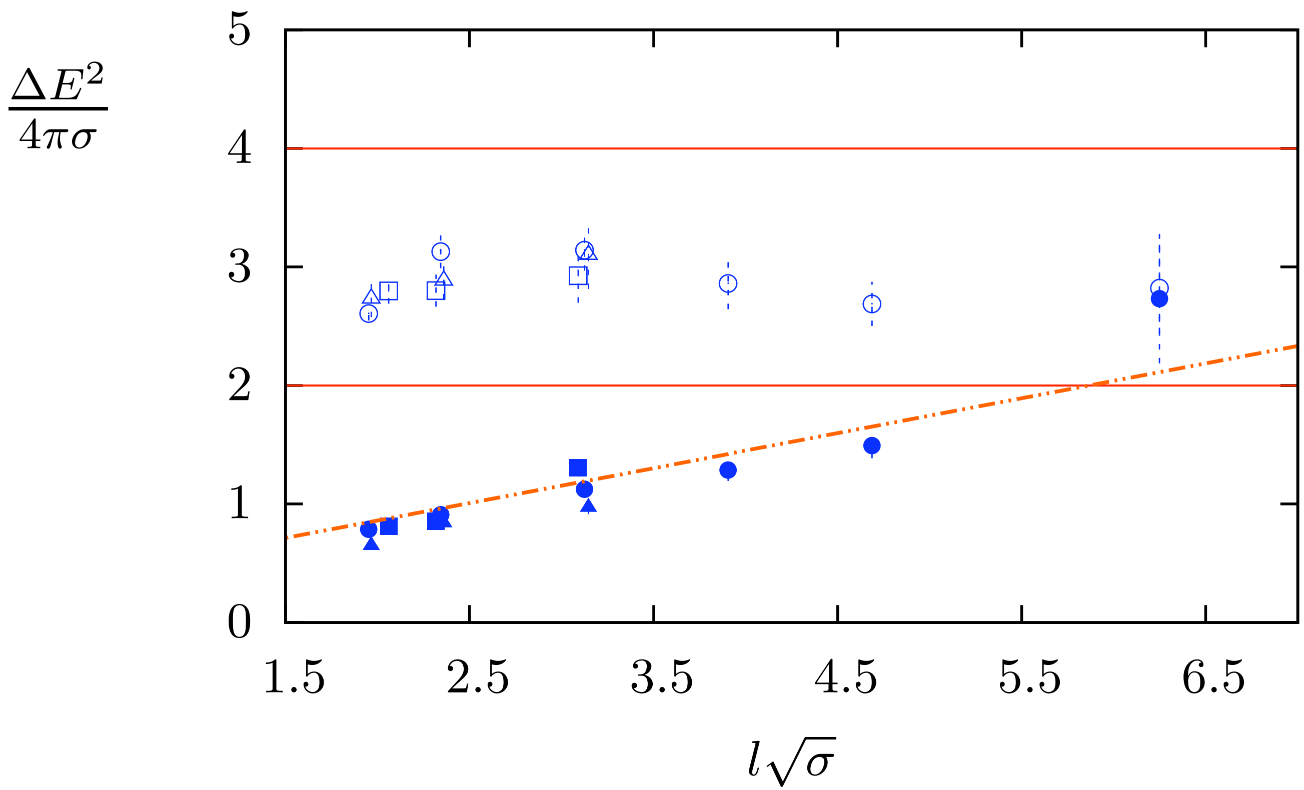
$\Longrightarrow$ anomalous 0^{--} state and also for $p = 2\pi/l$ states



states: $J^{P_t} = 0^+(\circ), 0^-(\bullet), 2^+(*), 2^-(+)$

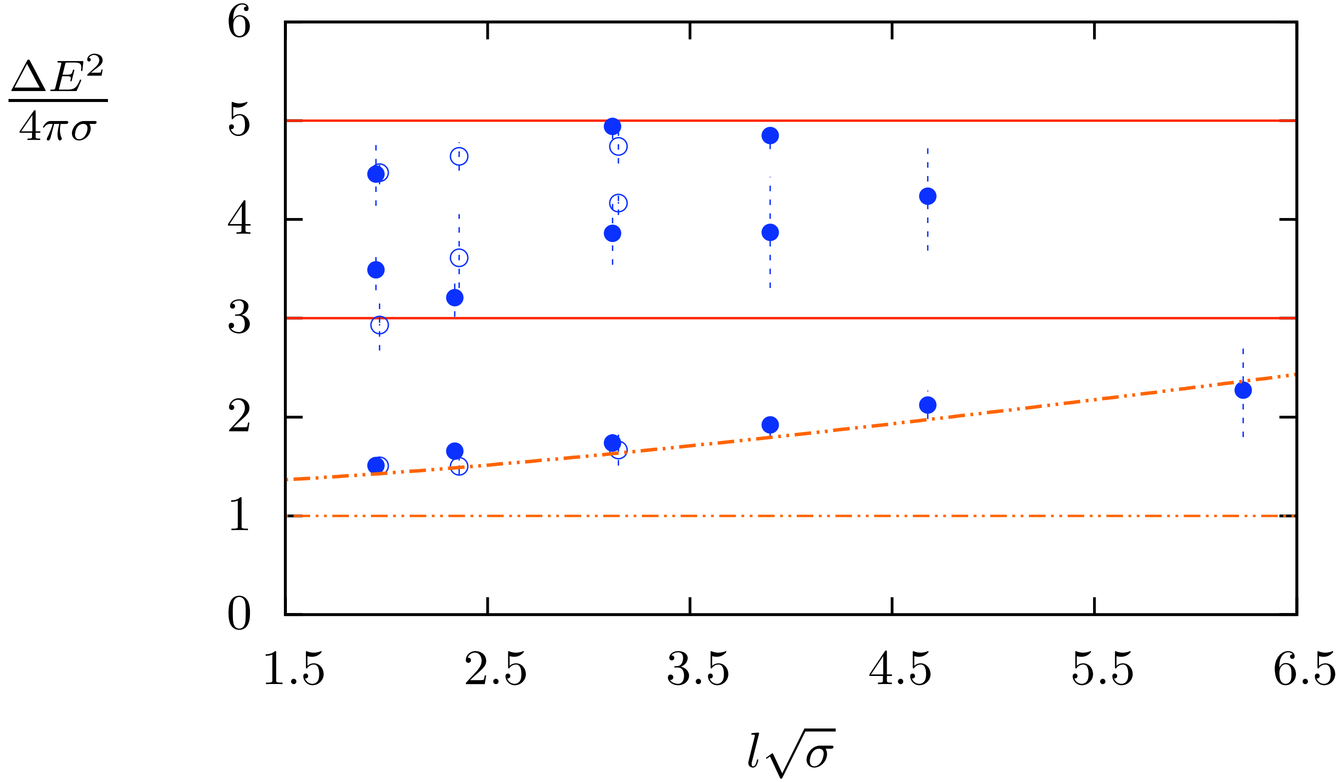
$\Longrightarrow$ anomalous 0^- state

$p = 0,\ 0^{--}$: is this an extra state – is there also a stringy state?



ansatz: $E(l) = E_0(l) + m$; $m = 1.85\sqrt{\sigma} \sim m_G/2$

similarly for $p = 1,\ 0^-$:   SU(3), ●; SU(5), ○



ansatz: $E(l) = E_0(l) + (m^2 + p^2)^{1/2}$; $m = 1.85\sqrt{\sigma} \sim m_G/2$

Some conclusions on confining flux tubes and strings

- flux tubes are like free Nambu-Goto strings, even when not much longer than they are wide

- this is so for all states in $D = 2 + 1$ and most in $D = 3 + 1$

- corrections to Nambu-Goto are consistent with known universal terms, and they resum to small contributions at smaller l for excited states

- in $D = 3 + 1$ we see extra states consistent with the excitation of massive modes

- in $D = 2 + 1$, despite the much greater accuracy, we see no such extra states