# CONFINEMENT IN MULTI-PARTON SECTORS OF TWO DIMENSIONAL GAUGE THEORIES

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## 1 How to calculate masses of particles ?

- Lattice
- Diagonalize Hamiltonian
- Light Cone Discretization
- QCD equations: coupled Bethe-Salpeter equations on the LC
- Simplifications: large N planar diagrams - single traces
- less dimensions
- even quantum mechanics (but at $N \to \infty$)
- supersymmetry

## 2 Planar gauge theory in 1+1 dimensions

- The history

FT on the light cone – C. Thorn ('77)
Warm-up: D=1+1, $QCD_2$ – 't Hooft ('74)
fermions in funamental irrep $\xrightarrow{\textbf{LargeN}}$ no multiparton states.

YM+with addjoint matter – Klebanov et al. ('93)
matter = fermions or *scalars* ( = reduced $YM_3$ )
$SYM_2$ – Matsumura et al. ('95)
D=4 Wilson and Glazek ('93)
Hiller et al. ('98)
$QCD_4$ on the light cone – Brodsky et al. (since '70)

## 2.1 One way: Light Cone Discretization

$$
\begin{aligned}
P^+ &= \sum_{n=2}^{\infty} \sum_{i=1}^{n} p_i^+, \quad p_i^+ > 0 \\
K &= \sum_{n=2}^{\infty} \sum_{i=1}^{n} r_i, \quad K, r_i - \text{natural},
\end{aligned}
$$

Cutoff $K \Longrightarrow$ partitions $\{r_1, r_2, \ldots\} \Longrightarrow$ states

$$
|\{r\}\rangle = Tr[a^\dagger(r_1) a^\dagger(r_2) ... a^\dagger(r_p)]|0\rangle \tag{1}
$$

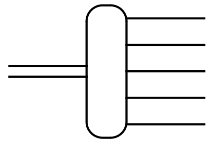
$|\{r\}\rangle \Longrightarrow \langle\{r\}|H|\{r'\}\rangle \Longrightarrow E_n$

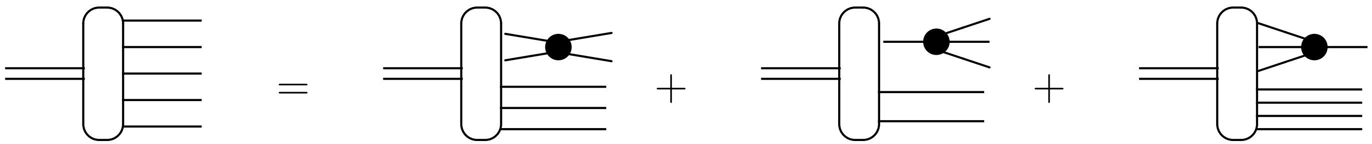
## 2.2 Second way: integral equations in the continuum

- Different cutoff – directly in the continuum

$$H|\Phi\rangle = M^2|\Phi\rangle \tag{2}$$

$|\Phi\rangle \rightarrow \Phi_n(x_1, x_2, \ldots, x_n) \qquad \leftrightarrow$





$$M^2\Phi_n(x_1 \ldots x_n) = A \otimes \Phi_n + B \otimes \Phi_{n-2} + C \otimes \Phi_{n+2} \tag{3}$$

- Interpretation: proton is invariant against elementary processes
- Fundamental: contain DGLAP and BFKL evolution eqns.
- Emission and absorption are present (parton recombination)

The cutoff:

$$n \leq n_{max} \tag{4}$$

$n_{max} = 2$ 't Hooft equation – exact for $QCD_2$ (with fundamental fermions)

• EQUATIONS

$$|\Phi\rangle = \sum_{n=2}^{\infty} \int [dx]\delta(1 - x_1 - x_2 - \ldots x_n)\Phi_n(x_1, x_2, \ldots x_n) Tr[a^\dagger(x_1)a^\dagger(x_2)\ldots a^\dagger(x_n)]|0\rangle$$

EXAMPLE 1: $QCD_2$ ( fundamental fermions )

$$M^2 f(x) = m^2 \left(\frac{1}{x} + \frac{1}{1-x}\right) f(x) + \frac{\lambda}{\pi}\int_0^1 \frac{dy}{(y-x)^2}\left[f(x) - f(y)\right]$$

$$f(x) = \Phi_2(x, 1-x)$$

EXAMPLE 2: $SYM_2$ restricted to the two-parton sector

There are two coupled equations in the bosonic sector

$$M^2\phi_{bb}(x) = m_b^2\left(\frac{1}{x}+\frac{1}{1-x}\right)\phi_{bb}(x)+\frac{\lambda}{2}\frac{\phi_{bb}(x)}{\sqrt{x(1-x)}}$$
$$-\frac{2\lambda}{\pi}\int_0^1\frac{(x+y)(2-x-y)}{4\sqrt{x(1-x)y(1-y)}}\frac{[\phi_{bb}(y)-\phi_{bb}(x)]}{(y-x)^2}dy+\frac{\lambda}{2\pi}\int_0^1\frac{1}{(y-x)}\frac{\phi_{ff}(y)}{\sqrt{x(1-x)}}dy$$

$$M^2\phi_{ff}(x) = m_f^2\left(\frac{1}{x}+\frac{1}{1-x}\right)\phi_{ff}(x)$$
$$-\frac{2\lambda}{\pi}\int_0^1\frac{[\phi_{ff}(y)-\phi_{ff}(x)]}{(y-x)^2}dy+\frac{\lambda}{2\pi}\int_0^1\frac{1}{(x-y)}\frac{\phi_{bb}(y)}{\sqrt{y(1-y)}}dy$$

and the single one in the fermionic sector

$$M^2\phi_{bf}(x) = \left(\frac{m_b^2}{x}+\frac{m_f^2}{1-x}\right)\phi_{bf}(x)+\frac{2\lambda}{\pi}\frac{\phi_{bf}(x)}{\sqrt{x}+x}$$
$$-\frac{2\lambda}{\pi}\int_0^1\frac{(x+y)}{2\sqrt{xy}}\frac{[\phi_{bf}(y)-\phi_{bf}(x)]}{(y-x)^2}dy-\frac{\lambda}{2\pi}\int_0^1\frac{1}{(1-y-x)}\frac{\phi_{bf}(y)}{\sqrt{xy}}dy \qquad (5)$$

Example 3: $YM_2$ with addjoint fermionc matter - all parton-number sectors

$$
\begin{aligned}
M^2\phi_n(x_1\dots x_n) &= \frac{m^2}{x_1}\phi_n(x_1\dots x_n) \\
&+ \frac{\lambda}{\pi}\frac{1}{(x_1+x_2)^2}\int_0^{x_1+x_2} dy\phi_n(y,x_1+x_2-y,x_3\dots x_n) \\
&+ \frac{\lambda}{\pi}\int_0^{x_1+x_2}\frac{dy}{(x_1-y)^2}\{\phi_n(x_1,x_2,x_3\dots x_n) \\
&\quad -\phi_n(y,x_1+x_2-y,x_3\dots x_n)\} \\
&+ \frac{\lambda}{\pi}\int_0^{x_1} dy\int_0^{x_1-y} dz\phi_{n+2}(y,z,x_1-y-z,x_2\dots x_n)\left[\frac{1}{(y+z)^2}-\frac{1}{(x_1-y)^2}\right] \\
&+ \frac{\lambda}{\pi}\phi_{n-2}(x_1+x_2+x_3,x_4\dots x_n)\left[\frac{1}{(x_1+x_2)^2}-\frac{1}{(x_1-x_3)^2}\right] \\
&\pm\ cyclic\ \ permutations\ \ of\ \ (x_1\dots x_n)
\end{aligned}
$$

## 3 Coulomb divergences

- IR divergencies (logarithmic) couple different multiplicity sectors
- Coulomb divergencies (linear), but they cancel within one multiplicity
- Can be done independently for each parton multiplicity $p$

A possibility

- $\longrightarrow$ Solve Coulomb problem first, and then successively add radiation

Simplified Hamiltonian, $SYM_2$ reduced from $SYM_4$ (Dorigoni), keeping only Coulomb terms

$$H_C^{quad} = \frac{\lambda}{\pi} \int_0^\infty dk \int_0^k \frac{dq}{q^2} \mathrm{Tr}[A_k^\dagger A_k] \tag{6}$$

$$H_C^{quartic} = -\frac{g^2}{2\pi} \int_0^\infty dp_1 dp_2 \left[ \int_0^{p_1} \frac{dq}{q^2} \mathrm{Tr}[A_{p_1}^\dagger B_{p_2}^\dagger B_{p_2+q} A_{p_1-q}] + \int_0^{p_2} \frac{dq}{q^2} \mathrm{Tr}(A_{p_2}^\dagger B_{p_1}^\dagger B_{p_1+q} A_{p_2-q}) \right] \tag{7}$$

# 4 Two partons

$$|k, K-k\rangle, \quad k = 1, .., K-1 \tag{8}$$

$$\langle k|H|k'\rangle \Rightarrow |\Phi_n\rangle \Rightarrow \Phi_n(k) \overset{FT}{\Rightarrow} \Phi_n(d_{12}) \tag{9}$$

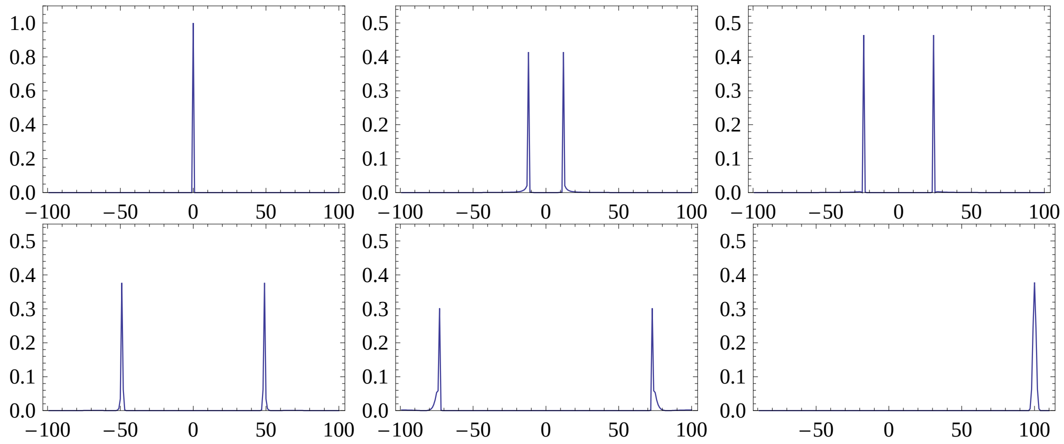


Figure 1: $\rho_n(d_{12}), p = 2, K = 200, n = 1, 25, 50, 100, 150, 199.$

## 5 Three partons - generalization of the 't Hooft solution to many bodies

$$|k_1, k_2, K - k_1 - k_2\rangle, \quad k_1 = 1, .., K-2, \quad k_2 = 1, .., K - k_1 - 1 \tag{10}$$

$$\langle k_1, k_2 | H | k_1', k_2' \rangle \Rightarrow |\Phi_n\rangle \Rightarrow \Phi_n(k_1, k_2) \overset{FT}{\Rightarrow} \Phi_n(d_{13}, d_{23}) \tag{11}$$

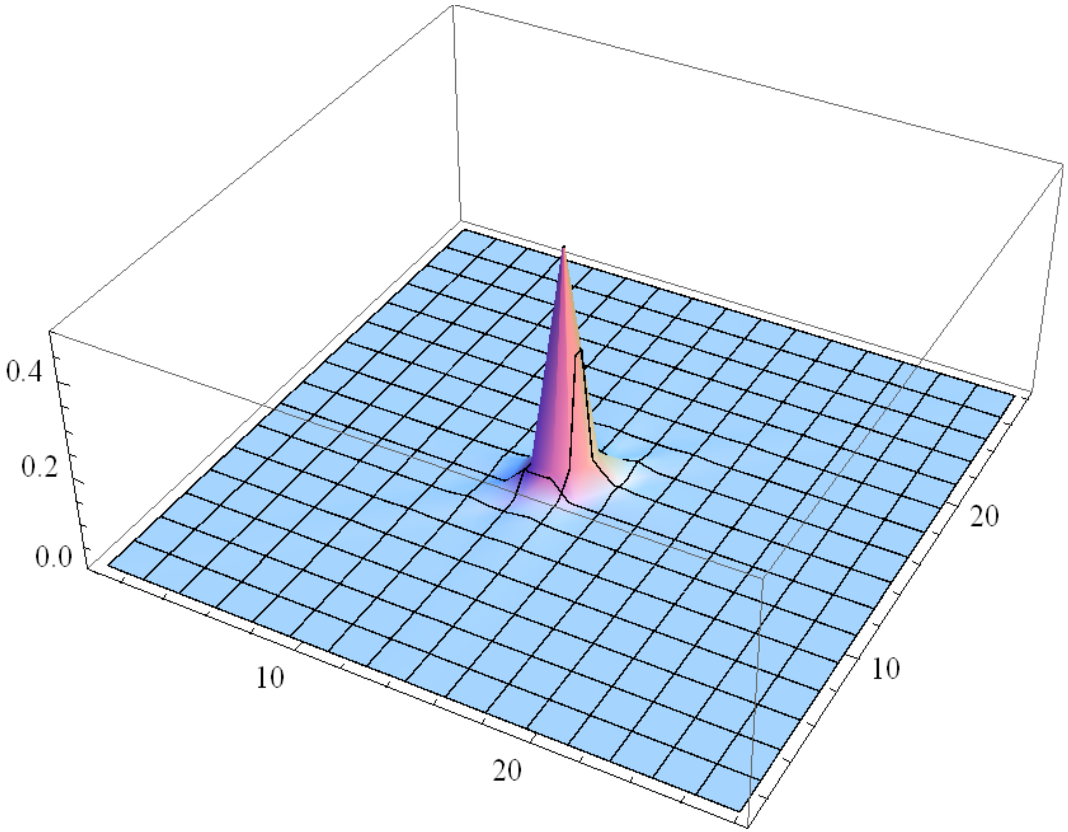



Figure 2: $\rho_1(d_{13}, d_{23})$

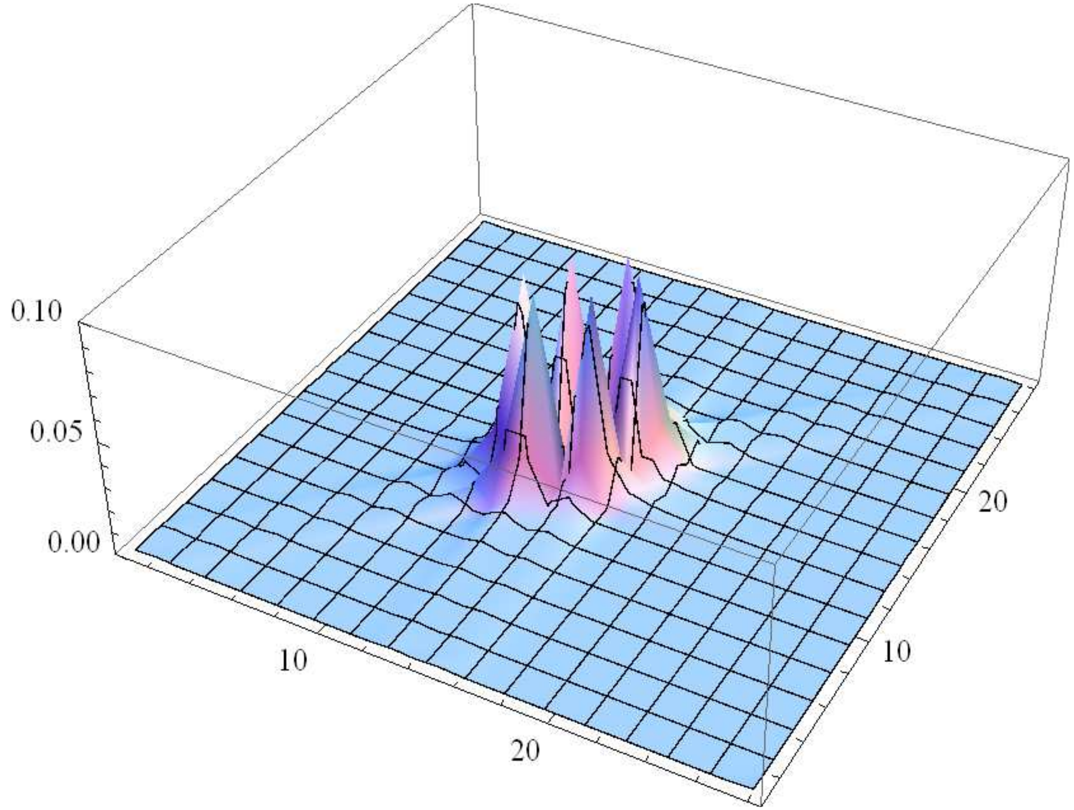



Figure 3: $|\rho_{10}(d_{13}, d_{23})$

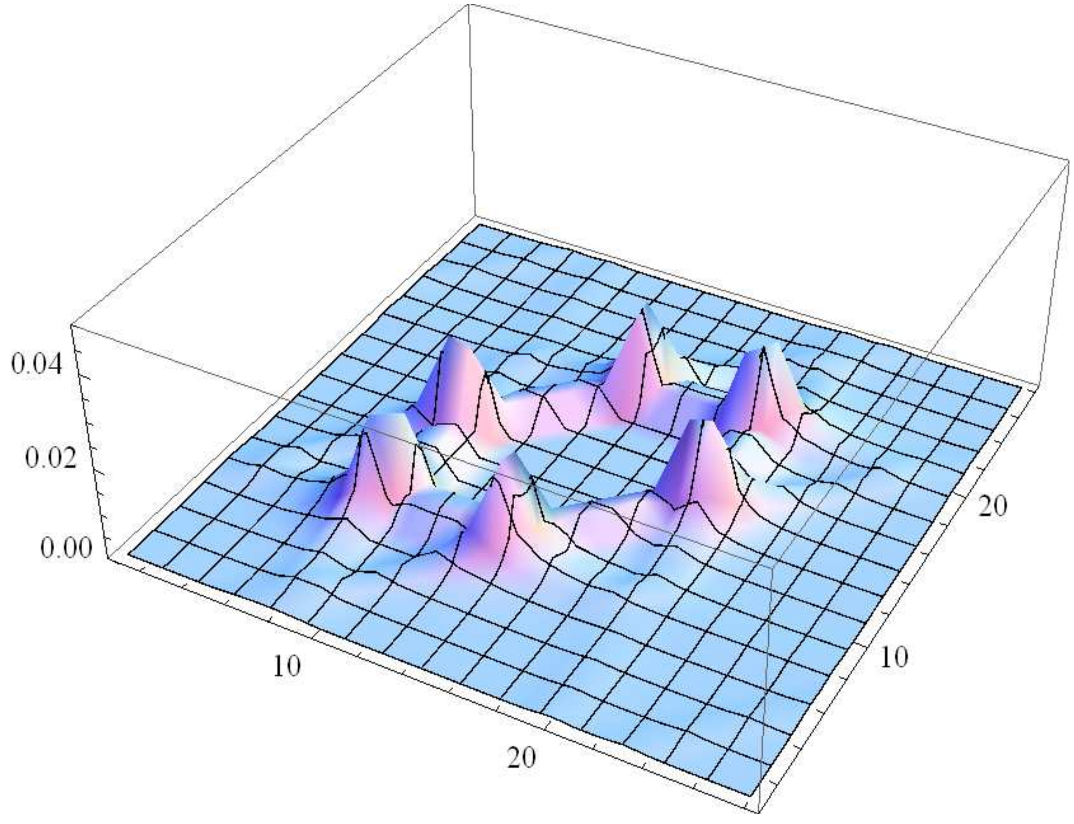



Figure 4: $\rho_{50}(d_{13}, d_{23})$

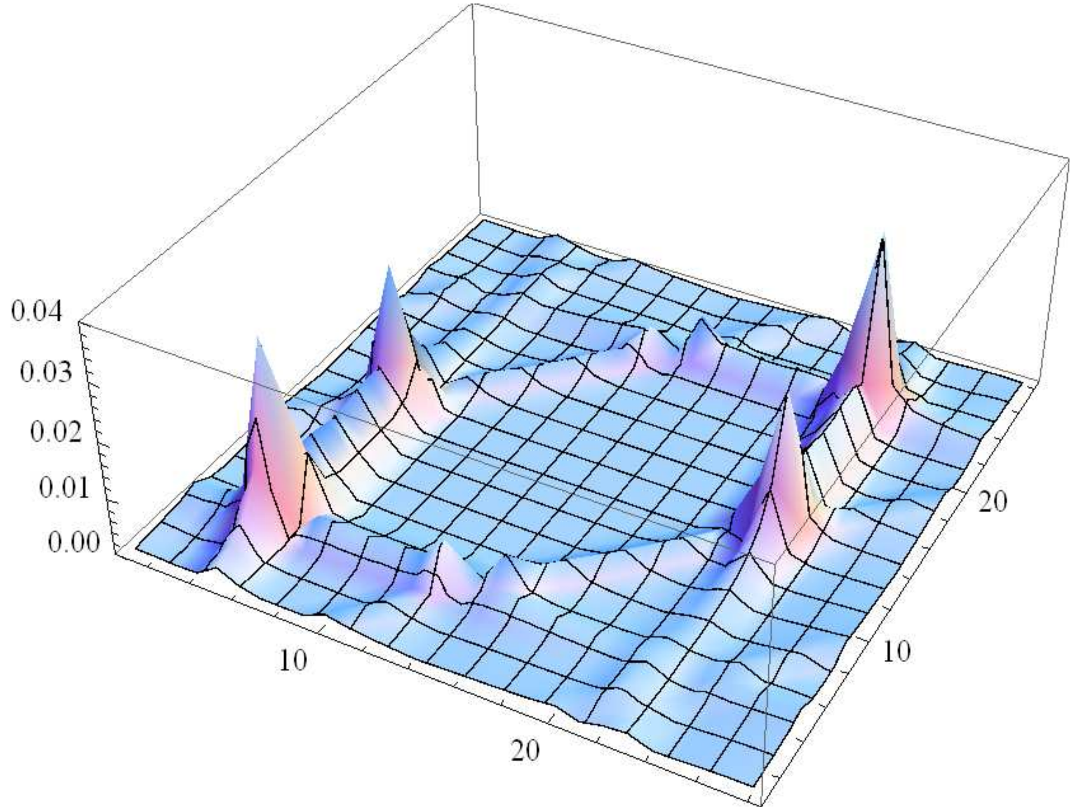



Figure 5: $\rho_{100}(d_{13}, d_{23})$

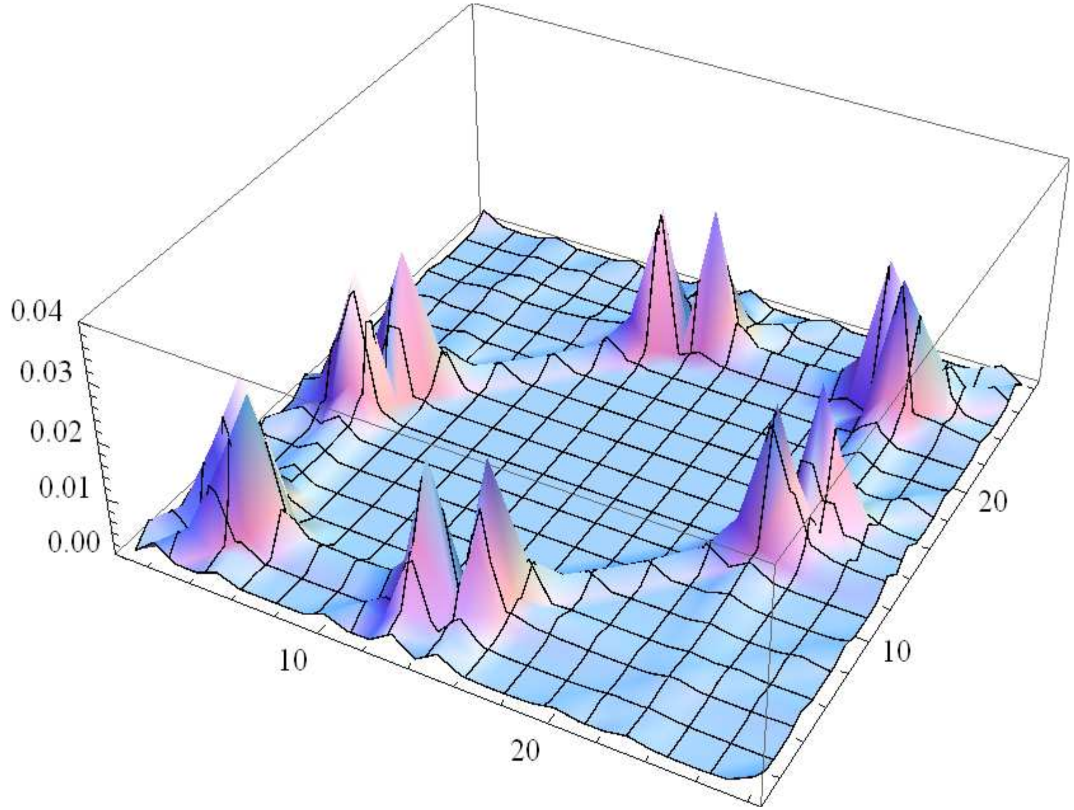



Figure 6: $\rho_{200}(d_{13}, d_{23})$

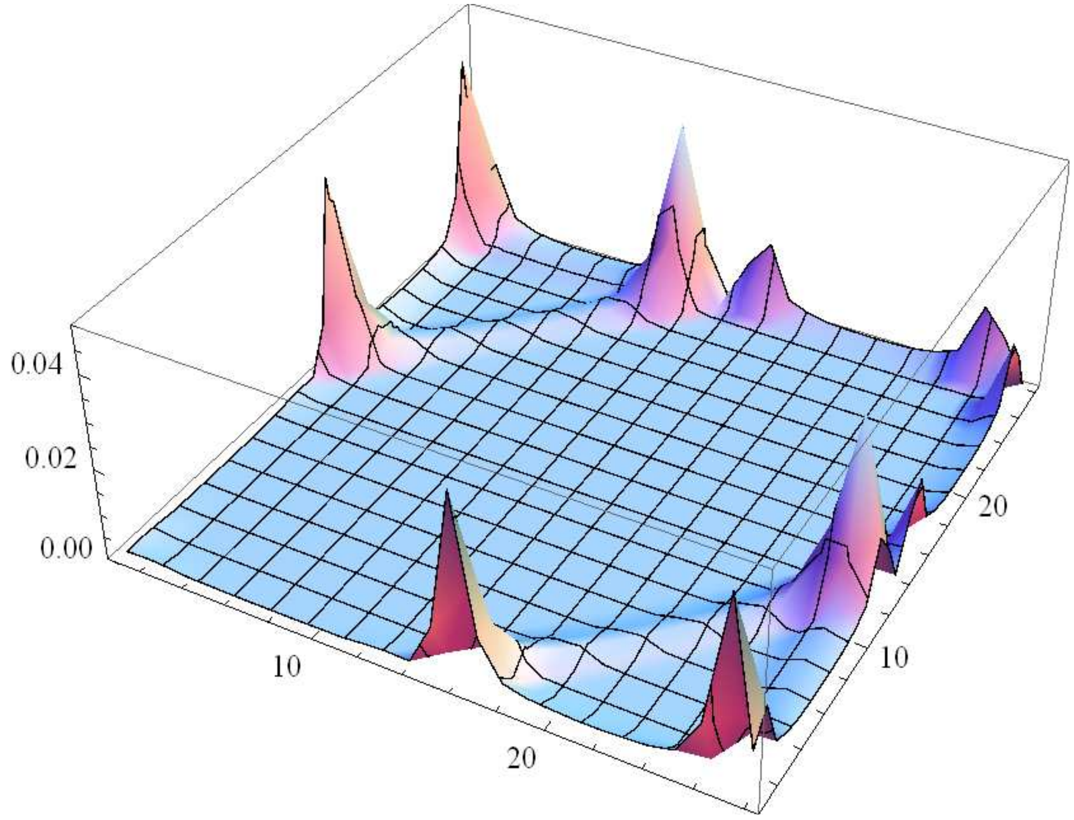



Figure 7: $\rho_{300}(d_{13}, d_{23})$

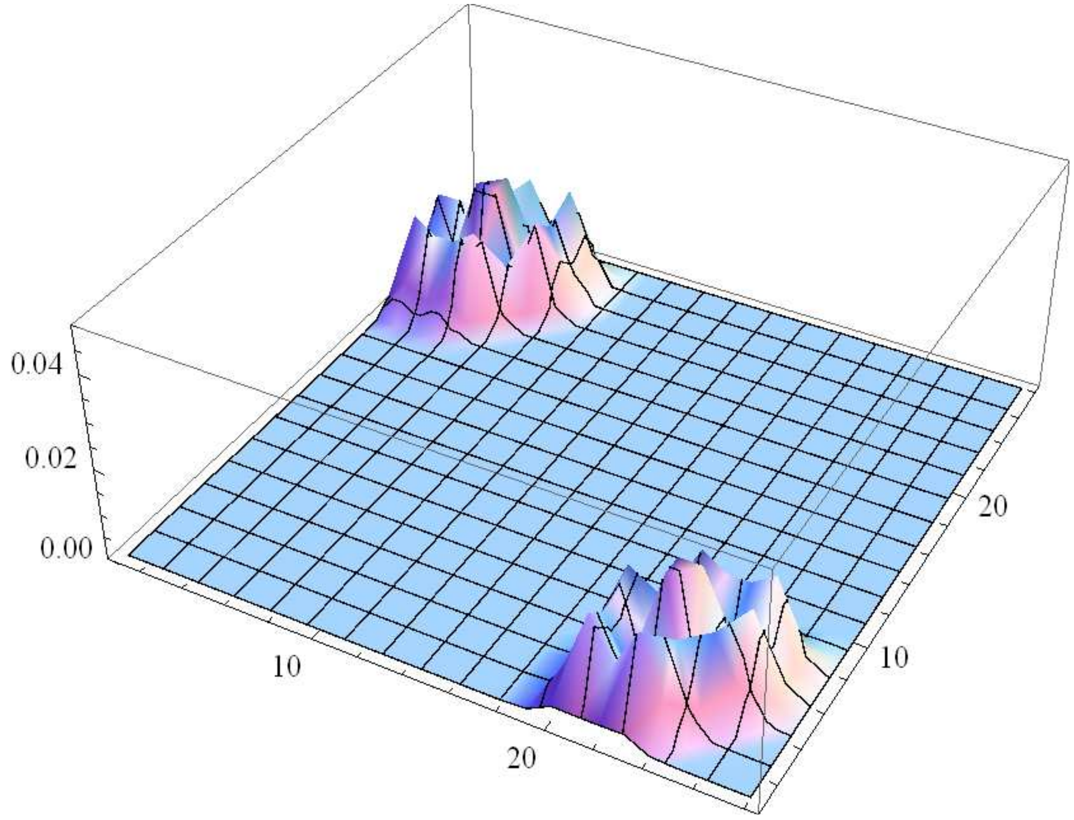



Figure 8: $\rho_{400}(d_{13}, d_{23})$

The highest state

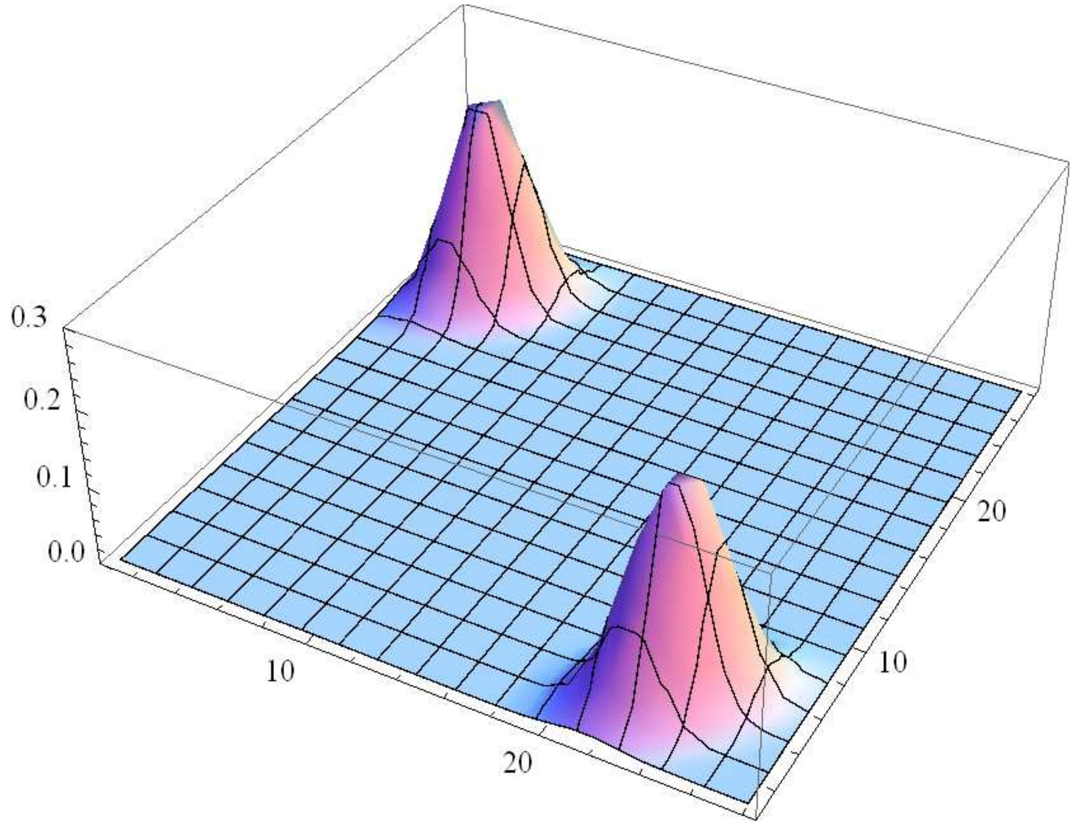



Figure 9: $\rho_{406}(d_{13}, d_{23})$

A "mercedes" configuration

## "Stringy" plot for two partons

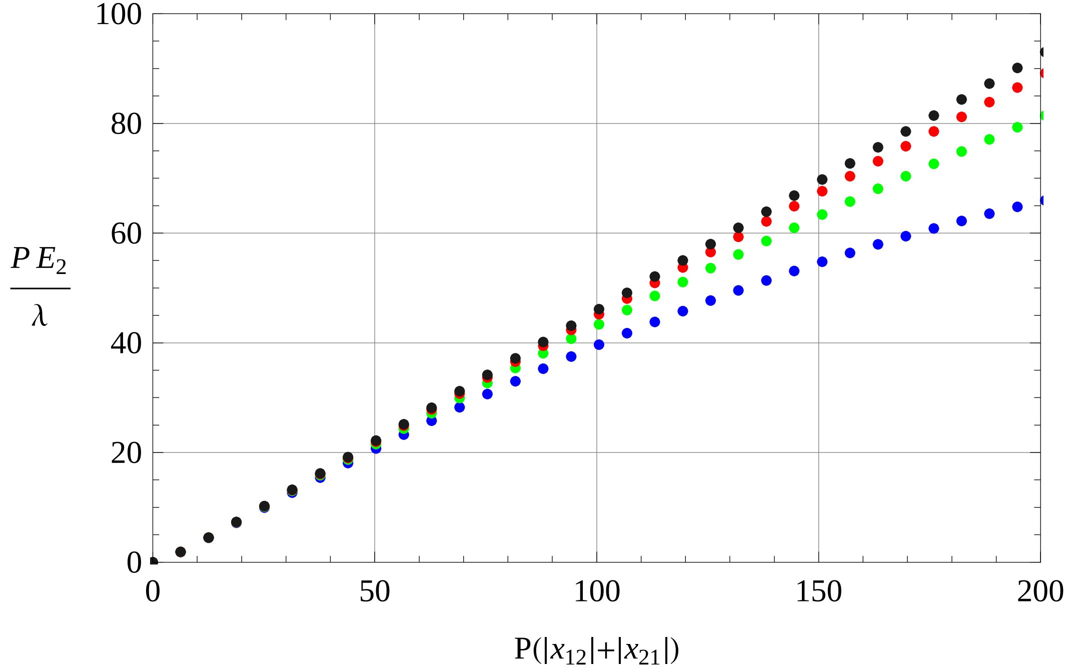



Figure 10: **Eigenenergies of the, p=2, excited states as a function of the relative separation between two partons,** $K = 30, 50, 100, 200$**.**

Extrapolation 1: in $K \to \infty$

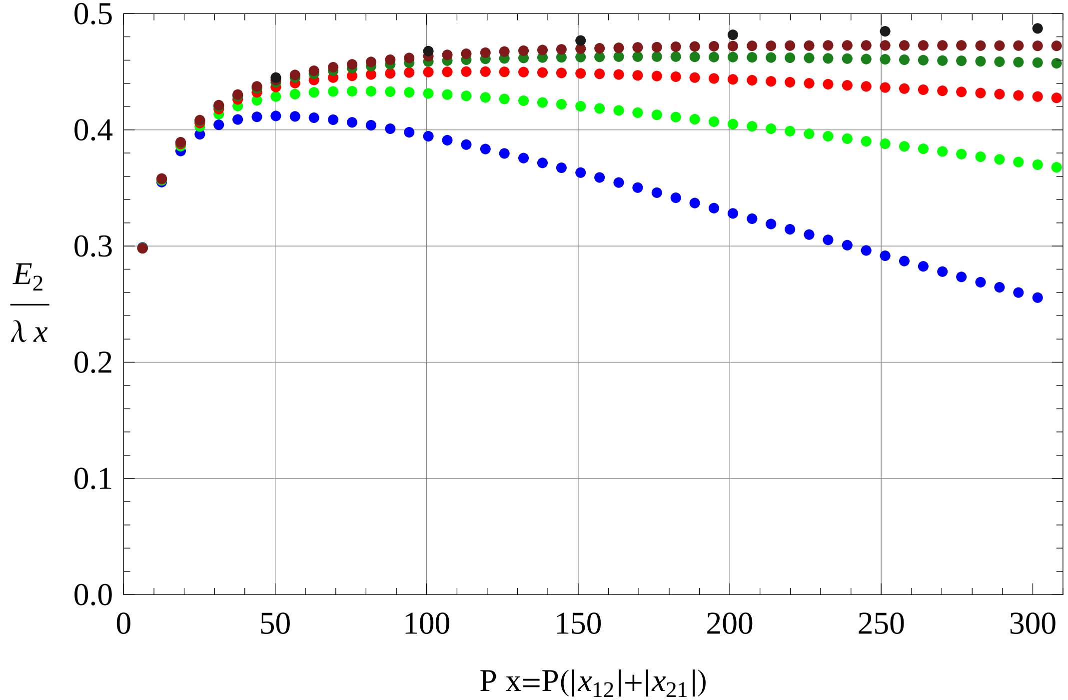



Figure 11:

Extrapolation 2: in $a = \frac{2\pi}{P} \to 0$

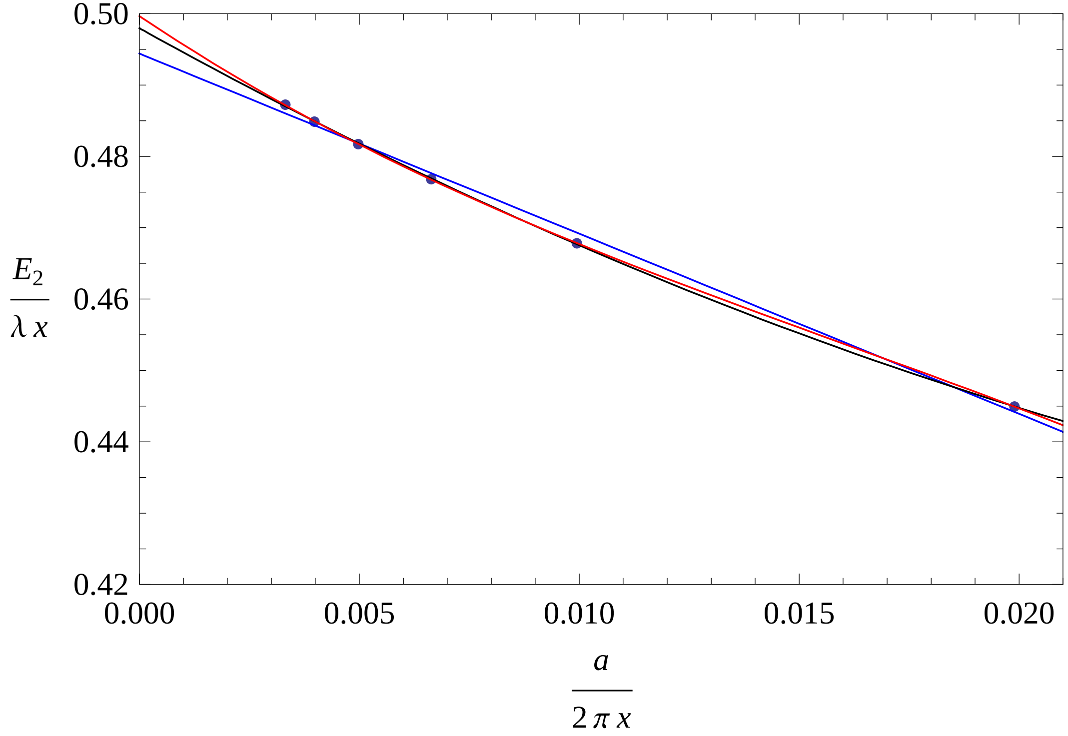



Figure 12:

## Families of states with three partons

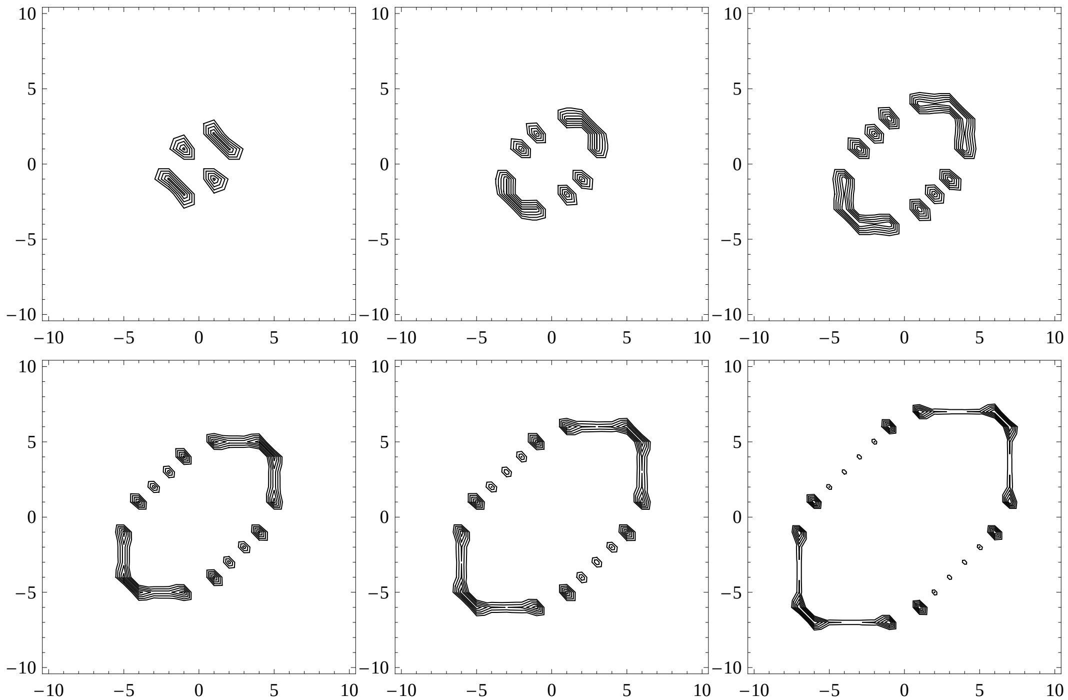



Figure 13: **Contour plots of $\rho_n(d_{13}, d_{23})$, as partons are moved further away.**
**Series A :** $n = 10, 19, 28, 41, 54, 72$**,** $4 \leq l = |d_{12}| + |d_{23}| + |d_{31}| \leq 14$**.**
**The minimal distance between partons = 1.**

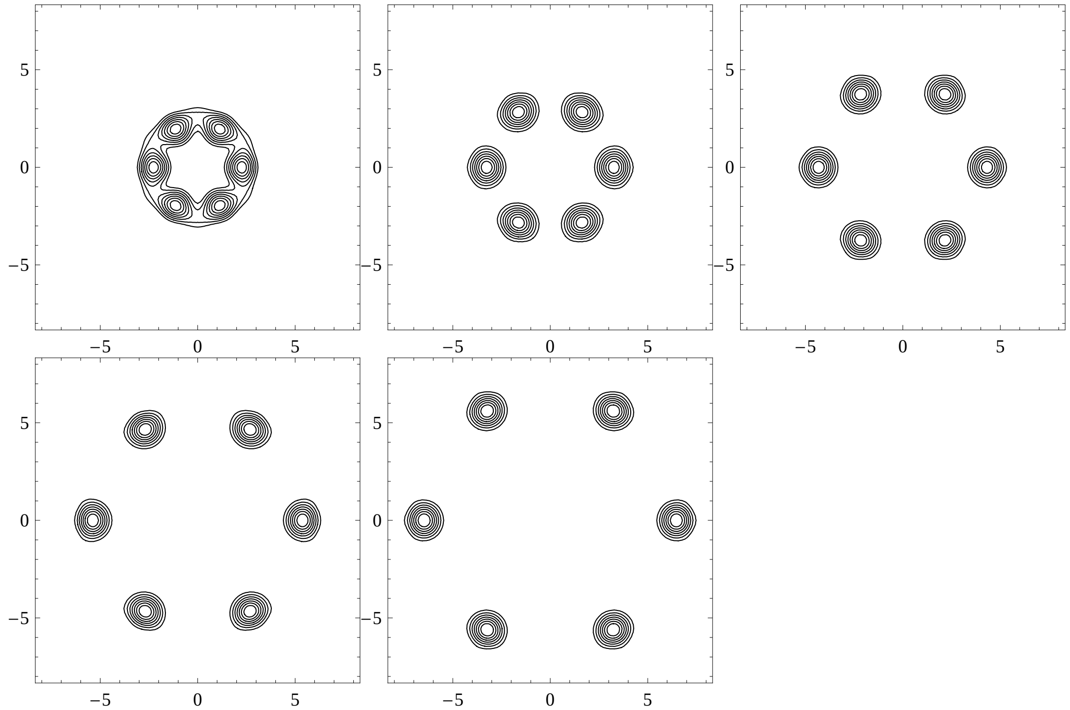



Figure 14: **Series B. As above but on the Dalitz plot. Now diquarks are allowed,** $d_{min} = 0$

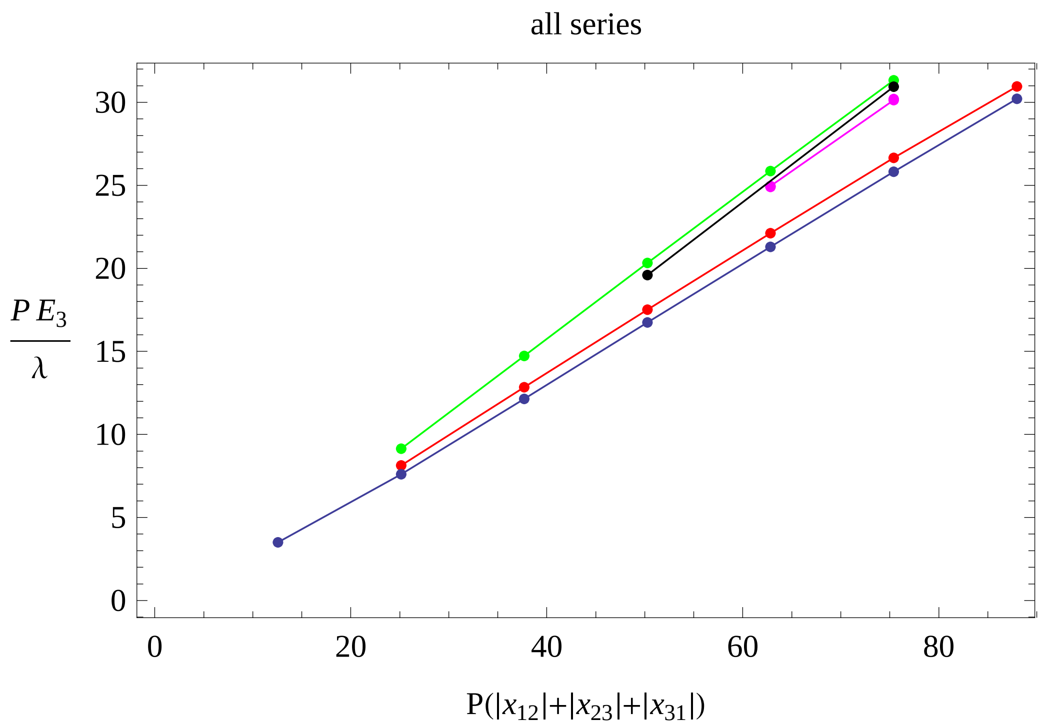



Figure 15: $\rho_{406}(d_{13}, d_{23})$

## "Stringy" plot for three partons

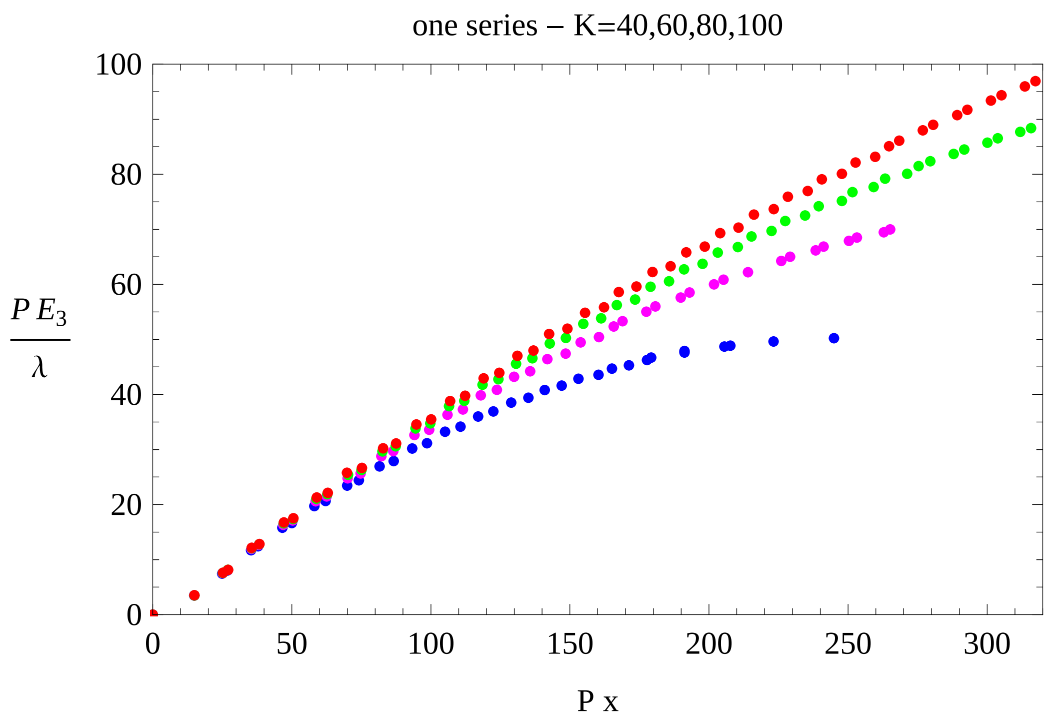



Figure 16: **Eigenenergies of the, p=3, excited states as a function of the combined length of strings stretching between three partons.**

$\Longrightarrow$ String tensions extracted from $E_2(l)$ and $E_3(l)$ seem to be consistent.

## Four partons

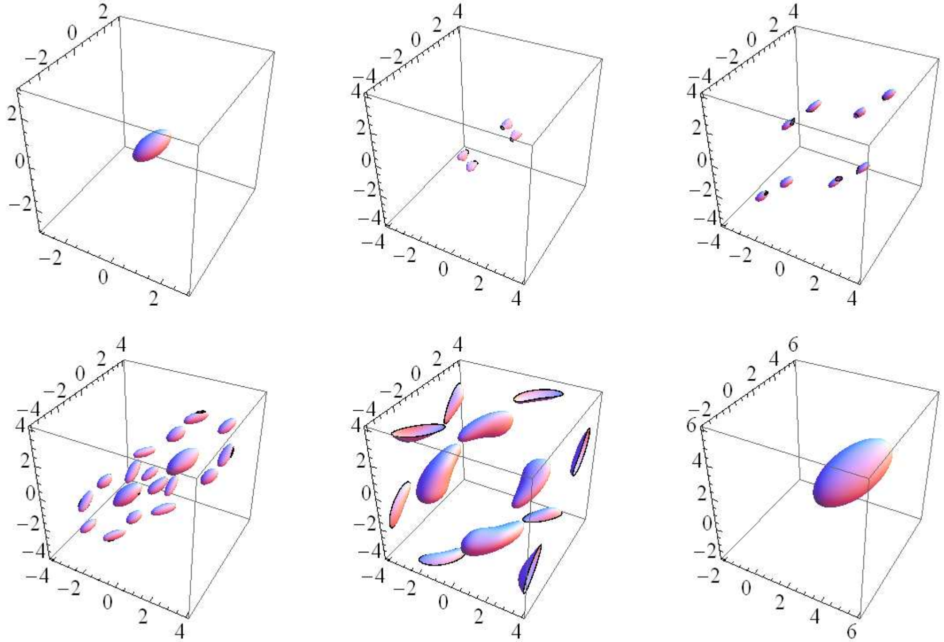


Figure 17: **Structure of eigenstates with four partons. Contour plots in three relative distances** $(d_{14}, d_{24}, d_{34})$ **for states no. 1,9,35,60,100,165 spanning the whole range of states for** $K = 12$**,** $r_{max} = 165$**.**

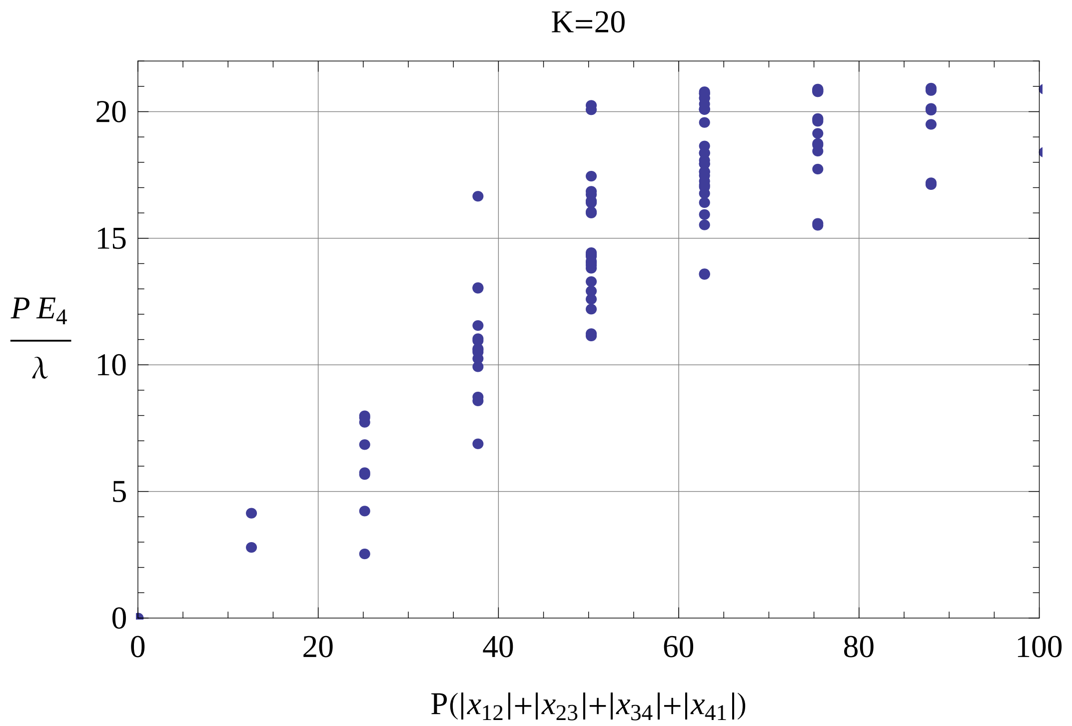



Figure 18: **Eigenenergies of the four parton states vs. the combined string length (all series).**

# 6 Inclusive distributions

## 6.1 Number of pairs at distance $\Delta$

$$D_r(\Delta) = \int d^{p-1}\vec{\Delta}_{p-1} \sum_{i=1}^{p-1} \delta(\Delta - d_{ip}) |\psi_r(\vec{\Delta}_{p-1})|^2, \tag{12}$$

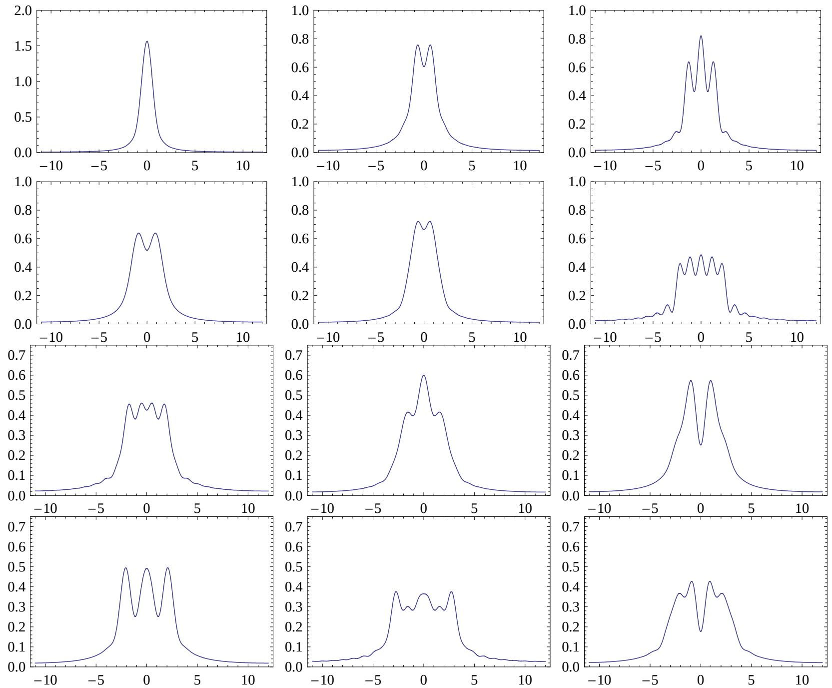


Figure 19: **Inclusive parton densities for four partons and for lower states** $r = 1, 4, 5, 6, 9, 12, 13, 14, 15, 20, 26, 29$**,** $K = 27, \;\; r_{max} = 2600$**.**

### 6.2 A simple application: massless Schwinger Model

What about the screening ? (Kutasov, Gross et al., Armoni and Sonnenschien)

SM: Exact solution in the two fermion sector - one free (composite) boson.

$$a^\dagger{}_n|0\rangle = \frac{1}{\sqrt{n}}\sum_{r=1}^{n-1} b_r^\dagger d_{n-r}^\dagger|0\rangle, \quad n = K. \tag{13}$$

Therefore the, normalized, p=4 component of the two boson eigenstates read ($K_2 = K/2$)

$$|m\rangle = a^\dagger{}_{K_2+m}a^\dagger{}_{K_2-m}|0\rangle = \frac{1}{\sqrt{K_2^2 - m^2}}\sum_{r=1}^{K_2+m-1} b_r^\dagger d_{K_2+m-r}^\dagger \sum_{s=1}^{K_2-m-1} b_s^\dagger d_{K_2-m-s}^\dagger|0\rangle. \tag{14}$$

The states are labeled by the relative momentum $2m$, $-(K_2 - 2) \leq m \leq (K_2 - 2)$ and have mass-squared eigenvalue

$$M_m^2 = \frac{e^2}{\pi}\frac{K^2}{K_2^2 - m^2}, \tag{15}$$

and have the following four parton Fock wave functions

$$\begin{aligned} f_K^{(m)}(k_1, k_2, k_3, k_4) &= f_K^{(m)}(k_1, k_2, K_2 + m - k_1, K_2 - m - k_2) &= \frac{1}{\sqrt{K_2^2 - m^2}}, \\ 1 \leq k_1 \leq K_2 + m - 1, & \qquad 1 \leq k_2 \leq K_2 - m - 1, \end{aligned} \tag{16}$$

which result in the following inclusive densities

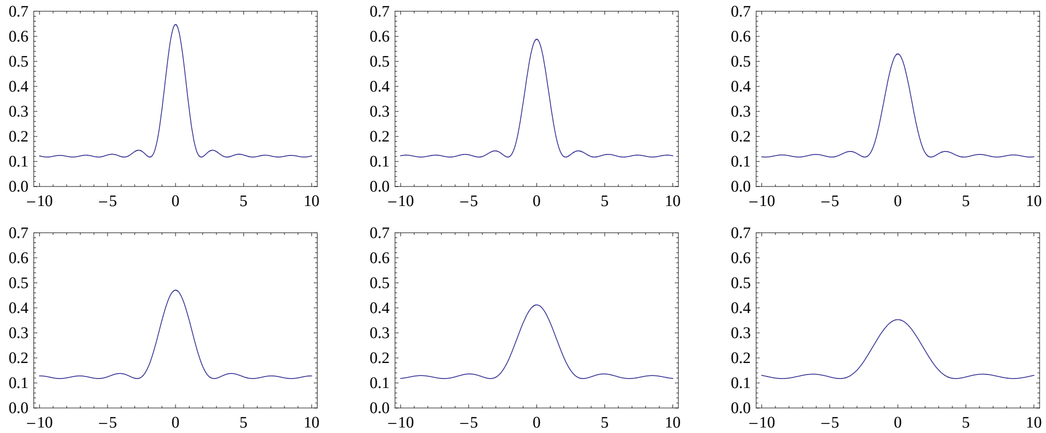



Figure 20: **Massless Schwinger Model: inclusive parton densities for four partons and for lower states** $r = 1, ..., 6$**,** $K = 20$**.**

## 7 Summary and the future

- 't Hooft solutions have a very simple interpretation in the configuration space.
- Generalization to more partons
  a) is readily possible, and
  b) also confirms a simple string picture (at fixed p).
- Future: generalizations of the (1+1) Coulomb problem
  supersymmetry
  high multiplicities
- Add radiation
- Mass gap in the 1+1 supersymmetric theory
- ASV equivalence
-