Unquenched matter in the gauge/gravity correspondence

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Large-N gauge theories

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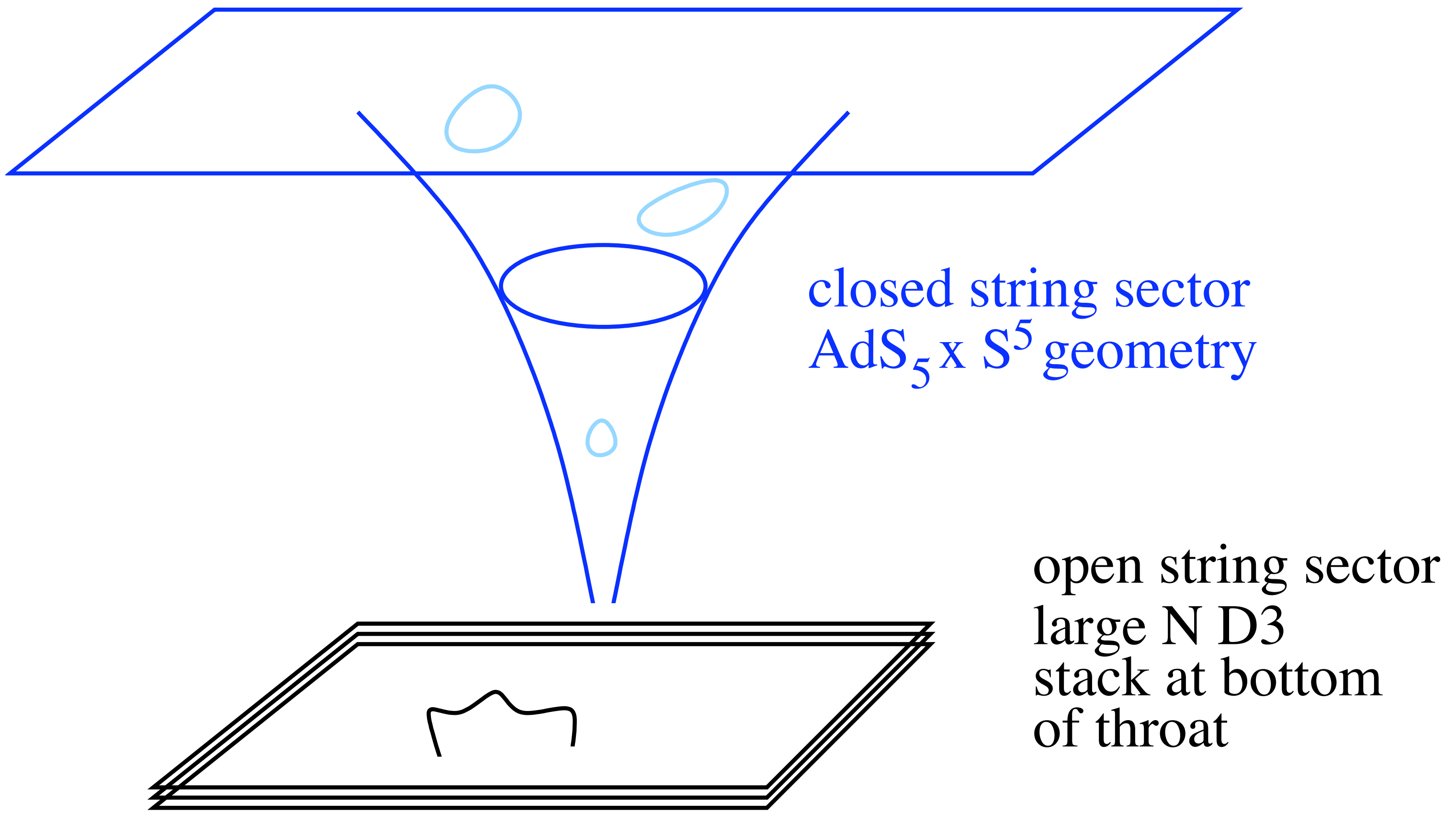
Plan of the talk

- Addition of flavor in AdS/CFT
- Quenched&Unquenched matter in AdS/CFT
- Smeared unquenched flavor in D3-D7
- Holographic flavor in Chern-Simons-matter systems
- Smeared unquenched flavor in ABJM
- Flavor effects in Chern-Simons-matter theories

AdS/CFT correspondence

Maldacena 97



$$\text{Sugra in } AdS_5 \times S^5 \quad \Longleftrightarrow \quad N_c \to \infty\,,\ \lambda >> 1\,,\ \mathcal{N} = 4 \text{ SYM}$$

Addition of flavor → flavor branes

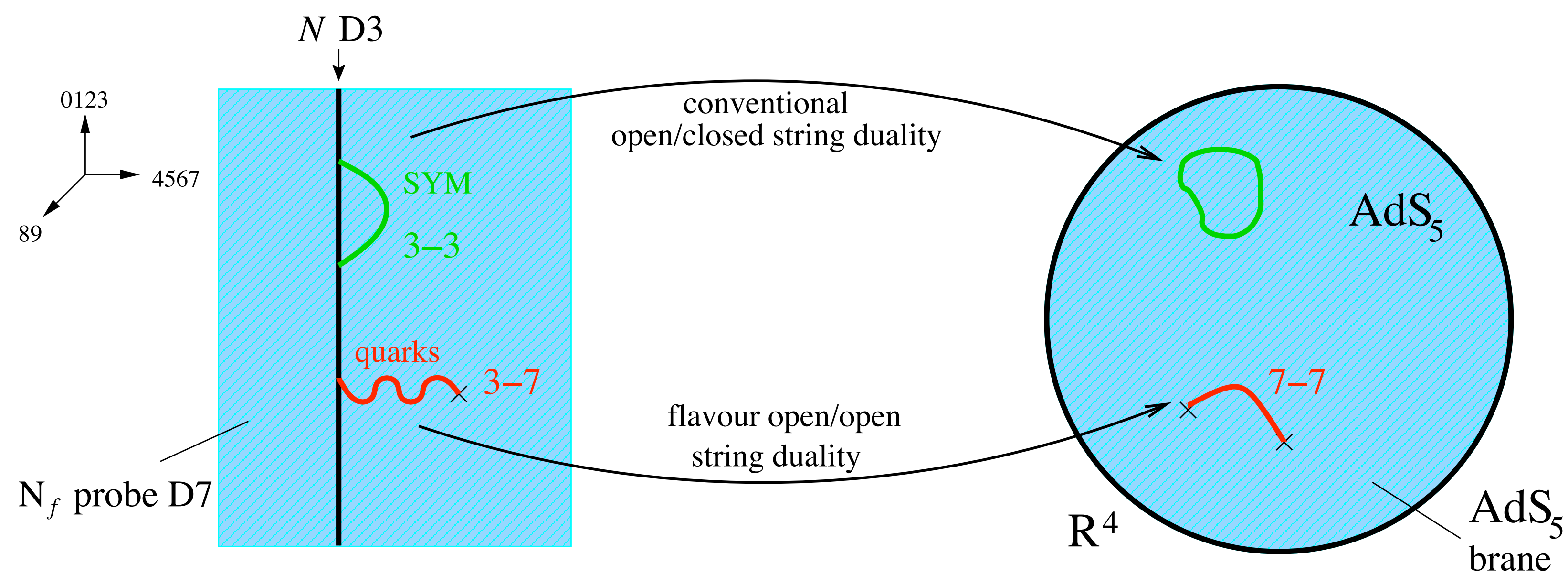
D3/D7 setup

Karch&Katz 02
Graña&Polchinski 01
Bertolini et al 01

	0	1	2	3	4	5	6	7	8	9
D3	X	X	X	X						
D7	X	X	X	X	X	X	X	X		

quark mass ⇓ separation in 89 directions

generalization of AdS/CFT



$$\lambda_{D7} = \lambda_{D3}\ (2\pi l_s)^4\ \frac{N_f}{N_c}$$

$$\lambda_{D7} \to 0 \text{ when } l_s \to 0 \text{ with } \lambda_{D3} \text{ fixed}$$

$U(N_f)$ → flavor symmetry

Quenched approximation

Neglect quark loops $\Rightarrow$ suppresed by factors $\dfrac{N_f}{N_c}$

Gravity side

Small number of D7s $\Rightarrow$ treat D7s as probes

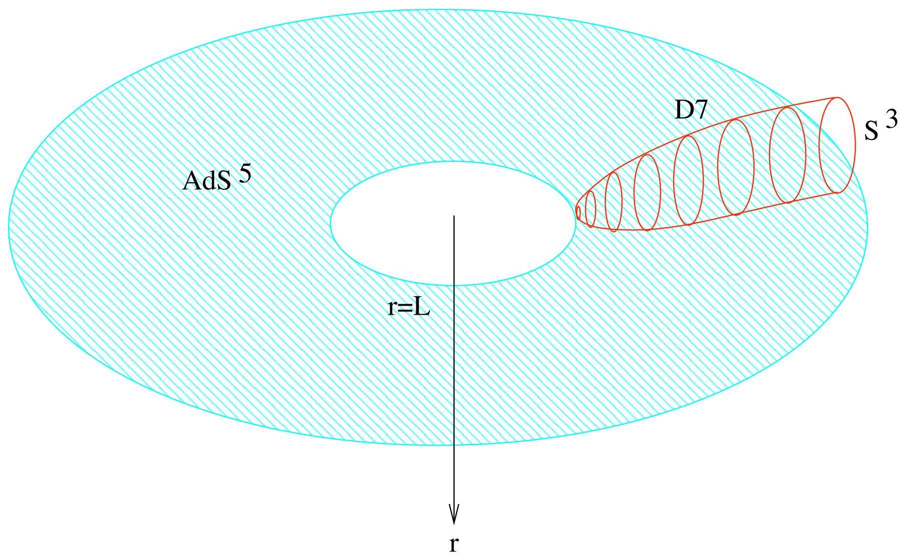
Fluctuations of D7 dual to "mesons"

-exact mass formulae

-matching fluctuations/operators

holography on the wv

(Kruczenski et al. 03)



Why going beyond the probe approximation?

- In real life $N_f \sim N_c$
- In $\mathcal{N} = 4$ SYM we want to capture the breaking of conformality due to the flavor
- Control of flavor dynamics
- QCD phase diagram
- Dualities in SUSY theories require $N_f \sim N_c$

$\vdots$

Including the backreaction

Action of gravity+branes

$$S = S_{IIB} + S_{fl}$$

$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g_{10}} \left[R - \frac{1}{2} \partial_M \Phi \partial^M \Phi - \frac{1}{2} e^{2\Phi} F_{(1)}^2 - \frac{1}{2}\frac{1}{5!} F_{(5)}^2 \right]$$

$$S_{fl} = -T_7 \sum_{N_f} \left(\int_{\mathcal{M}_8} d^8\xi \, e^{\Phi} \sqrt{-g_8} - \int_{\mathcal{M}_8} C_8 \right)$$ → sources of gravity fields

Rewrite the WZ term as:

$$S_{WZ} = T_{D7} \int_{\mathcal{M}_{10}} \Omega \wedge C_8 \qquad \Omega = \sum_{N_f} \delta^{(2)}(\mathcal{M}_8) \, \omega_2$$

ω_2 → transverse volume element

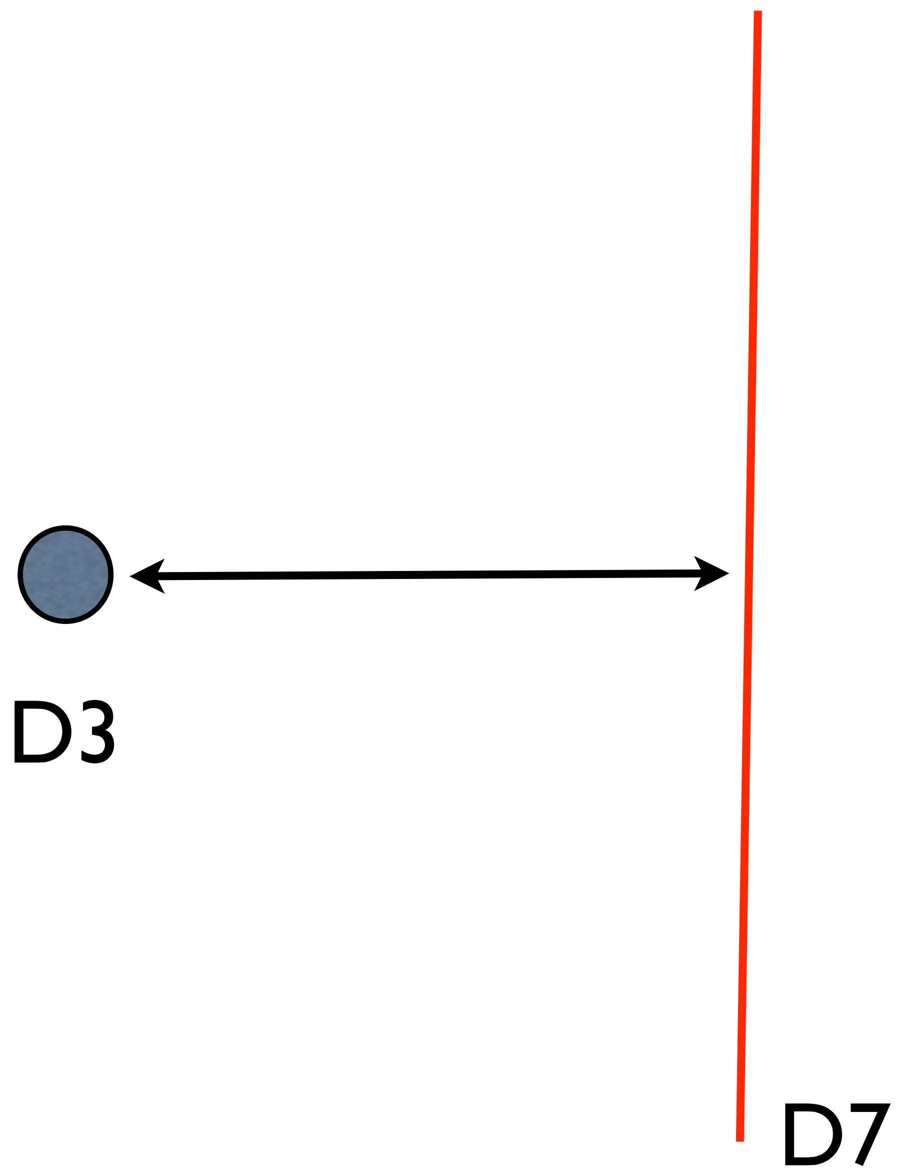
Ω → charge distribution two-form

S_{WZ} induces a violation of Bianchi identity of F_1

$$\boxed{dF_1 = -\Omega}$$ → δ-function source term

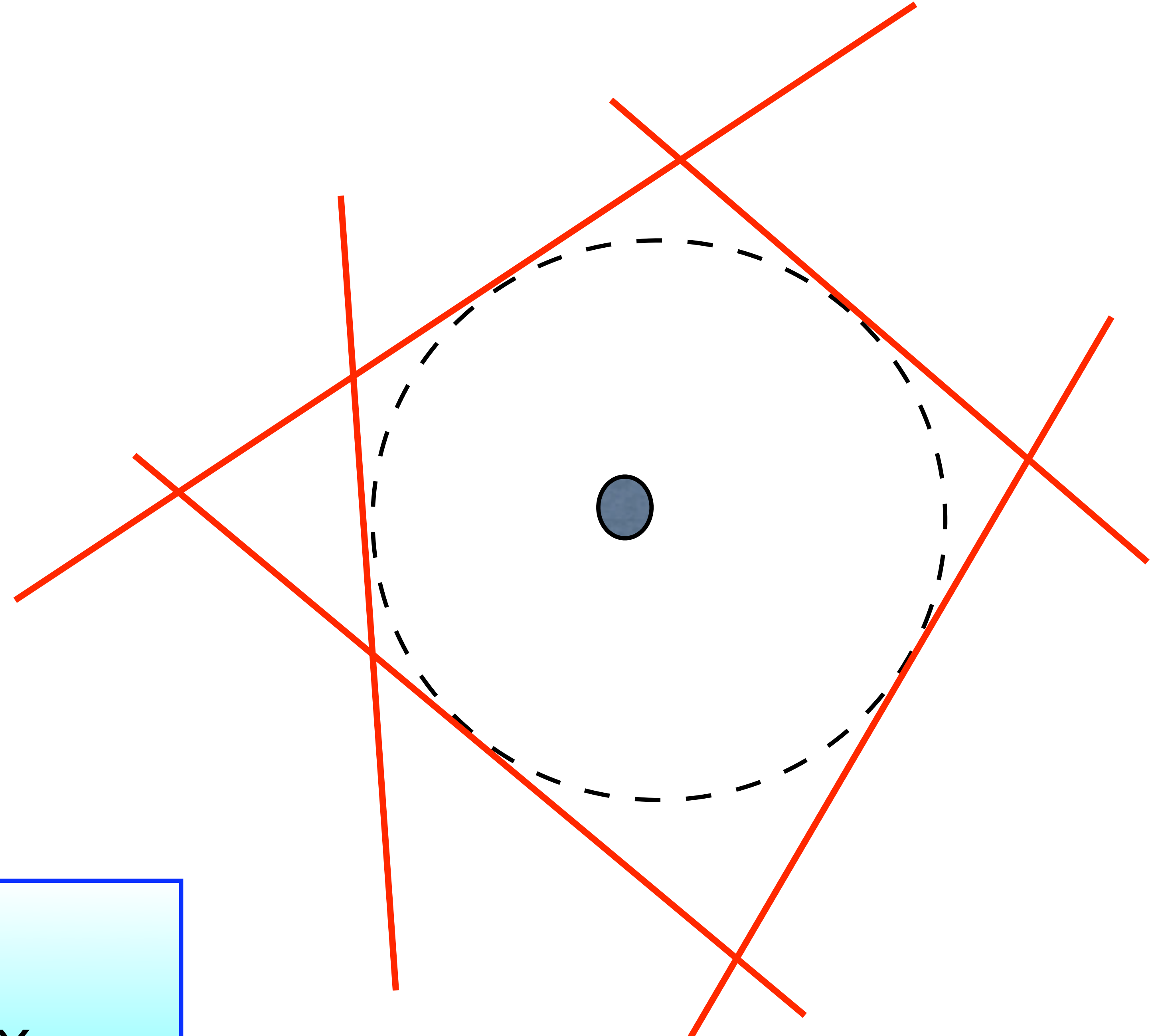
Einstein eqs. have also δ-function source terms

Localized embedding



density distribution form with delta-functions

Smeared sources



-no delta-function sources

-still can preserve (less) SUSY

-much simpler (analytic) solutions

$-$flavor symmetry : $U(1)^{N_f}$

what is the deformation of the metric due to smeared flavor?

S^5 as a $U(1)$ bundle

metric of $\mathbb{C}^3$

$$ds^2 = |dZ^1|^2 + |dZ^2|^2 + |dZ^3|^2$$

define

$$r^2 = \sum |Z^i|^2 \qquad Z^i = r z^i$$

$$|z^1|^2 + |z^2|^2 + |z^3|^2 = 1$$

$$\Longrightarrow \qquad ds^2 = dr^2 + r^2\, ds^2_{S^5}$$

parametrize

$$z^1 = \cos\frac{\chi}{2}\ \cos\frac{\theta}{2}\ e^{\frac{i}{2}\,(2\tau+\psi+\varphi)} \qquad z^2 = \cos\frac{\chi}{2}\ \sin\frac{\theta}{2}\ e^{\frac{i}{2}\,(2\tau+\psi-\varphi)} \qquad z^3 = \sin\frac{\chi}{2}\, e^{i\tau}$$

$$\boxed{ds^2_{S^5} = ds^2_{CP^2} + (d\tau + A)^2}$$

$$ds^2_{CP^2} = \frac{1}{4}d\chi^2 + \frac{1}{4}\cos^2\frac{\chi}{2}(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{1}{4}\cos^2\frac{\chi}{2}\sin^2\frac{\chi}{2}(d\psi + \cos\theta d\varphi)^2$$

$$A = \frac{1}{2}\cos^2\frac{\chi}{2}(d\psi + \cos\theta d\varphi)$$

Deformation of $\mathrm{AdS}_5 \times \mathrm{S}^5$ preserving four SUSYs (unflavored case)

$$ds^2 = h^{-\frac{1}{2}}\, dx_{1,3}^2 + h^{\frac{1}{2}} \left[\frac{dr^2}{F(r)} + r^2\, ds^2_{CP^2} + r^2\, F(r)\, (d\tau + A)^2 \right]$$

$$F(r) = 1 - \frac{b^6}{r^6}$$

The CP2 and the U(1) fiber are squashed differently

Hint: this is the type of deformation induced by the smeared flavor branes

Ansatz (massless quarks)

$$ds^2 = \Big[h(\rho)\Big]^{-\frac{1}{2}} dx_{1,3}^2 + \alpha' \Big[h(\rho)\Big]^{\frac{1}{2}} \Big[e^{2f(\rho)} d\rho^2 + e^{2g(\rho)}\, ds^2_{CP^2} + e^{2f(\rho)}\, \big(d\tau + A\big)^2\Big]$$

$$\Phi = \Phi(\rho) \qquad F_{(5)} = Q_c\,(1 + *)\varepsilon(S^5) \qquad F_1 = Q_f\,(d\tau + A)$$

$$Q_c \equiv \frac{(2\pi)^4 g_s N_c}{Vol(S^5)} = 16\pi g_s N_c \qquad Q_f = \frac{Vol(X_3) g_s\, N_f}{4 Vol(S^5)} = \frac{g_s\, N_f}{2\pi}$$

BPS equations

$$\dot{\Phi} = \frac{g_s\, N_f}{2\pi}\, e^{\Phi} \qquad \dot{g} = e^{2f-2g}$$

$$\dot{h} = -Q_c\, e^{-4g} \qquad \dot{f} = 3 - 2e^{2f-2g} - \frac{g_s\, N_f}{4\pi}\, e^{\Phi}$$

Integration of the BPS equations

Introduce a reference scale $\rho = \rho_* \;\rightarrow\; \phi_* = \phi(\rho = \rho_*)$

Deformation parameter

$$\epsilon_* = \frac{g_s N_f}{2\pi}\, e^{\phi_*} \;\Longrightarrow\; \boxed{\epsilon_* = \frac{1}{8\pi^2}\,\lambda_*\,\frac{N_f}{N_c}}$$

$$\boxed{e^{\phi-\phi_*} = \frac{1}{1+\epsilon_*\,(\rho_*-\rho)}}$$

$$\boxed{e^{g} = \sqrt{\alpha'}\; e^{\rho}\,\left(1+\epsilon_*\Big(\frac{1}{6}+\rho_*-\rho\Big)\right)^{\frac{1}{6}}}$$

$$\boxed{e^{f} = \sqrt{\alpha'}\; e^{\rho}\,\left(1+\epsilon_*(\rho_*-\rho)\right)^{\frac{1}{2}}\left(1+\epsilon_*\Big(\frac{1}{6}+\rho_*-\rho\Big)\right)^{-\frac{1}{3}}}$$

$$\boxed{\frac{dh}{d\rho} = -\frac{Q_c}{\alpha'^2}\; e^{-4\rho}\left(1+\epsilon_*\Big(\frac{1}{6}+\rho_*-\rho\Big)\right)^{-\frac{2}{3}}}$$

Properties of the solution

- dilaton blows up at $\rho = \rho_{LP} = \rho_* + \epsilon_*^{-1}$ (Landau pole)
- metric singular at $\rho = -\infty$ (IR)

  Good singularity that disappears when quarks are massive
- regime of validity $1 << N_c^{\frac{1}{3}} << N_f << N_c$

Matching the field theory

coupling constant

radius-energy relation $$\rho_{LP} - \rho = \log \frac{\Lambda_L}{Q}$$

$$g_{YM}^2 = 4\pi e^{\Phi} \Rightarrow \boxed{\frac{8\pi^2}{g_{YM}^2} = N_f \log \frac{\Lambda_L}{Q}}$$

same running as in F.T.

Perturbative solution $\longrightarrow$ expansion in powers of ϵ_*

change to a new radial coordinate such that: $h = \frac{R^4}{r^4}$ $\quad R^4 = \frac{1}{4} Q_c = 4\pi\, g_s\, \alpha'^2\, N_c$

$$e^g = r\Big[1 + \frac{\epsilon_*}{24}(1 - \frac{1}{3}\frac{r^4}{r_*^4}) + \frac{\epsilon_*^2}{1152}\Big(9 - \frac{106}{9}\frac{r^4}{r_*^4} + \frac{5}{9}\frac{r^8}{r_*^8} + 48\log(\frac{r}{r_*})\Big) + O(\epsilon_*^3)\Big]$$

$$e^f = r\Big[1 - \frac{\epsilon_*}{24}(1 + \frac{1}{3}\frac{r^4}{r_*^4}) + \frac{\epsilon_*^2}{1152}\Big(17 - \frac{94}{9}\frac{r^4}{r_*^4} + \frac{5}{9}\frac{r^8}{r_*^8} - 48\log(\frac{r}{r_*})\Big) + O(\epsilon_*^3)\Big]$$

$$\phi = \phi_* + \epsilon_* \log\frac{r}{r_*} + \frac{\epsilon_*^2}{72}\Big(1 - \frac{r^4}{r_*^4} + 12\log\frac{r}{r_*} + 36\log^2\frac{r}{r_*}\Big) + O(\epsilon_*^3)$$

deviation from $AdS_5 \times S^5$ order by order in ϵ_*

$r_* << r_{LP}$ UV scale (in a Wilsonian sense) far below the Landau pole

$\epsilon_* = \frac{1}{8\pi^2}\lambda_* \frac{N_f}{N_c} \sim g_{YM}^2(r_*)\, N_f$ $\longrightarrow$ measures internal flavor loop contributions

In computing observables we should be sure that the UV pathological region is decoupled

- One can study the effects of dynamical quarks in the screening of color charges (meson masses, quark potentials, screening lengths,..) (Biggazi et al., 0903.4747)

- Flavored black holes and hydrodynamics

  (Biggazi et al., 0909.2865, 0912.3256, 1101.3560)

- This method can be applied to add flavor to other backgrounds (Klebanov-Strassler, CVMN, ...)

  (Benini et al.,0706.1268, Casero, Nuñez&Paredes hep-th/0602027,...)

For further results on this and other unquenched backgrounds, see the review 1002.1088

Flavor in Chern-Simons-matter systems in 2+1

ABJM theory (Aharony et al. 0812.18)

CS with gauge group $U(N)_k \times U(N)_{-k}$ + bifundamental fields

$k \rightarrow$ CS level $\qquad \frac{1}{k} \sim$ coupling constant

M-theory description for large $N \rightarrow AdS_4 \times \mathbb{S}^7/\mathbb{Z}_k$

Sugra description in type IIA $\Longrightarrow$ $AdS_4 \times \mathbb{CP}^3 +$ fluxes

$$\boxed{ds^2 = L^2\, ds^2_{AdS_4} + 4\, L^2\, ds^2_{\mathbb{CP}^3}} \qquad L^4 = 2\pi^2 \frac{N}{k}$$

$$F_2 = 2k\, J \qquad F_4 = \frac{3\pi}{\sqrt{2}} \left(kN\right)^{\frac{1}{2}} \Omega_{AdS_4}$$

$$e^{\phi} = \frac{2L}{k} = 2\sqrt{\pi}\left(\frac{2N}{k^5}\right)^{\frac{1}{4}}$$

Effective description for $N^{\frac{1}{5}} << k << N$

Flavor branes (massless quarks)

Hohenegger&Kirsch 0903.1730
Gaiotto&Jafferis 0903.2175

D6-branes extended in AdS_4 and wrapping $\mathbb{RP}^3 \subset \mathbb{CP}^3$

Introduce quarks in the $(N,1)$ and $(1,N)$ representation

Backreaction

$$S_{WZ} = T_{D_6} \sum_{i=1}^{N_f} \int_{\mathcal{M}_7^{(i)}} \hat{C}_7 \to T_{D_6} \int_{\mathcal{M}_{10}} C_7 \wedge \Omega$$

Ω is a charge distribution 3-form

Modified Bianchi identity

$$\boxed{dF_2 = 2\pi\, \Omega}$$

Localized solution in 11d for coincident massless flavors

$AdS_4 \times \mathcal{M}_7$ with $\mathcal{M}_7$ a hyperkahler 3-Sasakian manifold

$\mathcal{N} = 3$ with $U(N_f)$ flavor symmetry

Backreaction with smearing

(E. Conde and AVR, to appear)

Write $\mathbb{CP}^3$ as an $\mathbb{S}^2$-bundle over $\mathbb{S}^4$

$$\boxed{ds^2_{\mathbb{CP}^3} = \frac{1}{4}\Big[ds^2_{\mathbb{S}^4} + \big(dx^i + \epsilon^{ijk}A^j x^k\big)^2\Big]}$$

$$\sum_i (x^i)^2 = 1$$

Fubini-Study metric

$A^i \rightarrow SU(2)$ instanton on $\mathbb{S}^4$

The RR two-form F_2 can be written as:

$$\boxed{F_2 = \frac{k}{2}\Big(E^1\wedge E^2 - (\mathcal{S}^4\wedge\mathcal{S}^3 + \mathcal{S}^1\wedge\mathcal{S}^2)\Big)}$$

$$\frac{1}{2\pi}\int_{\mathbb{CP}^1} F_2 = k$$

$\mathcal{S}^i \rightarrow$ (rotated) basis of one-forms along $\mathbb{S}^4$

$E^i \rightarrow$ one-forms along the $\mathbb{S}^2$ fiber

Some Killing spinors are constant in this basis $\Longrightarrow$ deform to preserve them

Prescription: squash F_2 and the metric

$$\boxed{F_2 = \frac{k}{2}\left[\,E^1\wedge E^2 - \eta\left(\mathcal{S}^4\wedge\mathcal{S}^3 + \mathcal{S}^1\wedge\mathcal{S}^2\right)\,\right]}$$

Induces violation of Bianchi identity

$$\boxed{\eta \equiv 1 + \frac{3N_f}{4k}}$$

Deformation parameter

$$\epsilon \equiv \frac{N_f}{k} = \frac{N_f}{N}\,\lambda$$

Flavored metric

$$\boxed{\begin{aligned} ds^2 &= L^2\, ds^2_{AdS_4} + ds^2_6 \\ ds^2_6 &= \frac{L^2}{b^2}\left[q\, ds^2_{\mathbb{S}^4} + \left(dx^i + \epsilon^{ijk}\,A^j\,x^k\right)^2\right]\end{aligned}}$$

$q \longrightarrow\ \mathbb{C}\,\mathbb{P}^3$ internal squashing

$b \longrightarrow$ relative $AdS_4/\mathbb{C}\,\mathbb{P}^3$ squashing

$\mathcal{N} = 1$ superconformal SUSY implies

$$q^2 - 3(1+\eta)\, q + 5\eta = 0$$

$$\boxed{q = 3 + \frac{9}{8}\,\frac{N_f}{k} - 2\sqrt{1 + \frac{3}{4}\,\frac{N_f}{k} + \left(\frac{3}{4}\right)^4 \left(\frac{N_f}{k}\right)^2}}$$

Also

$$\boxed{b = \frac{q(\eta + q)}{2(q + \eta q - \eta)}}$$

The new AdS_4 radius is:

$$\boxed{L^4 = 2\pi^2\,\frac{N}{k}\,\frac{(2-q)\,b^4}{q(q + \eta q - \eta)}}$$

Dilaton and F_4:

$$\boxed{e^{-\phi} = \frac{b}{4}\,\frac{\eta+q}{2-q}\,\frac{k}{L}}$$

$$\boxed{F_4 = \frac{3kb}{4}\,\frac{\eta+q}{2-q}\,L^2\,\Omega_{AdS_4}}$$

Regime of validity

$$L >> 1\,, \qquad e^{\phi} << 1$$

If $N_f/k \sim 1$ $\Rightarrow$ $\boxed{N^{\frac{1}{5}} << k << N}$ (same as in the unflavored case)

When $N_f >> k$

$$L^4 \sim \frac{N}{N_f} \qquad e^{\phi} \sim \left(\frac{N}{N_f^5}\right)^{\frac{1}{4}} \Rightarrow \boxed{N^{\frac{1}{5}} << N_f << N}$$

Flavor effects

Free energy on the 3-sphere (measures # dof's)

$$F(\mathbb{S}^3) = -\log|Z_{\mathbb{S}^3}| \;\longrightarrow\; \boxed{F(\mathbb{S}^3) = \frac{\pi L^2}{2G_N}} \;\longleftarrow\; \frac{1}{G_N} = \frac{1}{G_{10}}\, e^{-2\phi}\, Vol(\mathcal{M}_6)$$

In flavored ABJM

$$\boxed{F(\mathbb{S}^3) = \frac{\pi\sqrt{2}}{3}\, k^{\frac{1}{2}}\, N^{\frac{3}{2}}\, \xi\Big(\frac{N_f}{k}\Big)}$$

$$\xi\Big(\frac{N_f}{k}\Big) \equiv \frac{1}{16}\, \frac{q^{\frac{5}{2}}\,(\eta+q)^4}{(2-q)^{\frac{1}{2}}\,(q+\eta q-\eta)^{\frac{7}{2}}}$$

For small N_f/k

$$\xi = 1 + \frac{3}{4}\,\frac{N_f}{k} - \frac{9}{64}\Big(\frac{N_f}{k}\Big)^2 + \mathcal{O}\Big(\Big(\frac{N_f}{k}\Big)^3\Big)$$

$$\boxed{F(\mathbb{S}^3) = \frac{\pi\sqrt{2}}{3}\,\frac{N^2}{\sqrt{\lambda}} + \frac{\pi\sqrt{2}}{4}\, N_f\, N\,\sqrt{\lambda} - \frac{3\pi\sqrt{2}}{64}\, N_f^2\,\lambda^{\frac{3}{2}} + \cdots}$$

unflavored term $\sim N^{\frac{3}{2}}$ $\longrightarrow$ amazing field theory match by Drukker et al. (1007.3837) !

For large N_f/k $\Longrightarrow$ $$\xi \sim \frac{225}{256}\sqrt{\frac{5}{2}}\sqrt{\frac{N_f}{k}} \approx 1.389\sqrt{\frac{N_f}{k}}$$

Comparison with 3-Sasakian $(U(N_f), \mathcal{N}=3 \text{ flavors})$

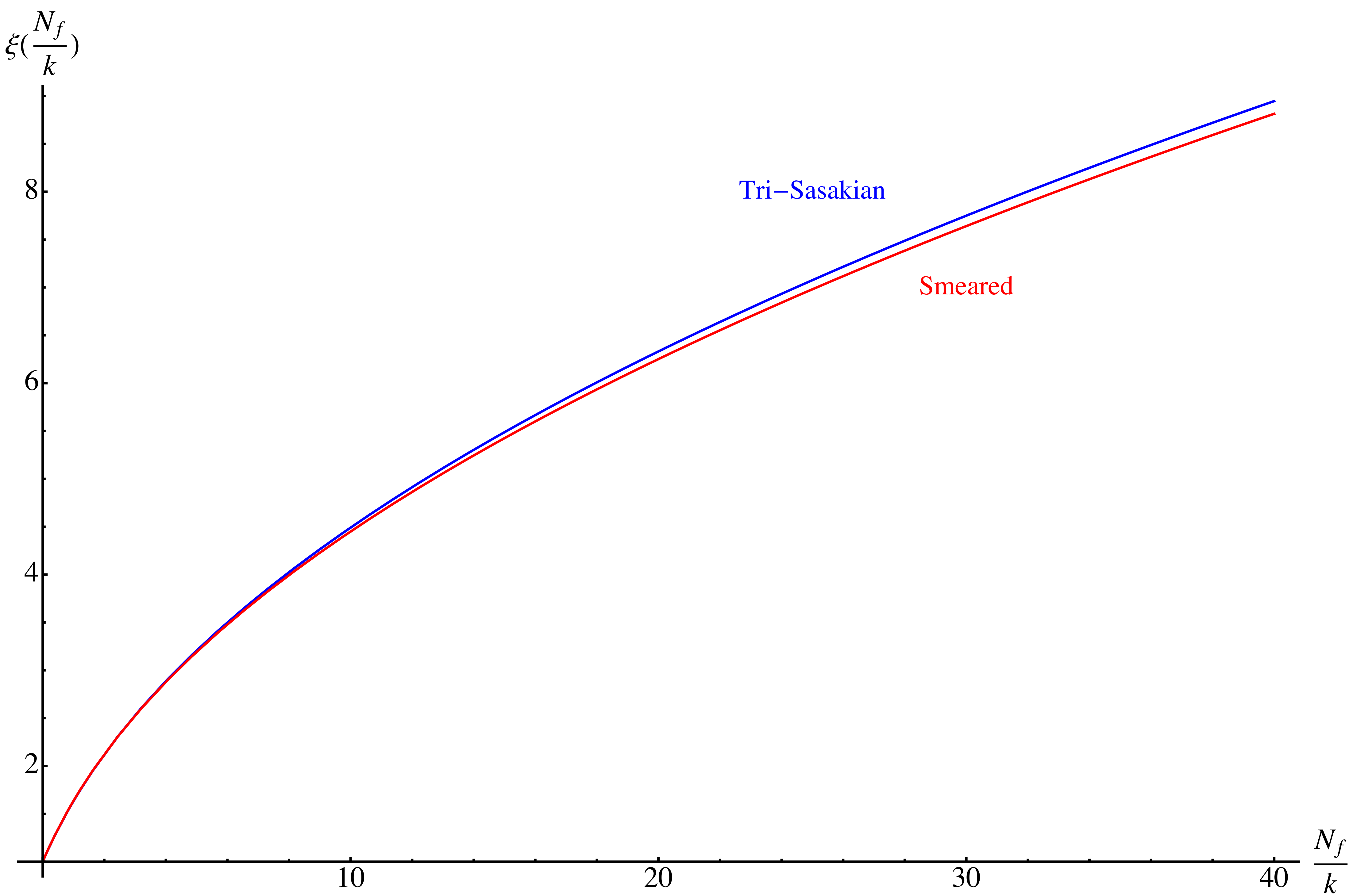
$$\boxed{\xi^{3-S} = \frac{1+\frac{N_f}{k}}{\sqrt{1+\frac{N_f}{2k}}}} \Longrightarrow$$

(Gaiotto&Jafferis 0903.2175)

$$\xi^{3-S} = 1 + \frac{3}{4}\frac{N_f}{k} - \frac{5}{32}\left(\frac{N_f}{k}\right)^2 + \mathcal{O}\left(\left(\frac{N_f}{k}\right)^3\right)$$

$$\xi^{3-S} \sim \sqrt{2}\sqrt{\frac{N_f}{k}} \qquad \text{when } N_f/k \to \infty$$

Field theory match: Couso-Santamaria et al. 1011.6281


$\xi(\frac{N_f}{k})$
2
4
6
8
10
20
30
40
$\frac{N_f}{k}$
Tri−Sasakian
Smeared

quark-antiquark energy

$$\boxed{V_{q\bar{q}} = -\frac{Q}{d}}$$

$$Q = \frac{4\pi^2 L^2}{\left[\Gamma\left(\frac{1}{4}\right)\right]^4}$$

(Maldacena, Rey)

In ABJM with flavor

$$\boxed{Q = \frac{4\pi^3\sqrt{2\lambda}}{\left[\Gamma\left(\frac{1}{4}\right)\right]^4}\,\sigma} \longleftarrow \sigma = \frac{1}{4}\,\frac{q^{\frac{3}{2}}\,(\eta+q)^2\,(2-q)^{\frac{1}{2}}}{(q+\eta q-\eta)^{\frac{5}{2}}}$$

Series expansion

$$\sigma = 1 - \frac{3}{8}\frac{N_f}{k} + \frac{9}{64}\left(\frac{N_k}{k}\right)^2 + \cdots$$

Dynamical quarks screen the Coulomb interaction

Scalar meson operators

From the normalizable fluctuations of the scalars transverse to the flavor D6-branes

$$\boxed{\dim(\bar\psi\psi) \;=\; 3-b} \quad \Longrightarrow \quad \left[\begin{array}{l} \dim(\bar\psi\psi) \;=\; 2 - \dfrac{3}{16}\,\dfrac{N_f}{k} + \dfrac{63}{512}\left(\dfrac{N_f}{k}\right)^2 + \cdots \\[2em] \dim(\bar\psi\psi) \to \dfrac{7}{4} \qquad \left(\dfrac{N_f}{k} \to \infty\right) \end{array}\right.$$

High Spin operators

$$\Delta - S \;=\; f(\lambda,\epsilon)\;\log S$$

(Gubser et al.)

$$f(\lambda,\epsilon) \;=\; \frac{L^2}{\pi}$$

cusp anomalous dimension

$$\boxed{f(\lambda,\epsilon) \;=\; \sqrt{2\lambda}\;\;\sigma}$$

Other flavor effects in meson masses, dimensions of monopole operators, k-string tensions, ...


Thank You!