The fermion sign problem and large N orbifold equivalence

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AC, M. Hanada, D. Robles-Llana, PRL106,2011
AC, B. Tiburzi, 1103.1623

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How does matter behave at extreme densities?

$$n_B \gtrsim \Lambda_{QCD}^{-3}$$

Critical for understanding e.g. neutron stars

Clean way to study finite n_B is to introduce a chemical potential for quark number (~baryon number):

$$\mathcal{L}_{\mathrm{QCD}} \to \mathcal{L}_{\mathrm{QCD}} + \mu_B \bar{\psi}\gamma^0\psi$$

Asymptotic freedom of QCD allows controlled calculations when $\mu_B/\Lambda_{QCD} \to \infty$.

Only known way to study QCD away from the perturbative regime is using lattice Monte Carlo methods.

Unfortunately, Monte Carlo methods do not work at finite μ_B !

Monte Carlo

$$\langle \mathcal{O} \rangle = \frac{\int dA_\mu d\psi d\bar{\psi} e^{-S[A_\mu,\psi,\bar{\psi}]} \mathcal{O}[A_\mu,\psi,\bar{\psi}]}{\int dA_\mu d\psi d\bar{\psi} e^{-S[A_\mu,\psi,\bar{\psi}]}}$$

$$= \frac{1}{Z} \int dA_\mu \det(\not{D}) e^{-S[A_\mu]} \mathcal{O}[A_\mu]$$

Monte Carlo method: generate random A_μ configurations using

$$\det(\not{D}) e^{-S[A_\mu]}$$

as a probability distribution, then evaluate the integral.

Works fine as long as distribution is > 0!

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QCD at μ_B=0: $\gamma_5 \not{D} \gamma_5 = \not{D}^\dagger$ → Eigenvalues of $\not{D}$ come in λ, λ* pairs

So then $\det(\not{D}) = \prod_i \lambda_i > 0$

The ~~sign~~ phase problem

Once $\mu_B > 0$, γ^5 symmetry breaks, and $\det(\not{D})$ becomes complex, with a rapidly fluctuating phase.

Can't use importance sampling anymore!

No known way to generically dodge this kind of problem.

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But maybe one just needs a clever algorithm to sum up the fluctuating phases?

Well...

Computational Complexity and Fundamental Limitations to Fermionic Quantum Monte Carlo Simulations

Matthias Troyer[1] and Uwe-Jens Wiese[2]
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(Received 11 August 2004; published 4 May 2005)

Quantum Monte Carlo simulations, while being efficient for bosons, suffer from the "negative sign problem" when applied to fermions—causing an exponential increase of the computing time with the number of particles. A polynomial time solution to the sign problem is highly desired since it would provide an unbiased and numerically exact method to simulate correlated quantum systems. Here we show that such a solution is almost certainly unattainable by proving that the sign problem is nondeterministic polynomial (NP) hard, implying that a generic solution of the sign problem would also solve all problems in the complexity class NP in polynomial time.

DOI: 10.1103/PhysRevLett.94.170201 PACS numbers: 02.70.Ss, 05.10.Ln

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P = NP

Clay Institute Prize

or

So how to make progress?

(1) Do **not** look for general solutions: exploit specific features of QCD.

(2) Forget real-world QCD with $N = 3$ colors - too hard!

Go to the large N limit!

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Go to the large N limit!

At large N, very different-looking gauge theories sometimes have the same correlation functions:

Large N orbifold equivalence!

Famous examples:

Eguchi-Kawai reduction and related ideas

Large N equivalence between $\mathcal{N} = 1$ SYM and $N_f = 1$ QCD

Armoni, Shifman, Veneziano

Goal: find a sign-problem-free theory which is orbifold-equivalent to large N QCD at $\mu_B > 0$.

Hands et al
and others

Do sign-problem-free theories exist?

Yes!

1. QCD with N=2 colors, and
2. QCD with adjoint representation quarks.

$\gamma_5 \not{D} \gamma_5 = \not{D}^\dagger$ still broken when $\mu_B > 0$

But now fermion representation is (pseudo)-real...

→ additional symmetry: even when $\mu_B > 0$! $C\gamma_5 \not{D} (C\gamma_5)^{-1} = \not{D}^*$

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But 1 & 2 have a number of major differences from *N*=3 QCD...

Goal is to use large N to get something equivalent to QCD.

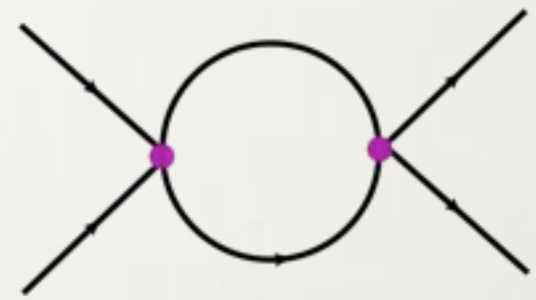
Large N in one slide

't Hooft large N limit: $N \to \infty$, keeping $g^2 N$ fixed, N_f fixed

Non-planar diagrams and quark loops suppressed

$\sim 1/N^{1/2}$

$\sim 1/N$



$\sim \frac{1}{N}\frac{N_f}{N} \sim \frac{1}{N^2}$

Mesons are stable, weakly-interacting; meson loops suppressed.

Good (10-30%) approx. to real world for many observables at $\mu_B = 0$.

Extent to which large N is good for $\mu_B > 0$ is an interesting question.

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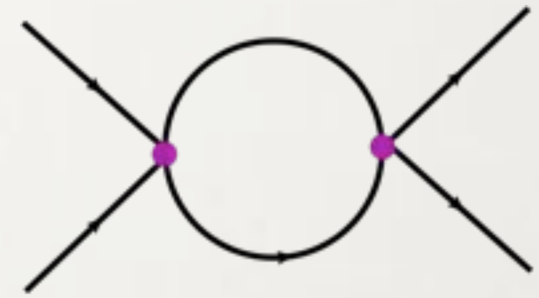

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Sign problem still present at large N:

Suppression of quark loops may allow dropping of $\det(\not{D})$ for generation of A_μ configurations

But $\det(\not{D})$ appears inside fermionic observables!

AC, M. Hanada, D. Robles-Llana, PRL 2011

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The proposal

1. SO(2N) gauge theory with N_f flavors of fundamental Dirac fermions $\cong$ SU(N) gauge theory with N_f flavors of fundamental Dirac fermions

Orbifold equivalence   Large N QCD

2. Equivalence can be made to hold even when $\mu_B > 0$.

→ Use deformation approach due to Unsal+Yaffe

3. The SO(2N) theory does not have a sign problem at finite μ_B.

→ Make sure D has enough symmetry, e.g.

$$C\gamma_5 \not{D} \,(C\gamma_5)^{-1} = \not{D}^{\,*}$$

A quick look at SO gauge theories

$$\mathcal{L} = \frac{1}{4g^2}\mathrm{tr}F_{\mu\nu}F^{\mu\nu} + \sum_{a=1}^{N_f}\bar{\psi}_a(\not{D} + m + \mu_B\gamma^4)\psi_a$$

Looks a lot like QCD: has both mesons and baryons

Still have $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$ symmetry.

Witten & Coleman, Peskin, 1980

But SO is real, so all fermion reps are real $\Rightarrow$ Flavor symmetry enhanced to $SU(2N_f)$

$$SU(2N_f) \xrightarrow{\langle\bar{\psi}\psi\rangle \neq 0} SO(2N_f)$$

$$N_f^2 - 1 \; + \; N_f(N_f - 1)$$

NG bosons

An embarrassment of riches: two ways to make color singlets

$$N_f^2 - 1$$

$\bar{\psi}_a \gamma^5 \psi_b$ P = -1

Pions!

+ all the other usual-looking mesons

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$$N_f(N_f - 1)$$

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P=+1

Baryonic pions...

NGBs with $U(1)_B$ charge!

Want simple way to refer to these particles:

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+ bmeson relatives of all the usual mesons

Ex.: bρ mesons

In what sense can such a weird theory be `equivalent' to QCD?

Orbifold Projection: the formal picture

Pick "mother" theory with a global symmetry G.

+

Pick a discrete cyclic subgroup $\mathbb{Z}_\Gamma \subset G$

The orbifold projection:

Set to zero all degrees of freedom in the mother not invariant under $\mathbb{Z}_\Gamma \subset G$

$\mathbb{Z}_\Gamma$ orbifold "daughter theory"

Looks violent, but...

Kachru, Silverstein 1998

Large N orbifold equivalence

If two theories are related by an $\mathbb{Z}_\Gamma$ orbifold projection

Kovtun, Unsal, Yaffe, 2003-4

and

$\mathbb{Z}_\Gamma$ symmetry is not spontaneously broken

then...

Correlation functions of neutral operators in mother and daughter theories will **coincide** in the large N limit.

Neutral operators in mother are the ones with non-zero image in daughter.

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Truth in advertising:

Existing proofs of large N equivalence require some generalizations for this application!

From SO(2N) to SU(N) QCD in one slide

SO theory has $SO(2N) \times U(1)_B$ symmetry

Pick group generators: $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \in SO(2N) \quad \omega = e^{i\pi/2} \in U(1)_B$

Group action: $A_\mu \longrightarrow J A_\mu J^T \qquad \psi \longrightarrow \omega J \psi$

$\mathbb{Z}_2$ symmetry

Result of orbifold: $\mathcal{L}^{SO} \longrightarrow \mathcal{L}^{SU}$

Survivors of projection

All gauge-invariant operators in pure-glue sector of SO theory

All mesons

$$\underbrace{\hspace{10em}}_{\text{neutral sector}}$$

Victims of projection

All bmesons

$$\underbrace{\hspace{10em}}_{\text{neutral sector}}$$

Reason bmesons are killed by the orbifold projection:

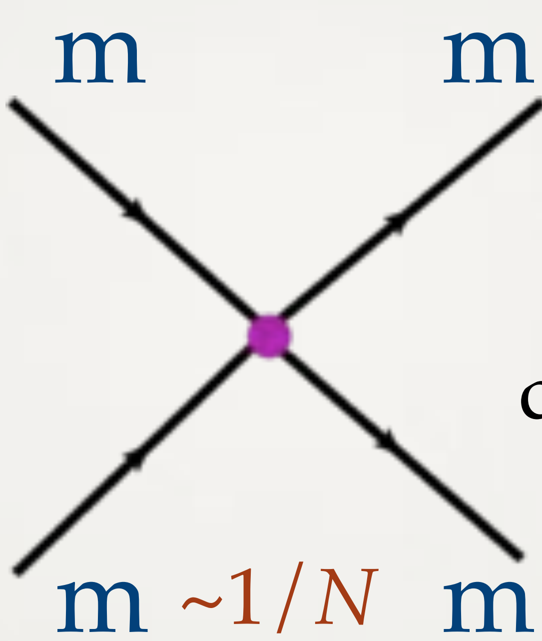
Operators of the form $\psi^T \psi$ have $\mathbb{Z}_2$ charge -1

Projection sets to zero all degrees of freedom not invariant under $\mathbb{Z}_2$

Baryons: orbifold projections get subtle, work in progress.

Cartoon picture of orbifold equivalence

Daughter:



+m tree diagrams

m = meson
b = bmeson

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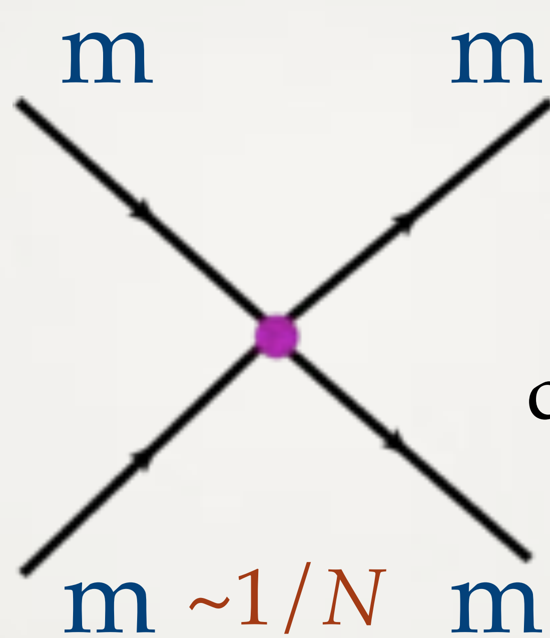

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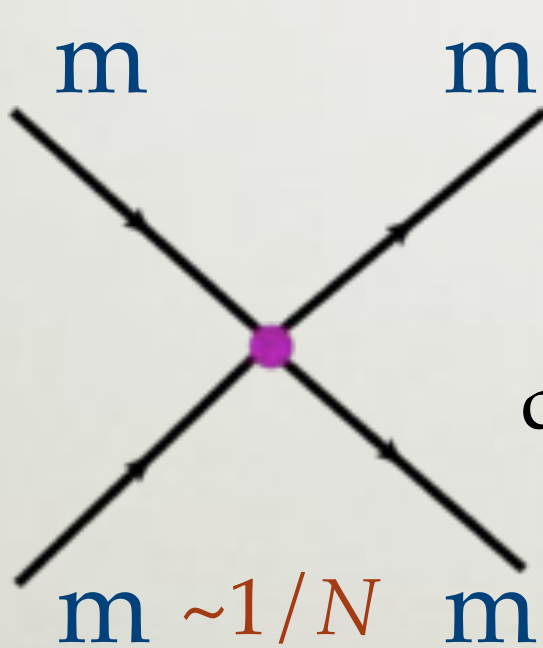
Mother:



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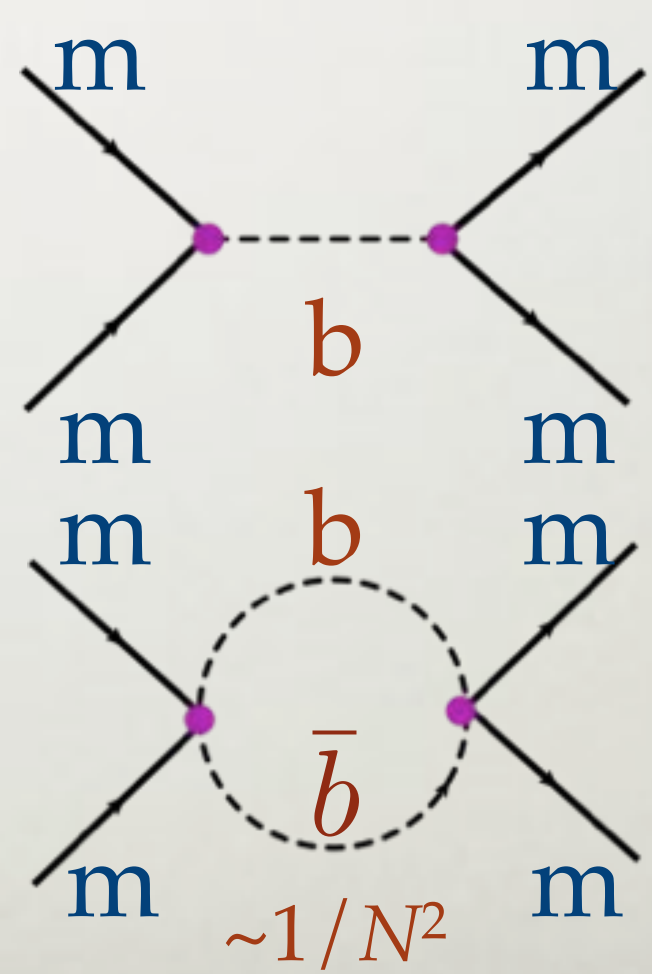

Not allowed if $U(1)_B$ unbroken

Allowed

The good news

No bmeson condensation at $\mu_B=0$. Vafa-Witten theorem

In fact, can show that there is no bmeson condensation at least for $\mu_B < m_\pi/2$.

So at least up to $\mu_B<m_\pi/2$, expect equivalence to hold.

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But large N QCD has a sign problem for *any* $\mu_B > 0$!

So orbifold equivalence gives a way to dodge the sign problem at least for $\mu_B < m_\pi/2$.

Already something... But not as much as we want!

The bad news

Once $\mu_B > m_\pi/2$ bpions condense : $\langle \psi^T C \gamma^5 \psi \rangle \neq 0$

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Protecting $U(1)_B$

inspired by double-trace deformations of Unsal and Yaffe, 2008.

Can we modify the SO theory so that

(1) the modified theory still maps to QCD, and

(2) prevent bpion condensation?

Yes!

$$\mathcal{L}_{SO} \longrightarrow \mathcal{L}_{SO} + \frac{c^2}{\Lambda^2_{QCD}} S^\dagger_{ab} S^{ab} \qquad S_{ab} = \psi_a^T C \gamma^5 \psi_b$$

New term orbifolds to zero.

Cartoon picture: free energy becomes

$$\langle F \rangle \longrightarrow \langle F \rangle + \frac{c^2}{\Lambda_{QCD}} \langle S \rangle \langle S^\dagger \rangle$$

This makes system pay a cost for condensing.

So use deformations to discourage bpion condensation.

Next step: make sure this is more than a cartoon.

Deformations and Effective Field Theory

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Hard to understand deformed theory analytically in general.

But if m_q, μ_B, and c are small compared to Λ_{QCD}, low-energy physics can be systematically described using effective field theory.

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Strategy:

(1) Construct chiral perturbation theory for the SO(2Nc) theory

(2) Use standard spurion analysis tricks to find new interaction terms for the pions, bpions induced by deformations.

(3) Determine the phase structure as a function of m_q, μ_B, c.

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XPT is written in terms of $\Sigma \in U(2N_f)/SO(2N_f)$

Σ contains all the NGBs:

$$\Sigma = \exp\left(\frac{i\eta'}{F_\Pi\sqrt{N_f}}\right)\exp\left(\frac{i\Pi}{F_\Pi}\right)\Sigma_0 \qquad \Pi = \begin{pmatrix} \pi & b \\ b^\dagger & \pi^T \end{pmatrix}$$

Note: at large N_c, chiral anomaly is suppressed and η' is light.

Low energy action

Without deformations, the EFT has the Lagrangian

$$\mathcal{L} = \frac{F_\Pi^2}{4}\mathrm{tr}\left[D_\mu \Sigma D_\mu \Sigma^\dagger\right] - \frac{\lambda F_\Pi^2}{4}\mathrm{tr}\left[\Sigma \mathcal{M} + \Sigma^\dagger \mathcal{M}^\dagger\right]$$

$$D_\mu \Sigma = \partial_\mu \Sigma + i B_\mu \Sigma + i \Sigma B_\mu^T \qquad B_\mu \sim \mu_B, \mathcal{M} \sim m_q$$

Higher-order terms are suppressed at large N_c

(also by low-energy expansion)

Deformations induces new terms in the low-energy action...

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In *XPT* turns it is easier to work with the deformations

$$V_\pm = \frac{c^2}{\Lambda_{QCD}^2} \sum_{a,b=1}^{N_f} \left(S_{ab}^\dagger S_{ab} \pm P_{ab}^\dagger P_{ab}\right) \quad \begin{aligned} P_{ab} &= \psi_a^T C \psi_b \\ S_{ab} &= \psi_a^T C \gamma^5 \psi_b \end{aligned}$$

V_+ is a chiral singlet, while V_- is not.

Will see that both deformations have identical effect
on neutral-sector physics, so long as $U(1)_B$ not broken.

Two deformations

To capture effects of deformations, use spurion analysis.

Same approach as used in XPT to understand lattice-spacing effects, inclusion of weak interaction effects, etc

V_+ produces just one new term in the EFT

$$c_+ \, F_\Pi^2 \sum_{a,b=1}^{N_f} \left(\mathrm{tr}\left[\Sigma L^{(ab)}\right] \mathrm{tr}\left[\Sigma^\dagger L^{(ab)\dagger}\right] + \mathrm{tr}\left[\Sigma R^{(ab)}\right] \mathrm{tr}\left[\Sigma^\dagger R^{(ab)\dagger}\right] \right)$$

V_- produces two new terms in the EFT

$$c_- F_\Pi^2 \sum_{a,b=1}^{N_f} \left(\mathrm{tr}[\Sigma L^{(ab)}]\mathrm{tr}[\Sigma R^{(ab)}] + \mathrm{tr}[\Sigma^\dagger L^{(ab)\dagger}]\mathrm{tr}[\Sigma^\dagger R^{(ab)\dagger}] \right)$$

$$+ \, d_- F_\Pi^2 \sum_{a,b=1}^{N_f} \left(\mathrm{tr}[\Sigma L^{(ab)} \Sigma R^{(ab)}] + \mathrm{tr}[\Sigma^\dagger L^{(ab)\dagger} \Sigma^\dagger R^{(ab)\dagger}] \right)$$

New low-energy constants $\boxed{c_+ , c_- , d_-}$

Spectrum of the deformed theory

Without symmetry breaking:

Mode	Mass with V_- deformation	Mass with V_+ deformation
π	$(m_\pi^2 + 4d_-)^{1/2}$	m_π
η'	$(m_\pi^2 + 4d_-)^{1/2}$	m_π
b	$(m_\pi^2 + 4c_-)^{1/2} + 2\mu$	$(m_\pi^2 + 4c_+)^{1/2} + 2\mu$
$b^\dagger$	$(m_\pi^2 + 4c_-)^{1/2} - 2\mu$	$(m_\pi^2 + 4c_+)^{1/2} - 2\mu$

Matching to microscopic theory gives N_c scaling of the new LECs

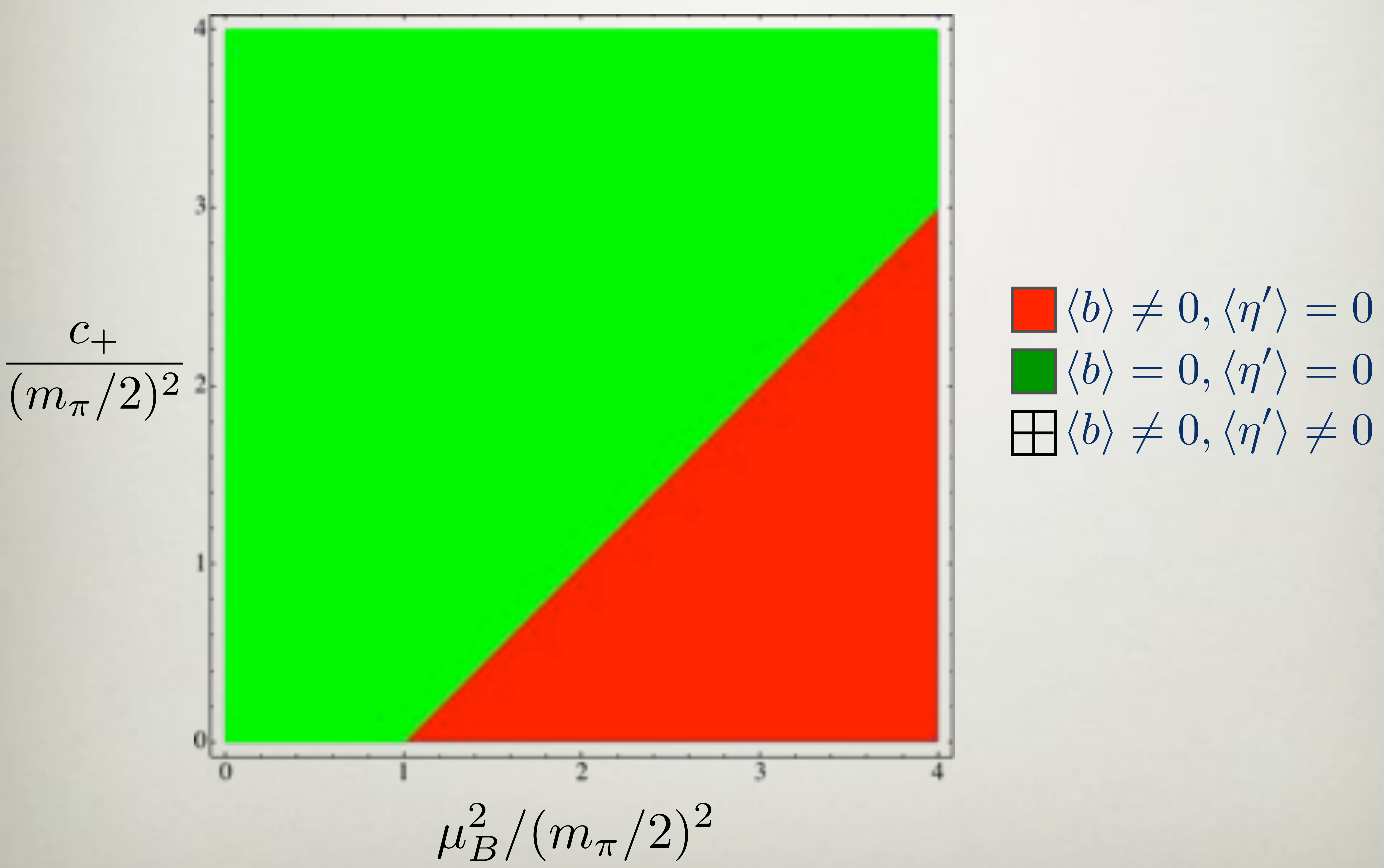
$c_-, c_+ \sim N_c^0$, $d_- \sim N_c^{-1}$

Can also show that the sign of c in microscopic theory controls the signs of the LECs in the EFT.

So both deformations raise the bpion mass, while leaving neutral-sector stuff alone.

To sort of symmetry realization pattern, minimize effective potential in deformed theory

Phase diagram of the V_+-deformed theory



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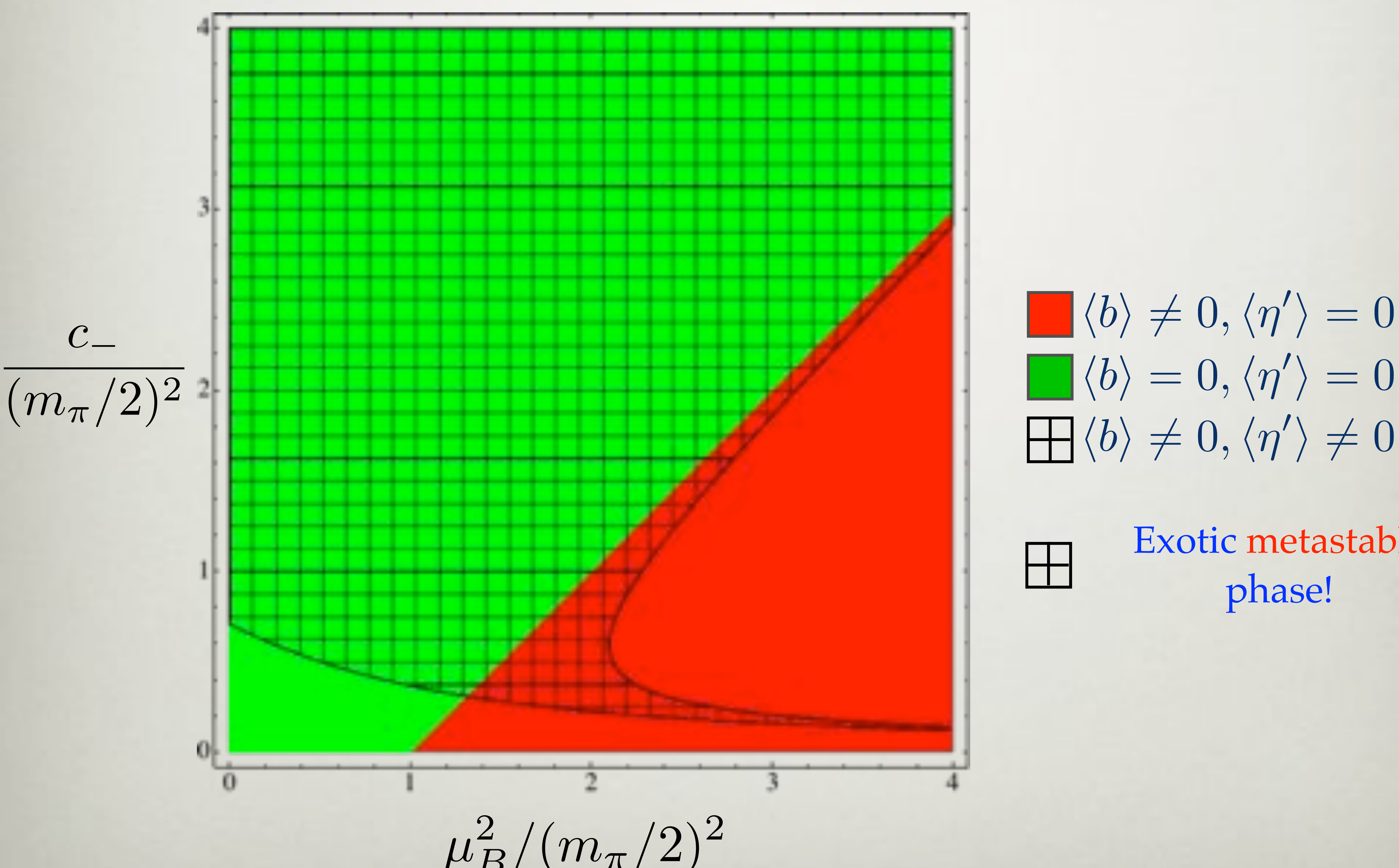

Exotic metastable phase!

Orbifold equivalence past $\mu_B = m_\pi/2$

With both deformations, the SO theory can be forced to stay in a $U(1)_B$-unbroken phase past $\mu_B = m_\pi/2$.

The correlation functions of neutral operators are identical with both deformations in the normal phase.

The V_--deformed theory has an exotic phase with η'-condensation. This phase is always metastable in our analysis.

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At level of EFT, large N-equivalence is `obvious':

$$\frac{U(N_f)_L \times U(N_F)_R}{U(N_f)_V} \subset \frac{SU(2N_f)}{SO(2N_f)}$$

Neutral correlators in $SU(2N_f)/SO(2N_f)$ EFT with given LECs coincide with correlators computed in $SU(N_f)_V$ EFT with the same LECs, so long as $U(1)_B$ is not broken.

AC, M. Hanada, D. Robles-Llana, 2010

AC, B. Tiburzi 2011

The proposal

1. SO(2N) gauge theory with N_f flavors of fundamental Dirac fermions $\cong$ SU(N) gauge theory with N_f flavors of fundamental Dirac fermions

Orbifold equivalence Large N QCD

2. Equivalence can be made to hold even when $\mu_B > m_\pi/2$.

→ Use deformation approach due to Unsal+Yaffe

3. The SO(2N) theory does not have a sign problem at finite μ_B.

→ Make sure D has enough symmetry, e.g.

$$C\gamma_5 \not{D} \, (C\gamma_5)^{-1} = \not{D}^{\,*}$$

Sign-free implementation of deformations

Deformations are four-quark operators, so must use auxiliary fields to put them on the lattice.

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For V_{-}, there is a rather baroque way to implement auxiliary fields that avoids reintroducing the sign problem

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Fierz rearrangement: $$S^{\dagger\, ab} S_{ab} = \sum_{\Gamma} (\bar{q}^i_a \Gamma q^j_a)^2$$

Can couple real auxiliary fields f_{ij} to

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Integration over f_{ij} gives original 4-quark terms

$$S^{\dagger\,ab} S_{ab} \longrightarrow \sum_{\Gamma} \left[\frac{1}{2} f^{\Gamma}_{ij} f^{ij\,\Gamma} + i c_{\Gamma} f^{\Gamma}_{ij} \bar{q}^i_a \Gamma q^j_a \right]$$

+ similar terms for $P^{\dagger\,ab} P_{ab}$

Sign-free implementation of V_- deformations

Result of integrating in auxiliary fields in flavor-singlet channel:

$$\frac{c^2}{\Lambda^2}\left(S^{\dagger ab}S_{ab} - P^{\dagger ab}P_{ab}\right) \Rightarrow \begin{aligned} &(f_{ij})^2/2 + (g_{ij})^2/2 + (h_{\mu\nu,ij})^2/2 \\ &+ ic_1 f_{ij}\bar{\psi}^i_a \psi^j_a + ic_2 g_{ij}\bar{\psi}^i_a \gamma^5 \psi^j_a \\ &+ ic_3 h_{\mu\nu,ij}\bar{\psi}^i_a \gamma^{\mu\nu}\psi^j_a \end{aligned}$$

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✔No sign problem in the chiral limit.

✔Large N equivalence to QCD kept past $\mu_B = m_\pi/2$

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The same trick does not work for V+. Are there other tricks that do?

Summary and open questions

Using SO theory, we can dodge sign problem even past $m_\pi/2$.

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Does equivalence hold through nuclear matter transition?

- Do bmesons with charge/mass less than lightest baryons exist, even in deformed theory?

- If so, expect condensation for big enough μ_B, killing equivalence.

We need non-perturbative tests!

Lattice, AdS/CFT, ...

To do:

Work out baryon-sector matching, extend equivalence proofs, look for sign-free way to work with V_+, try to get away from chiral limit, try to dodge other sign problems,...