Holographic flows and large-N equivalences

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Generalities

- Holography gives a lot interesting results for four-dimensional large-N gauge theories at strong coupling
- The best understood cases are conformal field theories, but the duality goes beyond them
- Some popular examples of non-CFT duals are: Witten QCD / Sakai-Sugimoto, Klebanov-Strassler, Maldacena-Nunez, $\mathcal{N}=2^*$ SYM, $\mathcal{N}=1^*$ SYM, ...

Generalities

- In non-conformal models usually there is an "interesting" IR theory and an "exotic" UV theory
- At weak coupling UV and IR physics decouple:

$$\Lambda_{IR} \sim \Lambda_{UV} e^{-\frac{1}{\lambda(\Lambda_{UV})}}$$

- However, at strong coupling they don't: $\Lambda_{IR} \sim \Lambda_{UV}$
- Deep in the IR this problem could be avoided ($Q^2 \ll \Lambda_{IR}^2$, $T \ll \Lambda_{IR}$)

A simple non-conformal theory

Let us choose as exotic theory $\mathcal{N}=4$ $SU(N)$ SYM (well-defined UV fixed point) and as interesting theory $\mathcal{N}=2$ $SU(N)$ SYM

- The field content of $\mathcal{N}=4$ SYM is:
  - Gauge field
  - Gauginos in **4** of $SU(4)_R$
  - Scalar fields in **6** of $SU(4)_R$
- The $\mathcal{N}=2^*$ SYM theory has the same field content but with a mass m for a hypermultiplet (4 real scalars and a Dirac fermion)
- $SU(4)_R \to SU(2)_R \times U(1)$

Comments about $\mathcal{N} = 2^*$ SYM

- At low energies $E \ll m$, the theory flows to $\mathcal{N} = 2$ SYM $(A_\mu, \lambda_\alpha, \Phi)$
- At very low energies $E \ll \Lambda_{IR}$ the effective IR theory is determined by the position on moduli space (eigenvalues of Φ). Generically $U(1)^{N-1}$ free theory.
- At low temperatures $T \ll \Lambda_{IR}$ the moduli space is lifted, but singular points with additional massless degrees of freedom may become the true IR theory. E.g. Paik and Yaffe '09 for $\mathcal{N} = 2$ $SU(2)$ SYM
- IR theories can be quite exotic themselves. In $\mathcal{N} = 2$ SYM dyons of charges (q_i, h_i) become massless at singular points. If

  $$h_1 \cdot q_2 - h_2 \cdot q_1 \neq 0 \mod N$$

  Then the IR becomes a strongly coupled CFT

  [Seiberg-Witten],[Argyres-Faraggi],[Klemm, Lerche, Theisen, Yankielowicz],[Argyres-Douglas]

Goal:

Describe the low T regime of (large-N, strong coupling) $\mathcal{N} = 2^* \; SU(N)$ SYM using holography

- The IR theory is not a CFT
- Bonus: we will learn about $\mathcal{N} = 4$ SYM with spontaneous breaking of conformal invariance

AdS/CFT duality

$d = 10$ **type IIB SUGRA in** $AdS_5 \times S^5$ **dual to** $\mathcal{N} = 4$ $SU(N_c)$ **SYM:**

- $N_c \to \infty$, $\lambda \gg 1$
- $g_s \sim 1/N_c \to 0$, $L^2/\alpha' \sim \sqrt{\lambda} \gg 1$
- The duality relates Kaluza-Klein modes on the S^5 with protected operators of dimension Δ depending on the momentum
- Momentum in $S^5 = SO(6) \simeq SU(4)_R$ R-charge

Dimensional reduction on S^5:

- Consistent truncation: only keep lowest Kaluza-Klein modes
- $d = 10$ type IIB SUGRA in $AdS_5 \times S^5 \to d = 5$ $\mathcal{N} = 8$ gauged SUGRA in AdS_5
- $SO(6)$ gauge charge $=$ $SO(6)$ R-charge
- mass in $AdS_5 \leftrightarrow$ dimension Δ

Scalar fields dual to marginal and relevant operators

m^2L^2	$SO(6)$	Δ	$\mathcal{O}$
-4	$\mathbf{20'}$	2	$\mathrm{tr}\left(\Phi_{\{i}\Phi_{j\}} - \frac{1}{6}\delta_{ij}\Phi_k^2\right)$
-3	$\mathbf{10} + \overline{\mathbf{10}}$	3	$\mathrm{tr}\left(\lambda_{\{a}\lambda_{b\}}\right) + h.c.$
0	$\mathbf{1}$	4	$\mathrm{tr}\left(F^2 + iF\tilde{F} + \cdots\right)$

Einstein + scalar $d = 5$ maximal supergravity action:

$$S = \frac{1}{4\pi G_5} \int d^5x \left[\frac{1}{4} R - \frac{1}{2} \sum_i (\partial X_i)^2 - V(X_i) \right].$$

- $V(X_i)$ has a local maximum at $X_i = 0 \Rightarrow AdS_5$ solution dual to $\mathcal{N} = 4$ SYM
- Classical solutions with $X_i \neq 0$ are dual to $\mathcal{N} = 4$ with relevant deformations or vevs
- If the classical solution ends at a different critical point of $V(X_i)$, there is an IR fixed point (not necessarily stable)
- Otherwise the solution is singular and the IR theory is not conformal

Dual to holographic flows

Conformal invariance is broken

- Explicit: mass terms for components of chiral multiplets
- Supersymmetry broken generically
- Two chiral multiplets with the same mass: $\mathcal{N} = 2$ supersymmetry ($\mathcal{N} = 2^*$ SYM)
- Scalar operators have a vev depending on the mass
- Spontaneous: scalar operators acquire a vev but masses are zero

For $\mathcal{N} \geq 2$ the vev of scalar operators determine a point in Coulomb moduli space

Coulomb moduli space

Moduli space $\mathcal{N}=4$ theory: $[\Phi_i,\Phi_j]=0$, $\mathcal{M}=\mathbb{R}^{6N}/\mathcal{W}$

Parametrization with eigenvalues: $\Phi_i=\mathrm{diag}\left(\varphi_i^{(1)},\cdots,\varphi_i^{(N)}\right)$

- Large-N limit: distribution of eigenvalues on $\mathbb{R}^6$

$$\sigma(\vec{y})=\frac{1}{N}\sum_{a=1}^{N}\delta(\vec{y}-\vec{\varphi}^{(a)})$$

- $\sigma(\vec{y})=$ distribution of D3 branes in transverse space
- Near-horizon limit:

$$ds^2=H^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu+H^{1/2}\sum_{i=1}^{6}(dy^i)^2,\quad H(\vec{y})=\int d^6w\,\frac{\sigma(\vec{w})}{|\vec{y}-\vec{w}|^4}$$

Holographic flows and Coulomb moduli space

$\mathcal{N} = 4$ **theory:**

- Eigenvalue distribution σ_n, $5 \geq n \geq 1$ has support on n-dimensional ball of radius Λ
- Symmetry: $SO(6) \to SO(n) \times SO(6-n)$
- Typical distance between eigenvalues: $\Delta\varphi \sim \Lambda/N^{1/n}$

$\mathcal{N} = 2^*$ **theory:**

- Two complex scalars massive: moduli space reduced to $\mathcal{M} = \mathbb{C}^N/\mathcal{W}$
- Distribution of eigenvalues on a disc: $\Delta\varphi \sim \Lambda/\sqrt{N}$
- Distribution of eigenvalues on a line: $\Delta\varphi \sim \Lambda/N$

Eigenvalues on a line: Wigner's semicircle distribution

Singularities in holographic flows

Gubser's classification of singularities: *the good, the bad and the naked*

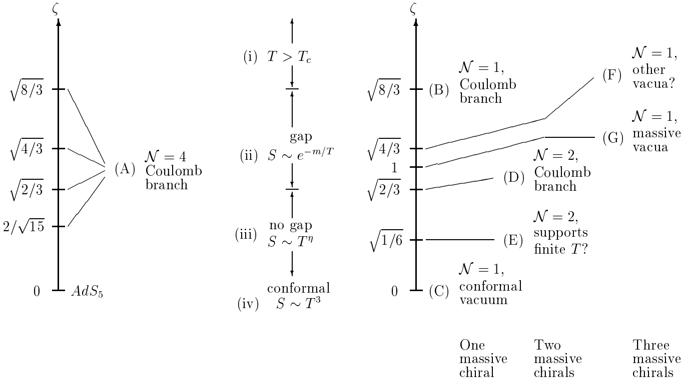
- (Zero temperature) solutions

$$ds^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + dr^2, \quad X_i = X_i(r).$$

- Asymptotic AdS_5 boundary: as $r \to \infty$, $A(r) \simeq r/L$, $X_i \simeq$ constant
- Null energy condition: $A''(r) \leq 0$ (analogous to "c-theorem")
- Singularities: $A(r) \to -\infty$ at $r = r_0$

$$e^{2A} \simeq (r - r_0)^{\frac{4}{3\sigma^2}}, \quad \sigma \leq \sqrt{\frac{8}{3}}$$

Thermodynamic behavior of singularities

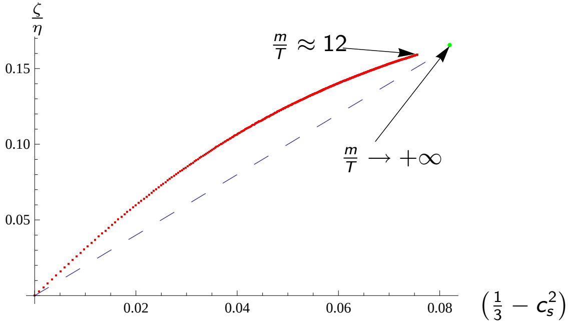


from hep-th/0002160

$$S \sim T^{\beta}, \quad \beta = \frac{6}{2-3\sigma^2},$$

σ	0	$1/\sqrt{6}$	$2/\sqrt{15}$
β	3	4	5

Buchel's calculation of speed of sound and bulk viscosity:



from 0708.xxxx

$$T \to 0: \; S \sim T^4, \; c_s^2 \simeq \frac{1}{4}, \; \frac{\zeta}{\eta} \simeq 2\left(\frac{1}{3} - c_s^2\right)$$

Bulk viscosity = reduction from 5d CFT [Buchel, Skenderis]

Conjecture

Given a linear distribution of eigenvalues in the Coulomb moduli space, at low T (large-N, strongly coupled) $\mathcal{N} = 2^*$ SYM flows to an effective 5d CFT, while $\mathcal{N} = 4$ SYM flows to a 6d CFT.

The metric dual to the of $SO(5)$ symmetric configuration is

$$ds^2_{4,1} = (gr)^2 H^{1/6} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{dr^2}{(gr)^2 H^{1/3}}, \quad H = 1 + \frac{\ell^2}{r^2}$$

Near-horizon limit $u^2 = 1/(g^2 \ell r) \to \infty$

$$ds^2_{4,1} \simeq \frac{(g\ell u)^{-4/3}}{u^2} \left[\eta_{\mu\nu} dx^\mu dx^\nu + \frac{4}{g^2} du^2 \right]$$

Scalar potential

$$V \simeq -\frac{15g^2}{2} X^2, \quad X \sim 2(g\ell u)^{2/3}$$

Proof for $\mathcal{N} = 4$ dual

Define

$$ds^2_{6,1} = e^{-2\phi} ds^2_{4,1} + e^{3\phi}(dy_1^2 + dy_2^2),$$

Then, with

$$X = e^{-\phi}, \quad \tilde{u} = 2^{3/2} g\ell u, \quad \tilde{x}^\mu = \sqrt{2} g\ell x^\mu .$$

we have an AdS_7 metric

$$ds^2_{6,1} = \frac{1}{\tilde{u}^2}\left[\eta_{\mu\nu} d\tilde{x}^\mu d\tilde{x}^\nu + \delta_{ab} dy^a dy^b + \frac{4}{g^2} d\tilde{u}^2\right]$$

and an uplifted action with scalar potential

$$V_{d=7} = -\frac{15g^2}{2}$$

Same as $d = 7$ $\mathcal{N} = 2$ SUGRA!

Uplift to $d = 11$ SUGRA $\Rightarrow$ near-horizon limit of a stack of M5 branes

Cvetic, Gubser, Lu, Pope '00

Proof for $\mathcal{N} = 2^*$ dual

The metric of duals to configurations in the Coulomb branch of $\mathcal{N} = 2^*$ SYM is [Pilch,Warner]

$$\begin{aligned} ds^2_{4,1} &= \frac{4}{g^2}\frac{dc^2}{\rho^8(c^2-1)^2} + k^2\frac{\rho^4}{c^2-1}\eta_{\mu\nu}dx^\mu dx^\nu \\ \rho^6 = e^{6\alpha} &= c + (c^2-1)\left[\gamma + \frac{1}{2}\log\left(\frac{c-1}{c+1}\right)\right] \\ c &= \cosh(2\chi) \end{aligned}$$

At low temperatures the geometry approaches the $\gamma = 0$ geometry (enhançon)
In the near-horizon limit $u \to \infty$

$$e^{2\chi} \simeq 2u, \quad e^{6\alpha} \simeq 2/(3u), \quad e^A \simeq 2^{1/3}ku^{-4/3}/3^{1/3}$$

Then, the metric becomes

$$ds^2_{4,1} \simeq \left(\frac{3}{2}\right)^{4/3} u^{-8/3}\left[\frac{4}{g^2}du^2 + \left(\frac{2k}{3}\right)^2 \eta_{\mu\nu}dx^\mu dx^\nu\right]$$

Proof for $\mathcal{N} = 2^*$ dual

Define

$$ds_{5,1}^2 = e^{-2\phi_2} ds_{4,1}^2 + e^{6\phi_2} dx_6^2 , \tag{1}$$

Then, with $\phi_1 = \frac{1}{2}(3\alpha + \chi)$, $\phi_2 = \frac{1}{2}(\alpha - \chi)$, $\phi_1 = -\phi + \log(4/3)/4$ we have an AdS_6 metric

$$ds_{5,1}^2 = \frac{3^{3/2}}{2u^2} \left[\frac{4}{g^2} du^2 + \left(\frac{2k}{3} \right)^2 \eta_{\mu\nu} dx^\mu dx^\nu + \frac{1}{9} dx_6^2 \right] .$$

and an uplifted action with scalar potential ($g_6 = 3m$ and $g_6^2 = \sqrt{3} g^2$)

$$V_{d=6}(\phi) = -\frac{1}{8} \left[g_6^2 e^{2\phi} + 4 g_6 m e^{-2\phi} - m^2 e^{-6\phi} \right]$$

Same as maximally supersymmetric solution of $d = 6$ $F(4)$ SUGRA!

Near-horizon limit of D4/D8/O8 intersection in type IIA Cvetic, Gubser, Lu, Pope '00

Large-N equivalences for IR theories

$\mathcal{N} = 4$: $(2,0)$ theory on M5 brane (6d SCFT)

$\mathcal{N} = 2^*$: $E_1 = SU(2)$ theory on D4/D8/O8 (actually D4/O8) intersection (5d SCFT)

It turns out we can understand why the 5d CFT should be equivalent to a D4/D8/O8 intersection

A large-N equivalence for $\mathcal{N}=4$

- Holographic dual to $\mathcal{N}=4$ SYM: $AdS_5\times S^5$
- Supersymmetric orientifold projection: $AdS_5\times S^5\to AdS_5\times S^5/\mathbb{Z}_2$+O7 plane+$N_f=4$ D7 branes
- $SU(4)_R\to SU(2)_R\times SU(2)$
- Same geometry: correlation functions in the common sector are the same $\Rightarrow$ large-N equivalence
- Holographic dual to orientifold: $\mathcal{N}=2$ $USp(N)$ SYM+antisymmetric hypermultiplet+$N_f=4$ fundamental hypermultiplets
- Fundamental hypermultiplets become massive in the Coulomb branch

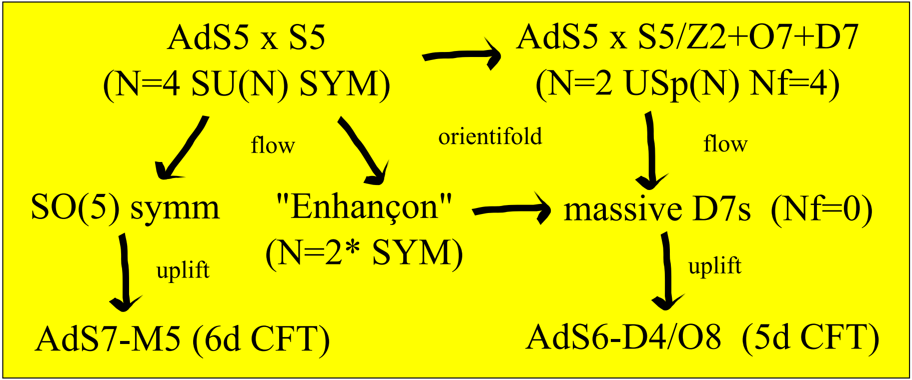
$$W=Q_i X\tilde{Q}_i$$

A large-N equivalence for $\mathcal{N} = 2^*$

- Mass in $\mathcal{N} = 4$ SYM for adjoint hypermultiplet ($\mathcal{N} = 2^*$ SYM)= Mass in orientifold theory for antisymmetric hypermultiplet

$$(\mathbf{1}, \mathbf{1})_0 \subset \mathbf{20}', \quad (\mathbf{3}, \mathbf{1})_{-2} \subset \mathbf{10}$$

- $SU(4)_R \to SU(2)_R \times U(1)$ or $SU(2)_R \times SU(2) \to SU(2)_R \times U(1)$
- Holographic dual of $\mathcal{N} = 2^*$ can be interpreted as $AdS_5 \times S^5/\mathbb{Z}_2$+O7+$N_f = 4$ *massive* D7 branes
- T-dual version: D4/D8/O8



What is the meaning of the uplift in the field theory?

Deconstruction

Let us start with the $\mathcal{N} = 4$ theory...

- Wigner's semicircle distribution of eigenvalues $\varphi \in (-\Lambda, \Lambda)$

$$\rho(\varphi) = \frac{2N}{\pi\Lambda^2}\sqrt{\Lambda^2 - \varphi^2}$$

- Change variables $\varphi = \Lambda x/N$, $x \in (-N, N)$

$$\rho(x) = \frac{2}{\pi}\sqrt{1 - \frac{x^2}{N^2}}$$

- Large-N limit: $x \in (-\infty, \infty)$

$$\rho(x) = \frac{2}{\pi}$$

Deconstruction

- Keep $N \gg 1$ finite but focus around $\varphi = 0$
- Separation between eigenvalues: $\Delta\varphi = \frac{\Lambda}{2}\left(\frac{\pi}{N}\right)$
- $U(1)^{(N-1)}$ gauge theory with 1/2 BPS vector multiplets of mass

$$m_n = \frac{g_{YM}\Lambda}{2}\left(\frac{\pi}{N}\right), \quad n = 1, 2, 3, \ldots$$

- same spectrum as tower of Kaluza-Klein modes (up to $n \sim O(N)$)
- Effective length

$$L_5 = \frac{4N}{g_{YM}\Lambda} \sim N^{3/2}(\lambda_{YM})^{-1/2}\Lambda^{-1}$$

- $SL(2,\mathbb{Z})$ symmetry of $\mathcal{N} = 4$: magnetically charged states with mass

$$M_n = \frac{\Lambda}{2g_{YM}}\left(\frac{\pi}{N}\right), \quad n = 1, 2, 3, \ldots$$

- KK modes for a circle of effective length

$$L_6 = \frac{4g_{YM}N}{\Lambda} \sim N^{1/2}(\lambda_{YM})^{1/2}\Lambda^{-1}$$

Deconstruction

- Dyonic states: KK momentum along both circles
- Spectrum of BPS states: six-dimensional theory compactified on a torus with 16 supercharges
- BPS spectrum of $(2,0)$ theory on *single* M5 brane Arkani-Hamed, Cohen, Kaplan, Karch
- $N \gg 1$ implies $L_5 \gg L_6 \gg \Lambda^{-1}$

Deconstruction

For the $\mathcal{N} = 2^*$ theory

- Same kind of arguments go through ($\Lambda = m$)
- There is no $SL(2, \mathbb{Z})$ duality: only one additional extra dimension
- Five-dimensional theory with 8 supercharges and coupling

$$g_5^2 = g_{YM}^2 L_5 = \frac{4 g_{YM} N}{km} \to \infty$$

- If g_5 is the bare coupling, there should be a conformal fixed point in this limit [Seiberg '96]
- No flavor: $E_1 = SU(2)$ 5d SCFT

Thank you!