Some extentions of the AdS/CFT correspondence

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Introduction

There are many ways to modify the original AdS/CFT correspondence. Here I will talk about two examples.

Example 1: Keeping the theory conformal but modifying 3-pt functions of T_{ab}. Achieved by considering higher derivative gravity in the bulk.

Example2: Introducing small number of defect fermions interacting with CFT. Leads to tachyon condensation in the bulk.

Outline

Introduction to higher derivative gravities

Gauss-Bonnet gravity Lagrangian:

$$\mathcal{L} = R + \frac{6}{L^2} + \lambda L^2 (R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd})$$

We will be interested in the $\lambda \sim 1$ regime.

More generally, we can add terms $\mathcal{O}(R^k)$ which are Euler densities in $2k$ dimensions:

$$\lambda_k L^{2k-2} \delta^{a_1 \ldots b_k}_{c_1 \ldots d_k} R^{c_1 d_1}{}_{a_1 b_1} \ldots R^{c_k d_k}{}_{a_k b_k}$$

They become non-trivial for gravity theories in AdS_D with $D > 2k$.

Special properties

- Equations of motion don't contain 3rd order derivatives g'''
- Metric and Palatini formulations are equivalent
- No ghosts around flat space
- Exact black hole solutions can be found

The last property allows one to study dual CFTs at finite temperature.
The first property implies that holographic dictionary is not modified.

Implications for AdS/CFT

The UV behavior is modified. Consider conformal rescaling

$$g_{ab}^{CFT} \to \exp(2\sigma)\, g_{ab}^{CFT}$$

CFT action is anomalous; there are two terms in 3+1 dimensions:

$$\delta\mathcal{W} = aE_4 + cW^2$$

Enstein-Hilbert (E-H) implies $a = c$. Lovelock implies $a \neq c$. Of course, the IR behavior is modified as well. E.g. values of transport coefficients are different from E-H.

Finite T; metastable states

Consider propagation of gravitons in the black hole background.

(D=5 Gauss-Bonnet: Brigante, Liu, Myers, Shenker, Yaida)

$$ds^2 = -\frac{f(r)}{\alpha}dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2}\left(\sum dx_i^2 + 2\phi(t,r,z)dx_1 dx_2\right)$$

Fourier transform: $\phi(t,r,z) = \int dwdqexp(-iwt + iqz)$
After substitutions and coordinate transformations, get
Schrodinger equation with $\hbar \to 1/\tilde{q} = T/q$:

$$-\frac{1}{\tilde{q}^2}\partial_y^2\Psi(y) + V(y)\Psi(y) = \frac{w^2}{q^2}\Psi(y)$$

Causality

Spectrum = states in finite T CFT.
In the $\tilde{q} \gg 1$ regime, there are stable states with $\partial w/\partial q > 1$ in some region of the parameter space. Causality places constraints on Lovelock couplings:

$$\sum_k [(d-2)(d-3) + 2d(k-1)]\lambda_k \alpha^{k-1} < 0$$

where α defines the AdS radius $L_{AdS}^2 = L^2/\alpha$ and satisfies $\sum_k \lambda_k \alpha^k = 0$.

This effect is absent at $T = 0$; appears as $\mathcal{O}(T/q)$ correction from the tails of black hole metric.

Positivity of energy flux = unitarity

Define $\varepsilon(\hat{n}) = \lim_{r\to\infty} r^2 \int dt T^0_i \hat{n}_i$
Conjecture (Hofman, Maldacena) $\langle\varepsilon(\hat{n})\rangle \geq 0$.
Consider a state created by $\epsilon^{ij} T_{ij}$

$$\langle\varepsilon(\hat{n})\rangle \sim 1 + t_2\left(\frac{\epsilon_{ij}\epsilon^{il}\hat{n}^j\hat{n}_l}{\epsilon_{ij}\epsilon^{ij}} - \frac{1}{d-1}\right) + t_4\left(\frac{(\epsilon^{ij}\hat{n}^i\hat{n}_j)^2}{\epsilon_{ij}\epsilon^{ij}} - \frac{2}{d^2-1}\right)$$

t_2 and t_4 are determined by the 2 and 3-point functions of T_{ab}.

- energy flux positivity in CFTs dual to Lovelock gravities is equivalent to causality at finite temperature!
- also equivalent to the absence of ghosts (at finite temperature).

Holographic entanglement entropy

Consider two systems A, B with Hilbert spaces consisting of two states $\{|1\rangle, |2\rangle\}$. Reduced density matrix of A is obtained by tracing over B; entanglement entropy is the resulting VN entropy.

$$\rho_A = \mathrm{tr}_B \rho; \qquad S_A = -\mathrm{tr}_A\, \rho_A \log \rho_A$$

Product state:

$$|1_A 1_B\rangle \Rightarrow S_A = 0$$

Pure (non product) state:

$$\frac{1}{\sqrt{2}} (|1_A 2_B\rangle - |2_A 1_B\rangle) \Rightarrow S_A = \ln 2$$

Holographic EE

Consider EE in CFT dual to Lovelock gravity in AdS. A proposal for holographic EE (Fursaev).

$$S(V) = \frac{1}{G_N^{(5)}} \int_\Sigma \sqrt{\sigma} \left(1 + \lambda_2 L^2 R_\Sigma\right)$$

Σ is the minimal surface ending on (∂V) which satisfies the e.o.m. derived from this action. R_Σ is the induced scalar curvature on Σ. Consider the case of a ball, bounded by the two-sphere of radius R. It is not hard to solve EOM near the boundary of AdS and extract the log-divergent term:

$$S(B) = \frac{R^2}{\epsilon^2} + \frac{a}{90} \ln R/\epsilon + \dots$$

a-theorem and AdS/CFT

Zamolodchikov's c-theorem in 1+1 dimensions: there is a c-function [made out of $\langle T_{ab} T_{ab} \rangle$] which is

- positive
- decreases along the RG flows
- equal to the central charge c at fixed points

Is there an analogous quantity in 3+1 dimensions? Conjecture: yes, and it is equal to a at fixed points.

Holographic a-theorem (Myers, Sinha) Consider a background which holographically describes the RG flow:

$$ds^2 = \exp(2A(r))(dx_\mu)^2 + dr^2$$

a-theorem and AdS/CFT

Then the quantity

$$a(r) = \frac{1}{l_p^3 A'(r)^3}\left(1 - 6\lambda L^2 A'(r)^2\right)$$

is equal to a at fixed points and satisfies

$$a'(r) = -\frac{1}{l_p^3 A'(r)^4}\left(T_0^0 - T_r^r\right)$$

The right hand side of this equation is proportional to the null energy condition.

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- Phenomenological applications in AdS/QCD, AdS/CMT and AdS/???
- Can we find a field theory dual to HDG? Can we find string theory which reduces to HDG?

Introduction: NJL model

Consider the following NJL model with the UV cutoff Λ:

$$\mathcal{L}=\bar{\psi}^i\gamma^\mu\partial_\mu\psi_i+\frac{g^2}{N}(\bar{\psi}^i\psi_i)^2=\bar{\psi}^i\gamma^\mu\partial_\mu\psi_i-\sigma\bar{\psi}^i\gamma^\mu\partial_\mu\psi_i-\frac{N\sigma^2}{2g^2}$$

Integrating out fermions produces effective potential for σ:

$$\frac{1}{N}V_{eff}=\frac{N\sigma^2}{2g^2}+\mathrm{tr}\log(\gamma^\nu\partial_\mu+\sigma);\ \frac{V'_{eff}}{N}=\frac{\sigma}{g^2}-\frac{\sigma}{\pi^2}\left(\Lambda-\sigma\arctan(\frac{\Lambda}{\sigma})\right)$$

For $g^2>g_c^2=\pi^2/\Lambda$, there is a mass gap $M\sim(1/g_c^2-1/g^2)$.

Introduction: D3/D7

Consider D3/D7 system with N D3 branes stretched along 0123 directions and a D7 brane stretched along 012 45678.

At energies $\ll 1/l_s$ the lagrangian is $U(N)$ $\mathcal{N} = 4$ SYM coupled to 3d dirac fermion.

At large t'Hooft coupling λ the dynamics of the matter is described by the DBI action for a D7 brane propagating in $AdS_5 \times S^5$. Note that the ground state of the fermionic system is described in terms of bosonic variables.

DBI action and EOM

At large t'Hooft coupling $\lambda \gg 1$ we are instructed to consider $AdS_5 \times S^5$ space $[r^2 = \rho^2 + (x^9)^2]$

$$ds^2 = r^2 dx_\mu dx^\mu + r^{-2} \left(d\rho^2 + \rho^2 d\Omega_4^2 + (dx^9)^2 \right)$$

The D7 brane embedding is specified by $x^9 = f(\rho)$, giving the DBI action

$$S_{D7} \sim \int d^3x \int d\rho \frac{\rho^4}{\rho^2 + f(\rho)^2} \sqrt{1 + f'(\rho)^2}$$

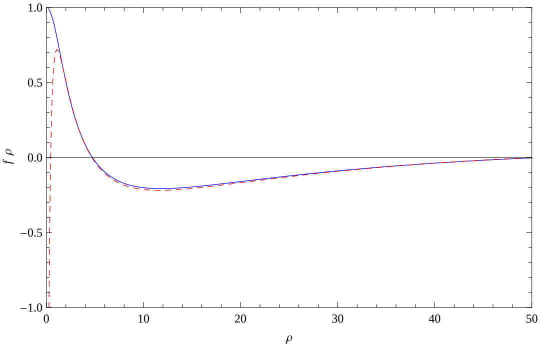
EOM for $f(\rho)$ are second order, non-linear differential equation. In the UV region, it gets linearized and can be solved.

Solutions of EOM

Namely, for $\rho \gg f(\rho)$,

$$f(\rho) \approx A\mu^{3/2}\rho^{-1/2}\sin(\frac{\sqrt{7}}{2}\log\rho/\mu + \varphi)$$

One can also solve the full EOM numerically starting from $f'(0) = 0$.



Solutions of EOM

To proceed, impose Dirichlet boundary conditions at the UV cutoff Λ. Solutions are labeled by the number of nodes in $(0, \Lambda)$.

In particular, $f_0(0) \sim \Lambda$. One can study spectrum of excitations around $f_n(\rho)$. There are n tachyons.

Solution $f_0(\rho)$ is energetically preferred and does not have tachyons living on it. Masses of excited states scale $\sim \Lambda$.

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- Interestingly, solution of SD equation in the rainbow approximation gives fermionic self-energy which has the same functional form as $f(\rho)$...
- It is desirable to separate the scales of physical masses and Λ. Work in progress...