

Flux tubes, domain walls and orientifold planar equivalence

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CERN

GGI, 5 May 2011

Orientifold planar equivalence

OrQCD

$SU(N)$ gauge theory ($\lambda = g^2 N$ fixed) with N_f Dirac fermions in the antisymmetric representation

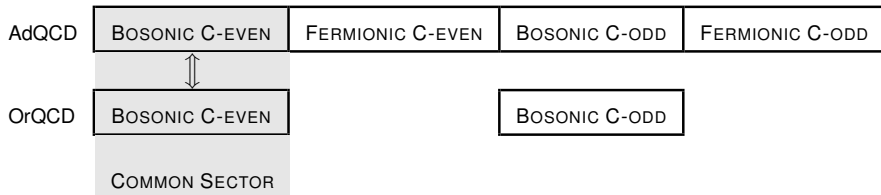
is equivalent in the large- N limit and in a common sector to

AdQCD

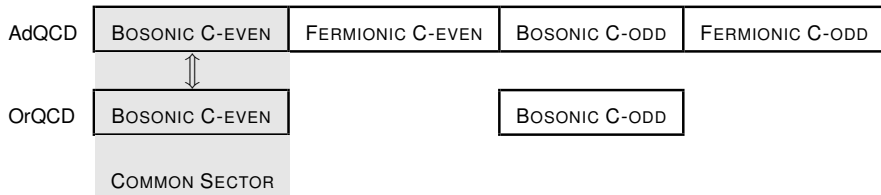
$SU(N)$ gauge theory ($\lambda = g^2 N$ fixed) with N_f Majorana fermions in the adjoint representation

if and only if C-symmetry is not spontaneously broken.

Gauge-invariant common sector



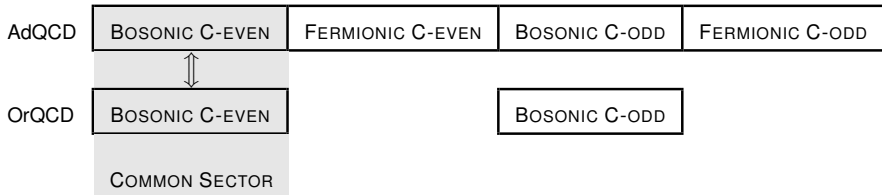
Gauge-invariant common sector



$$e^{-N^2 W(J)} = \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left\{ -S(A, \psi, \bar{\psi}) + N^2 \int J(x) O(x) d^4x \right\}$$

$$\lim_{N \rightarrow \infty} W_{Or}(J) = \lim_{N \rightarrow \infty} W_{Ad}(J)$$

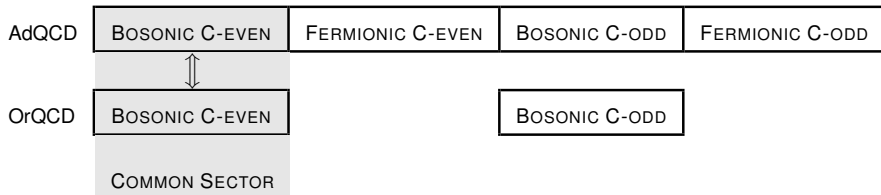
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$$\lim_{N \rightarrow \infty} \frac{\langle O(x_1) \cdots O(x_n) \rangle_{c, Or}}{N^{2-2n}} = \lim_{N \rightarrow \infty} \frac{\langle O(x_1) \cdots O(x_n) \rangle_{c, Ad}}{N^{2-2n}}$$

Gauge-invariant common sector

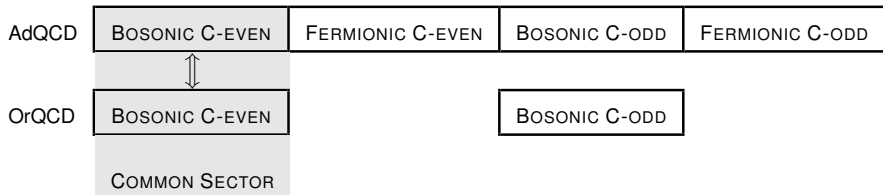


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$$\lim_{N \rightarrow \infty} \langle 0|O|0 \rangle_{Or} = \lim_{N \rightarrow \infty} \langle 0|O|0 \rangle_{Ad}$$

Gauge-invariant common sector

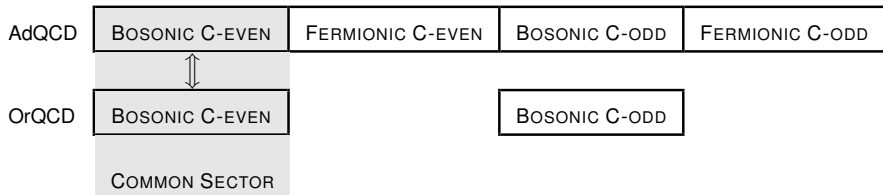


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Gauge-invariant common sector



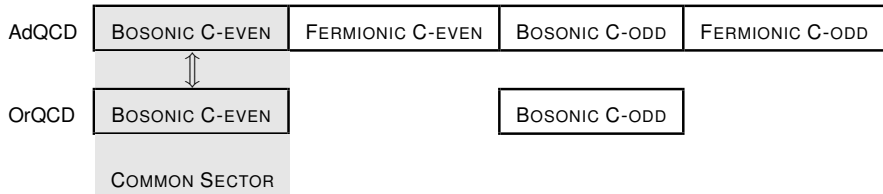
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$$\lim_{N \rightarrow \infty} \langle a | O | b \rangle_{Or} = \lim_{N \rightarrow \infty} \langle a | O | b \rangle_{Ad}$$

$$\lim_{N \rightarrow \infty} \langle a | e^{-tH} | b \rangle_{Or} = \lim_{N \rightarrow \infty} \langle a | e^{-tH} | b \rangle_{Ad}$$

Gauge-invariant common sector



$$e^{-N^2 W(J)} = \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp \left\{ -S(A, \psi, \bar{\psi}) + N^2 \int J(x) O(x) d^4x \right\}$$

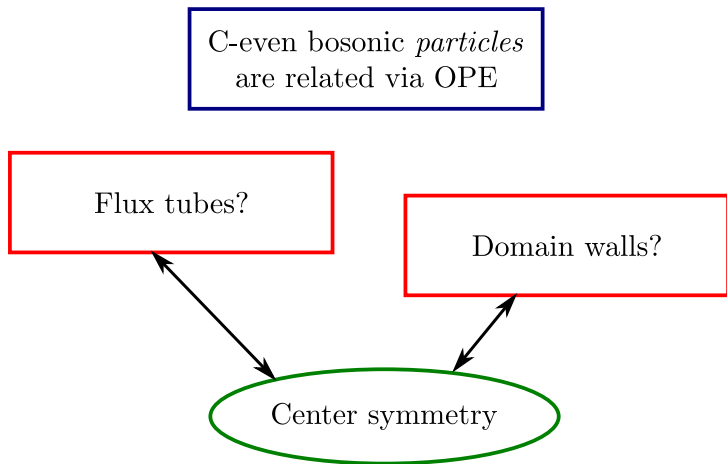
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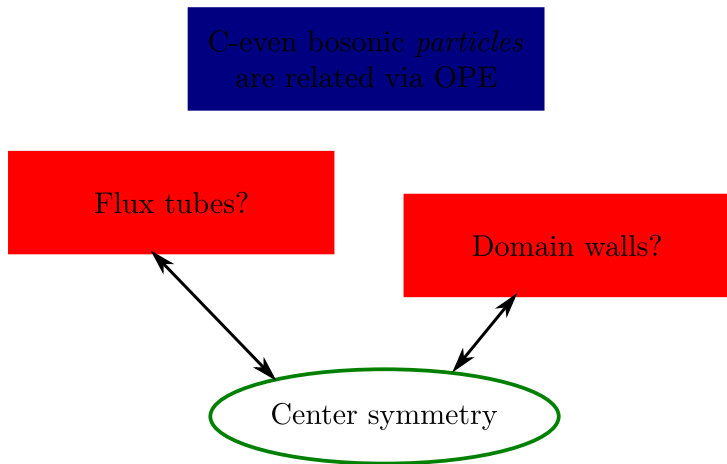
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Symmetries **inside** the common sector are the same in AdjQCD and OrientiQCD.

Overview



Overview



Symmetry (mis)matching

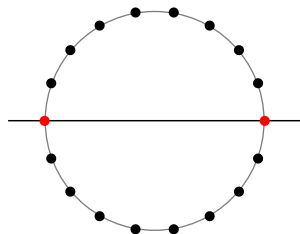
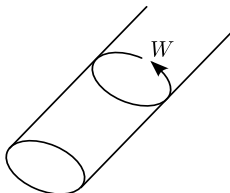
A center transformation around a compact dimension \hat{z} is a local gauge transformation, that is periodic modulo an element of the center Z_N .

$$A_\mu(x) \rightarrow \Omega(x)A_\mu(x)\Omega^\dagger(x) + i\Omega(x)\partial_\mu\Omega^\dagger(x)$$

$$\psi(x) \rightarrow R[\Omega(x)]\psi(x)$$

$$\Omega(x + L\hat{z}) = \Omega(x)e^{\frac{2\pi ik}{N}}$$

$$\text{tr } W \rightarrow e^{\frac{2\pi ik}{N}} \text{tr } W$$



Even- N OrQCD: Z_2

Odd- N OrQCD: $-$

AdQCD: Z_N

Only Z_2 maps the common sector into itself

OPE \Rightarrow OrQCD($N = \infty$) has at least a Z_2 symmetry

Symmetry (mis)matching

Z_N symmetry

$$\langle \text{tr } W \rangle = 0$$

$$\langle \text{tr } (W^2) \rangle = 0$$

$$\langle \text{tr } (W^3) \rangle = 0$$

\vdots

$$\langle \text{tr } (W^N) \rangle \neq 0$$

Symmetry (mis)matching

Z_N symmetry

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Symmetry (mis)matching

Z_N symmetry

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Z_2 symmetry

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\vdots

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OrQCD($N = \infty$)

$$\frac{1}{N} \langle \text{tr } W \rangle_{Or} = \frac{1}{N} \langle \text{tr } W \rangle_{Adj} = 0$$

$$\frac{1}{N} \langle \text{tr } (W^2) \rangle_{Or} = \frac{1}{N} \langle \text{tr } (W^2) \rangle_{Adj} = 0$$

$$\frac{1}{N} \langle \text{tr } (W^3) \rangle_{Or} = \frac{1}{N} \langle \text{tr } (W^3) \rangle_{Adj} = 0$$

\vdots

Is Z_N a symmetry for OrQCD($N = \infty$)?

A quantum mechanical analogy

$$H = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2$$

T_ℓ is an operator with defined angular momentum ℓ

$$\langle 0|T_0|0\rangle \neq 0$$

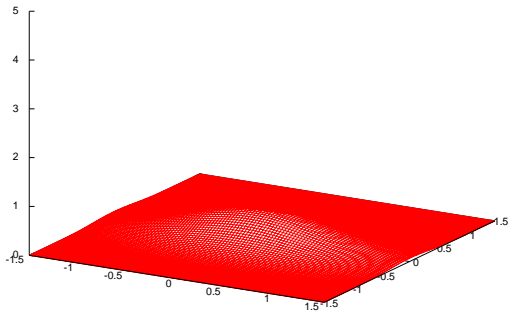
$$\langle 0|T_1|0\rangle = 0$$

$$\langle 0|T_2|0\rangle \neq 0$$

...

A quantum mechanical analogy

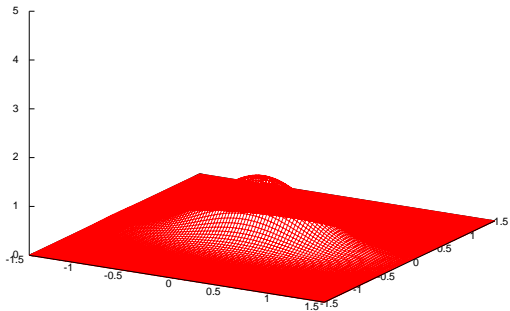
$$H = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2$$



Vacuum probability distribution in coordinate-space for $\hbar = 1$

A quantum mechanical analogy

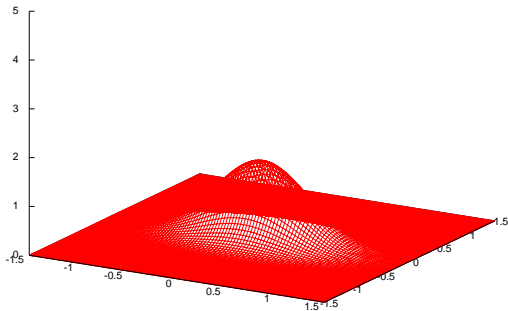
$$H = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2$$



Vacuum probability distribution in coordinate-space for $\hbar = 0.5$

A quantum mechanical analogy

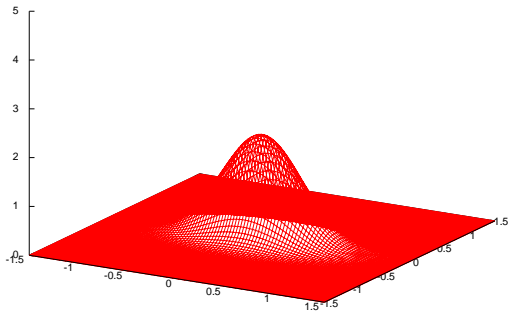
$$H = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2$$



Vacuum probability distribution in coordinate-space for $\hbar = 0.4$

A quantum mechanical analogy

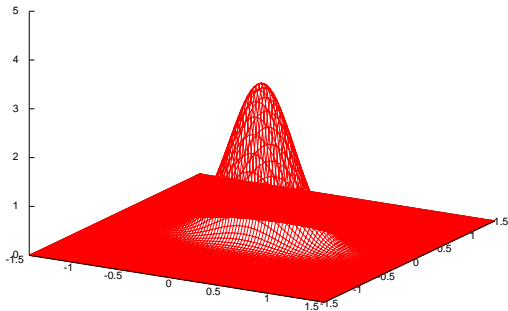
$$H = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2$$



Vacuum probability distribution in coordinate-space for $\hbar = 0.3$

A quantum mechanical analogy

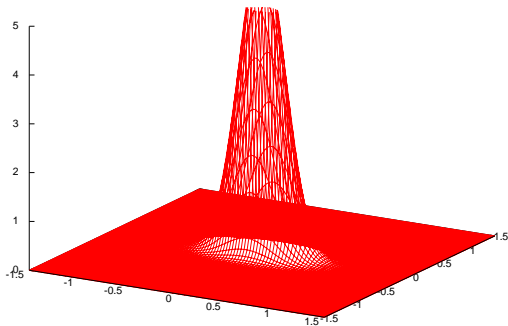
$$H = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2$$



Vacuum probability distribution in coordinate-space for $\hbar = 0.2$

A quantum mechanical analogy

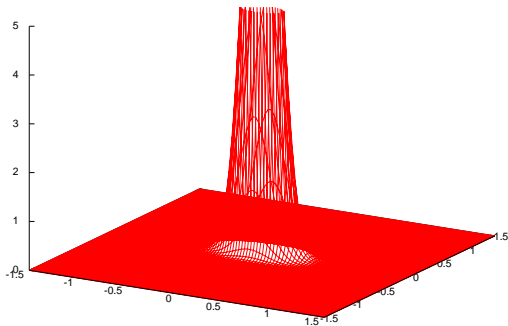
$$H = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2$$



Vacuum probability distribution in coordinate-space for $\hbar = 0.1$

A quantum mechanical analogy

$$H = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2$$



Vacuum probability distribution in coordinate-space for $\hbar = 0.05$

A quantum mechanical analogy

$$H = \frac{p_x^2 + p_y^2}{2m} + \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2$$

$$\lim_{\hbar \rightarrow 0} |\psi_0(x)|^2 = \delta^2(\mathbf{r})$$

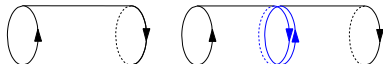
$$\lim_{\hbar \rightarrow 0} \langle 0 | T_\ell | 0 \rangle = 0 \quad \text{for } \ell \neq 0$$

The vacuum is invariant under rotations, but the Hamiltonian is not!

Two-point functions of Polyakov loops

AdQCD

$$\lim_{N \rightarrow \infty} \langle P(x)P^\dagger(y) \rangle_c \neq 0$$



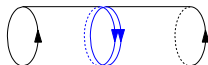
$$\langle P(x)P(y) \rangle_c = 0$$

OrQCD

$$\lim_{N \rightarrow \infty} \langle P(x)P^\dagger(y) \rangle_c \neq 0$$

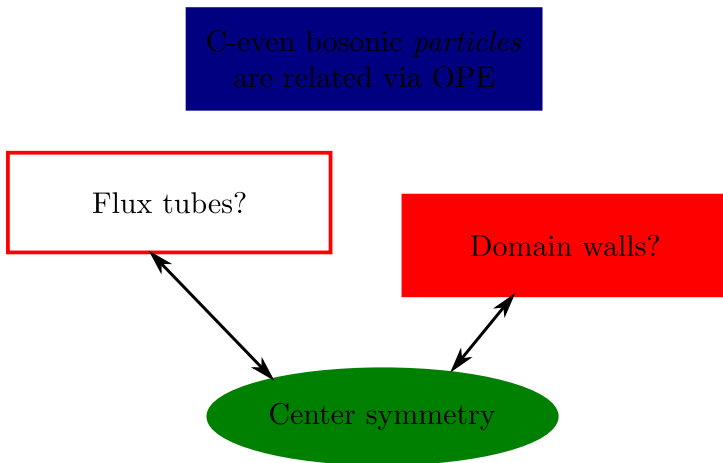


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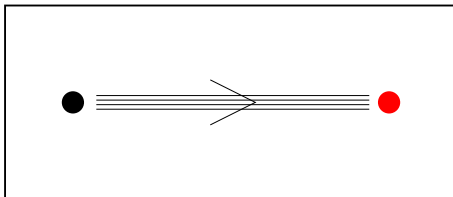


$$\begin{aligned} \langle \text{Re}P(0)\text{Re}P(x) \rangle_{c,Or} &= \langle \text{Re}P(0)\text{Re}P(x) \rangle_{c,Ad} \\ \langle P(0)P(x) \rangle_{c,Or} + \langle P(0)P(x)^\dagger \rangle_{c,Or} &= \langle P(0)P(x)^\dagger \rangle_{c,Ad} \end{aligned}$$

Overview

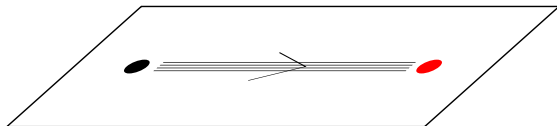


External charges



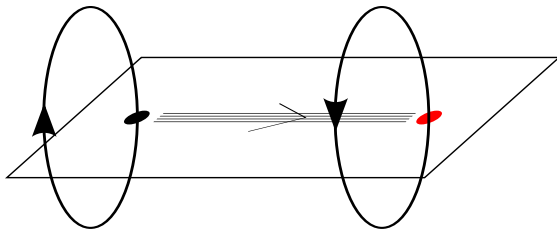
$$G^A(x)|open - string\rangle \equiv \left[\frac{1}{g^2} D_i E_i^A(x) - \psi^\dagger T_{\mathcal{R}}^A \psi(x) \right] |open - string\rangle = \rho_{ext}^A(x) |open - string\rangle$$

External charges



$$G^A(x)|open - string\rangle \equiv \left[\frac{1}{g^2} D_i E_i^A(x) - \psi^\dagger T_{\mathcal{R}}^A \psi(x) \right] |open - string\rangle = \rho_{ext}^A(x) |open - string\rangle$$

External charges



$$\begin{aligned}
 \langle P(\mathbf{x})P^\dagger(\mathbf{y}) \rangle_{YM} &\propto \text{partition function of YM coupled to a static quark in } \mathbf{x} \\
 &\quad \text{and a static antiquark in } \mathbf{y} = \\
 &= \sum_{\text{open-string states}} m_n e^{-\beta V_n(|\mathbf{x}-\mathbf{y}|)}
 \end{aligned}$$

$$G^A(x)|open-string\rangle \equiv \left[\frac{1}{g^2} D_i E_i^A(x) - \psi^\dagger T_{\mathcal{R}}^A \psi(x) \right] |open-string\rangle = \rho_{ext}^A(x) |open-string\rangle$$

Bosonic and fermionic open strings

$$\langle \text{Re}P(0)\text{Re}P(x) \rangle_{c,Or} = \langle \text{Re}P(0)\text{Re}P(x) \rangle_{c,Ad}$$

$$\langle P(0)P(x) \rangle_{c,Or} + \langle P(0)P(x)^\dagger \rangle_{c,Or} = \langle P(0)P(x)^\dagger \rangle_{c,Ad}$$

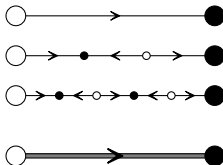
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bosonic oriented
 $\tilde{\sigma}_b$

Oriented bosonic open string in OrQCD



Bosonic and fermionic open strings

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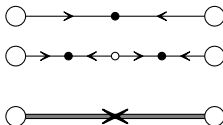
bosonic oriented

$\tilde{\sigma}_b$

fermionic unoriented

$\tilde{\sigma}_f$

Unoriented fermionic open string in OrQCD



Bosonic and fermionic open strings

$$\langle \text{Re}P(0)\text{Re}P(x) \rangle_{c,Or} = \langle \text{Re}P(0)\text{Re}P(x) \rangle_{c,Ad}$$

$\langle P(0)P(x) \rangle_{c,Or}$	+	$\langle P(0)P(x)^\dagger \rangle_{c,Or}$	=	$\langle P(0)P(x)^\dagger \rangle_{c,Ad}$
		bosonic oriented	=	bosonic oriented
		$\tilde{\sigma}_b$	=	σ_b
fermionic unoriented			=	fermionic oriented
$\tilde{\sigma}_f$			=	σ_f

Oriented open string in AdQCD



Bosonic and fermionic open strings

$$\langle \text{Re}P(0)\text{Re}P(x) \rangle_{c,Or} = \langle \text{Re}P(0)\text{Re}P(x) \rangle_{c,Ad}$$

$$\begin{array}{rclcl}
 \langle P(0)P(x) \rangle_{c,Or} & + & \langle P(0)P(x)^\dagger \rangle_{c,Or} & = & \langle P(0)P(x)^\dagger \rangle_{c,Ad} \\
 & & \text{bosonic oriented} & = & \text{bosonic oriented} \\
 & & \tilde{\sigma}_b & = & \sigma_b \\
 \text{fermionic unoriented} & & & = & \text{fermionic oriented} \\
 \tilde{\sigma}_f & & & = & \sigma_f
 \end{array}$$

Supersymmetry ($N_f = 1$)

- SUSY implies degeneracy between bosons and fermions.
- The external charges explicitly break SUSY.
- However SUSY breaking is a boundary effect.

$$\sigma_b = \sigma_f$$

Bosonic and fermionic string tensions in SYM

- The **Hamiltonian**.

$$H = \int \left[\frac{1}{2g^2} E_i^A E_i^A + \frac{1}{2g^2} B_i^A B_i^A + \frac{i}{2g^2} \bar{\lambda} \gamma_i D_i^{Adj} \lambda \right] d^3x$$

- The **supercharges** in the (on-shell) de Wit-Freedman formalism.

$$[S, A_i^A(x)] = \gamma_i \lambda^A(x)$$

$$\{S_\alpha, \lambda_\beta^A(x)\} = -\frac{1}{4} F_{\mu\nu}^A(x) [\gamma^\mu, \gamma^\nu]_{\alpha\beta}$$

$$[S, G^A(x)] = 0$$

$$\{S, \bar{S}\} = 2(\gamma^0 H - \gamma_k \Pi_k)$$

$$\sum_\alpha (S^\dagger)^\alpha S_\alpha = 4H$$

- The supercharges and the Hamiltonian do not commute in general.

$$[S, H] = \int \gamma^0 \lambda^A G^A d^3x$$

Bosonic and fermionic string tensions in SYM

$$\sum_{\alpha} (S^{\dagger})^{\alpha} S_{\alpha} = 4H \quad [S, H] = \int \gamma^0 \lambda^A G^A d^3x = \int \gamma^0 \lambda^A \rho_{ext}^A d^3x$$

- Consider the eigenstates in the string sector.

$$H |B, n\rangle = (\sigma^b R + O(R^0)) |B, n\rangle^b$$

$$H |F, n\rangle = (\sigma^f R + O(R^0)) |F, n\rangle^f$$

- Choose a bosonic state $|B, n\rangle$.

$$4\sigma^b R = \sum_{\alpha} \langle B, n | (S^{\dagger})^{\alpha} S_{\alpha} |B, n\rangle = \sum_{\alpha, n'} |\langle F, n' | S_{\alpha} |B, n\rangle|^2$$

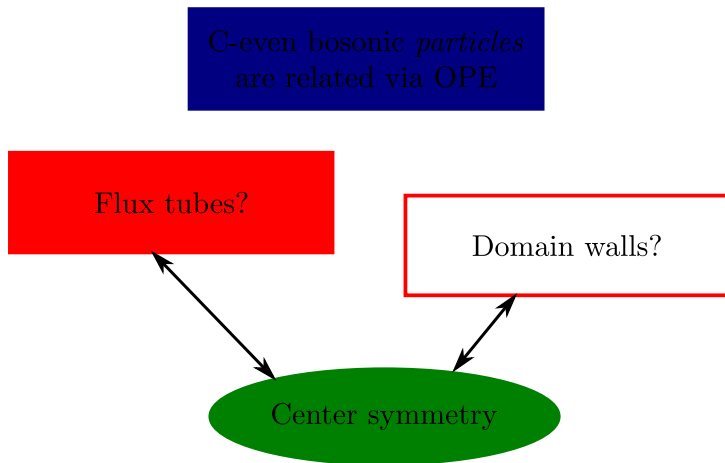
- At least one fermionic state $|F, n'\rangle$ and one index α exist with the following property:

$$\langle F, n' | S_{\alpha} |B, n\rangle = O(\sqrt{R})$$

- Computing the matrix element of $[S, H]$...

$$(\sigma^b - \sigma^f) \frac{\langle F, n' | S_{\alpha} |B, n\rangle}{R^{1/2}} + O\left(\frac{1}{R^{3/2}}\right) = \frac{1}{R^{3/2}} \langle F, n' | \int d^3x (\gamma_0 \lambda^A)_{\alpha} \rho_{ext}^A |B, n\rangle$$

Overview

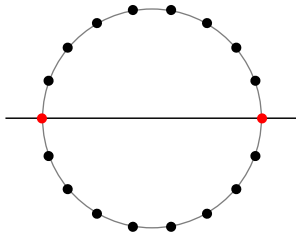


Deconfined phase

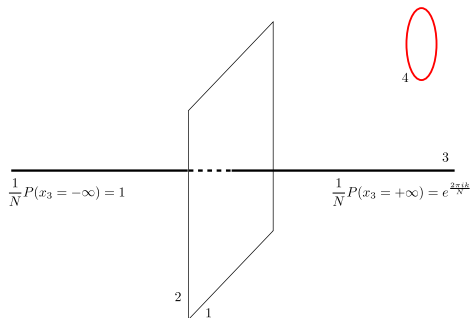
Effective potential for the Polyakov loop $W = e^{iv}\mathbf{1}_N$

$$\frac{V_{Ad}(e^{iv})}{N^2} = 0$$

$$\frac{V_{Or}(e^{iv})}{N^2} = \frac{\pi^2}{24\beta^4} - \frac{1}{24\pi^2\beta^4}(\pi^2 - [2v \bmod 2\pi]^2)^2$$



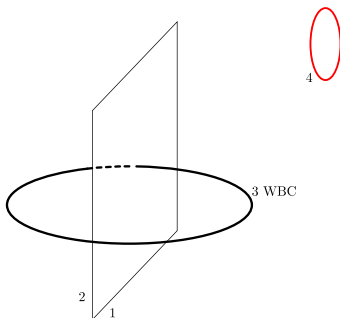
Domain wall, 't Hooft loop and Dirac string



$$\sigma(\beta) \propto k(N - k)$$

$$\text{for } k = \frac{N}{2} : \sigma(\beta) \propto N^2$$

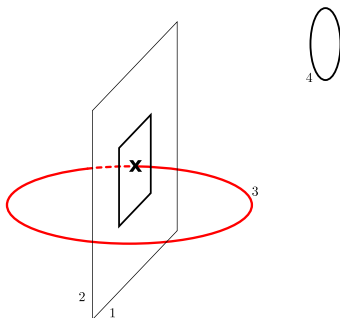
Domain wall, 't Hooft loop and Dirac string



$$\text{WBC: } P(x_1, x_2, x_3 + L) = e^{\frac{2\pi i k}{N}} P(x_1, x_2, x_3)$$

$$Z_{\text{wall}} = \lim_{L_3 \rightarrow \infty} \text{tr}_{L_3, \text{WBC}} \left[e^{-\beta H} \right] \propto e^{-L_1 L_2 \sigma(\beta)}$$

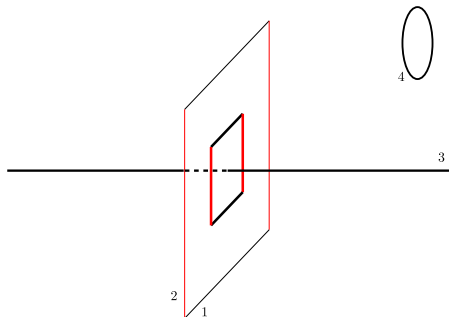
Domain wall, 't Hooft loop and Dirac string



$$Z_{\text{wall}} = \lim_{L_3 \rightarrow \infty} \text{tr}_{\beta, TBC} \left[e^{-L_3 H} e^{\frac{4\pi i}{g} \int \text{tr} E_4 Y_k dx_1 dx_2} \right] \propto e^{-L_1 L_2 \sigma(\beta)}$$

$$e^{2\pi i Y_k} = e^{\frac{2\pi i k}{N}}$$

Domain wall, 't Hooft loop and Dirac string

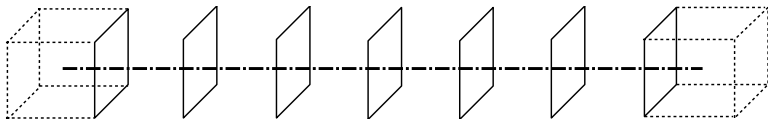


$$Z_{\text{wall}} = \langle 0 | e^{-L_2 H_{\text{Dirac}}(L_1, \beta)} | 0 \rangle \propto e^{-L_1 E_{\text{Dirac}}(L_2, \beta)} = e^{-L_1 L_2 \sigma(\beta)}$$

The operator $H_{\text{Dirac}}(L_1, \beta)/N^2$ is in the common sector and has a well defined large- N limit.

$$\lim_{N \rightarrow \infty} \frac{\sigma_{Or}(\beta)}{N^2} = \lim_{N \rightarrow \infty} \frac{\sigma_{Ad}(\beta)}{N^2}$$

Hamiltonian for the Dirac string



$$H_{Dirac} = \frac{a^2 N}{2\lambda} \sum_{\mathbf{x}} \text{tr} \vec{\mathcal{E}}(\mathbf{x})^2 + \frac{N}{2\lambda a} \sum_{\square} \Re \text{tr} (\mathbf{1} - z_{\square} U_{\square}) + H_F$$

$$z_{\square} = \begin{cases} -1 & \text{if } \square \text{ goes around the Dirac string} \\ 1 & \text{elsewhere} \end{cases}$$

Conclusions

- In the large- N limit, OrQCD has a Z_2 center symmetry. However its vacuum (in the confined phase) is symmetric under Z_N transformations.
- Orientifold planar equivalence holds in a larger sector than the particle sector.
- In the open-string sector, orientifold planar equivalence holds nontrivially both for bosonic and fermionic strings.
- Orientifold planar equivalence holds also for the domain wall interpolating between the two vacua $P/N = \pm 1$ (in the deconfined phase).