# How narrow is the sQGP transition?

## A simple non-perturbative approach to hot gluodynamics compared to lattice data

Chris Korthals Altes

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April 2011

Vith A. Dumitru, Y. Guo, Y. Hidaka, R. Pisarski, arXiv:1011.3820, Phys.Rev D


- We all want to understand the groundstate in RHIC experiments...
- In this modest building housing the RHIC VACUUM FACILITY the decision is made on a day to day basis whether to present the groundstate as AdS/CFT, monopole condensate, ...whatever the theorist likes.


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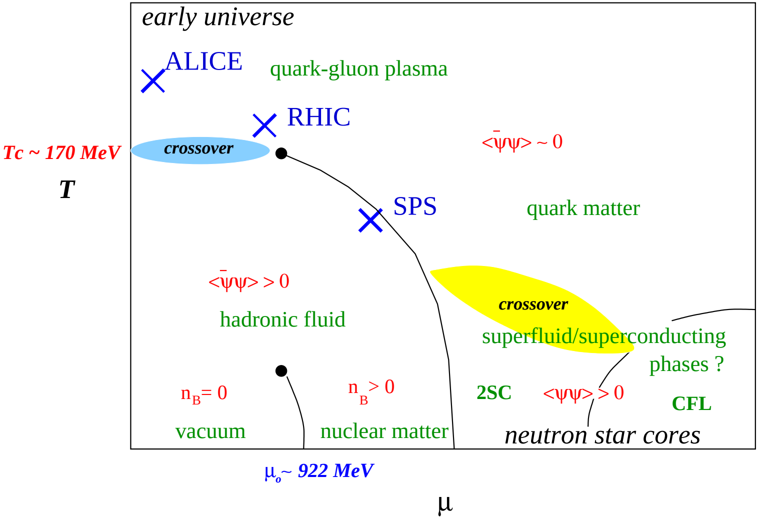


Figure: Proposed phase diagram for QCD. 2SC and CFL refer to the diquark condensates .

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- Facts from the lattice: EOS and flux loops
- Fancy: determining an Ansatz for the effective potential from EOS
- Predictions from effective potential.
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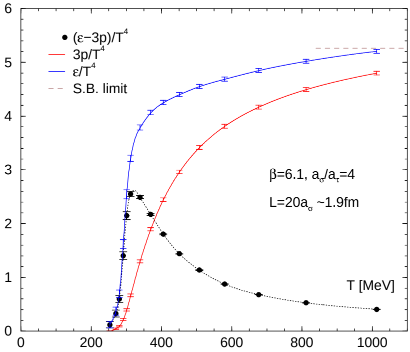
# Pressure, energy density and interaction measure




Fixed scale data by T.Umeda et al.., arXiv0809.2842.

- energy density much steeper than pressure, so is the interaction measure, with peak at $\sim 1.2 T_c$.
- interaction measure falls off like $1/T^2$ beyond $T = 1.2 T_c$ , not like $(1/\log T)^2$

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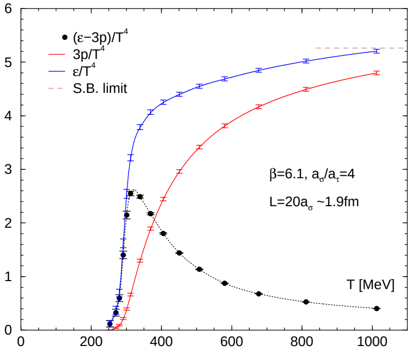


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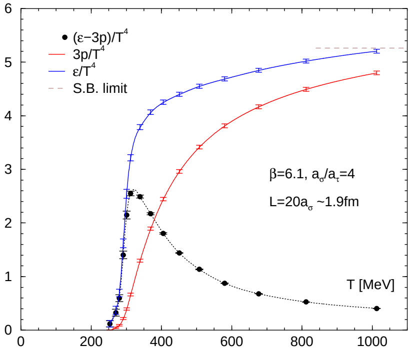


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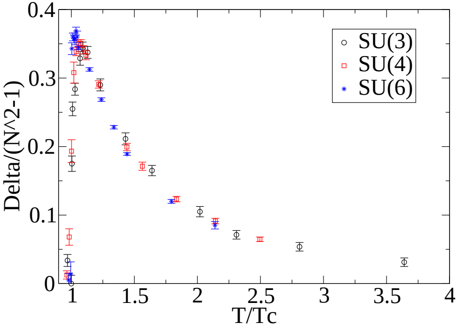
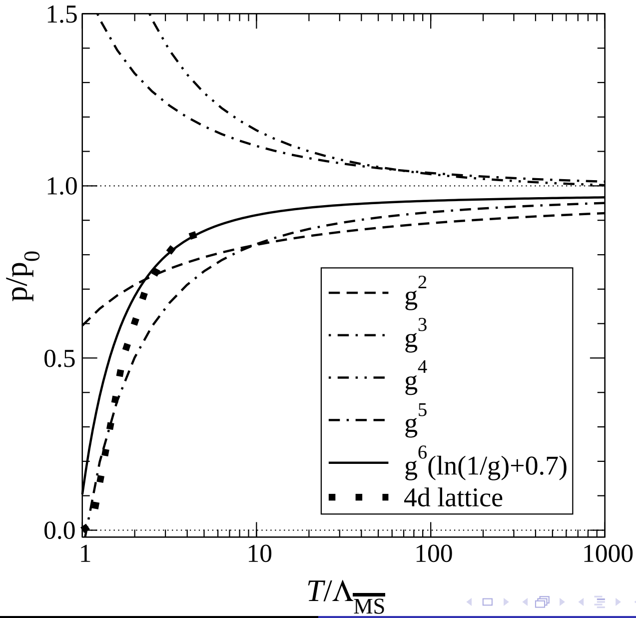
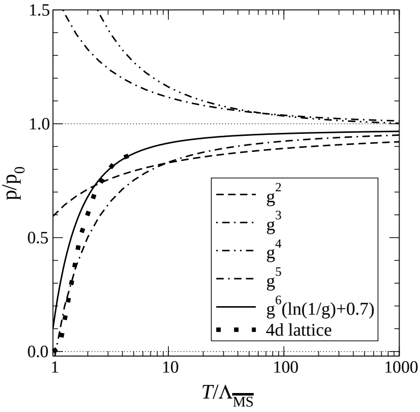



Figure: Interaction measure scaled by $N^2 - 1$, Panero 2009. Note the small reduced discontinuity at $T_c$



1.5
1.0
0.5
0.0
p/p0
1
10
100
1000
T/ΛMS
g2
g3
g4
g5
g6(ln(1/g)+0.7)
4d lattice

# Pressure of the plasma




- Comparison of perturbative results.The $O(g^3)$ has the wrong sign. Electric quasiparticles not good enough!
- The pressure gets contribution from magnetic sector starting from $g^6$. What is in this magnetic sector??

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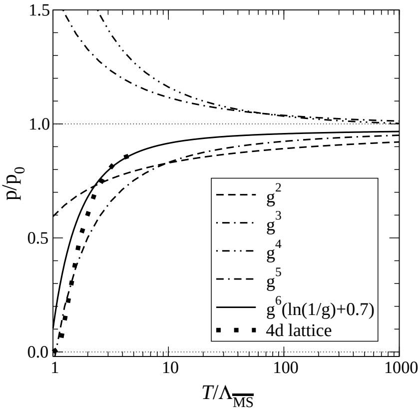


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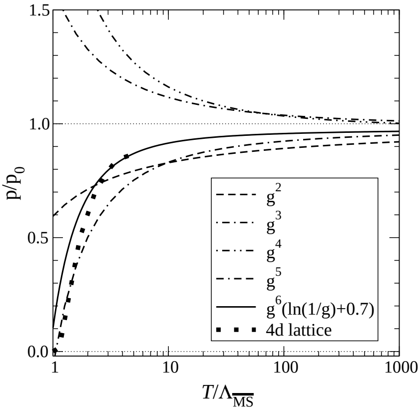


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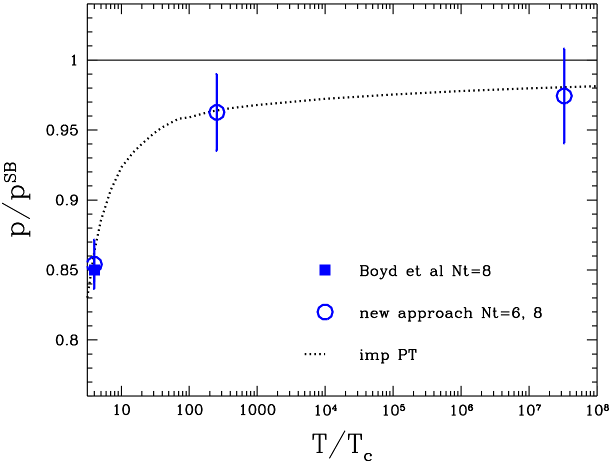
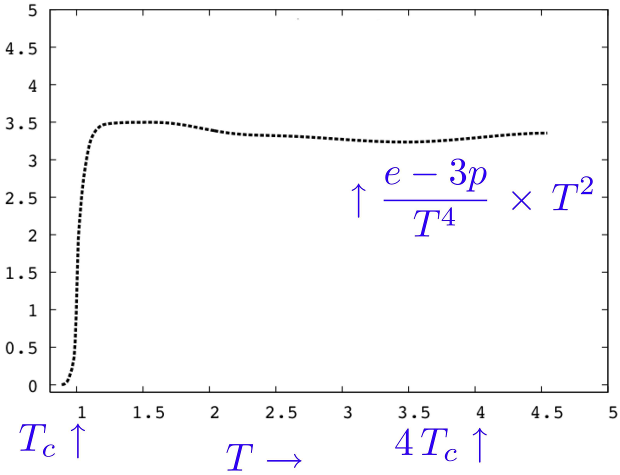



EQCD prediction for pressure relates ultrahigh T points (arXiv:0710.4197) data hep-lat/9602007. For HTL improvement see arXiv:1005.1603.

# The $1/T^2$ law for the interaction measure




RDP at Kyoto 2006 (also Meisinger et al.hep-phys/ 0108009); data hep-lat/9602007; arxiv.org/abs/0810.1570, /0809.2842 .

# What is the plasma consisting of, as seen by flux loops?

- electric colour flux as seen by a spatial 't Hooft loop ("e-loop")
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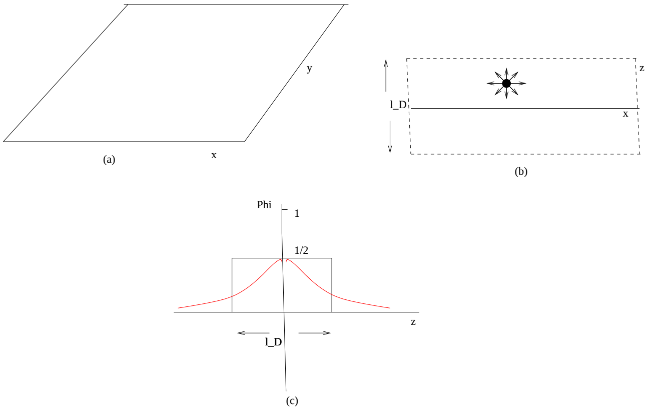
# Flux of Debye screened glue




- Spatial 't Hooft loop $V_k = \exp(i\frac{4\pi}{g}\int Tr\vec{E}Y_k.d\vec{S})$ in x-y plane.
- $Y_k$ generalized hypercharge $\exp(i2\pi Y_k) = \exp(ik2\pi/N)\mathbf{1}$.
- There are k(N-k) gluons with $Y_k$ charge 1, etc...

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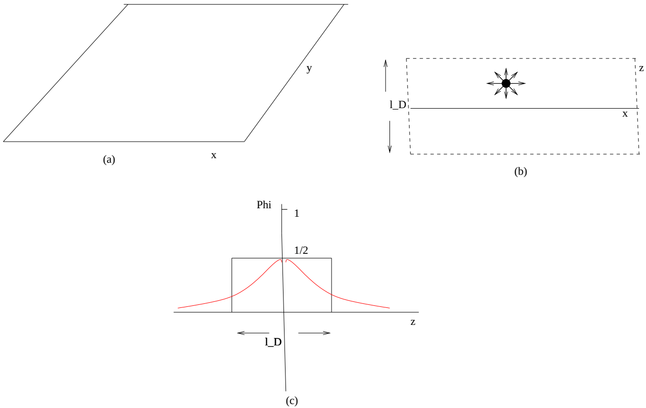


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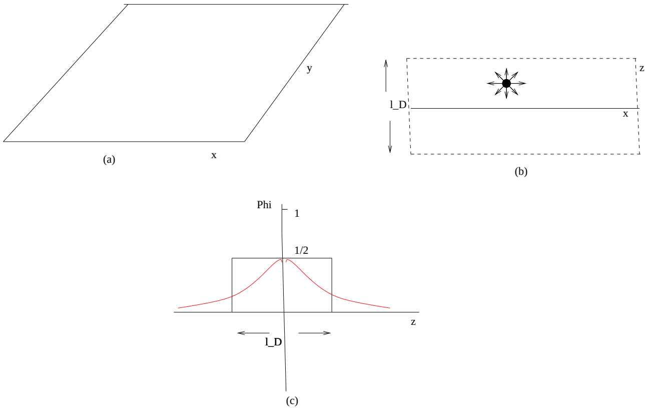


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- At $T >> T_c$ a gas of Debye screened gluons: $l_D \sim \frac{1}{gT} >> \frac{1}{T}$
- Any gluon species with charge $\pm 1$: contributes $\exp(i2\pi/2) = -1$.
- In the slab are on average $\bar{l} = n(T) l_D.Area$ gluons of hat species. Poisson distribution for average due to a charged species:
- $< V_k >_{one\ cs} = \sum_l \frac{\bar{l}^l}{l!}(-1)^l \exp(-\bar{l}) = \exp(-2\bar{l})$
- All $2k(N-k)$ charged gluon species (supposed independent):

$$< V_k >= \exp(-4k(N-k) l_D n(T).Area)$$

- Casimir scaling: $\rho_k(T) \sim k(N-k) l_D n(T)$

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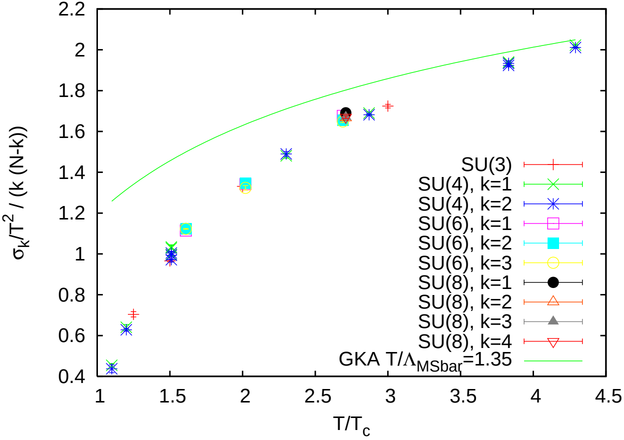
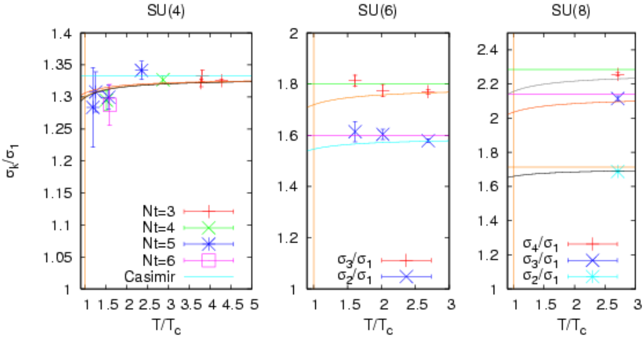
# Reduced electric flux tension in deconfined phase




$N_c \leq 8$, de Forcrand et al.,hep-lat/0510081, Bursa/Teper, hep-lat/0505025

GKA: field theory calculation to two loop order hep-ph/0102022, cubic order in hep-ph0412322. BGKAP: PRL66, 998, 1991.

# Electric flux tension in the deconfined phase




e-tension for $SU(N_c)$, $N_c \leq 8$, PdF et al.,hep-lat/051008

# Electric flux tension

- Casimir scaling good for ANY T above 1.15 $T_c$ in deconfined phase
- Two loop reduced tension does not match the lattice calculation
- Warranted: 3 or more loop calculation (Yannis Burnier, 'CPKA, York Schroeder, Aleksi Vuorinen).
- This talk: perhaps a more insightful way-beyond perturbation theory- to understand the Casimir scaling down to $\geq T_c$

# Casimir scaling of spatial Wilson loops

- GKA (hep-ph0102022): at high T a dilute gas of adjoint monopoles causes Casimir scaling for Wilson loops.
- Lucini Teper (2001....) and hep-lat/051008 :

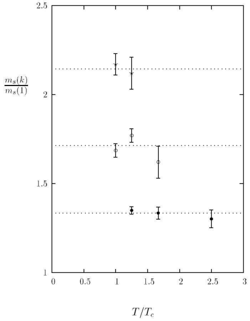



Figure 1: The mass ratio of the $k = 2$ to $k = 1$ spatial loops in SU(4) (•) and in SU(8) (∘), and of the $k = 3$ to $k = 1$ loop in SU(8) (⋆). All in the deconfined phase and for $a \simeq 1/5T_c$. High $T$ Casimir scaling predictions are shown for comparison.

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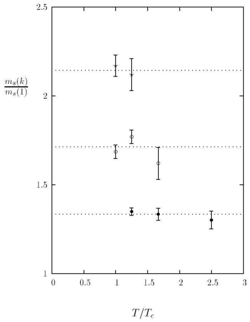


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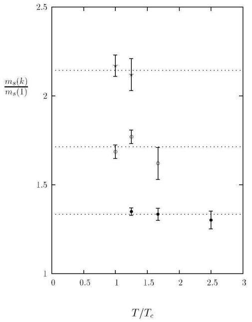


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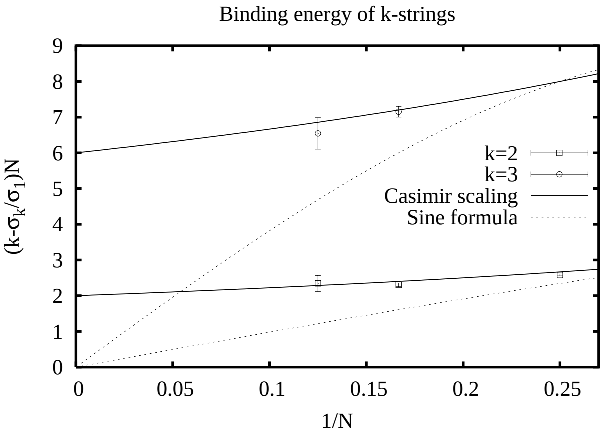
# m-flux tension at asymptotic T. Lattice results




flux-tension for $SU(N_c)$, $N_c \leq 8$, Meyer, hep-lat/0412021)

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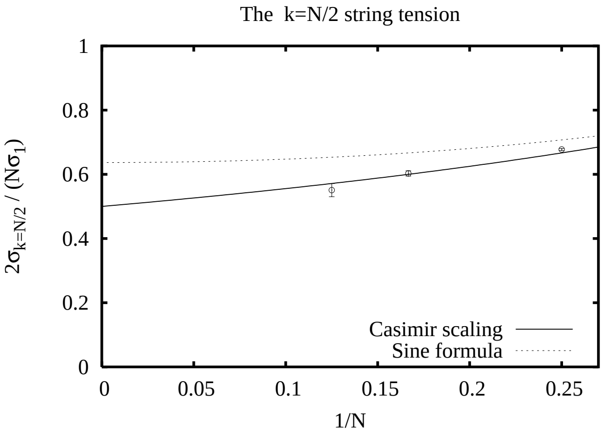


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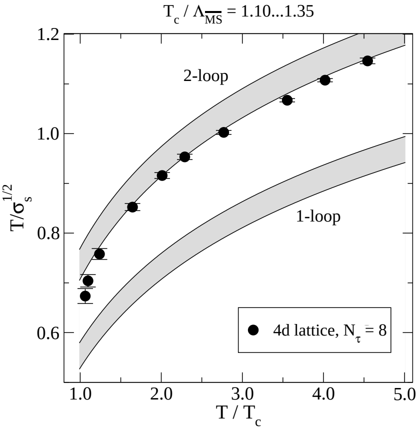
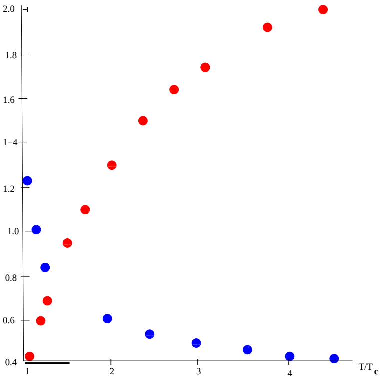
# 3d results propagate in ALL of the deconfined phase through the running coupling




m-tension, $N_c = 3$, hep-lat/0503003

$$\sigma(T) = c_{3d} g_M^4(T) = c_{3d} g_E^4(T)(1 + small) = c_{3d} g^4(T)(1 + .. + 3loop)T^2$$

# How do magnetic and electric flux compare?



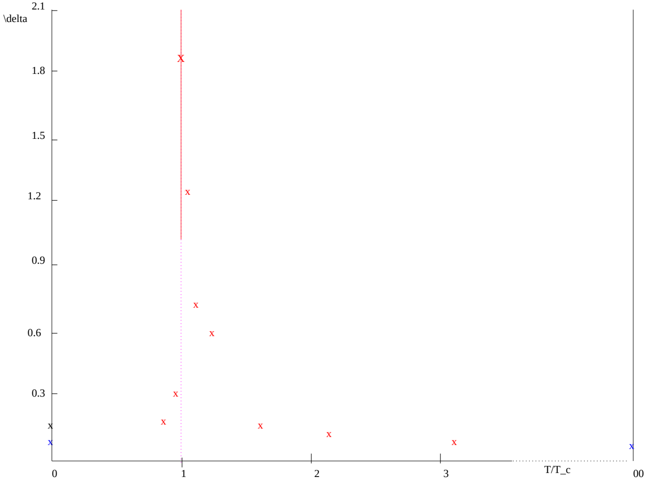

SU(3) colour electric flux versus SU(3) colour magnetic flux

Note: equality inside the peak of the interaction measure $T/T_c \sim 1.10$.
o peak might be due to a correlation of electric and magnetic quasi-particles

# Correlations between loops

- Measure correlation on the lattice between nearby, almost contingent 't Hooft and Wilson loop as function of temperature.
- For very high T: magnetic and electric populations are uncorrelaled, so expect no correlation between loops.
- For T in critical region around the peak of the conformal energy the correlation may become quite strong.
- The correlation is a key quantity for understanding the behaviour of the plasma components.
- Unfortunately it is subleading in in large N limit, so simplest AdS/CFT is not enought to access it.

# The ratio $\delta$ as function of T. SU(3) case




\delta=\sigma_s/(m_0^++)^2, colours as in previous figure.

The ratio $\sigma_1/m^2_{++}$, SU(3), Datta, Gupta,hep-lat/0208001

- At $T \sim 1.2T_c$ the ratio has risen with a factor 10
- From large T to $T_c$ the ratio increases with a factor 40! !
- SU(3) weakly first order, may explain the large ratio.
- $m_{--}$ is probably the inverse radius of the adjoint magnetic quasi particle, determines a much smaller ratio which would be the diluteness $l^3_{--}n_M$, but is not yet available for all T.

# Perturbation theory and the flux loops

- Once the non-perturbative 3d part of the magnetic loops is detemined on lattice, perturbation theory works, and they have Casimir scaling.
- Although the magnetic free energy scales as a gas of adjoint quasi-particles, no classical adjoint monopoles are known in QCD.
- The electric loops have Casimir scaling according to one two and two loop order. To three loop order the preliminary results suggest the same.
- "Precocious" QGP behaviour (see below) may be an alternative explanation

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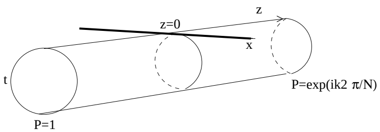
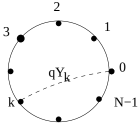
# Field theory calculation of loop average

$\langle V_k(L)\rangle = Tr_{phys} V_k(L)\exp(-H/T)/Tr_{phys}\exp(-H/T)$ By translation into path integral language: Domainwall at z=0 between domains where Polyakov loop takes different Z(N) values has energy $\rho_k(T)$.




Periodic time direction and z−direction orthogonal to (x,y) plane. Loop $L_x$ x $L_y$ at z=0.



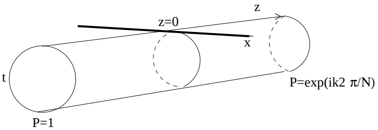

Mimima of effective potential numbered by k

Polyakov loop profile along z-direction, and Z(N) vacua.

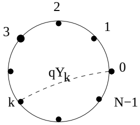
- Effective action: $U = K(q)q'^2 + V(q)$ in loop expansion.
- tunnneling along $qY_k$ between the minima gives $\rho_k$
- energy of wall/per unit length=$\rho_k(T)$.

# Field theory calculation of loop average

$\langle V_k(L)\rangle = Tr_{phys} V_k(L) \exp(-H/T)/Tr_{phys} \exp(-H/T)$ By translation into path integral language: Domainwall at z=0 between domains where Polyakov loop takes different Z(N) values has energy $\rho_k(T)$.




Periodic time direction and z−direction orthogonal to (x,y) plane. Loop $L_x$ x $L_y$ at z=0.

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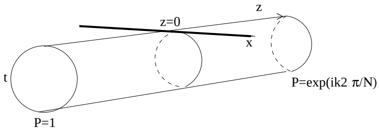


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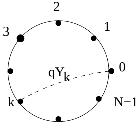


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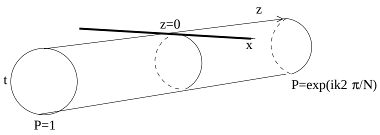


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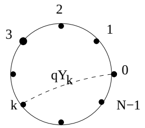


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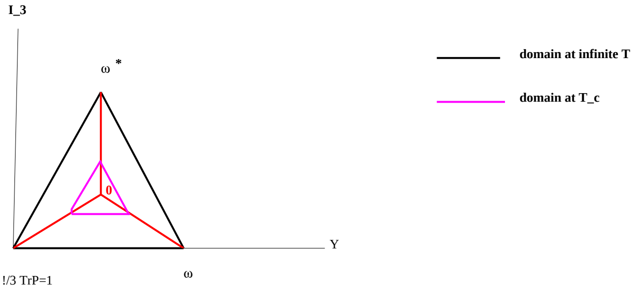


Domain of the SU(3) effective potential in Cartan space

- Infinite T see perturbative potential
- $T \geq T_c$ see histogram
- Thermodynamic functions live on the C invariant minima (red lines, Z(3) related copies)
- we want a model for the potential V in between these temperatures.

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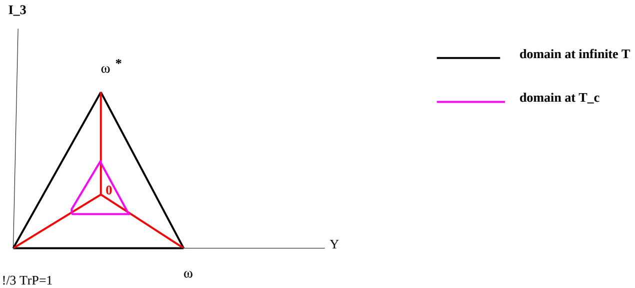


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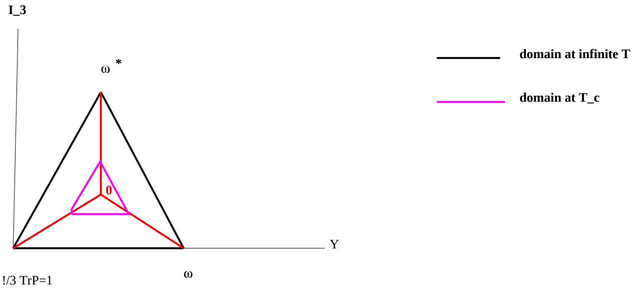


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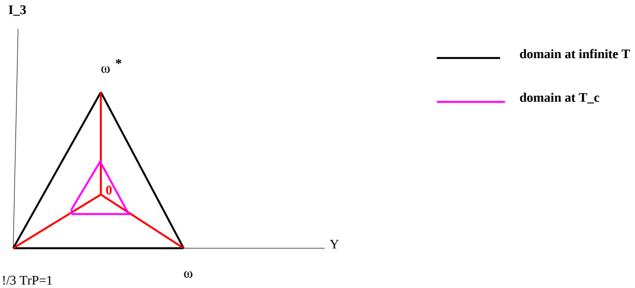


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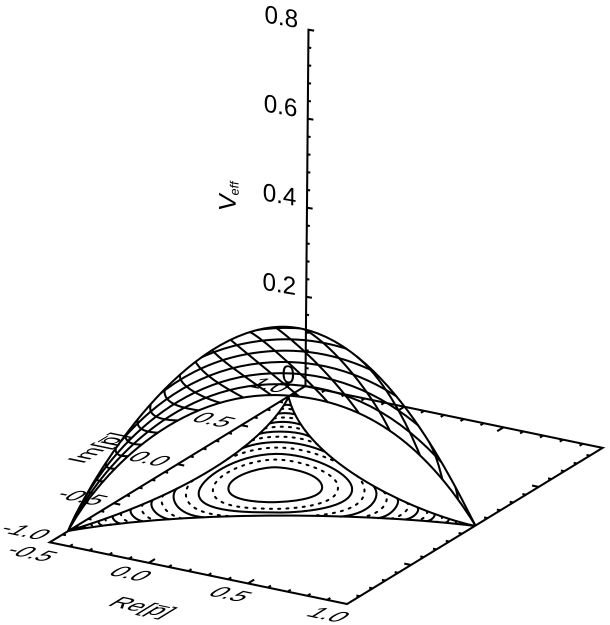
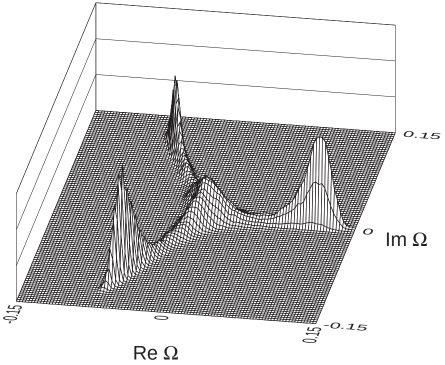



SU(3), lowest order perturbative effective potential.




Histogram of the Polyakov loop P in SU(3). It equals $\exp -(Vol)V(P)$.

The Z(3) minima have moved in towards the symmetric point.

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# Repulsive and attractive eigenvalues of the Wilson line

- SU(2): $P = \text{diagonal}(\exp(i\phi/2, \exp(-i\phi/2))$
- At high T in perturbation theory: phases cluster at centergroup values $\phi = 2\pi q = 0, \quad \pi$
- $T^4 V_{pert} = T^4(\pi^2/15) + \frac{2\pi^2}{3} q^2(1-|q|)^2)$ minima in q=0,1
- Adding a term $V_{nonpert} = -M^2 T^2 |q|(1-|q|)$ induces tendency to repulsion: minima are $\phi = \pm\pi$
- So our mean field like Ansatz is:

$$\begin{aligned} T^4 V &= T^4(V_{pert} + V_{nonpert}) \\ &= T^4\left(\frac{\pi^2}{15} + \frac{2}{3}\pi^2 q^2(1-|q|)^2 - (\frac{M}{T})^2(|q|(1-|q|) + d)\right) \end{aligned} \quad (1)$$

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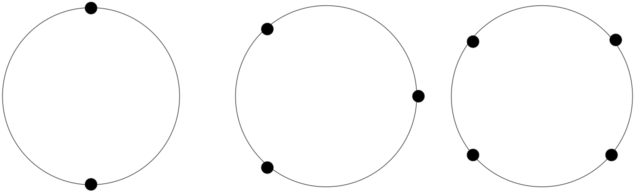
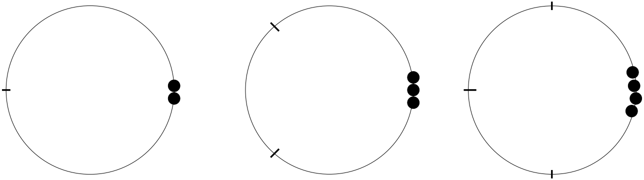


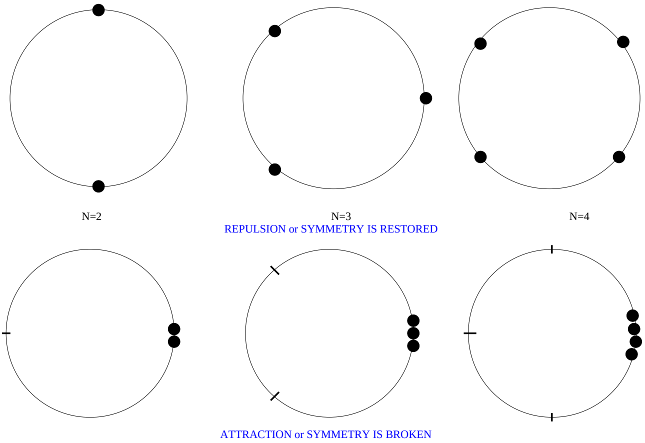
N=2 N=3 N=4

REPULSION or SYMMETRY IS RESTORED



ATTRACTION or SYMMETRY IS BROKEN

As T goes down the eigenvalues start to decluster and move out to the equal spacing positions. In all but SU(2) the transition is first order, so the eigenvalues stop short of the equal spacing positions.

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# Simple relations

To determine pressure from $V(q)$:
find the extrema $q_0$ of $V(q),\ V'(q_0) = 0$:

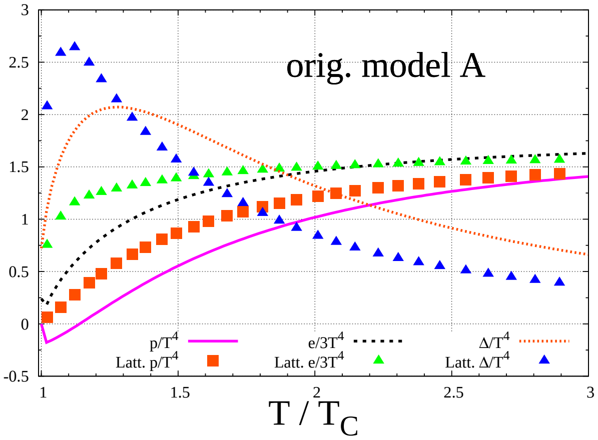
$$p = -V(q_0). \tag{2}$$

Now the relation of $\Delta$ to $V$ is immediate:

$$\begin{aligned} \frac{\Delta}{T^4} &= T\frac{\partial}{\partial T}\left(p/T^4\right) \\ &= -\frac{\partial V(q_0)}{\partial T} \\ &= -T\frac{\partial q_0}{\partial T}V'(q_0) + 2M^2/T^2 V_{nonpert}(q_0) \end{aligned} \tag{3}$$

So $\Delta$ relates only (not unexpected) to the non-perturbative potential.:

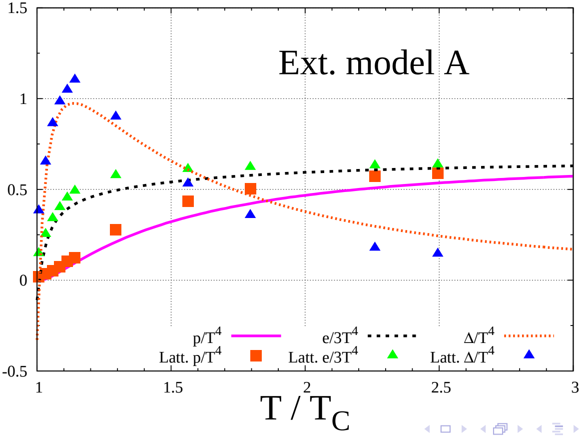
$$\frac{\Delta}{T^4} = 2M^2/T^2 V_{nonpert}(q_0). \tag{4}$$

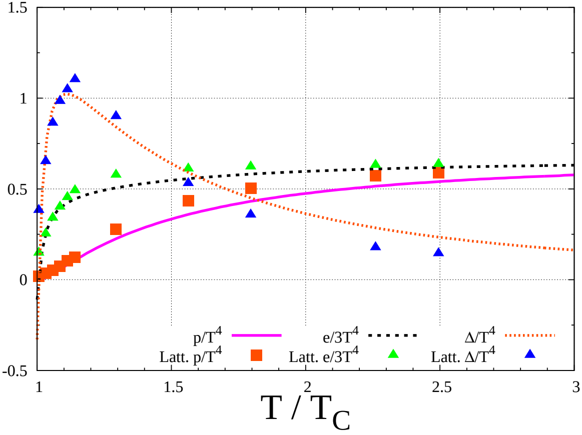

orig. model A
3
2.5
2
1.5
1
0.5
0
-0.5
1
1.5
2
2.5
3
T / T_C
p/T^4
Latt. p/T^4
e/3T^4
Latt. e/3T^4
Δ/T^4
Latt. Δ/T^4

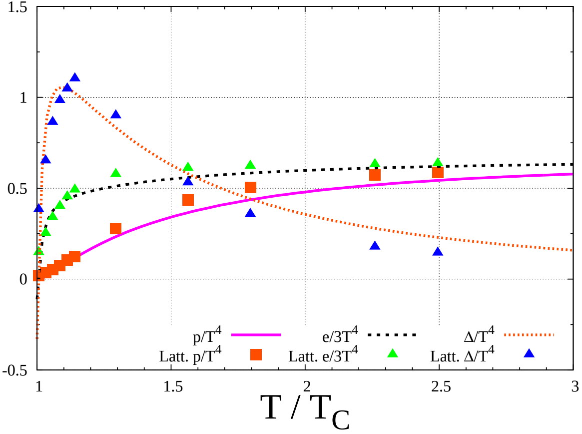
This Ansatz is good, but not good enough!

- The interaction measure not rising steep enough: the maximum is displaced to much too high $T$
- We need another parameter to fix this:

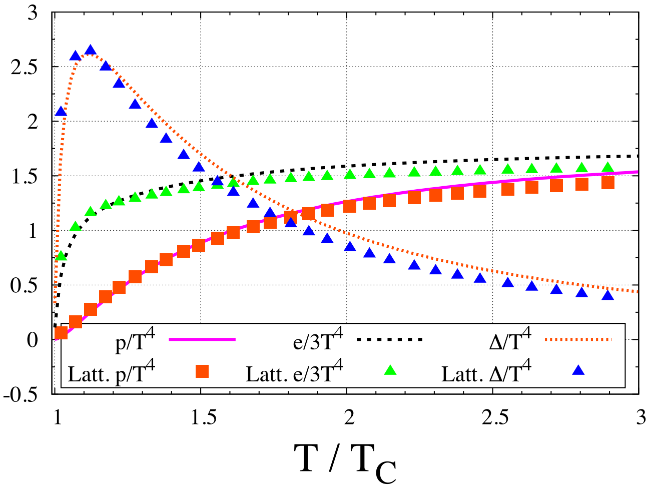
$$V_{nonpert} \to V_{nonpert} - c(M/T)^2(|q|^2(1-|q|)^2$$




1.5
1
0.5
0
-0.5
1
1.5
2
2.5
3
T / T$_C$
p/T$^4$
e/3T$^4$
Δ/T$^4$
Latt. p/T$^4$
Latt. e/3T$^4$
Latt. Δ/T$^4$




SU(2) thermodynamic functions, c=2.




SU(3),thermodynamic functions c=1 .

# Predictions

- The effective potential is now fixed. There are four predictions to be checked by lattice.
- Interface tension for $T \geq T_c$
- For SU(3) and higher $N_c$: tension at $T_c$ for coexisting phases.
- Polyakov loop average

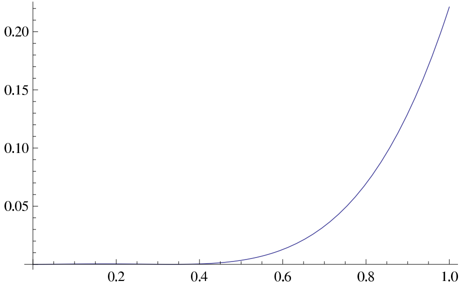



Figure: Potential at $T_c$, showing a a VERY weak first order transition, as function of 1-q. q=1 is the confined state. You see a very small maximum at $1 - q = 0.16$, i.e. $1 - q_c = 0.33$ is the minimum degenerate with the minimum at $1 - q = 0$, the confining vacuum.

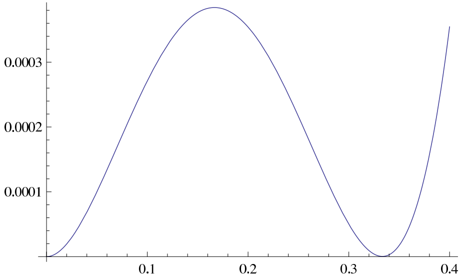



Figure: Vertical blow up of the first graph and now you see the first order transition, i.e. the degeneracy at $1 - q = 0$ and $1 - q = 0.33$

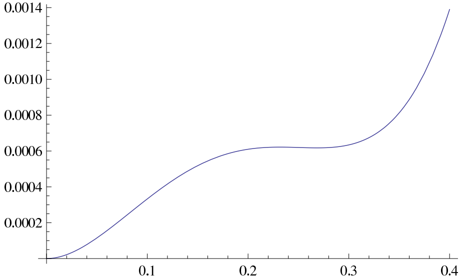



Figure: Potential at 0.99 $T_c$, as function of 1-q. The metastable minimum at non-zero 1-q has almost gone away.

- For coexisting phases the tension is (SU(3)) $= 0.0258012 T_c^2/\sqrt{g^2(T)}$ (smaller than Bursa-Teper result). Large latent heat means very broad flat potential at $T_c$
- SU(2) interface tension$= \frac{4\pi^2 T^2}{3\sqrt{6g^2(T)}}(1-(T_c/T)^2)^{3/2}$ if c=0, and kinetic energy is taken classical.
- SU(2): the fit is done with (obviously) c=1.5 (SU(2) and just the two loop corrections from the complete QGP. The latter are not the whole story. We have to include loop corrections from fluctuations around the SQGP minima. This is being done.
- For SU(3) the tunneling path is only for the QGP along the $\lambda_8$. Away from QGP there is a $\lambda_3$ component taken numerically into account, to obtain the minimal action.

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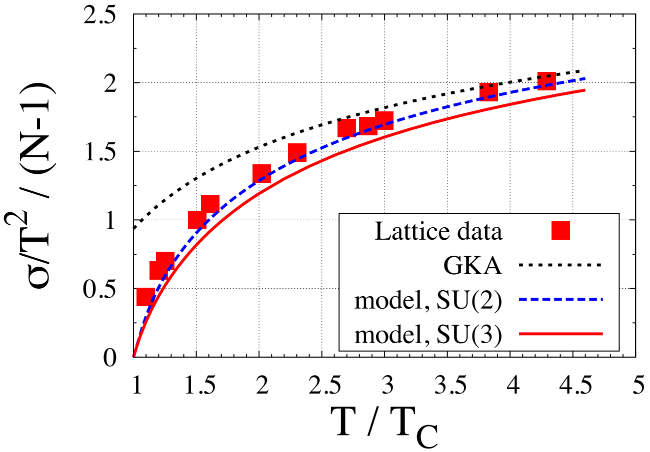
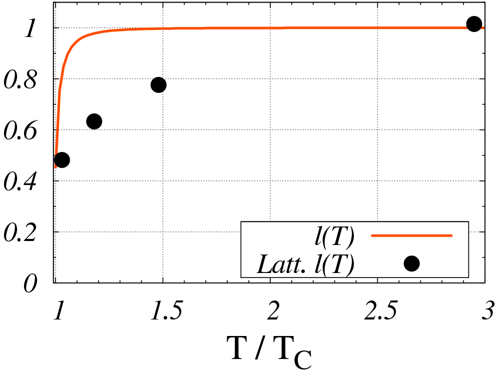


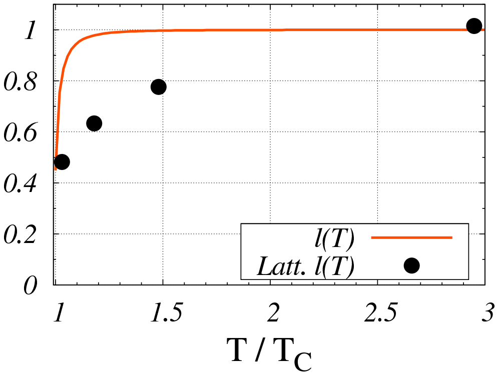

Interface SU(2), SU(3), de Forcrand, Noth, hep-lat/0506005

Interface tension $\sim ((T - T_c)/T_c)^{3/2}$, i.e. critical exponent =1.5 instead of universal 1.26.




SU(3), Polyakov loop average, Gupta et al., arXiv:0711.2251

- Narrowness of the sQGP ($T/T_c = 1$ to 1.2)) is closely related to narrowness of interaction measure
- Our result does contradict the data. $O(g^2)$ corrections unlikely to produce agreement. Data without fuzzing the loop.

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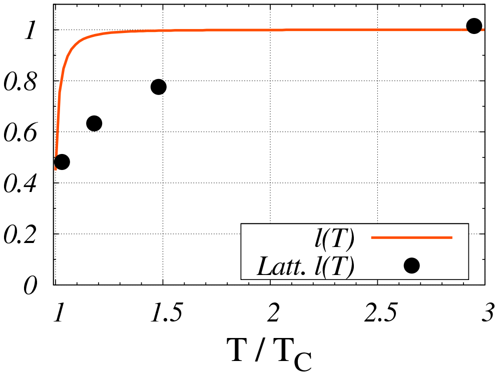


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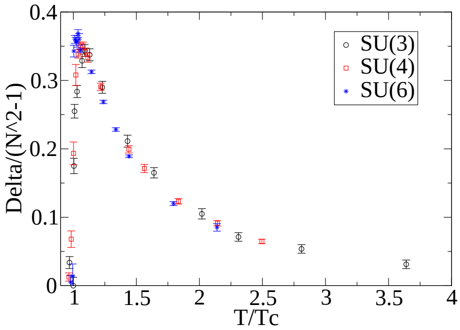



Figure: Interaction measure scaled by $N^2 - 1$, Panero 2009. Reduced discontinuity looks very small, like we find.

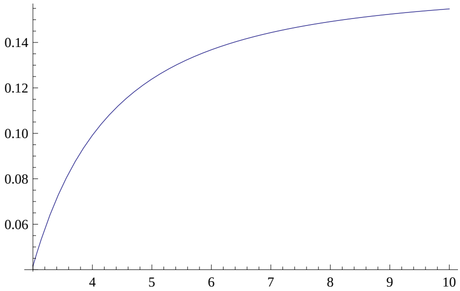



Figure: Latent heat in units of $(N^2 - 1)T_c^4$, Teper/Bursa: $0.744 - 0.34/N^2$

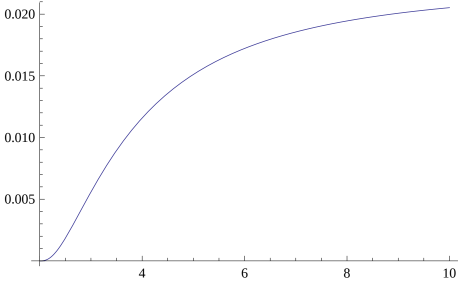



Figure: Order-disorder interface between coexisting phases, in units of $(N^2-1)T_c^2/\sqrt{g^2(T_c)N}$, as function of $N$

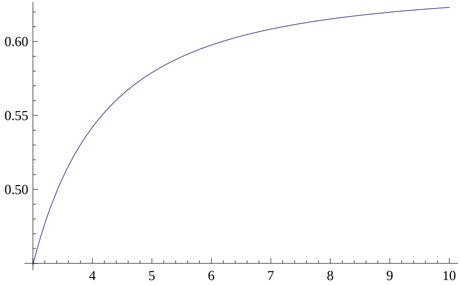



Figure: Jump of the normalized Polyakov loop at $T_c$, as function of $N$

# Masses induced by the presence of the loop

- The presence of the loop induces a shift in the time derivatives, or equivalently in the Matsubara frequencies of off diagonal fluctuations:
- $p_0 \rightarrow p_0 + 2\pi T(q_i - q_j)$
- So the corresponding inverse propagator is corrected not only by $m^2(q)$ a q dependent Debye mass (O(gT)), but also by an O(1) shift: $\vec{p}^2 + m_D^2(q) + (2\pi T(q_i - q_j))^2$

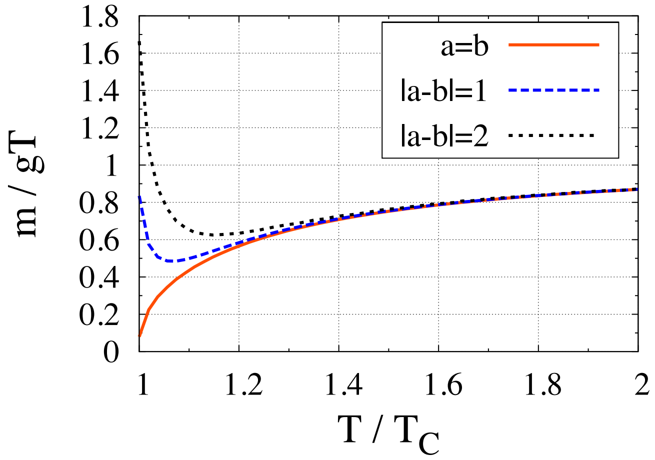
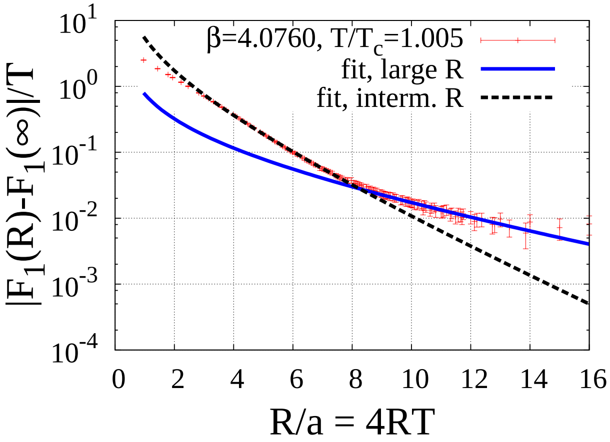



Illustration of the behaviour of the masses for SU(3).


β=4.0760, T/T_c=1.005
fit, large R
fit, interm. R
|F_1(R)-F_1(∞)|/T
R/a = 4RT
10^1
10^0
10^-1
10^-2
10^-3
10^-4
0
2
4
6
8
10
12
14
16

SU(3),

# Conclusions

- Model: EOS fully fixes effective potential
- Predicts surface tensions (o-o, o-d),Polyakov loop average, latent heat,
- Our model finds precocious QGP at $T = 1.20T_c$, beyond which P=1
- The precociousness is persisting for more colours $N_c = 4, 5, \ldots.$
- If so: the Casimir scaling of the e-tension down to $T \sim 1.2T_c$ may be understandable, and should be compared to the Teper/Bursa lattice data.
- Conspicuously absent is prediction for magnetic tension
- Introduce quarks!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