Phase transitions in holographic QCD with Dense Media

B. Gwak, M. Kim, BHL,Y.Seo, S.-J. Sin,   arXiv :1105.xxxx

Bum-Hoon Lee
Center for Quantum Spacetime
Sogang University
Seoul, Korea

서강대학교
SOGANG UNIVERSITY

Seminar @ GGI   April 26, 2011

CQUeST
Center for Quantum SpaceTime

Contents

I. Introduction
- Motivation
- AdS/CFT – Holography Principle, Dictionary
- AdS/QCD : Top-down & Bottom up approach

II. Dual Geometry
BHL, Park ,Sin JHEP 0907,(2009)
- Dual Geometry at finite temperature (w/o chemical potential)
- Dual Geometry at finite (arbitrary) density

III. Phases of Holographic QCD

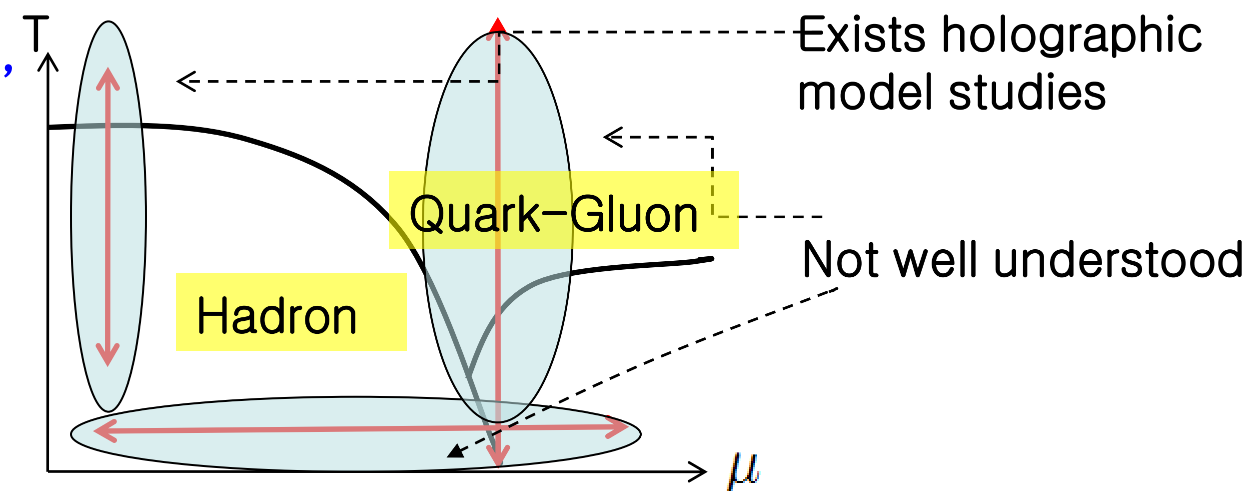
B. Gwak, M. Kim, B.-H.L.,Y.Seo, S.-J. Sin,
arXiv :1105.xxxx

IV. Summary

I. Introduction : Motivation & Basics

- AdS-CFT Holography : Useful tool for strongly interacting system such as QCD, Condensed Matter, etc.

- Holography idea of AdS/CFT applied to QCD is called AdS/QCD
- With AdS-QCD, how to explain properties such as
  confinement,
  χ-symm breaking,
  phases,
  etc. ?



Main idea on holography through the Dp branes

* Dp branes carry tension (energy) and charge (source for p+2 form)
  → Gravity in AdS space (dim = ((p+1)+1))
* Dp brane's low energy dynamics by fluctuating open strings
  → Yang-Mills in (p+1) dim. (CFT)

(Closed string picture) : Dp branes carry energy → AdS ((p+1)+1) dim)

and RR−charge → source for p+2 form flux H

$$S_{IIB} = \frac{1}{4\kappa_B^2}\int \sqrt{G}e^{-2\Phi}(2R_G + 8\partial_\mu\Phi\partial^\mu\Phi - |H_3|^2) - \frac{1}{4\kappa_B^2}\int \Big[\sqrt{G}(|F_1|^2 + |\tilde{F}_3|^2 + \frac{1}{2}|\tilde{F}_5|^2) + C_4 \wedge H_3 \wedge F_3 + \text{fermions}\Big]$$

$$\Rightarrow \begin{cases} ds^2 = f_p^{-1/2}(-dt^2 + dx_1^2 + \cdots + dx_p^2) + f_p^{1/2}(dx_{p+1}^2 + \cdots + dx_9^2)\ , \\ e^{-2\phi} = f_p^{\frac{p-3}{2}}\ , \qquad f_p = 1 + nc_p^{10}/r^{7-p} \quad \text{(harmonic function)} \\ A_{0\cdots p} = -\frac{1}{2}(f_p^{-1} - 1)\ , \end{cases}$$

Ex) D3 brane :

$$\begin{cases} ds^2 = f^{-1/2}dx_{||}^2 + f^{1/2}(dr^2 + r^2 d\Omega_5^2) \quad f = 1 + \frac{4\pi g N \alpha'^2}{r^4} \\ \phi = \text{constant} \qquad \text{(conformal symmetry)} \end{cases}$$

The near horizon limit gives AdS x S5 $\alpha' \to 0\ ,\quad U \equiv \frac{r}{\alpha'} = \text{fixed}$

$$ds^2 = \alpha'\Big[\underbrace{\frac{U^2}{\sqrt{4\pi g N}}dx_{||}^2 + \sqrt{4\pi g N}\frac{dU^2}{U^2}}_{\text{AdS5}} + \underbrace{\sqrt{4\pi g N}d\Omega_5^2}_{\text{S5}}\Big]$$

AdS5 x S5

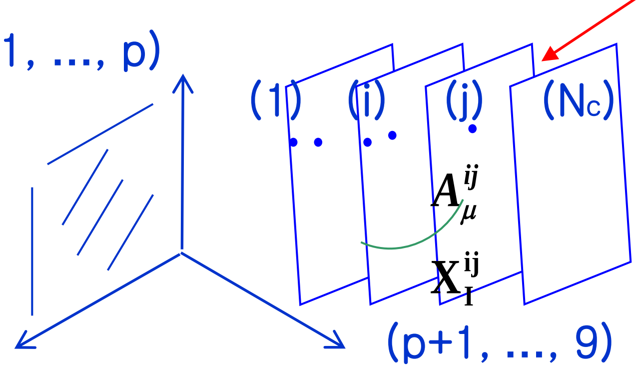
the radius of S_5
= the radius of AdS_5

$$R_{sph}^2/\alpha' = \sqrt{4\pi g N}$$

- Isometry : SO(4, 2) x SO (6)
- $gN \gg 1$ ⇨ SUGRA approach is reliable

(open string picture of Dp branes) Low Energy Dynamics
--> p+1 dim. SUSY SU(Nc) YM Theories

Neumann boundary condition



Dirichlet boundary condition

#Dp-branes = Nc $\quad g_s = e^{\phi} = g_{YM}^2$

$A_\mu, \mu = 0,1,\ldots.p$

$X_I, I = p+1,\ldots,q$

* BPS state Tension=Charge $(= \frac{1}{g_s l_s^{p+1}})$

* Preserves 1/2 of 2x16 SUSY

brane

$E_L Q_L + E_R + Q_R, R \quad E_L = \pm\Gamma^0\Gamma^1 \ldots \Gamma^P E_R$

Anti brane

* dynamical

10 dim. N=1 SYM $L = -\frac{1}{4g_{YM}^2} Tr F_{MN} F^{MN} + L_{fer}$

dim reduction

$$\boxed{g_{YM}^2 = g_s l_s^{p-3}, \Phi^I = \frac{X^I}{l_s^2}}$$

(p+1) dim. SYM (#SUSY = 16 =32/2)

Ex) # Nc D3 branes

A_μ, Φ_I, Ψ :

0, 1, ..., 3 $\quad$ 1, ..., 6

: Nc×Nc matirices, adj. repn. of U(Nc)

$$L = -\frac{1}{4g_{YM}^2} Tr(-\frac{1}{4} F_{\mu\mu} F^{\mu\mu} + \frac{1}{2} D_\mu \Phi^\Sigma D^\mu \Phi^I + \sum_{I,J} [\Phi^I, \Phi^J]^2) + Fermions$$

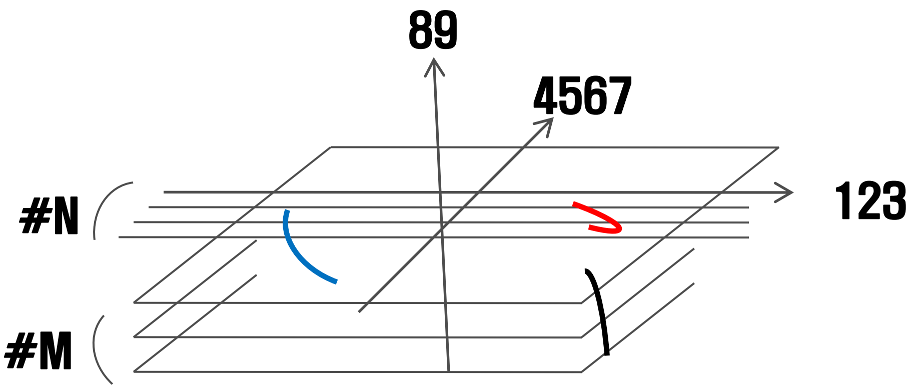
Conformal symmetry R-symmetry :

• global symmetry : SO(4,2) x SU(4) = SO(6)

• If $\lambda = g_{YM}^2 N_c << 1$, then perturbative

rotation perpendicular to D3 branes

Ex) D3-D7 -> 3+1 dim. N=2 SU(Nc) YM theory with Nf hypermultiplets



		0	1	2	3	4	5	6	7	8	9
#N	D3	–	–	–	–	X	X	X	X	X	X
#M	D7	–	–	–	–	–	–	–	–	X	X

rotations

Strings

CPX

3-3 : A_μ, Φ, λ, χ

0,1,2,3

(N=2 Vector multiplet)

Adjoint representation

3-7 : Q, $\tilde{Q}$, ψ, $\tilde{\psi}$

(N=2 hyper multiplet)

Matter in Fundamental

$\mathcal{N}=2$	components	spin	$SU(2)_\Phi \times SU(2)_\mathcal{R}$	$U(1)_\mathcal{R}$	Δ	$U(N_f)$	$U(1)_B$
(Φ_1, Φ_2)	X^4, X^5, X^6, X^7	0	$(\frac{1}{2}, \frac{1}{2})$	0	1	1	0
hyper	λ_1, λ_2	$\frac{1}{2}$	$(\frac{1}{2}, 0)$	-1	$\frac{3}{2}$	1	0
(Φ_3, W_α)	$X_V^A = (X^8, X^9)$	0	$(0,0)$	$+2$	1	1	0
vector	λ_3, λ_4	$\frac{1}{2}$	$(0, \frac{1}{2})$	$+1$	$\frac{3}{2}$	1	0
	v_μ	1	$(0,0)$	0	1	1	0
$(Q, \tilde{Q})$	$q^m = (q, \bar{\tilde{q}})$	0	$(0, \frac{1}{2})$	0	1	N_f	$+1$
fund. hyper	$\psi_i = (\psi, \tilde{\psi}^\dagger)$	$\frac{1}{2}$	$(0,0)$	∓ 1	$\frac{3}{2}$	N_f	$+1$

Note : 7-7 strings decouples

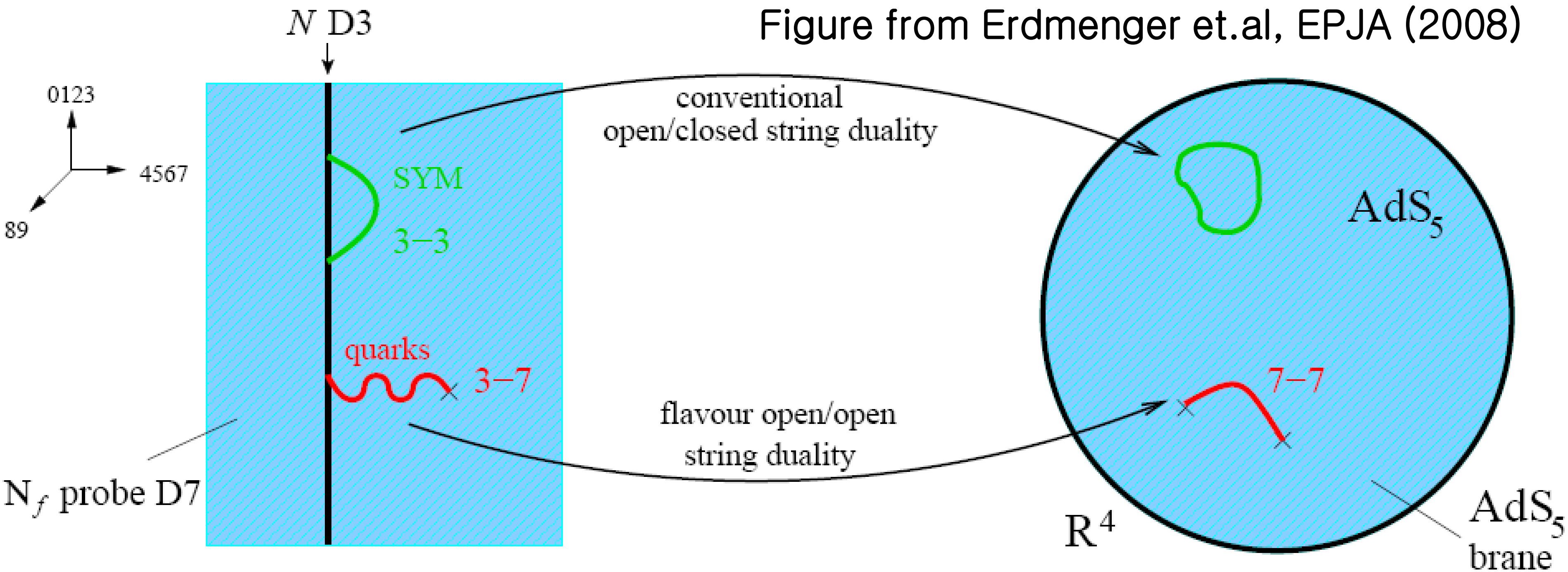
$$\mathcal{L} = \mathrm{Im}\left[\tau \int d^4\theta \left(\mathrm{tr}\,(\bar{\Phi}_I e^V \Phi_I e^{-V}) + Q_r^\dagger e^V Q^r + \tilde{Q}_r^\dagger e^{-V} \tilde{Q}^r\right) + \tau \int d^2\theta (\mathrm{tr}\,(W^\alpha W_\alpha) + W) + c.c.\right],$$

$$W = \mathrm{tr}\,(\varepsilon_{IJK} \Phi_I \Phi_J \Phi_K) + \tilde{Q}_r (m_q + \Phi_3) Q^r$$

$$\beta^\lambda_{\mathcal{N}=2} \equiv N_c \beta_{\mathcal{N}=2} = \frac{1}{2\pi}\left(\frac{\lambda}{4\pi}\right)^2 \frac{N_f}{N_c} \xrightarrow[\text{(quenching)}]{N_f/N_c \ll 1} 0$$

Toward the more realistic models
: AdS/CFT with flavors – Intersecting D-Branes

Figure from Erdmenger et.al, EPJA (2008)



"Boundary" QFT ⟺ "Bulk" Gravity

7-7 open strings : Low energy dynamics for D7 branes (DBI action)

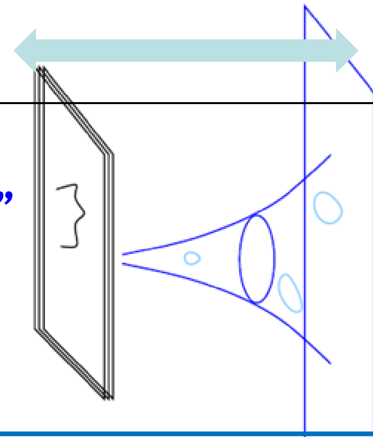
$$S_{D7} = -\mu_7 \int d^8\xi \sqrt{-\det\left(P[G]_{ab} + 2\pi\alpha' F_{ab}\right)} + \frac{(2\pi\alpha')^2}{2}\mu_7 \int P[C^{(4)}] \wedge F \wedge F$$

$$\mu_7 = [(2\pi)^7 g_s \alpha'^4]^{-1}$$

AdS/CFT Holography : simplest example

N=4 SU(Nc) SYM

SUGRA on AdS5 x S5



4D QFT (CFT) (open string)
on D-brane at the "boundary"

Classical Gravity(closed string)
in "bulk",

Conformal x R-symmetry I
SO(4,2) x SO(6)

Isometries of AdS x S5
SO(4,2) x SO(6)

Parameters (g_{YM}^2 , N)

$$\boxed{4\pi g_s N = \frac{R^4}{\alpha'^2} = \lambda = g_{YM}^2 N}$$

(g_s , R)

Extension of the AdS/CFT

- with $\beta(g^2) \to 0$ (asym. freedom) in asymptotic AdS
- less SUSY
- Finite T QFT Black Hole b.g.
- Chemical potential bulk gauge field
- Fundamental matters flavor brane
  - quenched approx. – w/o back reaction

  …
- QCD ? (phases, etc.) in ??

AdS/CFT Dictionary

Witten 98;
Gubser,Klebanov,Polyakov 98

Partition function of bulk gravity theory (semi-classial)

$$Z = \int_{\phi_0} \mathcal{D}\phi \exp\left(-S[\phi, \boxed{g_{\mu\nu}}]\right)$$

$$\phi(t, \mathbf{x};\ u=\infty) = u^{\Delta-4}\phi_0(t,\mathbf{x})$$

ϕ_0 bdry value of the bulk field ϕ

Generating functional of bdry QFT for operator $\mathcal{O}$

$$Z = \left\langle \exp \int_{boundary} d^d x \phi_0 \mathcal{O} \right\rangle$$

$$= \int \boxed{\mathcal{D}[\Phi]} \exp\{iS_4 + i\int \phi_0(x)\mathcal{O}\}$$

ϕ_0: source of the bdry op. $\mathcal{O}$

- ϕ: scalar → $S = \int d^4x du \sqrt{-g}\left(g^{ab}\partial_a\phi\partial_b\phi - m^2\phi^2\right)$ $\quad \phi(u) \sim u^{4-\Delta}\phi_0 + u^{\Delta}\langle\mathcal{O}\rangle$
- Correlation functions by $\dfrac{\delta^n Z_{\rm string}}{\delta\phi_0(t_1,\mathbf{x}_1)\cdots\delta\phi_0(t_n,\mathbf{x}_n)} = \left\langle T\,\mathcal{O}(t_1,\mathbf{x}_1)\cdots\mathcal{O}(t_n,\mathbf{x}_n)\right\rangle_{\rm field\ theory}$
- Radial coord. r in the bulk is proportional to the energy scale E of QFT
- 5D bulk field ϕ ←→ Operator $\mathcal{O}$
  - w/ 5D mass m_5 ←→ w/ Operator dimesion Δ
- 5D gauge symmetry ←→ Current (global symmetry)
- Large r (small z) ←→ Large Q

$\mathcal{O}$ (Operator in QFT) <-> ϕ (p-form Field in 5D)

$$(\Delta - p)(\Delta + p - 4) = m_5^2$$

Δ : Conformal dimension

m_5^2 : mass (squared)

Note : the fluctuation field ϕ on the bulk space corresponds to a source for the QCD Operator $\mathcal{O}$.

Ex)

4D: $\mathcal{O}(x)$	5D: $\phi(x, z)$	p	Δ	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3
$\langle \mathrm{Tr} G^2 \rangle$ Gluon cond .	dilaton	0	4	0
$\bar{q}_L \gamma^\mu q_L$ $\bar{q}_R \gamma^\mu q_R$ baryon density	vector w/ U(1)	1	3	0

fields in gravity

- massless dilaton
- scalar field with $m^2 = -\frac{3}{R^2}$
- m=0 vector field A_μ in the $SU(N_f)$ gauge group

dual

operators of QCD

- gluon condensation $\langle \mathrm{Tr} G^2 \rangle$
- chiral condensation $\bar{q}_R q_L$
- mesons in the $SU(N_f)$ flavor group

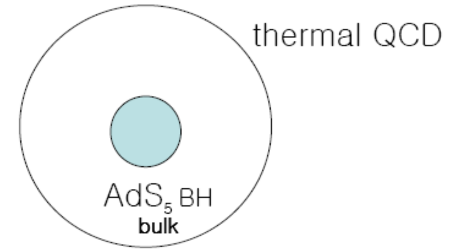
Temperatue

- Black hole gemometry
- $T = \frac{r_T}{\pi R^2}$

E. Witten (1998)

$$ds_5^2 = \frac{1}{z^2}\Big(f^2(z)dt^2 - (dx^i)^2 - \frac{1}{f^2(z)}dz^2\Big),$$

$$f^2(z) = 1 - (\frac{z}{z_T})^4 \qquad T = \frac{1}{\pi z_T}$$



Flavor degrees of freedom

- Adding probe brane
- $y(\rho) = M_q + \frac{<\bar{\psi}\psi>}{\rho^2} + \cdots \qquad (\rho >> 1)$

Chemical potential or Density

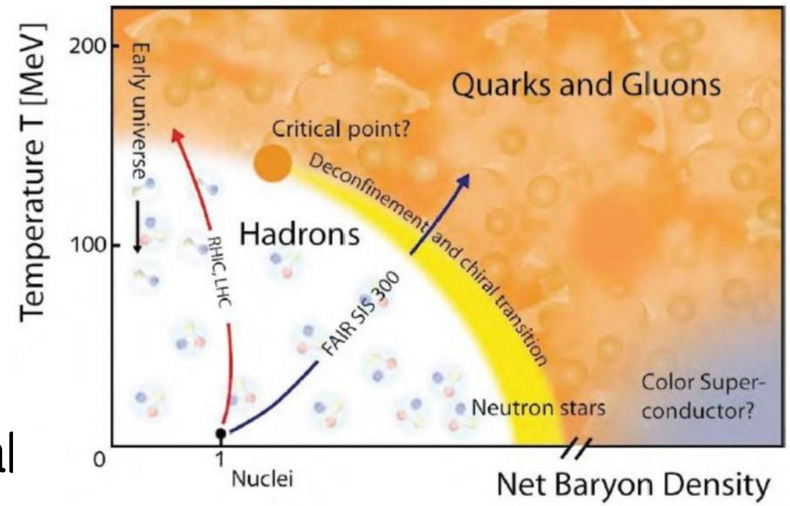
- Turning on $U(1)$ gauge field on probe brane
- $A_\mu \leftrightarrow < J^\mu >= \bar{\psi}\gamma^\mu\psi$
- $A_t = \mu + \frac{Q}{\rho^2} + \cdots \qquad (\rho >> 1)$

Source of gauge field

- End point of fundamental strings
- Physical object which carry $U(1)$ baryon charge
- Fundamental strings which connect probe brane and black hole $\rightarrow$ Quarks
- Fundamental strings which connect probe brane and baryon vertex $\rightarrow$ Baryons

AdS/QCD

Goal : Try to understand QCD using the 5 dimensional dual gravity theory (AdS/CFT correspondence)



Need the dual geometry of QCD.

1. Approaches :

Top-down Approach :

rooted in string theory

Find brane config. for the gravity dual

Bottom-up Approach : phenomenological

Introduce fields, etc. as needed based on the AdS/CFT

* Hard Wall Model – Introduce IR brane for confinement
* Soft Wall Model – dilaton running

Light-Front : radial direction of AdS <-> Parton momenta

(Brodsky, de Teramond, 2006)

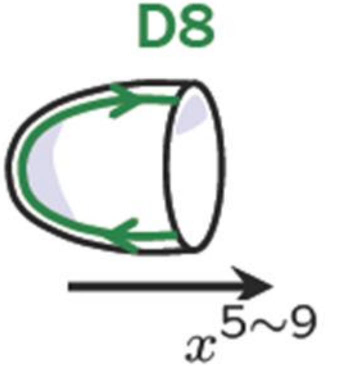
Top-Down Approach

Observation :

- Nc of D3 branes : AdS5 x S5 <-> N=4 SUSY YM
- Nc of D3 / orbifold, etc. : AdS5 x X <-> N=2, 1 YM

QFT with Asymptotic AdS SUGRA Duals

- N=1* Polchinski & Strassler,hep-th/0003136
- Cascading Gauge Theory Klebanov & Strassler, JHEP 2000
- Nc of D3 & N_f of D7 Kruczenski ,Mateos, Myers, Winters 2004
  N=2 SYM with quarks massive (in general): m_q
  Probe approximation (N_c>>N_f) (~ quenched approx.)
  (No back reaction to the bulk gometry from the flavor branes.)
  Free energy ~ Flavor-brane action
- Nc of D4 b.g. (Witten) + N_f of D8 system Sakai & Sugimoto 2005
  Topology : R(1,3) x R2 x S4
  The Effective Action : 5D U(Nf) YM CS theory
  $$S_{\text{5dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$
  -> closely related to QCD



- etc.

Bottom-Up Approach

Karch,Katz,Son,Stephanov, PRD(2006)

Introduce the contents (fields, etc.) as needed based on the AdS/CFT
– Phenomenological

- Kaluza-Klein modes – radial excitations of hadrons
  identified by the symmetry properties of the modes

- Confinement realized in
  - * Soft Wall Model – by dilaton running
  - * Hard Wall Model – by introducing IR brane for confinement

Ex) Confinement –Deconfinement

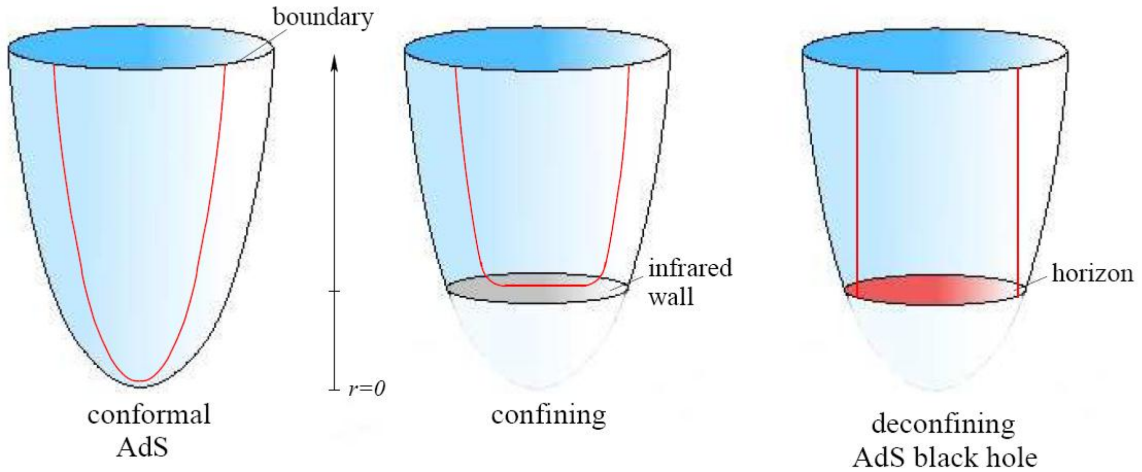


Figure from Erdmenger et.al, EPJA (2008)

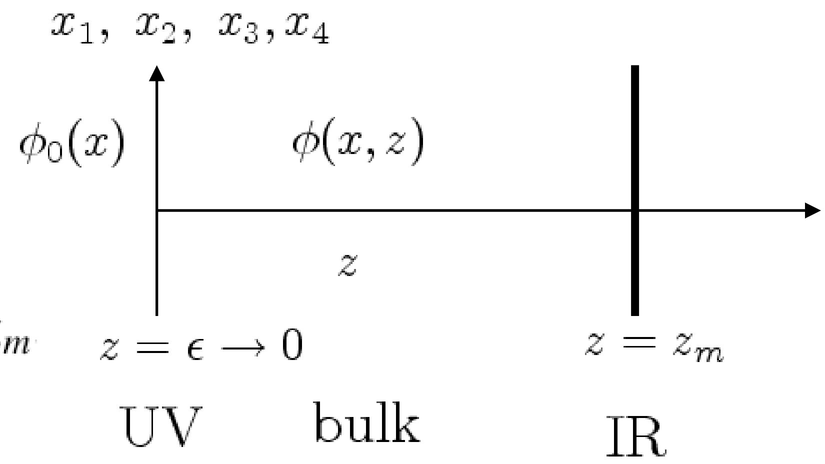
Erlich, Katz, Son, Stephanov, PRL (2005)
Da Rold, Pomarol, NPB (2005)

Ex) Hard wall Model

Infrared Brane at $z = z_m$
⟶ Confinement

Metric – Slice of AdS metric

$$ds^2 = \frac{1}{z^2}(-dz^2 + dx^\mu dx_\mu), \qquad 0 < z \leq z_m$$



5D action (Nf=2)

$$S = \int d^5x \sqrt{-g}\left(-\frac{1}{2g_5^2}\mathrm{Tr}\left(L_{MN}L^{MN} + R_{MN}R^{MN}\right) + \mathrm{Tr}\left(|D_M X|^2 + m_X^2 |X|^2\right)\right)$$

$$X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3$$

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

Observable	Measured (MeV)		
m_π	139.6±0.0004 [8]	139.6*	141
m_ρ	775.8±0.5 [8]	775.8*	832
m_{a_1}	1230±40 [8]	1363	1220
f_π	92.4±0.35 [8]	92.4*	84.0
$F_\rho^{1/2}$	345±8 [15]	329	353
$F_{a_1}^{1/2}$	433±13 [6, 16]	486	440
$g_{\rho\pi\pi}$	6.03±0.07 [8]	4.48	5.29

Parameters

$m_q \quad \sigma \quad z_m$

$$g_5^2 = \frac{12\pi^2}{N_c}$$

Witten '98

2. The Dual Geometry

(for the pure Yang-Mills theory without quark matters)

1) Low T (confining phase) : tAdS

(thermal) AdS space, no stable AdS black hole

2) High T (deconfining phase) : AdS BH

Schwarzschild-type AdS black hole

This geometry is described by the following action

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{g}\,(-\mathcal{R}+2\Lambda)$$

$\Lambda = -\frac{6}{R^2}$: cosmological constant

R : AdS radius

"confinement" phase ⟺ "de-confinement" phase

Thermal AdS (Low T) ⟺ AdS-BH (High T)

Hawking-Page transition

Transition of bulk geometry at temperature $\beta(=1/T)$.

Herzog [hep-th/0608151]

1) tAdS w/o IR cutoff → no confinement

AdS metric : $ds^2 = \frac{R^2}{z^2}\left(-dt^2 + d\vec{x}^2 + dz^2\right)$

the boundary located at z=0

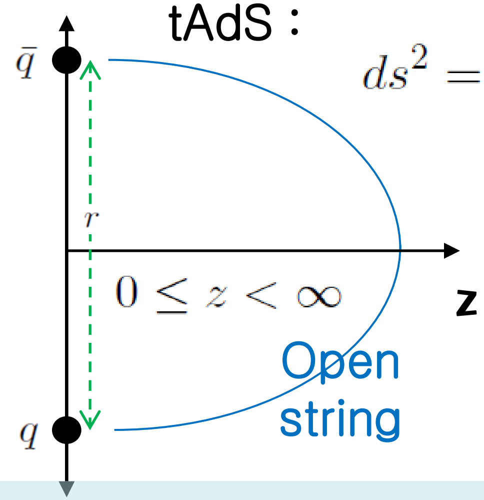
with the topology $R \times R^3$

Wick rotation

tAdS : $ds^2 = \frac{R^2}{z^2}\left(d\tau^2 + d\vec{x}^2 + dz^2\right)$

The periodicity of τ : $\beta = 1/T$

$S \times R^3$

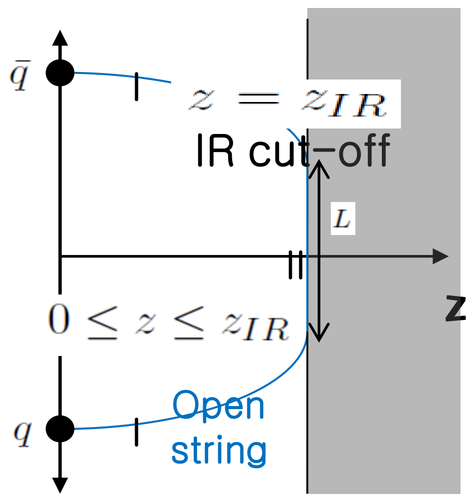


- the Coulomb potential $E \sim -\frac{1}{r}$

  -> no confining potential.

  [Maldacena, Phys.Rev.Lett. 80 (1998) 4859]

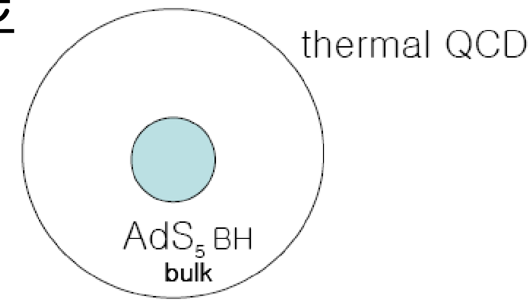
tAdS with IR cut-off (hard wall model) → confinement

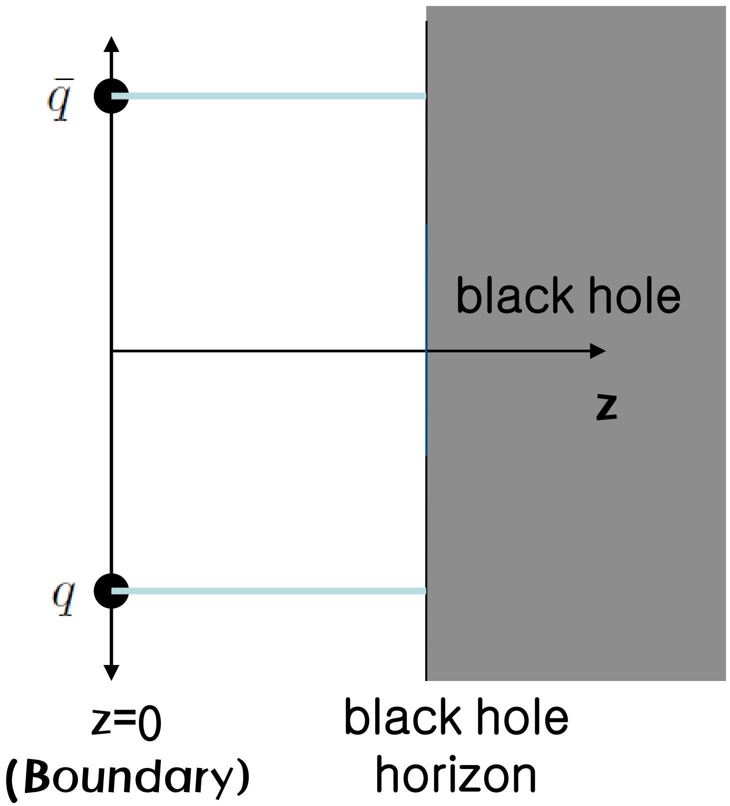


- In region I, the Coulomb-like potential. $E \sim -\frac{1}{r}$
- In region II, the confining potential. $E \sim T_s\, L$

2) AdS BH → deconfining phase

$$ds^2 = \frac{R^2}{z^2}\left((1 - mz^4)dt^2 + d\vec{x}^2 + \frac{1}{1 - mz^4}dz^2\right)$$





- For z<< z_h , AdS5 (x S5)
- an event horizon at $z_h = m^{-1/4}$
  smooth geometry if t periodic with $\pi \frac{R^2}{z_h}$
- The Hawking temperature : $T_H = \frac{1}{\pi z_h}$
  identified with the temperature of the boundary gauge theory.
- This black hole geometry corresponds to the deconfining phase of the boundary gauge theory, since there is no confining potential.

3) Hawking-Page phase transition [Herzog , Phys.Rev.Lett.98:091601,2007]

	Low T	β(=1/Tc)	High T
QCD Phase transition	confinement	⟷	de-confinement
dual			
Dual Geometry	tAdS w/ IR cutoff		AdS BH

Hawking-Page transition =Transition of bulk geometry at Tc

To investigate the Hawking-Page transition, we should calculate the free energy, which is proportional to the gravity on-shell action.

The regularized on-shell action

1) for the tAdS, $S_{tAdS} = \frac{4R^3}{\kappa^2}\int_0^{\beta'} dt \int_\epsilon^{z_{IR}} dz \frac{1}{z^5}$

β' : arbitrary

2) for the AdS BH $S_{AdSBH} = \frac{4R^3}{\kappa^2}\int_0^{\pi z_h} dt \int_\epsilon^{z_h} dz \frac{1}{z^5}$

To remove the divergence at $z=0$ introduce a UV cut-off ϵ

the period in the t-direction of tAdS β'
= the period in t-direction of AdS BH at z ϵ

$\Rightarrow \beta' = \pi z_h \sqrt{f(\epsilon)}$

Using this, the difference of two actions :

$$\Delta S = \lim_{\epsilon\to 0}(S_{AdSBH} - S_{tAdS}) = \frac{\pi z_h R^3}{\kappa^2}\left(\frac{1}{z_{IR}^4} - \frac{1}{2z_h^4}\right)$$

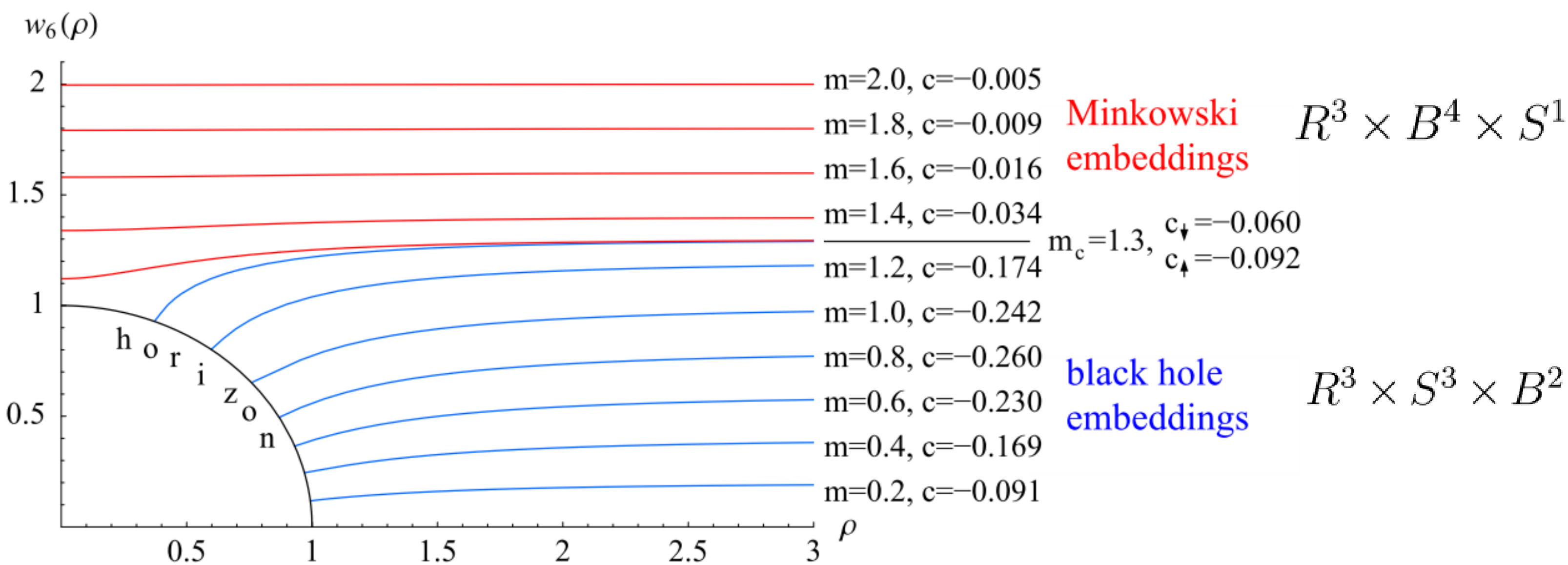
- The Hawking-Page (or deconfinement) transition occurs at

$$z_h = \frac{z_{IR}}{2^{1/4}} \quad \text{or} \quad T_c = \frac{2^{1/4}}{\pi z_{IR}}$$

- At the low temperature $T < T_c$
  the tAdS space is stable (confining phase) .

- At the high temperature $T > T_c$
  the AdS BH is more stable (deconfining phase)

Note : Phenomenology such as meson spectrum, etc. can be studied
by embedding D7, etc.



Dual geometry for finite chemical potential

bulk	boundary
field A_0	dual operator $\bar{\psi}\gamma_0\psi = \psi^\dagger\psi$ (quark number density)

Chamblin–Emparan–Johnson–Myers,1999
Cvetic–Gubser,1999

5–dimensional action dual to the gauge theory with quark matters

$$S = \int d^5x\sqrt{G}\left[\frac{1}{2\kappa^2}(-\mathcal{R}+2\Lambda)+\frac{1}{4g^2}F_{MN}F^{MN}\right]$$

Euclidean

Wick rotation $t \to -i\tau$

Equations of motion

1) Einstein equation $\mathcal{R}_{MN} - \frac{1}{2}G_{MN}\mathcal{R} + G_{MN}\Lambda = \frac{\kappa^2}{g^2}\left(F_{MP}F_N^P - \frac{1}{4}G_{MN}F_{PQ}F^{PQ}\right)$

2) Maxwell equation $0 = \partial_M\sqrt{-G}G^{MP}G^{NQ}F_{PQ}$

Ansatz :

$$\begin{cases} ds^2 = \dfrac{R^2}{z^2}\left(f(z)dt^2 + d\vec{x}^2 + \dfrac{1}{f(z)}dz^2\right) \\ A_0 = A(z) \quad \text{and other are zero.} \end{cases}$$

Solutions

S.-J. Sin, 2007

- most general solution, which is RNAdS BH (RN AdS black hole)

$$f(z) = 1 - mz^4 + q^2 z^6$$
$$A(z) = i\left(\mu - Qz^2\right)$$

corresponds to the deconfining phase (quark-gluon plasma)

m	black hole mass
q	black charge
μ	quark chemical potential
$Q = \sqrt{\frac{3g^2R^2}{2\kappa^2}}\, q$	quark number density

Note

1) The value of A_0 at the boundary ($z=0$) corresponds to the quark chemical potential μ of QCD.

2) The dual operator of A_0 is denoted by Q, which is the quark (or baryon) number density operator.

3) We use $\frac{1}{2\kappa^2} = \frac{N_c^2}{8\pi^2 R^3}$ and $\frac{1}{g^2} = \frac{N_c N_f}{4\pi^2 R}$

- What is the dual geometry of the confining (or hadronic) phase ?

find non-black hole solution (BHL, Park ,Sin JHEP 0907,(2009))

$$f(z) = 1 + q^2 z^6$$
$$A(z) = i\left(\mu - Qz^2\right)$$

We call it tcAdS (thermal charged AdS space)

- baryonic chemical potential

$$\mu_B = 3\mu$$

- baryon number density

$$Q_B = Q/3$$

Note : Solutions in both phases are valid for arbitrary densities

RNAdS BH (QGP)

$$ds^2 = \frac{R^2}{z^2}\left((1 - mz^4 + q^2z^6)d\tau^2 + d\vec{x}^2 + \frac{1}{1 - mz^4 + q^2z^6}dz^2\right)$$

- black hole horizon z_+ : $0 = f(z_+) = 1 - mz_+^4 + q^2z_+^6$
- Hawking temperature $T_{RN} = \frac{1}{\pi z_+}\left(1 - \frac{1}{2}q^2z_+^6\right)$
- For the norm of A_0 at the black hole horizon $||A(z)|| \equiv g^{\tau\tau}A_\tau A_\tau$

  to be regular, we should impose the Dirichlet boundary condition

$$A(z_+) = 0$$

- From this, we can obtain a relation between Q and μ

$$Q^2 = \frac{\mu^2}{z_+^4}$$

- Using these relations, can rewrite z_+ as a function of μ and T_{RN}

$$z_+ = \frac{3g^2R^2}{2\kappa^2\mu^2}\left(\sqrt{\pi^2T_{RN}^2 + \frac{4\kappa^2\mu^2}{3g^2R^2}} - \pi T_{RN}\right)$$

tcAdS (Hadronic phase)

- Impose the Dirichlet boundary condition at the IR cut-off

$$A(z_{IR}) = i\alpha\mu,$$

where α is an arbitrary constant and will be determined later.

- Using this, we can find the relation between μ and Q

$$Q = \frac{(1-\alpha)\mu}{z_{IR}^2}.$$

- After imposing the Dirichlet B.C at the UV cut-off, the renormalized on-shell action for the tcAdS

$$\begin{aligned}
\bar{S}^D_{tc} &= -\frac{V_3 R^3}{\kappa^2}\frac{1}{T_{tc}}\left(\frac{1}{z_{IR}^4} + \frac{2\kappa^2}{3g^2R^2}\frac{(1-\alpha)^2\mu^2}{z_{IR}^2}\right) \\
&\equiv \frac{\Omega_{tc}}{T_{tc}}
\end{aligned}$$

- From this renormalized action, the particle number is reduced to

$$N = \frac{2}{3}(1-\alpha)\frac{2R}{g^2}QV_3$$

- Using the Legendre transformation, N should satisfy the following relation $\mu N = S_b T_{tc}$

  where the boundary action for the tcAdS is given by

$$S_b = \frac{\mu}{T_{tc}}\frac{2R}{g^2}QV_3$$

- So, we see that α should be $-1/2$ for the consistency.

  Then, the renormalized on-shell action for the tcAdS

$$\bar{S}^D_{tc} = -\frac{V_3 R^3}{\kappa^2}\frac{1}{T_{tc}}\left(\frac{1}{z_{IR}^4} + \frac{3\kappa^2}{2g^2R^2}\frac{\mu^2}{z_{IR}^2}\right)$$

  with $\mu = \frac{2}{3}Qz_{IR}^2.$

Hawking-Page transition

- The difference of the on-shell actions for RN AdS BH and tcAdS

$$\Delta S = S^D_{RN} - S^D_{tc}$$
$$= \frac{V_3 R^3}{\kappa^2} \frac{1}{T_{RN}} \left(\frac{1}{z_{IR}^4} - \frac{1}{2 z_+^4} + \frac{3\kappa^2}{2g^2 R^2} \frac{\mu^2}{z_{IR}^2} - \frac{\kappa^2}{3g^2 R^2} \frac{\mu^2}{z_+^2} \right)$$

- Wher $\Delta S = 0$, Hawking-Page transition occurs

- Suppose that $\Delta S = 0$ at a critical point $z_+ = z_c$

  1) For $z_+ < z_c$, ΔS becomes negative. $\Longrightarrow$ deconfining phase

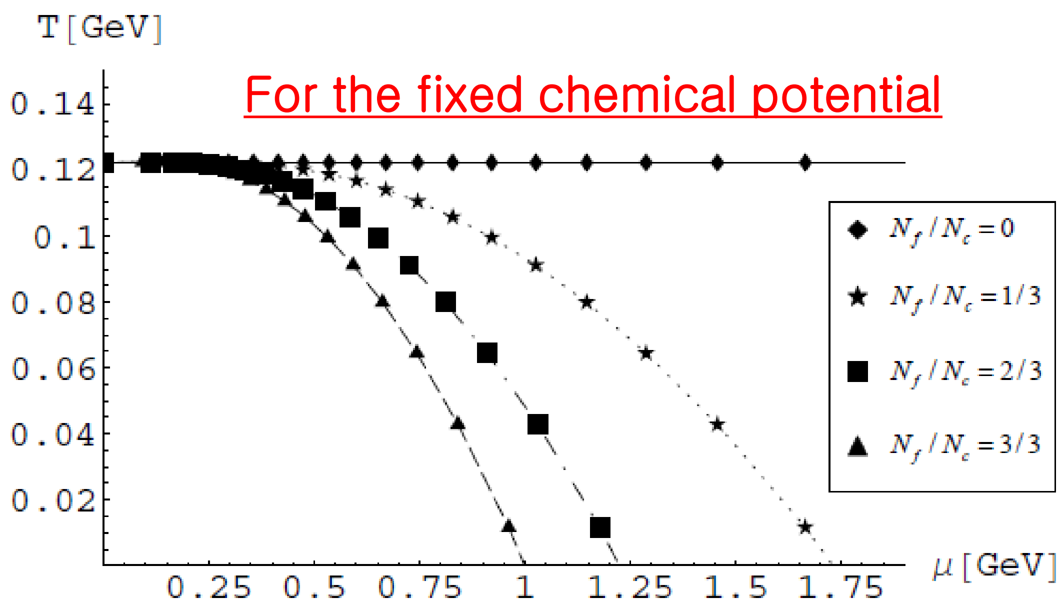
  2) For $z_c < z_+ \leq z_{IR}$, tcAdS is stable. $\Longrightarrow$ confining phase

- Introducing new dimensionless variables

$$
\begin{aligned}
\tilde{z}_c &\equiv \frac{z_c}{z_{IR}}, \\
\tilde{\mu}_c &\equiv \mu_c z_{IR}, \\
\tilde{T}_c &\equiv T_c z_{IR},
\end{aligned}
$$

the Hawking–Page transition occurs at

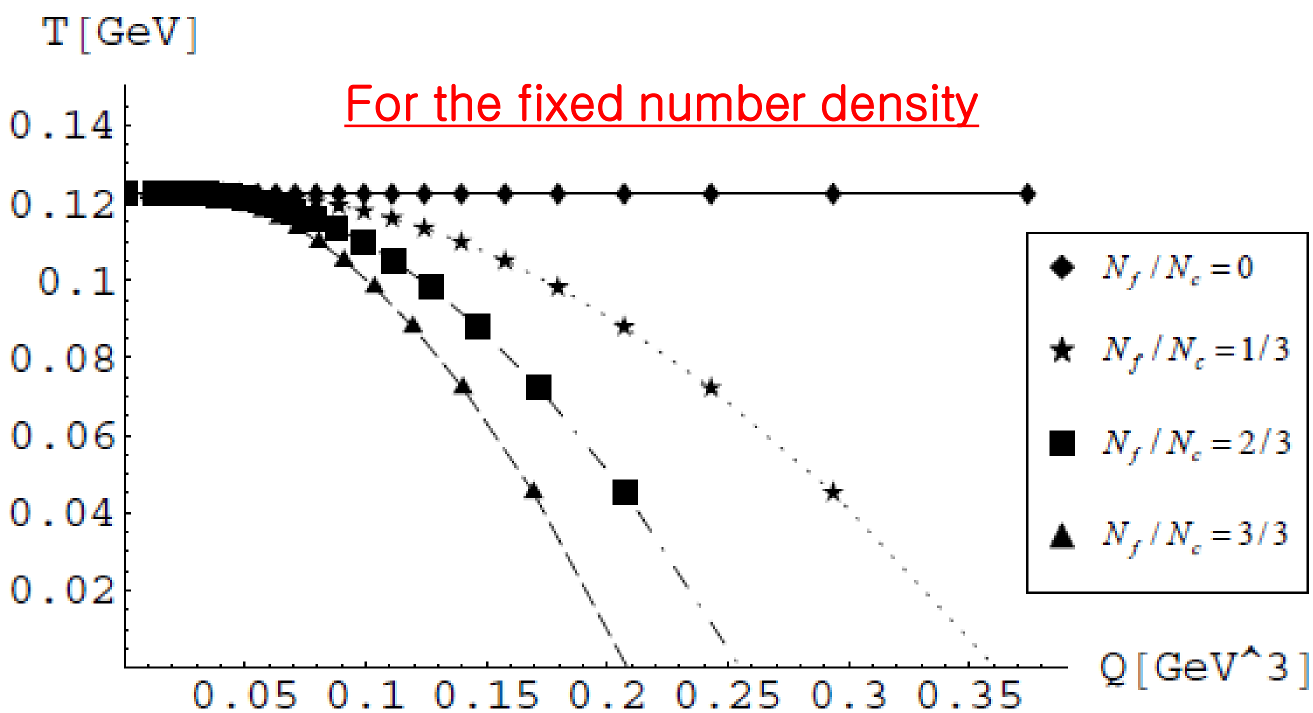
$$
\begin{cases}
\tilde{\mu}_c = \sqrt{\dfrac{3N_c}{N_f}\,\dfrac{(1-2\tilde{z}_c^4)}{\tilde{z}_c^2(9\tilde{z}_c^2-2)}}\,, \\[2ex]
\tilde{T}_c = \dfrac{1}{\pi\tilde{z}_c}\left(1-\dfrac{1-2\tilde{z}_c^4}{9\tilde{z}_c^2-2}\right).
\end{cases}
$$



● After the Legendre transformation, the Hawking-Page transition in the fixed quark number density case occurs at

$$\tilde{Q}_c = \sqrt{\frac{3N_c}{2N_f}\frac{(1-2\tilde{z}_c^4)}{\tilde{z}_c^4(5\tilde{z}_c^2-2)}},$$

$$\tilde{T}_c = \frac{1}{\pi\tilde{z}_c}\left[1-\frac{\tilde{z}_c^2}{2}\frac{(1-2\tilde{z}_c^4)}{(5\tilde{z}_c^2-2)}\right]$$

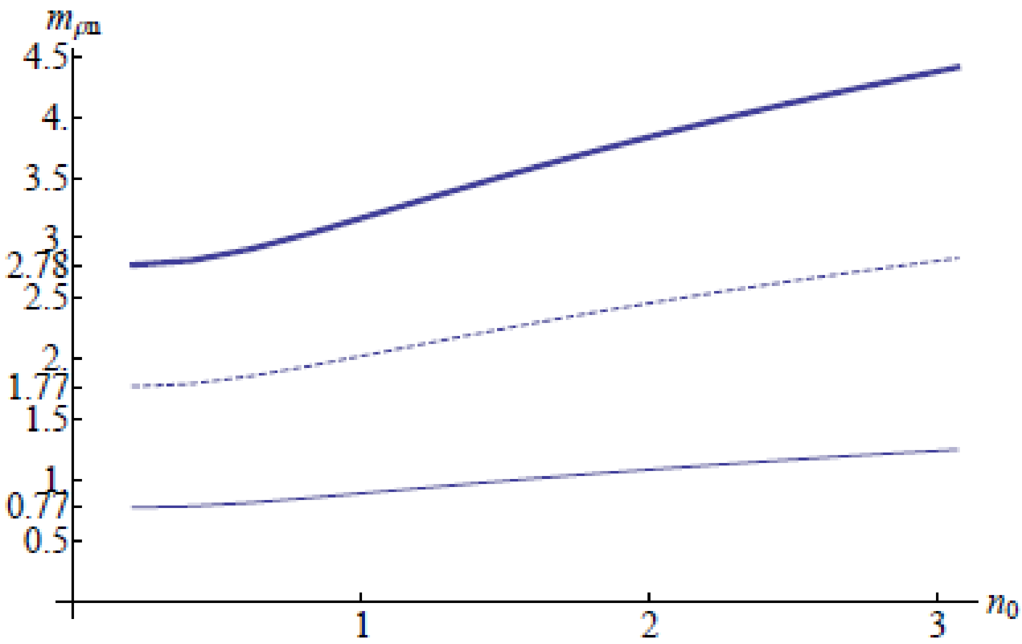


Light meson spectra in the hadronic phase

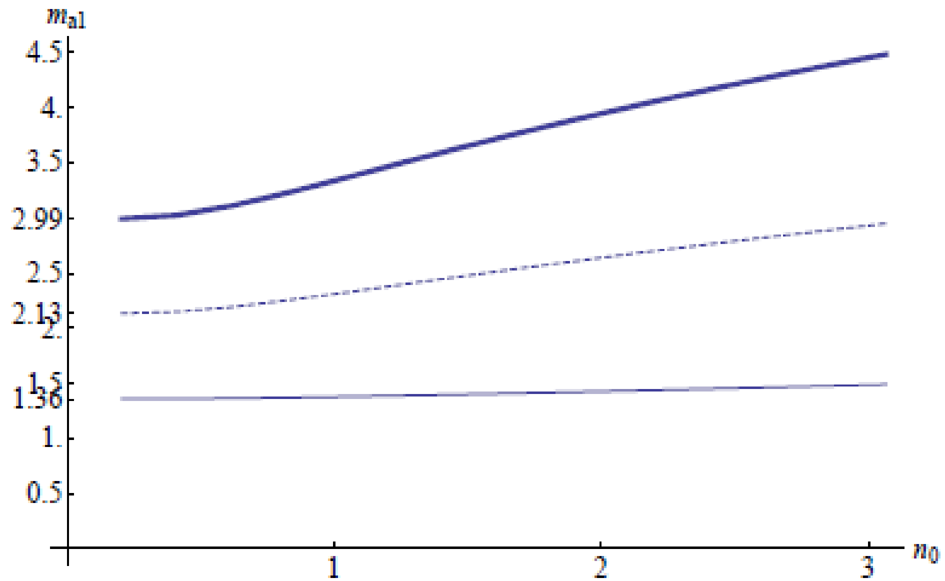
Turn on the fluctuation in bulk corresponding the meson spectra in QCD

$$\Delta S = \int d^5x\sqrt{G}\ \mathrm{Tr}\left[|DX|^2 - \frac{3}{R^2}|X|^2 + \frac{1}{4g_5^2}\left(F_L^2 + F_R^2\right)\right]$$

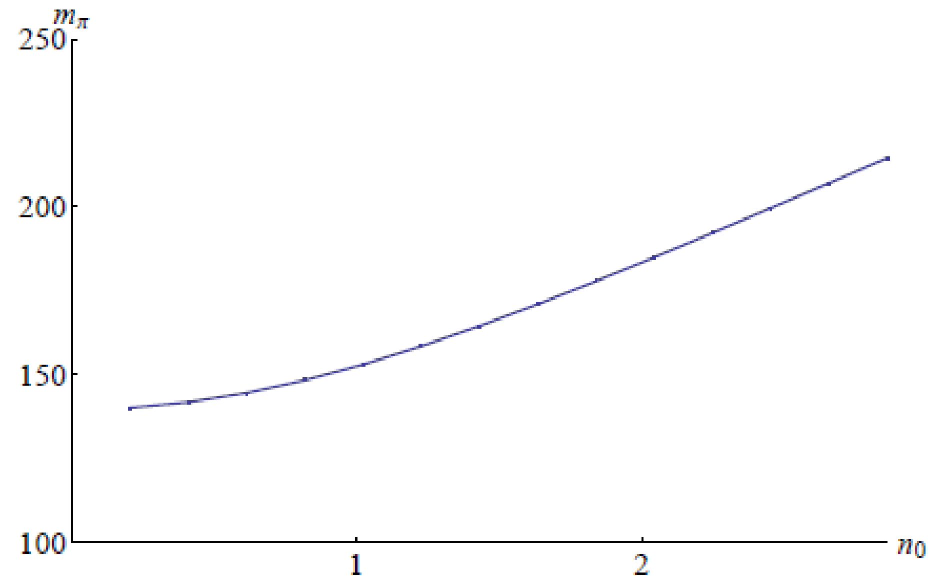
1. Vector meson



2. Axial vector meson



3. pion



III. AdS/QCD based on the D7 embedding in black D3/D-instanton geometry

B. Gwak, M. Kim, BHL,Y.Seo, S.-J. Sin, arXiv :1105.xxxx

Motivation

- Alternative to the Geometrical phase Transition for in AdS/CFT ?
- Baryon Vertex (phase) and confinement at finite T (Black Hole Background) ?

Finite Temperature with Dilaton background (Solution of Type IIB SUGRA)

$$ds_{10}^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f(r)^2 dt^2 + d\vec{x}^2\right) + \frac{1}{f(r)^2}\frac{R^2}{r^2}dr^2 + R^3 d\Omega_5^2\right], \qquad R^4 = 4\pi g_s N_c \alpha'^2$$

$$e^{\Phi} = 1 + \frac{q}{r_T^4}\log\frac{1}{f(r)^2}, \qquad \chi = -e^{-\Phi} + \chi_0,$$

$$f(r) = \sqrt{1 - \left(\frac{r_T}{r}\right)^4},$$

Hawking Temperature

$$T = r_T/\pi R^2$$

Zero Temperature Limit : becomes near horizon geometry of D3-D(-1)

$$ds^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(dt^2 + d\vec{x}^2\right) + \frac{R^2}{r^2}\left(dr^2 + r^2 d\Omega_5^2\right)\right],$$

(Liu & Tseytlin 9903091)

$$e^{\Phi} = 1 + \frac{q}{r^4}, \qquad \chi = -e^{-\Phi} + \xi_\infty.$$

- AdSxS5 at UV Flat at IR (w/ dilaton singular)
- N=2 (with gluon condensation)

Ex) Zero Temperature and without density

Background Metric by D3 & D−instantons

$$ds^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(dt^2+d\vec{x}^2\right)+\frac{R^2}{r^2}\left(dr^2+r^2d\Omega_5^2\right)\right],$$

$$e^{\Phi} = 1+\frac{q}{r^4}, \qquad \chi = -e^{-\Phi}+\xi_\infty .$$

(Liu & Tseytlin 9903091)

$$\frac{R^2}{r^2}(d\rho^2+\rho^2 d\Omega_3^2+dw_5^2+dw_6^2)$$

- AdSxS5 at UV
  Flat at IR (w/ dilaton singular)
- N=2 (with gluon condensation)

D7 Brane as a Probe

$$\rho^2 = w_1^2+\ldots+w_4^2$$

$$r^2 = \rho^2+w_5^2+w_6^2 = \rho^2+y^2$$

Induced metric on D7

$$ds^2_{D7} = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(dt^2+d\vec{x}^2\right)+\frac{R^2}{r^2}\left((1+y'^2)d\rho^2+\rho^2\Omega_3^2\right)\right]$$

DBI action of D7

$$S_{D7} = -\mu_7\int d^8\xi\,\sqrt{-\det\left(P[G]_{ab}+2\pi\alpha' F_{ab}\right)}+\frac{(2\pi\alpha')^2}{2}\mu_7\int P[C^{(4)}]\wedge F\wedge F$$

$$\mu_7 = [(2\pi)^7 g_s\alpha'^4]^{-1}$$

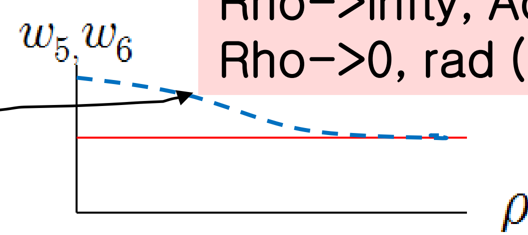
becomes (D7 wraps S3 of S5)

$$S_{D7} = -\tau_7\int dt d\rho\; e^{\Phi}\rho^3\sqrt{1+y'^2}$$

$$\tau_7 = \mu_7 V_3\Omega_3$$

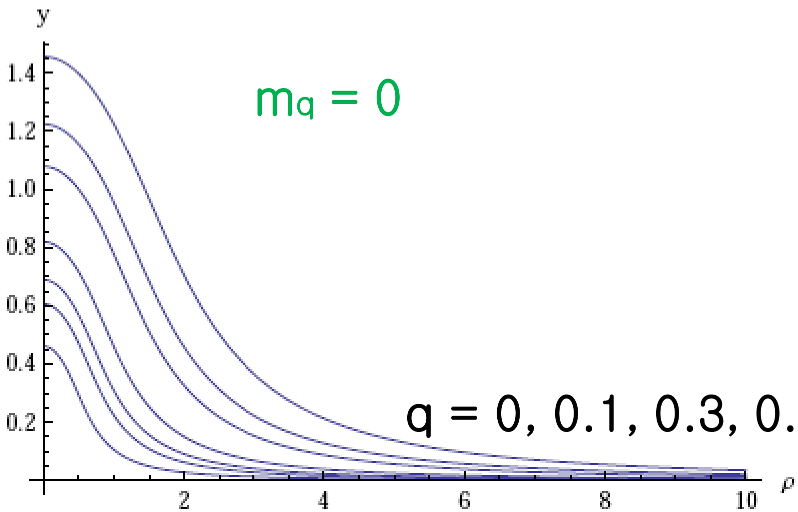
Embedding solution :

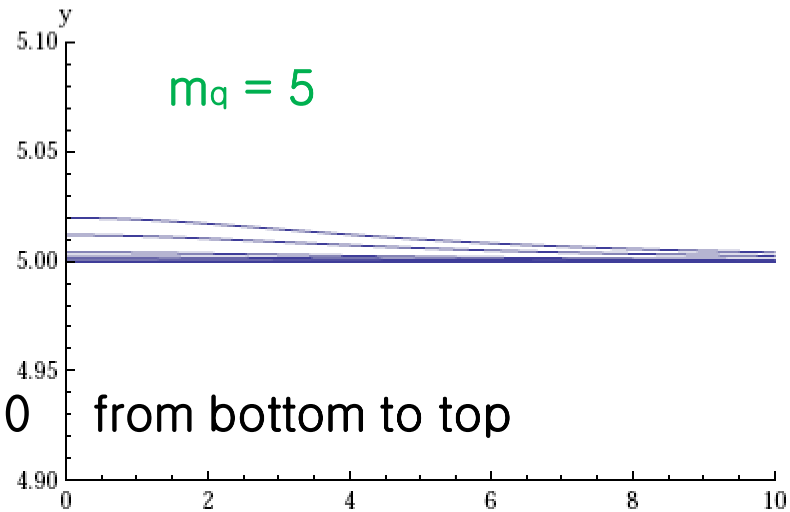
$$y(\rho) = M_q+\frac{<\bar\psi\psi>}{\rho^2}+\cdots \qquad (\rho >> 1)$$



Rho−>infty, AdS5xS3
Rho−>0, rad (S3)−>0

- solution of D7 brane embeddings

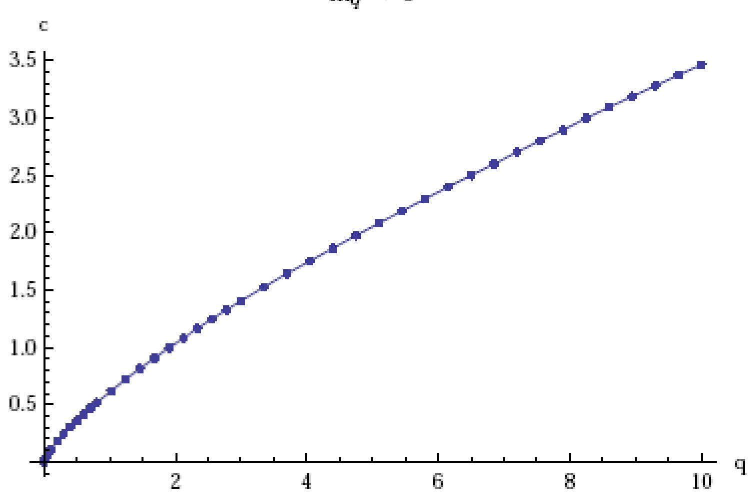




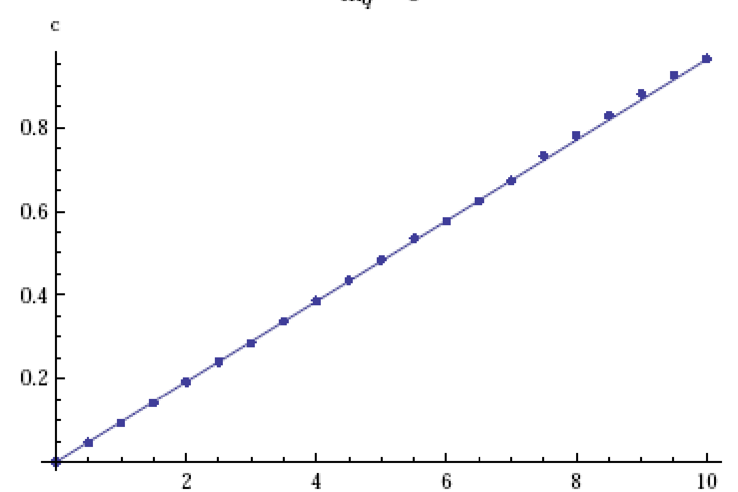
q = 0, 0.1, 0.3, 0.5, 1,3,5,10 from bottom to top

- q dependence of chiral condensation

$m_q \to 0$



$m_q = 5$



- In large quark mass limit, $< \bar{\psi}\psi > = \frac{\alpha_s N_f}{12\pi m_q} < \mathrm{Tr} F^2 >$

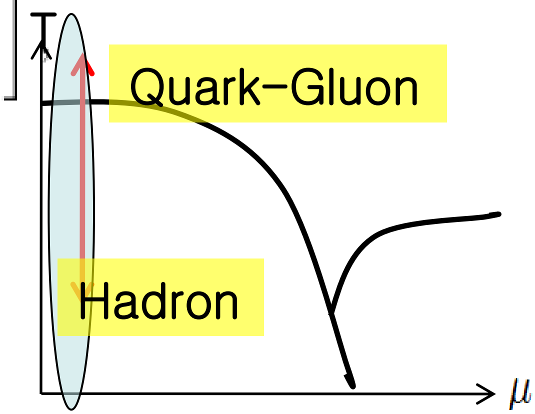
Ex) Finite Temperature and without density

Finite Temperature (Black Hole geometry) of D3/D-instanton system

$$ds_{10}^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f(r)^2 dt^2 + d\vec{x}^2\right) + \frac{1}{f(r)^2}\frac{R^2}{r^2}dr^2 + R^3 d\Omega_5^2\right]$$

$$e^{\Phi} = 1 + \frac{q}{r_T^4}\log\frac{1}{f(r)^2}, \qquad \chi = -e^{-\Phi} + \chi_0,$$

$$f(r) = \sqrt{1 - \left(\frac{r_T}{r}\right)^4}, \qquad T = r_T/\pi R^2.$$



Rewrite in terms of dimensionless parameter $\frac{d\xi^2}{\xi^2} = \frac{dr^2}{r^2 f^2(r)}$

$$ds^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f(r)^2 dt^2 + d\vec{x}^2\right) + \frac{R^2}{\xi^2}\left(d\xi^2 + \xi^2 d\Omega_5^2\right)\right],$$

or

$$ds^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f(r)^2 dt^2 + d\vec{x}^2\right) + \frac{R^2}{\xi^2}\left(d\rho^2 + \rho^2\Omega_3^2 + dy^2 + y^2 d\phi^2\right)\right].$$

where $\xi^2 = \rho^2 + y^2$

$$\left(\frac{r}{r_T}\right)^2 = \frac{1}{2}\left(\frac{\xi^2}{\xi_T^2} + \frac{\xi_T^2}{\xi^2}\right), \quad \text{and} \quad f = \left(\frac{1 - \xi_T^4/\xi^4}{1 + \xi_T^4/\xi^4}\right) \equiv \frac{\omega_-}{\omega_+}.$$

Induced metric on D7

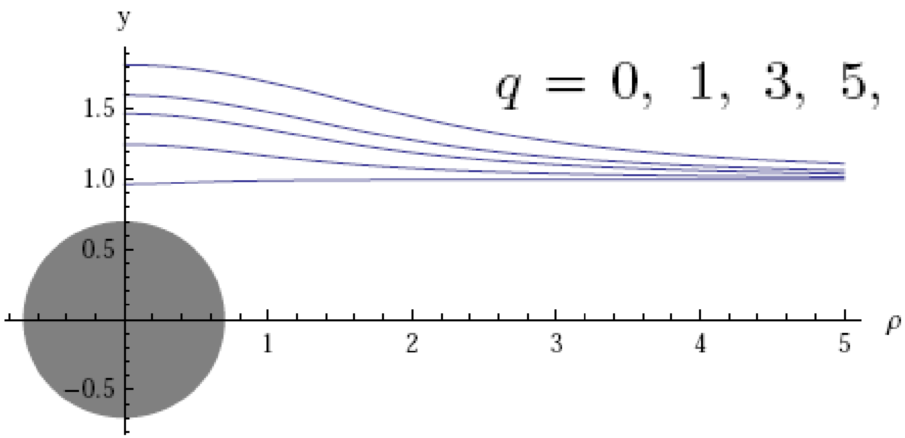
$$ds^2_{D7} = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f(r)^2dt^2 + d\vec{x}^2\right) + \frac{R^2}{\xi^2}\left((1+y'^2)d\rho^2 + \rho^2\Omega_3^2\right)\right]$$

DBI action of D7

$$S_{D7} = \tau_7 \int dt d\rho e^{\Phi} \rho^3 \omega_+ \omega_- \sqrt{1+y'^2} \qquad \tau_7 = \xi_T^4 \mu_7 V_3 \Omega_3$$

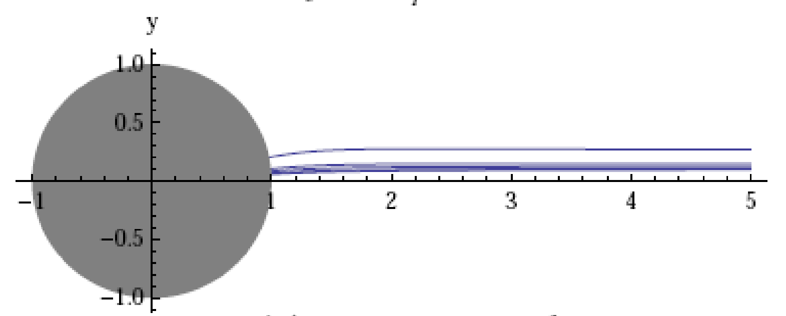
Minkowski and Black Hole embedding

$\xi_T=0.7,\ m_q=1$

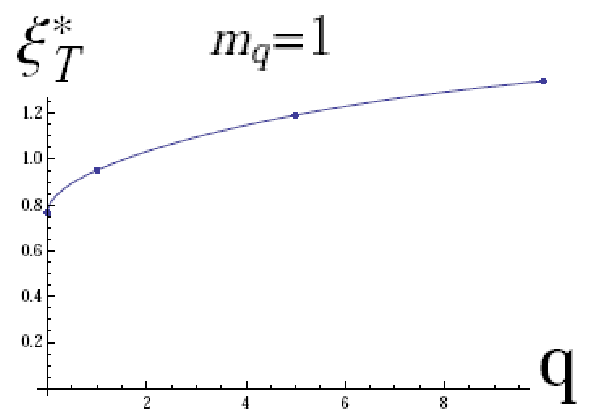


$q\ =\ 0,\ 1,\ 3,\ 5,\ 10$ (from below)

$\xi_T=1,\ m_q=0.1$

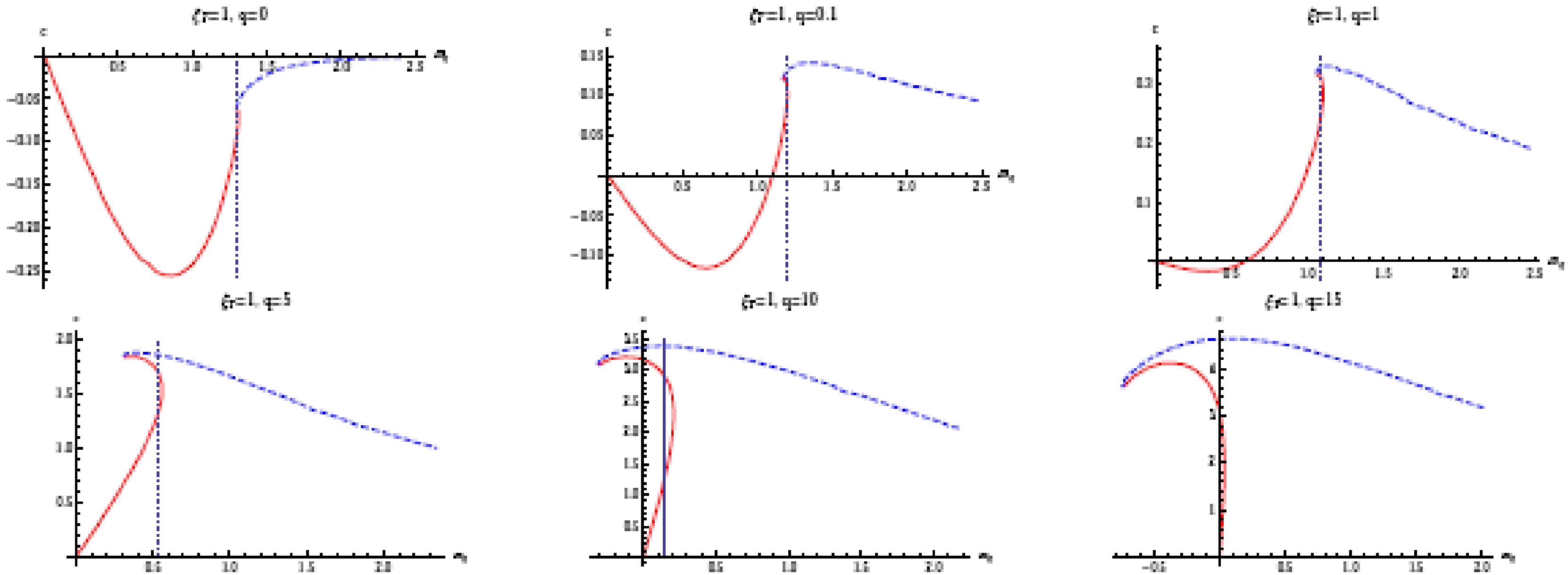


‘Repulsion’ by q > ‘attraction’ by BH

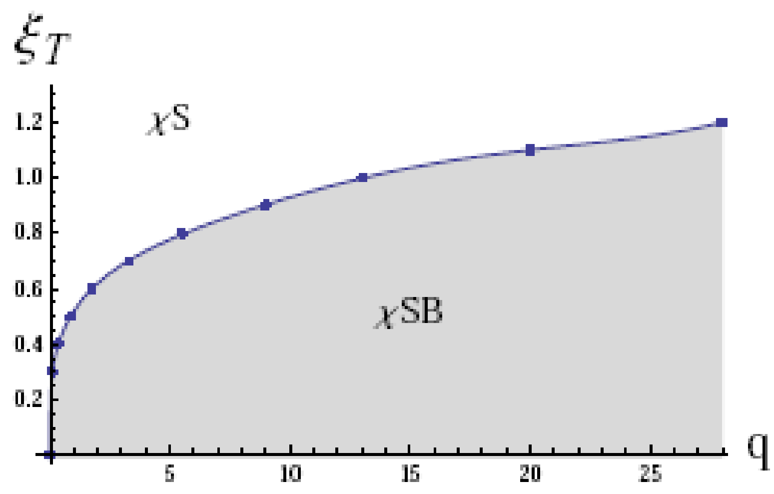


Phase transition temperature increases as q increases

- Finite temperature without density
  - Chiral condensation



  - q dependence of chiral symmetry restoration temperature



Finite Temperature and with finite density

Turn on U(1) gauge field on D7 brane

DBI action of D7

$$S_{D7} = -\tau_7 \int dt d\rho \rho^3 e^{\Phi/2} \omega_+^{3/2} \sqrt{e^{\Phi/2} \frac{\omega_-^2}{\omega_+}(1+\dot{y}^2) - \tilde{F}^2} := \int dt d\rho \mathcal{L}_{D7},$$

$$\tau_7 = \mu_7 V_4 \Omega_3, \quad \tilde{F} = 2\pi\alpha' F_{t\rho}$$

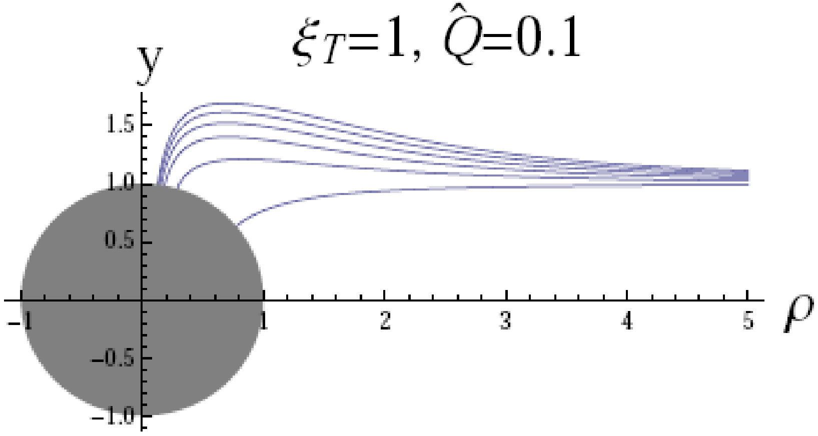
Minkowski and Black Hole embedding

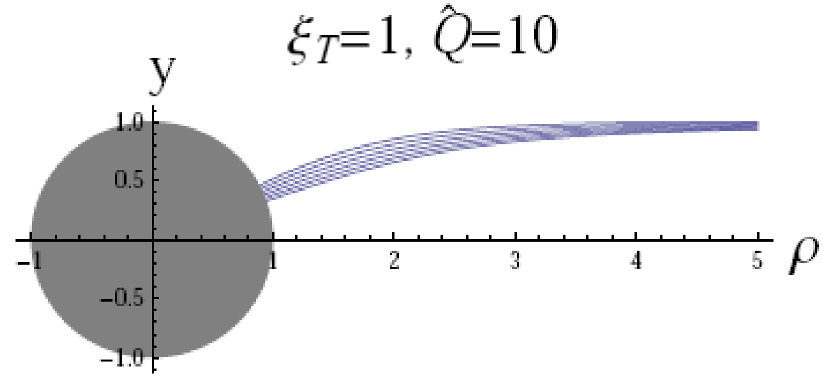
- Legendre transformation

$$\begin{aligned} \mathcal{H}_{D7} &= \tilde{F} \frac{\partial \mathcal{L}_{D7}}{\partial \tilde{F}} - \mathcal{L}_{D7} \\ &= \tau_7 \int d\rho \sqrt{e^{\Phi} \frac{\omega_-^2}{\omega_+}(1+\dot{y}^2)} \sqrt{\frac{\tilde{Q}^2}{\tau_7^2} + \rho^6 e^{\Phi} \omega_+^3}, \end{aligned}$$

- Source of $U(1)$ gauge field on D7 brane is endpoint of fundamental strings
- There are two way to attaching fundamental strings on D7 brane

Quark Phase

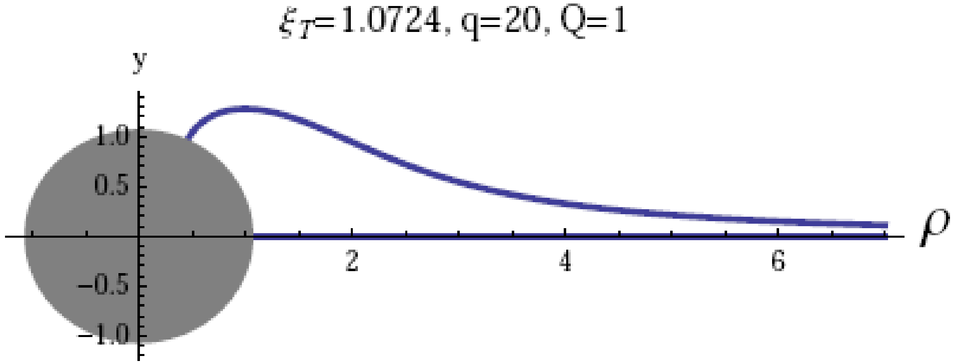


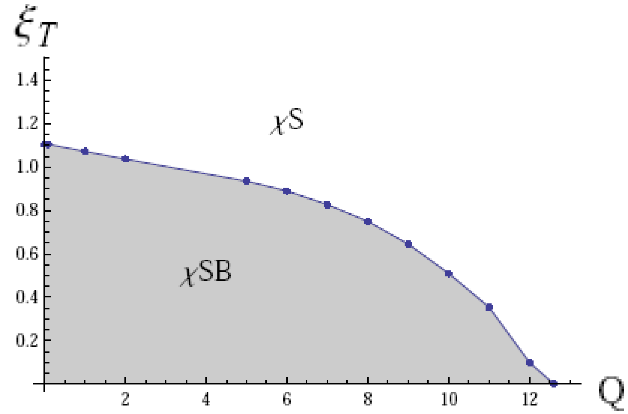


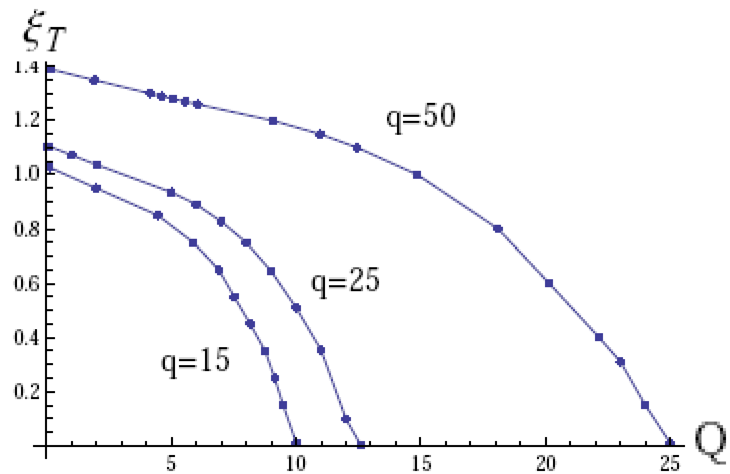
Regularity condition $\dot{y}(\rho_{min}) = \tan\theta$

- As q increases, the repulsion effect on D7 also increases.
- F1 strings connect BH horizon and probe brane
- Physical object is freely moving quark

- In $m_q \to 0$ limit, we have two phases

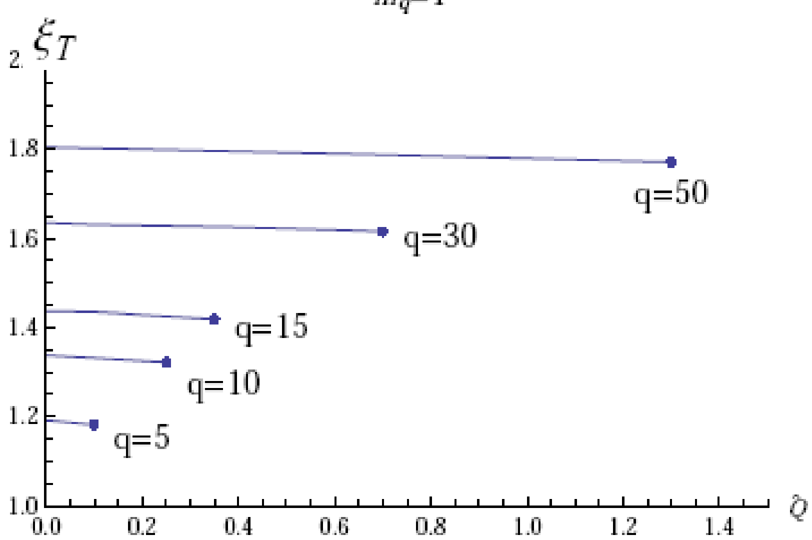






(Note : If q=0, then the trivial flat embedding is the unique solution for $m_q \to 0$.)

Finite quark mass $m_q \neq 0$



Baryon Phase

Background metric $F_{t\theta} \neq 0$

$$ds^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}\left(f(r)^2 dt^2 + d\vec{x}^2\right) + R^2\left(\frac{d\xi^2}{\xi^2} + d\theta^2 + \sin^2\theta d\Omega_4^2\right)\right]$$

Induced metric on D5

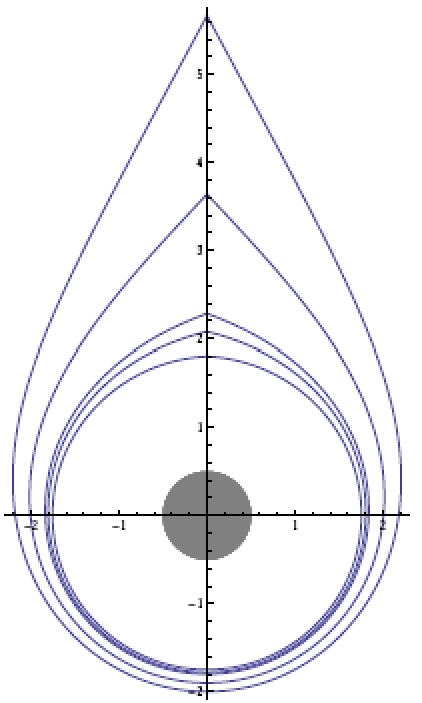
$$ds_{D5}^2 = e^{\Phi/2}\left[\frac{r^2}{R^2}f^2 dt^2 + R^2\left(\frac{\xi'^2}{\xi^2} + 1\right)d\theta^2 + R^2\sin^2\theta d\Omega_4^2\right]$$

DBI action

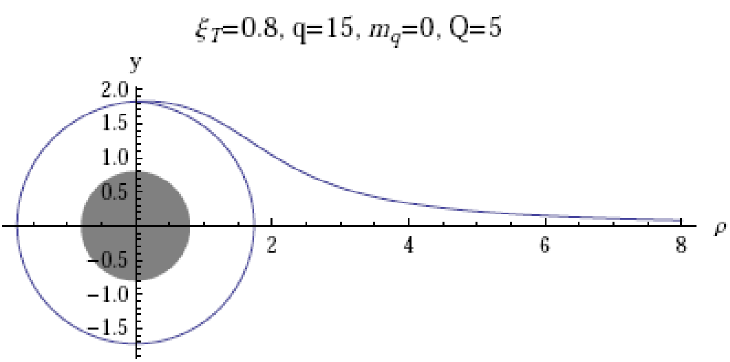
$$S_{D5} = -\mu_5\int e^{-\Phi}\sqrt{-\det(g + 2\pi\alpha' F)} + \mu_5\int A_{(1)}\wedge G_{(5)}$$

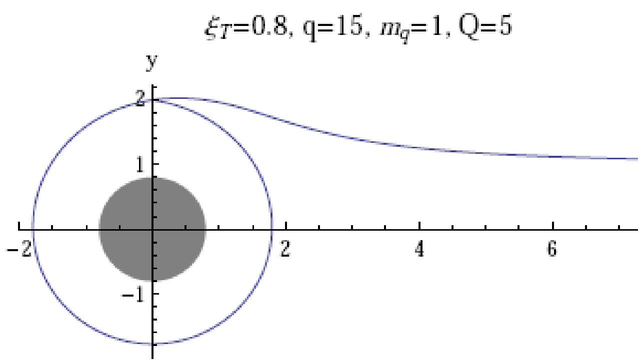
$$= \tau_5\int dt d\theta \sin^4\theta e^{\Phi}\left[-\sqrt{e^{\Phi}\frac{\omega_-^2}{\omega_+}(\xi^2 + \xi'^2) - \tilde{F}^2} + 4\tilde{A}_t\right]$$

- F1's connect spherical D5 & probe D7
- Phys. Ob. = baryon vtx (bd state of Nc quarks)
- χ−symm. broken



D7 brane at the tip of D5 with force balance condition



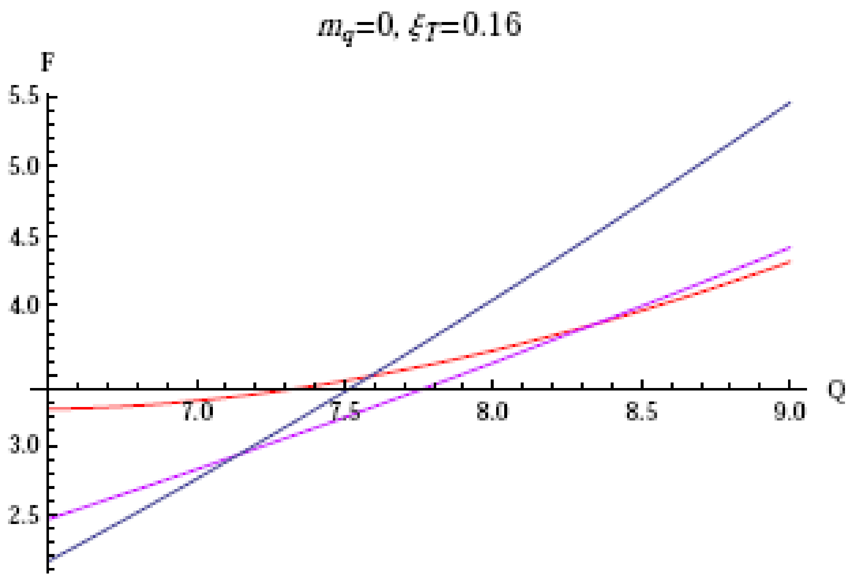


- Phase transition
  - for give m_q and q, there are two kind of embeddings in same temperature
  - we have to choose physical embedding by comparing free energy
  - free energy for quark phase

$$\mathcal{F}_{\rm quark}(\hat{Q}) = \tau_7 \int_{\rho_{min}}^{\infty} d\rho \hat{\mathcal{H}}_{D7}(\hat{Q})\Big|_{\rm quark\ phase}$$

  - free energy for baryon phase

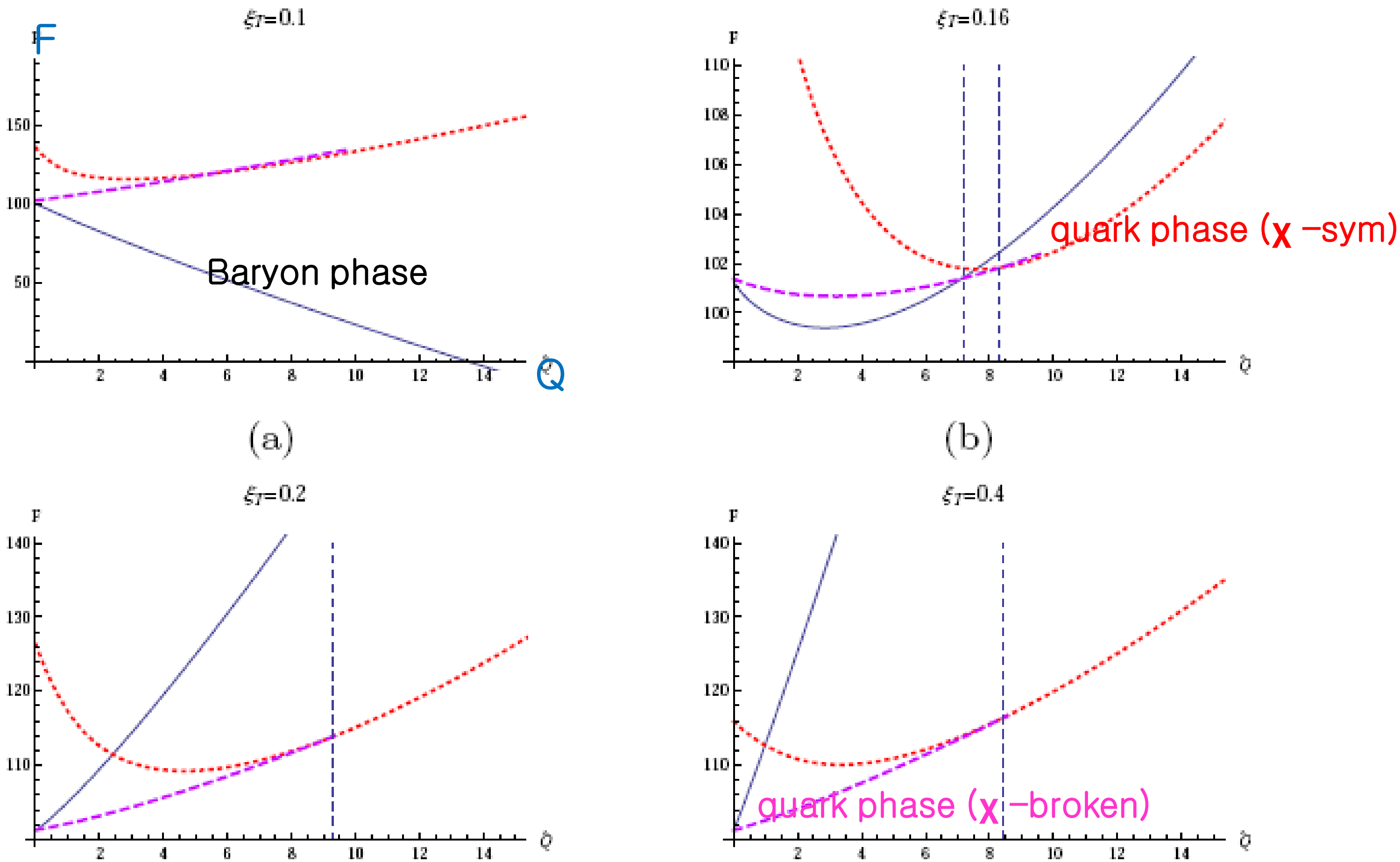
$$\mathcal{F}_{\rm baryon}(\hat{Q}) = \tau_7 \int d\rho \hat{\mathcal{H}}_{D7}(Q)\Big|_{\rm baryon\ phase} + \frac{Q}{N_C}\tau_5 \int d\theta \hat{\mathcal{H}}_{D5}$$



black: baryon phase

purple: quark phase without chiral symmetry, red: quark phase with chiral symmetry

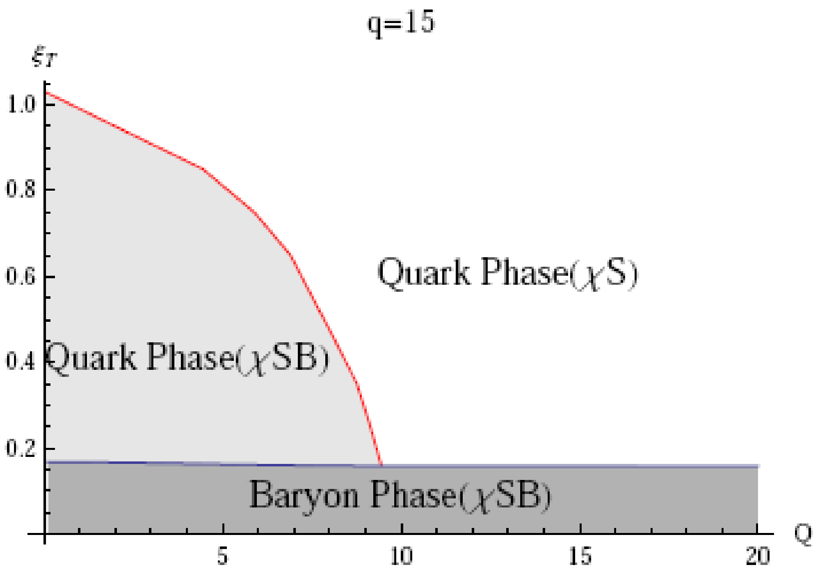
Density dependence of free energy (for $m_q = 0$ and q=15)

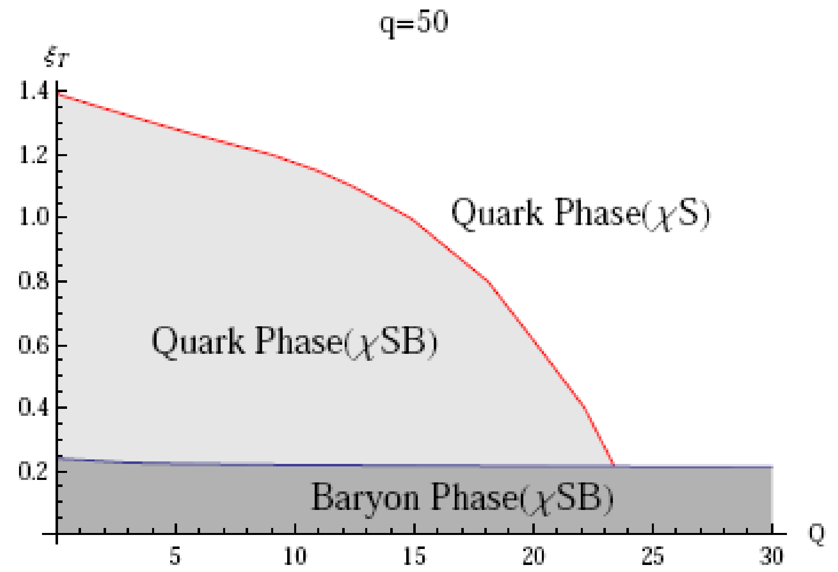


(a) (b) (c) (d)

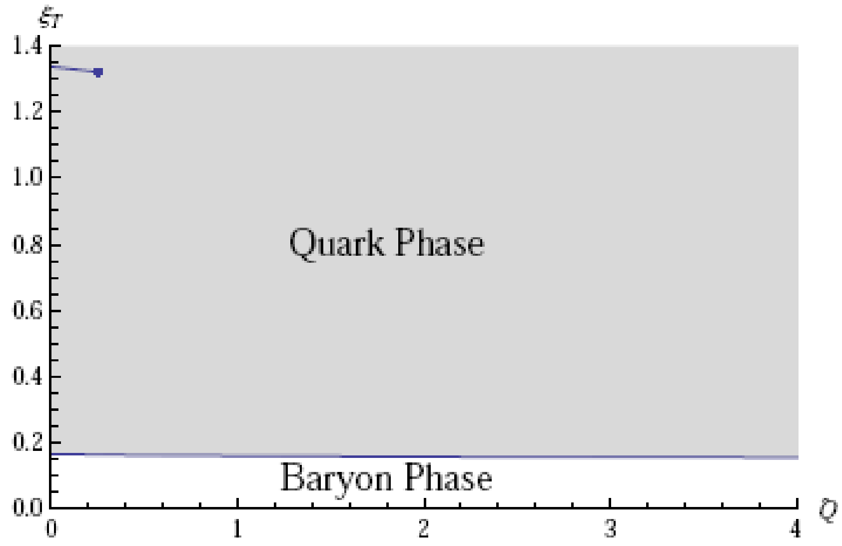
Phase Diagram

Zero quark mass





Finite quark mass



IV. Summary

- Holographic Principles :

  (d+1 dim.) (classical) Sugra ↔ (d dim.) (quantum) YM theories

- AdS/QCD – Top-down Approach & Bottom-up Approach

- QCD using Holographic dual Geometry

  – w/o chemical potential –

  phase : confined phase ↔ deconfined phase transition

  Geometry : thermal AdS ↔ AdS BH

  Hawking-Page transition

  – in dense matter – (U(1) chemical potential → baryon density)

  deconfined phase by RNAdS BH ↔ hadronic phase by tcAdS

  Hawking-Page phase transition

- In the hadronic phase, the quark density dependence of the light meson masses has been investigated.

IV. Summary – continued

- Holographic QCD model in D3/D-instanton background
- Two phases and phase transitions : for given T and density

  quark phase : physical objects : quarks

  baryon phase : baryon (vertex) as a physical object
- We study phase structure with and without quark mass
- We also study density dependence of chemical potential (eq. of state) and phase structure in grand canonical ensemble
- Future works : Meson spectrum & beyond probe approximation, etc.

Thank You !