UNQUENCHED HOLOGRAPHIC MAGNETIC CATALYSIS

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Large-N Gauge Theories.
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Magnetic catalysis of chiral symmetry breaking

Miranski et al. in hep-ph/9412257

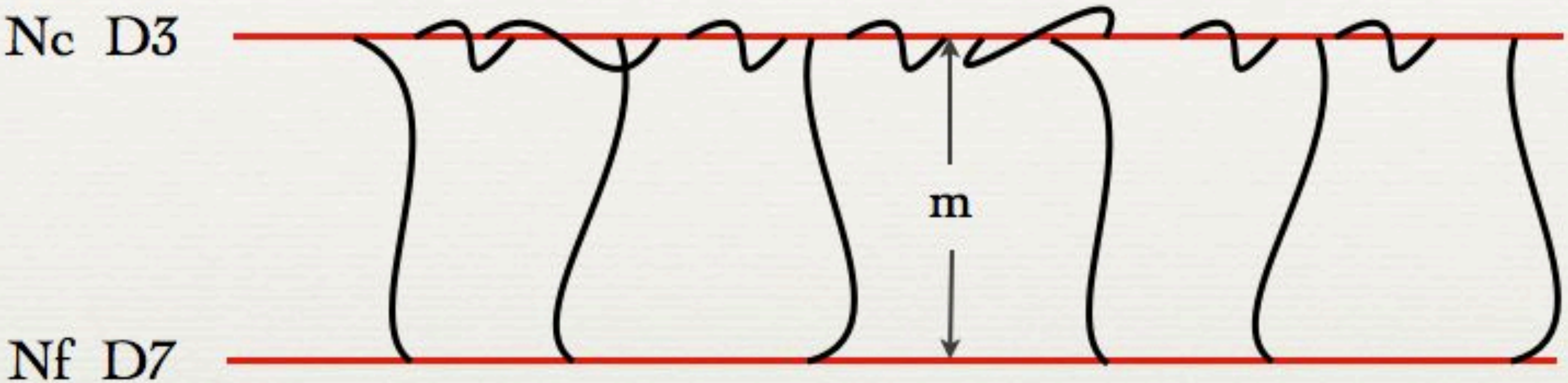
It is shown that in $3 + 1$ dimensions, a constant magnetic field is a catalyst of dynamical chiral symmetry breaking, leading to generating a fermion mass even at the weakest attractive interaction between fermions.

The essence of the effect is the dimensional reduction $D \to D - 2$ in the dynamics of fermion pairing in a magnetic field

Summary

- Brief definition
- Holographic set up. Introducing magnetic field
  - Mass generation in the D3/D7 system
- Unquenched flavor
  - Brief review
  - Introducing magnetic field

Holographic Flavored gauge theories (Karch, Katz, 2002)

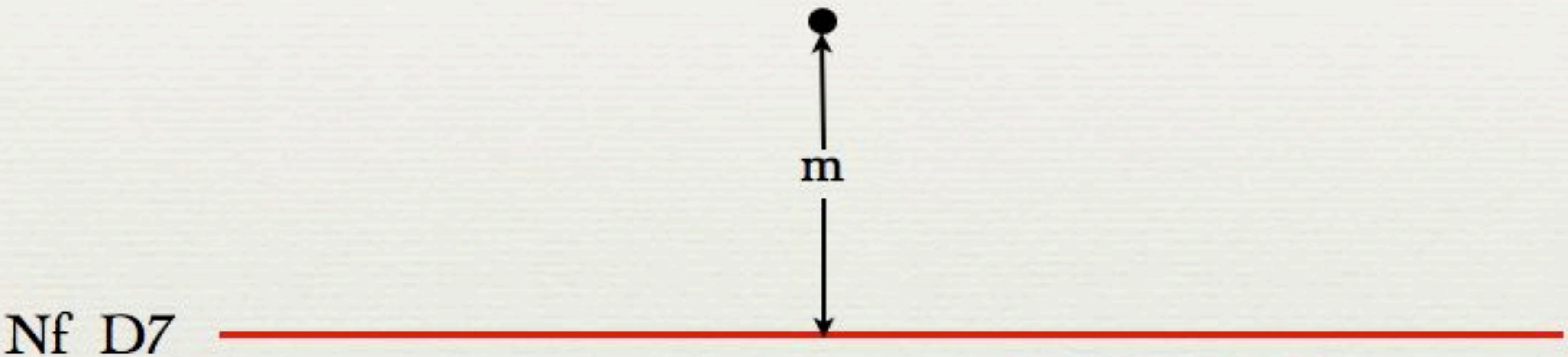


	0	1	2	3	4	5	6	7	8	9
D3	—	—	—	—	·	·	·	·	·	·
D7	—	—	—	—	—	—	—	—	·	·

$\mathcal{N} = 2$ Hypermultiplet

$$m_q \int d^2\theta \tilde{Q} Q \rightarrow \text{ SYM} \qquad m_q = m/2\pi\alpha'$$

$$ds^2 = \frac{\rho^2 + L^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{\rho^2 + L^2}(d\rho^2 + \rho^2 d\Omega_3^2 + dL^2 + L^2 d\phi^2)$$



$$L(\rho) = m + \frac{c}{\rho^2} + \dots$$

$m_q = m/2\pi\alpha'$

$\langle\bar{\psi}\psi\rangle \propto c$

The D7 are BPS

no fermionic condensate

$\langle\bar{\psi}\psi\rangle \equiv 0$

Introducing magnetic field (Filev, Johnson, Rashkov, 2007)

$$ds^2 = \frac{\rho^2 + L^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{\rho^2 + L^2}(d\rho^2 + \rho^2 d\Omega_3^2 + dL^2 + L^2 d\phi^2)$$

$$B_{(2)} = H dx_2 \wedge dx_3 \qquad L(\rho) \;\; \phi = const$$

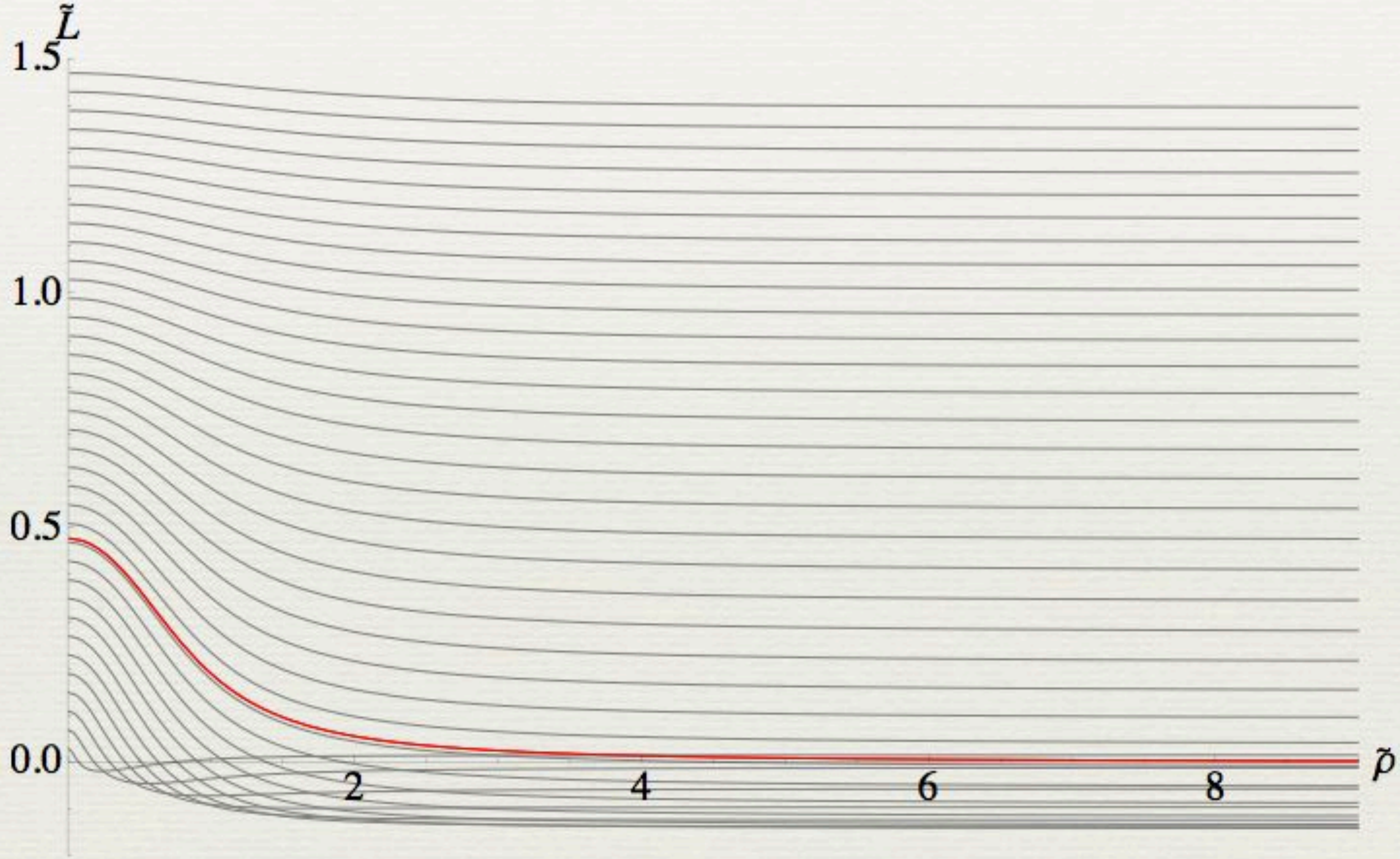
$$S_{DBI} = -\mu_7 \int_{\mathcal{M}_8} d^8\xi e^{-\Phi}\sqrt{|G_{ab} + B_{ab}|} \longrightarrow \text{EOM}$$

$$L(\rho) = m + \frac{c}{\rho^2} + \ldots$$

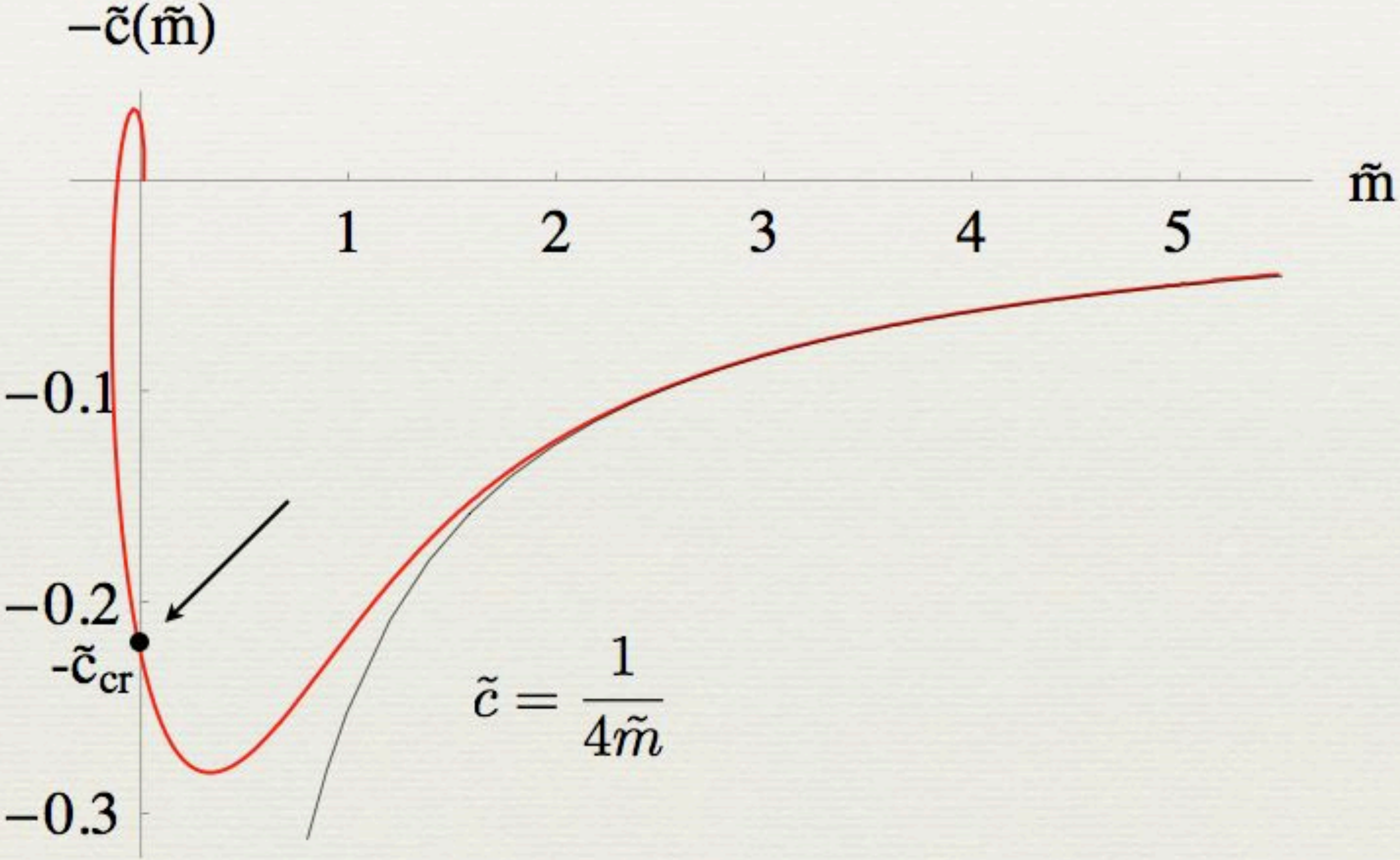
$$\tilde{\rho} = \rho / R\sqrt{H} \qquad \tilde{m} = m / R\sqrt{H}$$

$$\tilde{L} = L / R\sqrt{H} \qquad \tilde{c} = c / R^3 H^{3/2}$$

Classical embeddings



$$\langle\bar{\psi}\psi\rangle = -\frac{N_f N_c \tilde{c}_{\rm cr}}{(2\pi^2)^{3/4}\lambda^{1/4}}\left(\frac{H}{2\pi\alpha'}\right)^{3/2}$$



Going beyond the probe approximation. Motivation.

(Nunez, Paredes, Ramallo; 2010)

- In real life $N_f \sim N_c$

- Breaking conformality in $\mathcal{N} = 4$ SYM

- Introduce dynamical quarks in the Veneziano limit

$$N_c \to \infty \qquad N_f/N_c = \text{fixed}$$

Including the backreaction

Supergravity + branes $\qquad S_{IIB} + S_{fl}$

$$S_{IIB} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\Big[R - \frac{1}{2}\partial_M\Phi\partial^M\Phi - \frac{1}{2}e^{2\Phi}F_{(1)}^2 - \frac{1}{2}\frac{1}{3!}e^{\Phi}F_{(3)}^2 - \frac{1}{2}\frac{1}{5!}F_{(5)}^2 - \frac{1}{2}\frac{1}{3!}e^{-\Phi}H_{(3)}^2\Big] - \frac{1}{2\kappa_{10}^2}\int C_4\wedge H_3\wedge F_3$$

$$S_{fl} = -T_7\sum_{N_f}\left[\int d^8x\, e^{\Phi}\sqrt{-\det(\hat{G}+e^{-\Phi/2}\mathcal{F})} - \int_{D7}\hat{C}_q\wedge\left(e^{-\mathcal{F}}\right)_{8-q}\right]$$

$$S_{WZ} = \sum_{N_f}\int_{\mathcal{M}_8}\dots \quad\longrightarrow\quad \int_{\mathcal{M}_{10}}\Omega\wedge\dots \qquad \Omega = \sum_{N_f}\delta^{(2)}(\mathcal{M}_8)\omega_2$$

$\omega_2 \;\rightarrow$ transverse volume element

$\Omega \;\rightarrow$ charge distribution two-form

S_{WZ} Induces violation of Bianchi identities of F_1 and F_3

$$
\begin{aligned}
\mathrm{d}F_{(1)} &= -g_s\,\Omega_2\,, \\
\mathrm{d}F_{(3)} &= H_{(3)}\wedge F_{(1)} - g_s\,\mathcal{F}\wedge\Omega_2\,,
\end{aligned}
$$

$\rightarrow \delta-$function source term

The Einstein equations are difficult to solve

Smearing the sources in analogy to electrostatics ?

Upsides:

- PDEs $\rightarrow$ ODEs
- some singularities are avoided

Downsides:

- very particular subset of flavored theories
- flavor symmetry broken down to $U(1)^{N_f}$

Smeared Karch-Katz model

(Bigazzi, Cotrone, Paredes; 2008) (Benini, Canoura, Cremonesi, Nunez, Ramallo; 2006)

Write the metric of $\mathrm{AdS}_5 \times S^5$ as

$$ds^2 = h^{-\frac{1}{2}} dx_{1,3}^2 + h^{\frac{1}{2}} [|dZ^1|^2 + |dZ^2|^2 + |dZ^3|^2] \quad Z^i = u^i + iv^i \, , i = 1, 2, 3$$

Embed a D7-brane along x^μ, Z^1, Z^2 with $Z_3 = Z_3(Z^2, \bar{Z}^2)$

$Z^3 = \text{const} \propto m_q \quad \rightarrow$ Karch-Katz model

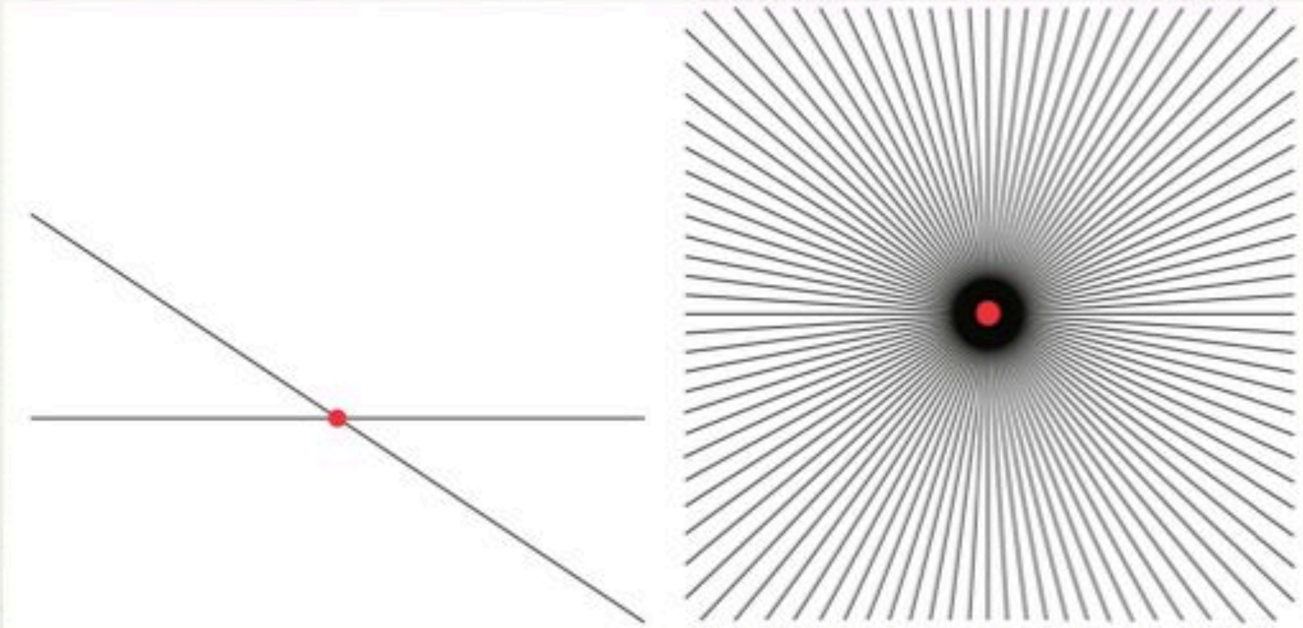
write in a more general form $\sum_{i=1}^{3} a_i Z^i = \text{const}$

and smear along all possible a_i. What is the metric ?

Intuitive picture

massless case ($m_q = 0$)

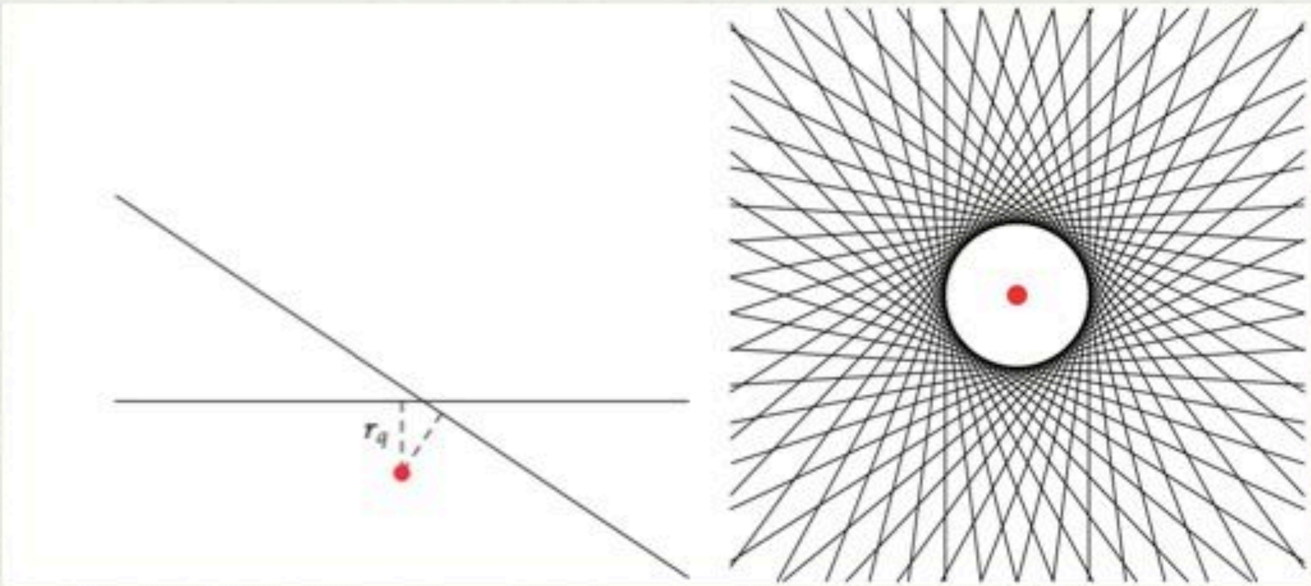
Singularity in the IR



massive case ($m_q \neq 0$)

the IR divergency

is regulated



Realization. Ansatz.

$$|dZ^1|^2+|dZ^2|^2+|dZ^3|^2=dr^2+r^2ds^2_{S^5} \qquad ds^2_{S^5}=ds^2_{CP^2}+(d\tau+A)^2$$

$$ds^2=h^{-\frac{1}{2}}dx^2_{1,3}+h^{\frac{1}{2}}[e^{2f}d\rho^2+e^{2g}ds^2_{CP^2}+e^2f(d\tau+A)^2]$$

$$\begin{aligned} ds^2_{CP^2} &= \frac{1}{4}d\chi^2+\frac{1}{4}\cos^2\frac{\chi}{2}(d\theta^2+\sin^2\theta d\varphi^2)+\frac{1}{4}\cos^2\frac{\chi}{2}\sin^2\frac{\chi}{2}(d\psi+\cos\theta d\varphi)^2 \quad \& \\ A_{CP^2} &= \frac{1}{2}\cos^2\frac{\chi}{2}(d\psi+\cos\theta d\varphi)\ . \qquad f(\rho),g(\rho),\phi(\rho) \end{aligned}$$

$$F_5=Q_c(1+*)\epsilon(S^5)$$

$$F_1=Q_fp(\rho)(d\tau+A)$$

$$N_c=\frac{Q_c Vol(S^5)}{(2\pi)^4 g_s\,\alpha'^2} \quad \& \quad N_f=\frac{4\,Q_f\,Vol(S^5)}{Vol(S^3)g_s}$$

$$p(\rho)=[1-e^{2(\rho_q-\rho)}]\Theta(\rho-\rho_q)$$

$$e^{\rho_q}\propto m_q$$

$\rightarrow$ radial distribution of the smeared sources

$\phi(\rho_{LP})=\infty \quad \rightarrow$ Landau pole at UV

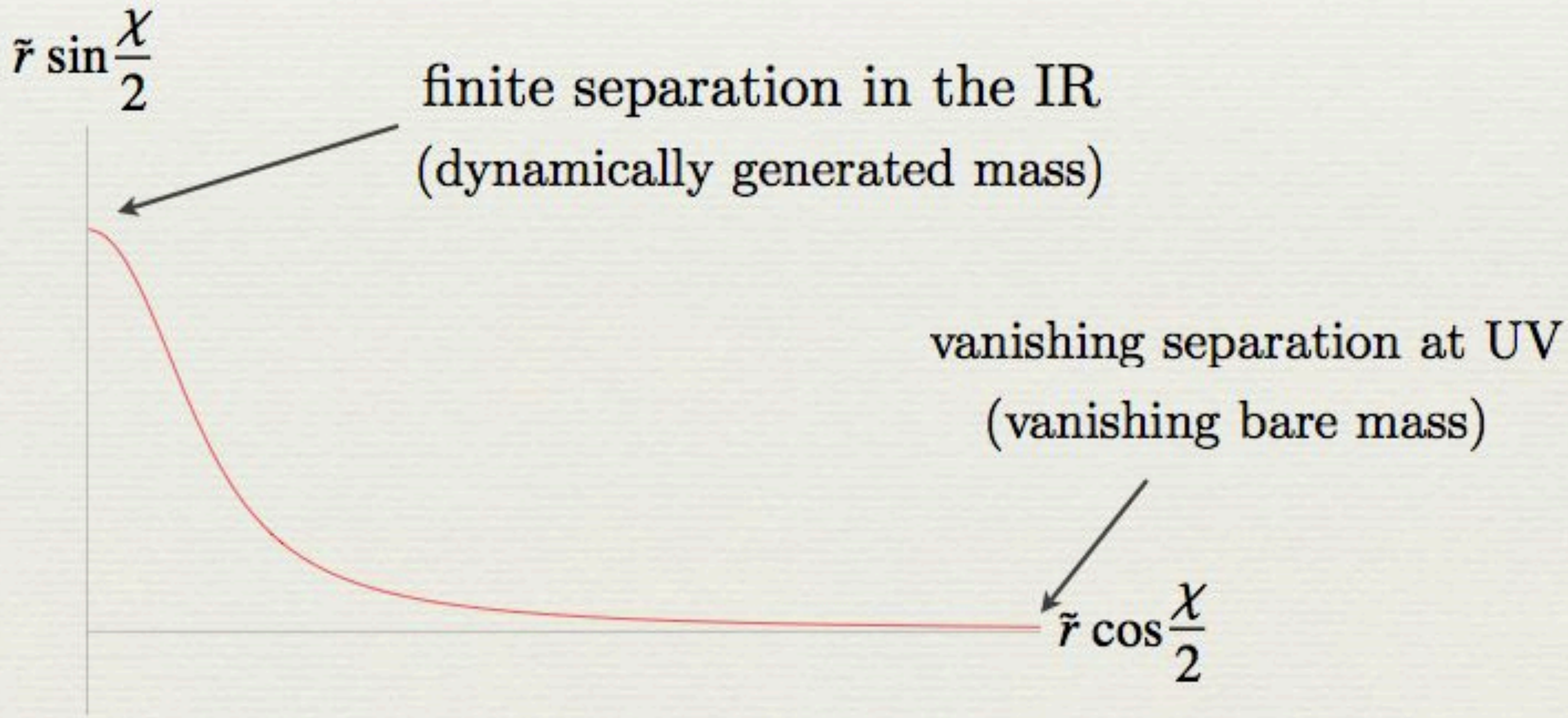
for $(m_q=0)$ singularity in the IR $(\rho=-\infty)$

Realizing mass generation ?

The smearing procedure can be generalized to non-supersymmetric "fiducial" embeddings

$\chi = \chi(\rho) \qquad p(\rho) = \cos^4 \chi(\rho)$ $\to$ radial distribution

(Bigazzi, Cotrone, Mas, Paredes, Ramallo, Tarrio; 2009)



Introducing external magnetic field. Ansatz.

inspired by (Bigazzi, Cotrone, Mas, Paredes, Ramallo, Tarrio; 2009)
(Bigazzi, Cotrone, Mas, Mayerson, Tarrio; 2011)

$$ds_{10}^2 = h^{-\frac{1}{2}}\left[-dt^2 + dx_1^2 + b(dx_2^2 + dx_3^2)\right] + h^{\frac{1}{2}}\left[b^2 S^8 F^2 d\sigma^2 + S^2 ds_{CP^2}^2 + F^2(d\tau + A_{CP^2})^2\right]$$

$$B_2 = H dx^2 \wedge dx^3\,, \quad C_2 = J\, dt \wedge dx^1\,,$$

$$F_5 = Q_c\,(1 + *)\varepsilon(S^5)\,, \quad F_1 = Q_f\,(d\tau + A_{CP^2})\,, \quad F_3 = dC_2 + B_2 \wedge F_1$$

The "fiducial" embedding is the trivial massless one. $\chi \equiv 0$

Equations of motion:

$$\partial_\sigma^2(\log b) = -\frac{4Q_f\,H^2\,hS^6F^2}{\beta_1} - e^{\Phi}\,H^2\,Q_f^2\,h\,S^8\,\beta_3$$

$$\partial_\sigma^2(\log h) = -Q_c^2\frac{b^2}{h^2} - \frac{2Q_f\,H^2\,hS^6F^2}{\beta_1} - \frac{1}{2}\,e^{\Phi}\,H^2\,Q_f^2\,h\,S^8\,\beta_3 + (1-\beta_2)\,\frac{e^{-\Phi}\,h\,H'^2}{b^2}$$

$$\partial_\sigma^2(\log S) = -2\,b^2F^4S^4 + 6\,b^2F^2S^6 - \frac{Q_f\,e^{\Phi}b^2F^2\,S^6}{\beta_1} + \frac{1}{4}\,e^{\Phi}\,H^2\,Q_f^2\,h\,S^8\,\beta_3$$

$$\partial_\sigma^2(\log F) = 4\,b^2F^4S^4 - \frac{1}{4}\,\left(1+\beta_1^2\right)\,Q_f^2\,e^{2\Phi}b^2S^8 + \frac{Q_f\,H^2\,hS^6F^2}{\beta_1} + \frac{1}{4}\,\frac{e^{-\Phi}\,h\,H'^2\,\beta_2}{b^2}$$

$$\partial_\sigma^2\Phi = \frac{1}{2}\,\left(1+\beta_1^2\right)\left[Q_f^2\,e^{2\Phi}\,b^2\,S^8 + \frac{4Q_f\,b^2\,e^{\Phi}S^6F^2}{\beta_1}\right] - \frac{1}{2}\,\frac{e^{-\Phi}\,h\,H'^2\,\beta_2}{b^2}$$

$$\partial_\sigma\left[\frac{e^{-\Phi}\,h\,H'}{b^2}\right] = e^{\Phi}\,Q_f^2\,H\,h\,S^8 - Q_c\,J' + \frac{4Q_f\,H\,hS^6F^2}{\beta_1}. \qquad J' = \frac{e^{-\Phi}\,Q_c}{h}\,(H_\star - H)$$

$$\beta_1 \equiv \sqrt{1+\frac{e^{-\Phi}\,H^2\,h}{b^2}}, \quad \beta_2 \equiv 1 + \frac{e^{2\Phi}\,J'^2\,b^2}{H'^2} \quad \& \quad \beta_3 \equiv 1 + \frac{e^{-2\Phi}\,H'^2\,\beta_2}{Q_f^2\,H^2\,b^2\,S^8}$$

We look for perturbative solution in Q_f near $\mathrm{AdS}_5 \times S^5$.

- We expect our solution to have both IR and UV divergencies
- We introduce a finite UV cut off $r(\sigma_*) = r_*$. Defined by:

$$\phi(r_*) = \phi_* \equiv \phi|_{Q_f=0} = \text{const.}$$

- We consider $H(r_*) = H_* = H|_{Q_f=0} = \text{const}$

H_* is the external magnetic field

- We fix the constants of integration by:

Requiring to reproduce the SUSY solution for $H_* = 0$

Requiring to match the SUSY solution at $r = r_*$

To first order in $\epsilon_* = Q_f e^{\Phi_*}$ we have:

$$\Phi = \Phi_\star - \frac{\epsilon_\star}{2}\left[\alpha_r - \alpha_{r_\star} - \frac{1}{2}\log\left[\frac{(\alpha_r+1)(\alpha_{r_\star}-1)}{(\alpha_r-1)(\alpha_{r_\star}+1)}\right]\right]$$

$$H = H_\star\left[1 - \frac{\epsilon_\star}{8}\left[\alpha_r\frac{\alpha_r^2+1}{\alpha_r^2-1} - \alpha_{r_\star}\frac{\alpha_{r_\star}^2+1}{\alpha_{r_\star}^2-1}\frac{r^4}{r_\star^4} - \frac{\alpha_r^2-1}{2}\log\left[\frac{\alpha_r+1}{\alpha_r-1}\right]\right.\right.$$
$$\left.\left. + \frac{\alpha_{r_\star}^2-1}{2}\log\left[\frac{\alpha_{r_\star}+1}{\alpha_{r_\star}-1}\right]\frac{r^4}{r_\star^4}\right]\right]$$

$$J' = -\epsilon_\star\frac{e^{-\Phi_\star}H_\star}{2r}\left[\alpha_r\frac{\alpha_r^2+1}{\alpha_r^2-1} - \alpha_{r_\star}\frac{\alpha_{r_\star}^2+1}{\alpha_{r_\star}^2-1}\frac{r^4}{r_\star^4} - \frac{\alpha_r^2-1}{2}\log\left[\frac{\alpha_r+1}{\alpha_r-1}\right]\right.$$
$$\left. + \frac{\alpha_r^2-1}{2}\log\left[\frac{\alpha_{r_\star}+1}{\alpha_{r_\star}-1}\right]\frac{r^8}{r_\star^8}\right]$$

$$b = 1 + \frac{\epsilon_\star}{2}\left[\alpha_r - \alpha_{r_\star} + \frac{1}{2}\left(\alpha_r^2-1\right)\log\left[\frac{\alpha_r+1}{\alpha_r-1}\right] - \frac{1}{2}\left(\alpha_{r_\star}^2-1\right)\log\left[\frac{\alpha_{r_\star}+1}{\alpha_{r_\star}-1}\right]\right]$$

$$
\begin{aligned}
\sigma &= \frac{1}{4r^4} + \epsilon_\star \left[-\frac{1}{72\,r^4} \left[\frac{\alpha_r}{\alpha_r^2 - 1} - \frac{r^4}{r_\star^4} \frac{\alpha_{r_\star}}{\alpha_{r_\star}^2 - 1} \right] + \frac{1}{96\,r^4} \left[\alpha_r \left(\alpha_r^2 - 1 \right) - \frac{r^4}{r_\star^4} \alpha_{r_\star} \left(\alpha_{r_\star}^2 - 1 \right) \right] \right. \\
&- \frac{1}{192\,r^4} \left[\left(\alpha_r^2 - 1 \right)^2 \log\left[\frac{\alpha_r + 1}{\alpha_r - 1} \right] - \frac{r^4}{r_\star^4} \left(\alpha_{r_\star}^2 - 1 \right)^2 \log\left[\frac{\alpha_{r_\star} + 1}{\alpha_{r_\star} - 1} \right] \right] - \frac{17}{144\,r^4} \left(\alpha_r - \alpha_{r_\star} \right) \\
&\left. - \frac{1}{16\,r^4} \left[\left(\alpha_r^2 - 1 \right) \log\left[\frac{\alpha_r + 1}{\alpha_r - 1} \right] - \left(\alpha_{r_\star}^2 - 1 \right) \log\left[\frac{\alpha_{r_\star} + 1}{\alpha_{r_\star} - 1} \right] \right] + \frac{\alpha_{r_\star}}{144\,r^4} \left(1 - \frac{r^4}{r_\star^4} \right) \right] ,
\end{aligned}
$$

$$
\begin{aligned}
F + 4S &= 5r + \frac{\epsilon_\star}{2} \left[-\frac{r}{16} \left[\alpha_r \left(\alpha_r^2 - 1 \right) - \frac{r^4}{r_\star^4} \alpha_{r_\star} \left(\alpha_{r_\star}^2 - 1 \right) \right] + \frac{r}{9} \frac{r^4}{r_\star^4} \right. \\
&+ \frac{r}{8} \left[\alpha_r - \frac{r^4}{r_\star^4} \alpha_{r_\star} \right] + \frac{r}{4} \left[\frac{\alpha_r}{\alpha_r^2 - 1} - \frac{r^4}{r_\star^4} \frac{\alpha_{r_\star}}{\alpha_{r_\star}^2 - 1} \right] \\
&\left. + \frac{r}{32} \left[\left(\alpha_r^2 - 1 \right)^2 \log\left[\frac{\alpha_r + 1}{\alpha_r - 1} \right] - \frac{r^4}{r_\star^4} \left(\alpha_{r_\star}^2 - 1 \right)^2 \log\left[\frac{\alpha_{r_\star} + 1}{\alpha_{r_\star} - 1} \right] \right] \right]
\end{aligned}
$$

$$
\begin{aligned}
F - S &= -\frac{\epsilon_\star}{12} \left[\frac{r}{4} \left[\alpha_r \left(\alpha_r^2 - 1 \right) - \frac{r^2}{r_\star^2} \alpha_{r_\star} \left(\alpha_{r_\star}^2 - 1 \right) \right] + r \frac{r^2}{r_\star^2} - \frac{r}{4} \left(\alpha_r^2 - 1 \right)^{3/2} \left(1 - \frac{r^8}{r_\star^8} \right) \right. \\
&\left. + \frac{5\,r}{8} \left[\alpha_r - \frac{r^2}{r_\star^2} \alpha_{r_\star} \right] + \frac{3}{8} \frac{r}{\sqrt{\alpha_r^2 - 1}} \log\left[\frac{\alpha_r + \sqrt{\alpha_r^2 - 1}}{\alpha_{r_\star} + \sqrt{\alpha_{r_\star}^2 - 1}} \right] \right] .
\end{aligned}
$$

Where the radial coordinate r is defined via

$$h = \frac{R^4}{r^4} \qquad \& \qquad R^4 \equiv \frac{1}{4} Q_c$$

and we have defined:

$$\alpha_r \equiv \sqrt{1 + \frac{e^{-\Phi_s} H_\star^2 Q_c}{4\, r^4}} \quad \& \quad \alpha_{r_\star} \equiv \sqrt{1 + \frac{e^{-\Phi_\star} H_\star^2 Q_c}{4\, r_\star^4}}$$

We expect our solution to be unstable under quantum fluctuations, because the "fiducial" embedding is unstable in external magnetic field.

Due to the IR singularity we cannot directly study the glueball spectrum However we could introduce additional flavor brane and analyze its stability

Introducing a probe D7-brane

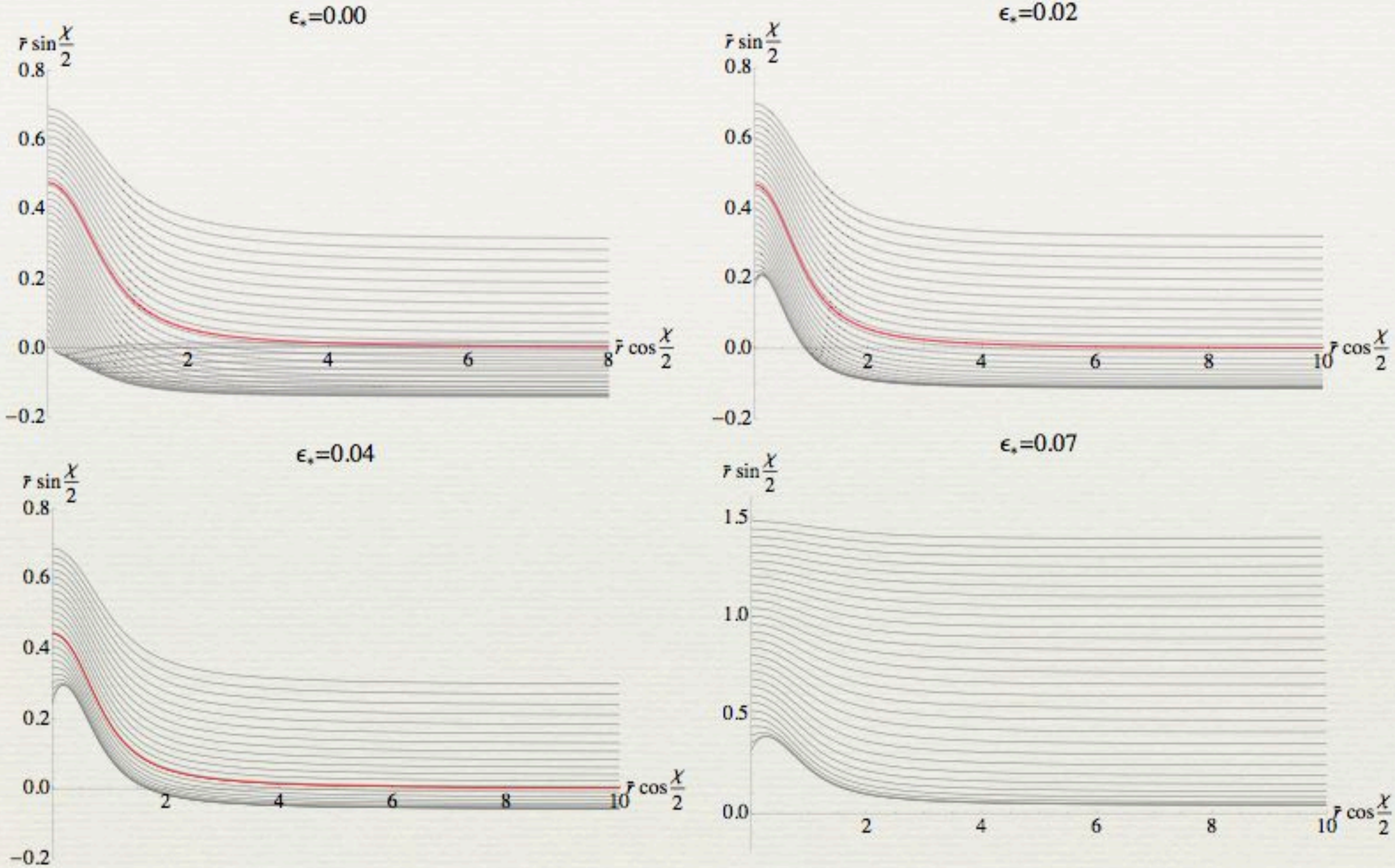
$$\tilde{\mathcal{L}}^{(0)} = -f_1(\tilde{r})\left(\cos^2\frac{\chi}{2} + f_2(\tilde{r})^2\sin^2\frac{\chi}{2}\right)^{\frac{1}{2}}\sqrt{1+f_3(\tilde{r})\chi'^2} + f_4(\tilde{r})\cos^4\frac{\chi}{2}$$

$$f_1(\tilde{r}) = \frac{1}{8}e^{\tilde{\Phi}}\tilde{b}^2\tilde{S}^6\tilde{F}^2\left(1+\frac{e^{-\tilde{\Phi}}\tilde{H}^2}{\tilde{r}^4\tilde{b}^2}\right)^{\frac{1}{2}}\left|\frac{\partial\tilde{\sigma}}{\partial\tilde{r}}\right|; \quad f_2(\tilde{r}) = \frac{\tilde{S}}{\tilde{F}};$$

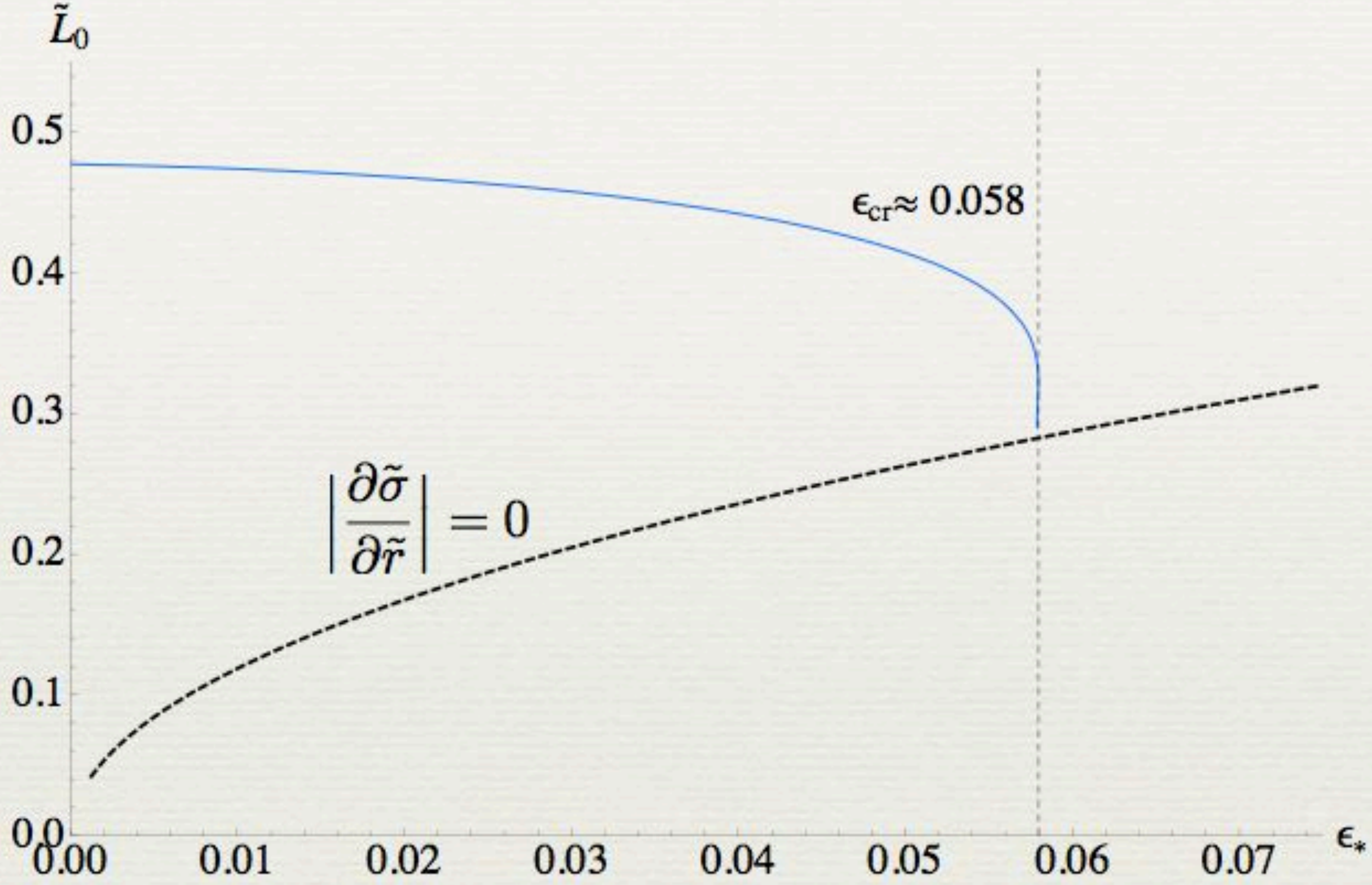
$$f_3(\tilde{r}) = \left(2\tilde{b}^2\tilde{S}^3\tilde{F}\left|\frac{\partial\tilde{\sigma}}{\partial\tilde{r}}\right|\right)^{-1}; \quad f_4(\tilde{r}) = \frac{\epsilon_*}{32}e^{2\tilde{\Phi}}\tilde{b}^2\tilde{S}^8\left(1+\frac{e^{-\tilde{\Phi}}\tilde{H}^2}{\tilde{r}^4\tilde{b}^2}\right)\left|\frac{\partial\tilde{\sigma}}{\partial\tilde{r}}\right|;$$

$$\tilde{r} = \frac{r}{r_m}\,, \quad r_m \equiv \frac{e^{-\Phi_*}Q_c H_*^2}{4}\,.$$

$$\chi(\tilde{r}) = \pi - \sqrt{a(\tilde{r}-\tilde{r}_{min})}\,, \quad a = \left.\frac{8f_1f_2}{f_3(-2f_4 + f_1f_3f_2' + f_2(f_3f_1' + f_1f_3'))}\right|_{\tilde{r}=\tilde{r}_{min}}$$



$$\left|\frac{\partial\tilde{\sigma}}{\partial\tilde{r}}\right| = 0 \quad \text{at } \tilde{r} = \tilde{r}_{IR}(\epsilon_*) > 0$$



For sufficiently strong magnetic fields the probe brane is unstable.

Can we identify tachyonic modes in the meson spectrum ?

Would this be reflecting an instability of the background ?

Meson Spectrum. Fluctuations along χ

The relevant pieces of the action are

$$\frac{S}{T_7} = -\int d^8x\, e^{\Phi}\sqrt{-\det\left[G_{ab} + e^{-\frac{\Phi}{2}}\left(B_{ab} + F_{ab}\right)\right]} + \int P[\mathcal{C}_{(8)} - F_{(2)} \wedge \mathcal{C}_{(6)} + \frac{1}{2} F_{(2)} \wedge F_{(2)} \wedge \mathcal{C}_{(4)}]\,, \tag{1}$$

where $\mathcal{C}_{(4)}, \mathcal{C}_{(6)}$ & $\mathcal{C}_{(8)}$ are defined by:

$$\mathcal{C}_{(4)} \equiv C_{(4)} - C_{(2)} \wedge B_{(2)}\;, \quad \mathcal{C}_{(6)} \equiv C_{(6)} - B_{(2)} \wedge \tilde{C}_{(4)}\;, \quad \mathcal{C}_{(8)} \equiv C_{(8)} - B_{(2)} \wedge C_{(6)} \tag{2}$$

and $\tilde{C}_{(4)}$ is the magnetic dual of the RR potential $C_{(4)}$

$$\tilde{C}_{(4)} \;=\; -\frac{1}{32}\, Q_c \sin\theta \cos^4\frac{\chi}{2}\, d\theta \wedge d\varphi \wedge d\psi \wedge d\tau\,, \tag{3}$$

We will consider fluctuations of the form

$$\chi \;=\; \chi_0(\sigma) + 2\pi\alpha'\,\delta\chi(\xi^a) \quad \& \quad \tau \;=\; 2\pi\alpha'\,\delta\tau(\xi^a), \tag{4}$$

where the indices $a, b \;=\; 0, 1, \ldots, 7$ run along the worldvolume of the D7-brane and expand to second order in α'.

We consider the ansatz:

$$\delta\chi = e^{i\omega t}\eta(\sigma); \quad \delta\tau = 0; \quad A_\mu = 0;$$

$$\partial_{\tilde{r}}\left(\frac{\tilde{\mathcal{L}}^{(0)}_{\rm DBI} f_3^2}{(1+f_3^2\chi'^2)^2}\partial_{\tilde{r}}\eta\right) + \left(\frac{\tilde{\mathcal{L}}^{(0)}_{\rm DBI} f_5^2}{1+f_3^2\chi'^2}\tilde{\omega}^2 - \left[\partial_\chi^2\tilde{\mathcal{L}}^{(0)} - \partial_{\tilde{r}}\left(\frac{f_3^2\chi'}{1+f_3^2\chi'^2}\partial_\chi\tilde{\mathcal{L}}^{(0)}_{\rm DBI}\right)\right]\right)\eta = 0\,,$$

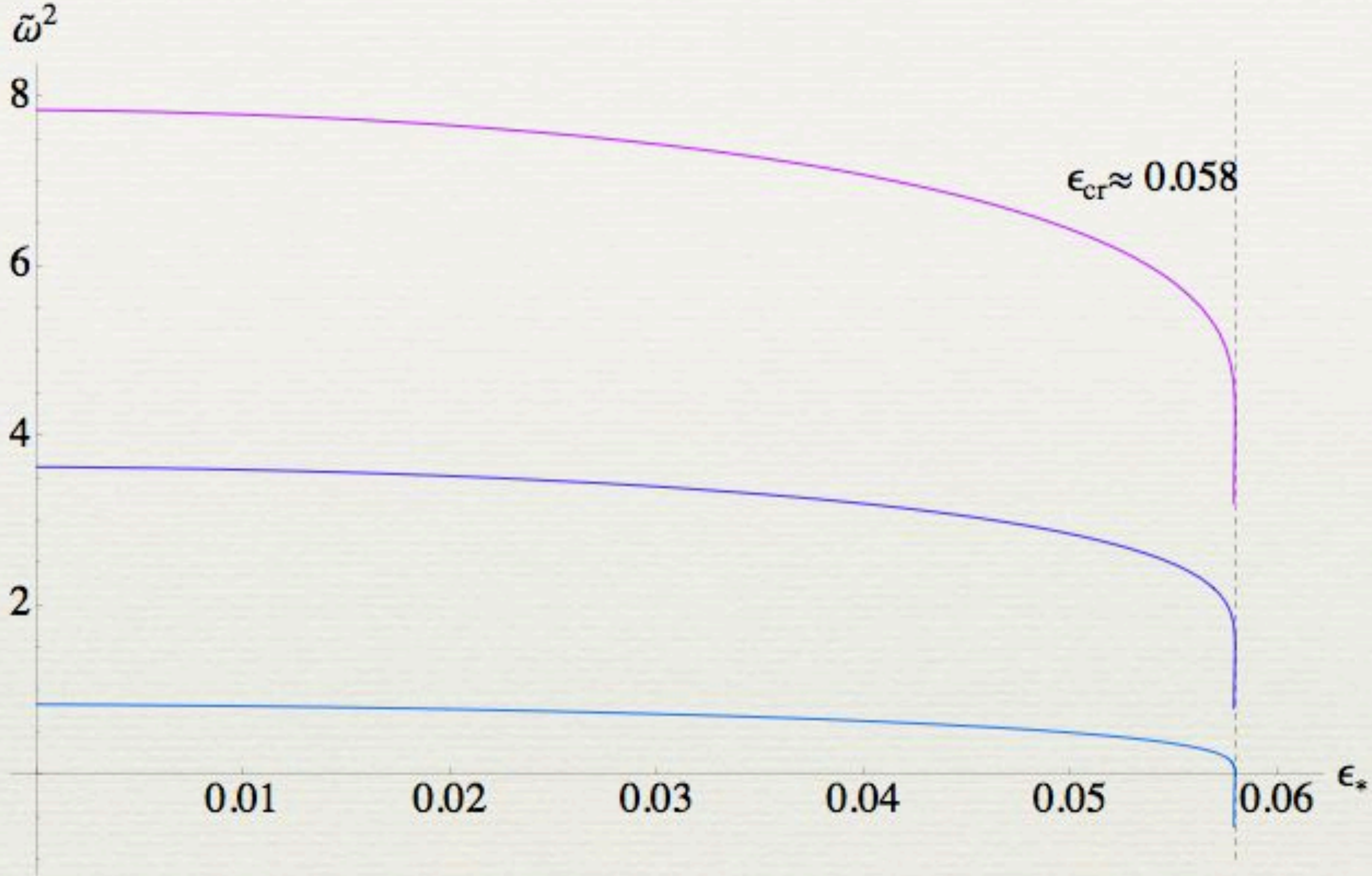
$$\tilde{\omega}^2 \equiv \frac{Q_c}{4r_m^2}\omega^2; \quad f_5(\tilde{r}) \equiv \frac{\tilde{S}(\tilde{r})}{2\tilde{r}^2}; \; . \qquad \int_{\tilde{r}_{min}}^{\tilde{r}_*} d\tilde{r}\sqrt{-\tilde{E}^{(0)}}\left|\eta(\tilde{r})\right|^2 < \infty\,.$$

$$\eta(\tilde{r}) = \frac{\zeta(\tilde{r})}{\sqrt{\tilde{r}-\tilde{r}_{min}}}$$

$$\zeta''(\tilde{r}) + \left(C_1 - \frac{1}{\tilde{r}-\tilde{r}_{min}}\right)\zeta'(\tilde{r}) + \left(C_0 + B^2\tilde{\omega}^2 - \frac{C_1}{\tilde{r}-\tilde{r}_{min}} + \frac{3}{4}\frac{1}{(\tilde{r}-\tilde{r}_{min})^2}\right)\zeta(\tilde{r}) = 0$$

predetermined boundary conditions at $\tilde{r}_{min}$

regularity at $r = r_*$ quantizes $\tilde{\omega}$



We see that indeed the ground states is tachyonic at $\epsilon_* = \epsilon_{\rm cr}$.

Possible extensions

- Smear non-trivial embeddings.

  -realize dynamical mass generation (technical difficulties)

- Construct the finite temperature solution (work in progress)

  -regular at IR

  -one can study quasi-normal modes of the background

  -One can analyze competition of scales

Thank you !