Towards NNLO Corrections for Jet Observables at LHC

Thomas Gehrmann

Universität Zürich

GGI workshop “High-Energy QCD after the Start of the LHC”, 12.9.2011

Precision observables in QCD

Processes measured to few per cent accuracy

- $e^+e^- \to 3j$
- $ep \to (2+1)j$
- $pp \to j + X$
- $pp \to (V = W, Z)$
- $pp \to (V = W, Z) + j$
- $pp \to t\bar{t}$

Processes with potentially large perturbative corrections

- $pp \to H$
- $pp \to H + j$
- $pp \to (\gamma\gamma, WW, ZZ)$

Need NNLO QCD predictions for

- meaningful interpretation of experimental data
- precise determination of fundamental parameters (including parton distributions)

Precision observables in QCD

Processes measured to few per cent accuracy

- $e^+e^- \to 3j$ ✓
- $ep \to (2+1)j$ ✘
- $pp \to j + X$ ✘
- $pp \to (V = W, Z)$ ✓
- $pp \to (V = W, Z) + j$ ✘
- $pp \to t\bar{t}$ ✘

Processes with potentially large perturbative corrections

- $pp \to H$ ✓
- $pp \to H + j$ ✘
- $pp \to (\gamma\gamma, WW, ZZ)$ ✘

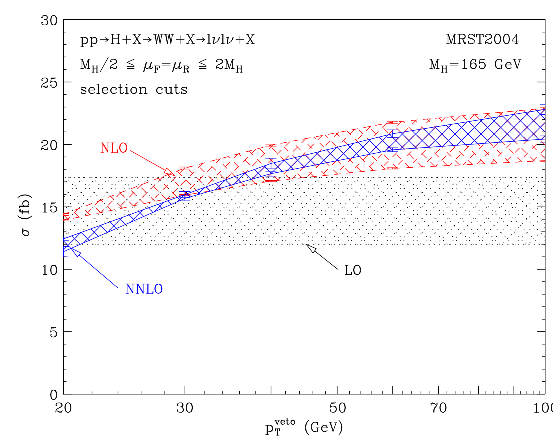
Need NNLO QCD predictions for

- meaningful interpretation of experimental data
- precise determination of fundamental parameters (including parton distributions)

Precision Observables in QCD

NNLO corrections known for

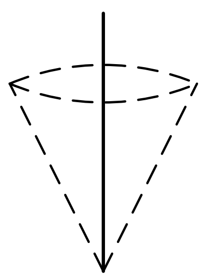
- vector boson production
  K. Melnikov, F. Petriello; S. Catani, L. Cieri, G. Ferrera, D. de Florian, M. Grazzini
  - fully exclusive calculations
  - including vector boson decay
  - allowing arbitrary final-state cuts
- Higgs boson production
  C. Anastasiou, K. Melnikov, F. Petriello; S. Catani, M. Grazzini
  - fully exclusive calculations
  - including Higgs boson decay to $\gamma\gamma$, VV
- Associated VH production
  G. Ferrera, M. Grazzini, F. Tramontano
  - fully exclusive calculation
  - including Higgs boson decay to $\gamma\gamma$, VV



Jets in Perturbation Theory

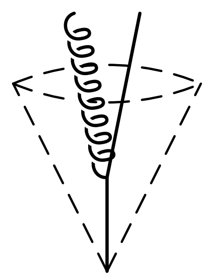
Jet Description

- Partons are combined into jets using the same jet algorithm as in experiment



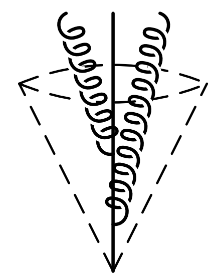
LO

each parton forms 1 jet on its own



NLO

2 partons in 1 jet, 1 parton experimentally unresolved



NNLO

3 partons in 1 jet, 2 partons experimentally unresolved

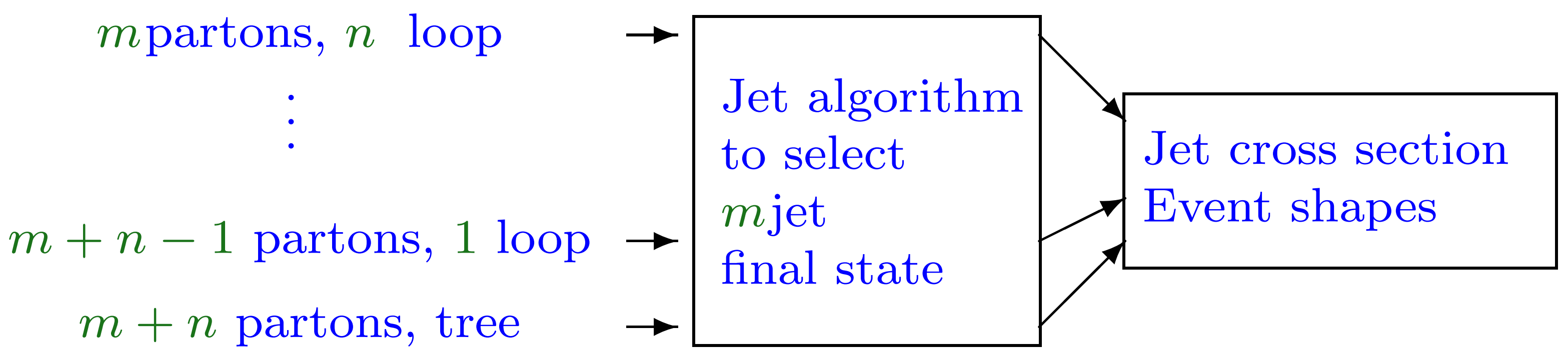
Improvement at higher orders:

- reduce error on theory prediction
- reliable error estimate
- better matching of parton level and hadron level jet algorithm
- account for kinematics of initial state radiation

Jets in Perturbation Theory

General structure:

m jets, n–th order in perturbation theory



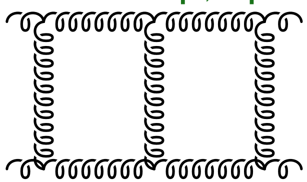
- Jet algorithm acts differently on different partonic final states
- Divergencies from soft and collinear real and virtual contributions must be extracted before application of jet algorithm

consider $pp \to 2$ jets

Ingredients to NNLO 2-jets

- Two-loop matrix elements

$|\mathcal{M}|^2_{\text{2-loop},2 \text{ partons}}$



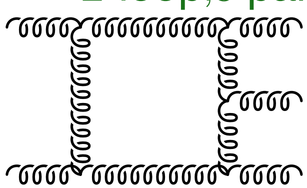
explicit infrared poles from loop integrals

C. Anastasiou, N. Glover, C. Oleari, M. Tejeida-Yeomans

Z. Bern, L. Dixon, A. De Freitas

- One-loop matrix elements

$|\mathcal{M}|^2_{\text{1-loop},3 \text{ partons}}$



explicit infrared poles from loop integral and

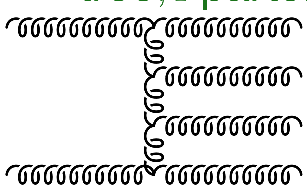
implicit infrared poles due to single unresolved radiation

Z. Kunszt, A. Signer, Z. Trocssanyi;

Z. Bern, L. Dixon, D. Kosower

- Tree level matrix elements

$|\mathcal{M}|^2_{\text{tree},4 \text{ partons}}$



implicit infrared poles due to double unresolved radiation

Infrared Poles cancel in the sum

Virtual Corrections at NNLO

Virtual two-loop corrections feasible due to:

- algorithms to reduce the ~ 10000's of integrals to a few $(10-30)$ master integrals
  - Integration-by-parts (IBP)
    K. Chetyrkin, F. Tkachov
  - Lorentz Invariance (LI)
    E. Remiddi, TG
  - and their implementation in computer algebra
    S. Laporta
- New methods to compute master integrals
  - Mellin-Barnes Transformation V. Smirnov, O. Veretin; B. Tausk;
    MB: M. Czakon; AMBRE: J. Gluza, K. Kajda, T. Riemann
  - Differential Equations E. Remiddi, TG
  - Sector Decomposition (numerically) T. Binoth, G. Heinrich
  - Nested Sums S. Moch, P. Uwer, S. Weinzierl

Virtual Corrections at NNLO

Reduction to master integrals

Identities:

- Integration-by-parts (IBP)
  K. Chetyrkin, F. Tkachov

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{d}^d l}{(2\pi)^d} \frac{\partial}{\partial a^\mu} \left[b^\mu f(k,l,p_i) \right] = 0$$

  with: $a^\mu = k^\mu, l^\mu$ and $b^\mu = k^\mu, l^\mu, p_i^\mu$

- Lorentz Invariance (LI)
  E. Remiddi, TG

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\mathrm{d}^d l}{(2\pi)^d} \delta\varepsilon_\nu^\mu \left(\sum_i p_i^\nu \frac{\partial}{\partial p_i^\mu} \right) f(k,l,p_i) = 0$$

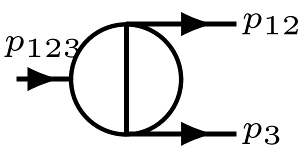
For each two-loop four-point integral, one has 10 IBP and 3 LI identities.

Virtual Corrections at NNLO

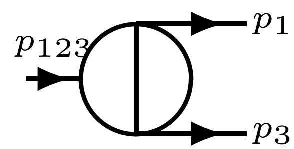
Master Integrals from differential equations

Example: two-loop off-shell vertex function

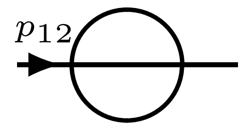
$$s_{123}\frac{\partial}{\partial s_{123}}$$



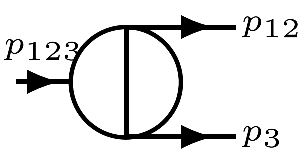
$$= +\frac{d-4}{2}\,\frac{2s_{123}-s_{12}}{s_{123}-s_{12}}$$



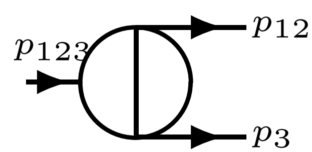
$$-\frac{3d-8}{2}\,\frac{1}{s_{123}-s_{12}}$$



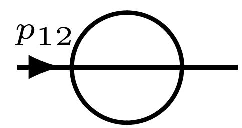
$$s_{12}\frac{\partial}{\partial s_{12}}$$



$$= -\frac{d-4}{2}\,\frac{s_{12}}{s_{123}-s_{12}}$$



$$+\frac{3d-8}{2}\,\frac{1}{s_{123}-s_{12}}$$



- is a hypergeometric differential equation
- boundary conditions are two-point functions
- Laurent-series: expansion of hypergeometric functions in their parameters
  HypExp: T. Huber, D. Maître; XSummer: S. Moch, P. Uwer
- yields (generalized) harmonic polylogarithms
  E. Remiddi, J. Vermaseren; A. Goncharov; HPL: D. Maître

Virtual Corrections at NNLO

Virtual two-loop matrix elements have been computed for:

- Bhabha-Scattering: $e^+e^- \to e^+e^-$
  Z. Bern, L. Dixon, A. Ghinculov
- Hadron-Hadron 2-Jet production: $qq' \to qq'$, $q\bar{q} \to q\bar{q}$, $q\bar{q} \to gg$, $gg \to gg$
  C. Anastasiou, N. Glover, C. Oleari, M. Yeomans-Tejeda
  Z. Bern, A. De Freitas, L. Dixon [SUSY-YM]
- Photon pair production at LHC: $gg \to \gamma\gamma$, $q\bar{q} \to \gamma\gamma$
  Z. Bern, A. De Freitas, L. Dixon
  C. Anastasiou, N. Glover, M. Yeomans-Tejeda
- Three-jet production: $e^+e^- \to \gamma^* \to q\bar{q}g$
  L. Garland, N. Glover, A.Koukoutsakis, E. Remiddi, TG
  S. Moch, P. Uwer, S. Weinzierl
- DIS (2+1) jet production: $\gamma^* g \to q\bar{q}$, Hadronic (V+1) jet production: $qg \to Vq$
  E. Remiddi, TG

Virtual Corrections at NNLO

Ongoing two-loop matrix element calculations:

- Higgs-plus-jet production: $gg \to Hg$, $q\bar{q} \to Hg$
  N. Glover, M. Jaquier, A. Koukoutsakis, TG
- Vector boson pair production: $q\bar{q} \to (V = W, Z)\gamma$
  L. Tancredi, TG
- Vector boson pair production: $q\bar{q} \to (VV = WW, ZZ)$
  G. Chachamis, M. Czakon; L. Tancredi, TG
- Top Quark pair production: $q\bar{q} \to Q\bar{Q}$, $gg \to Q\bar{Q}$
  M. Czakon, A. Mitov, S. Moch
  R. Bonciani, A. Ferroglia, D. Maître, A. von Manteuffel, C. Studerus, TG

Real corrections at NNLO

Double real radiation

$$\mathrm{d}\sigma^{(m+2)} = |\mathcal{M}_{m+2}|^2 \mathrm{d}\Phi_{m+2} J_m^{(m+2)}(p_1,\ldots,p_{m+2}) \sim \frac{1}{\epsilon^4}$$

with $J_m^{(m+2)}$ jet definition for combining m+2 partons into m jets

- expression is too complicated to be evaluated analytically
- want to study multiple observables and different jet definitions
- need method to extract divergencies

$\longrightarrow$ Evaluation with subtraction term

NLO Subtraction

Structure of NLO m-jet cross section (subtraction formalism):
Z. Kunszt, D. Soper

$$\mathrm{d}\sigma_{NLO} = \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^R_{NLO} - \mathrm{d}\sigma^S_{NLO}\right) + \left[\int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^S_{NLO} + \int_{\mathrm{d}\Phi_m} \mathrm{d}\sigma^V_{NLO}\right]$$

- $\mathrm{d}\sigma^S_{NLO}$: local counter term for $\mathrm{d}\sigma^R_{NLO}$
- $\mathrm{d}\sigma^R_{NLO} - \mathrm{d}\sigma^S_{NLO}$: free of divergences, can be integrated numerically

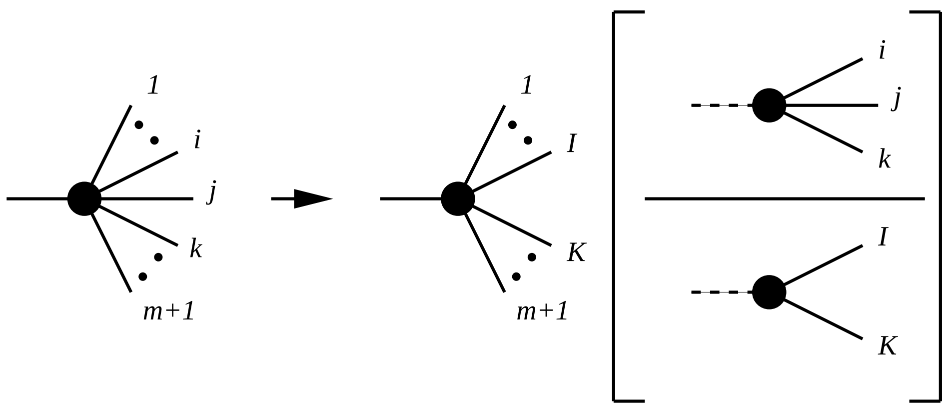
General methods at NLO

- Dipole subtraction S. Catani, M. Seymour
- $\mathcal{E}$-prescription S. Frixione, Z. Kunszt, A. Signer;
  NNLO: S. Frixione, M. Grazzini; V. Del Duca, G. Somogyi, Z. Trocsanyi
- Antenna subtraction
  D. Kosower; J. Campbell, M. Cullen, N. Glover; A. Daleo, D. Maître, TG
  NNLO: A. Gehrmann-De Ridder, N. Glover, TG
- q_T subtraction(NNLO) S. Catani, M. Grazzini

NLO Antenna Subtraction

Building block of $\mathrm{d}\sigma^S_{NLO}$: NLO-Antenna function X^0_{ijk}

Contains all singularities of parton j emitted between partons i and k



$$X^0_{ijk} = S_{ijk,IK} \frac{|M^0_{ijk}|^2}{|M^0_{IK}|^2}$$

$$\mathrm{d}\Phi_{X_{ijk}} = \frac{\mathrm{d}\Phi_3}{P_2}$$

Phase space factorisation

$$\mathrm{d}\Phi_{m+1}(p_1,\ldots,p_{m+1};q) = \mathrm{d}\Phi_m(p_1,\ldots,\tilde{p}_I,\tilde{p}_K,\ldots,p_{m+1};q)\cdot \mathrm{d}\Phi_{X_{ijk}}(p_i,p_j,p_k;\tilde{p}_I+\tilde{p}_K)$$

Integrated subtraction term (analytically)

$$|\mathcal{M}_m|^2 J_m^{(m)}\, \mathrm{d}\Phi_m \int \mathrm{d}\Phi_{X_{ijk}}\, X^0_{ijk} \sim |\mathcal{M}_m|^2 J_m^{(m)}\, \mathrm{d}\Phi_m \int \mathrm{d}\Phi_3 |M^0_{ijk}|^2$$

can be combined with $\mathrm{d}\sigma^V_{NLO}$

NNLO Infrared Subtraction

Structure of NNLO m-jet cross section:

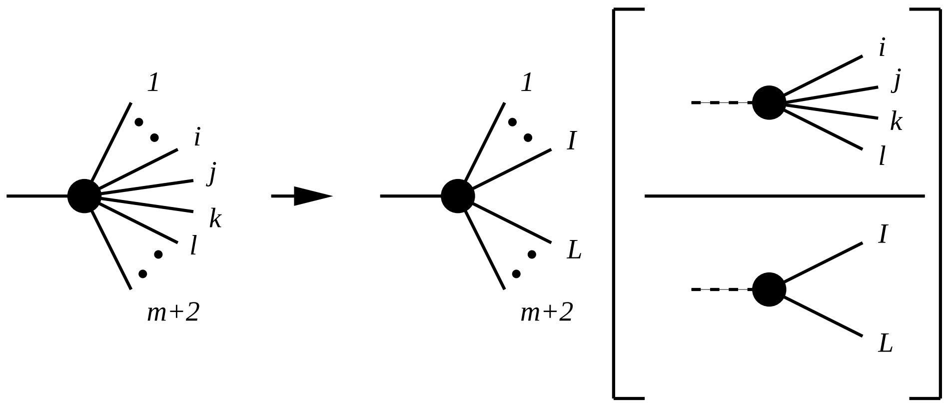
$$\begin{aligned} \mathrm{d}\sigma_{NNLO} &= \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S}_{NNLO}\right) \\ &+ \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) \\ &+ \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V,2}_{NNLO} + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{S}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO}\,, \end{aligned}$$

- $\mathrm{d}\sigma^{S}_{NNLO}$: real radiation subtraction term for $\mathrm{d}\sigma^{R}_{NNLO}$
- $\mathrm{d}\sigma^{VS,1}_{NNLO}$: one-loop virtual subtraction term for $\mathrm{d}\sigma^{V,1}_{NNLO}$
- $\mathrm{d}\sigma^{V,2}_{NNLO}$: two-loop virtual corrections

Each line above is finite numerically and free of infrared ϵ-poles $\longrightarrow$ numerical programme

Double Real Subtraction

Two colour-connected unresolved partons



$$X^0_{ijkl} = S_{ijkl,IL} \frac{|M^0_{ijkl}|^2}{|M^0_{IL}|^2}$$

$$\mathrm{d}\Phi_{X_{ijkl}} = \frac{\mathrm{d}\Phi_4}{P_2}$$

Phase space factorisation

$$\mathrm{d}\Phi_{m+2}(p_1,\ldots,p_{m+2};q) = \mathrm{d}\Phi_m(p_1,\ldots,\tilde{p}_I,\tilde{p}_L,\ldots,p_{m+2};q)\cdot\mathrm{d}\Phi_{X_{ijkl}}(p_i,p_j,p_k,p_l;\tilde{p}_I+\tilde{p}_L)$$

Integrated subtraction term (analytically)

$$|\mathcal{M}_m|^2\, J_m^{(m)}\, \mathrm{d}\Phi_m \int \mathrm{d}\Phi_{X_{ijkl}}\, X^0_{ijkl} \sim |\mathcal{M}_m|^2\, J_m^{(m)}\, \mathrm{d}\Phi_m \int \mathrm{d}\Phi_4 |M^0_{ijkl}|^2$$

Four-particle inclusive phase space integrals are known

A. Gehrmann-De Ridder, G. Heinrich, TG

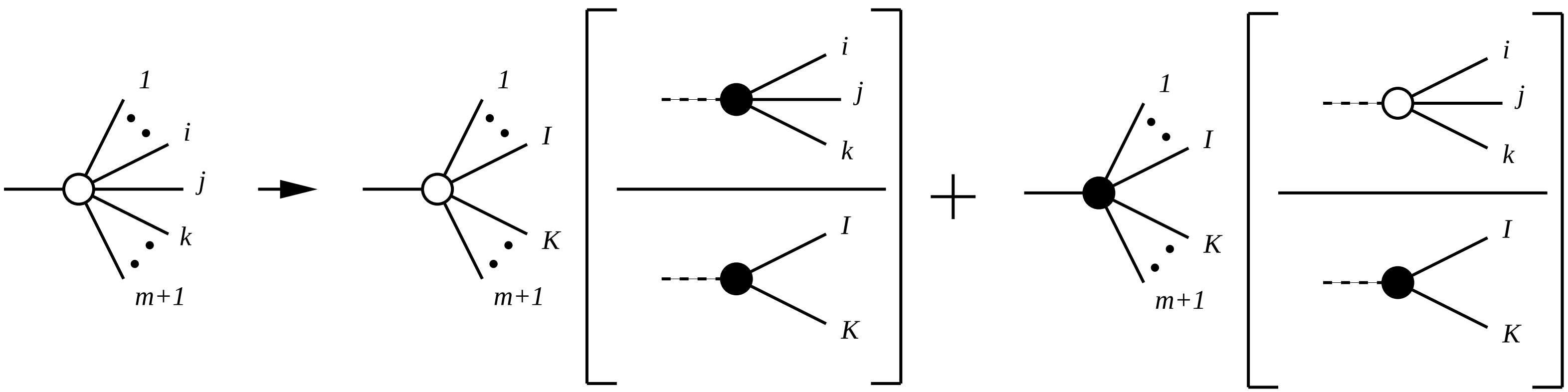
One-loop Real Subtraction

Single unresolved limit of one-loop amplitudes

$$Loop_{m+1} \stackrel{j\ unresolved}{\longrightarrow} Split_{tree} \times Loop_m + Split_{loop} \times Tree_m$$

Z. Bern, L.D. Dixon, D. Dunbar, D. Kosower; S. Catani, M. Grazzini; D. Kosower, P. Uwer
Z. Bern, V. Del Duca, W.B. Kilgore, C.R. Schmidt
Z. Bern, L.D. Dixon, D. Kosower; S. Badger, E.W.N. Glover

Accordingly: $Split_{tree} \to X^0_{ijk}$, $Split_{loop} \to X^1_{ijk}$



$$X^1_{ijk} = S_{ijk,IK} \frac{|\mathcal{M}^1_{ijk}|^2}{|\mathcal{M}^0_{IK}|^2} - X^0_{ijk} \frac{|\mathcal{M}^1_{IK}|^2}{|\mathcal{M}^0_{IK}|^2}$$

Colour-ordered antenna functions

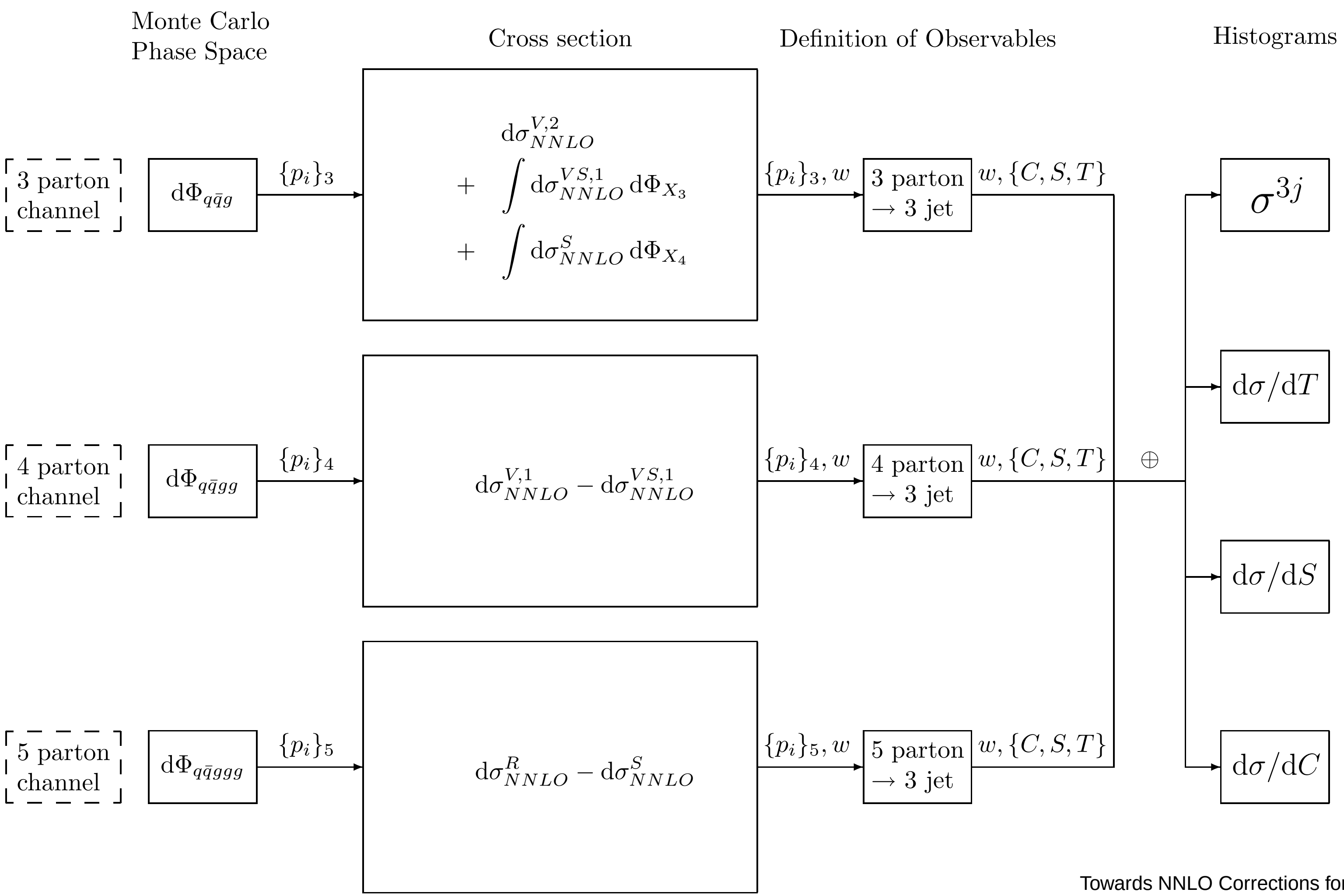
Antenna Functions

- colour-ordered pair of hard partons (radiators) with radiation in between
  - hard quark-antiquark pair: A, B, C
  - hard quark-gluon pair: D, E
  - hard gluon-gluon pair: F, G, H
- three-parton antenna $\longrightarrow$ one unresolved parton
- four-parton antenna $\longrightarrow$ two unresolved partons
- can be at tree level or at one loop
- all three-parton and four-parton antenna functions can be derived from physical matrix elements, normalised to two-parton matrix elements
  - $q\bar{q}$ from $\gamma^* \to q\bar{q} + X$
  - qg from $\tilde{\chi} \to \tilde{g}g + X$
  - gg from $H \to gg + X$

$e^+e^- \to 3$ jets at NNLO

Structure of $e^+e^- \to 3$ jets program:

EERAD3: A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, TG

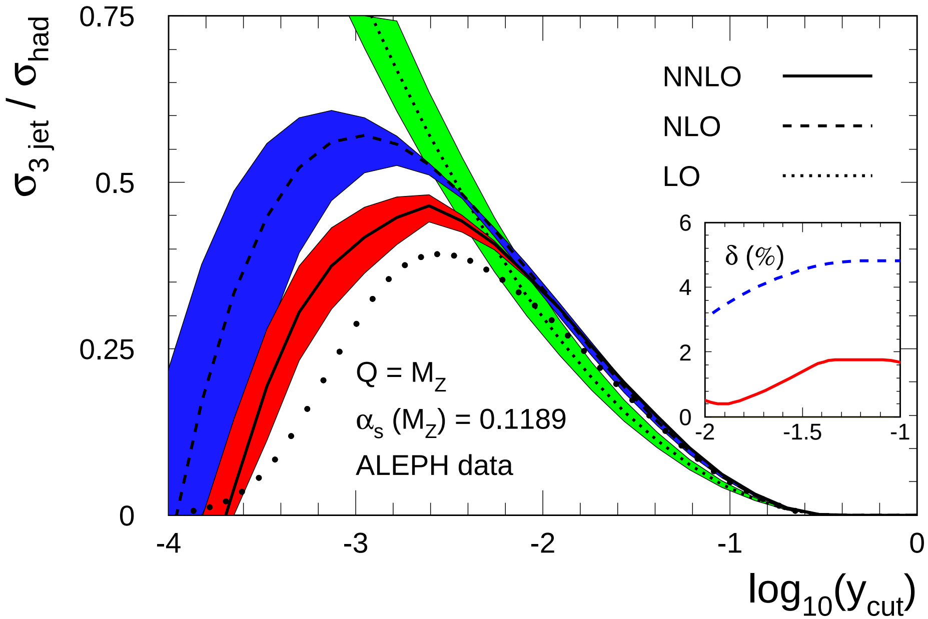


Three-jet cross section at NNLO

NNLO corrections: jet rates

Three-jet fraction in Durham jet algorithm

$$y_{i,j,D} = \frac{2\min(E_i^2, E_j^2)\,(1-\cos\theta_{ij})}{E_{vis}^2}$$



- vary $\mu = [M_Z/2\,;\,2\,M_Z]$
- NNLO corrections small
- substantial reduction of scale dependence
- better description towards lower jet resolution
- comparison with data yields

$$\alpha_s(M_Z) = 0.1175 \pm 0.0020(exp) \pm 0.0015(th)$$

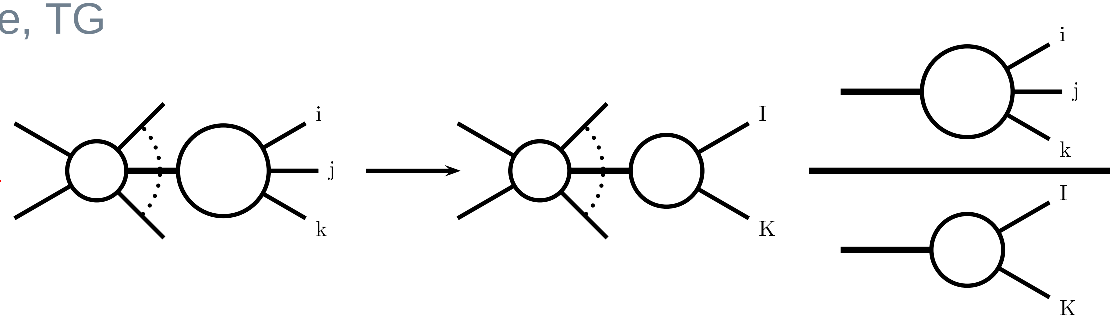
G. Dissertori, A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, H. Stenzel, TG

Incoming hadrons

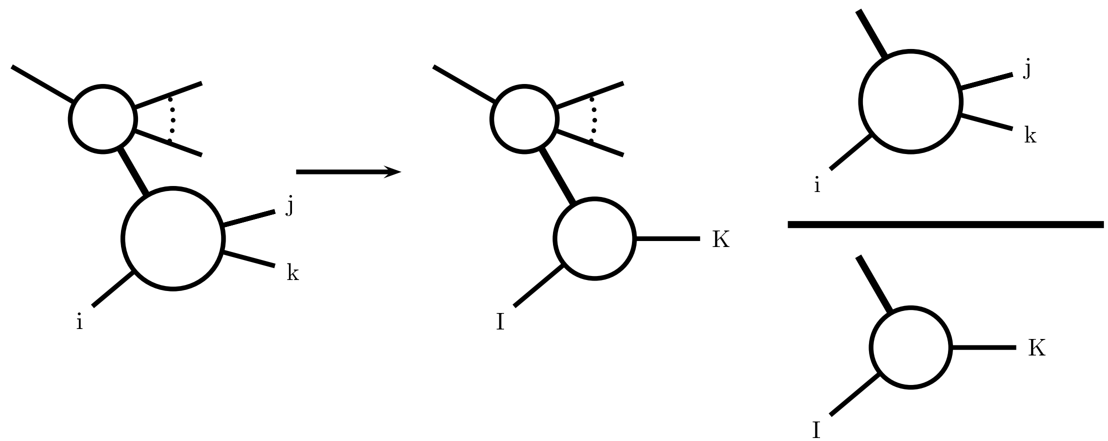
Three antenna types

NLO: A. Daleo, D. Maître, TG

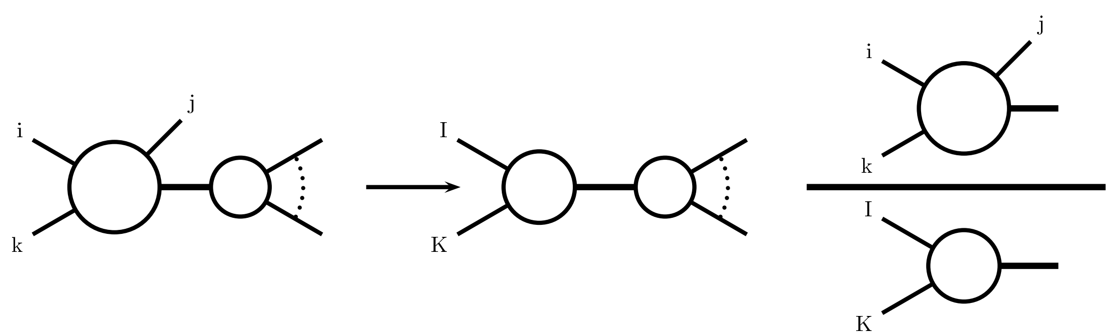
- final-final antenna



- initial-final antenna



- initial-initial antenna



Initial–final antenna functions

Real Radiation: $2 \to 3$

- obtain antenna functions by crossing $1 \to 4$ NNLO antennae
- kinematics: $q + p \to k_1 + k_2 + k_3$, with $q^2 < 0$.
- phase space factorization:

$$
\begin{aligned}
&\mathrm{d}\Phi_{m+2}(k_1, \ldots, k_j, k_k, k_l, \ldots, k_{m+1}; p, r) = \\
&\qquad \mathrm{d}\Phi_m(k_1, \ldots, K_L, \ldots, k_{m+2}; xp, r)\,\frac{Q^2}{2\pi}\,\mathrm{d}\Phi_3(k_j, k_k, k_l; p, q)\,\frac{\mathrm{d}x}{x}
\end{aligned}
$$

A. Daleo, D. Maître, TG

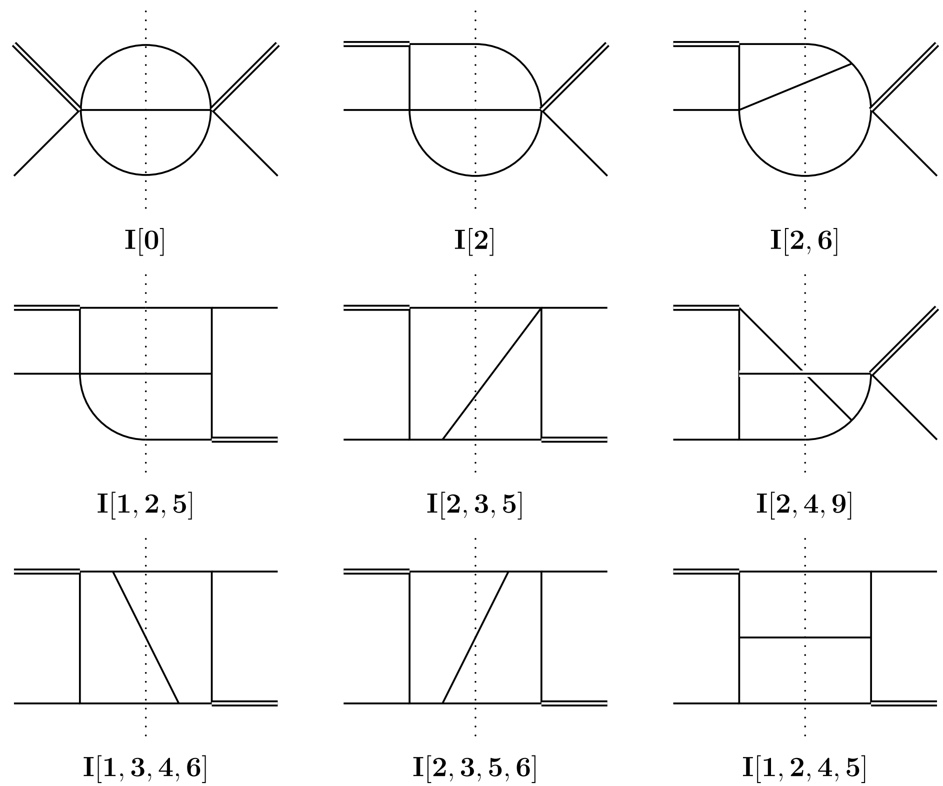
- integrated antenna functions: inclusive three-particle phase space integrals with q^2 and $z = -q^2/(2q \cdot p)$ fixed
- similar to NNLO deep-inelastic coefficient functions
  W.L. van Neerven, E.B. Zijlstra; S. Moch, G. Soar, J. Vermaseren, A. Vogt

Initial–final antenna functions

Real Radiation: $2 \to 3$

A. Daleo, A. Gehrmann-De Ridder, G. Luisoni, TG

- reduce phase space integrals to master integrals
  C. Anastasiou, K. Melnikov
- compute using differential equations



Initial–final antenna functions

Real Radiation at One Loop: $2 \to 2$

- obtain antenna functions by crossing one-loop $1 \to 3$ NNLO antennae
- kinematics: $q + p \to k_1 + k_2$, with $q^2 < 0$.
- phase space factorization:

$$\mathrm{d}\Phi_{m+1}(k_1, \ldots, k_j, k_k, \ldots, k_{m+1}; p, r) = \mathrm{d}\Phi_m(k_1, \ldots, K_K, \ldots, k_{m+2}; xp, r)\frac{Q^2}{2\pi}\mathrm{d}\Phi_2(k_j, k_k; p, q)\frac{\mathrm{d}x}{x}$$

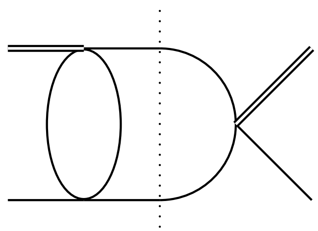
- integrated antenna functions: inclusive two-particle phase space integrals of one-loop matrix elements with q^2 and $z = -q^2/(2q \cdot p)$ fixed

Initial–final antenna functions

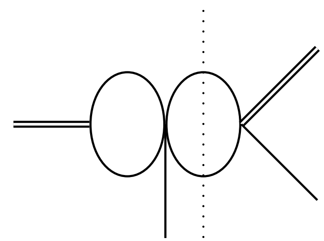
Real Radiation at One Loop: $2 \to 2$

A. Daleo, A. Gehrmann-De Ridder, G. Luisoni, TG

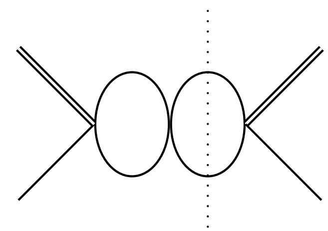
- reduce to master integrals
- most yield trivial Γ-functions
- non-trivial ones computed using differential equations



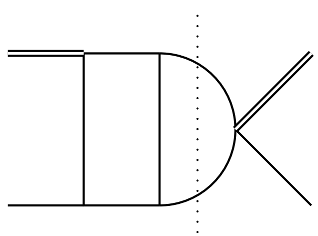
$\mathbf{V[1,3]}$



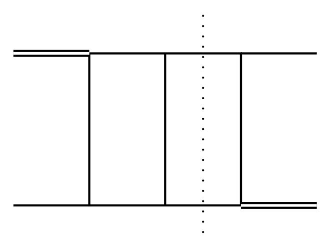
$\mathbf{V[1,4]}$



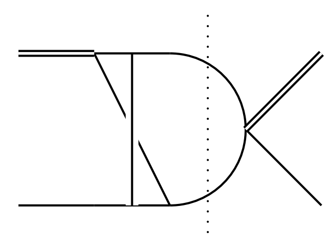
$\mathbf{V[2,4]}$



$\mathbf{V[1,2,3,4]}$



$\mathbf{V[1,2,3,4,5]}$



$\mathbf{C[1,2,3,4]}$

Initial–initial antenna functions

Real Radiation: $2 \to 3$

- obtain antenna functions by crossing $1 \to 4$ NNLO antennae
- kinematics: $p_a + p_b \to k_1 + k_2 + q$, with $q^2 > 0$.
- phase space factorization: (A. Daleo, D. Maître, TG)

$$\mathrm{d}\Phi_{m+2}(k_1,\ldots,k_{m+2};p_1,p_2) = \mathrm{d}\Phi_m(\tilde{k}_1,\ldots,\tilde{k}_i,\tilde{k}_l,\ldots,\tilde{k}_{m+1};x_1p_1,x_2p_2)$$
$$\delta(x_1-\hat{x}_1)\,\delta(x_2-\hat{x}_2)\,[\mathrm{d}k_j]\,[\mathrm{d}k_k]\,\mathrm{d}x_1\,\mathrm{d}x_2$$

$$\hat{x}_1 = \left(\frac{s_{12}-s_{j2}-s_{k2}}{s_{12}}\,\frac{s_{12}-s_{1j}-s_{1k}-s_{j2}-s_{k2}+s_{jk}}{s_{12}-s_{1j}-s_{1k}}\right)^{\frac{1}{2}}$$
$$\hat{x}_2 = \left(\frac{s_{12}-s_{1j}-s_{1k}}{s_{12}}\,\frac{s_{12}-s_{1j}-s_{1k}-s_{j2}-s_{k2}+s_{jk}}{s_{12}-s_{j2}-s_{k2}}\right)^{\frac{1}{2}}$$

- integration: inclusive three-particle phase space integrals with q^2 and x_1, x_2 fixed
- similar to NNLO coefficient functions for differential Drell-Yan production
  C. Anastasiou, L.J. Dixon, K. Melnikov, F. Petriello

Initial–initial antenna functions

Real Radiation: $2 \to 3$

Integration of antenna functions

R. Boughezal, A. Gehrmann-De Ridder, M. Ritzmann

- express phase space integrals as master integrals with two constraints: x_1, x_2
- distinguish
  - hard region: $x_1, x_2 \neq 1$: need ϵ^2
  - collinear regions: $x_1 = 1$ or $x_2 = 1$: need ϵ^3
  - soft region: $x_1 = x_2 = 1$: need ϵ^4
- full set of antenna functions contains 32 master integrals
- antenna functions with secondary fermion pair contain only 12 of them, already completed
- full set in progress

Initial–initial antenna functions

Real Radiation at One Loop: $2 \to 2$

- obtain antenna functions by crossing one-loop $1 \to 3$ NNLO antennae
- kinematics: $p_a + p_b \to k_1 + q$, with $q^2 > 0$.
- phase space factorization

$$\mathrm{d}\Phi_{m+1}(k_1,\ldots,k_{m+1};p_1,p_2) = \mathrm{d}\Phi_m(\tilde{k}_1,\ldots,\tilde{k}_i,\tilde{k}_k,\ldots,\tilde{k}_{m+1};x_1p_1,x_2p_2)$$
$$\delta(x_1-\hat{x}_1)\,\delta(x_2-\hat{x}_2)\,[\mathrm{d}k_j]\,\mathrm{d}x_1\,\mathrm{d}x_2$$

$$\hat{x}_1 = \left(\frac{s_{12}-s_{j2}}{s_{12}}\,\frac{s_{12}-s_{1j}-s_{j2}}{s_{12}-s_{1j}}\right)^{\frac{1}{2}}$$

$$\hat{x}_2 = \left(\frac{s_{12}-s_{1j}}{s_{12}}\,\frac{s_{12}-s_{1j}-s_{j2}}{s_{12}-s_{j2}}\right)^{\frac{1}{2}}$$

- phase space integral overconstrained, expand in distributions

P.F. Monni, TG

Integrated antenna functions

Three-parton tree-level

$\mathcal{X}_3^0$	Final-Final	Initial-Final	Initial-Initial
A	✓ [1]	✓ [2]	✓ [2]
D	✓ [1]	✓ [2]	✓ [2]
E	✓ [1]	✓ [2]	✓ [2]
F	✓ [1]	✓ [2]	✓ [2]
G	✓ [1]	✓ [2]	✓ [2]

[1] A. Gehrmann-De Ridder, N. Glover, TG

[2] A. Daleo, D. Maitre, TG

$\mathcal{S}$	Final-Final	Initial-Final	Initial-Initial
S	✓ [1]	✓ [2]	✘

[1] A. Gehrmann-De Ridder, N. Glover, G. Heinrich, TG

[2] A. Daleo, A. Gehrmann-De Ridder, G. Luisoni, TG

Integrated antenna functions

Four-parton tree-level

$\mathcal{X}_4^0$	Final-Final	Initial-Final	Initial-Initial
$A, \tilde{A}$	✓ [1]	✓ [2]	✘
B	✓ [1]	✓ [2]	✓ [3]
C	✓ [1]	✓ [2]	✘
D	✓ [1]	✓ [2]	✘
$E, \tilde{E}$	✓ [1]	✓ [2]	✓ [3]
F	✓ [1]	✓ [2]	✘
$G, \tilde{G}$	✓ [1]	✓ [2]	✘
H	✓ [1]	✓ [2]	✓ [3]

[1] A. Gehrmann-De Ridder, N. Glover, TG

[2] A. Daleo, A. Gehrmann-De Ridder, G. Luisoni, TG

[3] R. Boughezal, A. Gehrmann-De Ridder, M. Ritzmann

Remaining Initial-Initial functions depend on further 20 master integrals

Integrated antenna functions

Three-parton one-loop

$\mathcal{X}_3^1$	Final-Final	Initial-Final	Initial-Initial
A, $\tilde{A}$, $\hat{A}$	✓ [1]	✓ [2]	✓ [3]
D, $\hat{D}$	✓ [1]	✓ [2]	✓ [3]
F, $\hat{F}$	✓ [1]	✓ [2]	✓ [3]
E, $\tilde{E}$, $\hat{E}$	✓ [1]	✓ [2]	✓ [3]
G, $\tilde{G}$, $\hat{G}$	✓ [1]	✓ [2]	✓ [3]

[1] A. Gehrmann-De Ridder, N. Glover, TG

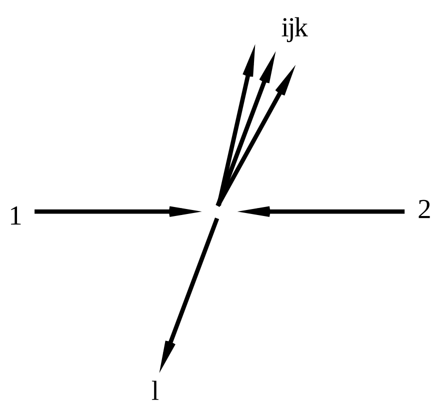
[2] A. Daleo, A. Gehrmann-De Ridder, G. Luisoni, TG

[3] P.F. Monni, TG

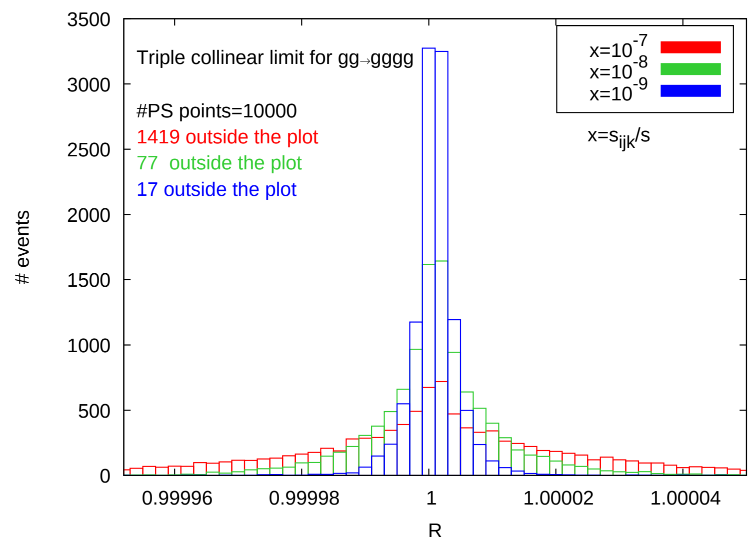
Implementation: $pp \to 2j$ at NNLO

Aim: “leading colour gluons-only" $pp \to 2$ jets to demonstrate proof of concept

- Double unresolved subtraction terms for leading colour six-gluon process tested
  N. Glover, J. Pires



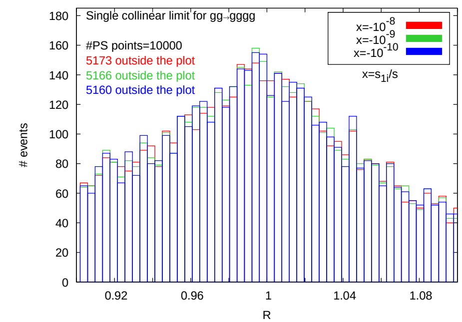
Example configuration of a triple collinear event with $s_{ijk} \to 0$.

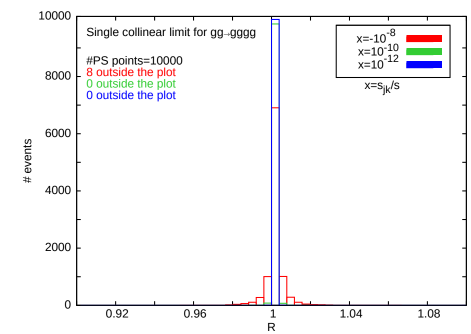


Distribution of $\mathrm{d}\hat{\sigma}^{R}_{NNLO}/\mathrm{d}\hat{\sigma}^{S}_{NNLO}$ for 10000 triple collinear phase space points.

Implementation: $pp \to 2j$ at NNLO

- Evidence of non-local azimuthal terms in collinear limits
  e.g. configuration of a single collinear event with $s_{1i} \to 0$.





- Solution: Combine events with momenta of collinear pair rotated by 90 degrees
  N. Glover, J. Pires

- Automatic generation of phase space points related by rotations
  A. Gehrmann-De Ridder, N. Glover, J. Pires, TG

- Implementation for jet production in deep inelastic scattering
  P. Jimenez-Delgado, G. Luisoni, TG

Implementation: $pp \to 2j$ at NNLO

Aim: "leading colour gluons-only" $pp \to 2$ jets to demonstrate proof of concept

- Single unresolved subtraction terms for leading colour one-loop five-gluon process in progress
  A. Gehrmann-De Ridder N. Glover, J. Pires
- involves:
  - integrated three-parton tree-level antenna functions
  - unintegrated three-parton one-loop antenna functions
  - integrated soft antenna functions
  - interplay of antenna functions with parton distribution counterterms
- local cancellation of singularities accomplished

Top quark pairs at NNLO

Two-loop matrix elements: $q\bar{q} \to t\bar{t}$ and $g\bar{g} \to t\bar{t}$

- known in high-energy limit (M. Czakon, A. Mitov, S. Moch)
- quark-initiated process: known numerically (M. Czakon)
- fermionic contributions and leading-colour terms confirmed analytically (R. Bonciani, A. Ferroglia, D. Maitre, A. von Manteuffel, C. Studerus, TG)

require: method to handle NNLO real radiation

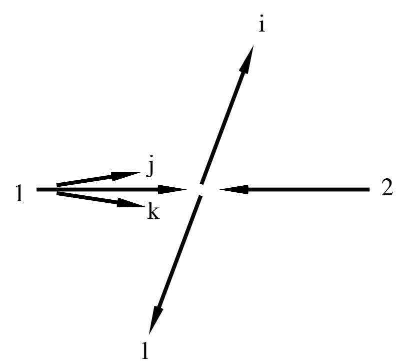
- combination of residue subtraction and sector decomposition
  - successfully applied to double real radiation (M. Czakon)
  - requires massive soft current up to one loop (I. Bierenbaum, M. Czakon, A. Mitov)
- massive antenna subtraction
  - massive antenna functions (G. Abelof, A. Gehrmann-De Ridder, M. Ritzmann; W. Bernreuther, O. Dekkers)
  - implementation of double real radiation (G. Abelof, A. Gehrmann-De Ridder)

Top quark pairs at NNLO

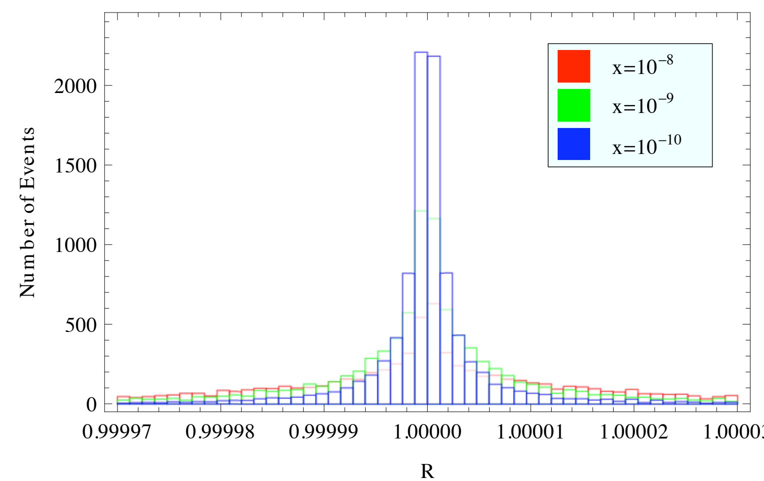
Implementation of antenna subtraction: double real radiation

A. Gehrmann-De Ridder, G. Abelof

- Types of double unresolved singularities
  - initial state radiation: soft or collinear
  - final state radiation: only soft



Example configuration of a triple collinear initial state radiation.



Distribution of $\mathrm{d}\hat{\sigma}^{R}_{NNLO}/\mathrm{d}\hat{\sigma}^{S}_{NNLO}$ for 10000 triple collinear phase space points.

Summary and Conclusions

- High precision data on jet observables demand theoretical accuracy beyond NLO
- Principal ingredients to NNLO jet calculations
  - two-loop virtual corrections
  - generic algorithm for real emission: antenna subtraction
- Development for NNLO jets at hadron colliders
  - antenna subtraction for initial state radiation
  - proof-of-concept on NNLO corrections to $gg \to gg$
- Precision calculations for jet observables at HERA/Tevatron/LHC in progress