Strong lensing and dark energy

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Summary

1. Lensing in brief
   - Deflection of light
   - From weak to strong lensing (in clusters)
2. The dark energy-strong lensing connection
   - Arc statistics
   - Geometrical constraints from multiple arcs

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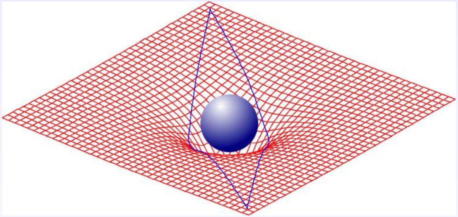
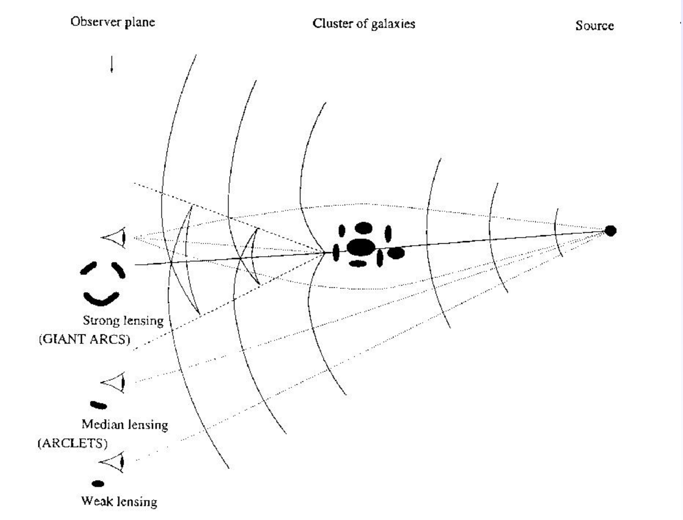
Deflection of light

- The General Theory of Relativity explains gravity in terms of assemblies of mass and energy curving space-time
- Masses deflect light in a way similar to a convex lens
- The deflection of light rays from their otherwise straight paths can give rise to multiple images because light rays can now pass masses in multiple ways
- Light bundles can be deformed and focused as they pass masses



$$\hat{\vec{\alpha}}(\vec{\xi}) = \frac{4G}{c^2}\frac{M}{\xi^2}\vec{\xi}$$

Galaxy clusters as gravitational lenses



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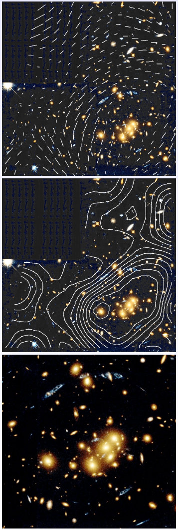

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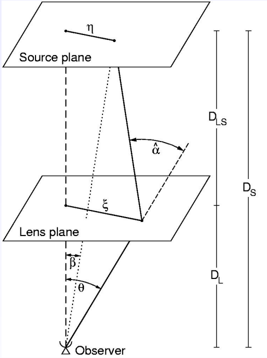
First order lensing



The lensing transformation is described by the lens equation:

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta}) \quad ; \quad \vec{\alpha} = \frac{D_{LS}}{D_S}\hat{\vec{\alpha}}$$

if $\alpha \sim 0$, this equation is approximated (at first order) by

$$\vec{\beta} \approx \left(1 - \frac{\partial \vec{\alpha}}{\partial \vec{\theta}}\right)\vec{\theta} = A\vec{\theta}$$

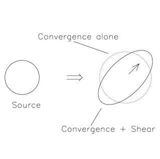
the Jacobian matrix is

$$A_{ij} = \left(1 - \frac{\partial \alpha_i}{\partial \theta_j}\right)$$

Convergence and shear

The Jacobian matrix can be written as:

$$A = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} = (1-\kappa)I - \gamma R(2\phi)$$



where

$$\kappa(\vec{\theta}) = \frac{\Sigma}{\Sigma_{\rm cr}}; \Sigma_{\rm crit} = \frac{c^2}{4\pi G}\frac{D_S}{D_L D_{LS}}$$

is the convergence and

$$\vec{\gamma}(\vec{\theta}) = (\gamma_1, \gamma_2)$$

is the shear

$$\begin{aligned} \kappa &= \frac{1}{2}\left(\frac{\partial\alpha_1}{\partial\theta_1} + \frac{\partial\alpha_2}{\partial\theta_2}\right) \\ \gamma_1 &= \frac{1}{2}\left(\frac{\partial\alpha_1}{\partial\theta_1} - \frac{\partial\alpha_2}{\partial\theta_2}\right) \\ \gamma_2 &= \left(\frac{\partial\alpha_1}{\partial\theta_2}\right) \end{aligned}$$

Induced ellipticity

$$\vec{e} = \vec{\gamma}/(1-\kappa) \approx \vec{\gamma}, \kappa \ll 1$$

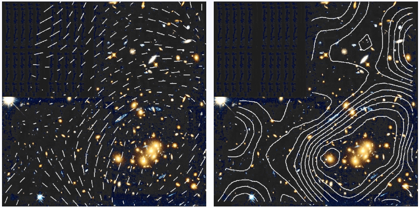
The principle of weak lensing

$$\text{obs.ellipticity} \approx \text{shear}$$

$$\text{shear} \leftrightarrow \partial\text{defl.angle} \leftrightarrow \text{convergence}$$

$$\Downarrow$$

$$\text{obs.ellipticity} \Rightarrow \text{mass}$$



$$\kappa(\vec{\theta}) = \frac{1}{\pi} \int d^2\theta' \, D(\vec{\theta} - \vec{\theta}')\gamma(\vec{\theta}')$$

Second order lensing

Expanding the lens equation at the second order

$$\beta_i \simeq \frac{\partial \beta_i}{\partial \theta_j}\theta_j + \frac{1}{2}\frac{\partial^2 \beta_i}{\partial \theta_j \partial \theta_k}\theta_j\theta_k \ .$$

$$D_{ijk} = \frac{\partial^2 \beta_i}{\partial \theta_j \partial \theta_k} = \frac{\partial A_{ij}}{\partial \theta_k} \ .$$

$$\beta_i \simeq A_{ij}\theta_j + \frac{1}{2}D_{ijk}\theta_j\theta_k$$

$$\begin{aligned} D_{ij1} &= \begin{pmatrix} -2\gamma_{1,1} - \gamma_{2,2} & -\gamma_{2,1} \\ -\gamma_{2,1} & -\gamma_{2,2} \end{pmatrix} , \\ D_{ij2} &= \begin{pmatrix} -\gamma_{2,1} & -\gamma_{2,2} \\ -\gamma_{2,2} & 2\gamma_{1,2} - \gamma_{2,1} \end{pmatrix} . \end{aligned}$$

The second order term can be written in terms of the first and of the second flexions:

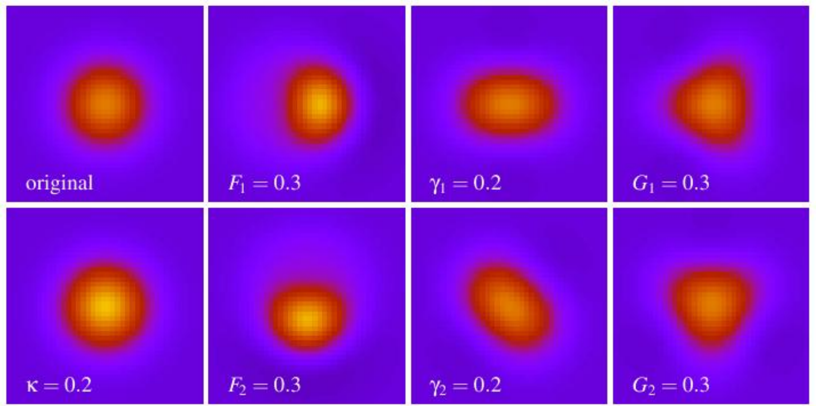
$$\begin{aligned} \vec{F} &= [(\gamma_{1,1} + \gamma_{2,2}), (\gamma_{2,1} - \gamma_{1,2})] \\ \vec{G} &= [(\gamma_{1,1} - \gamma_{2,2}), (\gamma_{2,1} + \gamma_{1,2})] \end{aligned}$$

Big advantage:

$$\vec{F} = \vec{\nabla}\kappa \ .$$

Since flexion depends on the spatial derivatives of the shear, it introduces further distortions in the images.

First and second order lensing... in practice!



Higher orders lensing: strong lensing

As soon as we approach the high density region of the lens, higher order terms become important...

$\kappa, \gamma \sim 1$; α is large

$$\beta_i = A_{ij}\theta_j + \frac{1}{2}D_{ijk}\theta_j\theta_k + O(>2)$$

Several consequences... Among them:

1. multiple images: lens equation have more than one solution!
2. large distortions: arclets, arcs, etc.

Where do gravitational arcs form?

- lens equation: $\delta\beta^2 \rightarrow \delta\theta^2$;
- locally, the lens equation can be linearized to yield:

$$\delta\beta^2 = \det A \cdot \delta\theta^2 \;;$$

- since lensing conserves the surface brightness, the source is magnified:

$$\mu = \frac{\delta\theta^2}{\delta\beta^2} = \frac{1}{\det A}$$

- using the definitions of A, κ and γ:

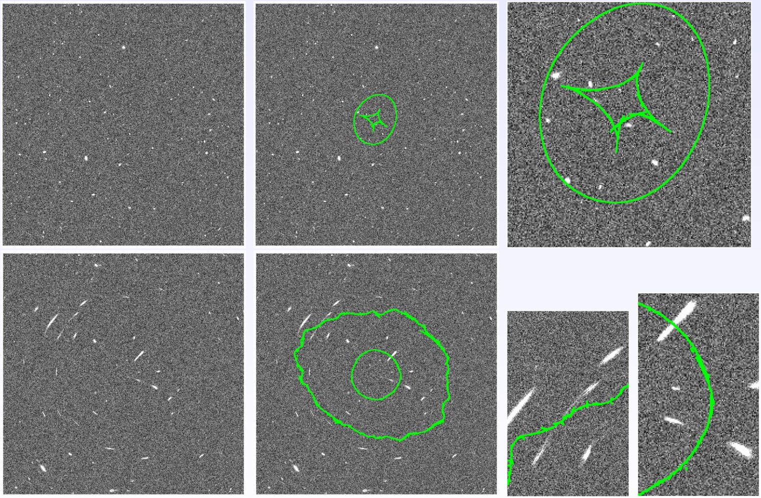
$$\det A = (1 - \kappa - \gamma)(1 - \kappa + \gamma)$$

- thus, $\kappa + \gamma = 1$ or $\kappa - \gamma = 1$ imply infinite magnification!

Critical lines and caustics

- the points $\vec{\theta}_c$ on the lens plane where $1-\kappa(\vec{\theta}_c)-\gamma(\vec{\theta}_c)=0$ and $1-\kappa(\vec{\theta}_c)+\gamma(\vec{\theta}_c)=0$ form the *critical lines* (tangential and radial, respectively)
- along the tangential (radial) critical line, the magnification is infinite in the tangential (radial) direction
- the corresponding points $\vec{\beta}_c=\vec{\theta}_c-\vec{\alpha}(\vec{\theta}_c)$ on the source plane form the *caustics*
- a source close to the caustics is thus *tangentially or radially* distorted across the critical lines

Example



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Arc statistics

The frequency at which arcs occur in clusters depends on

- the geometry of the Universe;
- the number of potential lenses at the relevant redshifts;
- the inner structure of the lenses (concentration, dynamical status, etc.)

Arc statistics: Bartelmann et al. (1998)

The number of *giant* arcs ($L/W > 10$) expected to be seen on the sky in a flat universe, where dark energy is in the form of a cosmological constant ($\Omega_m = 0.3, \Omega_\Lambda = 0.7$), is

- a factor of ten larger than in a flat universe without dark energy ($\Omega_m = 1, \Omega_\Lambda = 0$);
- a factor of ten smaller than in an open universe without dark energy ($\Omega_m = 0.3, \Omega_\Lambda = 0$).

In numbers:

model	number
SCDM	~ 10
ΛCDM	~ 100
OCDM	~ 1000

[see also Li et al. (2006)]

Some dark energy cosmologies

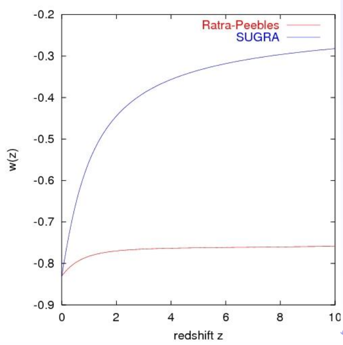
Dark energy

Component of smooth energy with equation of state $p_{DE} = w(t)\rho_{DE}c^2$ and large negative pressure

A few models have be already tested with numerical simulations (see e.g. Klypin et al. 2003; Dolag et al., 2004; Macció et al., 2004) $\Rightarrow$ important for testing lensing!

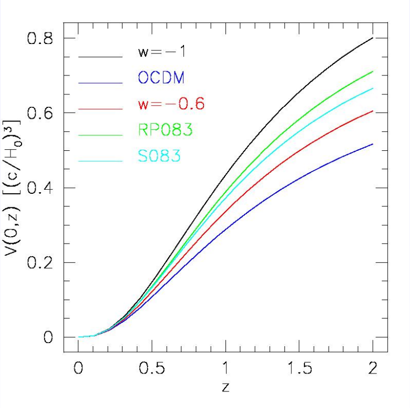
For the following discussion we focus on

- w=-1 $\Longrightarrow$ ΛCDM;
- w=const.> -1
- w=w(t) $\Longrightarrow$ SUGRA or Ratra-Peebles potentials (Brax & Martin, 2000; Peebles & Ratra, 2002)



DE and geometry

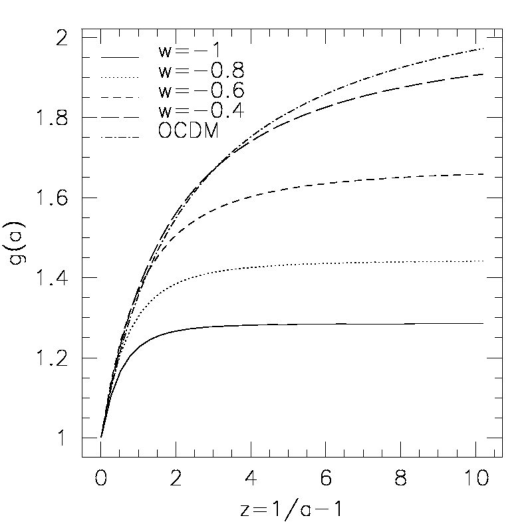
Dark energy obviously affects the distances and the volumes



Linear growth of density fluctuations

The linear growth of density fluctuations is more rapid in DE cosmologies

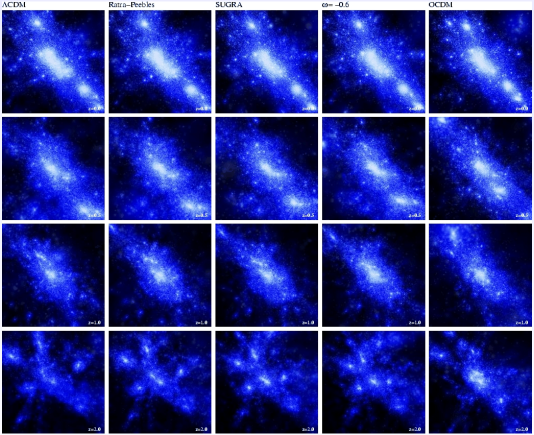
The earlier DE becomes dominant, the earlier structures must form



Structure formation in DE models

The same cluster appears already formed at higher redshifts in these DE models

Dolag et al. (2004): DE models interpolate between ΛCDM and OCDM models



Halo concentration in DE models

Dark matter halos develop NFW density profiles:

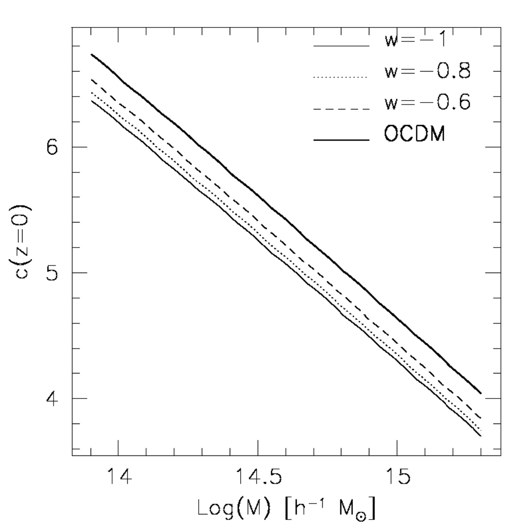
$$\rho(r) = \frac{\rho_s}{r/r_s(1 + r/rs)^2}$$

The concentration is

$$c = \frac{r_{vir}}{r_s}$$

Numerical simulations show that halo concentration keeps memory of the density of the universe at the epoch of formation

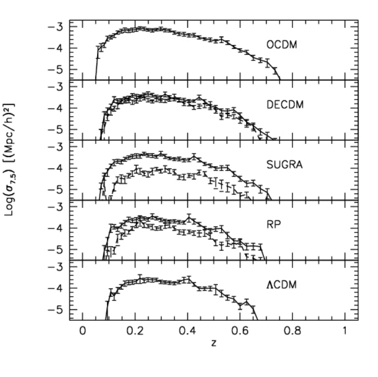
If a cluster form earlier, it will result to be more concentrated



Arc statistics in DE models

17 halos evolved at different redshifts (52 snaps between $z = 1$ and $z = 0$), simulated in

- ▷ ΛCDM ($\Omega_M = 0.3, \Omega_{DE} = 0.7$)
- ▷ DECDM ($w = -0.6$)
- ▷ Ratra-Peebles ($w_0 = -0.83$)
- ▷ SUGRA ($w_0 = -0.83$)
- ▷ OCDM ($\Omega_M = 0.3, \Omega_{DE} = 0$)

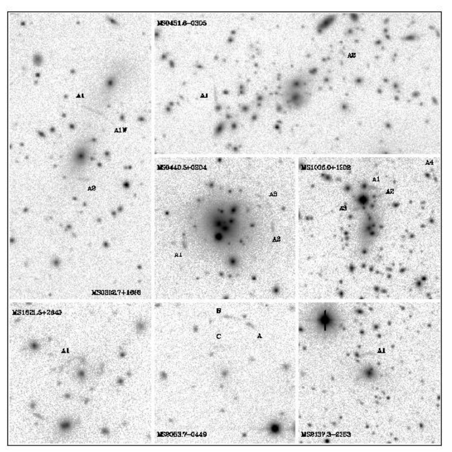


(Meneghetti, Bartelmann, Dolag, Perrotta, Baccigalupi, Moscardini, Tormen, 2005)

Studied using ray-tracing techniques [see also Macció (2005) and Fedeli & Bartelmann (2006) for other DE models]

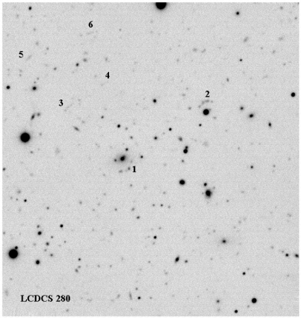
Observational results: EMSS

The reference work has been for many years the arc survey by Luppino et al. (1998).
They selected a sample of 38 X-ray bright clusters from the EMSS and found a lensing incidence of
$0.2 \div 0.3$
giant arcs per massive cluster ($L/W > 10$, $B < 21.5$).
Extrapolating to the whole sky, the observed number of giant arcs is ~ 3000.

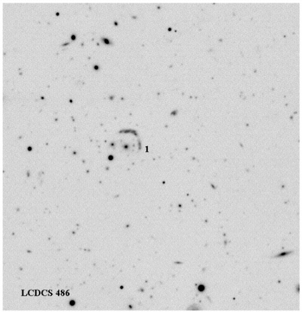


Observational results: LCDCS

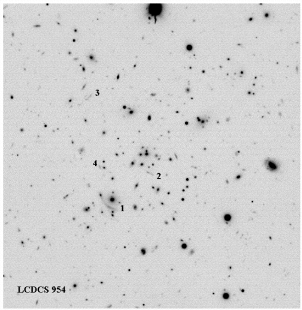
Zaritsky & Gonzales (2003): the lensing efficiency remains large at high redshift ($z \gtrsim 0.5$).



$z = 0.54$



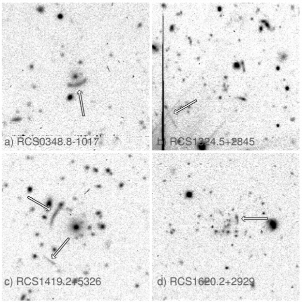
$z = 0.61$

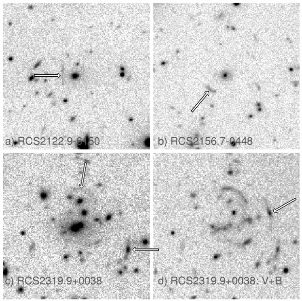


$z = 0.67$

Observational results: RCS

Gladders et al. (2003): "...the high incidence of multiple arc systems and their overall high redshift suggests that a sub-population of the global cluster population is responsable for much of the observed lensing."
A property associated with clusters at early times results in the boosted lensing cross sections.



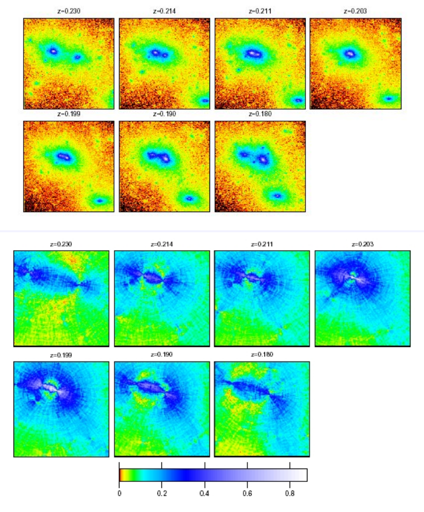


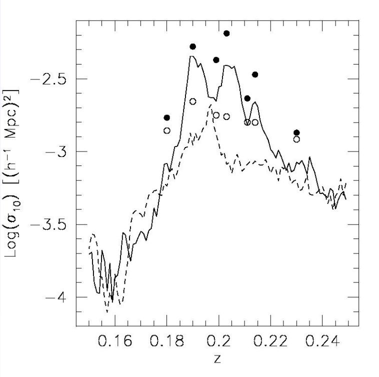
The arc statistics problem

1. arc statistics studies find that the number of arcs theoretically expected in the framework of ΛCDM cosmology is too low compared to observations ($5-10$ times smaller)
2. all other cosmological tests support the "concordance model" (ΛCDM)
3. the strong lensing efficiency is expected to decline at high redshift

Dark energy in form different from a cosmological constant represents a way alleviate the problem...

Sensitivity of arc statistics to cluster dynamics





Torri et al. (2004) [see also Fedeli et al. (2006)]

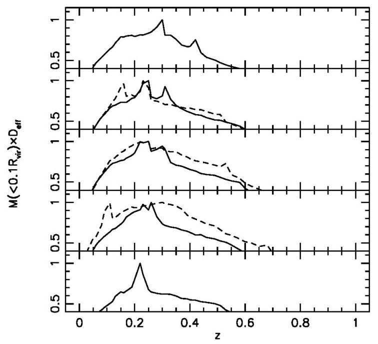
Cluster dynamics and DE

If a cluster has to reach a particular configuration "today" but form at different epochs, its assembling history has to be diluted or compressed in different time intervals.

Clusters evolve through accretion of smaller clumps of matter or *mergers*.

Thus DE changes the timing of mergers.

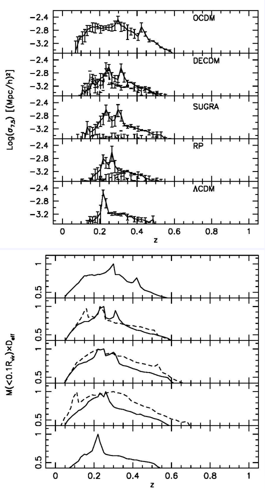
This combines with the concept of *effective distance for lensing*: the lens has to be at $\sim$ half way betwen the observer and the source to be efficient



Arc efficiency boosts at high redshifts

Diluting the assembling of clusters to higher redshifts, dark-energy in a form different from a cosmological allows peaks of strong lensing efficiency at higher redshifts

This is combined with the more extended "lensing window" due to a more favorable geometry of the universe, with a larger concentration of the lenses, etc.



Arc statistics & DE: conclusions

- Dark-energy may be a possible solution of the arc statistics problems.
- large number of arcs (expecially at high redshift) may indicate $w \neq -1$

Weaknesses of the method

Can we put constraints on w and w'?

- large sensitivity to ellipticity, asymmetries, substructures
- need realistic cluster models (simulations in many dark energy models)
- still few observations for statistical comparison

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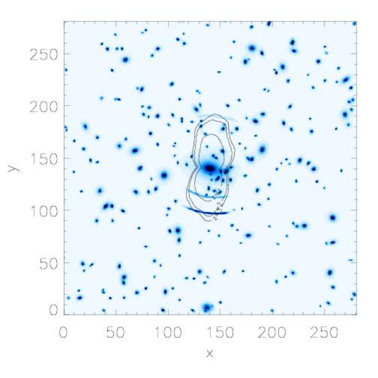
Multiple arcs in clusters



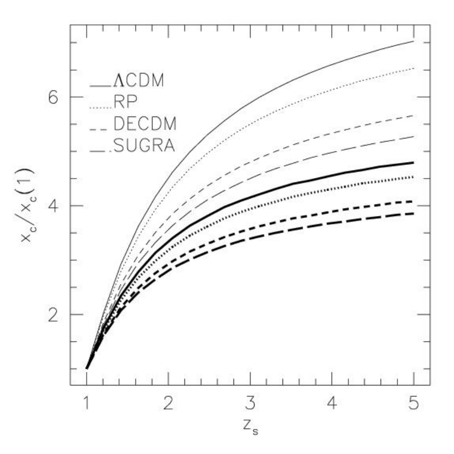
Arcs from sources at different redshifts can be used for constraining the positions of the lens critical curves

The growth of the critical lines as function of the source redshift depends on geometrical factors

$$\theta_t = \sqrt{\frac{4GM(\theta_t)}{c^2} \frac{D_{LS}}{D_L D_S}}$$

(Link & Pierce, 1998; Golse, Kneib & Soucail, 2003; Sereno, 2003)

Growth of the critical lines: analytic models



Assuming a density profile in the form

$$\rho(r) = \frac{\rho_c}{r/r_s(1 + r/r_s)^2}$$

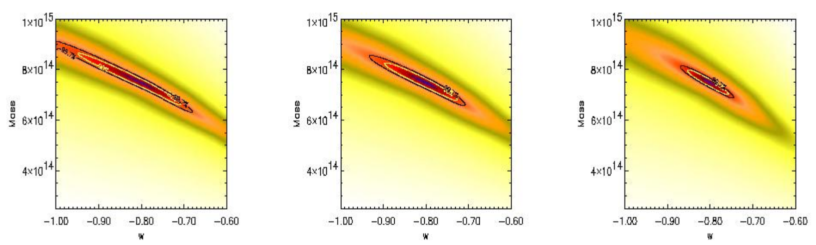
Different behaviours depending on:

- lens ellipticity
- lens mass
- lens redshift

[see also Dalal et al. (2004)]

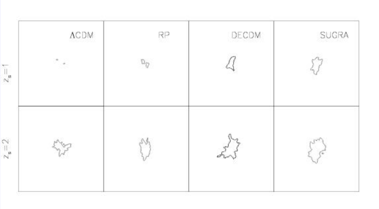
Difficulties in fitting a single lens

(Meneghetti, Jain, Bartelmann, Dolag, 2005)

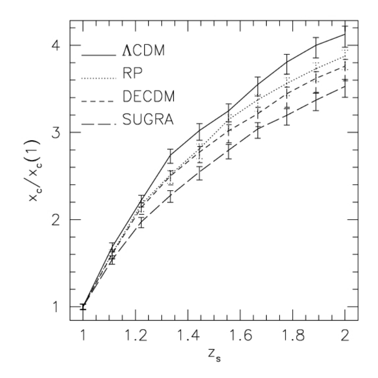


Growth of the critical lines: numerical models

(Meneghetti, Jain, Bartelmann, Dolag, 2005)



Averaging over a sample of numerical clusters, we reproduce the trend expected from analytic estimates!



Geometrical constraints: conclusions

- the so called "golden lens" approach is very unlikely to give any constraints on the equation of state of dark matter
- good news: averaging over $\sim$ 1000 clusters may work (future missions from space)