Topological Bandstructures for Ultracold Atoms

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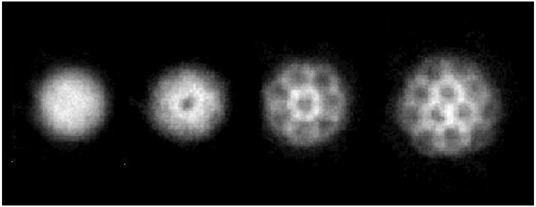
New quantum states of matter in and out of equilibrium
GGI, Florence, 12 April 2012

NRC, PRL **106**, 175301 (2011)
Benjamin Béri & NRC, PRL **107**, 145301 (2011)
NRC & Jean Dalibard, EPL **95**, 66004 (2011)

Motivation: fractional quantum Hall regime

Rotating BECs

$$n_\phi = \frac{2M\Omega}{h}$$



[K. W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, Phys. Rev. Lett. **84**, 806 (2000)]

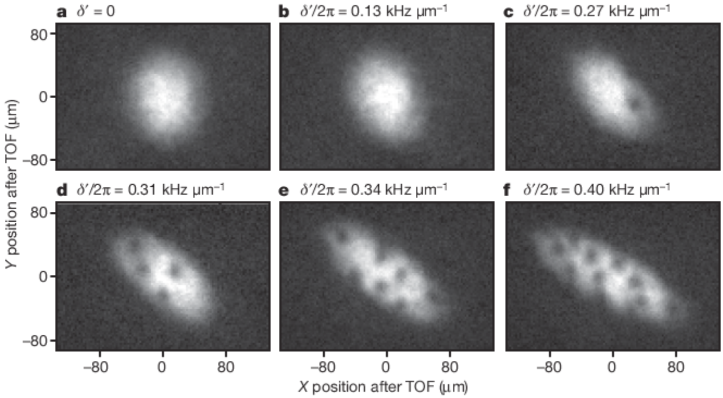
FQH states of bosons for $\frac{n_{2\mathrm{D}}}{n_\phi} \lesssim 6$ [NRC, Wilkin & Gunn, PRL (2001)]

[Laughlin, composite fermion, Moore-Read and Read-Rezayi]

$$\Omega \simeq 2\pi \times 100\mathrm{Hz} \Rightarrow n_\phi \lesssim 2 \times 10^7 \mathrm{cm}^{-2}$$

Optically Induced Gauge Fields

[Y.-J. Lin, R.L. Compton, K. Jiménez-García, J.V. Porto and I.B. Spielman, Nature **462**, 628 (2009)]



"Optical Flux Lattices"

[NRC, PRL **106**, 175301 (2011); NRC & Jean Dalibard, EPL **95**, 66004 (2011)]

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

- Landau levels: Narrow bands with unit Chern number
  $n_\phi \simeq 10^9\mathrm{cm}^{-2} \Rightarrow$FQH states at high particle densities

- Distinct from previous tight-binding proposals

  [Jaksch & Zoller (2003); Mueller (2004); Sørensen, Demler & Lukin (2005); Gerbier & Dalibard (2010)]

- Generalizes to $\mathbb{Z}_2$ topological invariant

  [Benjamin Béri & NRC, PRL **107**, 145301 (2011)]

- "Nearly free electron" approach to topological bands

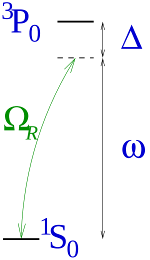
Outline

Optically Induced Gauge Fields

[J. Dalibard, F. Gerbier, G. Juzeliūnas, P. Öhberg, RMP **83**, 1523 (2011)]

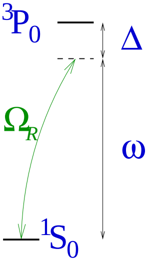
$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

$\hat{V}(\mathbf{r})$: optical coupling of N internal states



e.g. $^1S_0$ and $^3P_0$ for Yb or alkaline earth atom

[F. Gerbier & J. Dalibard, NJP **12**, 033007 (2010)]



e.g. $^1S_0$ and $^3P_0$ for Yb or alkaline earth atom

[F. Gerbier & J. Dalibard, NJP **12**, 033007 (2010)]

$$\begin{aligned}\hat{V} &= \hbar \begin{pmatrix} 0 & \frac{1}{2}\left(\Omega_R e^{i\omega t} + \Omega_R^* e^{-i\omega t}\right) \\ \frac{1}{2}\left(\Omega_R^* e^{-i\omega t} + \Omega_R e^{i\omega t}\right) & \omega_0 \end{pmatrix} \\ &\to \hbar \begin{pmatrix} -\frac{\Delta}{2} & \frac{1}{2}\left(\Omega_R + \Omega_R^* e^{-2i\omega t}\right) \\ \frac{1}{2}\left(\Omega_R^* + \Omega_R e^{2i\omega t}\right) & \frac{\Delta}{2} \end{pmatrix}\end{aligned}$$

RWA $\omega \gg \Delta, \Omega_R$ $\qquad \hat{V} \to \frac{\hbar}{2} \begin{pmatrix} -\Delta & \Omega_R(\mathbf{r}) \\ \Omega_R^*(\mathbf{r}) & \Delta \end{pmatrix}$

[J. Dalibard, F. Gerbier, G. Juzeliūnas, P. Öhberg, RMP **83**, 1523 (2011)]

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \hat{V}(\mathbf{r})$$

$\hat{V}(\mathbf{r}) \Rightarrow$local spectrum $E_n(\mathbf{r})$ and dressed states $|n_\mathbf{r}\rangle$

$$|\psi(\mathbf{r})\rangle = \sum_n \psi_n(\mathbf{r})|n_\mathbf{r}\rangle$$

Adiabatic motion $H_n\psi_n = \langle n_\mathbf{r}|\hat{H}\psi_n|n_\mathbf{r}\rangle$

$$H_n = \frac{(\mathbf{p} - q\mathbf{A})^2}{2M} + V_n(\mathbf{r}) \qquad q\mathbf{A} = i\hbar\langle n_\mathbf{r}|\boldsymbol{\nabla} n_\mathbf{r}\rangle$$

Maximum flux density: Back of the envelope

Vector potential $q\mathbf{A} = i\hbar\langle 0_{\mathbf{r}}|\boldsymbol{\nabla} 0_{\mathbf{r}}\rangle \Rightarrow |q\mathbf{A}| \lesssim \frac{h}{\lambda}$

Cloud of radius $R \gg \lambda$

$$
\begin{aligned}
N_\phi &\equiv \int n_\phi d^2\mathbf{r} = \frac{q}{h}\int \boldsymbol{\nabla}\times\mathbf{A}\cdot d\mathbf{S} = \frac{q}{h}\oint \mathbf{A}\cdot d\mathbf{r} \lesssim \frac{1}{\lambda}(2\pi R) \\
\Rightarrow \bar{n}_\phi &\equiv \frac{N_\phi}{\pi R^2} \lesssim \frac{1}{R\lambda} \simeq 2\times 10^7 \mathrm{cm}^{-2}
\end{aligned}
$$

$[R \simeq 10\mu\mathrm{m}\ \ \lambda \simeq 0.5\mu\mathrm{m}]$

Maximum flux density: Carefully this time!

~~Optical wavelength~~ $\cancel{\lambda \Rightarrow |q\mathbf{A}| \lesssim \frac{h}{\lambda}}$

$\mathbf{A}$ can have *singularities* – if the optical fields have vortices.

e.g. $\Omega_R(\mathbf{r}) \sim (x + iy)$

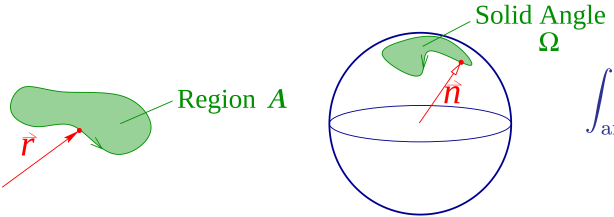
Vanishing net flux. Can be removed by a gauge transformation.

[cf. "Dirac strings"]

Gauge-independent approach (two-level system)

Bloch vector $\vec{n}(\mathbf{r}) = \langle 0_\mathbf{r}|\hat{\vec{\sigma}}|0_\mathbf{r}\rangle$

$$n_\phi = \frac{1}{8\pi}\epsilon_{ijk}\epsilon_{\mu\nu}n_i\partial_\mu n_j\partial_\nu n_k \qquad |n_\phi| \lesssim \frac{1}{\lambda^2}$$



$$\int_{\text{area } A} n_\phi d^2 r = \frac{\Omega}{4\pi}$$

The number of flux quanta in region A is the number of times the Bloch vector wraps over the sphere.

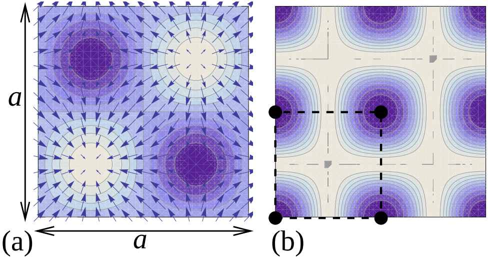
Optical flux lattices

[NRC, Phys. Rev. Lett. **106**, 175301 (2011)]

Spatially periodic light fields which cause the Bloch vector to wrap the sphere a nonzero integer number, N_ϕ, times in each unit cell.

$$\bar{n}_\phi = \frac{N_\phi}{A_{\rm cell}} \sim \frac{1}{\lambda^2} \simeq 10^9 \text{cm}^{-2}$$

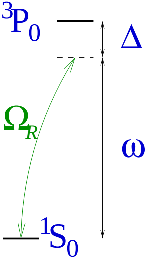
vectors (n_x, n_y)
contours n_z
$N_\phi = 2$



contours n_ϕ

Optical Flux Lattice: One-Photon Implementation

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \mathcal{V}\hat{M}(\mathbf{r}) \qquad \hat{M} = \vec{M}(\mathbf{r})\cdot\hat{\vec{\sigma}}$$



e.g. $^1S_0$ and $^3P_0$ for Yb or alkaline earth atom

[Gerbier & Dalibard, New Journal of Physics **12**, 033007 (2010)]

M_x, M_y: Rabi coupling, $\omega \simeq \omega_0$
M_z: standing waves at "anti-magic" frequency, $\omega_{\rm am}$

$$\mathcal{V}M = \begin{pmatrix} -\frac{\hbar\Delta}{2} - V_{\rm am}(\mathbf{r}) & \frac{\hbar\Omega(\mathbf{r})}{2} \\ \frac{\hbar\Omega^*(\mathbf{r})}{2} & \frac{\hbar\Delta}{2} + V_{\rm am}(\mathbf{r}) \end{pmatrix}$$

Square Lattice

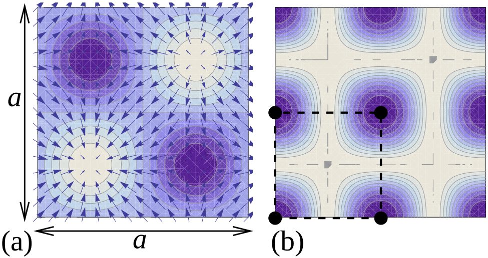
$$\hat{M}_{\rm sq} = \begin{pmatrix} \sin(\kappa x)\sin(\kappa y) & \cos(\kappa x) - i\cos(\kappa y) \\ \cos(\kappa x) + i\cos(\kappa y) & -\sin(\kappa x)\sin(\kappa y) \end{pmatrix}$$

where $\kappa \equiv 2\pi/a$.

vectors (n_x, n_y)

contours n_z

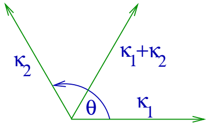
$N_\phi = 2$



contours n_ϕ

Triangular lattice

$$\hat{M}_{\mathrm{tri}} = \begin{pmatrix} \cos[\mathbf{r}\cdot(\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)] & \cos(\mathbf{r}\cdot\boldsymbol{\kappa}_1) - i\cos(\mathbf{r}\cdot\boldsymbol{\kappa}_2) \\ \cos(\mathbf{r}\cdot\boldsymbol{\kappa}_1) + i\cos(\mathbf{r}\cdot\boldsymbol{\kappa}_2) & -\cos[\mathbf{r}\cdot(\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)] \end{pmatrix}$$

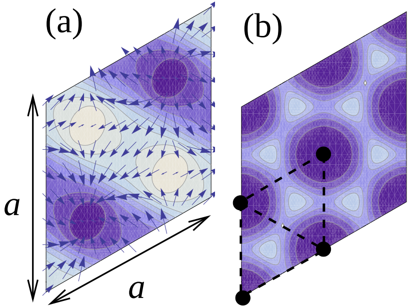


$\theta \simeq 2\pi/3$

vectors: (n_x, n_y)

contours: n_z

$N_\phi = 2$

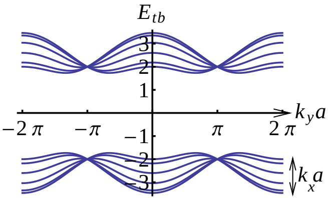


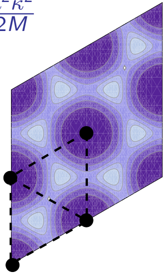
Bandstructure (Triangular Lattice)

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}} + \mathcal{V}\left[c_1\hat{\sigma}_x + c_2\hat{\sigma}_y + c_{12}\hat{\sigma}_z\right]$$

$$c_i \equiv \cos(\boldsymbol{\kappa}_i \cdot \mathbf{r}),\ c_{12} \equiv \cos[(\boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2)\cdot\mathbf{r}]$$

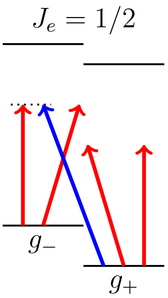
Tight-binding limit $\quad \mathcal{V} \gtrsim E_R \equiv \frac{\hbar^2\kappa^2}{2M}$





Lowest energy band has narrow width and Chern number of 1.

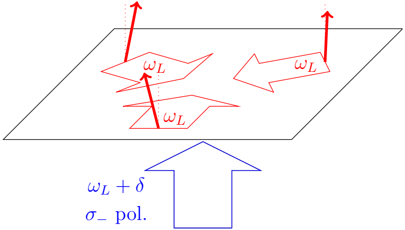
Two-Photon Dressed States

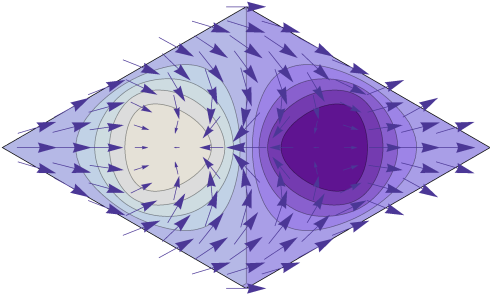


[NRC & Jean Dalibard, EPL **95**, 66004 (2011)]

Light at two frequencies:

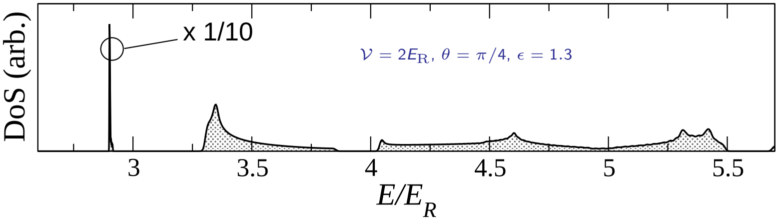
- ω_L with Rabi freqs. κ_m $(m = 0, \pm 1)$
- $\omega_L + \delta$ with Rabi freq. E in σ_-





Triangular lattice with $N_\phi = 1$ per unit cell.

Bandstructure, $J_g = 1/2$



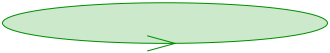
- Narrow lowest energy band, with Chern number of 1
- Can also be applied to bosons $J_g = 1$ (e.g. ^{87}Rb)

Experimental Consequences

Non-interacting fermions (IQHE)

- Filled band has chiral edge state:

  Precession of collective modes



- Bloch oscillations

[Hannah Price & NRC, PRA **85**, 033620 (2012)]

Interacting fermions/bosons

Strongly correlated phases if interactions large compared to bandwidth: likely candidates for FQHE states.

- Incompressible states (density plateaus)
- Chiral edge modes

Outline

Topological Insulators

[Hasan & Kane, RMP **82**, 3045 (2010); Qi & Zhang, RMP **83**, 1057 (2011)]

TI: Band insulator with gapless surface states.

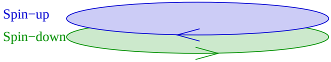
- IQHE: 2D, broken time reversal symmetry (TRS)

  Chern number $\Rightarrow$number of chiral edge states

- $\mathbb{Z}_2$ TI: fermions ($S = \frac{1}{2}, \frac{3}{2}, \ldots$) *with* TRS (Kramers' deg.)

  Band insulators are: trivial; or non-trivial (metallic surface)

  2D: counterpropagating edge channels of opposite spin;



  3D: relativistic (Dirac) 2D surface state.

$\mathbb{Z}_2$ Topological Insulators

$$\hat{H} = \frac{\mathbf{p}^2}{2M}\hat{\mathbb{I}}_N + \mathcal{V}\hat{M}(\mathbf{r})$$

[Benjamin Béri & NRC, PRL **107**, 145301 (2011)]

Time-reversal $\hat{\theta} = i\hat{\sigma}_y\hat{K}$ TRS: $\hat{M} = \hat{\theta}^{-1}\hat{M}\hat{\theta} \Rightarrow N = 4$

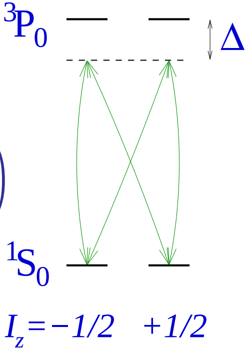
$$\begin{aligned} \hat{M} &= \begin{pmatrix} (A+B)\hat{\mathbb{I}}_2 & C\hat{\mathbb{I}}_2 - i\hat{\vec{\sigma}}\cdot\vec{D} \\ C\hat{\mathbb{I}}_2 + i\hat{\vec{\sigma}}\cdot\vec{D} & (A-B)\hat{\mathbb{I}}_2 \end{pmatrix} \\ &= A\hat{\mathbb{I}}_4 + B\hat{\Sigma}_3 + C\hat{\Sigma}_1 + \vec{D}\hat{\Sigma}_2\hat{\vec{\sigma}} \end{aligned}$$

[$A, B, C, \vec{D} = (D_x, D_y, D_z)$ real]

Dressed states are Kramers doublets $\Rightarrow$non-Abelian gauge field.

^{171}Yb has nuclear spin $I = 1/2$

$$\mathcal{V}\hat{M} = \begin{pmatrix} -\left(\frac{\hbar}{2}\Delta + V_{\rm am}\right)\hat{\mathbb{I}}_2 & -i\hat{\vec{\sigma}}\cdot\vec{\mathcal{E}}d_r \\ i\hat{\vec{\sigma}}\cdot\vec{\mathcal{E}}^* d_r & \left(\frac{\hbar}{2}\Delta + V_{\rm am}\right)\hat{\mathbb{I}}_2 \end{pmatrix}$$

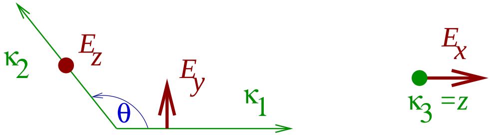


TRS preserved if all components of $\vec{\mathcal{E}}$ have a common phase.

Two Dimensions

$$d_r\vec{\mathcal{E}} = \mathcal{V}\left(\delta, \cos(\mathbf{r}\cdot\boldsymbol{\kappa}_1), \cos(\mathbf{r}\cdot\boldsymbol{\kappa}_2)\right)$$

$$\boldsymbol{\kappa}_1 = (1,0,0)\kappa$$
$$\boldsymbol{\kappa}_2 = (\cos\theta, \sin\theta, 0)\kappa$$



$$\frac{\hbar}{2}\Delta + V_{\rm am}(\mathbf{r}) = -\mathcal{V}\cos[\mathbf{r}\cdot(\boldsymbol{\kappa}_1+\boldsymbol{\kappa}_2)]$$

For Yb, $\theta \simeq 2\pi/3$

$\hat{U} = 2^{-1/2}(\hat{\mathbb{I}}_4 - i\hat{\Sigma}_3\hat{\sigma}_2)$

$$\hat{M}' = \hat{U}^\dagger \hat{M} \hat{U} = c_1\hat{\Sigma}_1 + c_2\hat{\Sigma}_2\hat{\sigma}_3 + c_{12}\hat{\Sigma}_3 + \delta\hat{\Sigma}_2\hat{\sigma}_1.$$

$$c_i \equiv \cos(\boldsymbol{\kappa}_i \cdot \mathbf{r}),\, c_{12} \equiv \cos[(\boldsymbol{\kappa}_1 + \boldsymbol{\kappa}_2) \cdot \mathbf{r}]$$

(i) Decoupled spins, $\delta = 0$

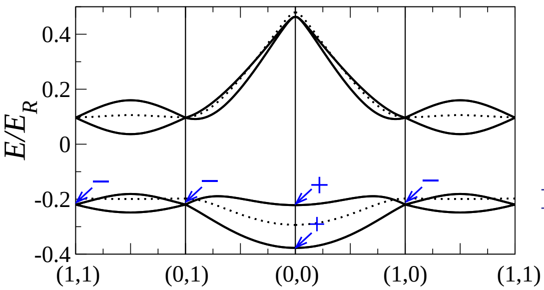
$$\hat{M}' = c_1\hat{\Sigma}_1 \pm c_2\hat{\Sigma}_2 + c_{12}\hat{\Sigma}_3$$

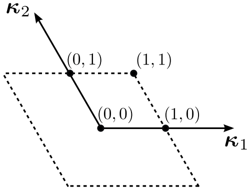
OFLs of opposite flux for spin $\sigma_3 = \pm 1$.

$\sigma_3 = \pm 1$ bands are degenerate, but with opposite Chern numbers.

$\Rightarrow$ "quantum spin Hall" system [e.g. Levin & Stern, PRL (2009)]

(ii) "Spin-orbit coupling", $\delta \neq 0$





$$\Gamma_{nm} = \frac{1}{2}(n\boldsymbol{\kappa}_1 + m\boldsymbol{\kappa}_2)$$

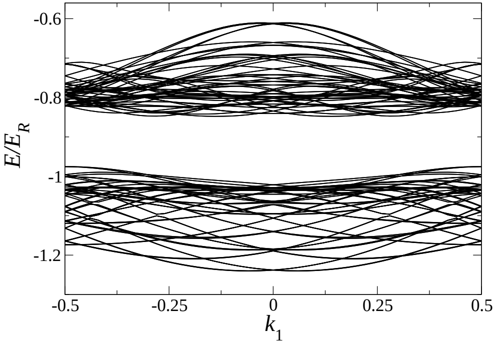
Inversion symmetry [Fu & Kane, PRB (2007)]

$$\prod_{n,m=0,1} \prod_{\alpha \in \text{filled}} \xi_{nm}^{(\alpha)} = -1$$

Three Dimensions

This nearly-free electron viewpoint leads to a general method to construct $\mathbb{Z}_2$ non-trivial bands in 3D. [Benjamin Béri & NRC, PRL **107**, 145301 (2011)]

e.g. $\delta \to \delta_0 \cos(\boldsymbol{\kappa}_3 \cdot \mathbf{r}) \qquad c_{12} \to c_{12} + \delta_0(\mu + c_{13} + c_{23})$



$\mathcal{V} = 0.9 E_{\mathrm{R}}$
$\delta_0 = 1, \mu = -0.4$

Summary

- Simple forms of optical dressing lead to "optical flux lattices": periodic magnetic flux with high mean density, $n_\phi \sim 1/\lambda^2$.
- The low energy bands are analogous to the lowest Landau level of a charged particle in a uniform magnetic field.
- The approach can be generalized to generate $\mathbb{Z}_2$ nontrivial bandstructures in 2D and 3D.
- Ultracold atomic gases can readily be used to explore strong correlation phenomena in topological bands.