Topological phases of matter: from bulk model wave functions to the edge theory

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Joint work with N. Read and E. Rezayi

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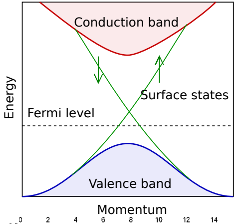
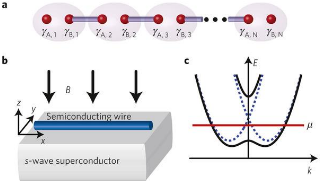
$$\text{"topological phase"} \quad \leftrightarrow \quad \text{gapped phase } E_{exc.} > 0 \text{ (massive)}$$
$$\text{"topological" quantity} \quad = \quad \text{invariant as long as } E_{exc.} > 0$$

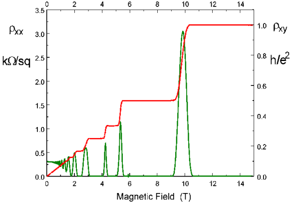
Examples (electronic):



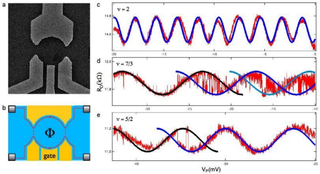
insulator



super-
conductor



integer
QHE



fractional
QHE

Topological phases: a simple example

BCS theory (Bardeen-Cooper-Schrieffer) of superconductivity:

$$\hat{H} = \sum_{\mathbf{k}} \left[\frac{\mathbf{k}^2}{2m} c_{\mathbf{k}}^\dagger c_{\mathbf{k}} + \frac{1}{2} \left(\overline{\Delta}_{\mathbf{k}} c_{-\mathbf{k}} c_{\mathbf{k}} + \Delta_{\mathbf{k}} c_{\mathbf{k}}^\dagger c_{-\mathbf{k}}^\dagger \right) \right]$$

chemical potential μ:

$$\hat{H} \to \hat{H} - \mu \hat{N} = \hat{H} - \mu \sum_{\mathbf{k}} c_k^\dagger c_k$$

in 2D, with T et P broken:

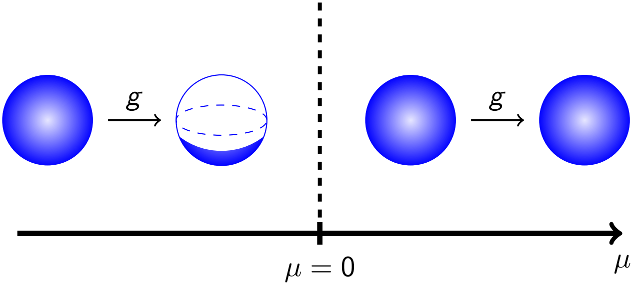
$$\boxed{\Delta_{\mathbf{k}} \sim \Delta_0 (k_x - i k_y)}$$

Topological phases: a simple example

$$\boxed{\Delta_{\mathbf{k}} \sim \Delta_0(k_x - ik_y)} \qquad (\text{“}p_x + ip_y\text{” superconductivity})$$

ground state: $|\psi\rangle = e^{\frac{1}{2}\sum_{\mathbf{k}} g(\mathbf{k})\, c^\dagger_{\mathbf{k}} c^\dagger_{-\mathbf{k}}} |0\rangle$

$$\begin{array}{rcl} S^2 \simeq \mathbb{C} \cup \{\infty\} & \longrightarrow & \mathbb{C} \cup \{\infty\} \simeq S^2 \\ \mathbf{k} & \longmapsto & g(\mathbf{k}) \end{array}$$

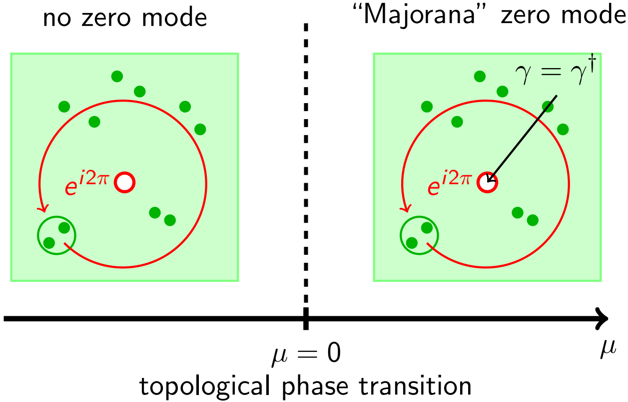


topological quantum phase transition

Topological phases: a simple example

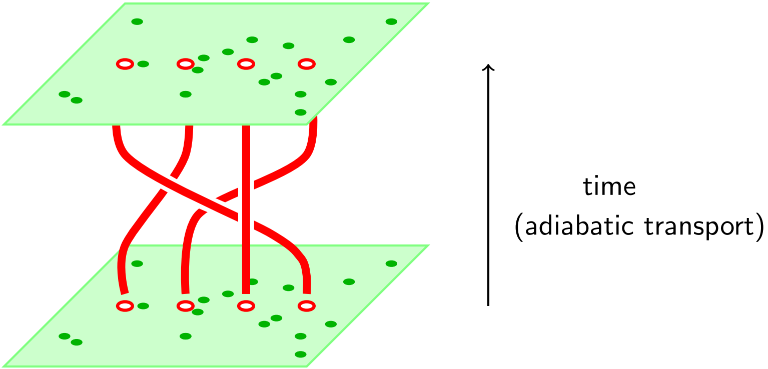
$\boxed{\Delta_{\mathbf{k}} \sim \Delta_0(k_x - ik_y)}$ ("$p_x + ip_y$" superconductivity)

topological excitation: vortex



Topological phases: a simple example

vortices and non-abelian statistics [Read and Green, 2000]



- same physics in the FQHE at $\nu = \frac{5}{2}$ [Moore and Read, 1991]
  $\rightarrow$ possible experimental observation (interferometry)
- topologically protected qubits [Kitaev, 1997]

Topologically non-trivial phases:

free particles + topology (topological insulators+superconductors)	quadratic Hamiltonian +topology (+perturbations)

⟶ well-known methods are available

interacting particles (FQHE, ???)	difficult problem

⟶ requires new methods

Hamiltonian with interactions

→ diagonalization

possible (difficult) numerically

privileged method (at least for FQHE):

model wave function

ground state + excitations

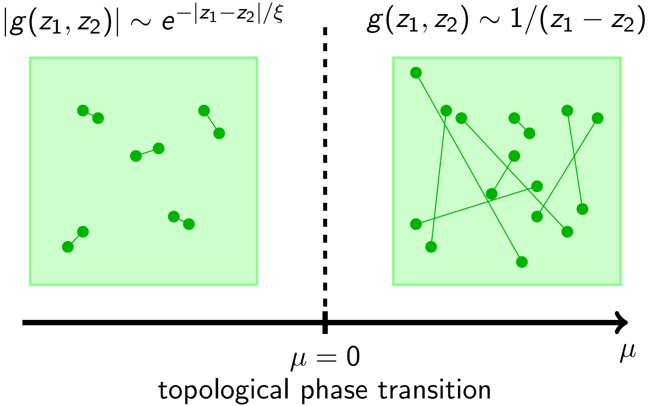
[Laughlin 1983]

⟶ a powerful method: “Moore-Read construction”

Moore-Read construction

$\Delta_{\mathbf{k}} \sim \Delta_0(k_x - ik_y)$ ("$p_x + ip_y$" superconductivity)

ground state: $|\psi\rangle = e^{\frac{1}{2}\int d^2z_1 \int d^2z_2\, g(z_1,z_2)\, c^\dagger(z_1) c^\dagger(z_2)} |0\rangle$



Moore-Read construction

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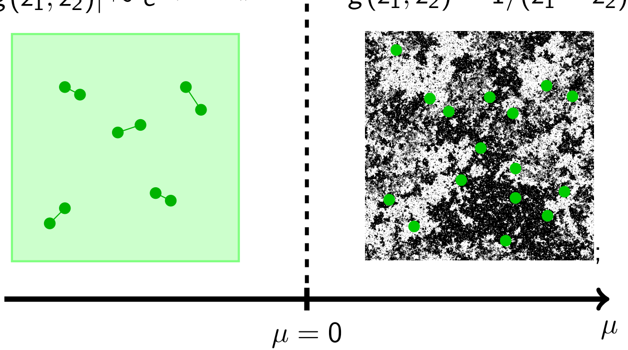
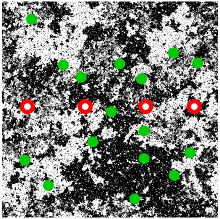
$$|g(z_1,z_2)| \sim e^{-|z_1-z_2|/\xi} \qquad\qquad g(z_1,z_2) \sim 1/(z_1-z_2)$$



topological phase transition

Moore-Read construction

with vortices



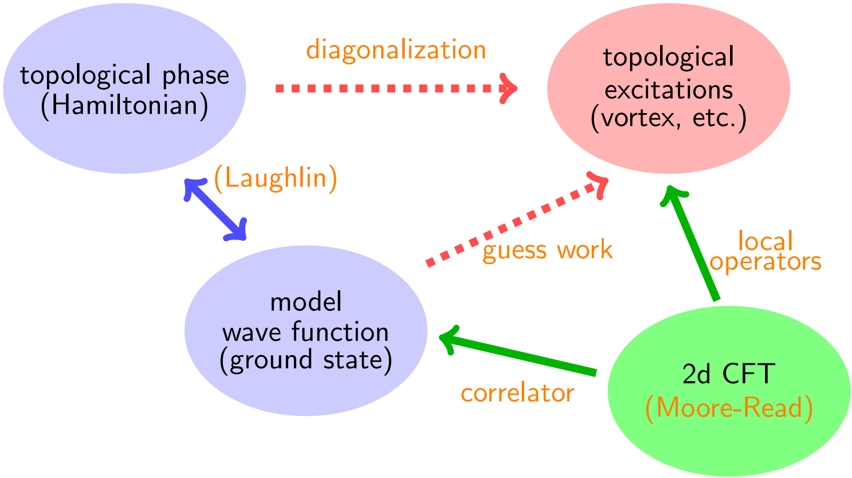
$$\psi_{\text{vortex}}(w_1, w_2, w_3, w_4, z_1, .., z_N)$$
$$= \langle \sigma(w_1)\sigma(w_2)\sigma(w_3)\sigma(w_4)\varepsilon(z_1)..\varepsilon(z_N)\rangle_{\text{Ising}}$$

local magnetization in the Ising CFT:

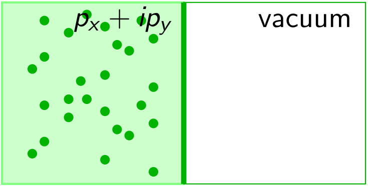
$$\sigma(w, \overline{w}) = \sigma(w) \times \overline{\sigma(w)}$$

correspondence local operators (CFT) $\leftrightarrow$ topological excitations

Moore-Read construction



Moore-Read construction: what about the edge excitations?



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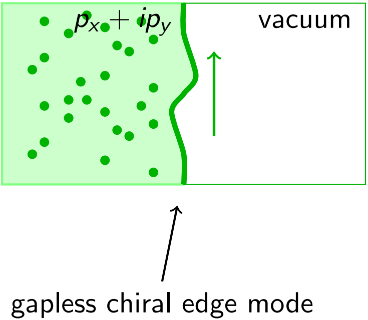
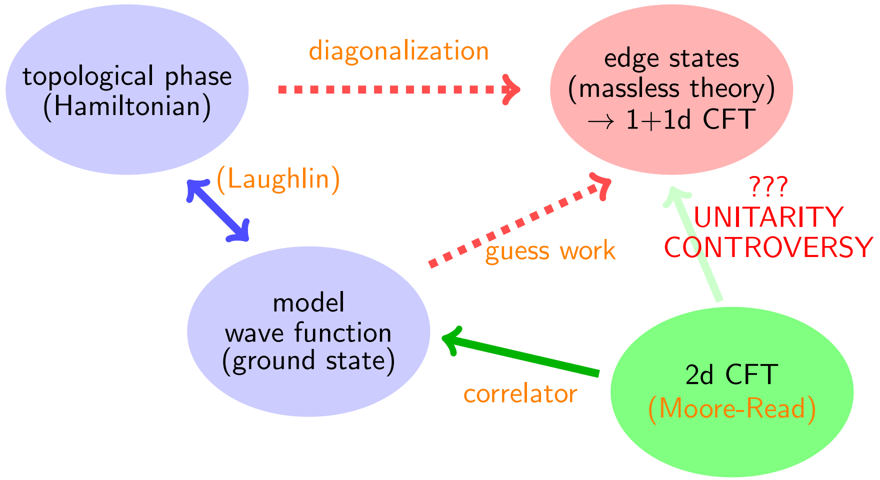


- gapless edge excitations described by a 1+1d CFT

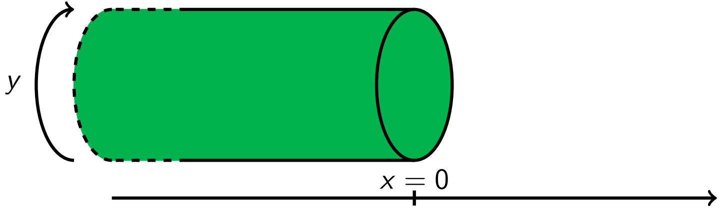
Moore-Read construction: what about the edge excitations?



Plan of the talk

- a simple example: $p_x + ip_y$, massive Ising and edge theory
- FQHE, conformal blocks and screening hypothesis
- application to the entanglement spectrum

Edge states for $p_x + ip_y$ paired superfluids



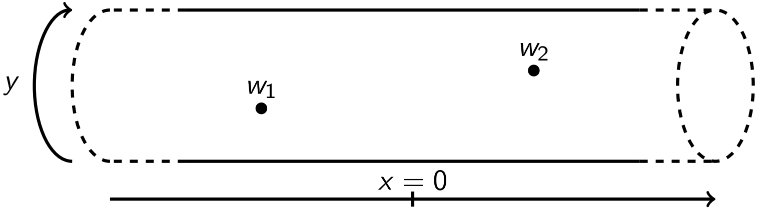
$$|\psi\rangle\rangle = \frac{1}{\sqrt{Z}} \exp\left(\int d^2 w_1 \int d^2 w_2 \, g(w_1, w_2) \, c^+_{w_1} c^+_{w_2} \right) |0\rangle\rangle$$

where $w = x + iy$

$$g(w_1, w_2) = \frac{\mu}{\frac{L}{2\pi} \sinh\left(2\pi \frac{w_1 - w_2}{L}\right)}$$

μ is an inverse length, related to the density

Edge states for $p_x + ip_y$ paired superfluids



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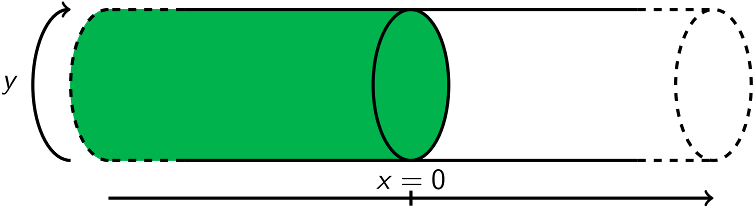
where $w = x + iy$

$$g(w_1, w_2) = \mu \, \langle \Psi(w_1) \Psi(w_2) \rangle$$

$\Psi(w)$ is a free (Majorana) fermion field

Edge states for $p_x + ip_y$ paired superfluids



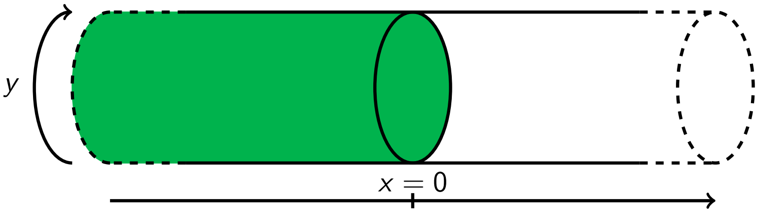
$$|\psi\rangle\rangle = \frac{1}{\sqrt{Z}} \left\langle \exp\left(\sqrt{\mu} \int d^2w \; \Psi(w) \; c_w^+\right) \right\rangle |0\rangle\rangle$$

where $w = x + iy$

$$\langle \Psi(w_1)\Psi(w_2)\rangle = \frac{1}{\frac{L}{2\pi} \sinh\left(2\pi \frac{w_1 - w_2}{L}\right)}$$

is the fermion propagator (then use Wick's theorem)

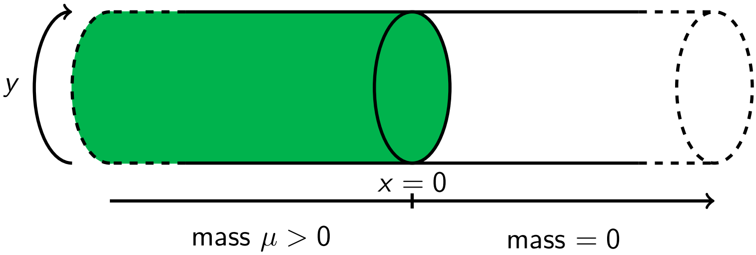
Edge states for $p_x + ip_y$ paired superfluids



Now look at the norm of $|\psi\rangle\rangle$

$$
\begin{aligned}
1 &= \langle\langle\psi|\psi\rangle\rangle \\
&= \frac{1}{Z}\langle\langle 0|\left\langle e^{\sqrt{\mu}\int d^2w\,\overline{\Psi(w)}\,c_w}\, e^{\sqrt{\mu}\int d^2w\,\Psi(w)\,c_w^+}\right\rangle|0\rangle\rangle \\
&= \frac{1}{Z}\left\langle e^{\mu\int d^2w\,\Psi(w)\overline{\Psi(w)}}\right\rangle
\end{aligned}
$$

Edge theory for $p_x + ip_y$ paired superfluids

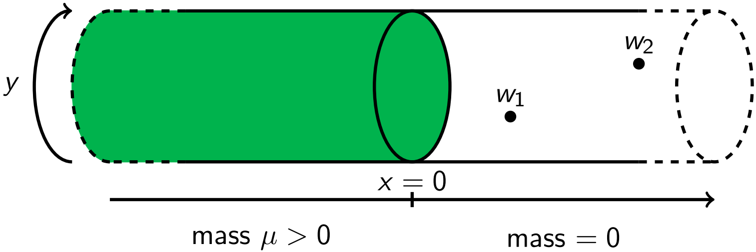


$$Z = \left\langle e^{\mu \int d^2w \, \Psi(w)\overline{\Psi(w)}} \right\rangle$$

note that μ is the mass is the action

$$S = \int d^2w \left[\Psi \partial_{\overline{w}} \Psi + \overline{\Psi} \partial_w \overline{\Psi} + \mu \, \Psi \overline{\Psi} \right]$$

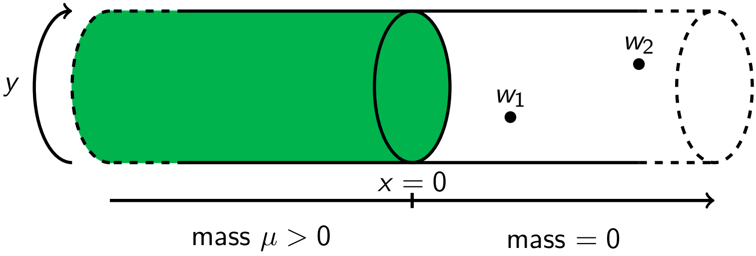
Edge theory for $p_x + ip_y$ paired superfluids



Model wave function for the edge excitations:

$$|\psi, w_1, w_2\rangle\rangle = \frac{1}{\sqrt{Z}} \left\langle \psi(w_1)\psi(w_2) \exp\left(\sqrt{\mu}\int d^2w\,\psi(w)c^+(w)\right)\right\rangle |0\rangle\rangle$$

Edge theory for $p_x + ip_y$ paired superfluids



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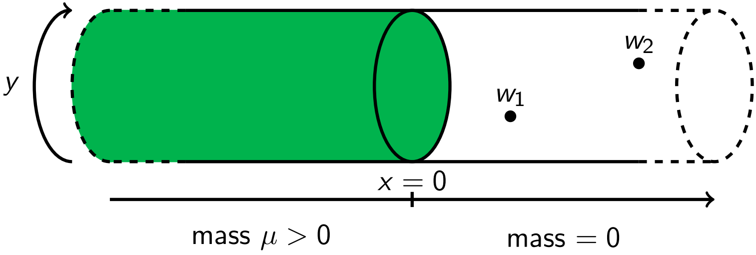
Question: what's the overlap between two edge states?

$$\langle\langle \psi | \psi, w_1, w_2\rangle\rangle = ? \qquad \langle\langle \psi, w_1 | \psi, w_2\rangle\rangle = ?\ ,\ etc.$$

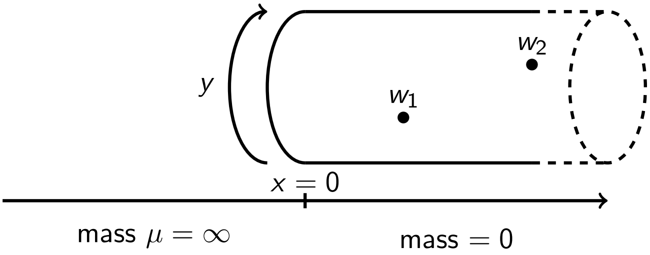
Edge theory for $p_x + ip_y$ paired superfluids



Question: what's the propagator in the massless phase?

$$\langle \Psi(w_1)\Psi(w_2) \rangle \qquad \left\langle \Psi(w_1)\overline{\Psi(w_2)} \right\rangle \qquad \left\langle \overline{\Psi(w_1)}\,\overline{\Psi(w_2)} \right\rangle$$

Edge theory for $p_x + ip_y$ paired superfluids



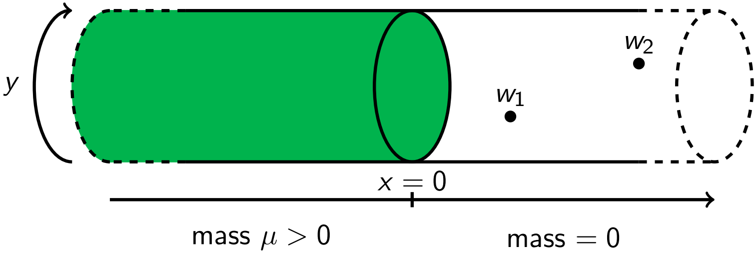
Answer when $\mu \to \infty$:

$$\left\langle \Psi(w_1)\overline{\Psi(w_2)} \right\rangle = \langle \Psi(x_1, y_1)\Psi(-x_2, y_2) \rangle$$

corresponding to the conformal boundary condition

$$\Psi(0, y) = \overline{\Psi}(0, y) \qquad (x = 0)$$

Edge theory for $p_x + ip_y$ paired superfluids



Answer at finite μ:

$$\left\langle \Psi(w_1)\overline{\Psi(w_2)} \right\rangle = \left\langle \Psi(x_1, y_1)\; e^{-S_{\rm pert}(\mu)}\; \Psi(-x_2, y_2) \right\rangle$$

corresponding to a perturbed conformal boundary condition

$$S_{\rm pert} \;=\; \lambda \int_0^L dy\, \phi(0, y) \;+\; \ldots$$

Conclusion for $p_x + ip_y$

- Edge excitations natural in the MR construction
- Overlaps between edge excitations
  $\Leftrightarrow$ massive/massless interface in the 2d Ising model

FQHE, screening and the edge theory

FQHE: trial wave functions as conformal blocks

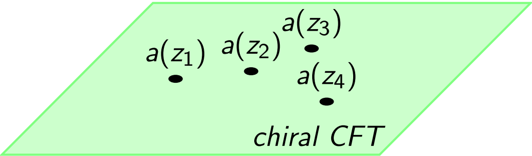
- N particles in the LLL

$$\psi(z_1, \ldots, z_N)$$

analytic in the z_i's, and antisymetric (Fermi statistics)

- Moore-Read: trial wave function given by conformal block

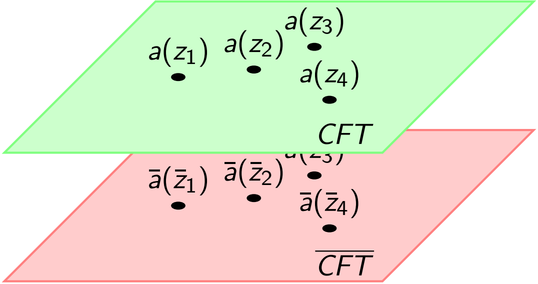
$$\psi(z_1, \ldots, z_N) = \langle a(z_N) \ldots a(z_1) \rangle$$



Screening hypothesis (I)

$$Z_N = \frac{1}{N!} \int_{\mathbb{C}^N} \prod_{i=1}^{N} e^{V(z_i, \bar{z}_i)} d^2 z_i \; |\psi(z_1, \ldots, z_N)|^2$$

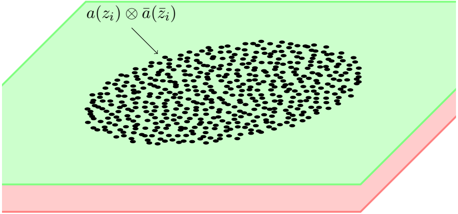
$$|\psi(z_1, \ldots, z_N)|^2 = \langle a(z_1) \ldots a(z_N) \rangle \times \langle \bar{a}(\bar{z}_1) \ldots \bar{a}(\bar{z}_N) \rangle$$



Screening hypothesis (II)

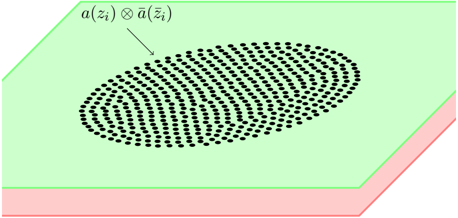
$$Z_N = \frac{1}{N!} \int \prod e^{V(z_i,\bar{z}_i)} d^2 z_i \quad \langle a(z_1) \dots a(z_N) \rangle \times \langle \bar{a}(\bar{z}_1) \dots \bar{a}(\bar{z}_N) \rangle$$

Screening



short-range correl. only

No screening

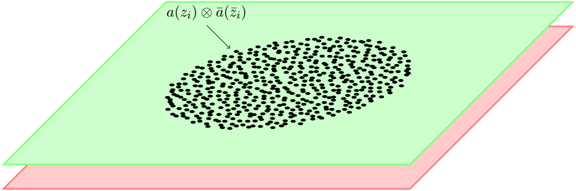


some long-range correl.
$\Rightarrow$ NOT a gapped
phase of matter

Screening hypothesis (III)

- thermodynamic limit $N \to \infty$, rescale the droplet to keep radius fixed (say radius$= 1$)
- at the boundary of the droplet ($|z| = 1$), screening implies

$$a(z) \;=\; \bar{a}^{\dagger}(\bar{z}) \;\times \text{ phase factor}$$



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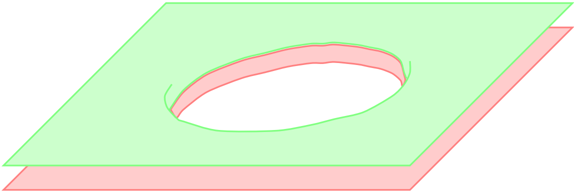

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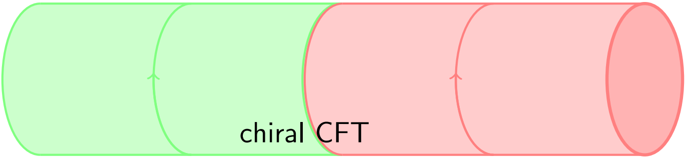


- conformal mapping $z \mapsto w(z)$ from the exterior of the droplet to the cylinder: $\quad a(w) \;=\; \bar{a}^{\dagger}(w)$

Edge states (I)

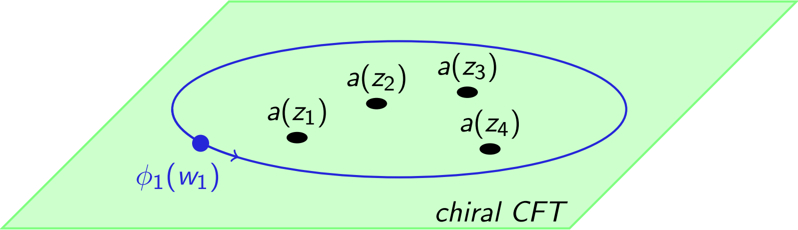
- Reminder: the ground state w.f. is defined as

$$\psi_{g.s}(z_1,\ldots,z_N) = \langle a(z_1)\ldots a(z_N)\rangle$$

- to construct the edge states w.f., we insert contour integrals

$$\left\langle \oint w_1^{n_1}\phi(w_1)dw_1 \ldots \oint w_k^{n_k}\phi(w_k)dw_k \; a(z_1)\ldots a(z_N) \right\rangle$$

$\phi_j(w_j)$ is in the chiral algebra generated by $a(w)$, $a^\dagger(w)$



Edge states (I)

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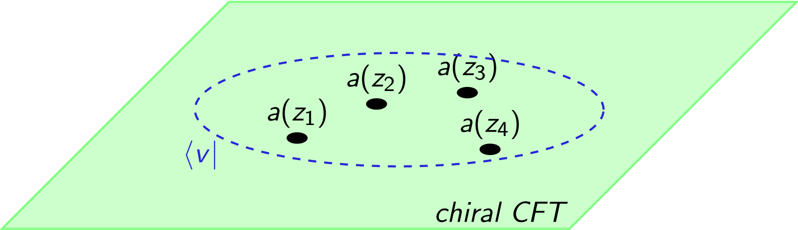
$$\psi_{g.s}(z_1, \ldots, z_N) = \langle a(z_1) \ldots a(z_N) \rangle$$

- in other words (radial quantization):

(*dual*) *CFT Hilbert sp.*	$\longrightarrow$	*edge theory Hilbert sp.*
$\langle v \rvert$	$\longmapsto$	$\psi_{\langle v \rvert} = \langle v \rvert a(z_1) \ldots a(z_N) \lvert 0 \rangle$



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- in other words (radial quantization):

$$\boxed{\begin{array}{ccc} (\textit{dual})\ \textit{CFT Hilbert sp.} & \longrightarrow & \textit{edge theory Hilbert sp.} \\ \langle v| & \longmapsto & \psi_{\langle v|} = \langle v\,| a(z_1) \ldots a(z_N) |\, 0 \rangle \end{array}}$$

- straightforward generalization of the previous constructions of edge states for specific w.f.
  an example (Laughlin):

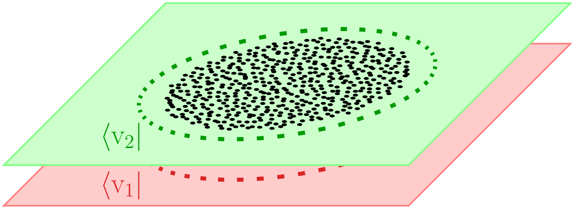
$$\psi_{\langle J_{n_1} J_{n_2}|}(\{z_i\}) \propto \sum_i z_i^{n_1} \sum_j z_j^{n_2} \times \prod_{k<l} (z_k - z_l)^{1/\nu}$$

Edge states (II)

- quantum-mechanical inner product

$$\langle\langle\, \psi_{\langle v_1|} \,|\, \psi_{\langle v_2|} \,\rangle\rangle \;=\; \frac{1}{Z_N\, N!} \int_{\mathbb{C}^N} \prod_{i=1}^{N} e^{V} d^2 z_i \left[\psi_{\langle v_1|}(\{z_i\})\right]^* \psi_{\langle v_2|}(\{z_i\})$$

- $\psi_{\langle v_2|}(\{z_i\}) \left[\psi_{\langle v_1|}(\{z_i\})\right]^* = \langle v_2|\, a(z_1) \ldots |0\rangle \overline{\langle v_1|\, a(z_1) \ldots |0\rangle}$



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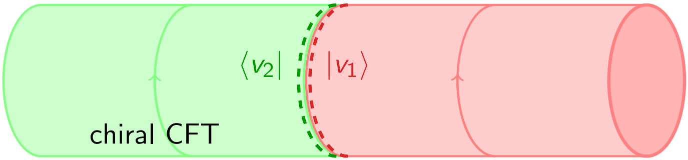
- $\psi_{\langle v_2|}(\{z_i\}) \left[\psi_{\langle v_1|}(\{z_i\})\right]^* = \langle v_2|\, a(z_1) \ldots |0\rangle \overline{\langle v_1|\, a(z_1) \ldots |0\rangle}$



- Consequence: $\langle\langle\, \psi_{\langle v_1|} \,|\, \psi_{\langle v_2|} \,\rangle\rangle = \langle v_2 \,|v_1\rangle$ when $N \to \infty$.

Bulk/edge correspondence

Screening implies that

(*dual*) *CFT Hilbert sp.* *inner product* $\langle . \,|. \rangle$	$\longrightarrow$	*edge theory Hilbert sp.* *inner product* $\langle\langle . \,|. \rangle\rangle$
$\langle v|$	$\longmapsto$	$\psi_{\langle v|} = \langle v \,| a(z_1) \ldots a(z_N) |\, 0 \rangle$

is a Hilbert sp. isomorphism in the thermodynamic limit $N \to \infty$:

$$\left\langle\left\langle \, \psi_{\langle v_1|} \,\middle|\, \psi_{\langle v_2|} \, \right\rangle\right\rangle \; = \; \langle v_2 \,| v_1 \rangle$$

(for the Laughlin w.f., similar argument appeared first in [Wen, 1992])

Entanglement spectrum: what are we talking about?

Entanglement spectrum: what are we talking about?

Starting point:

- quantum many-body systems: different phases matter

$$|g.s.1\rangle = |\cdots \otimes \Uparrow \otimes \Uparrow \otimes \Uparrow \otimes \Uparrow \otimes \ldots\rangle$$

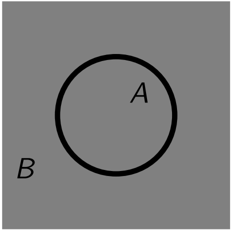
$|g.s.2\rangle = | \cdots$  $\cdots \rangle$

- some phases are "very entangled", some are "less entangled"

- study entanglement in the g.s. and use it as (theoretical/numerical) diagnostic tool

Entanglement spectrum: what are we talking about?

$|\psi\rangle \in \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$



$H|\psi\rangle = 0$

- Schmidt decomposition:

$$|\psi\rangle = \sum_{k=1}^{r} e^{-\xi_k/2} |k\rangle_A \otimes |k\rangle_B$$

$${}_A\langle k|k'\rangle_A = {}_B\langle k, k'\rangle_B = \delta_{k,k'}$$

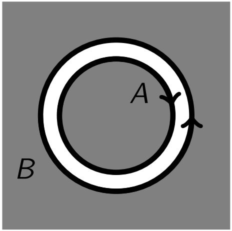
$$r \leq min(dim\mathcal{H}_A, dim\mathcal{H}_B)$$

$\{\xi_k\}$ is the entanglement spectrum

ES in quantum Hall systems

Physical scenario (part of [Qi, Katsura, Ludwig, 2011])

- gapped bulk
- gapless, chiral edge modes



$$H = H_A + H_B$$

ES in quantum Hall systems

Physical scenario (part of [Qi, Katsura, Ludwig, 2011])

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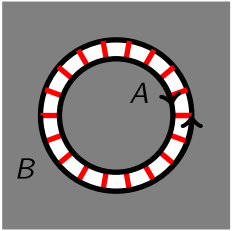
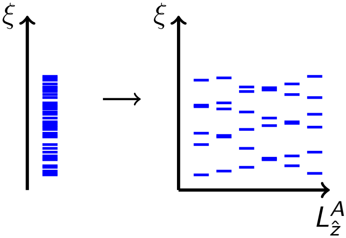
- gapless, chiral edge modes



$$H = H_A + H_B + H_{AB}$$

- rotational invariance: $[H_{AB}, L^A_{\hat{z}} + L^B_{\hat{z}}] = [H_A, L^A_{\hat{z}}] = [H_B, L^B_{\hat{z}}] = 0$

- plot the ES vs. angular momentum $L^A_{\hat{z}}$



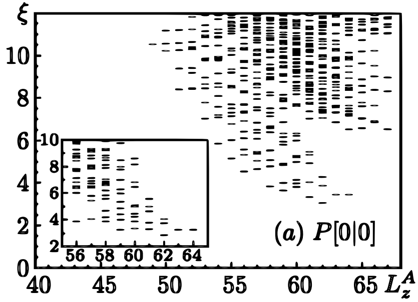
"level counting": in each $L^A_{\hat{z}}$ subsector

$$\#\{\xi_k(L^A_{\hat{z}})\} \leq dim\mathcal{H}^{edge}_{A,L^A_{\hat{z}}}$$

(by construction)

ES in quantum Hall systems

- relation between counting of Schmidt eigenvalues and edge states conjectured by [Li and Haldane, 2008]
- coined the term “entanglement spectrum”, and proposed it as a numerical diagnostic tool

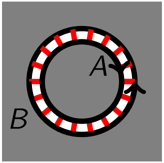


- anticipated in the last few lines of [Kitaev and Preskill, 2006]

ES in quantum Hall systems

. . . in summary:

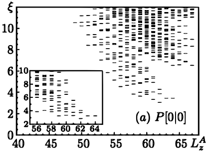
cut/glue argument (edge only)



trial wave-functions (bulk $\to$ edge)

$$\psi(z_1, \ldots, z_N)$$

real-life system (complicated)



ES in quantum Hall systems

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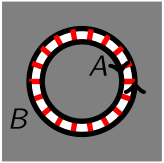

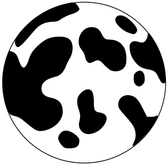

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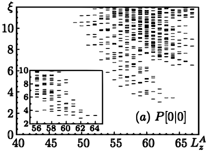

ES in quantum Hall systems

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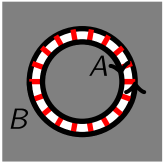

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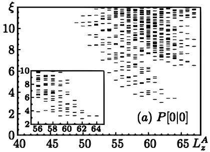
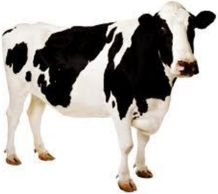
real-life system (complicated)

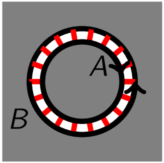




ES in quantum Hall systems

…in summary:

cut/glue argument (edge only)



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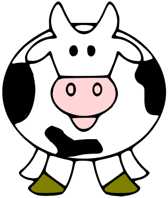

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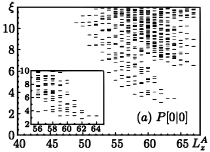
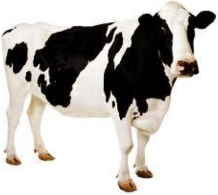
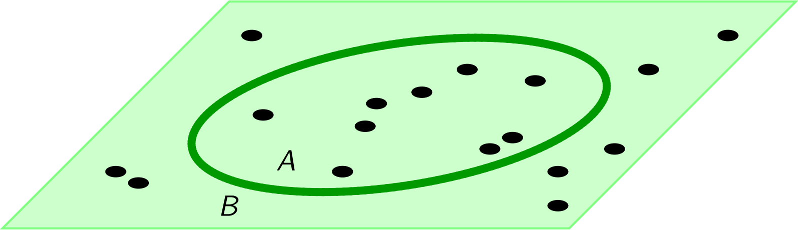




Application to the entanglement spectrum (I)

- what's the Schmidt decomp. of $\psi_{g.s}(z_1, \dots, z_N)$?

$$|\psi_{g.s}\rangle\rangle = \sum_{N_A=0}^{N} \sum_{n} e^{-\xi_n/2} \left|\psi_n^A\right\rangle\rangle \otimes \left|\psi_n^B\right\rangle\rangle$$



- we focus on Real Space Partition

Application to the entanglement spectrum (II)

- divide the $N = N_A + N_B$ coordinates z_i's into two sets

$$z_1, \ldots, z_{N_A} \qquad w_1 = z_{N_A+1}, \ \ldots, \ w_{N_B} = z_{N_A+N_B}$$

- because $\psi_{g.s}$ is a conformal correlator, one has

$$\begin{aligned}\psi_{g.s}(\{z_j\},\{w_k\}) &= \langle a(z_1)..a(z_{N_A})\, a(w_1)..a(w_{N_B})\rangle \\ &= \sum_{|v_n\rangle} \langle\, a(z_1)..a(z_{N_A})|\, v_n\rangle \,\langle v_n\, | a(w_1)..a(w_{N_B})\,\rangle \\ &= \sum_{|v_n\rangle} \psi_{\langle v_n|}(\{z_j\})\; \tilde{\psi}_{\langle v_n|}(\{w_k\})\end{aligned}$$

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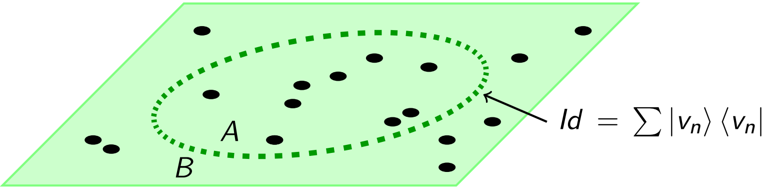

Application to the entanglement spectrum (II)

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$$\begin{aligned}
\psi_{g.s}(\{z_j\},\{w_k\}) &= \langle a(z_1)..a(z_{N_A})\, a(w_1)..a(w_{N_B})\rangle \\
&= \sum_{|v_n\rangle} \langle\, a(z_1)..a(z_{N_A})|\, v_n\rangle\, \langle v_n\, |a(w_1)..a(w_{N_B})\,\rangle \\
&= \sum_{|v_n\rangle} \psi_{\langle v_n|}(\{z_j\})\, \tilde{\psi}_{\langle v_n|}(\{w_k\})
\end{aligned}$$

- we get the entanglement spectrum using the overlaps

$$\big\langle\big\langle \psi_{\langle v_n|} \,\big|\psi_{\langle v_m|}\big\rangle\big\rangle = \langle v_m\,|v_n\rangle \ + \ O(1/\sqrt{N})$$

Application to the entanglement spectrum (II)

Result: the entanglement spectrum is given by

$$\xi(\Delta N_A, \Delta L_{\hat{z}}^A) = \alpha R - \gamma_{\text{topo}} + O(1/R)$$

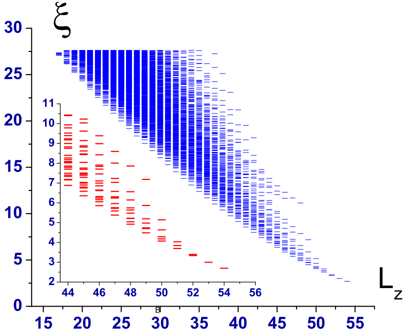
in other words, the spectrum is infinitely degenerate (!!!) up to $O(1/R)$ corrections

Application to the entanglement spectrum (II)

Result: the entanglement spectrum is given by

$$\xi(\Delta N_A, \Delta L^A_{\hat{z}}) = \alpha\, R - \gamma_{\rm topo} + O(1/R)$$

more seriously. . .



Laughlin $\nu = 1/3$

Overlaps beyond the leading order

- finite N effects captured by a perturbation of the CFT action at the boundary of the droplet

$$S_b = \sum_a \lambda_a \int_{|z|=R} \phi_a(z)|dz|$$

- ϕ_a has scaling dimension $h_a \Rightarrow \lambda_a \sim (\frac{1}{\sqrt{N}})^{h_a - 1}$
- the overlaps become

$$\langle\langle \psi_{\langle v_1|} \, | \, \psi_{\langle v_2|} \rangle\rangle = \langle v_2| \frac{e^{-S_b}}{\langle e^{-S_b} \rangle} |v_1\rangle$$

- in many cases, the stress-tensor $T(z)$ is the least irrelevant boundary perturbation

$$\Rightarrow \quad \langle\langle \psi_{\langle v_1|} \, | \, \psi_{\langle v_2|} \rangle\rangle = \langle v_2| \, e^{-\frac{const.}{\sqrt{N}} L_0} \, |v_1\rangle + O(1/N)$$

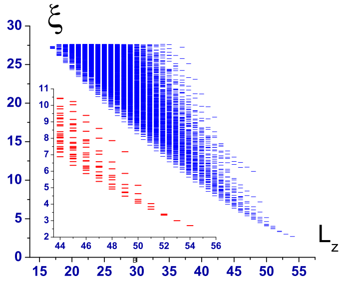
Application to the entanglement spectrum (III)

This proves the so-called "scaling property" [JD-Read-Rezayi 2012]

the entanglement spectrum is the spectrum of the Hamiltonian of a 1+1d CFT perturbed by local operators

$$H_{ES} = \alpha R - \gamma_{\mathrm{topo}} + \frac{v_F}{R} L_0 + \sum_a \lambda_a \int_{|z|=R} \phi_a(z) |dz|$$

locality of the entanglement Hamiltonian



Summary

- provided screening, we are left with a CFT outside the droplet with a (perturbed) conformal boundary condition
- allows to compute overlaps between edge states when $N \to \infty$

$$\left\langle\left\langle \psi_{\langle v_1|} \,\middle|\, \psi_{\langle v_2|} \right\rangle\right\rangle \underset{N\to\infty}{\longrightarrow} \langle v_2 | v_1 \rangle$$

- precise formulation of the bulk/boundary correspondence: isomorphism between bulk CFT/edge theory Hilbert spaces
- proof of the locality of the entanglement Hamiltonian

Thank you.